

One-pion production in neutrino-nucleon and neutrino-nucleus collisions

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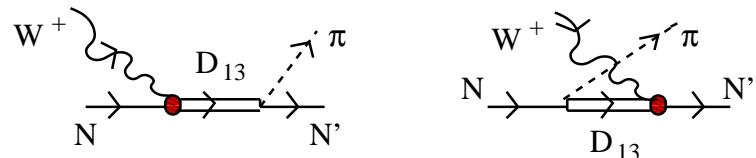
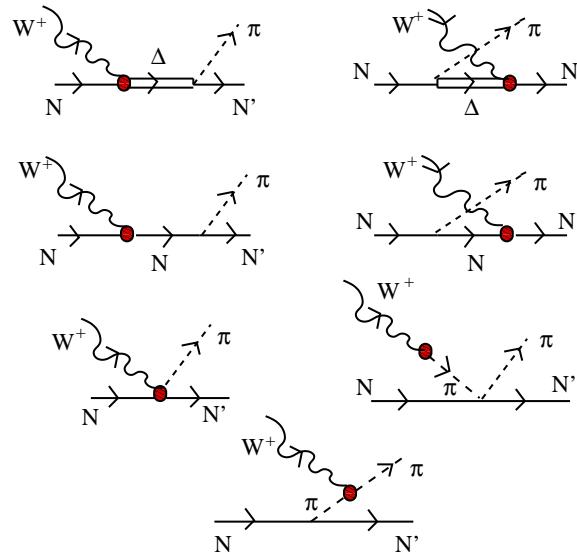
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HNV pion production model for one pion production in one slide



Phys. Rev. D87, 113009 (2013)

Phys. Rev. D76, 033005(2007)

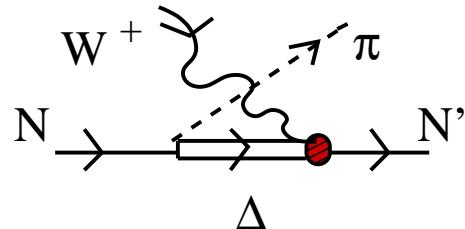
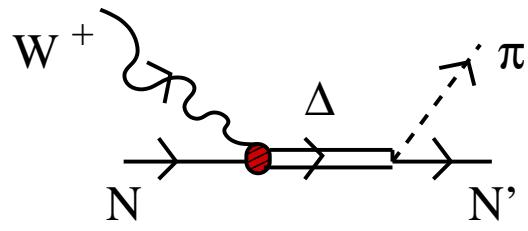
Partial unitarization through Olsson's method: $T_B + T_{\Delta P} \rightarrow T_B + e^{i\delta_V} T_{\Delta P}^V + e^{i\delta_A} T_{\Delta P}^A$
 Phys. Rev. D93, 014016 (2016)

Modified Delta propagator:

$$\frac{P_{\mu\nu}}{p_\Delta^2 - M_\Delta^2 + iM_\Delta\Gamma_\Delta} \rightarrow \frac{P_{\mu\nu}}{p_\Delta^2 - M_\Delta^2 + iM_\Delta\Gamma_\Delta} + c\delta P_{\mu\nu}, \quad \delta P_{\mu\nu} = \frac{P_{\mu\nu} - \frac{p_\Delta^2}{M_\Delta^2} P_{\mu\nu}^{\frac{3}{2}}}{p_\Delta^2 - M_\Delta^2}$$

We get better results for $\nu_\mu n \rightarrow \mu^- n \pi^+$ by fitting c [Phys. Rev. D95, 053007 (2017)]

Δ contribution to one pion production



$$j_{cc+}^{\mu} \Big|_{\Delta P} = i C^{\Delta} \frac{f^*}{m_{\pi}} \sqrt{3} \cos \theta_C \frac{k_{\pi}^{\alpha}}{p_{\Delta}^2 - M_{\Delta}^2 + i M_{\Delta} \Gamma_{\Delta}} \bar{u}(\vec{p}') P_{\alpha\beta}(p_{\Delta}) \Gamma^{\beta\mu}(p, q) u(\vec{p}), \quad p_{\Delta} = p + q,$$

and $C^{\Delta} = \begin{pmatrix} 1 & \text{for} & p\pi^+ \\ 1/3 & \text{for} & n\pi^+ \end{pmatrix}$

$$j_{cc+}^{\mu} \Big|_{C\Delta P} = i C^{C\Delta} \frac{f^*}{m_{\pi}} \frac{1}{\sqrt{3}} \cos \theta_C \frac{k_{\pi}^{\beta}}{p_{\Delta}^2 - M_{\Delta}^2 + i M_{\Delta} \Gamma_{\Delta}} \bar{u}(\vec{p}') \hat{\Gamma}^{\mu\alpha}(p', q) P_{\alpha\beta}(p_{\Delta}) u(\vec{p}), \quad p_{\Delta} = p' - q,$$

and $C^{C\Delta} = \begin{pmatrix} 1 & \text{for} & p\pi^+ \\ 3 & \text{for} & n\pi^+ \end{pmatrix}, \quad \hat{\Gamma}^{\mu\alpha}(p', q) = \gamma^0 [\Gamma^{\alpha\mu}(p', -q)]^\dagger \gamma^0$

Δ contribution to one pion production II

$$P^{\mu\nu}(p_\Delta) = -(p_\Delta + M_\Delta) \left[g^{\mu\nu} - \frac{1}{3} \gamma^\mu \gamma^\nu - \frac{2}{3} \frac{p_\Delta^\mu p_\Delta^\nu}{M_\Delta^2} + \frac{1}{3} \frac{p_\Delta^\mu \gamma^\nu - p_\Delta^\nu \gamma^\mu}{M_\Delta} \right]$$

$$\begin{aligned} \Gamma^{\alpha\mu}(p, q) &= \left[\frac{C_3^V}{M} (g^{\alpha\mu} q - q^\alpha \gamma^\mu) + \frac{C_4^V}{M^2} (g^{\alpha\mu} q \cdot p_\Delta - q^\alpha p_\Delta^\mu) + \frac{C_5^V}{M^2} (g^{\alpha\mu} q \cdot p - q^\alpha p^\mu) + C_6^V g^{\mu\alpha} \right] \gamma_5 \\ &\quad + \left[\frac{C_3^A}{M} (g^{\alpha\mu} q - q^\alpha \gamma^\mu) + \frac{C_4^A}{M^2} (g^{\alpha\mu} q \cdot p_\Delta - q^\alpha p_\Delta^\mu) + C_5^A g^{\alpha\mu} + \frac{C_6^A}{M^2} q^\mu q^\alpha \right] \end{aligned}$$

Vector form factors taken from O. Lalakulich et al., PRD74,014009 (2006)

$$\begin{aligned} C_3^V &= \frac{2.13}{(1 - q^2/M_V^2)^2} \times \frac{1}{1 - \frac{q^2}{4M_V^2}}, \quad C_4^V = \frac{-1.51}{(1 - q^2/M_V^2)^2} \times \frac{1}{1 - \frac{q^2}{4M_V^2}}, \\ C_5^V &= \frac{0.48}{(1 - q^2/M_V^2)^2} \times \frac{1}{1 - \frac{q^2}{0.776M_V^2}}, \quad C_6^V = 0 \text{ (CVC)} \quad \text{with } M_V = 0.84 \text{ GeV} \end{aligned}$$

For the axial form factors we use Adler's model [Ann. Phys. 50, 189 (1968)] in which

$C_4^A(q^2) = -\frac{C_5^A(q^2)}{4}$, $C_3^A(q^2) = 0$, and we further take

$$C_5^A(q^2) = \frac{C_5^A(0)}{(1 - q^2/M_{A\Delta}^2)^2}, \quad C_6^A(q^2) = C_5^A(q^2) \frac{M^2}{m_\pi^2 - q^2} \quad (\text{PCAC})$$

From our latest fit we obtain $C_5^A(0) = 1.18 \pm 0.07$, $M_{A\Delta} = 950 \pm 60$ MeV.

Nonresonant production

Interaction Lagrangian

We use a SU(2) nonlinear sigma model. Up to $\mathcal{O}(1/f_\pi^3)$ the interaction Lagrangian reads

$$\begin{aligned}\mathcal{L}_{\text{int}}^\sigma &= \frac{g_A}{f_\pi} \bar{\Psi} \gamma^\mu \gamma_5 \frac{\vec{\tau}}{2} (\partial_\mu \vec{\phi}) \Psi - \frac{1}{4f_\pi^2} \bar{\Psi} \gamma_\mu \vec{\tau} (\vec{\phi} \times \partial^\mu \vec{\phi}) \Psi \\ &\quad - \frac{1}{6f_\pi^2} \left(\vec{\phi}^2 \partial_\mu \vec{\phi} \partial^\mu \vec{\phi} - (\vec{\phi} \partial_\mu \vec{\phi})(\vec{\phi} \partial^\mu \vec{\phi}) \right) \\ &\quad + \frac{m_\pi^2}{24f_\pi^2} (\vec{\phi}^2)^2 - \frac{g_A}{6f_\pi^3} \bar{\Psi} \gamma^\mu \gamma_5 \left[\vec{\phi}^2 \frac{\vec{\tau}}{2} \partial_\mu \vec{\phi} - (\vec{\phi} \partial_\mu \vec{\phi}) \frac{\vec{\tau}}{2} \vec{\phi} \right] \Psi,\end{aligned}$$

where $\Psi = \begin{pmatrix} p \\ n \end{pmatrix}$ is the nucleon field, $\vec{\phi}$ is the isovector pion field.

To evaluate the different contributions we also need the coupling to the W boson, i.e. we need the vector and axial currents.

Nonresonant production

Vector and axial currents I

To the corresponding order the vector and axial currents are

$$\begin{aligned}\vec{V}^\mu &= \vec{\phi} \times \partial^\mu \vec{\phi} + \bar{\Psi} \gamma^\mu \frac{\vec{\tau}}{2} \Psi + \frac{g_A}{2f_\pi} \bar{\Psi} \gamma^\mu \gamma_5 (\vec{\phi} \times \vec{\tau}) \Psi - \frac{1}{4f_\pi^2} \bar{\Psi} \gamma^\mu \left[\vec{\tau} \vec{\phi}^2 - \vec{\phi} (\vec{\tau} \cdot \vec{\phi}) \right] \Psi \\ &\quad - \frac{\vec{\phi}^2}{3f_\pi^2} (\vec{\phi} \times \partial^\mu \vec{\phi}) + \mathcal{O}\left(\frac{1}{f_\pi^3}\right)\end{aligned}$$

$$\begin{aligned}\vec{A}^\mu &= f_\pi \partial^\mu \vec{\phi} + g_A \bar{\Psi} \gamma^\mu \gamma_5 \frac{\vec{\tau}}{2} \Psi + \frac{1}{2f_\pi} \bar{\Psi} \gamma^\mu (\vec{\phi} \times \vec{\tau}) \Psi + \frac{2}{3f_\pi} \left[\vec{\phi} (\vec{\phi} \cdot \partial^\mu \vec{\phi}) - \vec{\phi}^2 \partial^\mu \vec{\phi} \right] \\ &\quad - \frac{g_A}{4f_\pi^2} \bar{\Psi} \gamma^\mu \gamma_5 \left[\vec{\tau} \vec{\phi}^2 - \vec{\phi} (\vec{\tau} \cdot \vec{\phi}) \right] \Psi + \mathcal{O}\left(\frac{1}{f_\pi^3}\right)\end{aligned}$$

The normalization is such that $-\sqrt{2} \cos \theta_C (V_{+1}^\mu - A_{+1}^\mu)$ provides the $W^+ n \rightarrow p$ vertex

Nonresonant production

Vector and axial currents II

The pure nucleonic part of the vector and axial currents are further modified by the inclusion of form factors. For the neutron to proton transition which fixes our normalization we have

$$V_N^\alpha(q) = 2 \times \left(F_1^V(q^2) \gamma^\alpha + i\mu_V \frac{F_2^V(q^2)}{2M} \sigma^{\alpha\nu} q_\nu \right),$$
$$A_N^\alpha(q) = G_A(q^2) \times \left(\gamma^\alpha \gamma_5 + \frac{q}{m_\pi^2 - q^2} q^\alpha \gamma_5 \right).$$

The magnetic part in $V_N^\alpha(q)$ is not provided by the sigma model neither the q^2 dependence of the form factors.

Vector current conservation requires that all other terms in the vector current are multiplied by the $F_1^V(q^2)$ form factor.

