

Theoretical Status of Neutrino Oscillation Physics

NuFact 2019

The 21st International Workshop on Neutrinos from Accelerators

Daegu, Republic of Korea

August 25 - 31, 2019

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Outline

- Current status of neutrino oscillations
- Origin of neutrino mixing matrix
- Origin of neutrino masses
- **New Physics** in Neutrino Oscillations
- Conclusion

Current status of neutrino oscillations

- Two big discoveries over past two decades :

- Neutrinos are massive

- Leptons mix



T. Kajita



A. McDonald

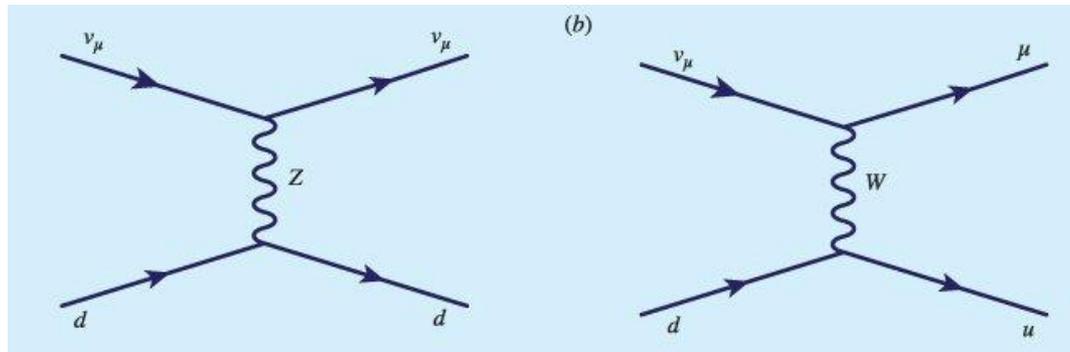
- They have been achieved by the observation of neutrino oscillations

(lots of sources: the sun, atmospheric, reactors and accelerators)



2015
Nobel
Prize

- Why leptons mix ?
 - Neutrino eigenstates involved in weak interactions



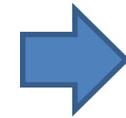
are **not mass eigenstates**:

• 3-flavor paradigm

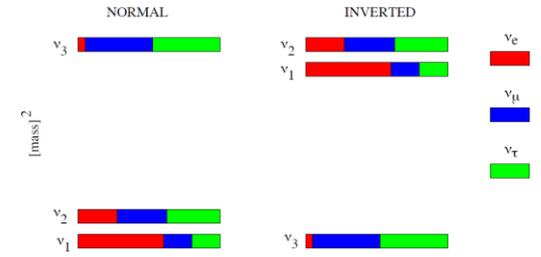
$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \quad \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L \quad \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_L : \\ \text{weak eigenstates}$$



$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$



$$\text{mass eigenstates}$$



- Specific parameterization of lepton mixing matrix

$$U_{PMNS} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & -s_{23} \\ 0 & s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & -s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & -s_{12} & 0 \\ s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e^{i\alpha} & 0 & 0 \\ 0 & e^{i\beta} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

- How do we probe neutrino mixing ?
 → neutrino oscillation

In vacuum, $\nu_\alpha \rightarrow \nu_\beta$ transition probability :

$$P_{\alpha\beta} = \delta_{\alpha\beta} - 4 \sum_{j \neq i}^n \text{Re}[U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^*] \sin^2 \left(\frac{\Delta_{ij}}{2} \right) + 2 \sum_{j \neq i} \text{Im}[U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^*] \sin(\Delta_{ij})$$

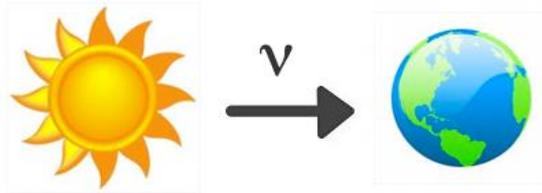
=0 if $\delta = 0$ in U,
 $\alpha = \beta$

$$\frac{\Delta_{ij}}{2} = \frac{(E_i - E_j)L}{2} = 1.27 \frac{(m_i^2 - m_j^2)}{\text{eV}^2} \frac{L/E}{\text{Km/GeV}}$$

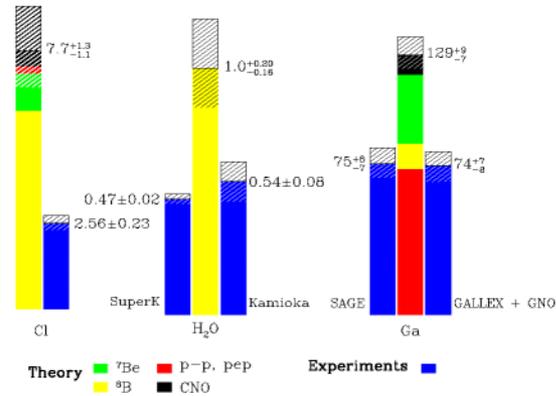
- From ν oscillation expts. we can determine
 - $\Delta m_{21}^2, \Delta m_{31}^2$
 - $\theta_{12}, \theta_{23}, \theta_{13}$
 - δ

θ_{12} & Δm_{21}^2

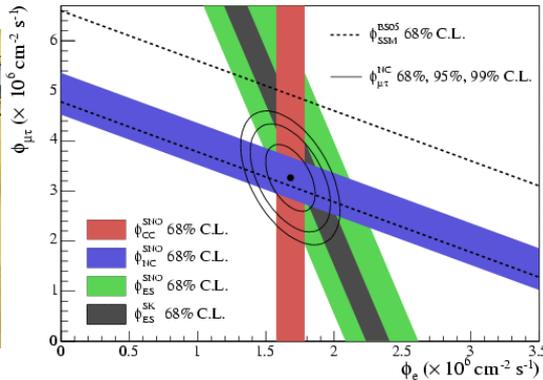
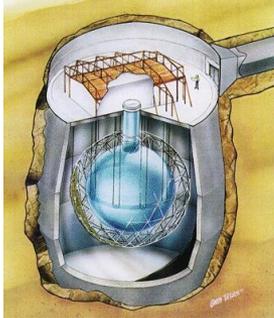
- Well measured by solar neutrino experiments and KamLAND



Total Rates: Standard Model vs. Experiment
Bahcall-Pinsonneault 2000

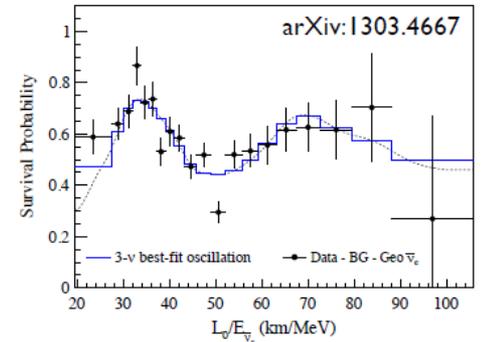
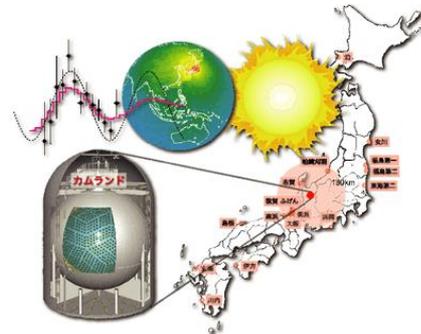


SNO



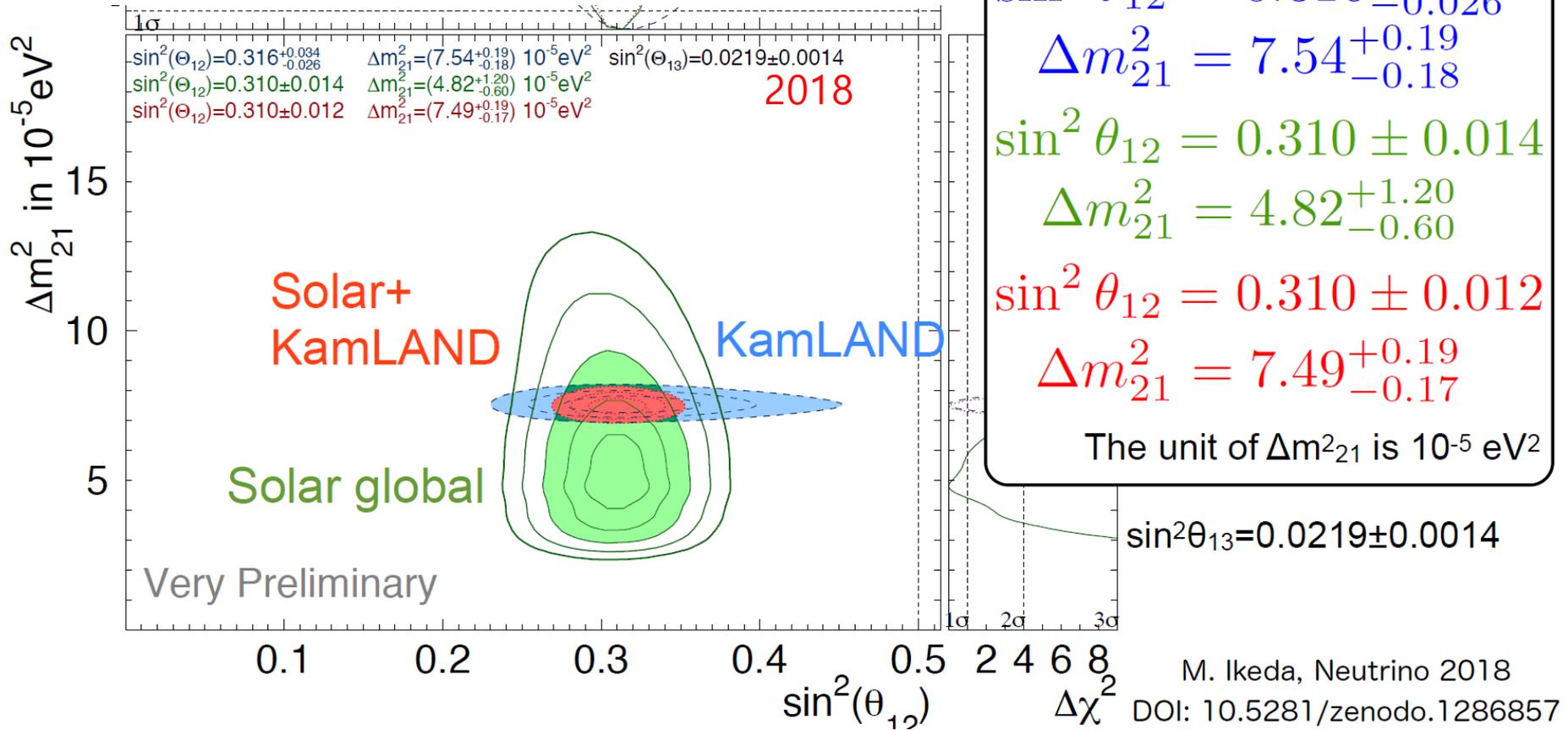
$$\frac{\Phi_{\text{CC}}}{\Phi_{\text{NC}}} = 0.301 \pm 0.033$$

KamLAND



θ_{12} & Δm_{21}^2

~2 σ tension between solar global and KamLAND in Δm_{21}^2



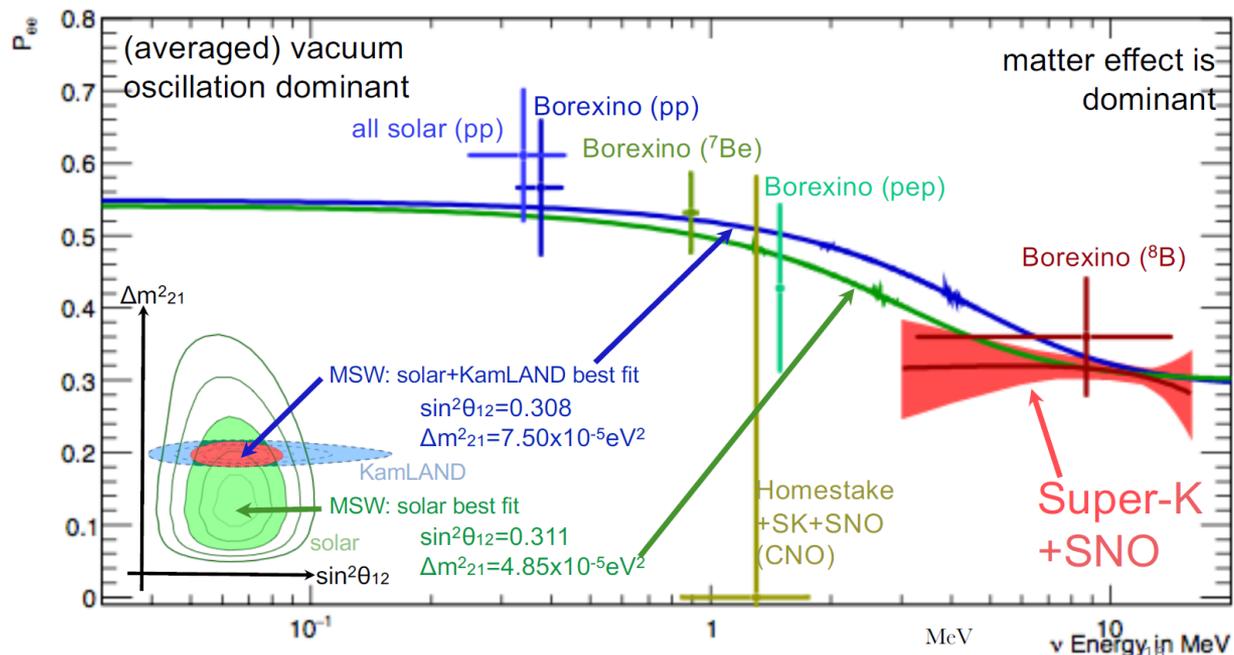
• MSW matter effect

(Minakata, Pena-garay, 2012)

$$P_{ee}^D = \cos^4 \theta_{13} \left[1 - \frac{1}{2} \sin^2 2\theta_{12} (1 + \cos 2\theta_{12} \xi_S) \right] + \sin^4 \theta_{13}, \text{ Low } E$$

$$P_{ee}^D = \cos^4 \theta_{13} \left[\sin^2 \theta_{12} + \frac{1}{4} \sin^2 2\theta_{12} \cos 2\theta_{12} \left(\frac{1}{\xi_S} \right)^2 \right] + \sin^4 \theta_{13} \quad \text{High } E$$

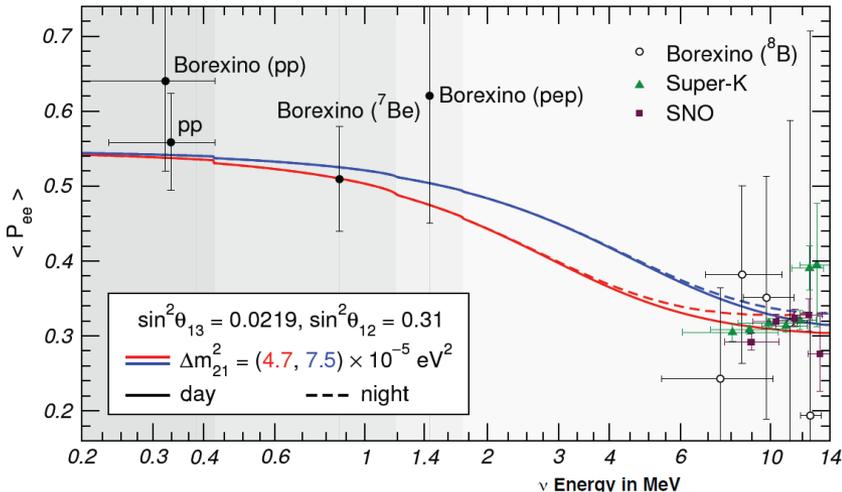
$$\xi_S \equiv \frac{l_\nu}{l_0} = 0.203 \times A_{\text{MSW}} \cos^2 \theta_{13} \left(\frac{E}{1 \text{ MeV}} \right) \left(\frac{\rho_S Y_e}{100 \text{ g cm}^{-3}} \right)$$



