"Tell me that you have found no sign of New Physics again, I dare you. I double dare you. Tell me one more goddamn **time!**"

Electroweak Precision Observables (at Future Colliders)

Sven Heinemeyer, IFT/IFCA (CSIC, Madrid/Santander)

Madrid, 06/2019

1. Motivation
2. Electroweak Precision Observables (EWPOs)
3. EWPO status
4. EWPO future
5. SM Higgs
6. BSM Higgs
7. Conclusions

1. Introduction

Experimental situation:

(HL-)LHC/ILC/CLIC/FCC-ee/CEPC/. . .
will provide (high!) accuracy measurements!

Theory situation:

– Measurements are performed using theory predictions

– measured observables have to be compared with theoretical predictions (in various models: SM, MSSM, . . .)

Full uncertainty is given by the (linear) sum of experimental and theoretical uncertainties!

$\Rightarrow$ Experimetal precision can only fully be exploited with theory uncertainties at the same level of accurady!

Many results shown here based on: [*arXiv:1906.05379*]
Write-up for FCC-ee physics WG2 – Precision EW Calculations

Theoretical uncertainties for electroweak and Higgs-boson precision measurements at FCC-ee

A. Freitas*[1], S. Heinemeyer*[2], M. Beneke[3], A. Blondel[4], S. Dittmaier[5], J. Gluza[6,7], A. Hoang[8], S. Jadach[9], P. Janot[10], J. Reuter[11], T. Riemann[6,12], C. Schwinn[13], M. Skrzypek[8], and S. Weinzierl[14]

⇒ Here: focus on e^+e^- precision

⇒ should be taken into account by "exp groups"!

⇒ Here: current status and future of EWPO/Higgs TH calculations
what may be achievable in TH calc. in $\mathcal{O}(20)$ years

Where we need theory prediction:

1. Prediction of the measured quantity
   Example: M_W, $\Gamma(H \to b\bar{b})$
   $\to$ at the same level or better as the experimental precision

2. Prediction of the measured process to extract the quantity
   Example: $e^+e^- \to W^+W^-$, $e^+e^- \to ZH$
   $\to$ better than then "pure" experimental precision

Where we need theory prediction:

1. Prediction of the measured quantity
   Example: M_W, $\Gamma(H \to b\bar{b})$
   $\to$ at the same level or better as the experimental precision

2. Prediction of the measured process to extract the quantity
   Example: $e^+e^- \to W^+W^-$, $e^+e^- \to ZH$
   $\to$ better than then "pure" experimental precision

Two types of theory uncertainties:

1. intrinsic: missing higher orders

2. parametric: uncertainty due to exp. uncertainty in SM input parameters
   Example: m_t, m_b, α_s, $\Delta\alpha_{\text{had}}$, ...

Options for the evaluation of intrinsic uncertainties:

1. Determine all prefactors of a certain diagram class (couplings, group factors, multiplicities, mass ratios) and assume the loop is $\mathcal{O}(1)$

2. Take the known contribution at n-loop and $(n-1)$-loop and thus estimate the $n+1$-loop contribution:

$$\frac{(n+1)^{(\text{estimated})}}{n^{(\text{known})}} \approx \frac{n^{(\text{known})}}{(n-1)^{(\text{known})}}$$

   $\Rightarrow$ simplified example! Has to be done
   "coupling constant by coupling constant"

3. Variation of $\mu^{\overline{\text{MS}}}$ (QCD!, EW?)

4. Compare different renormalizations

$\Rightarrow$ Mostly used here: 1 & 2

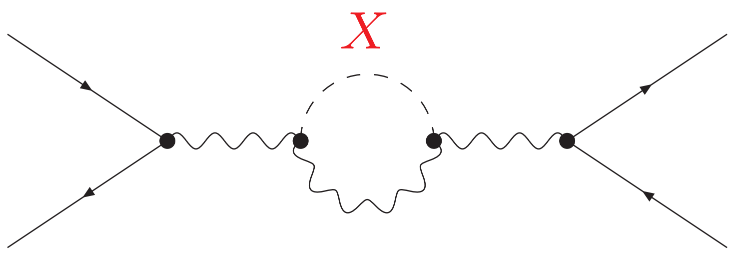
2. Electroweak Precision Observables

Comparison of observables with theory:

Precision data:		Theory:
$M_W, \sin^2\theta_{\text{eff}}, a_\mu,\ M_h$	$\leftrightarrow$	SM, MSSM , . . .

$\Downarrow$

Test of theory at quantum level: Sensitivity to loop corrections, e.g. X



$\Downarrow$

SM: limits on M_H,   BSM: limits on M_X

Very high accuracy of measurements and theoretical predictions needed
$\Rightarrow$ only models "ready" so far: SM, MSSM

Precision observables in the SM and the MSSM

M_W, $\sin^2\theta_{\rm eff}$, M_h, $(g-2)_\mu$, b physics, ...

A) Theoretical prediction for M_W in terms

of $M_Z, \alpha, G_\mu, \Delta r$:

$$M_W^2\left(1-\frac{M_W^2}{M_Z^2}\right)=\frac{\pi\,\alpha}{\sqrt{2}\,G_\mu}(1+\Delta r)$$

$\Updownarrow$

loop corrections

Evaluate Δr from μ decay $\Rightarrow M_W$

One-loop result for M_W in the SM:

[*A. Sirlin '80*] , [*W. Marciano, A. Sirlin '80*]

$$\Delta r_{1-\text{loop}} = \Delta\alpha - \frac{c_W^2}{s_W^2}\Delta\rho + \Delta r_{\text{rem}}(M_H)$$

$$\sim \log\frac{M_Z}{m_f} \qquad \sim m_t^2 \qquad \log(M_H/M_W)$$

$$\sim 6\% \qquad \sim 3.3\% \qquad \sim 1\%$$

Precision observables in the SM and the MSSM

M_W, $\sin^2\theta_{\rm eff}$, M_h, $(g-2)_\mu$, b physics, . . .

A) Theoretical prediction for M_W in terms

of $M_Z, \alpha, G_\mu, \Delta r$:

$$M_W^2 \left(1 - \frac{M_W^2}{M_Z^2}\right) = \frac{\pi\,\alpha}{\sqrt{2}\,G_\mu} (1 + \Delta r)$$

⇕

loop corrections

B) Effective mixing angle:

$$\sin^2\theta_{\rm eff} = \frac{1}{4\,|Q_f|}\left(1 - \frac{\mathrm{Re}\,g_V^f}{\mathrm{Re}\,g_A^f}\right)$$

Higher order contributions:

$$g_V^f \to g_V^f + \Delta g_V^f, \quad g_A^f \to g_A^f + \Delta g_A^f$$

Corrections to M_W, $\sin^2\theta_{\rm eff}$ → approximation via the ρ-parameter:

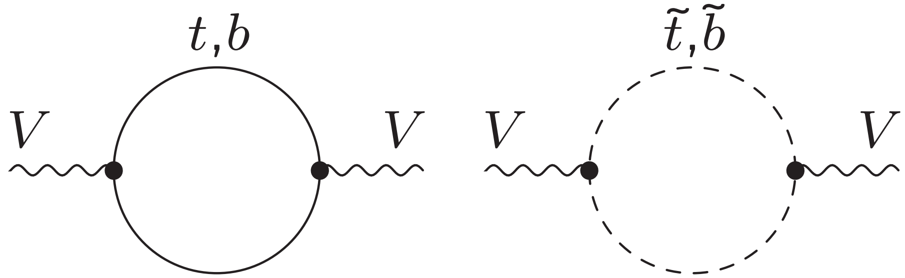
ρ measures the relative strength between
neutral current interaction and charged current interaction

$$\rho = \frac{1}{1 - \Delta\rho} \qquad\qquad \Delta\rho = \frac{\Sigma_Z(0)}{M_Z^2} - \frac{\Sigma_W(0)}{M_W^2}$$

(leading, process independent terms)

$\Delta\rho$ gives the main contribution to EW observables:

$$\Delta M_W \approx \frac{M_W}{2}\frac{c_W^2}{c_W^2 - s_W^2}\Delta\rho, \qquad\qquad \Delta\sin^2\theta_W^{\rm eff} \approx -\frac{c_W^2 s_W^2}{c_W^2 - s_W^2}\Delta\rho$$



$\Delta\rho^{\rm SUSY}$ from $\tilde{t}/\tilde{b}$ loops > 0 $\quad\Rightarrow M_W^{\rm SUSY} \gtrsim M_W^{\rm SM}$

All the EWPO:

M_W (best from threshold scan)

$$\sigma^0_{\rm had} = \sum_q \sigma_q(M_Z^2),$$

$$\Gamma_Z = \sum_f \Gamma[Z \to f\bar{f}],$$ (from a fit to $\sigma_f(s)$ at various values of s)

$$R_\ell = \left[\sum_q \sigma_q(M_Z^2)\right]/\sigma_\ell(M_Z^2),$$ $(\ell = e, \mu, \tau)$