Nonresonant production

Vector and axial currents III

For the vector form factors we use the parametrization of S. Galster et al in NPB 32, 221 (1971)

$$F_1^N = \frac{G_E^N + \kappa G_M^N}{1 + \kappa}, \quad \mu_N F_2^N = \frac{G_M^N - G_E^N}{1 + \kappa},$$
$$G_E^p = \frac{G_M^p}{\mu_p} = \frac{G_M^n}{\mu_n} = -(1 + \lambda_n \kappa) \frac{G_E^n}{\mu_n \kappa} = \left(\frac{1}{1 - q^2/M_D^2} \right)^2$$

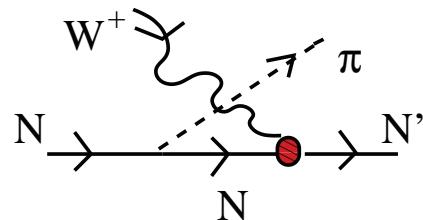
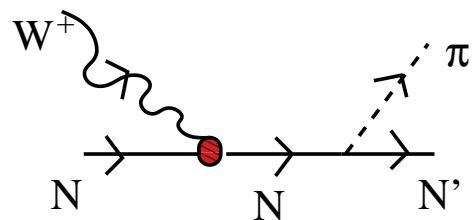
with $\kappa = -q^2/4M^2$, $M_D = 0.843$ GeV, $\mu_p = 2.792847$, $\mu_n = -1.913043$ and $\lambda_n = 5.6$.
Besides,

$$F_1^V(q^2) = \frac{1}{2} (F_1^p(q^2) - F_1^n(q^2)), \quad \mu_V F_2^V(q^2) = \frac{1}{2} (\mu_p F_2^p(q^2) - \mu_n F_2^n(q^2)).$$

The axial form factor we take from the book of Ericson & Weise (The International Series of Monographs on Physics 74)

$$G_A(q^2) = \frac{g_A}{(1 - q^2/M_A^2)^2}, \quad g_A = 1.26, \quad M_A = 1.05 \text{ GeV}$$

One-pion nonresonant production



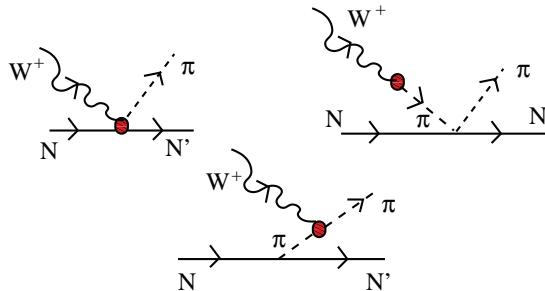
$$j_{cc+}^\mu \Big|_{NP} = -i C^{NP} \frac{g_A}{\sqrt{2} f_\pi} \cos \theta_C \bar{u}(\vec{p}') \not{k}_\pi \gamma_5 \frac{\not{p} + \not{q} + M}{(p + q)^2 - M^2 + i\epsilon} [V_N^\mu(q) - A_N^\mu(q)] u(\vec{p}),$$

$$C^{NP} = \begin{pmatrix} 0 & \text{for} & p\pi^+ \\ 1 & \text{for} & n\pi^+ \end{pmatrix}$$

$$j_{cc+}^\mu \Big|_{CNP} = -i C^{CNP} \frac{g_A}{\sqrt{2} f_\pi} \cos \theta_C \bar{u}(\vec{p}') [V_N^\mu(q) - A_N^\mu(q)] \frac{\not{p}' - \not{q} + M}{(p' - q)^2 - M^2 + i\epsilon} \not{k}_\pi \gamma_5 u(\vec{p}),$$

$$C^{CNP} = \begin{pmatrix} 1 & \text{for} & p\pi^+ \\ 0 & \text{for} & n\pi^+ \end{pmatrix}$$

One-pion nonresonant production II



$$j_{cc+}^{\mu} \Big|_{CT} = -i C^{CT} \frac{1}{\sqrt{2} f_{\pi}} \cos \theta_C \bar{u}(\vec{p}') \gamma^{\mu} \left(g_A F_{CT}^V(q^2) \gamma_5 - F_{\rho} \left((q - k_{\pi})^2 \right) \right) u(\vec{p}),$$

$$C^{CT} = \begin{pmatrix} 1 & \text{for} & p\pi^+ \\ -1 & \text{for} & n\pi^+ \end{pmatrix}$$

$$j_{cc+}^{\mu} \Big|_{PP} = -i C^{PP} F_{\rho} \left((q - k_{\pi})^2 \right) \frac{1}{\sqrt{2} f_{\pi}} \cos \theta_C \frac{q^{\mu}}{q^2 - m_{\pi}^2} \bar{u}(\vec{p}') \not{u}(\vec{p}),$$

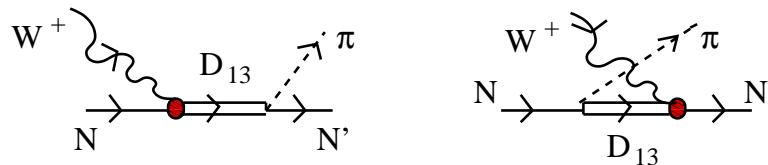
$$C^{PP} = \begin{pmatrix} 1 & \text{for} & p\pi^+ \\ -1 & \text{for} & n\pi^+ \end{pmatrix}$$

$$j_{cc+}^{\mu} \Big|_{PF} = -i C^{PF} F_{PF}(q^2) \frac{g_A}{\sqrt{2} f_{\pi}} \cos \theta_C \frac{(2k_{\pi} - q)^{\mu}}{(k_{\pi} - q)^2 - m_{\pi}^2} 2M \bar{u}(\vec{p}') \gamma_5 u(\vec{p}),$$

$$C^{PF} = \begin{pmatrix} 1 & \text{for} & p\pi^+ \\ -1 & \text{for} & n\pi^+ \end{pmatrix}$$

$$F_{CT}^V = F_{PF} = 2F_1^V, F_{\rho}(1) = 1/(1 - t/m_{\rho}^2) \text{ with } m_{\rho} = 0.7758 \text{ GeV.}$$

D_{13} contribution to one pion production



$$j_{cc+}^{\mu}|_{DP} = i C^{DP} g_D \sqrt{\frac{2}{3}} \cos \theta_C \frac{k_{\pi}^{\alpha}}{p_D^2 - M_D^2 + i M_D \Gamma_D} \bar{u}(\vec{p}') \gamma_5 P_{\alpha\beta}^D(p_D) \Gamma_D^{\beta\mu}(p, q) u(\vec{p}), \quad p_D = p + q,$$

$$C^{DP} = \begin{cases} 0 & p\pi^+ \\ 1 & n\pi^+ \end{cases}$$

$$j_{cc+}^{\mu}|_{CDP} = -i C^{CDP} g_D \sqrt{\frac{2}{3}} \cos \theta_C \frac{k_{\pi}^{\alpha}}{p_D^2 - M_D^2 + i M_D \Gamma_D} \bar{u}(\vec{p}') \hat{\Gamma}_D^{\mu\beta}(p', -q) P_{\beta\alpha}^D(p_D) \gamma_5 u(\vec{p}), \quad p_D = p' - q$$

$$C^{CDP} = \begin{cases} 1 & p\pi^+ \\ 0 & n\pi^+ \end{cases}$$

$$M_D = 1520 \text{ MeV}, \quad g_D = 20 \text{ GeV}^{-1}. \quad \Gamma_D = \Gamma_D^{N\pi} + \Gamma_D^{\Delta\pi}$$

$$\Gamma_D^{N\pi} = \frac{g_D^2}{8\pi} \frac{1}{3s} [(\sqrt{s} - M)^2 - m_{\pi}^2] |\vec{p}_{\pi}|^3 \theta(\sqrt{s} - M - m_{\pi}),$$

$$\Gamma_D^{\Delta\pi} = 0.39 \times 115 \text{ MeV} \frac{|\vec{p}'_{\pi}|}{|\vec{p}'_{\pi}{}^{o-s}|} \theta(\sqrt{s} - M - m_{\pi}),$$

$$\text{with } |\vec{p}'_{\pi}| = \frac{\lambda^{1/2}(s, M_{\Delta}^2, m_{\pi}^2)}{2\sqrt{s}} \text{ and } |\vec{p}'_{\pi}{}^{o-s}| = \frac{\lambda^{1/2}(M_D^2, M_{\Delta}^2, m_{\pi}^2)}{2M_D}.$$

D₁₃ contribution to one pion production II

$$\begin{aligned}\Gamma^{\beta\mu}(p, q) &= \left[\frac{\tilde{C}_3^V}{M} \left(g^{\beta\mu} q - q^\beta \gamma^\mu \right) + \frac{\tilde{C}_4^V}{M^2} \left(g^{\beta\mu} q \cdot p_D - q^\beta p_D^\mu \right) + \frac{\tilde{C}_5^V}{M^2} \left(g^{\beta\mu} q \cdot p - q^\beta p^\mu \right) + \tilde{C}_6^V g^{\beta\mu} \right] \\ &\quad + \left[\frac{\tilde{C}_3^A}{M} \left(g^{\beta\mu} q - q^\beta \gamma^\mu \right) + \frac{\tilde{C}_4^A}{M^2} \left(g^{\beta\mu} q \cdot p_D - q^\beta p_D^\mu \right) + \tilde{C}_5^A g^{\beta\mu} + \frac{\tilde{C}_6^A}{M^2} q^\beta q^\mu \right] \gamma_5.\end{aligned}$$

The axial form factors are taken from O. Lalakulich et al., PRD74, 014009 (2006)

$$\tilde{C}_3^A = \tilde{C}_4^A = 0, \quad \tilde{C}_5^A = \frac{-2.1}{(1 - q^2/M_A^2)^2} \frac{1}{1 - q^2/(3M_A^2)}, \quad \tilde{C}_6^A(q^2) = \tilde{C}_5^A(q^2) \frac{M^2}{m_\pi^2 - q^2}, \quad M_A = 1 \text{ GeV},$$

while for the vector ones we fitted the form factor results in the Thesis of T. Leitner, from where we got

$$\tilde{C}_3^V = \frac{-2.98}{[1 - q^2/(1.4M_V^2)]^2}, \quad \tilde{C}_4^V = \frac{4.21/D_V}{1 - q^2/(3.7M_V^2)}, \quad \tilde{C}_5^V = \frac{-3.13/D_V}{1 - q^2/(0.42M_V^2)}, \quad \tilde{C}_6^V = 0,$$

with $M_V = 0.84 \text{ GeV}$ and $D_V = (1 - q^2/M_V^2)^2$

A copy of T. LeitnerThesis can be retrieved from

"<https://www.uni-giessen.de/fbz/fb07/fachgebiete/physik/einrichtungen/theorie/inst/theses/dissertation/neutrino-nucleus-interactions-in-acoupled-channel-hadronic-transport-model>"

Δ propagator modification

The $\nu_\mu n \rightarrow \mu^- n \pi^+$ channel gets a large contribution from the crossed Delta term being very sensitive to the spin 1/2 components in the Δ propagator. In the zero width limit, the Δ propagator is given by

$$\begin{aligned}
G_{\mu\nu}(p_\Delta) &= \frac{P_{\mu\nu}(p_\Delta)}{p_\Delta^2 - M_\Delta^2 + i\epsilon} \\
P^{\mu\nu}(p_\Delta) &= -(p_\Delta + M_\Delta) \left[g^{\mu\nu} - \frac{1}{3} \gamma^\mu \gamma^\nu - \frac{2}{3} \frac{p_\Delta^\mu p_\Delta^\nu}{M_\Delta^2} + \frac{1}{3} \frac{p_\Delta^\mu \gamma^\nu - p_\Delta^\nu \gamma^\mu}{M_\Delta} \right] \\
&= P_{\mu\nu}^{\frac{3}{2}}(p) + \underbrace{(p^2 - M_\Delta^2) \left[\frac{2}{3M_\Delta^2} (p + M_\Delta) \frac{p_\mu p_\nu}{p^2} - \frac{1}{3M_\Delta} \left(\frac{p^\rho p_\nu \gamma_{\mu\rho}}{p^2} + \frac{p^\rho p_\mu \gamma_{\rho\nu}}{p^2} \right) \right]}_{\text{spin-1/2}},
\end{aligned}$$

with $\gamma_{\mu\nu} = \frac{1}{2} [\gamma_\mu, \gamma_\nu]$ and the true spin-3/2 projection operator being

$$P_{\mu\nu}^{\frac{3}{2}}(p) = -(p + M_\Delta) \left[g_{\mu\nu} - \frac{1}{3} \gamma_\mu \gamma_\nu - \frac{1}{3p^2} (p \gamma_\mu p_\nu + p_\mu \gamma_\nu p) \right].$$

Due to the $(p^2 - M_\Delta^2)$ factor that cancels the corresponding factor in the propagator denominator, the spin-1/2 component do not propagate giving rise to contact interactions. Its contribution is small for the direct- Δ term while it is large for the crossed- Δ term.