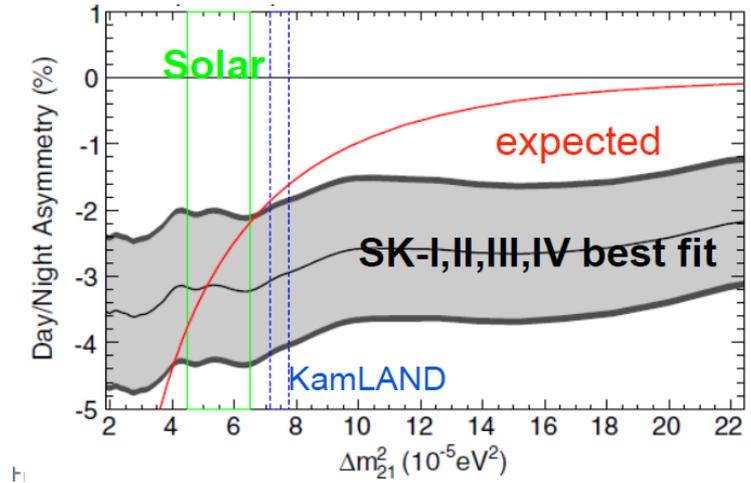
BOREXINO (Barbara Caccianiga 2019)

- Δm_{21}^2 preferred by KamLAND predicts **steeper upturn** at solar spectrum and **smaller A(D/N)**

Spectrum distortion



Maltoni, Smirnov EJP A52(2016)



Koshino (SK, 2019)

θ_{23} & Δm_{31}^2

- Measured by atmospheric ν experiments (SK), ν telescope (IceCube, ANTARES), accelerator experiments (MINOS, T2K, NovA)

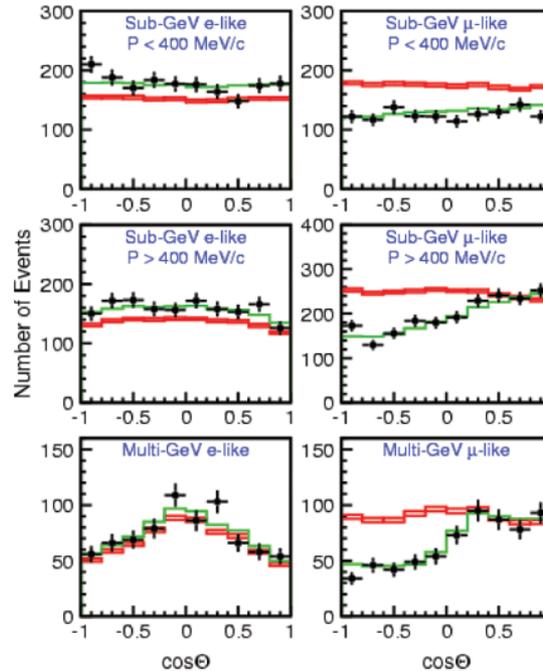
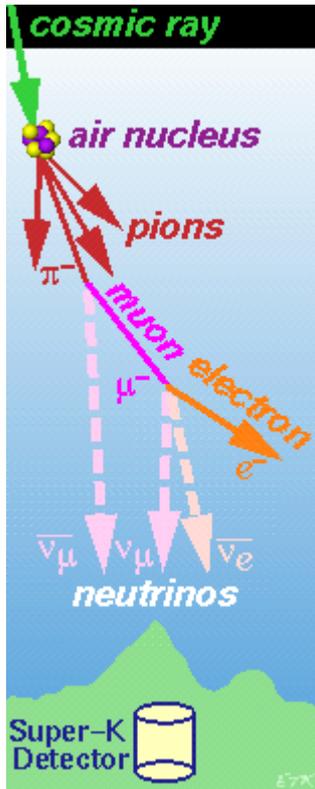
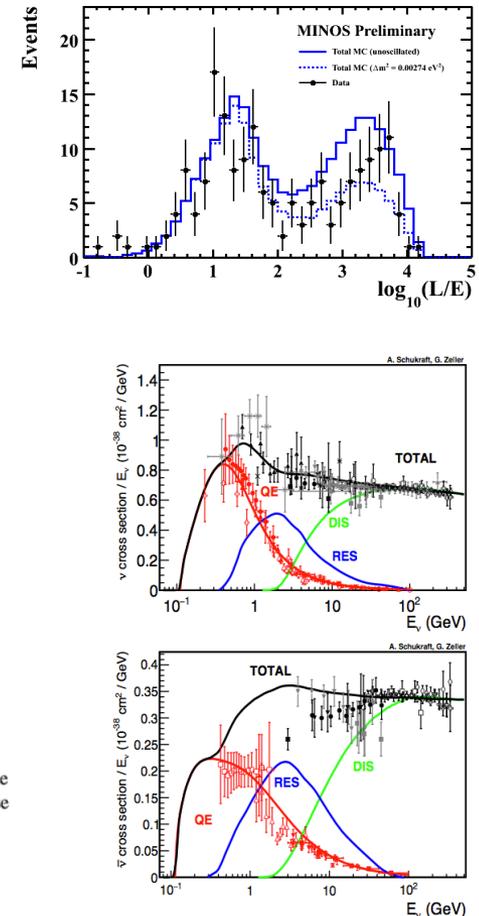


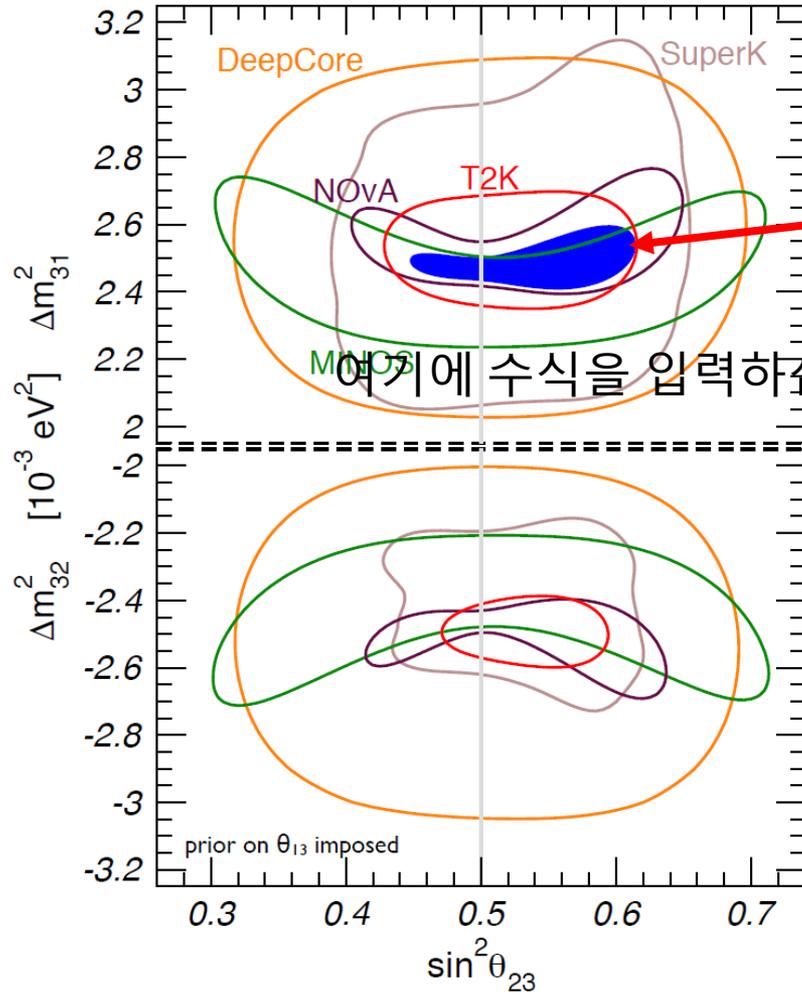
FIG. 1: Atmospheric neutrino data from Super-Kamiokande experiment, the left panel is the electron type events and the right panel is the muon type events. The energy range increase from top to down. From Ref. [8].



θ_{23} & Δm_{31}^2

NuFIT 4.0 (2018)

NO



IO

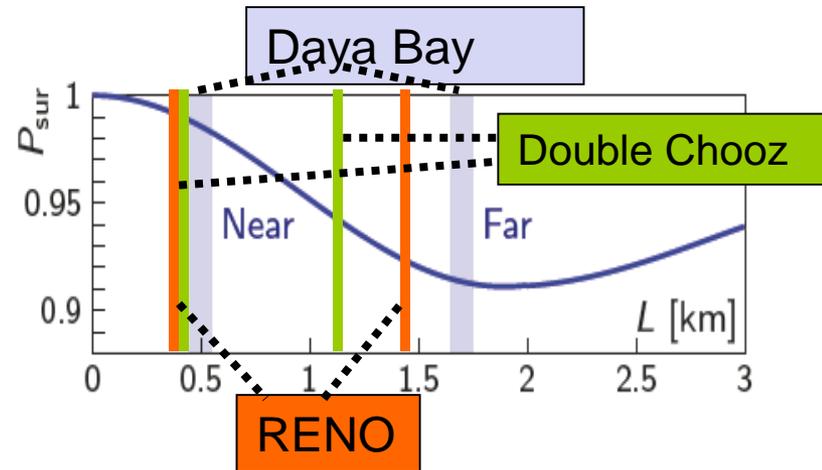
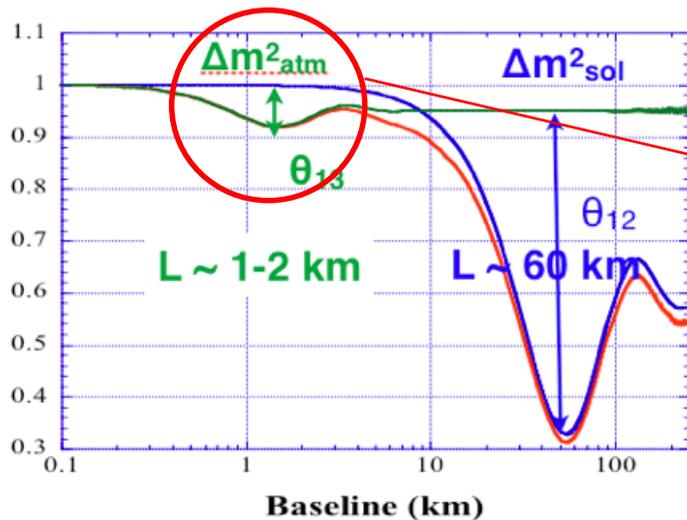
θ_{13}

- Measuring θ_{13} important role in determining CP violation & mass hierarchy

$$J_{CP} = \text{Im}(U_{\mu 3} U_{e 3}^* U_{e 2} U_{\mu 2}^*) = \frac{1}{8} \sin 2\theta_{12} \sin 2\theta_{23} \sin 2\theta_{13} \cos \theta_{13} \sin \delta$$

- Using reactor antineutrino oscillation

$$P(\nu_e \rightarrow \nu_e) \approx 1 - \cos^4 \theta_{13} \sin^2 2\theta_{12} \sin^2 \left(\frac{1.27 \Delta m_{12}^2 L}{E_\nu} \right) - \sin^2 2\theta_{13} \sin^2 \left(\frac{1.27 \Delta m_{13}^2 L}{E_\nu} \right)$$

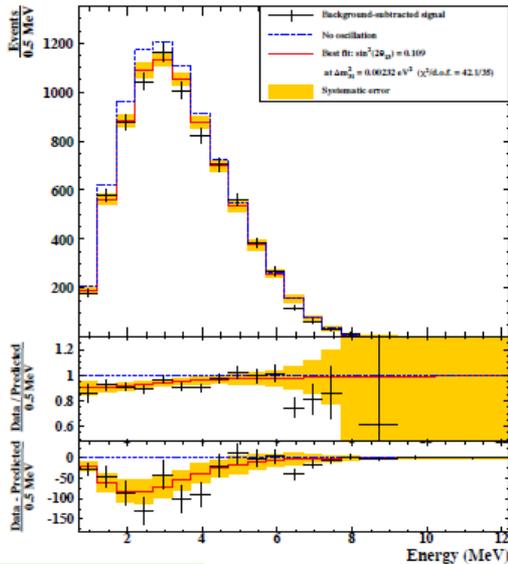


θ_{13}



Double Chooz

(Gd+H combined, EPS 2013, July)



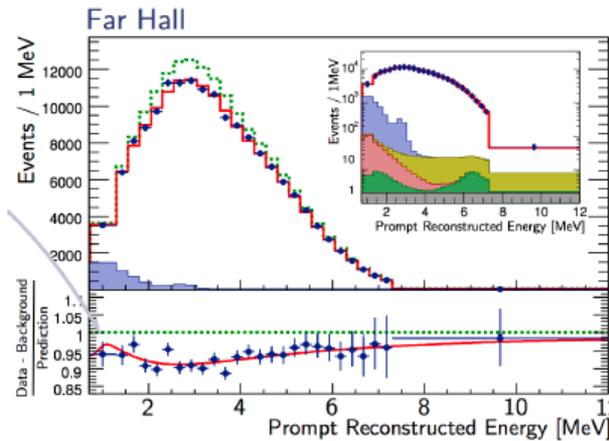
$$\sin^2 2\theta_{13}$$

$$0.109 \pm 0.035$$



Daya Bay

(NuFact 2013, August)