$$R_q = \sigma_q(M_Z^2)/\left[\sum_q \sigma_q(M_Z^2)\right],$$ $(q = b, c)$

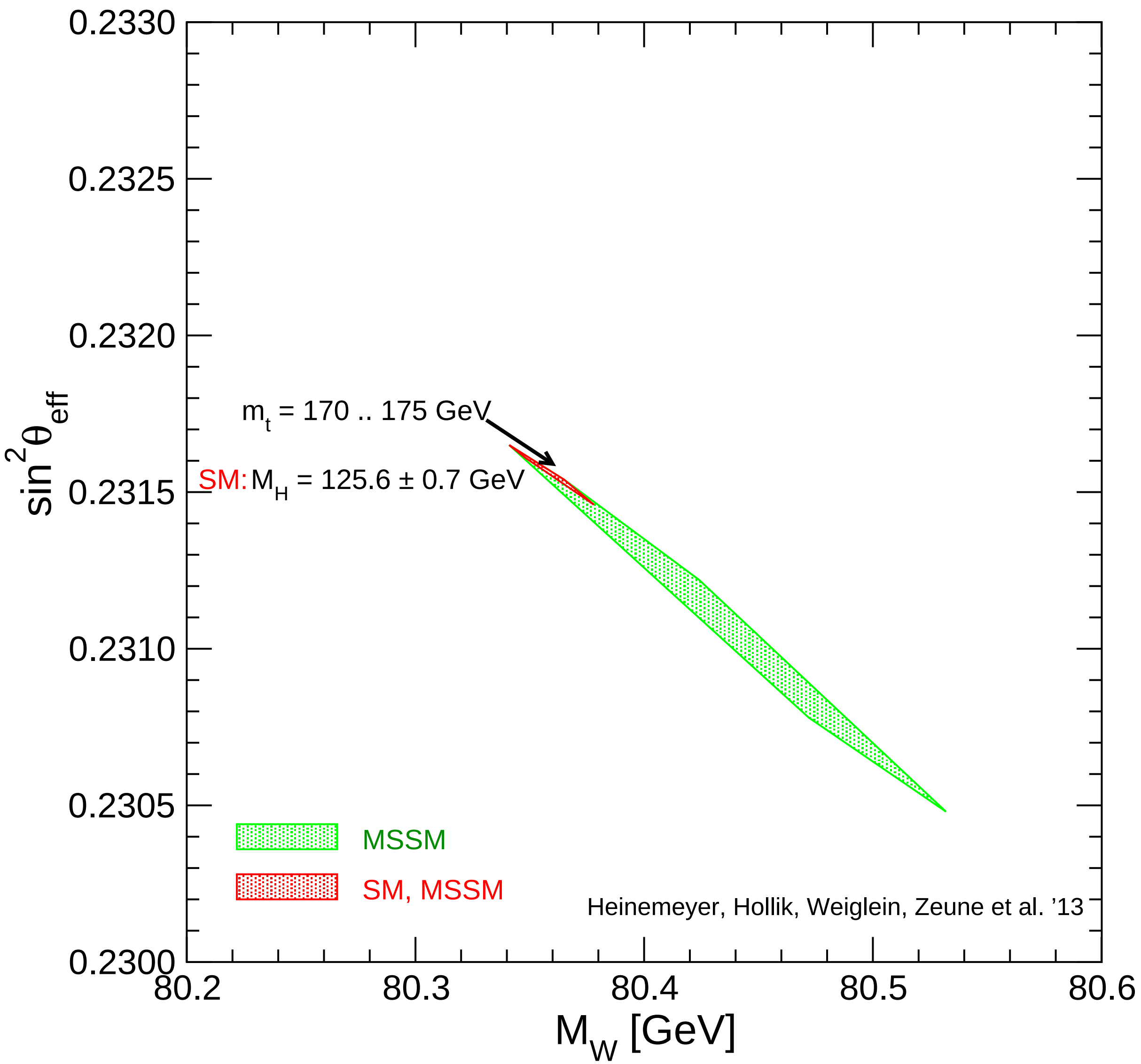
$$A^f_{\rm FB} = \frac{\sigma_f(\theta < \frac{\pi}{2}) - \sigma_f(\theta > \frac{\pi}{2})}{\sigma_f(\theta < \frac{\pi}{2}) + \sigma_f(\theta > \frac{\pi}{2})} \equiv \frac{3}{4}\mathcal{A}_e\mathcal{A}_f,$$

$$A^f_{\rm LR} = \frac{\sigma_f(P_e < 0) - \sigma_f(P_e > 0)}{\sigma_f(P_e < 0) + \sigma_f(P_e > 0)} \equiv \mathcal{A}_e|P_e|$$

$$\mathcal{A}_f = 2\frac{g_{V_f}/g_{A_f}}{1 + (g_{V_f}/g_{A_f})^2} = \frac{1 - 4|Q_f|\sin^2\theta^f_{\rm eff}}{1 - 4|Q_f|\sin^2\theta^f_{\rm eff} + 8(|Q_f|\sin^2\theta^f_{\rm eff})^2}$$ $(f = \ell, b, \ldots)$

What M_W and $\sin^2\theta_{\rm eff}$ precision do we want?

[*S.H., W. Hollik, G. Weiglein, L. Zeune et al. '13*]



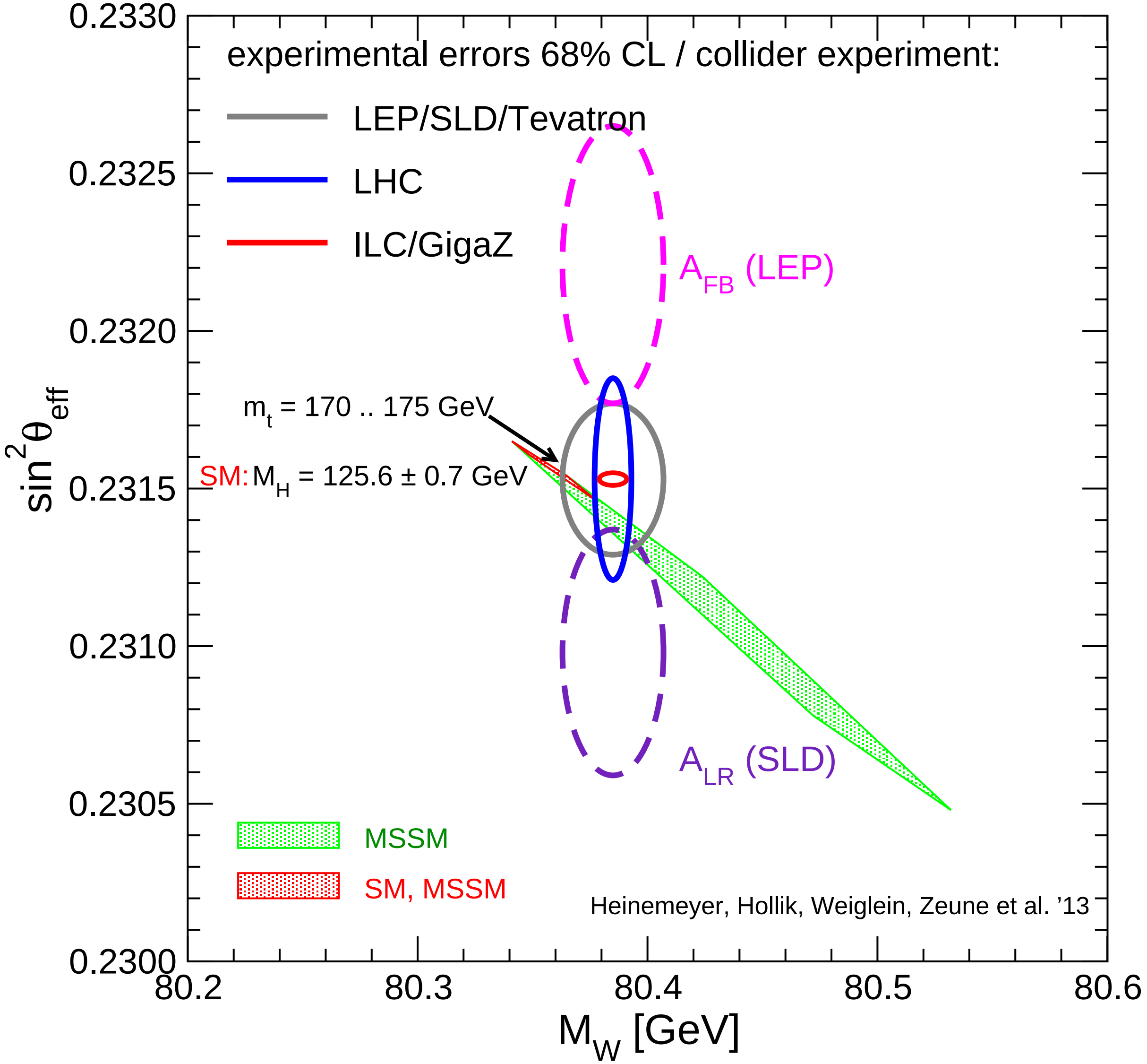
MSSM band:
scan over
SUSY masses

overlap:
SM is MSSM-like
MSSM is SM-like

SM band:
variation of m_t

What M_W and $\sin^2\theta_{\text{eff}}$ precision do we want?

[*S.H., W. Hollik, G. Weiglein, L. Zeune et al. '13*]



MSSM band:
scan over
SUSY masses

overlap:
SM is MSSM-like
MSSM is SM-like

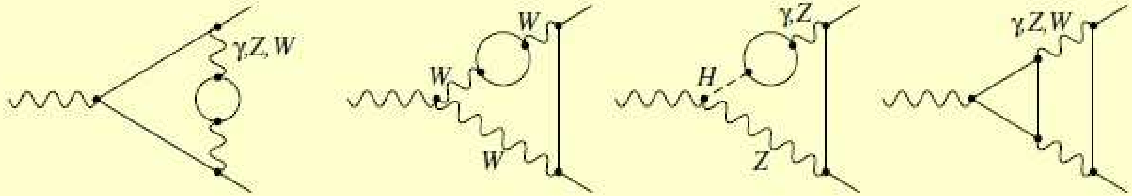
SM band:
variation of m_t

3. EWPO Status

Existing higher-order corrections to the EWPO

[*taken from A. Freitas '16*]

Known corrections to Δr, $\sin^2\theta^f_{\rm eff}$, g_{Vf}, g_{Af}:



- Complete NNLO corrections (Δr, $\sin^2\theta^\ell_{\rm eff}$) Freitas, Hollik, Walter, Weiglein '00
  Awramik, Czakon '02; Onishchenko, Veretin '02
  Awramik, Czakon, Freitas, Weiglein '04; Awramik, Czakon, Freitas '06
  Hollik, Meier, Uccirati '05,07; Degrassi, Gambino, Giardino '14
- "Fermionic" NNLO corrections (g_{Vf}, g_{Af}) Czarnecki, Kühn '96
  Harlander, Seidensticker, Steinhauser '98
  Freitas '13,14
- Partial 3/4-loop corrections to ρ/T-parameter
  $\mathcal{O}(\alpha_t\alpha_s^2)$, $\mathcal{O}(\alpha_t^2\alpha_s)$, $\mathcal{O}(\alpha_t\alpha_s^3)$ Chetyrkin, Kühn, Steinhauser '95
  Faisst, Kühn, Seidensticker, Veretin '03
  Boughezal, Tausk, v. d. Bij '05
  Schröder, Steinhauser '05; Chetyrkin et al. '06
  Boughezal, Czakon '06

  ($\alpha_t \equiv \frac{y_t^2}{4\pi}$)

Intrinsic uncertainties:

Quantity	current experimental unc.	current intrinsic unc.	
M_W [MeV]	15	4	$(\alpha^3, \alpha^2\alpha_s)$
$\sin^2\theta^\ell_{\rm eff}$ [10^{-5}]	16	4.5	$(\alpha^3, \alpha^2\alpha_s)$
Γ_Z [MeV]	2.3	0.5	$(\alpha^2_{\rm bos}, \alpha^3, \alpha^2\alpha_s, \alpha\alpha_s^2)$
R_b [10^{-5}]	66	15	$(\alpha^2_{\rm bos}, \alpha^3, \alpha^2\alpha_s)$
R_l [10^{-3}]	25	5	$(\alpha^2_{\rm bos}, \alpha^3, \alpha^2\alpha_s)$

Intrinsic uncertainties:

Quantity	current experimental unc.	current intrinsic unc.	
M_W [MeV]	15	4	$(\alpha^3, \alpha^2\alpha_s)$
$\sin^2\theta^\ell_{\rm eff}$ [10^{-5}]	16	4.5	$(\alpha^3, \alpha^2\alpha_s)$
Γ_Z [MeV]	2.3	0.5	$(\alpha^2_{\rm bos}, \alpha^3, \alpha^2\alpha_s, \alpha\alpha_s^2)$
R_b [10^{-5}]	66	15	$(\alpha^2_{\rm bos}, \alpha^3, \alpha^2\alpha_s)$
R_l [10^{-3}]	25	5	$(\alpha^2_{\rm bos}, \alpha^3, \alpha^2\alpha_s)$

Parametric uncertainties:

Quantity	$\delta m_t = 0.9$ GeV	$\delta(\Delta\alpha_{\rm had}) = 10^{-4}$	$\delta M_Z = 2.1$ MeV
$\delta M_W^{\rm para}$ [MeV]	5.5	2	2.5
$\delta\sin^2\theta^{\ell,{\rm para}}_{\rm eff}$ [10^{-5}]	3.0	3.6	1.4

Intrinsic uncertainties:

Quantity	current experimental unc.	current intrinsic unc.	
M_W [MeV]	15	4	$(\alpha^3, \alpha^2\alpha_s)$
$\sin^2\theta^\ell_{\text{eff}}$ $[10^{-5}]$	16	4.5	$(\alpha^3, \alpha^2\alpha_s)$
Γ_Z [MeV]	2.3	0.5	$(\alpha^2_{\text{bos}}, \alpha^3, \alpha^2\alpha_s, \alpha\alpha_s^2)$
R_b $[10^{-5}]$	66	15	$(\alpha^2_{\text{bos}}, \alpha^3, \alpha^2\alpha_s)$
R_l $[10^{-3}]$	25	5	$(\alpha^2_{\text{bos}}, \alpha^3, \alpha^2\alpha_s)$

Parametric uncertainties:

Quantity	$\delta m_t = 0.9$ GeV	$\delta(\Delta\alpha_{\text{had}}) = 10^{-4}$	$\delta M_Z = 2.1$ MeV
δM_W^{para} [MeV]	5.5	2	2.5
$\delta \sin^2\theta^{\ell,\text{para}}_{\text{eff}}$ $[10^{-5}]$	3.0	3.6	1.4

$\Rightarrow$ Current intrinsic/parametric uncertainties are substantially smaller than current experimental uncertainties :-)

Intrinsic uncertainties: NEW: α^2_{bos} calc. [*Dubovyka et al. '18*]

Quantity	current experimental unc.	current intrinsic unc.	
M_W [MeV]	15	4	$(\alpha^3, \alpha^2\alpha_s)$
$\sin^2\theta^\ell_{\text{eff}}$ $[10^{-5}]$	16	4.5	$(\alpha^3, \alpha^2\alpha_s)$
Γ_Z [MeV]	2.3	0.5	$(\alpha^2_{\text{bos}}, \alpha^3, \alpha^2\alpha_s, \alpha\alpha_s^2)$
R_b $[10^{-5}]$	66	15	$(\alpha^2_{\text{bos}}, \alpha^3, \alpha^2\alpha_s)$
R_l $[10^{-3}]$	25	5	$(\alpha^2_{\text{bos}}, \alpha^3, \alpha^2\alpha_s)$

Parametric uncertainties:

Quantity	$\delta m_t = 0.9$ GeV	$\delta(\Delta\alpha_{\text{had}}) = 10^{-4}$	$\delta M_Z = 2.1$ MeV
δM_W^{para} [MeV]	5.5	2	2.5
$\delta \sin^2\theta^{\ell,\text{para}}_{\text{eff}}$ $[10^{-5}]$	3.0	3.6	1.4

$\Rightarrow$ Current intrinsic/parametric uncertainties are substantially smaller than current experimental uncertainties :-)

Additional uncertainty for M_W from threshold scan:

Not only $e^+e^- \to W^{(*)}W^{(*)}$, but $e^+e^- \to WW \to$ 4f needed

Current status:

full one-loop for $2 \to 4$ process

[*A. Denner, S. Dittmaier, M. Roth, D. Wackeroth '99-'02*]

$\Rightarrow$ extraction of M_W at the level of $\sim$ 6 MeV

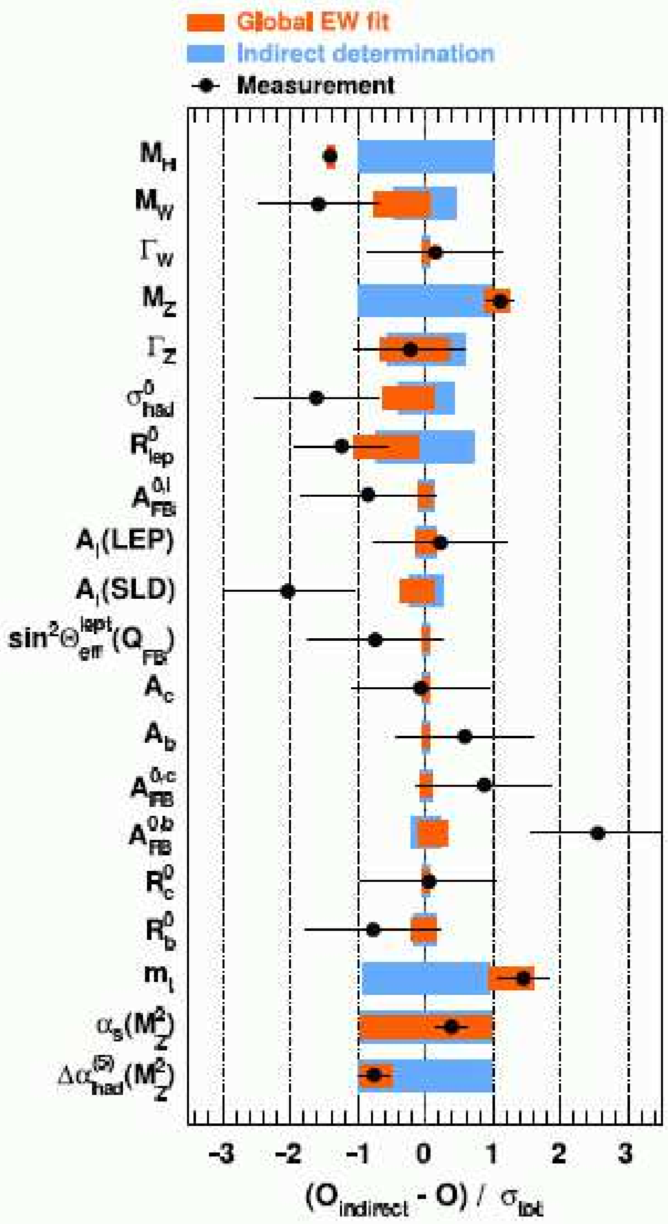
Most recent improvement:

leading 2L corrections from EFT

[*Actis, Beneke, Falgari, Schwinn '08*]

$\Rightarrow$ impact on M_W at the level of $\sim$ 3 MeV

$\Rightarrow$ well under control for LEP data



Surprisingly good agreement:

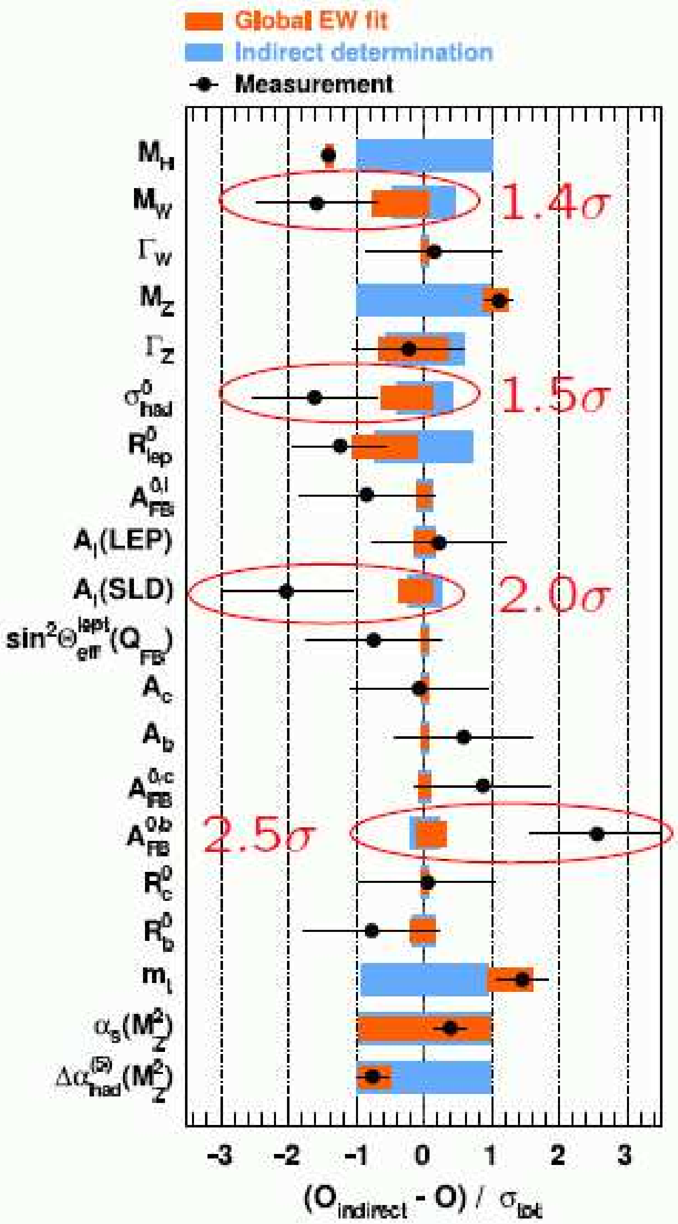
χ^2/d.o.f. $= 18.1/14$ ($p = 20\%$)