For some authors [See for instance V. Pascalutsa, Phys. Lett. B 503, 85 (2001)] the use of this spin-1/2 part should be avoided. One way of achieving this goal is by the use of “consistent couplings”

Δ propagator modification. Consistent Δ couplings

Consistent couplings are the ones that respect the gauge symmetry

$$\Psi_\mu \rightarrow \Psi_\mu + \partial_\mu \epsilon$$

present in the free-massless Rarita-Schwinger lagrangian. This symmetry requires that in any linear interaction term the Δ field couples only to conserved currents

$$\mathcal{L}_{\text{int}} = g \bar{\Psi}_\beta J^\beta + H.c., \quad \partial_\beta J^\beta = 0.$$

Couplings not respecting that symmetry are called inconsistent.

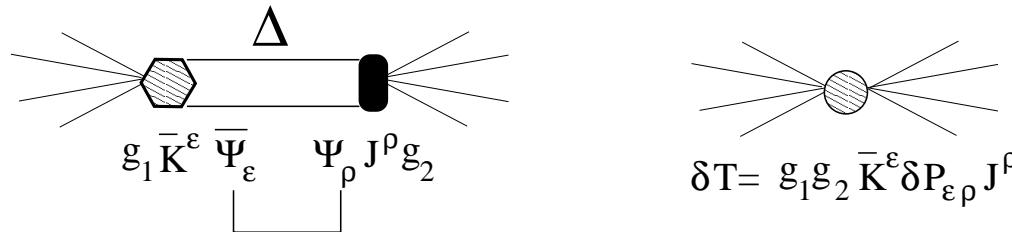
As shown in V. Pascalutsa, Phys. Lett. B 503, 85 (2001), inconsistent couplings can be transformed into consistent ones by a redefinition of the Δ field resulting in a new consistent interaction lagrangian

$$\mathcal{L}'_{\text{int}} = g \bar{\Psi}_\beta \mathcal{J}^\beta + H.c.,$$

plus an additional contact interaction lagrangian. Thus, as far as all relevant contact interactions are taken into account, descriptions using consistent or inconsistent couplings are equivalent. Is is only the coupling constants of the contact terms that differ. But those constants have to be fitted to experimental data.

Δ propagator modification. Consistent Δ couplings II

For process mediated by an intermediate Δ



$$T = g_1 g_2 \bar{K}^\epsilon \frac{P_{\epsilon\rho}}{p_\Delta^2 - M_\Delta^2} J^\rho, \rightarrow T_{consistent} = g_1 g_2 \bar{K}^\epsilon \frac{P_{\epsilon\rho}}{p_\Delta^2 - M_\Delta^2} \mathcal{J}^\rho = g_1 g_2 \bar{K}^\epsilon \frac{p_\Delta^2}{M_\Delta^2} \frac{P_{\epsilon\rho}^{\frac{3}{2}}}{p_\Delta^2 - M_\Delta^2} J^\rho.$$

Then, one has $T = T_{consistent} + \delta T$ with $\delta T = g_1 g_2 \bar{K}^\epsilon \frac{P_{\epsilon\rho} - \frac{P_\Delta^2}{M_\Delta^2} P_{\epsilon\rho}^{\frac{3}{2}}}{p_\Delta^2 - M_\Delta^2} J^\rho$. Since,

$$\begin{aligned} P_{\epsilon\rho} - \frac{P_\Delta^2}{M_\Delta^2} P_{\epsilon\rho}^{\frac{3}{2}} &= (p_\Delta^2 - M_\Delta^2) \delta P_{\epsilon\rho}, \\ \delta P_{\epsilon\rho} &= \frac{1}{M_\Delta^2} (p_\Delta + M_\Delta) \left(g_{\epsilon\rho} - \frac{1}{3} \gamma_\epsilon \gamma_\rho \right) + \frac{1}{3 M_\Delta^2} (p_{\Delta\epsilon} \gamma_\rho - p_{\Delta\rho} \gamma_\epsilon) \end{aligned}$$

the δT contributions amount to a contact (nonpropagating) interaction.

Δ propagator modification. Consistent Δ couplings III

Aiming at improving the description of the $\nu_\mu n \rightarrow \mu^- n\pi^+$ channel we supplement the model with additional contact terms by modifying

$$\begin{aligned}
 \frac{P_{\mu\nu}(p_\Delta)}{p_\Delta^2 - M_\Delta^2 + i\epsilon} &\rightarrow \frac{P_{\mu\nu}(p_\Delta) + c \left(P_{\mu\nu}(p_\Delta) - \frac{p_\Delta^2}{M_\Delta^2} P_{\mu\nu}^{\frac{3}{2}}(p_\Delta) \right)}{p_\Delta^2 - M_\Delta^2 + i\epsilon} = \frac{P_{\mu\nu}(p_\Delta)}{p_\Delta^2 - M_\Delta^2 + i\epsilon} + c \delta P_{\mu\nu}(p_\Delta) \\
 &\rightarrow \frac{P_{\mu\nu}(p_\Delta)}{p_\Delta^2 - M_\Delta^2 + iM_\Delta\Gamma_\Delta} + c \delta P_{\mu\nu}(p_\Delta) \\
 &= \frac{p_\Delta^2}{M_\Delta^2} \frac{P_{\mu\nu}^{\frac{3}{2}}(p_\Delta)}{p_\Delta^2 - M_\Delta^2 + iM_\Delta\Gamma_\Delta} + \frac{(1+c)(p_\Delta^2 - M_\Delta^2) + i c M_\Delta \Gamma_\Delta}{p_\Delta^2 - M_\Delta^2 + iM_\Delta\Gamma_\Delta} \delta P_{\mu\nu}(p_\Delta)
 \end{aligned}$$

Due to the presence of Γ_Δ , a value of $c = -1$ does not correspond exactly to the use of a consistent $\pi N \Delta$ coupling.

Our final value is $c = -1.11 \pm 0.21$

Partial unitarization

Writing the S matrix as

$$S = I - iT$$

the unitarity condition $S^\dagger S = I$ that guarantees the conservation of probability implies

$$i(T - T^\dagger) = T^\dagger T$$

For a given transition between asymptotic states $|I\rangle$, $|F\rangle$ one has

$$i(\langle F|T|I\rangle - \langle F|T^\dagger|I\rangle) = \langle F|T^\dagger T|I\rangle = \sum_N \langle F|T^\dagger|N\rangle \langle N|T|I\rangle = \sum_N \langle N|T|F\rangle^* \langle N|T|I\rangle$$

For the case of $|I\rangle = |F\rangle$ one has

$$\text{Im}\langle I|T|I\rangle = -\frac{1}{2} \sum_N |\langle N|T|I\rangle|^2$$

which constitutes the optical theorem

Partial unitarization II

Time reversal invariance states that (Time reversal operator \mathcal{T} is antilinear)

$$\langle F|S|I\rangle = \langle I_{\mathcal{T}}|S|F_{\mathcal{T}}\rangle = \langle F_{\mathcal{T}}|S^{\dagger}|I_{\mathcal{T}}\rangle^* = \langle F|\mathcal{T}^{\dagger}S^{\dagger}\mathcal{T}|F\rangle$$

from where

$$\mathcal{T}^{\dagger}S^{\dagger}\mathcal{T} = S \implies \mathcal{T}^{\dagger}T^{\dagger}\mathcal{T} = T$$

and then

$$\begin{aligned} \sum_N \langle N|T|F\rangle^* \langle N|T|I\rangle &= i(\langle F|T|I\rangle - \langle F|T^{\dagger}|I\rangle) = i(\langle F|T|I\rangle - \langle I|T|F\rangle^*) \\ &= i(\langle F|T|I\rangle - \langle I|\mathcal{T}^{\dagger}T^{\dagger}\mathcal{T}|F\rangle^*) = i(\langle F|T|I\rangle - \langle I_{\mathcal{T}}|T^{\dagger}|F_{\mathcal{T}}\rangle) \\ &= i(\langle F|T|I\rangle - \langle F_{\mathcal{T}}|T|I_{\mathcal{T}}\rangle^*) \end{aligned}$$

For the case in which $\langle F|T|I\rangle = \langle F_{\mathcal{T}}|T|I_{\mathcal{T}}\rangle$ and there is only one intermediate state $|N\rangle = |F\rangle$ contributing to the sum one arrives at

$$\langle N|T|N\rangle^* \langle N|T|I\rangle = -2\text{Im}\langle F|T|I\rangle \in \mathbb{R}$$

so that the phases of $\langle N|T|I\rangle$ and $\langle N|T|N\rangle$ coincide. This result constitutes Watson theorem.

Partial unitarization II

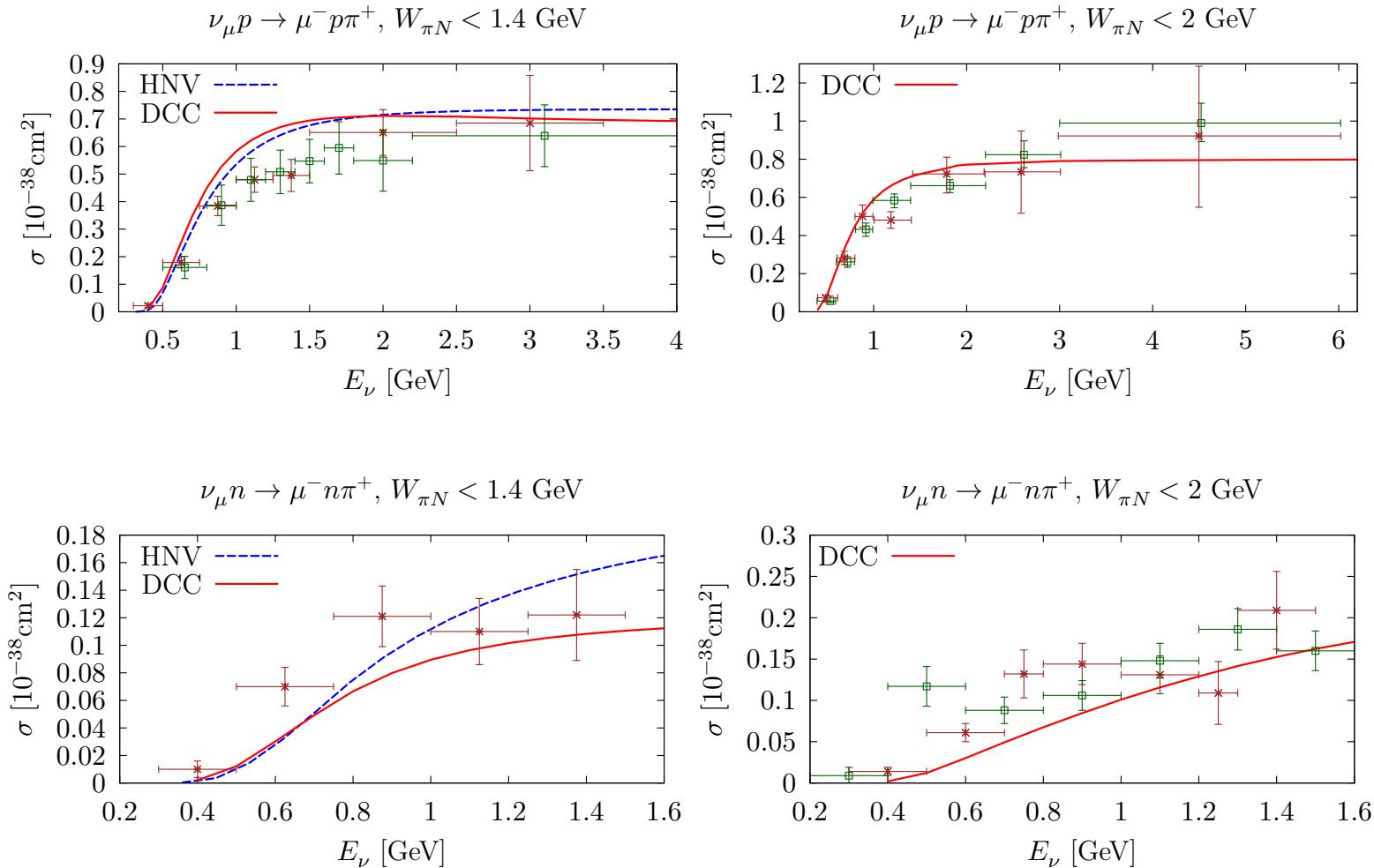
We partially implement Watson theorem. We follow the procedure suggested by M.G. Olsson in NPB78,55 (1974) and change

$$T_B + T_{\Delta P} \rightarrow T_B + e^{i\varphi_V} T_{\Delta P}^V + e^{i\varphi_A} T_{\Delta P}^A$$

The idea was to choose φ_V, φ_A in such a way that each multipole contributing to the total amplitude had the right πN strong phase.