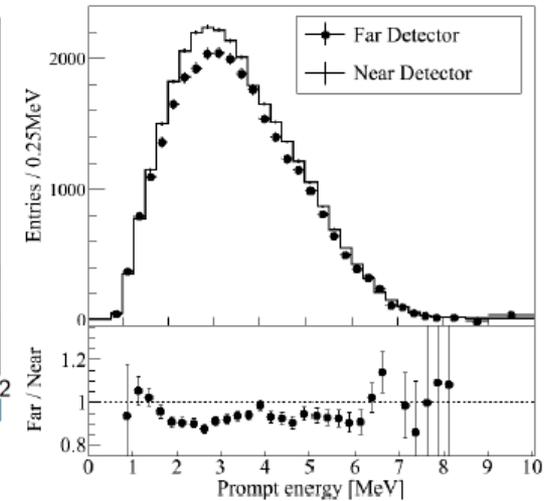
New results from Daya Bay
Rate+Shape fit just delivered

$$0.090^{+0.008}_{-0.009}$$



RENO

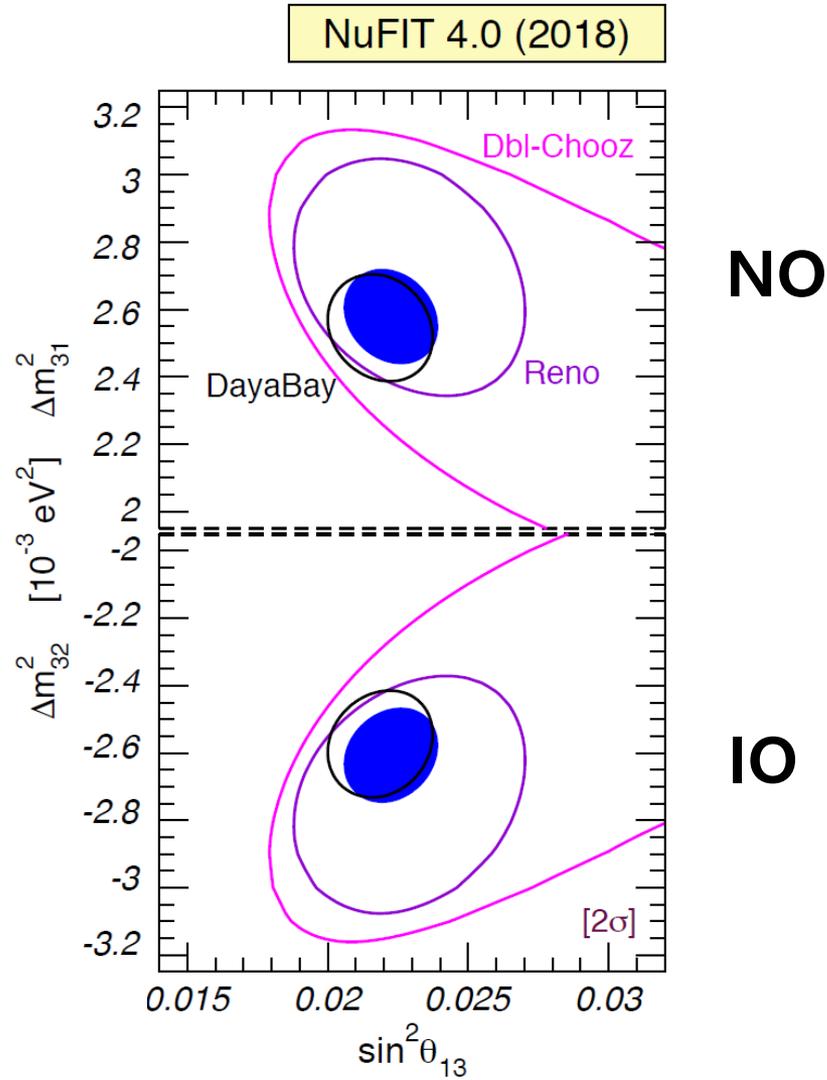
(NuTel 2013, March)



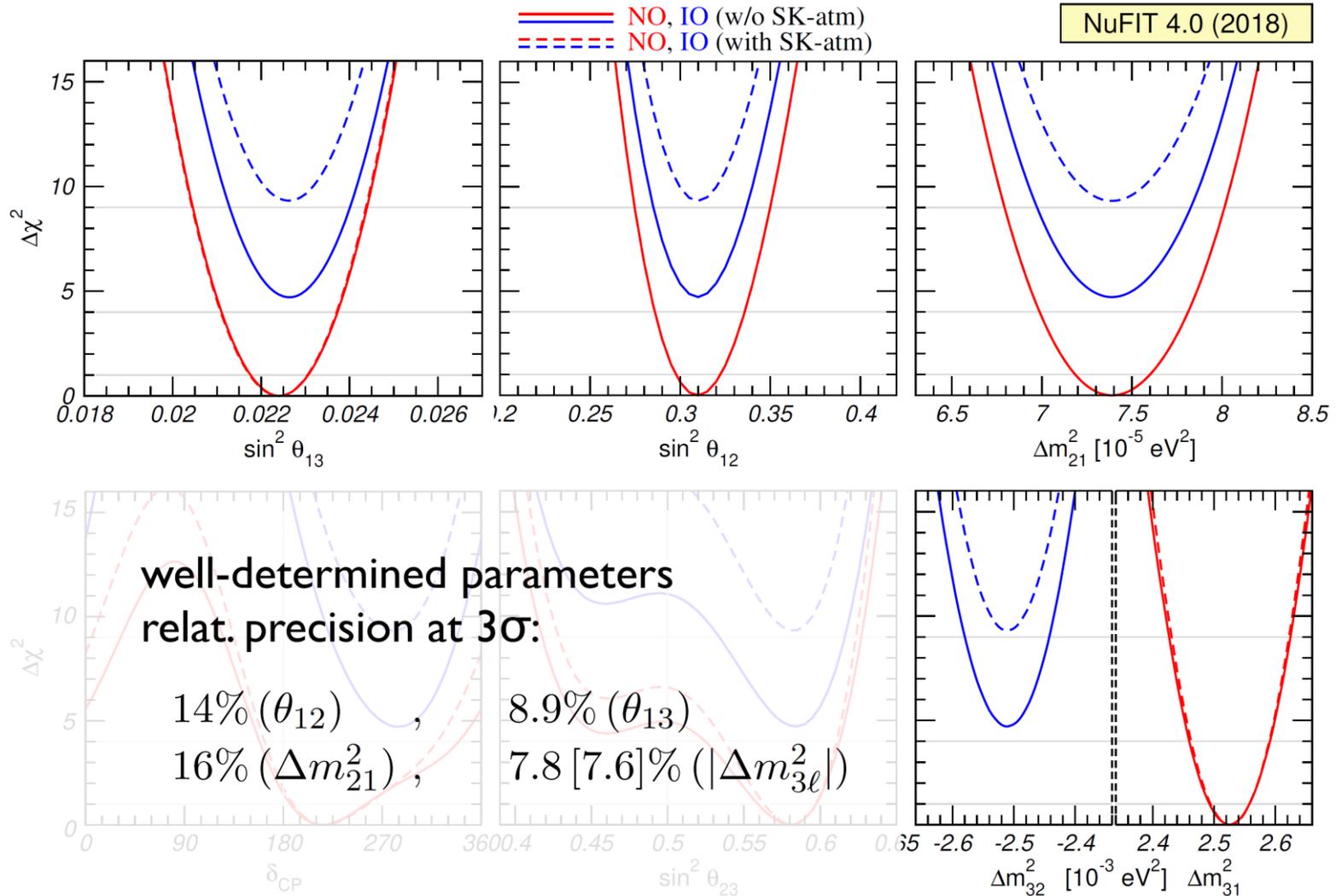
$$0.100 \pm 0.018$$

Precision already < 10%!

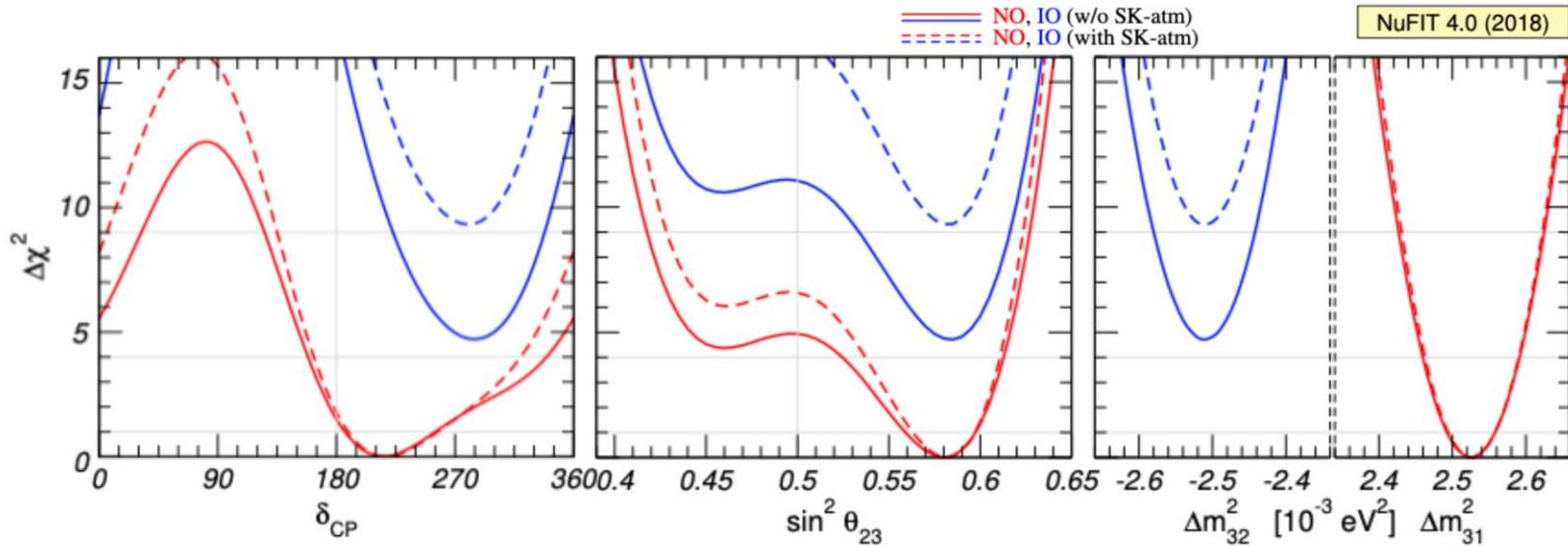
θ_{13}



How precisely determined?



Not-so-well measured



- CP conservation allowed at $\Delta\chi^2 = 1.8$ but bf at $\delta = 217^\circ$
- Octant of θ_{23} : 2nd octant preferred, bf at $\sin^2 \theta_{23} = 0.58$
- Mass ordering : NO is preferred over IO. (adding SK I-IV to the global fit \rightarrow IO disfavored at 3σ)

Precision Measurements

parameter	best fit $\pm 1\sigma$	3σ range	
Δm_{21}^2 [10^{-5}eV^2]	$7.55^{+0.20}_{-0.16}$	7.05–8.14	2.4%
$ \Delta m_{31}^2 $ [10^{-3}eV^2] (NO)	2.50 ± 0.03	2.41–2.60	1.3%
$ \Delta m_{31}^2 $ [10^{-3}eV^2] (IO)	$2.42^{+0.03}_{-0.04}$	2.31–2.51	
$\sin^2 \theta_{12}/10^{-1}$	$3.20^{+0.20}_{-0.16}$	2.73–3.79	5.5%
$\sin^2 \theta_{23}/10^{-1}$ (NO)	$5.47^{+0.20}_{-0.30}$	4.45–5.99	4.7%
$\sin^2 \theta_{23}/10^{-1}$ (IO)	$5.51^{+0.18}_{-0.30}$	4.53–5.98	4.4%
$\sin^2 \theta_{13}/10^{-2}$ (NO)	$2.160^{+0.083}_{-0.069}$	1.96–2.41	3.5%
$\sin^2 \theta_{13}/10^{-2}$ (IO)	$2.220^{+0.074}_{-0.076}$	1.99–2.44	
δ/π (NO)	$1.32^{+0.21}_{-0.15}$	0.87–1.94	10%
δ/π (IO)	$1.56^{+0.13}_{-0.15}$	1.12–1.94	9%

relative 1 σ uncertainty

(de Salas, Forero, Ternes, Tortola, Valle, PLB782 , 1708.01186)

- Implications of global fit:

- ✓ $\theta_{12} + \theta_C = \pi/4$ satisfied within 2σ .

- quark-lepton complementarity (Raidal, Smirnov, Minakata, SK, Kim,...'04)

- ✓ Non-maximal θ_{23} is favored at 2 (1.5) σ level for NO (IO)

- could be related to $\sqrt{m_2/m_3}$ similar to Gatto-Sartori-Tonin

- ✓ Zero θ_{13} is excluded at 10 σ . → test for flavor models

- ✓ Two large angles → hint for discrete flavor symmetry?

- ✓ $\delta \approx 3\pi/2$ is favored by LBL exps.

- could be related with mixing angles, flavor symmetries etc. ?

Theoretical Issues

- Origins of neutrino mixing & CP violation
 - flavor symmetry
 - predictions
- Origins of tiny neutrino mass
 - seesaw variants
 - radiative generation
- New physics in neutrino oscillation

I. Origin of mixing pattern

- From fit to neutrino data in 3-neutrino paradigm

$$|U_{PMNS}| = \begin{pmatrix} 0.800 - 0.844 & 0.515 - 0.581 & 0.139 - 0.155 \\ 0.229 - 0.516 & 0.438 - 0.699 & 0.614 - 0.790 \\ 0.249 - 0.528 & 0.462 - 0.715 & 0.595 - 0.776 \end{pmatrix}$$

Looks different from quark mixing matrix !!

$$|V_{CKM}| = \begin{pmatrix} 0.97434 & 0.22506 & 0.00357 \\ 0.22492 & 0.97351 & 0.0414 \\ 0.00875 & 0.0403 & 0.99915 \end{pmatrix}$$

PDG(2016)

- How do we understand ν mixing matrix ?

Before measuring θ_{13} , tri-bimaximal mixing hypothesis :

Harrison & Perkins & Scott (2002)

$$- U^{TBM} = \begin{pmatrix} \frac{2}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$\theta_{13} \approx 0; \quad \theta_{23} \approx 45^\circ; \quad \theta_{12} = \sin^{-1}\left(\frac{1}{\sqrt{3}}\right) \approx 35.3^\circ$$

- generates specific neutrino mass matrix

$$UM_\nu^D U^T = \begin{pmatrix} m_1 & m_2 & m_2 \\ \cdot & \frac{1}{2}(m_1 + m_2 + m_3) & \frac{1}{2}(m_1 + m_2 - m_3) \\ \cdot & \cdot & \frac{1}{2}(m_1 + m_2 + m_3) \end{pmatrix}$$

$$= \frac{m_1 + m_3}{2} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \frac{m_2 - m_1}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \frac{m_1 - m_3}{2} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

A_4 symmetric

- Integer matrix elements suggest non-Abelian discrete symmetry

- **Mixing understanding from discrete symmetries**

-setting $U_{PMNS} = (\vec{u}_1, \vec{u}_2, \vec{u}_3)$, we construct group generators:

$$S_1 = \vec{u}_1 \vec{u}_1^+ - \vec{u}_2 \vec{u}_2^+ - \vec{u}_3 \vec{u}_3^+$$

$$S_2 = -\vec{u}_1 \vec{u}_1^+ + \vec{u}_2 \vec{u}_2^+ - \vec{u}_3 \vec{u}_3^+$$

(CSLam'06)

$$S_3 = -\vec{u}_1 \vec{u}_1^+ - \vec{u}_2 \vec{u}_2^+ + \vec{u}_3 \vec{u}_3^+$$

- $S_1 = \frac{1}{3} \begin{pmatrix} 1 & -2 & -2 \\ -2 & -2 & -1 \\ 2 & -1 & -2 \end{pmatrix} \quad S_2 = \frac{1}{3} \begin{pmatrix} -1 & 2 & -2 \\ 2 & -1 & -2 \\ -2 & -2 & -1 \end{pmatrix} \quad S_3 = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$

then, $S^T \bar{M}_\nu S = \bar{M}_\nu$

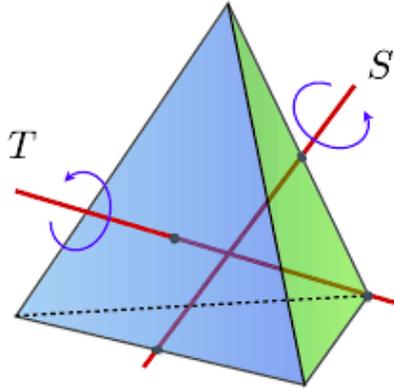
C.S.Lam, PRD98(2008)
arXiv:0809.1185

- For charged lepton, $T^+ \bar{M}_e T = \bar{M}_e$ with $\bar{M}_e = M_e^+ M_e$ & $T^n = 1$