Most quantities measured with 1%–0.1% precision

GFitter coll. '14

Overview about all EWPO:

[taken from A. Freitas '16]



Surprisingly good agreement:

χ^2/d.o.f. $= 18.1/14$ ($p = 20\%$)

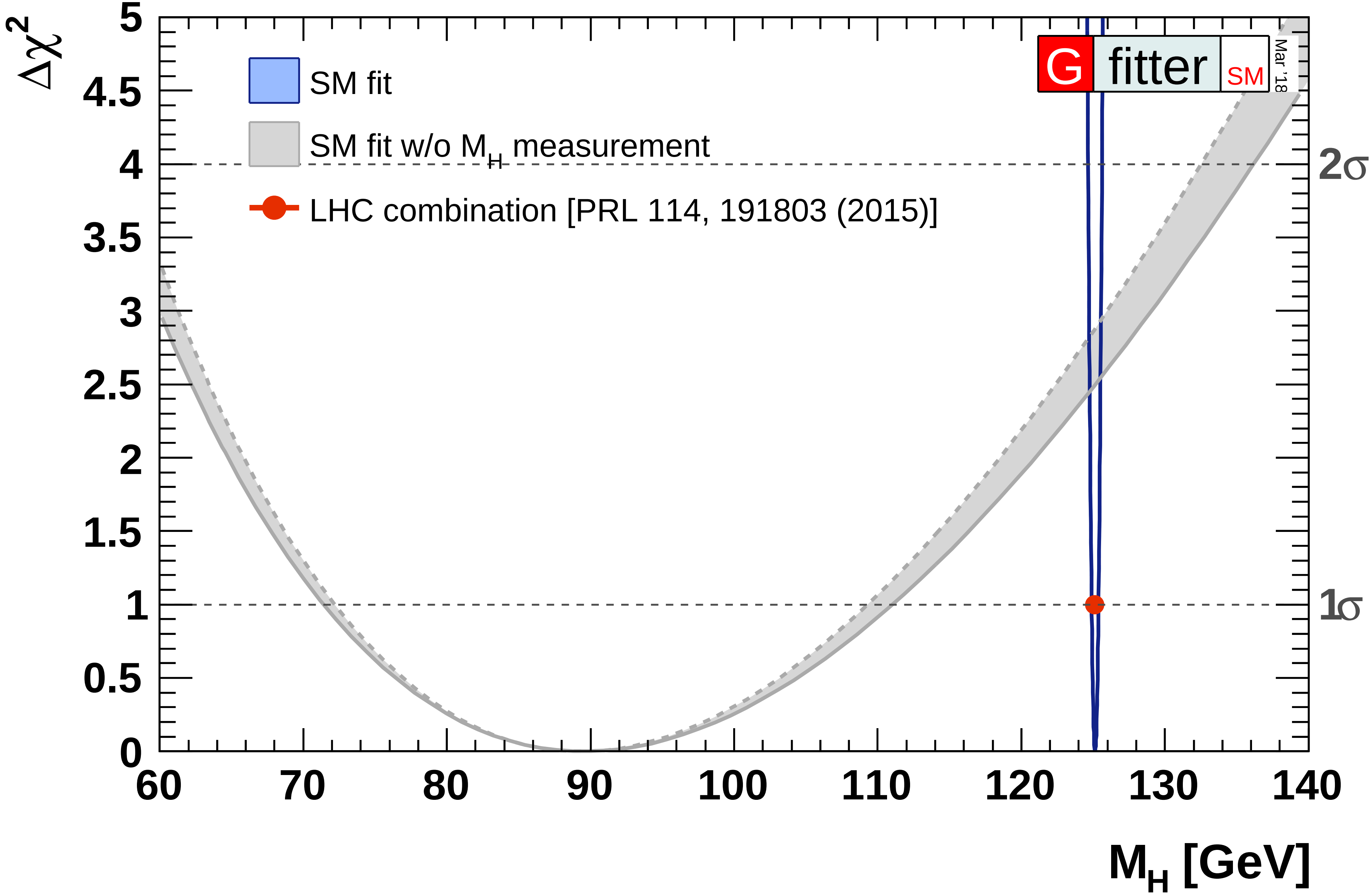
Most quantities measured with 1%–0.1% precision

A few interesting deviations:

M_W	$(\sim 1.4\sigma)$
σ^0_{had}	$(\sim 1.5\sigma)$
A_ℓ(SLD)	$(\sim 2\sigma)$
A^b_{FB}	$(\sim 2.5\sigma)$
$(g_\mu - 2)$	$(\sim 3\sigma)$

GFitter coll. '14

Current fit to M_H: [GFitter '18]



4. EWPO Future

Our future estimates:

– assume to go substantially beyond what is known now

– assume that many theorists will put many^2 hours of work into it (motivation?)

– do not assume that magically new calculational methods are invented

– are overall optimistic

⇒ they should be taken seriously!

⇒ An honest evaluation of theory uncertainties will increase the robustness of a future collider physics case!

What is needed to match the FC precision?

Compare:

1. FC (pure) experimental (anticipated) precision

2. Intrinsic uncertainties

3. Parametric uncertainties
   → taking into account the improved precision of SM paramters at the FC

Combined uncertainty:

$$\text{total} = \sqrt{\text{experimental}^2 + \text{parametric}^2} + \text{intrinsic}$$

Intrinsic uncertainties: $\Rightarrow$ can be the limiting factor!

Quantity	ILC	FCC-ee	Current intrinsic unc.		Projected unc.
M_W [MeV]	3	0.5	4	$(\alpha^3, \alpha^2\alpha_s)$	1
$\sin^2\theta^\ell_{\rm eff}$ [10^{-5}]	1.3	0.6	4.5	$(\alpha^3, \alpha^2\alpha_s)$	1.5
Γ_Z [MeV]	1	0.1	0.5	$(\alpha^3, \alpha^2\alpha_s, \alpha\alpha_s^2)$	0.2 (?)
R_b [10^{-5}]	15	6	15	$(\alpha^3, \alpha^2\alpha_s)$	7 (?)
R_l [10^{-3}]	10??	1	5	$(\alpha^3, \alpha^2\alpha_s)$	1.5 (?)

These calculations are required for the projection:

– complete $\mathcal{O}\left(\alpha\alpha_s^2\right)$ corrections
– fermionic $\mathcal{O}\left(\alpha^2\alpha_s\right)$ corrections
– double-fermionic $\mathcal{O}\left(\alpha^3\right)$ corrections
– leading four-loop corrections enhanced by the top Yukawa coupling
– the $\mathcal{O}\left(\alpha^2_{\rm bos}\right)$ corrections are done now [*Dubovyka et al. '18*]

For these calculations, qualitatively new developments of existing loop integration techniques will be required, but no conceptual paradigm shift.

Parametric uncertainties:

1. M_H: better than 50 MeV $\Rightarrow$ negligible

2. M_Z: $\sim$ 0.1 MeV with negligible theory uncertainties $\Rightarrow$ negligible

3. $\alpha_s(M_Z)$: from (mainly) R_ℓ
   $\delta\alpha_s^{\text{exp}} \sim 10^{-4}$, $\delta\alpha_s^{\text{theo}} \sim 1.5 \times 10^{-4}$

4. m_t: from threshold scan
   $\delta m_t^{\text{exp}} \sim \mathcal{O}\,(10\text{ MeV})$
   $\delta m_t^{\text{theo}} \sim 50\text{ MeV}$ (NNNLO/NNLL $\oplus$ $1S \to \overline{\text{MS}} \oplus \delta\alpha_s$)

5. m_b: from lattice calculations $\Rightarrow$ negligible for EWPO
   $\delta m_b \sim 10$ MeV (still under discussion, too optimistic?)

6. $\Delta\alpha_{\text{had}}$: BES III and Belle II: $\delta(\Delta\alpha_{\text{had}}) \sim 5 \times 10^{-5}$
   better from measurements "around the Z pole?

Uncertainty budget for m_t:

[*talk by A. Hoang '15*]

$\delta\alpha_s(M_z) = 0.001$

Msbar mass error budget (from threshold scan)

$(\delta M_t^{\mathrm{SD-low}})^{\mathrm{exp}}$	$(\delta M_t^{\mathrm{SD-low}})^{\mathrm{theo}}$	$(\delta\overline{m}_t(\overline{m}_t))^{\mathrm{conversion}}$	$(\delta\overline{m}_t(\overline{m}_t))^{\alpha_s}$
40 MeV	50 MeV	7 – 23 MeV	70 MeV

$\Rightarrow$ improvement in α_s crucial

e^+e^- collider: precision measurement:

$$R_l := \frac{\Gamma(Z \to \text{ hadrons})}{\Gamma(Z \to l^+l^-)}$$

Improvement down to $\delta^{\mathrm{exp}}\alpha_s \sim 0.001 - 0.0001$ possible?!

Note: TH uncertainty (assuming fermionic 3-loop corrections):

$\delta R_l^{\mathrm{theo}} \sim 0.0015 \Rightarrow \delta\alpha_s^{\mathrm{theo}} \sim 0.00015$ $\Rightarrow$ hard to beat . . .