In practice, we were only able to recover Watson theorem for the dominant vector and the dominant axial multipoles with the Δ quantum numbers.

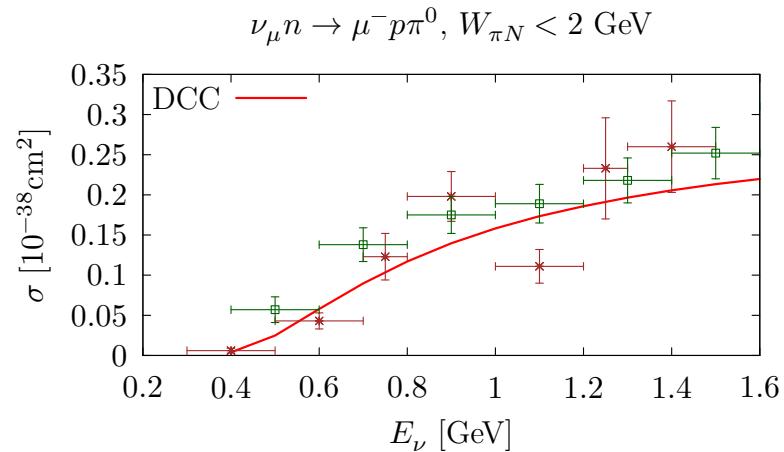
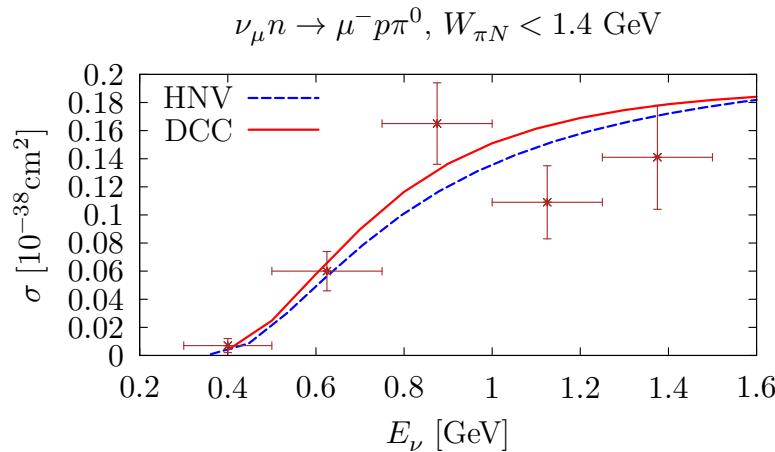
Total cross section results



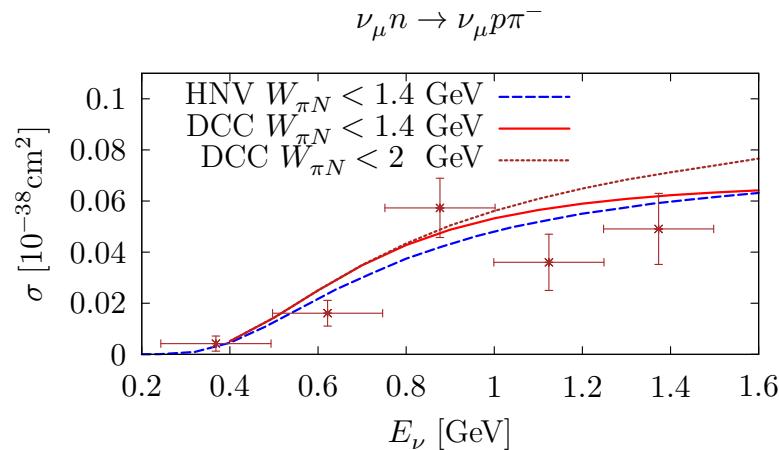
Data from P. Rodrigues, C. Wilkinson, and K. McFarland, Eur. Phys. J. C76, 474 (2016) and C. Wilkinson, P. Rodrigues, S. Cartwright, L. Thompson, and K. McFarland, Phys. Rev. D90, 112017 (2014).

DCC stands for the dynamical coupled channel model described in A. Matsuyama, T. Sato, and T. S. H. Lee, Phys. Rep. 439, 193 (2007) and H. Kamano, S. X. Nakamura, T. S. H. Lee, and T. Sato, Phys. Rev. C 88, 035209 (2013).

Total cross section results II

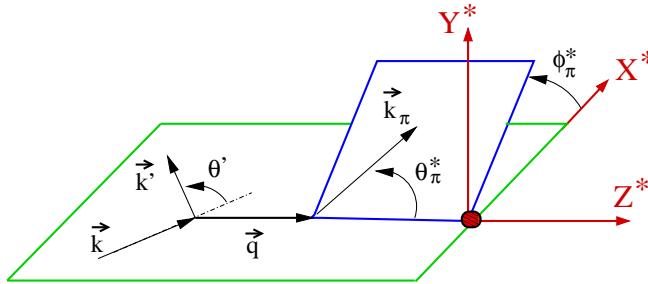


Data P. Rodrigues, C. Wilkinson, and K. McFarland, Eur. Phys. J. C76, 474 (2016).



Data from M. Derrick et al., Phys. Lett. 92B, 363 (1980).

Pion angular distribution



Using Lorentz covariance one can write

$$\frac{d\sigma_{CC\pm}}{dQ^2 dW_{\pi N} d\Omega_\pi^*} = \frac{G_F^2 W_{\pi N}}{4\pi M |\vec{k}|^2} \left(A^* \cdot \mathbf{1} + B^* \cdot \cos \phi_\pi^* + C^* \cdot \cos 2\phi_\pi^* + D^* \cdot \sin \phi_\pi^* + E^* \cdot \sin 2\phi_\pi^* \right)$$

$$A^* = \int \frac{|\vec{k}_\pi^*|^2 d|\vec{k}_\pi^*|}{E_\pi^*} [L^{00} W_{00}^{(s)} + 2L^{03} W_{03}^{(s)} + L^{33} W_{33}^{(s)} + \frac{1}{2}(L^{11} + L^{22}) (W_{11}^{(s)} + W_{22}^{(s)}) + 2iL^{12} W_{12}^{(a)}]_{\phi_\pi^*=0},$$

$$B^* = \int \frac{|\vec{k}_\pi^*|^2 d|\vec{k}_\pi^*|}{E_\pi^*} 2[L^{01} W_{01}^{(s)} + L^{13} W_{13}^{(s)} + iL^{02} W_{02}^{(a)} + iL^{23} W_{23}^{(a)}]_{\phi_\pi^*=0},$$

$$C^* = \int \frac{|\vec{k}_\pi^*|^2 d|\vec{k}_\pi^*|}{E_\pi^*} \frac{1}{2} [(L^{11} - L^{22}) (W_{11}^{(s)} - W_{22}^{(s)})]_{\phi_\pi^*=0},$$

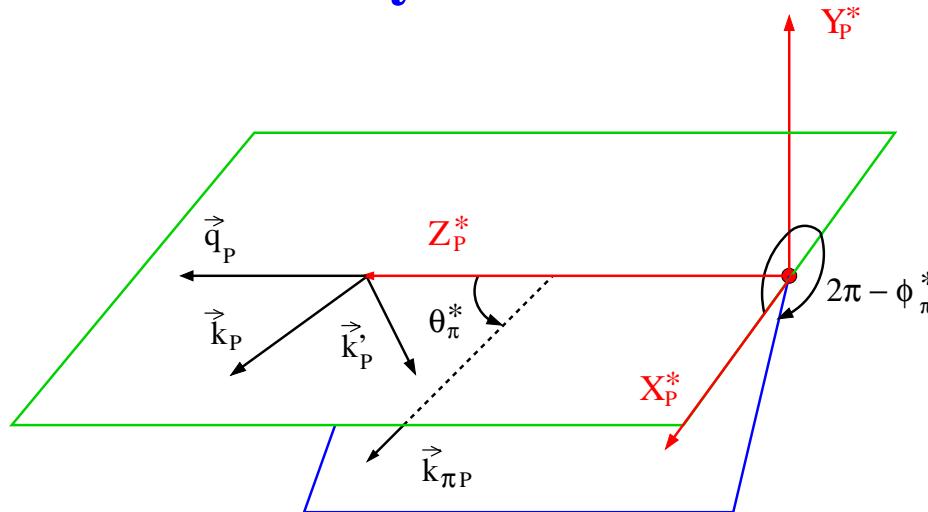
$$D^* = \int \frac{|\vec{k}_\pi^*|^2 d|\vec{k}_\pi^*|}{E_\pi^*} 2[-L^{01} W_{02}^{(s)} - L^{13} W_{23}^{(s)} + iL^{02} W_{01}^{(a)} + iL^{23} W_{13}^{(a)}]_{\phi_\pi^*=0},$$

$$E^* = \int \frac{|\vec{k}_\pi^*|^2 d|\vec{k}_\pi^*|}{E_\pi^*} [(L^{22} - L^{11}) W_{12}^{(s)}]_{\phi_\pi^*=0},$$

s and *a* stand for symmetric and antisymmetric.

Besides, B^* , D^* have a multiplicative $\sin \theta_\pi^*$ factor and E^* a $\sin^2 \theta_\pi^*$ one.

Parity violation



Under parity all three-momenta get reversed but so do the Z^* and X^* axes. As a result the components of $k_P^*, k'^*_P, q_P^*, p_P^*$ in the new system remain unchanged. Only $k_{\pi P}^*$ is modified

$$\theta_\pi^* \rightarrow \theta_\pi^* , \quad \phi_\pi^* \rightarrow 2\pi - \phi_\pi^*.$$

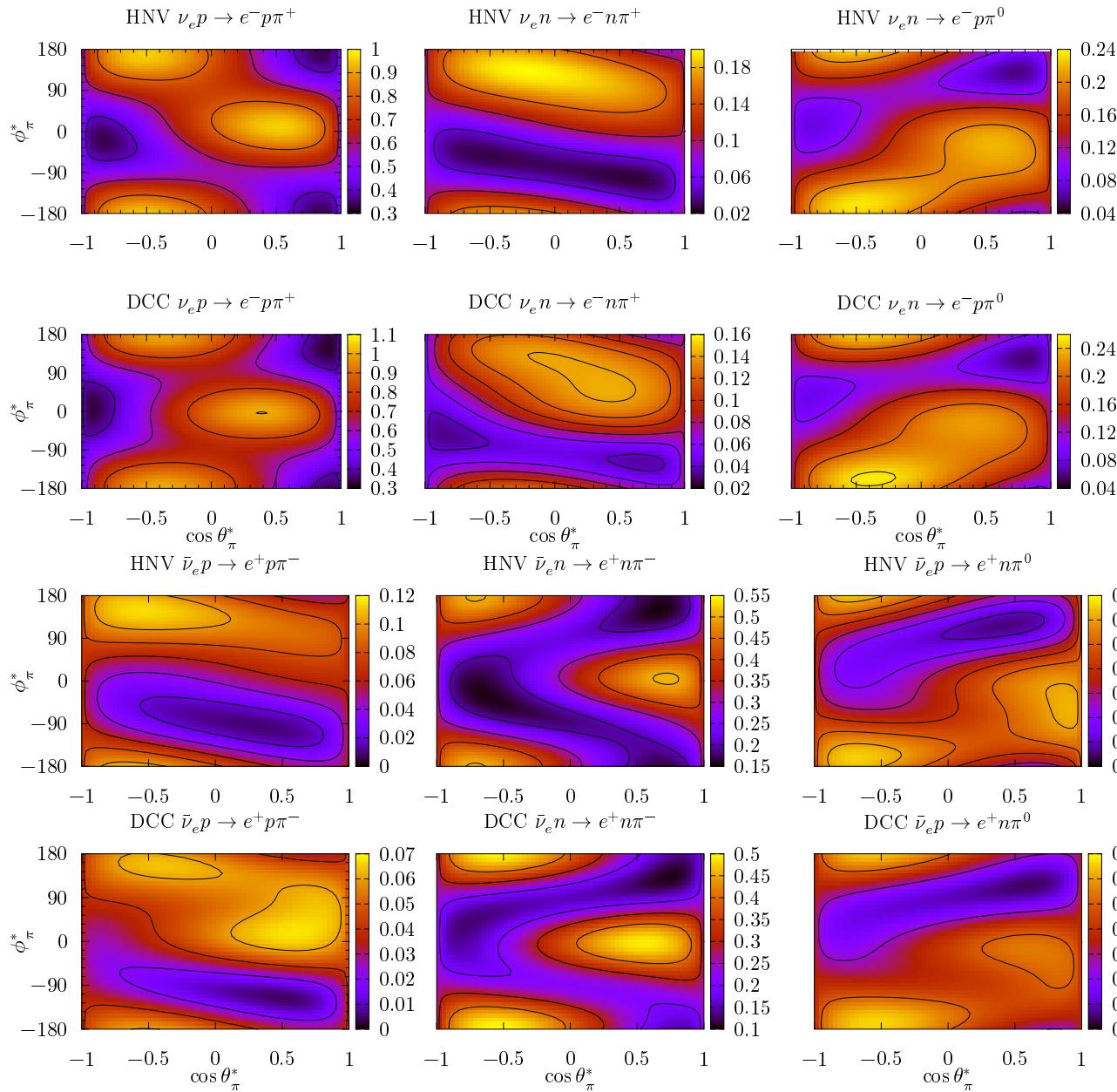
Thus, the terms in $\sin\phi_\pi^*$, $\sin 2\phi_\pi^*$ change sign.