 $\{S, T\}$ forms a discrete group (flavor symmetry)

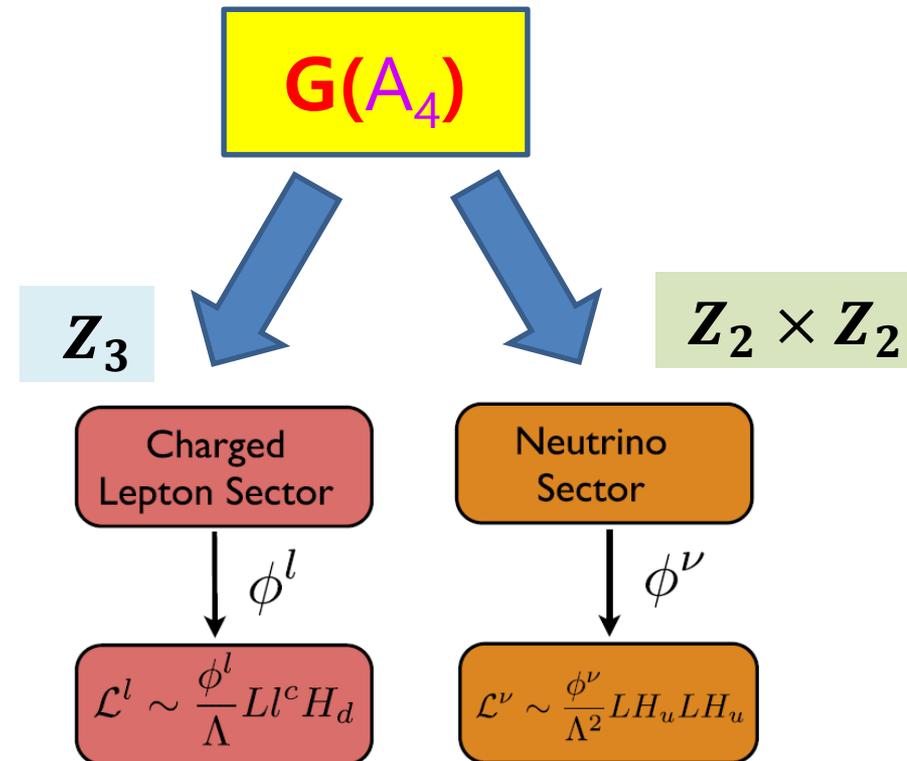
(Mixing matrices diagonalize M_ν and \bar{M}_e also diagonalize S and T)

- Simplest group with a triplet representation: A_4
 A_4 has subgroups: three Z_2 , four Z_3 , one $Z_2 \times Z_2$



- A_4 is spontaneously broken to subgroups:
 Neutrino sector preserves, $Z_2 \times Z_2$:
 Charged lepton sector preserves, Z_3 :

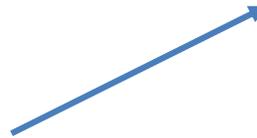
arXiv: 1402.4271 King, Merle, Morisi,
 Simizu, Tanimoto



- Many discrete groups reproducing TBM mixing

Group	d	Irr. Repr.'s	Presentation
$D_3 \sim S_3$	6	1, 1', 2	$A^3 = B^2 = (AB)^2 = 1$
D_4	8	$1_1, \dots, 1_4, 2$	$A^4 = B^2 = (AB)^2 = 1$
D_7	14	1, 1', 2, 2', 2''	$A^7 = B^2 = (AB)^2 = 1$
A_4	12	1, 1', 1'', 3	$A^3 = B^2 = (AB)^3 = 1$
$A_5 \sim PSL_2(5)$	60	1, 3, 3', 4, 5	$A^3 = B^2 = (BA)^5 = 1$
T'	24	1, 1', 1'', 2, 2', 2'', 3	$A^3 = (AB)^3 = R^2 = 1, B^2 = R$
S_4	24	1, 1', 2, 3, 3'	$BM: A^4 = B^2 = (AB)^3 = 1$ $TB: A^3 = B^4 = (BA^2)^2 = 1$
$\Delta(27) \sim Z_3 \times Z_3$	27	$1_1, \dots, 1_9, 3, \bar{3}$	
$PSL_2(7)$	168	1, 3, $\bar{3}, 6, 7, 8$	$A^3 = B^2 = (BA)^7 = (B^{-1}A^{-1}BA)^4 = 1$
$T_7 \sim Z_7 \times Z_3$	21	1, 1', $\bar{1}, 3, \bar{3}$	$A^7 = B^3 = 1, AB = BA^4$

(Altarelli, Feruglio, 1002.0211)



Each group has many models!

(Barry, Rodejohann, PRD81(2010))

Type	L_i	ℓ_i^c	ν_i^c	Δ
A1	$\underline{3}$	$\underline{1}, \underline{1}', \underline{1}''$
A2				$\underline{1}, \underline{1}', \underline{1}'', \underline{3}$
B1	$\underline{3}$	$\underline{1}, \underline{1}', \underline{1}''$	$\underline{3}$...
B2				$\underline{1}, \underline{3}$
C1				...
C2	$\underline{3}$	$\underline{3}$...	$\underline{1}$
C3				$\underline{1}, \underline{3}$
C4				$\underline{1}, \underline{1}', \underline{1}'', \underline{3}$
D1				...
D2	$\underline{3}$	$\underline{3}$	$\underline{3}$	$\underline{1}$
D3				$\underline{1}'$
D4				$\underline{1}', \underline{3}$
E	$\underline{3}$	$\underline{3}$	$\underline{1}, \underline{1}', \underline{1}''$...
F	$\underline{1}, \underline{1}', \underline{1}''$	$\underline{3}$	$\underline{3}$	$\underline{1}$ or $\underline{1}'$
G	$\underline{3}$	$\underline{1}, \underline{1}', \underline{1}''$	$\underline{1}, \underline{1}', \underline{1}''$...
H	$\underline{3}$	$\underline{1}, \underline{1}, \underline{1}$
I	$\underline{3}$	$\underline{1}, \underline{1}, \underline{1}$	$\underline{1}, \underline{1}, \underline{1}$...
J	$\underline{3}$	$\underline{1}, \underline{1}, \underline{1}$	$\underline{3}$...

- **Modification of Tri-Bimaximal Mixing**

- Simple possible forms :

$$\begin{cases} U_{TBM} U_{ij}(\theta) \\ U_{ij}^+(\theta) U_{TBM} \end{cases}$$

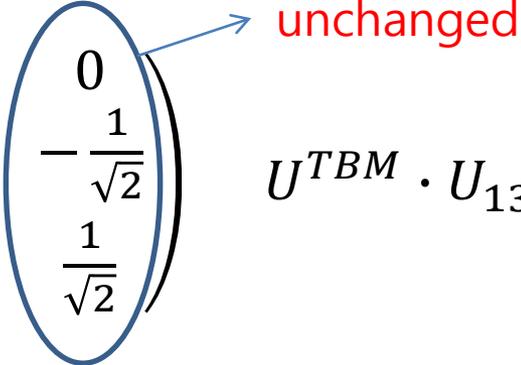
- θ possibly gives rise to non-zero θ_{13} and deviation from maximal θ_{23}

(He & Zee, PLB645(2007), SK & Kim PRD90(2014)
See also, Goswami, Petcov, Ray, Rodejohann, PRD80(2009))

- Best fit achieved by (SK & Kim, PRD90(2014))

$$U_{TBM} \cdot U_{23} \sim \begin{pmatrix} \frac{\sqrt{2}}{\sqrt{3}} & -\frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}}\lambda \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{2}}\lambda & \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}}\lambda \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{2}}\lambda & \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}}\lambda \end{pmatrix} \quad (c_{23} \sim 1, s_{23} \sim \lambda)$$

- $$U^{TBM} \cdot U_{12} = \begin{pmatrix} \blacksquare & \blacksquare & 0 \\ \blacksquare & \blacksquare & -\frac{1}{\sqrt{2}} \\ \blacksquare & \blacksquare & \frac{1}{\sqrt{2}} \end{pmatrix} \quad U^{TBM} \cdot U_{13} = \begin{pmatrix} \blacksquare & -\frac{1}{\sqrt{3}} & \blacksquare \\ \blacksquare & \frac{1}{\sqrt{3}} & \blacksquare \\ \blacksquare & \frac{1}{\sqrt{3}} & \blacksquare \end{pmatrix}$$



- $$U^{TBM} \cdot U_{23} = \begin{pmatrix} \frac{\sqrt{2}}{\sqrt{3}} & \blacksquare & \blacksquare \\ \frac{1}{\sqrt{6}} & \blacksquare & \blacksquare \\ \frac{1}{\sqrt{6}} & \blacksquare & \blacksquare \end{pmatrix}$$

Unchanged columns may reflect the remnants of flavor symmetry \rightarrow residual symmetry

Including Phases

- Any forms of neutrino mixing matrix should be equivalent to U_{PMNS} presented in the standard parameterization :

- $U_{PMNS} = U_{23}(\theta_{23})U_{13}(\theta_{13}, \delta_D)U_{12}(\theta_{12})P_\phi$

$$= \begin{pmatrix} c_{12}c_{13} & -s_{12}c_{13} & s_{13}e^{i\delta_D^*} \\ * & * & -s_{23}c_{13} \\ * & * & c_{23}c_{13} \end{pmatrix} \cdot \begin{pmatrix} e^{i\phi_1} & & \\ & e^{i\phi_2} & \\ & & e^{i\phi_3} \end{pmatrix}$$

$$= P_\alpha \cdot V \cdot P_\beta \quad , V = U^{TBM} \cdot U_{23(13)}(\theta, \xi)$$



$$V_{ij}e^{i(\alpha_i+\beta_j)} = (U_{PMNS})_{ij}$$

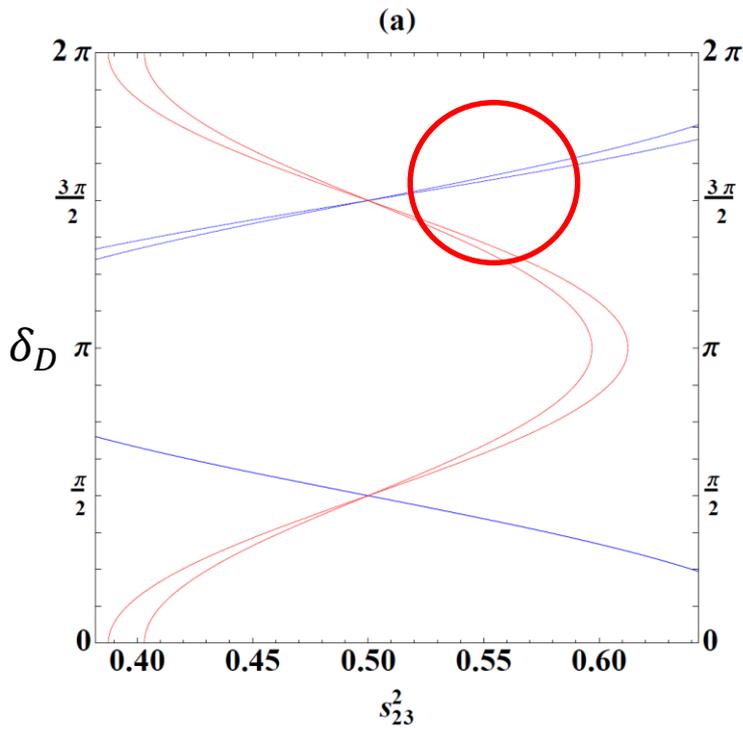
- Predictions : (SK & CSKim, PRD90(2014), SK & Tanimoto, PRD91(2015))

$$s_{12}^2 = 1 - \frac{2}{3(1-s_{13}^2)}$$

$$s_{12}^2 = \frac{1}{3(1-s_{13}^2)}$$

$$\cos \delta_D = \frac{1}{2 \tan 2\theta_{23}} \cdot \frac{1 - 5s_{13}^2}{s_{13}\sqrt{2 - 6s_{13}^2}}$$

$$\cos \delta_D = \frac{1}{\tan 2\theta_{23}} \cdot \frac{1 - 2s_{13}^2}{s_{13}\sqrt{2 - 3s_{13}^2}}$$



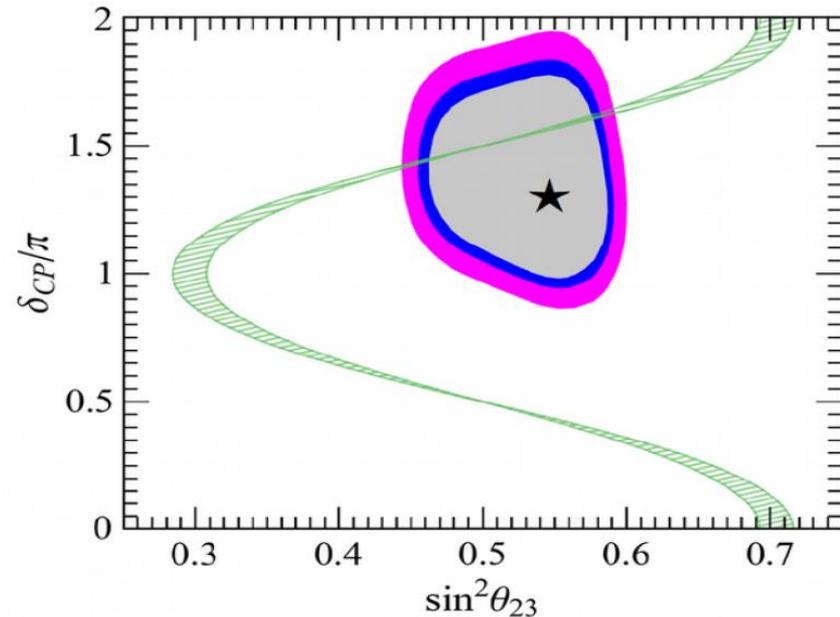
The relations can be obtained from A4 model
(SK & Tanimoto, PRD91 (2015))

Alternatively,

$$\begin{bmatrix} \sqrt{\frac{2}{3}} & \frac{e^{-i\rho} \cos \theta}{\sqrt{3}} & -\frac{ie^{-i\rho} \sin \theta}{\sqrt{3}} \\ -\frac{e^{i\rho}}{\sqrt{6}} & \frac{\cos \theta}{\sqrt{3}} - \frac{ie^{-i\sigma} \sin \theta}{\sqrt{2}} & \frac{e^{-i\sigma} \cos \theta}{\sqrt{2}} - \frac{i \sin \theta}{\sqrt{3}} \\ \frac{e^{i(\rho+\sigma)}}{\sqrt{6}} & -\frac{e^{i\sigma} \cos \theta}{\sqrt{3}} - \frac{i \sin \theta}{\sqrt{2}} & \frac{\cos \theta}{\sqrt{2}} + \frac{ie^{i\sigma} \sin \theta}{\sqrt{3}} \end{bmatrix}$$

$\sin^2 \theta_{12} = \frac{\cos^2 \theta}{\cos^2 \theta + 2},$	$\sin^2 \theta_{23} = \frac{1}{2} + \frac{\sqrt{6} \sin 2\theta \sin \sigma}{2\cos^2 \theta + 4}$
$\sin^2 \theta_{13} = \frac{\sin^2 \theta}{3},$	$\tan \delta_{CP} = \frac{(\cos^2 \theta + 2) \cot \sigma}{5\cos^2 \theta - 2},$
PHYSICAL REVIEW D 98 , 055019 (2018)	