M_W parametric:

parametric today: $\delta m_t = 0.9$ GeV, $\delta(\Delta\alpha_{\text{had}}) = 10^{-4}$, $\delta M_Z = 2.1$ MeV

$$\delta M_W^{\text{para},m_t} = 5.5 \text{ MeV}, \quad \delta M_W^{\text{para},\Delta\alpha_{\text{had}}} = 2 \text{ MeV}, \quad \delta M_W^{\text{para},M_Z} = 2.5 \text{ MeV}$$

parametric future: $\delta m_t^{\text{fut}} = 0.05$ GeV, $\delta(\Delta\alpha_{\text{had}})^{\text{fut}} = 5 \times 10^{-5}$, $\delta M_Z^{\text{ILC/FCC-ee}} = 1/0.1$ MeV

$$\Delta M_W^{\text{para,fut},m_t} = 0.5 \text{ MeV}, \; \Delta M_W^{\text{para,fut},\Delta\alpha_{\text{had}}} = 1 \text{ MeV}, \; \Delta M_W^{\text{para,fut},M_Z} = 0.2/0.02 \text{ MeV}$$

$\sin^2\theta_{\text{eff}}$ parametric: $[10^{-5}]$

parametric today: $\delta m_t = 0.9$ GeV, $\delta(\Delta\alpha_{\text{had}}) = 10^{-4}$, $\delta M_Z = 2.1$ MeV

$$\delta \sin^2\theta_{\text{eff}}^{\text{para},m_t} = 3.0, \quad \delta \sin^2\theta_{\text{eff}}^{\text{para},\Delta\alpha_{\text{had}}} = 3.6, \quad \delta \sin^2\theta_{\text{eff}}^{\text{para},M_Z} = 1.4$$

parametric future: $\delta m_t^{\text{fut}} = 0.05$ GeV, $\delta(\Delta\alpha_{\text{had}})^{\text{fut}} = 5 \times 10^{-5}$, $\delta M_Z^{\text{ILC/FCC-ee}} = 1/0.1$ MeV

$$\Delta \sin^2\theta_{\text{eff}}^{\text{para,fut},m_t} = 0.2, \; \Delta \sin^2\theta_{\text{eff}}^{\text{para,fut},\Delta\alpha_{\text{had}}} = 1.8, \; \Delta \sin^2\theta_{\text{eff}}^{\text{para,fut},M_Z} = 0.65/0.07$$

Additional uncertainty for M_W from threshold scan:

Not only $e^+e^- \to W^{(*)}W^{(*)}$, but $e^+e^- \to WW \to$ 4f needed

Current status:

full one-loop for $2 \to 4$ process

[*A. Denner, S. Dittmaier, M. Roth, D. Wackeroth '99-'02*]

$\Rightarrow$ extraction of M_W at the level of ~ 6 MeV

Most recent improvement:

leading 2L corrections from EFT

[*Actis, Beneke, Falgari, Schwinn '08*]

$\Rightarrow$ impact on M_W at the level of ~ 3 MeV

$\Rightarrow$ full 2L for $2 \to 4$ process not foreseeable

Potentially possible:

2L resummed higher-order terms for $e^+e^- \to WW$ and $W \to ff'$

$\Rightarrow$ extraction of M_W at ~ 1 MeV?? $\oplus$ pure exp. uncertainty of $\sim 3/0.5$ MeV

Summary of future parametric uncertainties:

Quantity	ILC	FCC-ee	future parametric unc.	Main source
M_W [MeV]	$3 \oplus 1$	$0.5 \oplus 1$	1	$\delta(\Delta\alpha_{\text{had}})$
$\sin^2\theta^{\ell}_{\text{eff}}$ [10^{-5}]	1.3	0.6	2	$\delta(\Delta\alpha_{\text{had}})$
Γ_Z [MeV]	1	0.1	0.5	
R_b [10^{-5}]	15	6	< 1	$\delta\alpha_s$

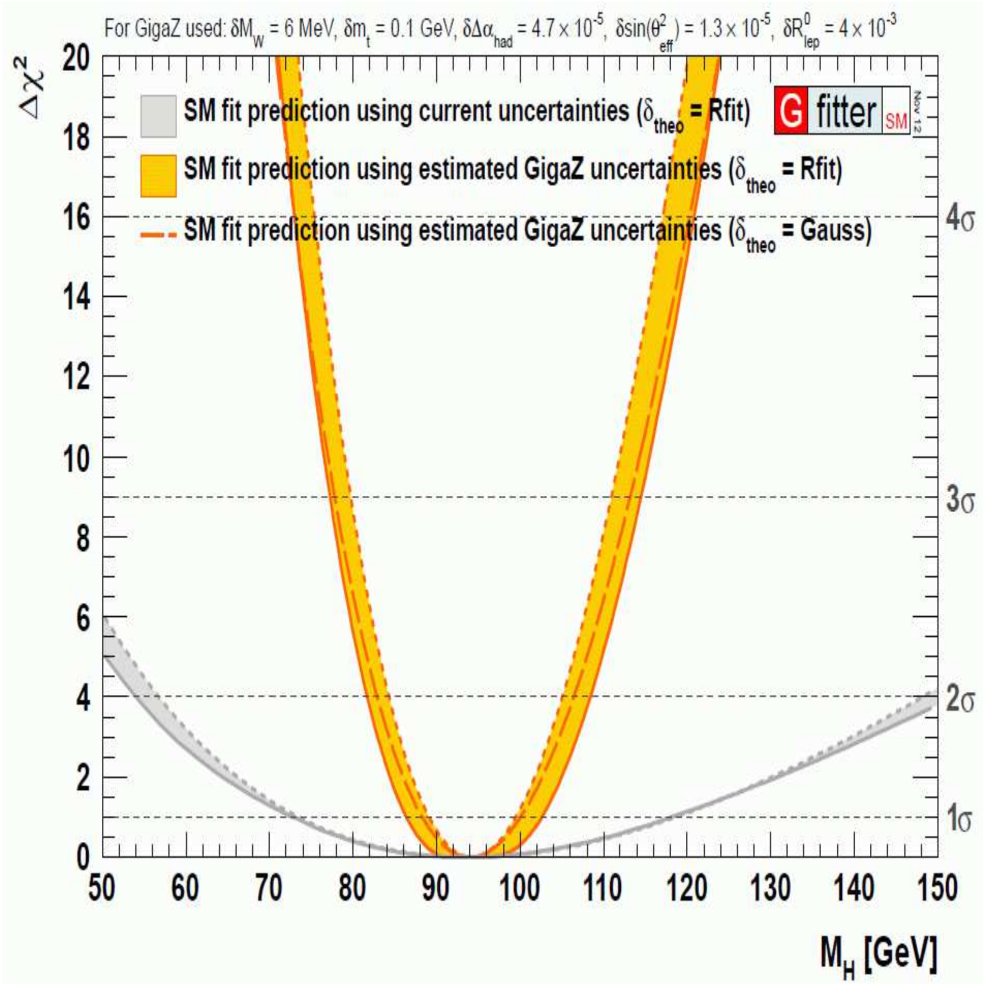
⇒ add quadratic to experimental uncertainties!

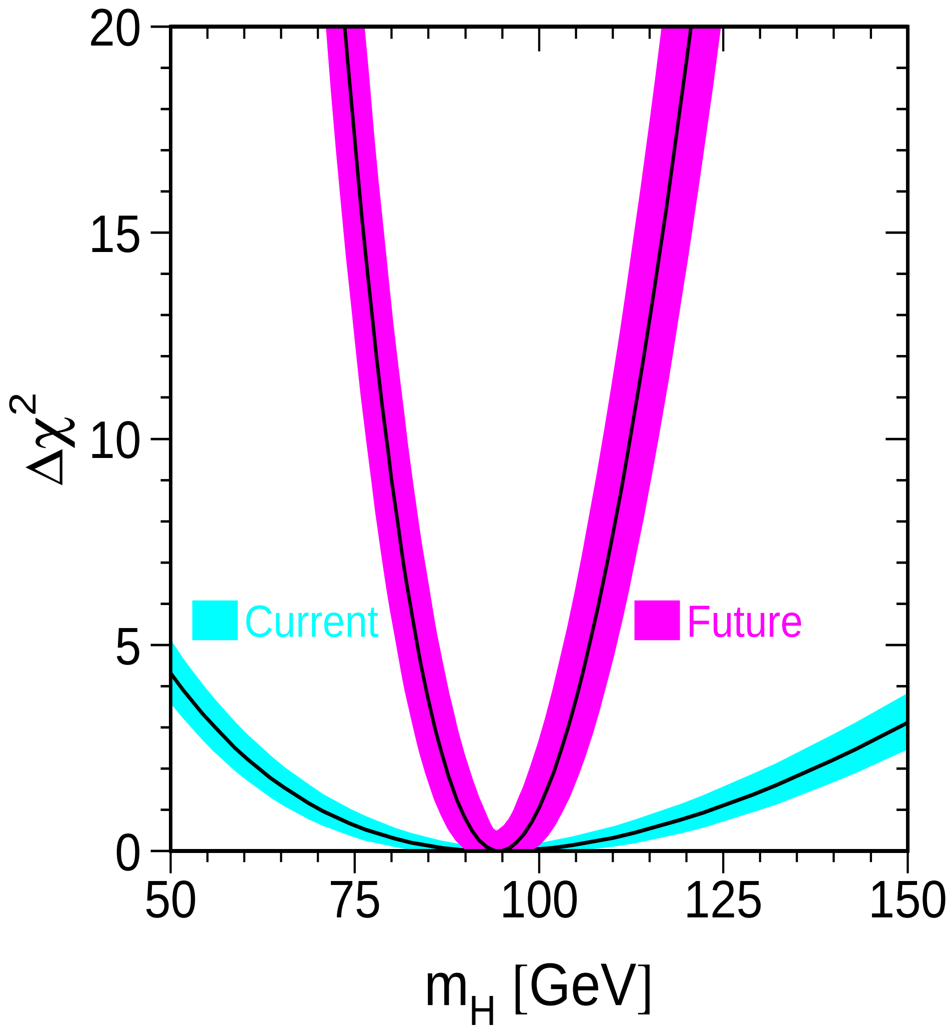
⇒ add linearly to intrinsic uncertainties!

$$\text{total} = \sqrt{\text{experimental}^2 + \text{parametric}^2} + \text{intrinsic}$$

Precise M_H test with the ILC precision:

[*GFitter '13*] [*LEPEWWG '13*]





$\Rightarrow \delta M_H^{\text{ind}} \lesssim 6$ GeV $\Leftarrow$ to be redone incl. all TH unc.

$\Rightarrow$ extremely sensitive test of SM (and BSM) possible

One more word of caution:

The above numbers have all been obtained assuming the SM as calculational framework.

The SM constitutes the model in which highest theoretical precision for the predictions of EWPO can be obtained.

We know that BSM physics must exist! (DM, gravity, . . .)

As soon as BSM physics will be discovered, an evaluation of the EWPO in any preferred BSM model will be necessary.

The corresponding theory uncertainties, both intrinsic and parametric, can then be larger (as known for the MSSM).

A dedicated theory effort (beyond the SM) would be needed in this case.