Parity violation reflects itself in the fact that the pion distributions measured above and below the scattering plane are different.

It originates from the interference between multipoles that have different phases. Below the two pion threshold, all multipoles with given total angular momentum, parity and pion orbital angular momentum quantum numbers have the same phase fixed by Watson theorem. Thus, a proper unitarization is essential to get a good reproduction of the parity violation observables.

Pion angular distribution III

CC processes



$$d\sigma/dQ^2 dW_{\pi N} d\Omega_\pi^*$$

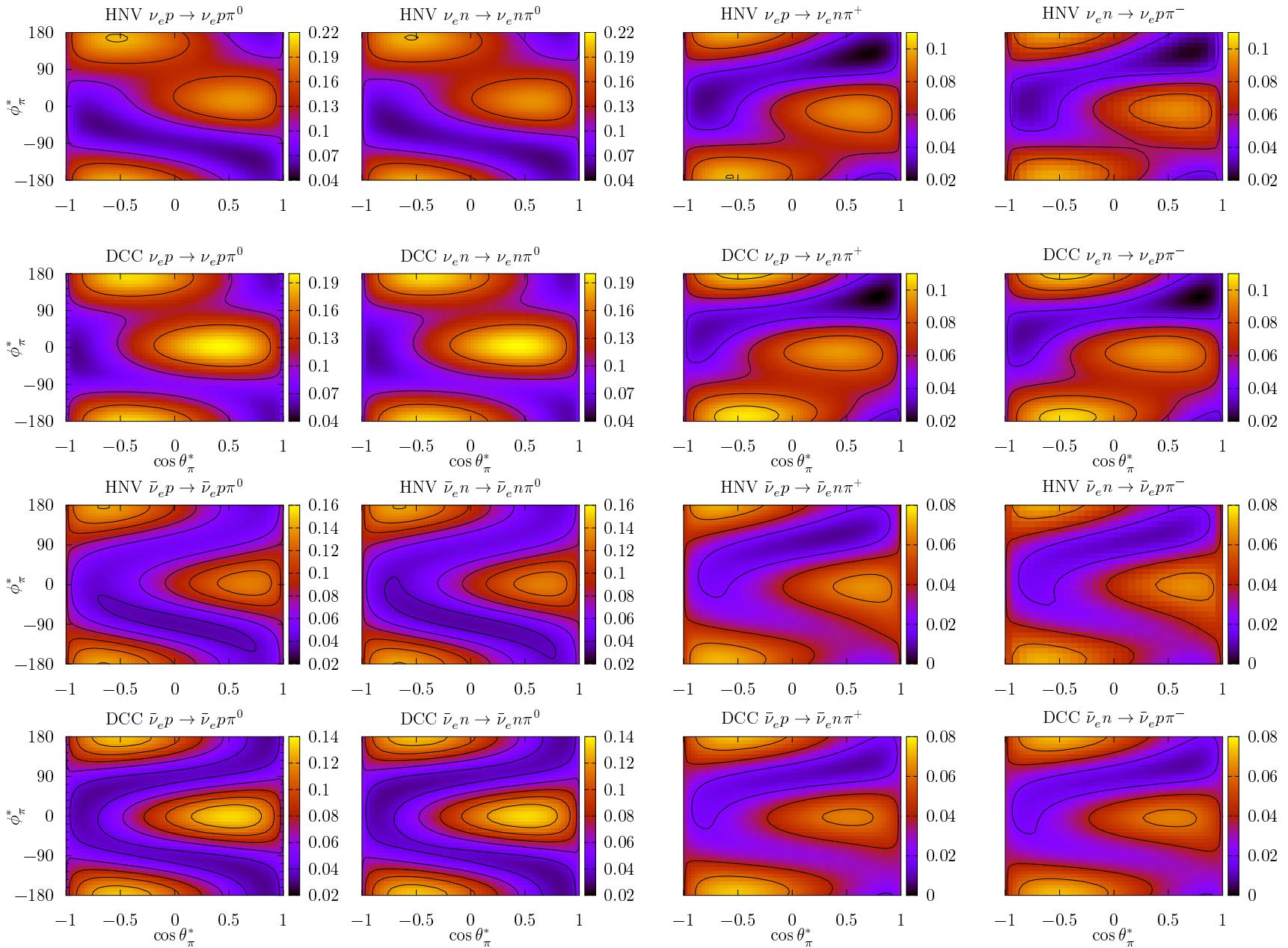
$$[10^{-38} \text{cm}^2 c^2/\text{GeV}^2]$$

$$E_\nu = 1 \text{ GeV}, Q^2 = 0.1 \text{ GeV}^2/c^2$$

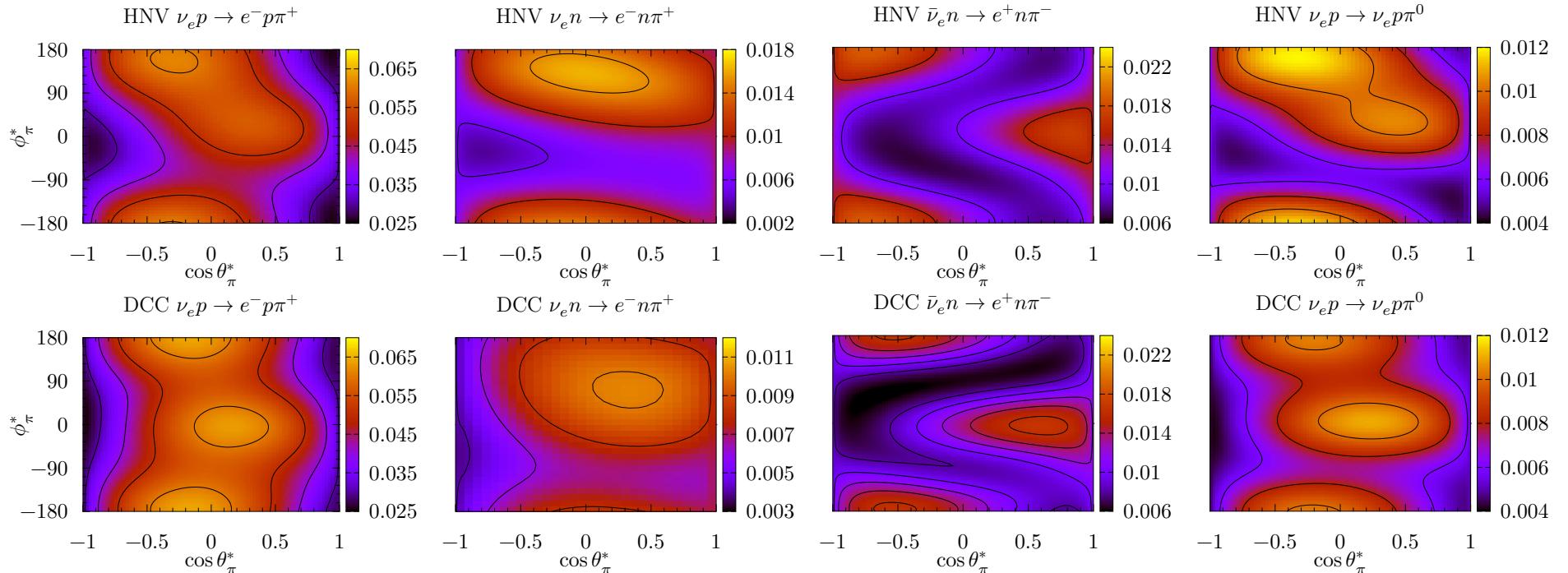
$$W_{\pi N} = 1.23 \text{ GeV}$$

Pion angular distribution IV

Same as before for NC processes



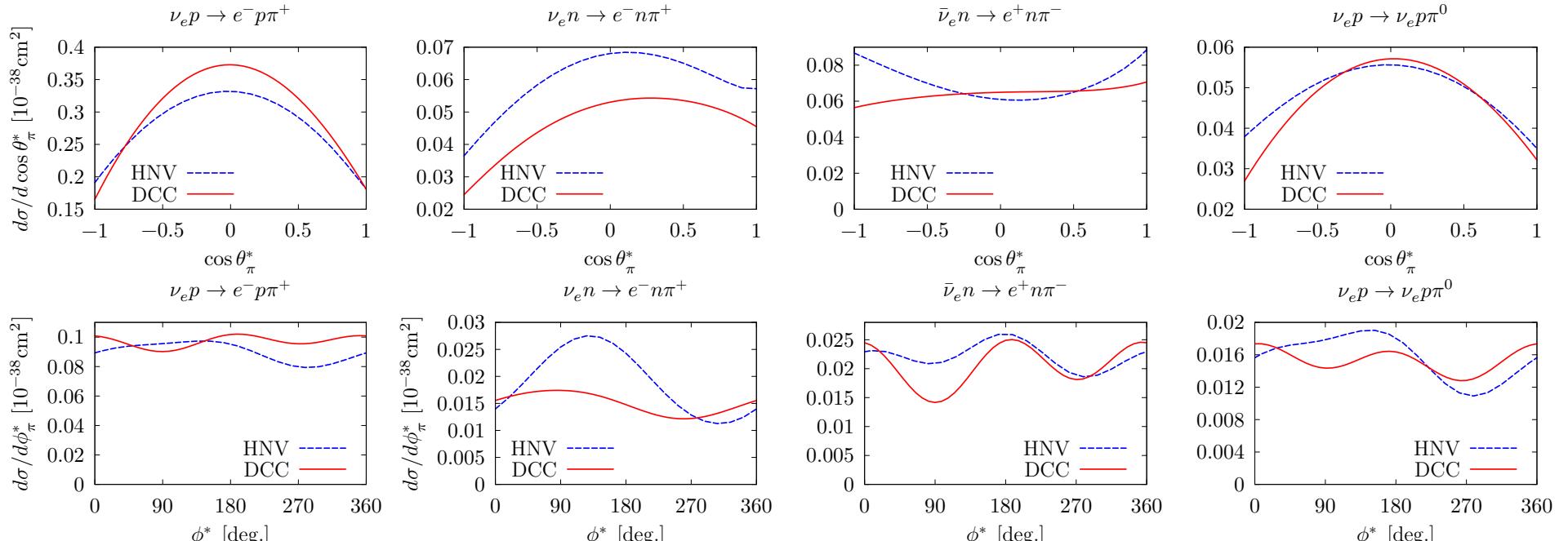
Pion angular distributions V



$d\sigma/d\Omega_\pi^*[10^{-38}\text{cm}^2]$, evaluated at $E_\nu = 1 \text{ GeV}$ and with a $W_{\pi N} < 1.4 \text{ GeV}$ cut.