(Cheng, Chulia, Ding, Srivastava, Valle)



- A_4 model easily realizes non-vanishing θ_{13} & CPV

Ahn, SK, PRD86 (2012)

Ahn, SK, CSKim, PRD87 (2013)

SK, Shimizu, Takagi, Shunya Takahashi, Tanimoto, PTEP(2018)

Additional Matrix

$$M_\nu = a \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + b \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + c \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} + d \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$a = \frac{y_{\phi\nu}^\nu \alpha_\nu v_u^2}{\Lambda}, \quad b = -\frac{y_{\phi\nu}^\nu \alpha_\nu v_u^2}{3\Lambda}, \quad c = \frac{y_\xi^\nu \alpha_\xi v_u^2}{\Lambda}, \quad d = \frac{y_{\xi'}^\nu \alpha_{\xi'} v_u^2}{\Lambda} \quad a = -3b$$

Both normal and inverted mass hierarchies are possible.

$$M_\nu = V_{\text{tri-bi}} \begin{pmatrix} a + c - \frac{d}{2} & 0 & \frac{\sqrt{3}}{2}d \\ 0 & a + 3b + c + d & 0 \\ \frac{\sqrt{3}}{2}d & 0 & a - c + \frac{d}{2} \end{pmatrix} V_{\text{tri-bi}}^T \quad \text{Tri-maximal mixing: TM2}$$

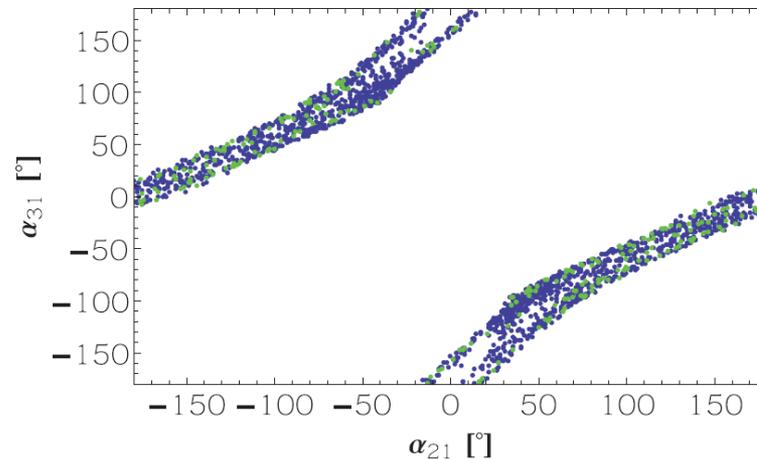
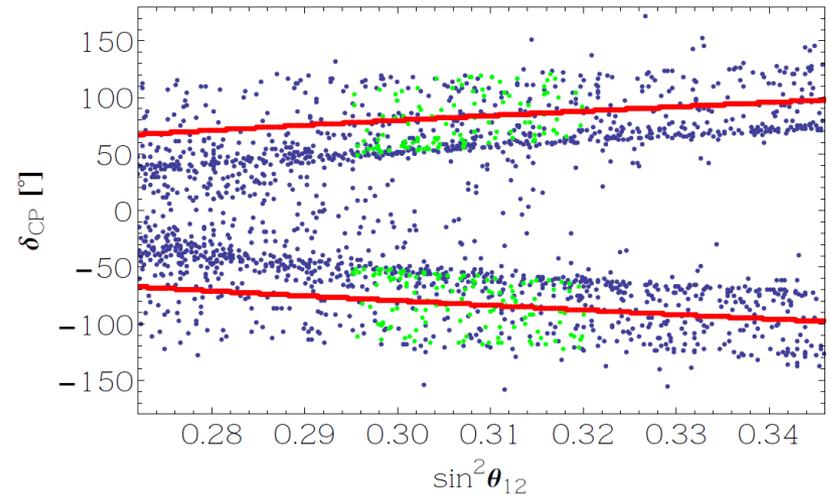
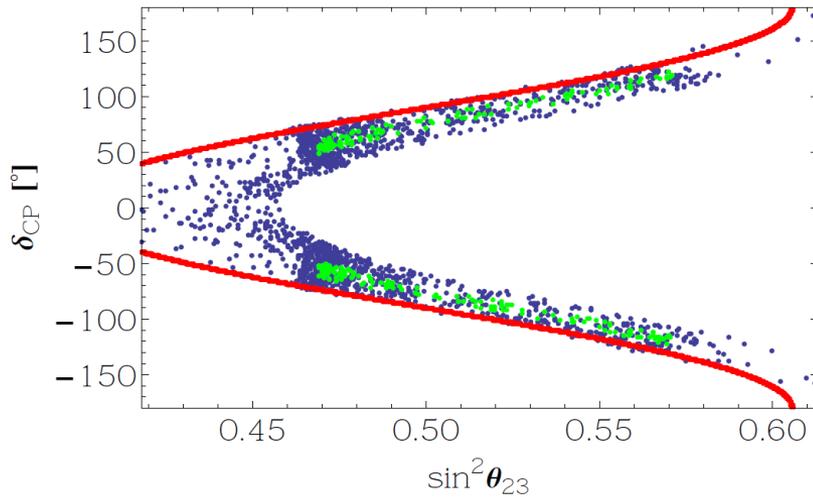
$$\Delta m_{31}^2 = -4a\sqrt{c^2 + d^2 - cd}, \quad \Delta m_{21}^2 = (a + 3b + c + d)^2 - (a + \sqrt{c^2 + d^2 - cd})^2$$

- A_4 model easily realizes non-vanishing θ_{13} & CPV

Ahn, SK, PRD86 (2012)

Ahn, SK, CSKim, PRD87 (2013)

SK, Shimizu, Takagi, Shunya Takahashi, Tanimoto, PTEP(2018)



Prediction of CP phase

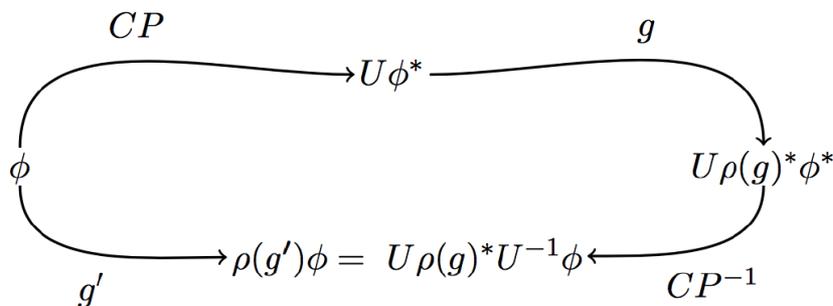
- $\mu - \tau$ reflection symmetry : $\nu_e \leftrightarrow \nu_e^C, \nu_\mu \leftrightarrow \nu_\tau^C$

$$m_\nu = \begin{pmatrix} x & z_1 & z_1^* \\ \cdot & z_2 & y \\ \cdot & \cdot & z_2^* \end{pmatrix} \Rightarrow \delta = \pm \frac{\pi}{2} \quad \& \quad \theta_{23} = \frac{\pi}{4}$$

(Grimus, Lavoura, PRB578(2004))

- General CP transformation :

- combining CP with flavor symmetry $\Rightarrow \delta = \pm \frac{\pi}{2}, \pm\pi, 0$



(Grimus; Chen; Feruglio, Hagedorn, Ziegler; Holthausen, Schmidt, Lindner; Ding, King, Stuart; Meroni, Petcov; Branco, King, Varzielas,...)

Prediction of CP phase

- Example $G_f = S_4 \times Z_3 \times CP$ broken to $G_\nu = Z_2 \times CP$,
 $G_l = Z_3$ (Hagedorn, Feruglio, Ziegler, EPJ74)

$$S = \frac{1}{3} \begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix} \quad T = \frac{1}{3} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix} \quad X_{3'} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$Sm_\nu S = m_\nu \quad \& \quad X_{3'} m_\nu X_{3'} = m_\nu^*$$

 leads to $\delta = \pm \frac{\pi}{2}$

$$U_{PMNS} = \frac{1}{\sqrt{6}} \begin{pmatrix} 2 \cos \theta & \sqrt{2} & 2 \sin \theta \\ -\cos \theta + i\sqrt{3} \sin \theta & \sqrt{2} & -\sin \theta - i\sqrt{3} \cos \theta \\ -\cos \theta - i\sqrt{3} \sin \theta & \sqrt{2} & -\sin \theta + i\sqrt{3} \cos \theta \end{pmatrix} K$$

$$\sin^2 \theta_{13} = \frac{2}{3} \sin^2 \theta$$

$$\sin^2 \theta_{12} = \frac{1}{2 + \cos 2\theta}$$

$$\sin^2 \theta_{23} = \frac{1}{2}$$

How to test Flavor Symmetry

- UV theories giving rise to flavor symmetry in lepton sector contains **new scalars** → probe of signal be test of FlaSy.
- Mixing angle sum rules:

Example:

$$\sin^2 \theta_{23} = \frac{1}{2} \frac{1}{\cos^2 \theta_{13}} \geq \frac{1}{2}, \quad \sin^2 \theta_{12} \simeq \frac{1}{\sqrt{3}} - \frac{2\sqrt{2}}{3} \sin \theta_{13} \cos \delta_{CP} + \frac{1}{3} \sin^2 \theta_{13} \cos 2\delta_{CP}$$

$$\sin^2 \theta_{12} = \frac{1}{3} \frac{1}{\cos^2 \theta_{13}} \geq \frac{1}{3}, \quad \cos \delta_{CP} \tan 2\theta_{23} \simeq \frac{1}{\sqrt{2} \sin \theta_{13}} \left(1 - \frac{5}{4} \sin^2 \theta_{13} \right)$$

$$s_{12}^2 = 1 - \frac{2}{3(1-s_{13}^2)}$$

- Neutrino mass sum rules in FLaSy ⇔ different **$0\nu\beta\beta$**

- Prediction of CP phase

(Girardi, Petcov, Titov, NPB894(2015))

$$(e.g.) \cos \delta_D = \frac{1}{\tan 2\theta_{23}} \cdot \frac{1 - 5s_{13}^2}{s_{13}\sqrt{2 - 6s_{13}^2}}$$

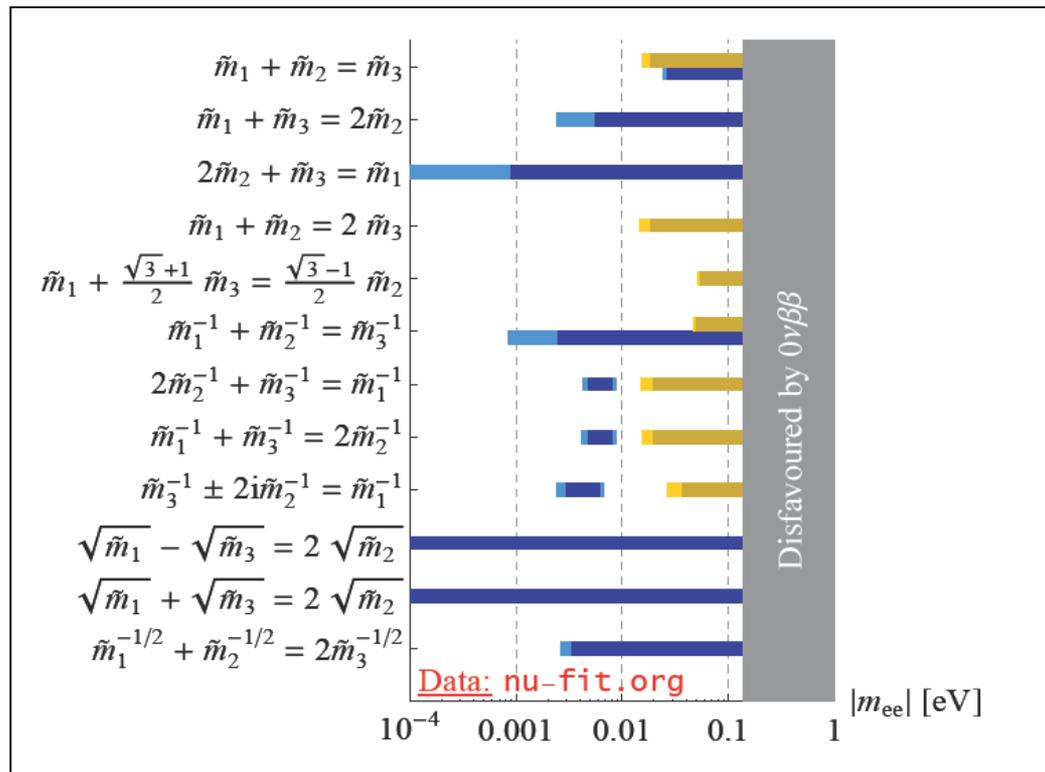
Neutrino mass sum rules

(King, Merle, Morisi, Shimizu, Tanimoto, New J. Phys. 2014)

Sum Rule	Group	Seesaw Type	Matrix
$\tilde{m}_1 + \tilde{m}_2 = \tilde{m}_3$	A_4 [167]([175, 178–181]); S_4 ([182]); A_5 [69] ^a	Weinberg	m_{LL}^ν
$\tilde{m}_1 + \tilde{m}_2 = \tilde{m}_3$	$\Delta(54)$ [183]; S_4 ([163])	Type II	M_L
$\tilde{m}_1 + 2\tilde{m}_2 = \tilde{m}_3$	S_4 [120]	Type II	M_L
$2\tilde{m}_2 + \tilde{m}_3 = \tilde{m}_1$	A_4 [165, 167] ([36, 37, 188–194, , , , , , 178–181]) S_4 ([45, 124]) ^b ; T' [195, 196] ([46, 134, 197, 198]); T_7 ([199])	Weinberg	m_{LL}^ν
$2\tilde{m}_2 + \tilde{m}_3 = \tilde{m}_1$	A_4 ([200])	Type II	M_L
$\tilde{m}_1 + \tilde{m}_2 = 2\tilde{m}_3$	S_4 [201] ^c	Dirac ^c	m^D
$\tilde{m}_1 + \tilde{m}_2 = 2\tilde{m}_3$	$L_e - L_\mu - L_\tau$ ([202])	Type II	M_L
$\tilde{m}_1 + \frac{\sqrt{3}+1}{2}\tilde{m}_3 = \frac{\sqrt{3}-1}{2}\tilde{m}_2$	A_5 ([203])	Weinberg	m_{LL}^ν
$\tilde{m}_1^{-1} + \tilde{m}_2^{-1} = \tilde{m}_3^{-1}$	A_4 [167]; S_4 ([163, 175]); A_5 [176, 177]	Type I	M_R
$\tilde{m}_1^{-1} + \tilde{m}_2^{-1} = \tilde{m}_3^{-1}$	S_4 ([163])	Type III	M_Σ
$2\tilde{m}_2^{-1} + \tilde{m}_3^{-1} = \tilde{m}_1^{-1}$	A_4 [135, 164, 165, 167, 204] ([37, 137, 145, 205–211]); T' [196]	Type I	M_R
$\tilde{m}_1^{-1} + \tilde{m}_3^{-1} = 2\tilde{m}_2^{-1}$	A_4 ([212–214]); T' [215]	Type I	M_R
$\tilde{m}_3^{-1} \pm 2i\tilde{m}_2^{-1} = \tilde{m}_1^{-1}$	$\Delta(96)$ [66]	Type I	M_R
$\tilde{m}_1^{1/2} - \tilde{m}_3^{1/2} = 2\tilde{m}_2^{1/2}$	A_4 ([162])	Type I	m^D
$\tilde{m}_1^{1/2} + \tilde{m}_3^{1/2} = 2\tilde{m}_2^{1/2}$	A_4 ([216])	Scotogenic	h_ν
$\tilde{m}_1^{-1/2} + \tilde{m}_2^{-1/2} = 2\tilde{m}_3^{-1/2}$	S_4 [217]	Inverse	M_{RS}