5. SM Higgs (the easy case)

Initial measurement: $\sigma \times$ BR

recoil method: $e^+e^- \to ZH$, $Z \to e^+e^-, \mu^+\mu^-$

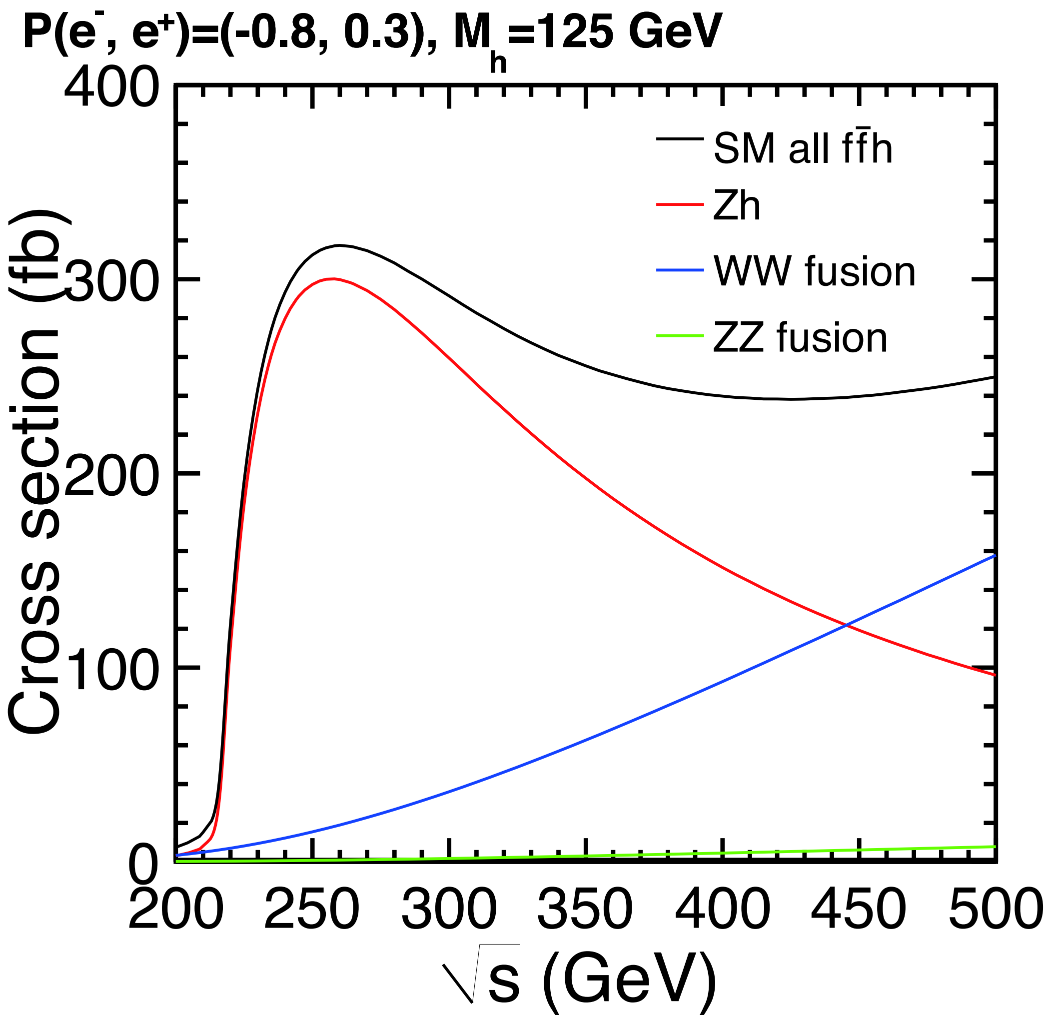
$\Rightarrow$ measurement of the Higgs production cross section

$\Rightarrow$ NO additional theoretical assumptions needed for absolute determination of partial widths

$\Rightarrow$ indirect measurement of total width

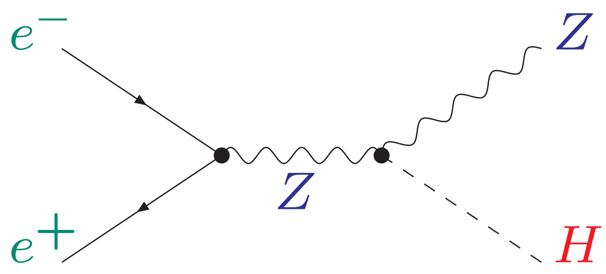
$\Rightarrow$ direct extraction of partial widths (couplings)

Higgs production cross sections:



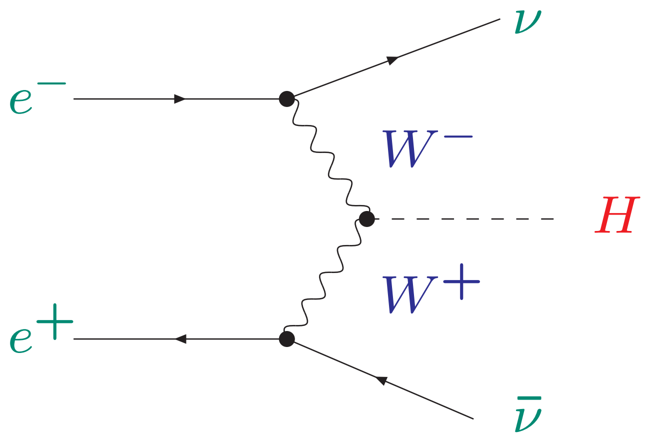
Higgs-strahlung:

$e^+e^- \to Z^* \to ZH$



weak boson fusion (WBF):

$e + e- \to \nu\bar{\nu}H$



$\sqrt{s} \sim 250$ GeV, Higgs-strahlung dominated

$e^+e^- \to ZH$:

$\delta\sigma_{HZ}^{\text{exp}} \sim 0.4\%$

full one-loop available, corrections of 5-10%

rough estimate: $\delta\sigma_{HZ}^{\text{theo}} \sim 1\%$ from missing two-loop corrections

Two-loop corrections for $2 \to 2$ can in principle be done . . .
$\mathcal{O}(\alpha_t\alpha_s)$ corrections: 1.3% *[Y. Gong, Z. Li, X. Xu, L. Yang '16]*

$\Rightarrow$ theory uncertainties sufficiently small
$\Rightarrow$ full two-loop for $2 \to 2$ should be done!

$e^+e^- \to \nu\bar{\nu}H$:

small contribution . . .

Partial two-loop calculation (with closed fermion loops)
can in principle be done . . .

$\Rightarrow$ theory uncertainties sufficiently small

Decay width theoretical uncertainties: General recipe:

[LHCHXSWG BR group '15]

1. Parametric Uncertainties: $p \pm \Delta p$

   – Evaluate partial widths and BRs with p, $p + \Delta p$, $p - \Delta p$ and take the differences w.r.t. central values

   – Upper ($p + \Delta p$) and lower ($p - \Delta p$) uncertainties summed in quadrature to obtain the Combined Parametric Uncertainty

2. Theoretical Uncertainties:

   – Calculate uncertainty for partial widths and corresponding BRs for each theoretical uncertainty

   – Combine the individual theoretical uncertainties linearly to obtain the Total Theoretical Uncertainty

   ⇒ estimate based on "what is included in the codes"!

3. Total Uncertainty:

   Linear sum of the Combined Parametric Uncertainty and the Total Theoretical Uncertainties

Intrinsic uncertainties for decay widths: [arXiv:1905.03764]

"ILC/FCC-ee" = expected precision on g^2_{Hxx} (incl. HL-LHC meas.)

Partial width	QCD	electroweak	total	future	ILC/FCC-ee
$H \to b\bar{b}$	$\sim 0.2\%$	$< 0.3\%$	$< 0.4\%$	$\sim 0.2\%$	1.4/1.5%
$H \to c\bar{c}$	$\sim 0.2\%$	$< 0.3\%$	$< 0.4\%$	$\sim 0.2\%$	2.6/2.6%
$H \to \tau^+\tau^-$	–	$< 0.3\%$	$< 0.3\%$	$< 0.1\%$	1.5/1.5%
$H \to \mu^+\mu^-$	–	$< 0.3\%$	$< 0.3\%$	$< 0.1\%$	7.8/7.8%
$H \to gg$	$\sim 3\%$	$\sim 1\%$	$\sim 3.2\%$	$\sim 1\%$	2.0/2.0%
$H \to \gamma\gamma$	$< 0.1\%$	$< 1\%$	$<1\%$	$< 1\%$	2.6/2.6%
$H \to Z\gamma$	$\lesssim 0.1\%$	$\sim 5\%$	$\sim 5\%$	$\sim 1\%$	22/22%
$H \to WW \to$ 4f	$< 0.5\%$	$< 0.3\%$	$\sim 0.5\%$	$\lesssim 0.4\%$	1.0/1.2%
$H \to ZZ \to$ 4f	$< 0.5\%$	$< 0.3\%$	$\sim 0.5\%$	$\lesssim 0.3\%$	0.5/0.5%
$\Gamma_{\rm tot}$				$\sim 0.3\%$	1.1/1.2%

$\Rightarrow$ non-negligible for $H \to WW/ZZ \to$ 4f

Future parametric uncertainties for decay widths:

decay	fut. intr.	fut. para. m_q	para. α_s	para. M_H	ILC/FCC-ee
$H \to b\bar{b}$	$\sim 0.2\%$	0.6%	$< 0.1\%$	–	1.4/1.5%
$H \to c\bar{c}$	$\sim 0.2\%$	$\sim 1\%$	$< 0.1\%$	–	2.6/2.6%
$H \to \tau^+\tau^-$	$< 0.1\%$	–	–	–	1.5/1.5%
$H \to \mu^+\mu^-$	$< 0.1\%$	–	–	–	7.8/7.8%
$H \to gg$	$\sim 1\%$		0.5%	–	2.0/2.0%
$H \to \gamma\gamma$	$< 1\%$	–	–	–	2.6/2.6%
$H \to Z\gamma$	$\sim 1\%$	–	–	$\sim 0.1\%$	22/22%
$H \to WW$	$\lesssim 0.4\%$	–	–	$\sim 0.1\%$	0.5/0.5%
$H \to ZZ$	$\lesssim 0.3\%$	–	–	$\sim 0.1\%$	0.4/0.5%
Γ_{tot}	$\sim 0.3\%$	$\sim 0.4\%$	$< 0.1\%$	$< 0.1\%$	1.1/1.2%

Γ_{tot} applies "to all" (partial cancelations . . .)
$\Rightarrow$ non-negligible in particular for $H \to WW/ZZ \to$ 4f (δm_b optimistic?)