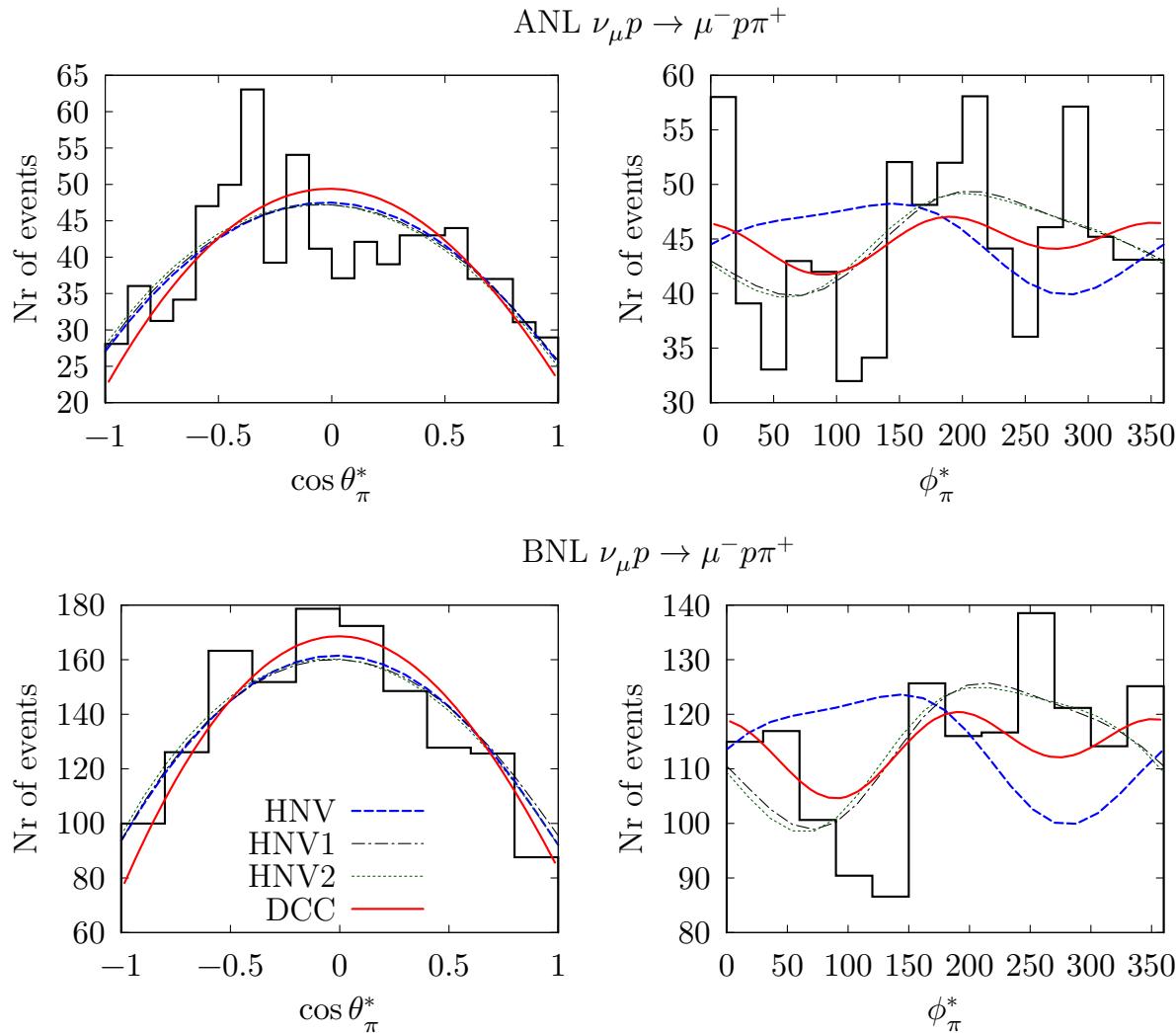
A clear anisotropy is still seen and the distribution is channel dependent

Pion angular distribution VI



$d\sigma/d\cos \theta_\pi^*$ and $d\sigma/d\phi_\pi^*$ in units of 10^{-38}cm^2 at $E_\nu = 1 \text{ GeV}$ and with $W_{\pi N} < 1.4 \text{ GeV}$

Pion angular distribution VII



Unnormalized, flux-averaged $d\sigma/d\cos \theta_\pi^*$ and $d\sigma/d\phi_\pi^*$. $W_{\pi N} < 1.4 \text{ GeV}$.

ANL data: G. M. Radecky et al, Phys. Rev. D 25, 1161 (1982).

BNL data: T. Kitagaki, Phys. Rev. D 34, 2554 (1986).

Incoherent pion production inside the nucleus I

PRD 87,113009 (2013)

We assume the nucleus can be described by its density and we shall use the local density approximation

The cross section at the nucleus level for initial pion production (prior to any FSI) is then

$$\begin{aligned} \frac{d\sigma}{d \cos \theta_\pi dE_\pi} &= \int d^3r \sum_{N=n,p} 2 \int \frac{d^3p_N}{(2\pi)^3} \theta(E_F^N(r) - E_N) \theta(E_N + q^0 - E_\pi - E_F^{N'}(r)) \\ &\quad \times \frac{d\sigma(\nu N \rightarrow l^- N' \pi)}{d \cos \theta_\pi dE_\pi} \end{aligned}$$

To compare with experiment, we have to convolute it with the neutrino flux $\Phi(|\vec{k}|)$

$$\begin{aligned} \frac{d\sigma}{d \cos \theta_\pi dE_\pi} &= \int d|\vec{k}| \Phi(|\vec{k}|) 4\pi \int dr r^2 \sum_{N=n,p} 2 \int \frac{d^3p_N}{(2\pi)^3} \theta(E_F^N(r) - E_N) \theta(E_N + q^0 - E_\pi - E_F^{N'}(r)) \\ &\quad \times \frac{d\sigma(\nu N \rightarrow l^- N' \pi)}{d \cos \theta_\pi dE_\pi} \end{aligned}$$

Incoherent pion production inside the nucleus II

From there we obtain

$$\frac{d\sigma}{d|\vec{k}| 4\pi r^2 dr d\cos\theta_\pi dE_\pi} = \Phi(|\vec{k}|) \sum_{N=n,p} 2 \int \frac{d^3 p_N}{(2\pi)^3} \theta(E_F^N(r) - E_N) \theta(E_N + q^0 - E_\pi - E_F^{N'}(r)) \\ \times \frac{d\sigma(\nu N \rightarrow l^- N' \pi)}{d\cos\theta_\pi dE_\pi}$$

Apart from modifications discussed in what follows, the above differential cross section is used in our simulation code to generate, in a given point inside the nucleus and by neutrinos of a given energy, pions with a certain charge, energy and momentum direction.

Defining $P = q - k_\pi$ (the four momentum transferred to the nucleus) and writing $d^3 p_N = d\cos\vartheta_N d\phi_N |\vec{p}_N| E_N dE_N$, where the angles are referred to a system in which the Z axis is along \vec{P} , we can integrate in the ϑ_N variable using the energy delta function present in $\frac{d\sigma(\nu N \rightarrow l^- N' \pi)}{d\cos\theta_\pi dE_\pi}$

Medium corrections I

Δ properties are strongly modified in the nuclear medium.

Its imaginary part changes due to

- Pauli blocking of the final nucleon which reduces the free width.
- In medium modification of the pionic decay width other than Pauli blocking
- Absorption processes $\Delta N \rightarrow NN$ and $\Delta NN \rightarrow NNN$.

We thus modify the Δ propagator of the direct Δ contribution approximating

$$\frac{1}{p_\Delta^2 - M_\Delta^2 + iM_\Delta\Gamma_\Delta} \approx \frac{1}{\sqrt{p_\Delta^2} + M_\Delta} \frac{1}{\sqrt{p_\Delta^2} - M_\Delta + i\Gamma_\Delta/2}$$

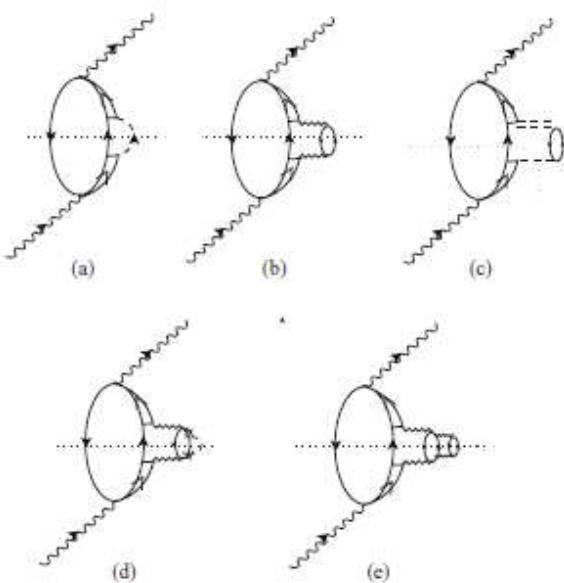
and substituting

$$\frac{\Gamma_\Delta}{2} \rightarrow \frac{\Gamma_\Delta^{\text{Pauli}}}{2} - \text{Im } \Sigma_\Delta$$

while keeping M_Δ in the propagator unchanged.

Medium corrections II

Delta width in a nuclear medium



The double dashed line represents the effective spin-isospin interaction originated by π and ρ exchange in the presence of short range correlations.

The wavy line includes an RPA sum with particle-hole and Delta-hole excitations.

The evaluation of $\text{Im } \Sigma_\Delta$ was done by E. Oset and L.L. Salcedo [Nuc. Phys. A468 (1987) 631].

Medium corrections III

The imaginary part can be parameterized as

$$-\text{Im } \Sigma_\Delta = C_Q \left(\frac{\rho}{\rho_0} \right)^\alpha + C_{A2} \left(\frac{\rho}{\rho_0} \right)^\beta + C_{A3} \left(\frac{\rho}{\rho_0} \right)^\gamma$$

with $\rho_0 = 0.17 \text{ fm}^{-3}$.

- The C_Q term corrects the pionic decay in the medium.
- The C_{A2} term corresponds to the process $\Delta N \rightarrow NN$
- The C_{A3} term corresponds to the $\Delta NN \rightarrow NNN$ process

The $C_Q, \alpha, C_{A2}, \beta$ and C_{A3}, γ coefficients are parametrized as a function of the kinetic energy of a pion that would excite a Δ of the corresponding invariant mass and are valid in the range $85 \text{ MeV} < T_\pi < 315 \text{ MeV}$.

Below 85 MeV the contributions from C_Q and C_{A3} are rather small and we take them from Nieves et al. [Nuc. Phys. A 554 (1993) 554], where the model was extended to lower energies. The term with C_{A2} shows a very mild energy dependence and we still use the original parameterization even at low energies.

For T_π above 315 MeV, we have kept these self-energy terms constant and equal to their values at the bound. The uncertainties in these pieces are not very relevant there because the $\Delta \rightarrow N\pi$ decay becomes very large and dominant.

Incoherent pion production inside the nucleus IV

The C_Q term not only modifies the Δ propagator but it also gives rise to a new source of pion production in the nuclear medium that has to be taken into account.

This new contribution has to be added incoherently and we implement it in an approximate way by taking as amplitude square for this process the amplitude square of the ΔP contribution multiplied by

$$\frac{C_Q(\rho/\rho_0)^\alpha}{\Gamma_\Delta^{\text{Free}}/2}$$

Its effect increases the total pion production cross section by less than 10%

Final state interaction

Once the pions are produced, we follow their path on its way out of the nucleus.

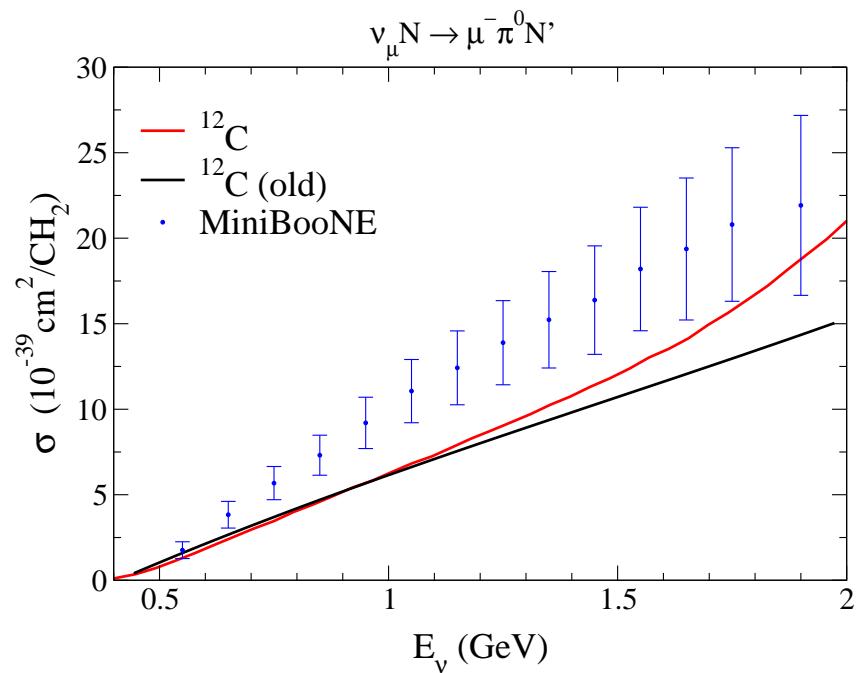
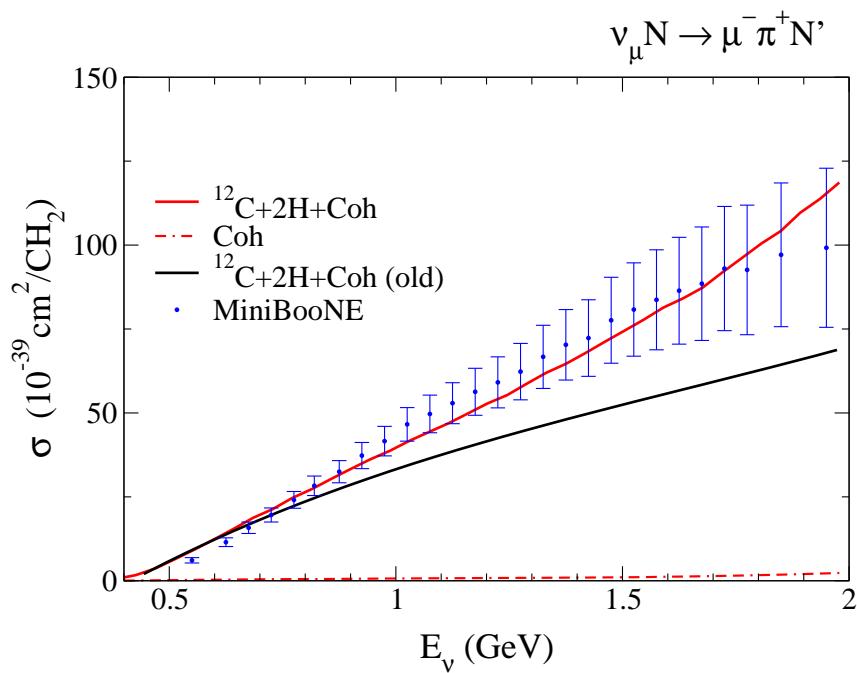
We use, with slight modifications, the model of L.L. Salcedo et al. [Nuc. Phys. A484 (1988) 557]

- P and S-wave pion absorption.
- P and S-wave quasielastic scattering on a nucleon.
 - Pions change energy and direction.
 - Pions could change charge.
- Pion propagate on straight lines in between collisions.