Neutrino mass sum rules

Restrictions on $|m_{ee}|$ by mass sum rules



King, Merle, Stuart, JHEP2013

King, Merle, Morisi, Shimizu, Tanimoto, New J. Phys. 2014

II. Origin of Neutrino Mass

- Why neutrinos are massless in SM ?
 - no right-handed neutrinos
 - only SU(2) doublet Higgs scalars
 - prohibiting non-renormalizable terms
- How can neutrinos have mass ?
 - breaking those restrictions

Seesaw Origins

Type-I Seesaw

- Introducing L-conserving right-handed neutrinos

$$Y_\nu \Phi \bar{\nu}_L \nu_R \rightarrow Y_\nu \langle \phi^0 \rangle \bar{\nu}_L \nu_R \sim 0.2 \text{ eV}$$

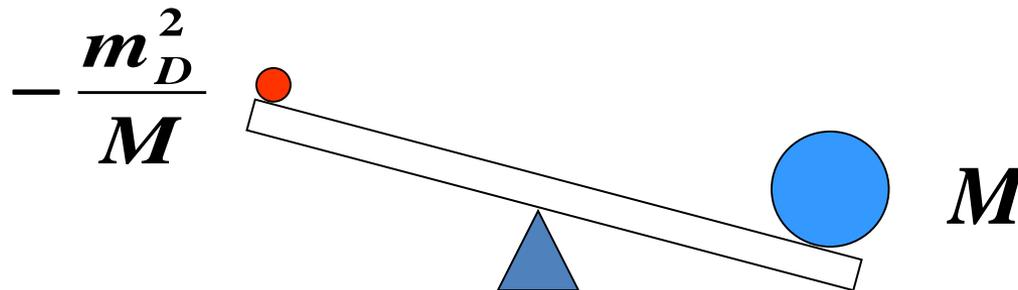
→ $Y_\nu \sim 10^{-12}$: why so small?

- No principle prohibit $M_R \bar{\nu}_R^c \nu_R$
- Seesaw mechanism :

ν_R can have large mass (L-violation: Type-I)

$$L_{mass} = \frac{1}{2} (\nu_L^T N_R^T) C^{-1} \begin{pmatrix} 0 & m_D \\ m_D & M \end{pmatrix} \begin{pmatrix} \nu_L \\ N_R \end{pmatrix} + c.c.$$

(Minkowski '77 Gellman
 Ramond Slansky '80
 Glashow, Yanagida '79
 Mohapatra Senjanovic '80
 Lazarides Shafi Weterrich '81
 Schechter-Valle '80 & '82)



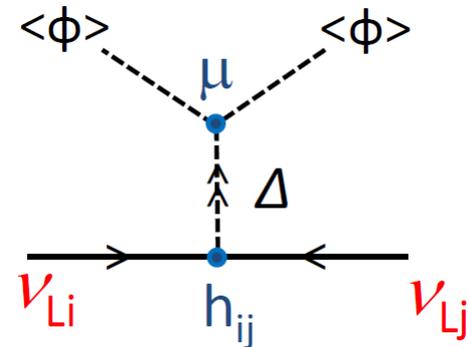
Type-II Seesaw

- Introducing $SU(2)$ triplet Higgs (Δ) (type-II):
 $hLL\Delta \leftarrow \langle \Delta \rangle < 8 \text{ GeV}$ from ρ parameter.
majorana mass

- Due to additional possible terms:

$$\mu\Phi\Delta^+\Phi + M_{\Delta}^2\text{Tr}[\Delta^+\Delta]$$

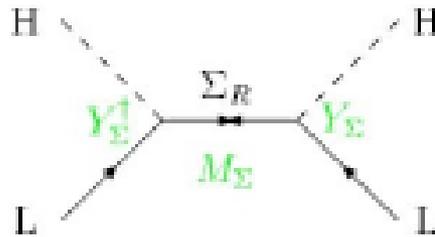
$$\rightarrow \langle \Delta \rangle = \frac{\mu \langle \Phi^0 \rangle^2}{M_{\Delta}^2}$$



(Magg, Wetterich; Lazarides, Shafi;
Mohapatra, Senjanovic; Schechter, Valle)

Type-III Seesaw

- Introducing SU(2) triplet fermions



$$m_\nu = Y_\Sigma^T \frac{1}{M_\Sigma} Y_\Sigma v^2$$

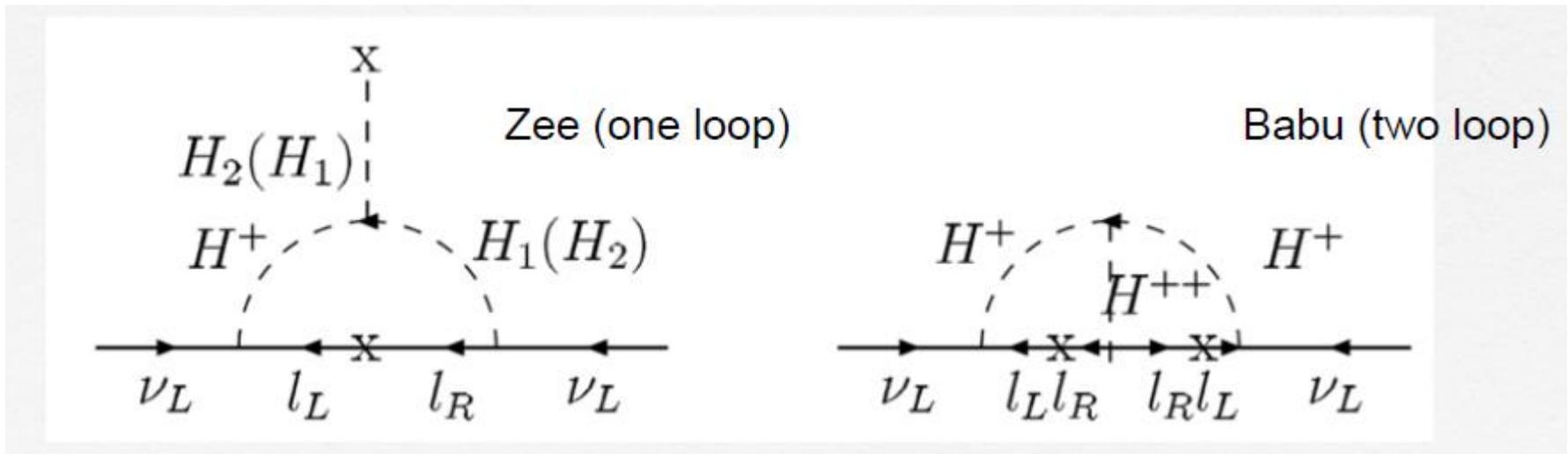
Foot, Lew, He, Joshi; Ma; Ma, Roy; T.H., Lin,
Notari, Papucci, Strumia; Bajc, Nemevsek,
Senjanovic; Dorsner, Fileviez-Perez;....

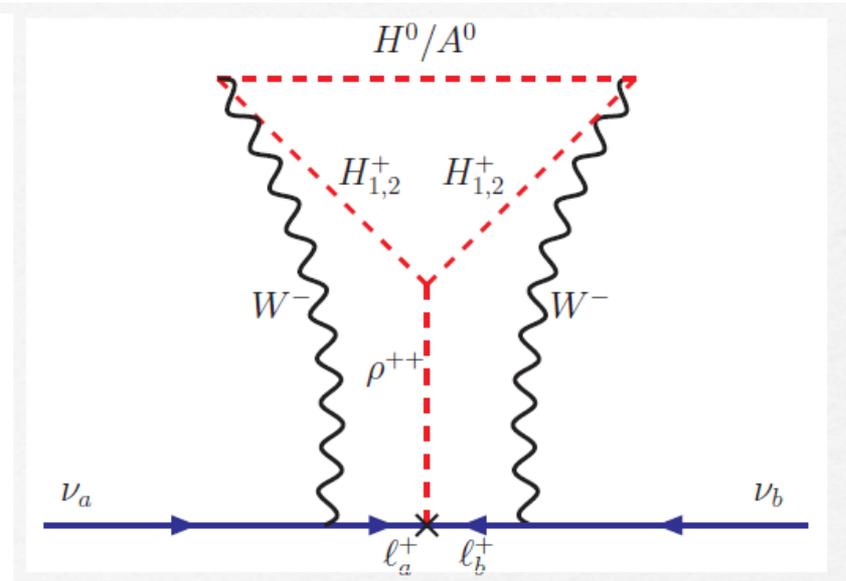
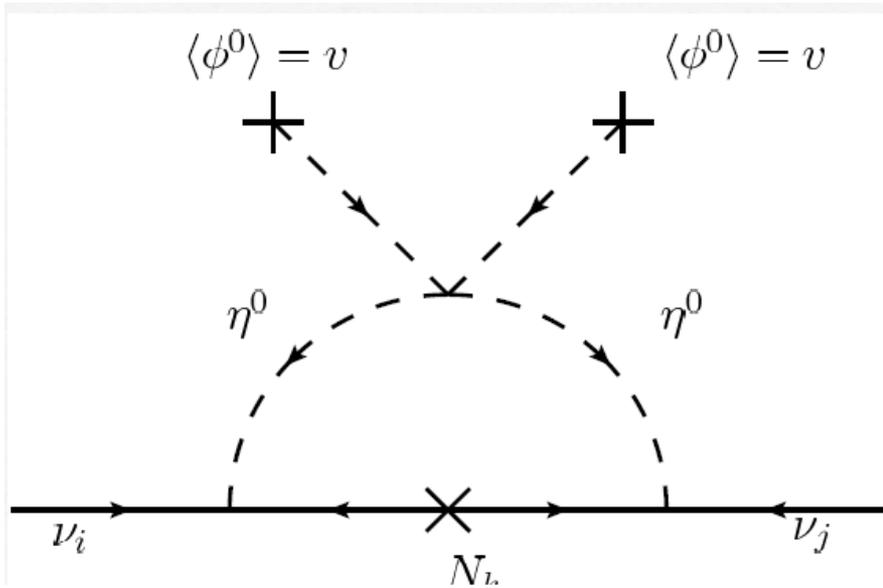
- Non-renormalizable term

$$\frac{\lambda}{M} LL\Phi\Phi \rightarrow \frac{\lambda}{M} \langle \phi^0 \rangle^2 \quad \text{same as type-I seesaw}$$

- Radiative generation of neutrino masses

➡ talk by Ramond Volkas





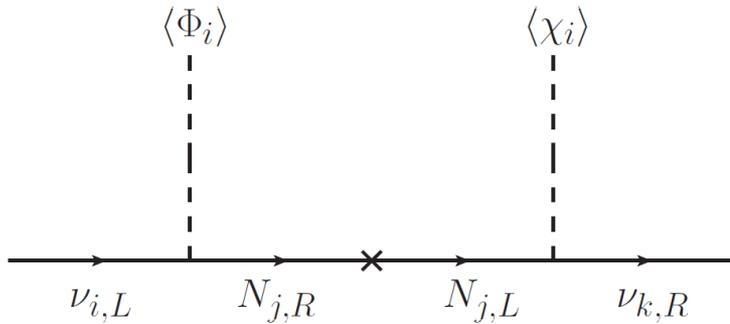
- Scotogenic (Ma)

Cocktail (Gustafsson et al.)

- R-parity violating SUSY model

Seesaw for Dirac Neutrino

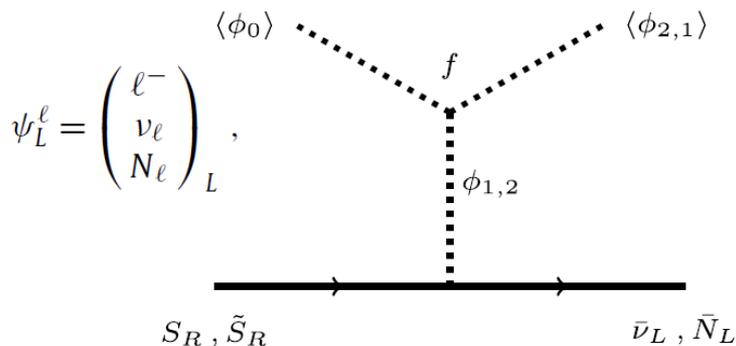
- Type-I Seesaw



Chulia, Srivastava, Valle, PLB761 (2016),
Chulia, Ma, Srivastava, Valle, PLB767 (2017)

The Dirac type-I seesaw mechanism. Φ_i and χ_i are triplets under $\Delta(27)$

Type-II Seesaw



(Valle, Vaquera-Araujo, PLB755(2016),

Addazi et al PLB759 (2016))

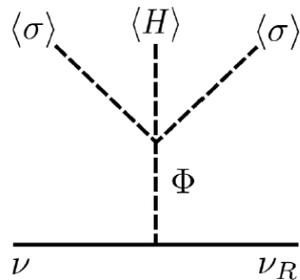
Anomaly free $SU(3)_C \times SU(3)_L \times U(1)_X$

Matter content of the model, where $\hat{u}_R \equiv (u_R, c_R, t_R, U_R)$ and $\hat{d}_R \equiv (d_R, s_R, b_R, D_R, D'_R)$.

	ψ_L^ℓ	ℓ_R	$S_R^\ell, \tilde{S}_R^\ell$	$Q_L^{1,2}$	Q_L^3	\hat{u}_R	\hat{d}_R	ϕ_0	ϕ_1	ϕ_2
$SU(3)_C$	1	1	1	3	3	3	3	1	1	1
$SU(3)_L$	3*	1	1	3	3*	1	1	3*	3*	3*
$U(1)_X$	$-\frac{1}{3}$	-1	0	0	$+\frac{1}{3}$	$+\frac{2}{3}$	$-\frac{1}{3}$	$+\frac{2}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$
\mathcal{L}	$-\frac{1}{3}$	-1	1	$-\frac{2}{3}$	$+\frac{2}{3}$	0	0	$+\frac{2}{3}$	$-\frac{4}{3}$	$-\frac{4}{3}$
$\mathbb{Z}_3^{\text{aux}}$	ω	ω	ω	ω^2	ω^2	ω^2	ω^2	1	1	1

Seesaw for Dirac Neutrino

- Type-II Seesaw

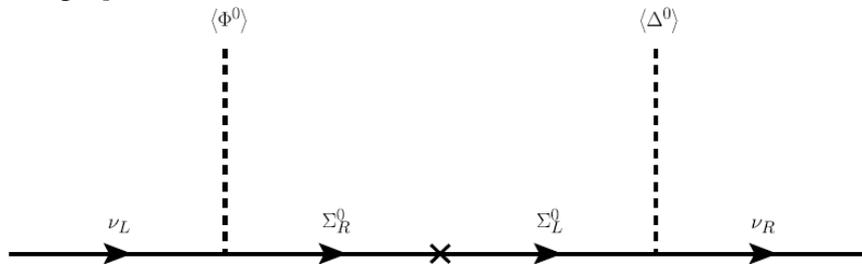


Bonilla, Valle, PLB762(2016)
Reig et al., PRD94(2016)

	\bar{L}	ℓ_R	ν_R	H	Φ	σ
$SU(2)_L$	2	1	1	2	2	1
Z_5	ω	ω^4	ω	1	ω^3	ω
Z_3	α^2	α	α	1	1	1

Neutrino mass generation in type-II Dirac seesaw mechanism

- Type-III Seesaw



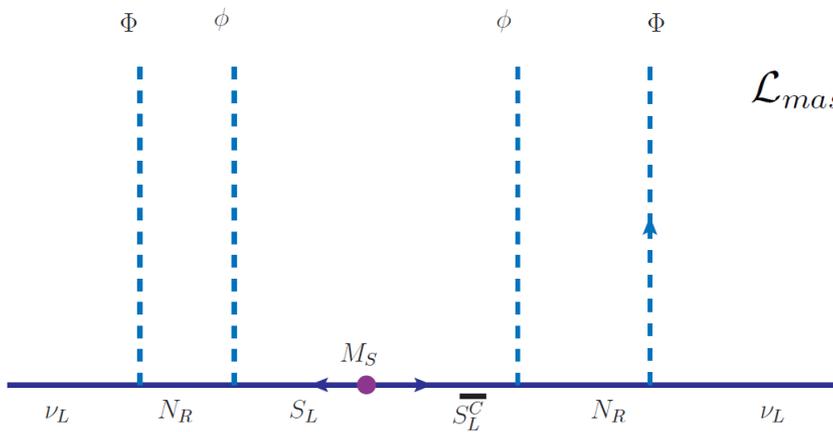
Chulia, Srivastava, Valle, PLB781(2018)

Neutrino mass generation in type-III Dirac seesaw

There can be d=5 op. leading to tiny Dirac mass.