Future theory uncertainties?

Intrinsic uncertainties:

$H \to b\bar{b}, H \to c\bar{c}$: higher-order EW corrections ??

$H \to \tau^+\tau^-, H \to \mu^+\mu^-$: higher-order EW corrections ?

$H \to gg$: improvement difficult

$H \to \gamma\gamma$: already very precise . . .

$H \to Z\gamma$: EW corrections could help . . .

$H \to WW^{(*)}, H \to ZZ^{(*)}$: already very precise, two-loop corrections unclear

$\Rightarrow$ intrinsic uncertainty can/will be sufficiently under control?!

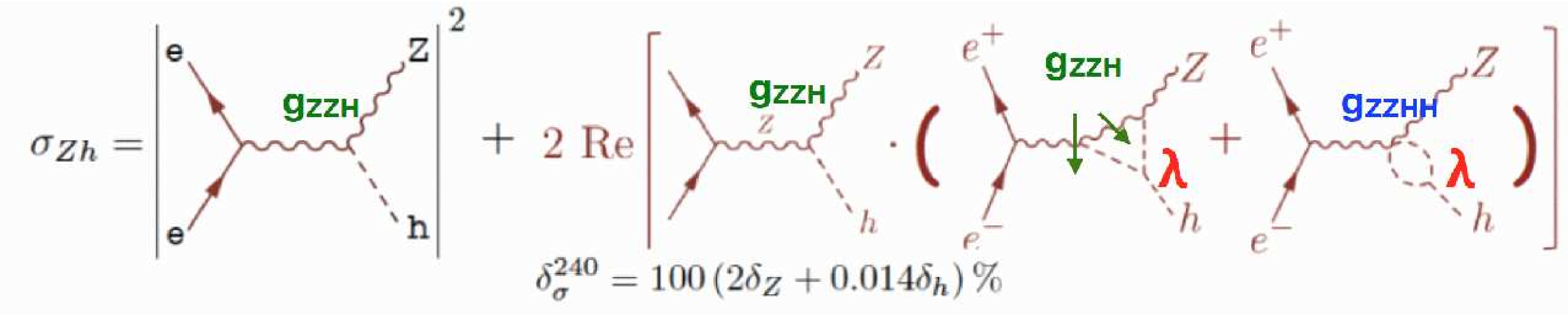
Parametric uncertainties:

– largely driven by $\delta m_b \Rightarrow$ improvement unclear (to me)
  lattice community does not seem to agree

– some improvement in α_s possible

Higgs self-coupling from loop corrections?

[arXiv:1312.3322]



⇒ sensitivity to λ_{HHH} goes down for higher $\sqrt{s}$

⇒ percent precision possible on σ_{ZH}, λ_{HHH}

⇒ indirect and model dependent measurement
(to be included in a global coupling fit - within a model)

⇒ $\mathcal{O}$ (10%) measurement of λ_{HHH} needed
to measure σ_{HZ} at the percent level!

⇒ higher $\sqrt{s}$ needed!

One word of caution:

The above numbers have all been obtained assuming the SM as calculational framework.

The SM constitutes the model in which highest theoretical precision for the predictions of EWPO can be obtained.

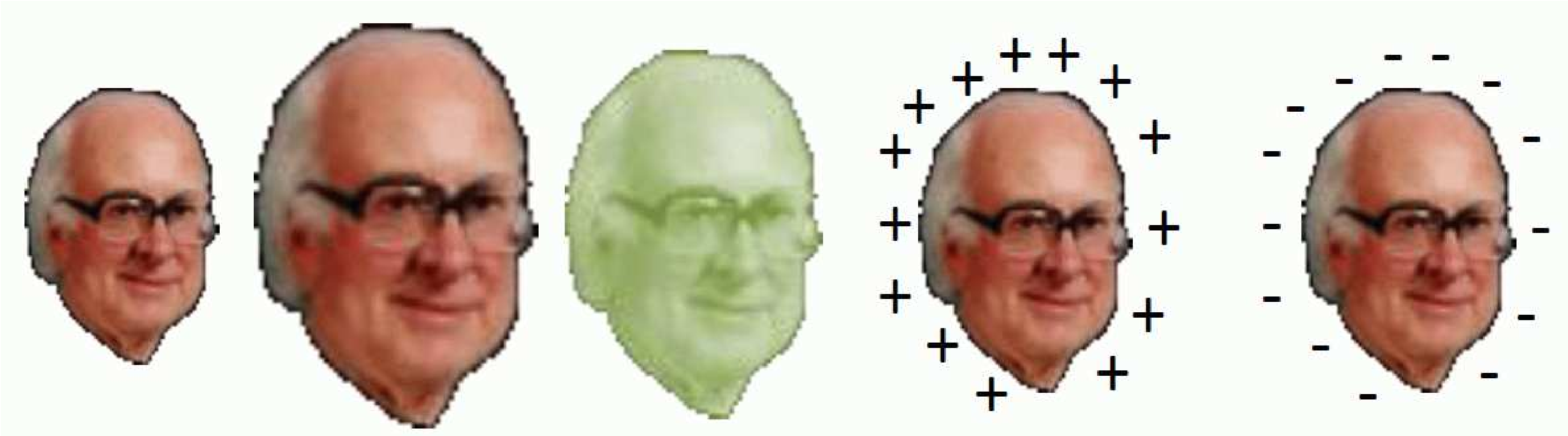
We know that BSM physics must exist! (DM, gravity, . . .)

As soon as BSM physics will be discovered, an evaluation of the Higgs predictions in any preferred BSM model will be necessary.

The corresponding theory uncertainties, both intrinsic and parametric, can then be larger (as known for the MSSM).

A dedicated theory effort (beyond the SM) would be needed in this case.

6. BSM Higgs (the difficult case)



Required precision for Higgs couplings?

MSSM example:

$$\kappa_V \approx 1 - 0.5\% \left(\frac{400 \text{ GeV}}{M_A}\right)^4$$

$$\kappa_t = \kappa_c \approx 1 - \mathcal{O}(10\%) \left(\frac{400 \text{ GeV}}{M_A}\right)^2 \cot^2\beta$$

$$\kappa_b = \kappa_\tau \approx 1 + \mathcal{O}(10\%) \left(\frac{400 \text{ GeV}}{M_A}\right)^2$$

Composite Higgs example:

$$\kappa_V \approx 1 - 3\% \left(\frac{1 \text{ TeV}}{f}\right)^2$$

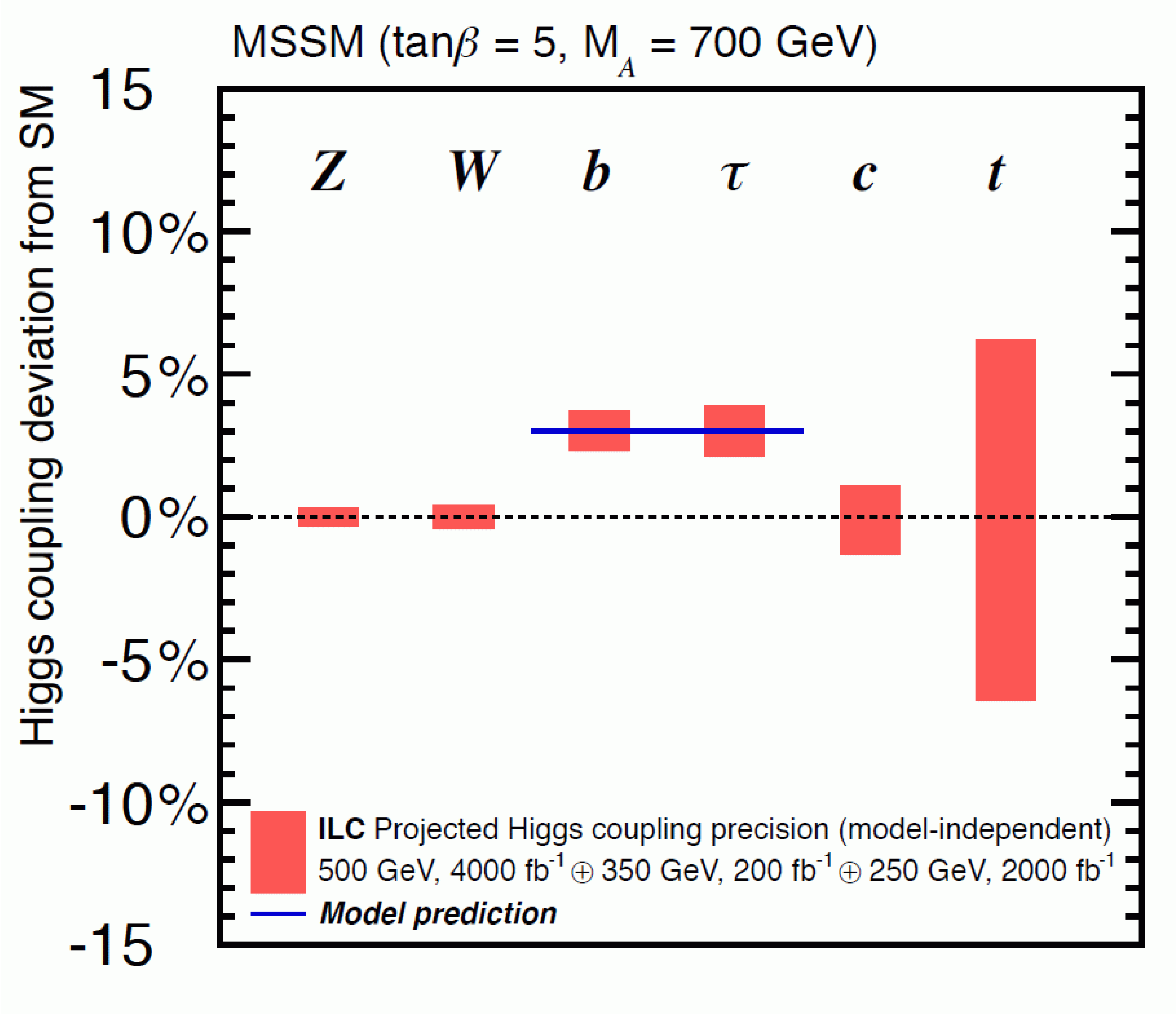
$$\kappa_F \approx 1 - (3-9)\% \left(\frac{1 \text{ TeV}}{f}\right)^2$$