The P- wave interaction is mediated by the Δ resonance excitation where the different contributions to the imaginary part of its self-energy give rise to pion two- and three-nucleon absorption and quasielastic processes.

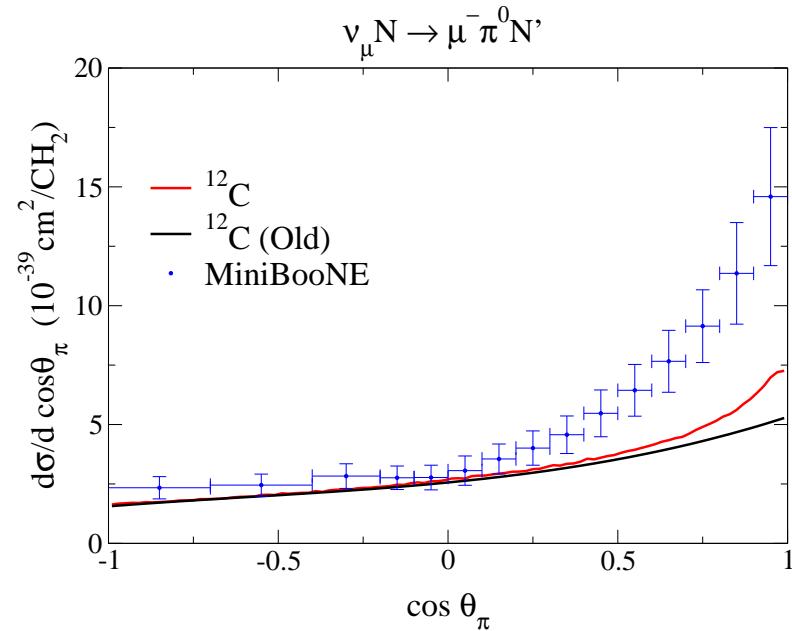
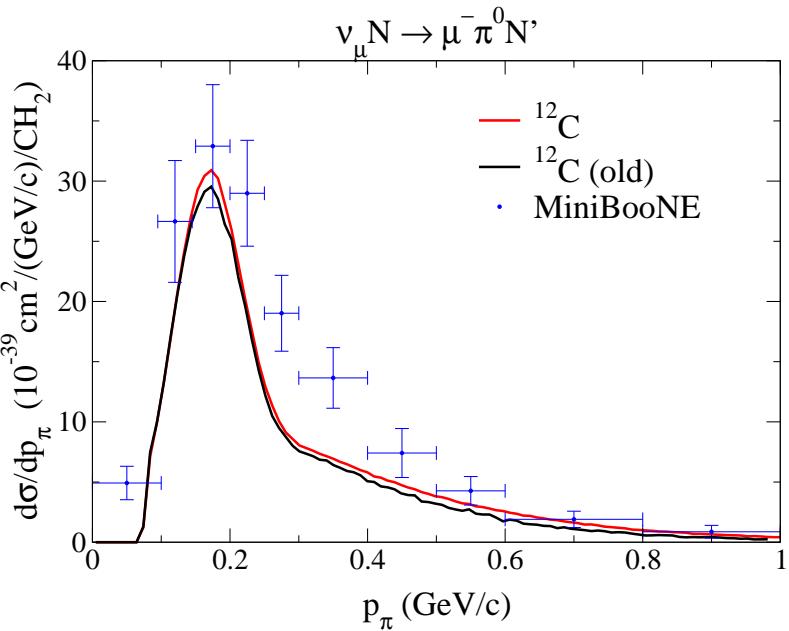
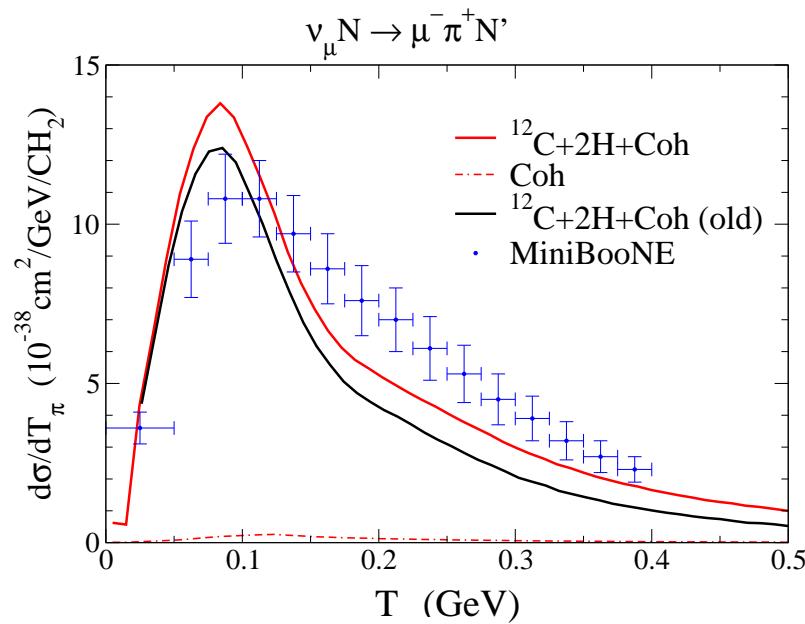
The intrinsic probabilities for each of the above mentioned reactions are evaluated microscopically as a function of the density and we use the local density approximation to evaluate them in finite nuclei.

(preliminary) Results for MiniBooNE (CH_2 target) I

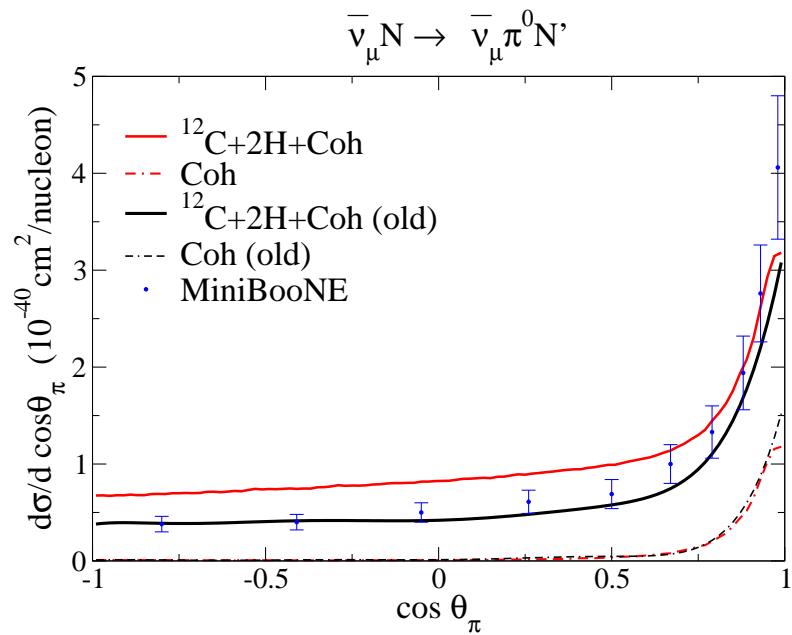
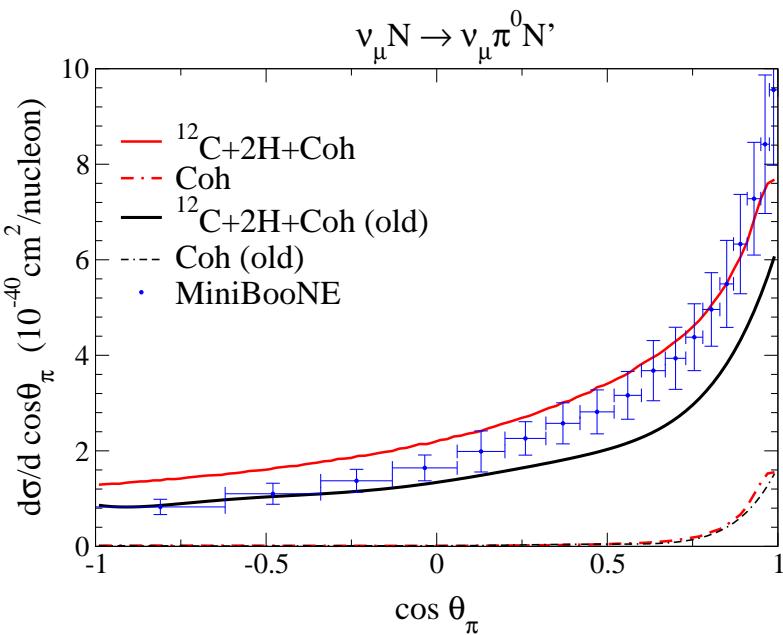
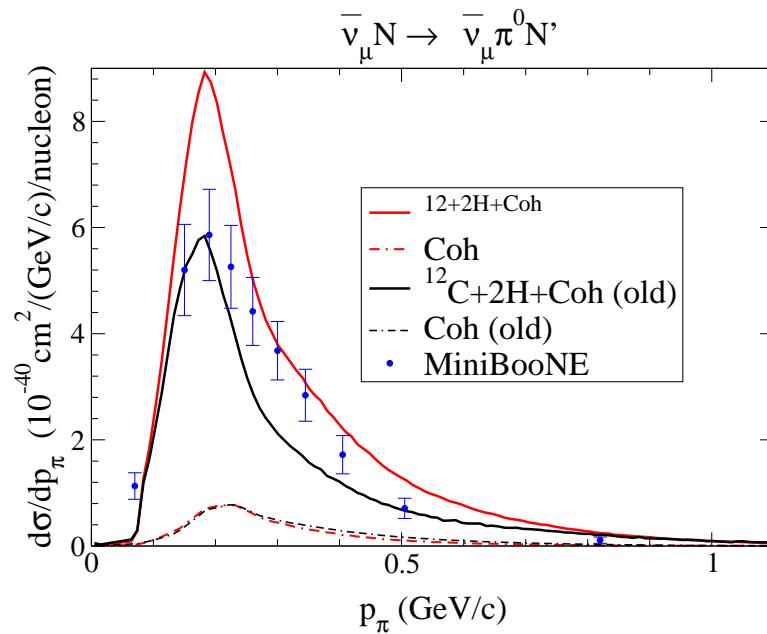
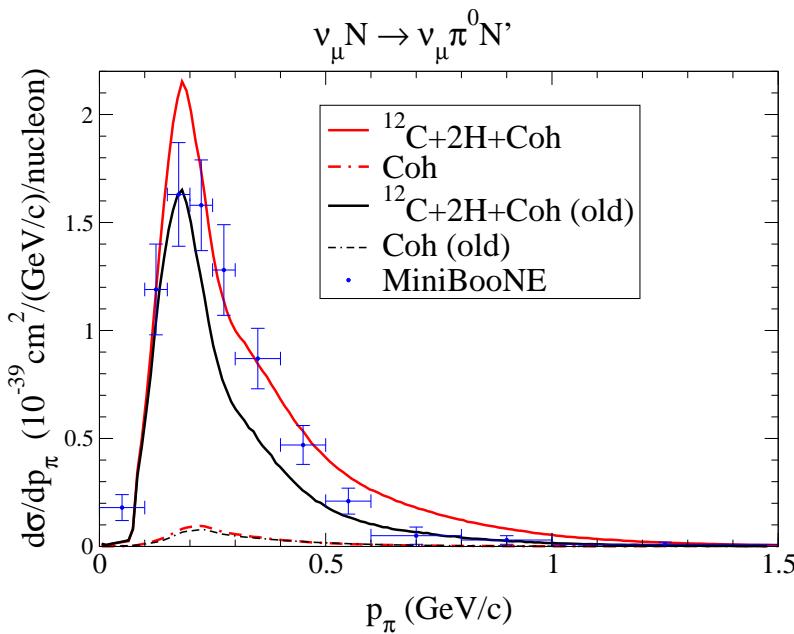


We get a better agreement at higher energies than before.