Inverse seesaw

- Inverse seesaw

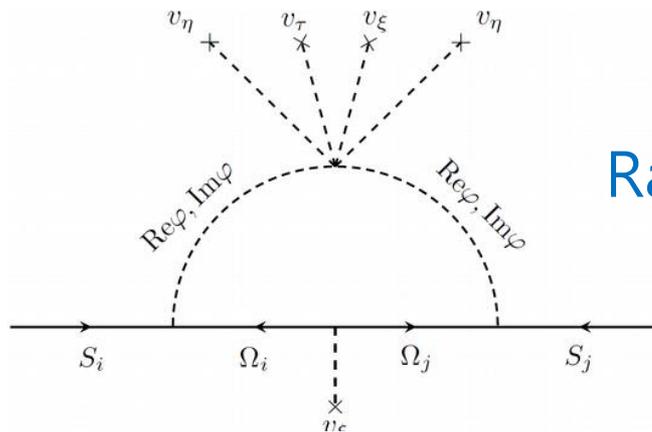


$$\mathcal{L}_{mass} = \left(\overline{\nu}_L^C \quad \overline{N}_R \quad \overline{S}_L^C \right) \begin{pmatrix} 0 & m_D^* & 0 \\ m_D^\dagger & 0 & m_{NS} \\ 0 & m_{NS}^T & M_S \end{pmatrix} \begin{pmatrix} \nu_L \\ N_R^C \\ S_L \end{pmatrix}$$

$$m_\nu = (m_D^* m_{NS}^{-1}) M_S (m_D^* m_{NS}^{-1})^T$$

Mohapatra, PRL56(1986)

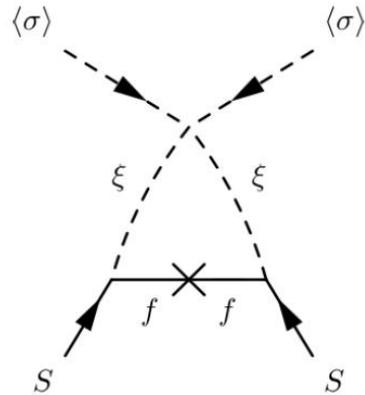
Mohapatra, Valle, PRD34(1986)



Radiative Inverse seesaw

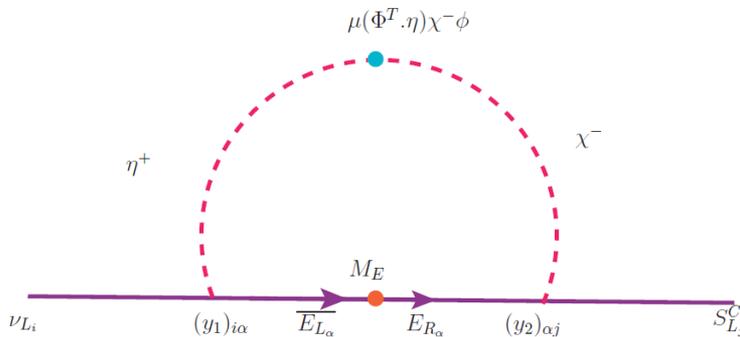
Carcamo Hernandez et al JHEP 1902 (2019)

- Scotogenic inverse seesaw



arXiv:1907.07728

- Inverse seesaw+1-loop (A. Das et al, 1704.02078)

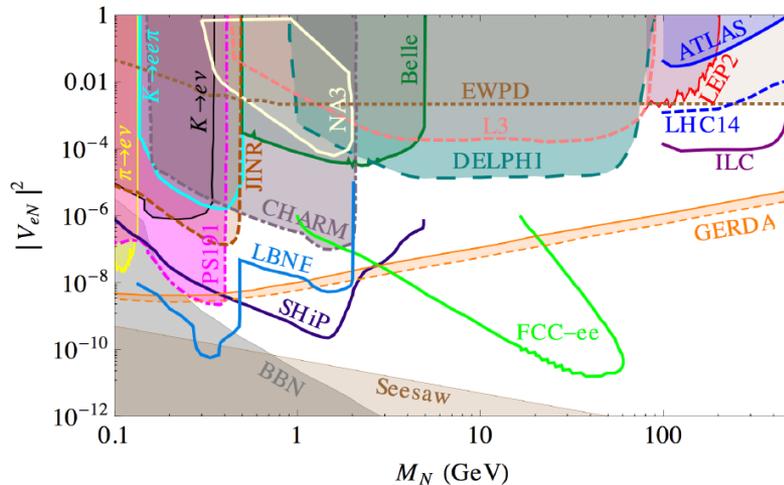


$$m_{\nu}^{\text{tree}+1\text{-loop}} = \begin{pmatrix} 0 & m_D^* & \delta_1^* \\ m_D^\dagger & 0 & m_{NS} \\ \delta_1^\dagger & m_{NS}^T & M_S \end{pmatrix}$$

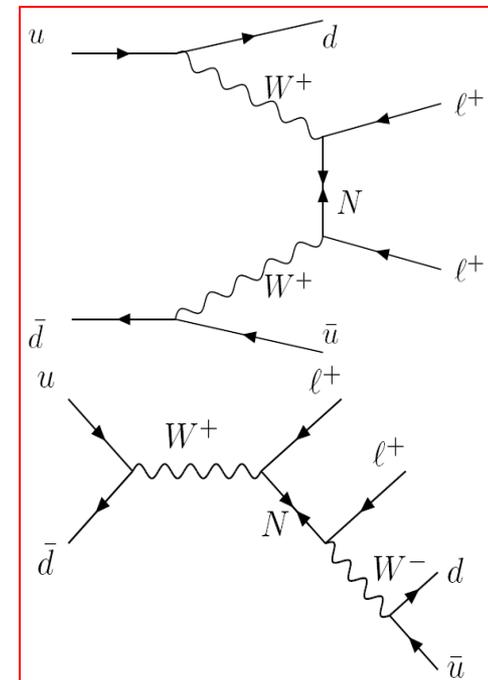
- Dirac Inverse seesaw (Borah, Karmakar, PLB780(2018))

What is the Seesaw scale ?

- For $m_D \sim m_t$, neutrino mass of $m_\nu \leq 1$ eV implies $M_R \sim 10^{14}$ GeV
 - close to the scale of Grand Unification $\sim 10^{16}$ GeV
- For $m_D \sim m_e$, neutrino mass of $m_\nu \leq 1$ eV implies $M_R \sim 1$ TeV.
 - potentially testable at collider



Deppisch, Dev, Pilaftsis, 1502.06541



What is the Seesaw scale ?

- ν MSM (Asaka, Blanchet, Shaposhnikov, PLB631(2005)):
 - $M_{R1} \sim \text{keV}$ scale warm dark matter
 - $M_{R2(R3)} \sim \text{few GeV}$ with tiny Yukawa couplings
- Minimal SM accommodating DM, baryogenesis at the price of fine tuning.

III. New Physics in ν Oscillation

- What causes deviation of standard oscillations
 - Non-standard Interactions(NSI)
 - Unitarity violation in U_{PMNS}
 - light sterile neutrinos
 - long-range forces
 - Lorentz/CPT violation
 - General neutrino interactions
 - decay etc.

NSI

- Existence of NSI indicates new physics beyond the SM

$$\delta\mathcal{L}_{\text{NSI}} = -2\sqrt{2}G_F \sum_{f,P} \epsilon_{\alpha\beta}^{fP} (\bar{\nu}_\alpha \gamma^\mu P_L \nu_\beta) (\bar{f} \gamma_\mu P f)$$

- effect of NSI in propagation can be presented through modification of matter potential

$$\epsilon_{\alpha\beta}^f = \epsilon_{\alpha\beta}^{fL} + \epsilon_{\alpha\beta}^{fR}$$

$$H_{\text{mat}} = \sqrt{2}G_F N_e(x) \begin{pmatrix} 1 + \epsilon_{ee}(x) & \epsilon_{e\mu}(x) & \epsilon_{e\tau}(x) \\ \epsilon_{e\mu}^*(x) & \epsilon_{\mu\mu}(x) & \epsilon_{\mu\tau}(x) \\ \epsilon_{e\tau}^*(x) & \epsilon_{\mu\tau}^*(x) & \epsilon_{\tau\tau}(x) \end{pmatrix}$$

- Even if no mixing in vacuum, $\nu_\alpha \rightarrow \nu_\beta$ can occur in matter
- Complex phases of off-diag. could be new source of CPV

Current conservative model independent bounds

$$\left(\begin{array}{lll} |\epsilon_{ee}| < 4.2 & |\epsilon_{e\mu}| < 0.33 & |\epsilon_{e\tau}| < 3.0 \\ & |\epsilon_{\mu\mu}| < 0.07 & |\epsilon_{\mu\tau}| < 0.33 \\ & & |\epsilon_{\tau\tau}| < 21 \end{array} \right)$$

Deepthi, Goswami, Nath, PLB936 (2018)

- NSI may affect neutrinos at the production point as well as detection point.
- To see those effects, we use different parameters ;

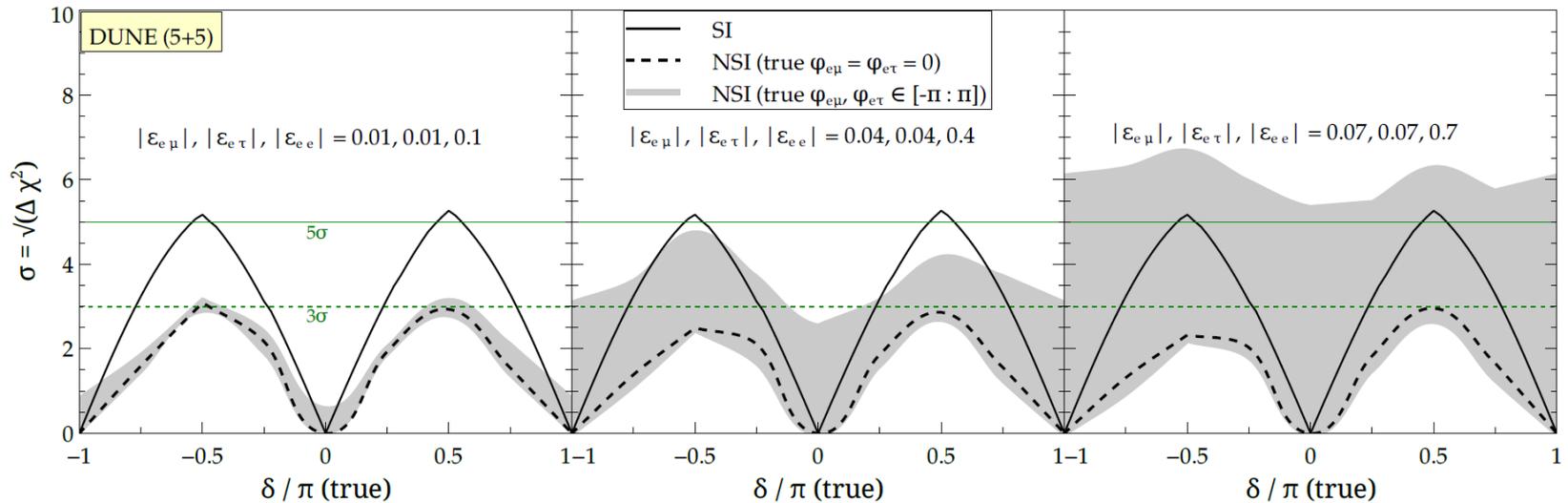
(ex) for production, $2G_F \sum_{\alpha} \epsilon_{l\alpha}^{CC} \left[\bar{l} (1 - \gamma_5) \gamma^{\rho} \nu_{\alpha} \right]$

Results from global fit to solar data and KamLAND

	LMA	LMA \oplus LMA-D
$\varepsilon_{ee}^u - \varepsilon_{\mu\mu}^u$	$[-0.020, +0.456]$	$\oplus[-1.192, -0.802]$
$\varepsilon_{\tau\tau}^u - \varepsilon_{\mu\mu}^u$	$[-0.005, +0.130]$	$[-0.152, +0.130]$
$\varepsilon_{e\mu}^u$	$[-0.060, +0.049]$	$[-0.060, +0.067]$
$\varepsilon_{e\tau}^u$	$[-0.292, +0.119]$	$[-0.292, +0.336]$
$\varepsilon_{\mu\tau}^u$	$[-0.013, +0.010]$	$[-0.013, +0.014]$
$\varepsilon_{ee}^d - \varepsilon_{\mu\mu}^d$	$[-0.027, +0.474]$	$\oplus[-1.232, -1.111]$
$\varepsilon_{\tau\tau}^d - \varepsilon_{\mu\mu}^d$	$[-0.005, +0.095]$	$[-0.013, +0.095]$
$\varepsilon_{e\mu}^d$	$[-0.061, +0.049]$	$[-0.061, +0.073]$
$\varepsilon_{e\tau}^d$	$[-0.247, +0.119]$	$[-0.247, +0.119]$
$\varepsilon_{\mu\tau}^d$	$[-0.012, +0.009]$	$[-0.012, +0.009]$
$\varepsilon_{ee}^p - \varepsilon_{\mu\mu}^p$	$[-0.041, +1.312]$	$\oplus[-3.328, -1.958]$
$\varepsilon_{\tau\tau}^p - \varepsilon_{\mu\mu}^p$	$[-0.015, +0.426]$	$[-0.424, +0.426]$
$\varepsilon_{e\mu}^p$	$[-0.178, +0.147]$	$[-0.178, +0.178]$
$\varepsilon_{e\tau}^p$	$[-0.954, +0.356]$	$[-0.954, +0.949]$
$\varepsilon_{\mu\tau}^p$	$[-0.035, +0.027]$	$[-0.035, +0.035]$