(preliminary) Results for MiniBooNE (CH_2 target) II

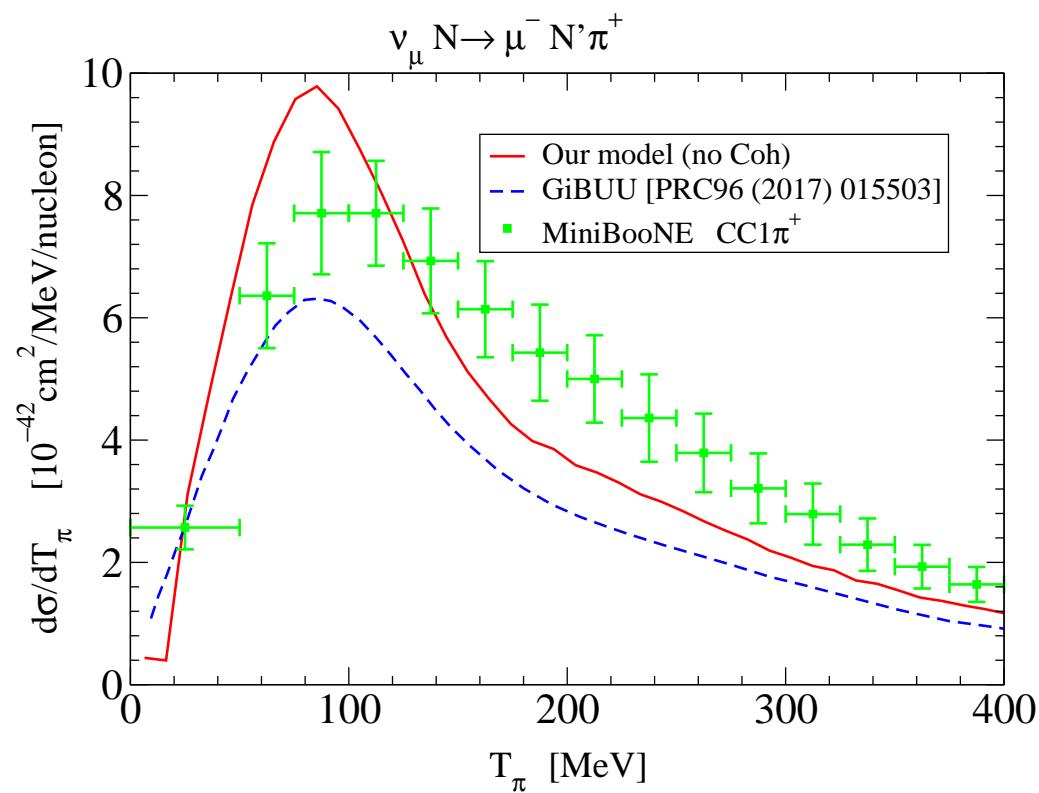


(preliminary) Results for MiniBooNE (CH_2 target) II



(preliminary) Results for MiniBooNE (CH_2 target)III

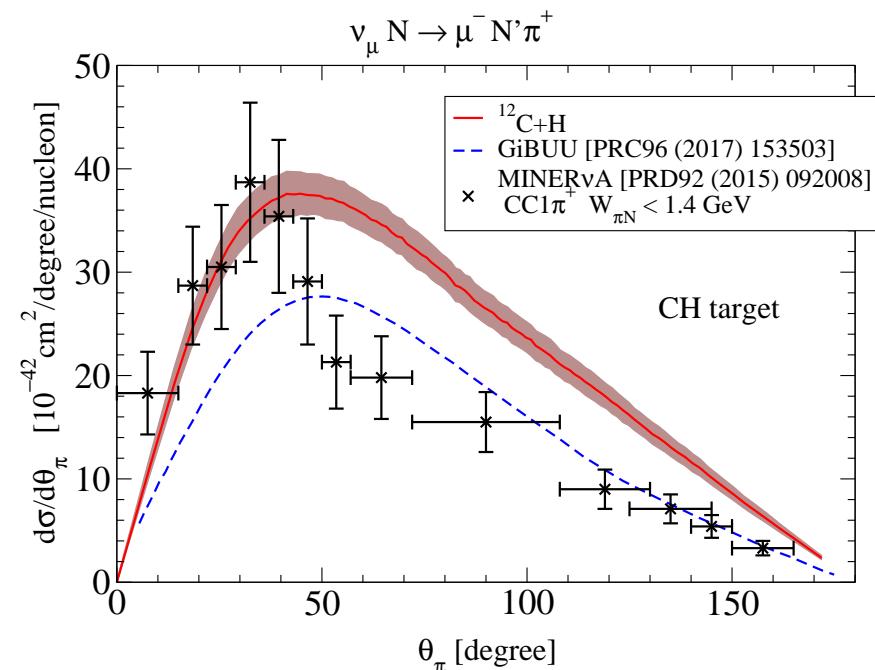
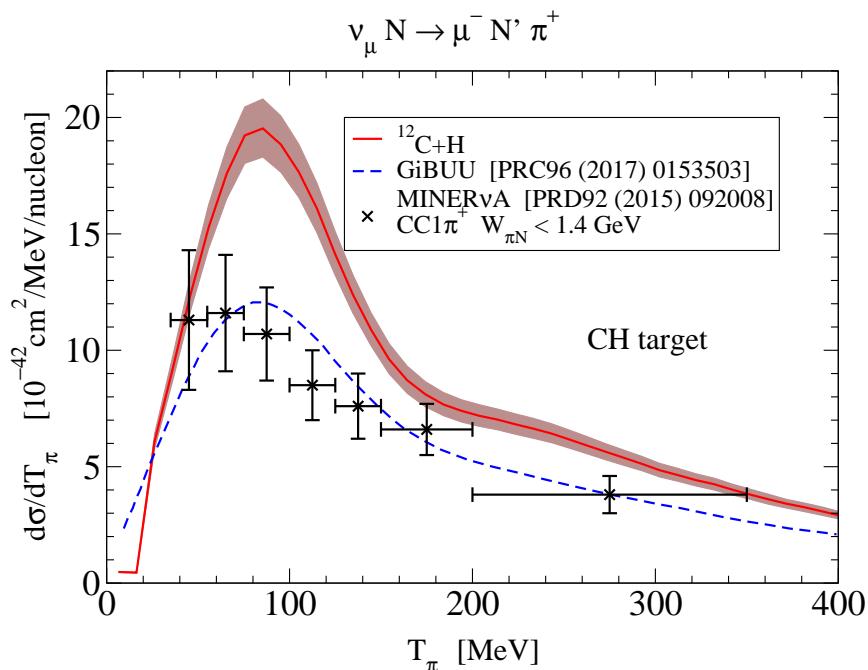
Comparison with GiBUU [PRC96 (2017) 015503]



Our results for high energy pions are in better agreement with experiment.

(preliminary) Results for MINER ν A (CH target)

We integrate the MINER ν A flux up to $E_\nu = 5$ GeV.



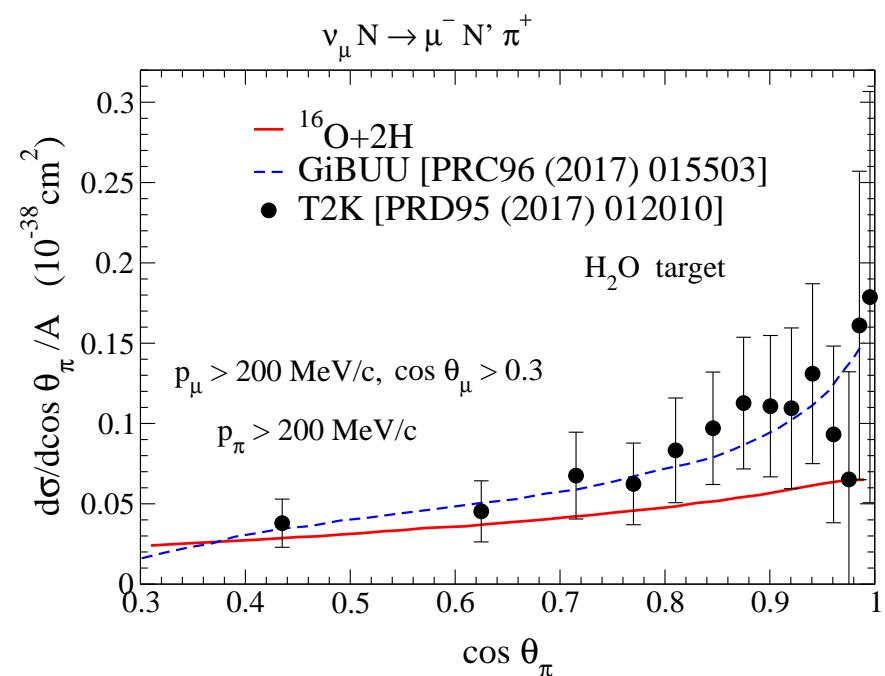
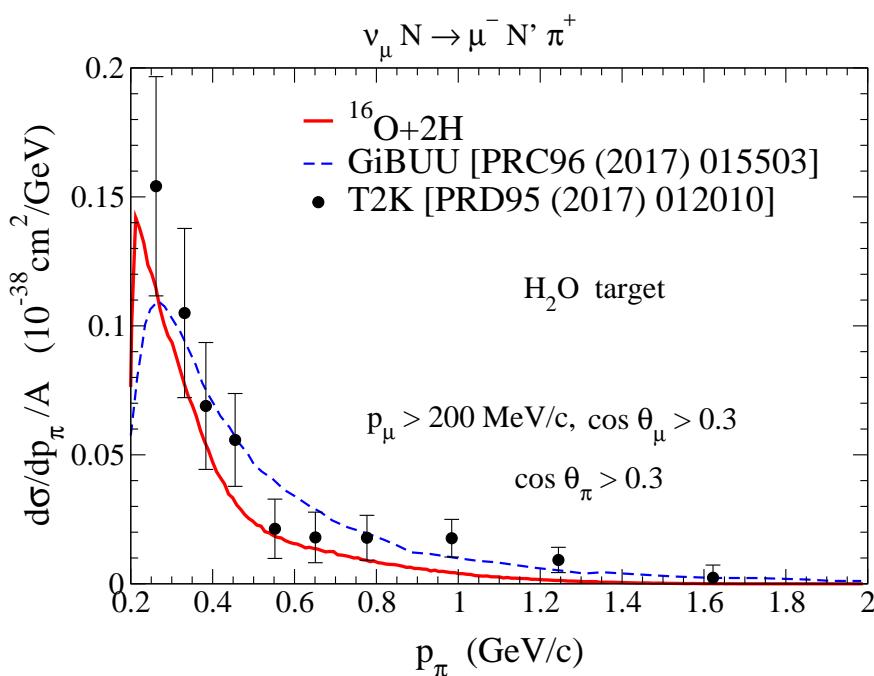
We produce too many pions in the backward direction while GiBUU underestimates the production of forward pions.

GiBUU gives a good reproduction of the $d\sigma/dT_\pi$ differential cross section

(preliminary) Results for T2K (H_2O target)

We integrate the T2K flux up to $E_\nu = 2 \text{ GeV}$.

We implement the cuts on the muon momentum on production (neglecting FSI for the muon)



$$\langle \sigma \rangle_\phi^{\text{exp.}} = 4.25 \pm 0.48 \pm 1.56 \times 10^{-40} \text{ cm}^2$$

$$\langle \sigma \rangle_\phi^{\text{GiBUU}} \approx 4.0 \times 10^{-40} \text{ cm}^2$$

$$\langle \sigma \rangle_\phi^{\text{ours}} \approx 2.8 \times 10^{-40} \text{ cm}^2$$

Summary

- I have shown our model for one-pion production by neutrinos.
- We get good agreement with data at the nucleon level.
- In good qualitative agreement with the DCC model, we predict anisotropic pion distributions (measured in the final πN center of mass).
- For pion production in nuclei the situation is not so clear. We get a reasonable reproduction of MiniBooNE data but we overestimate the production in the case of *Minerva* data.