(Esteban et al., 1805.04530)

NSI

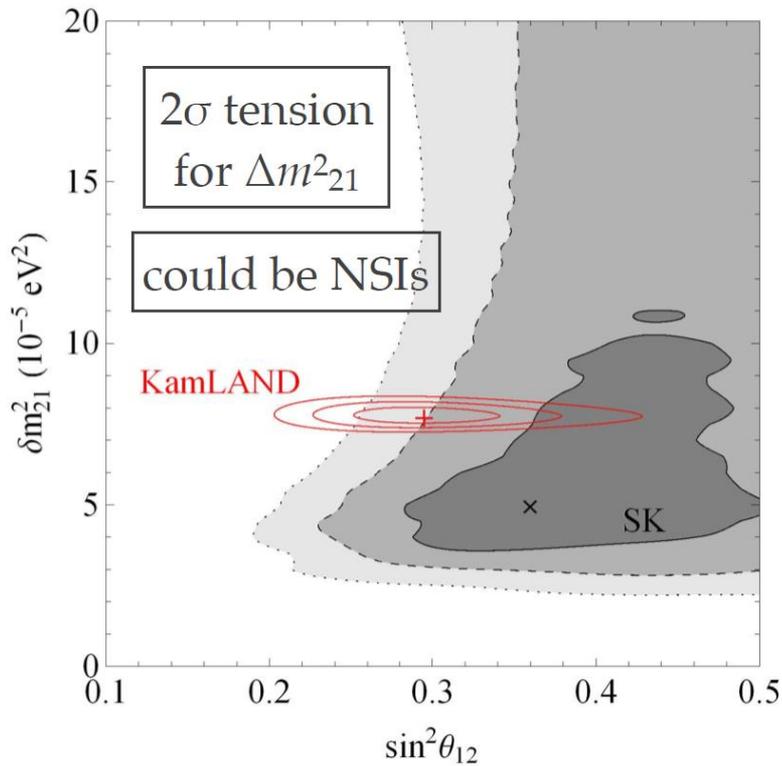


- NSI can prevent determination of CP violation

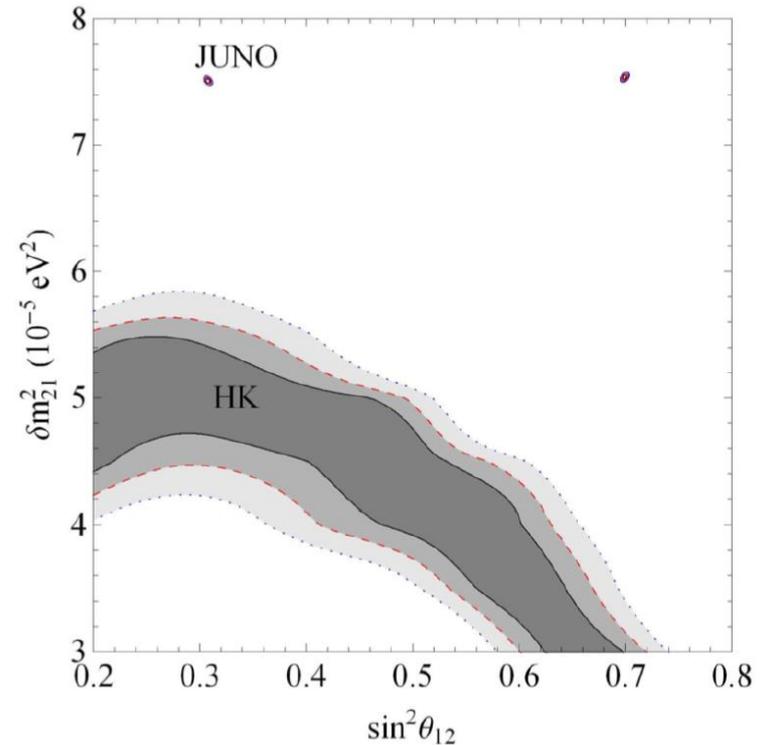
Masud, Mehta, PRD94(2016)

NSI

- 2σ tension for Δm_{21}^2 could be due to NSI



Liao, Marfatia, Whisnant, 1704.04711



A(D/N) consistent with SK Δm_{21}^2 is also
Consistent with KL Δm_{21}^2 and $\epsilon_{ee}^{u(d)} \sim 0.1$

(JUNO and HyperK would reject no NSI-solution by 7σ)

Origin of NSI

- ε from integrating out **scalar of type II seesaw**: $\varepsilon_{\alpha\beta}^e \propto (m_\nu)_{\alpha\beta}$ (Malinsky, Ohlsson, Zhang, 0811.3346)
- ε from integrating out **leptoquarks** (Wise, Zhang, 1404.4663)
- ε from integrating out **charge +1 scalar singlet**:
- ε from **loop effects**, including secret neutrino interactions (Bischer, Rodejohann, Xu, 1807.08102)
- ε from **higher dimensional operators** (Gavela et al., 0809.3451); within **flavor symmetry models** have information on flavor symmetry (Wang, Zhou, 1801.05656)
- ε from integrating out **Z'** (Heeck, Lindner, Rodejohann, Vogl, 1812.04067)

Non-unitarity

- Source of non-unitary : sterile neutrino, effective op... (minimal unitarity violation: Antusch et al, 2006)
- **Parametrization** (Z. Xing, PLB 2008, Escrihuela et al. PRD92(2015))

$$N = N^{NP}U = \begin{pmatrix} \alpha_{11} & 0 & 0 \\ \alpha_{21} & \alpha_{22} & 0 \\ \alpha_{31} & \alpha_{32} & \alpha_{33} \end{pmatrix} U$$

- **Constraints from experimental data:** ν oscillations, W & Z decays, rare lepton-flavor-violating decays, lepton universality tests,
- Sensitivity to CPV in LBL exps. can be affected by the presence of non-unitarity.

One parameter (1 d.o.f.)		All parameters (6 d.o.f.)		
90% C.L.	3σ	90% C.L.	3σ	
Neutrinos + charged leptons				
$\alpha_{11} >$	0.9974	0.9963	0.9961	0.9952
$\alpha_{22} >$	0.9994	0.9991	0.9990	0.9987
$\alpha_{33} >$	0.9988	0.9976	0.9973	0.9961
$ \alpha_{21} <$	1.7×10^{-3}	2.5×10^{-3}	2.6×10^{-3}	4.0×10^{-3}
$ \alpha_{31} <$	2.0×10^{-3}	4.4×10^{-3}	5.0×10^{-3}	7.0×10^{-3}
$ \alpha_{32} <$	1.1×10^{-3}	2.0×10^{-3}	2.4×10^{-3}	3.4×10^{-3}
Neutrinos only				
$\alpha_{11} >$	0.98	0.95	0.96	0.93
$\alpha_{22} >$	0.99	0.96	0.97	0.95
$\alpha_{33} >$	0.93	0.76	0.79	0.61
$ \alpha_{21} <$	1.0×10^{-2}	2.6×10^{-2}	2.4×10^{-2}	3.6×10^{-2}
$ \alpha_{31} <$	4.2×10^{-2}	9.8×10^{-2}	9.0×10^{-2}	1.3×10^{-1}
$ \alpha_{32} <$	9.8×10^{-3}	1.7×10^{-2}	1.6×10^{-2}	2.1×10^{-2}

Escrivuela, Forero, Miranda, Tortola, Valle, New.J.Phys.19(2017)

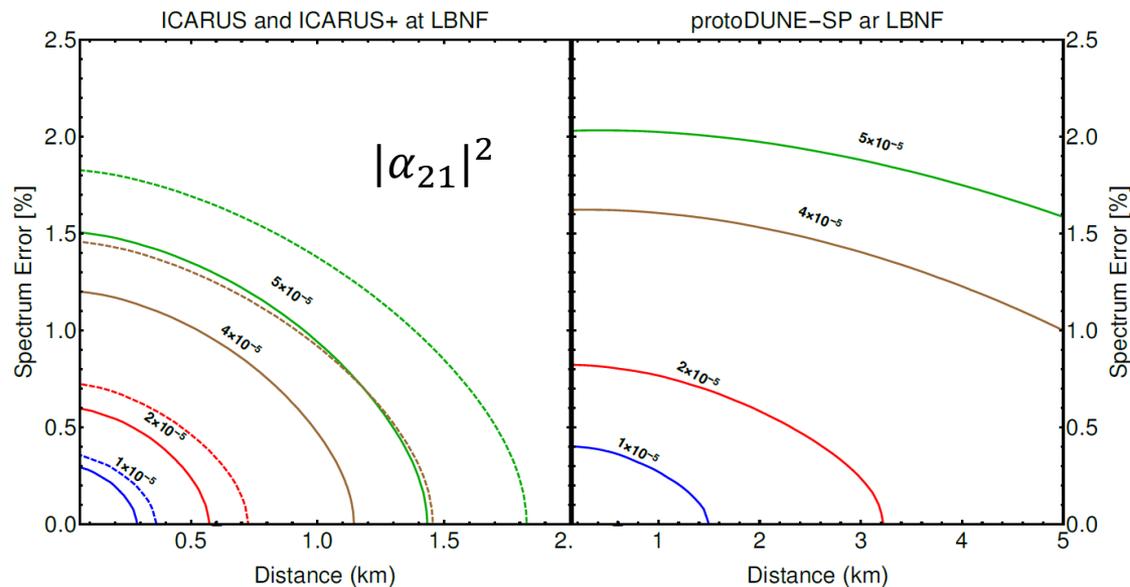
- Non-unitarity predicts “zero-distance effect”

$$P(\nu_\mu \rightarrow \nu_e) = \alpha_{11} |\alpha_{21}|^2$$

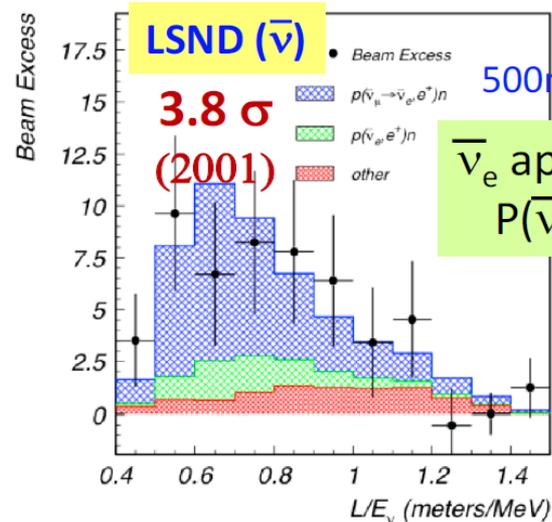
- Thus, at very short distances from the neutrino source, # of detected electron neutrinos, N_e , is given by

$$N_e = \phi_{\nu_e}^0 + |\alpha_{21}|^2 \phi_{\nu_\mu}^0$$

- Capabilities of SBL as well as LBL as a probe of the unitarity of lepton mixing : (Miranda et al. PRD97(2018); Escribuela et al ,New.J.Phys.19(2017)



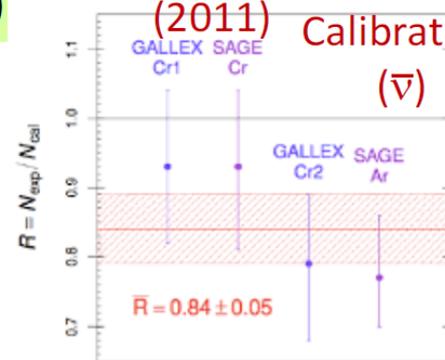
Sterile Neutrinos at $\sim eV$?



ν_e disappearance
 $P(\nu_e \rightarrow \nu_e)$

GALLEX/SAGE

3 σ Source Calibration ($\bar{\nu}$)



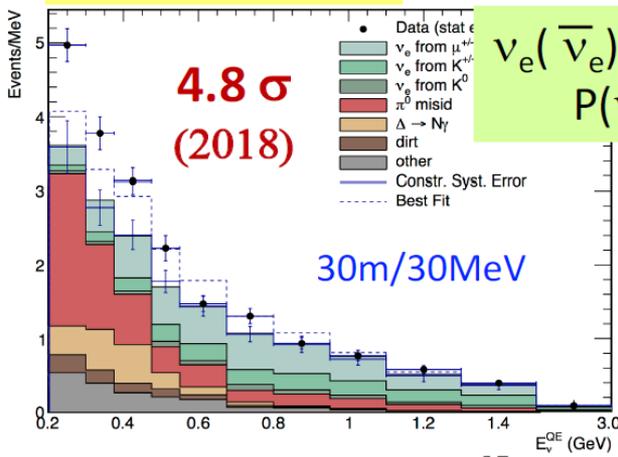
Phys. Rev. C83, 065504 (2011)

3 $\sigma \sim 4\sigma$ evidences

$\bar{\nu}_e$ appearance
 $P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$

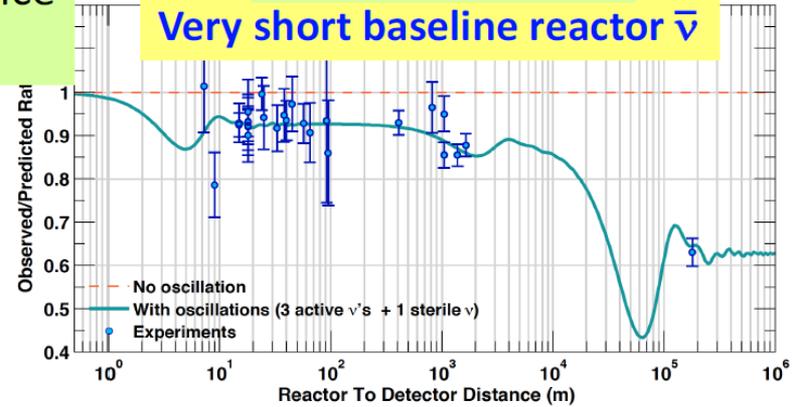
$\bar{\nu}_e$ disappearance
 $P(\bar{\nu}_e \rightarrow \bar{\nu}_e)$ 3 σ (2011)

MiniBooNE ($\nu, \bar{\nu}$)



ν_e ($\bar{\nu}_e$) appearance
 $P(\nu_\mu \rightarrow \nu_e)$

Very short baseline reactor $\bar{\nu}$



Conclusion

- Lots of progress in neutrino physics in the past years
 - PMNS parameters approach CKM precision
- Still lots to learn about neutrino
 - mass ordering, CP violation, Majorana or Dirac etc.
- Lots of theoretical idea proposed to understand our universe via neutrino
 - More idea will emerge in future
- Lots of experimental programs and proposals exist
 - New era of neutrino physics

Minimal Seesaw (type-I)

- **2 RH neutrinos** : Frampton, Glashow, Yanagida, PLB548(2002), Endoh, SK, Kaneko, Morozumi, Tanimoto, PRL89(2002)
- **Littlest Seesaw** : Dirac texture zero & 2 RH ν
(S.F. King, JHEP1307(2013))
- **Littlest Seesaw from S_4** (Chen, Ding, King, Li : 1906.1141)

$$m_\nu = m_a \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix} + m_s e^{i\eta} \begin{pmatrix} 1 & 2-x & x \\ 2-x & (x-2)^2 & (2-x)x \\ x & (2-x)x & x^2 \end{pmatrix}$$

$$(x, \eta) = (-1/2, -\pi/2)$$

$$0.593 \leq \sin^2 \theta_{23} \leq 0.609.$$

$$-0.358 \leq \delta_{CP}/\pi \leq -0.348$$