$\Rightarrow$ couplings to bosons in the per mille range
$\Rightarrow$ couplings to fermions in the per cent range

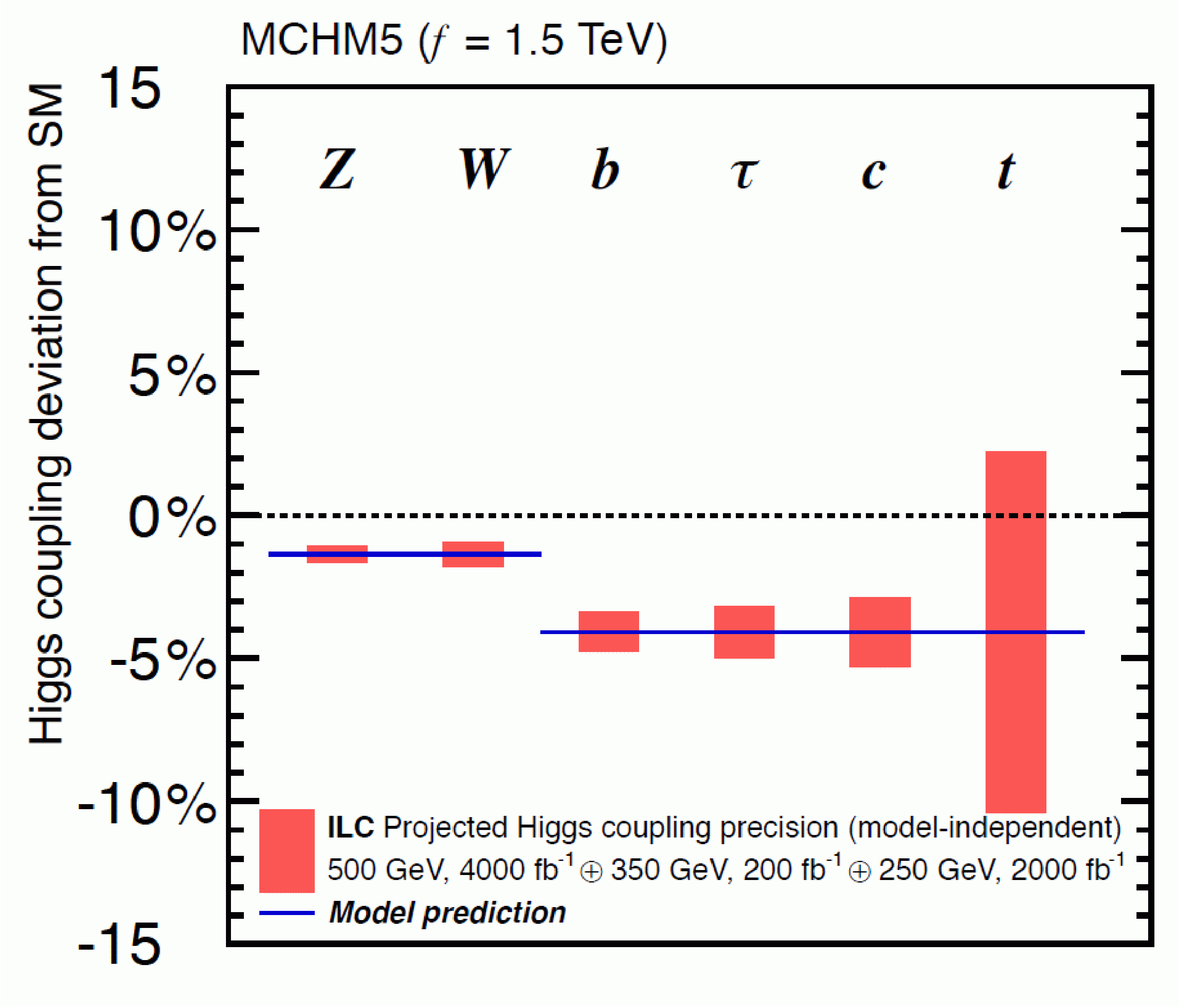
$\Rightarrow$ the more precise the better

$\Rightarrow$ theory match?

ILC precision vs. MSSM prediction:



ILC precision vs. Composite Higgs prediction:



Required precision for $\mathcal{CP}$-admixture?

$$\boxed{H = \cos\alpha\ \mathcal{CP}\text{-even} + \sin\alpha\ \mathcal{CP}\text{-odd}}$$

$$\mathcal{A}(X \to VV) = \frac{1}{v}\left(a_1 m_V^2 \varepsilon_1^* \varepsilon_2^* + a_2 f_{\mu\nu}^{*(1)} f^{*(2),\mu\nu} + a_3 f_{\mu\nu}^{*(1)} \tilde{f}^{*(2),\mu\nu}\right)$$

$$\mathcal{A}(X \to f\bar{f}) = \frac{m_f}{v}\bar{u}_2\,(b_1 + i b_2 \gamma_5)\,u_1$$

$$f_{\mathcal{CP}} = \frac{|a_3|^2\sigma_3}{\sum |a_i|^2\sigma_i}$$

Desired precision:

gauge bosons: $f_{\mathcal{CP}} \lesssim 10^{-5}$ (loop suppressed)

fermions: $f_{\mathcal{CP}} \lesssim 10^{-2}$

Taking the MSSM Higgs production as show case:

⇒ "best case" of "difficult case"!

Neutral Higgs production:

$$e^+e^- \to h_i Z,\ h_i\gamma,\ h_i h_j,\ h_i \nu\bar{\nu},\ h_i e^+e^-,\ h_i t\bar{t},\ h_i b\bar{b},\ \ldots \qquad (i,j = 1,2,3)\,.$$

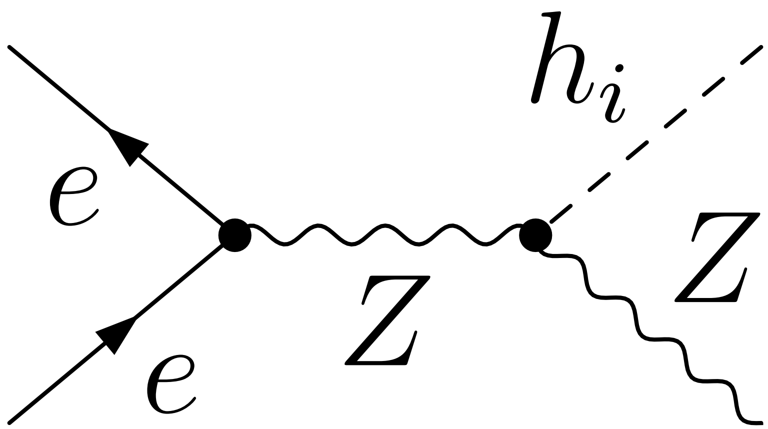
Now available in the cMSSM at the full one-loop level:
[*S.H., C. Schappacher '15*] [*F. Arco, S.H., C. Schappacher '18*]

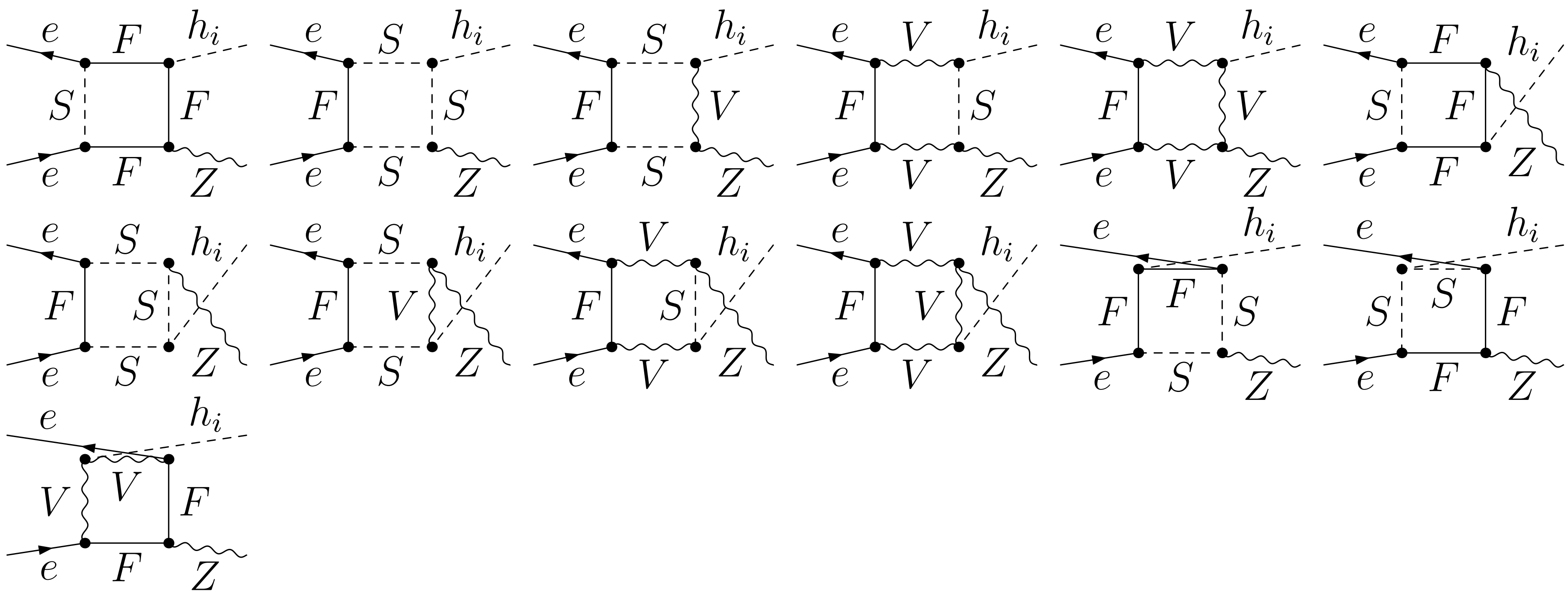
$$\sigma(e^+e^- \to h_i h_j)$$
$$\sigma(e^+e^- \to h_i Z)$$
$$\sigma(e^+e^- \to h_i \gamma)$$

⇒ no dedicated two-loop corrections available yet
⇒ as in SM full two-loop would be needed (possible . . .)

Remember: more neutral Higgs production channels
⊕ charged Higgs production channels!

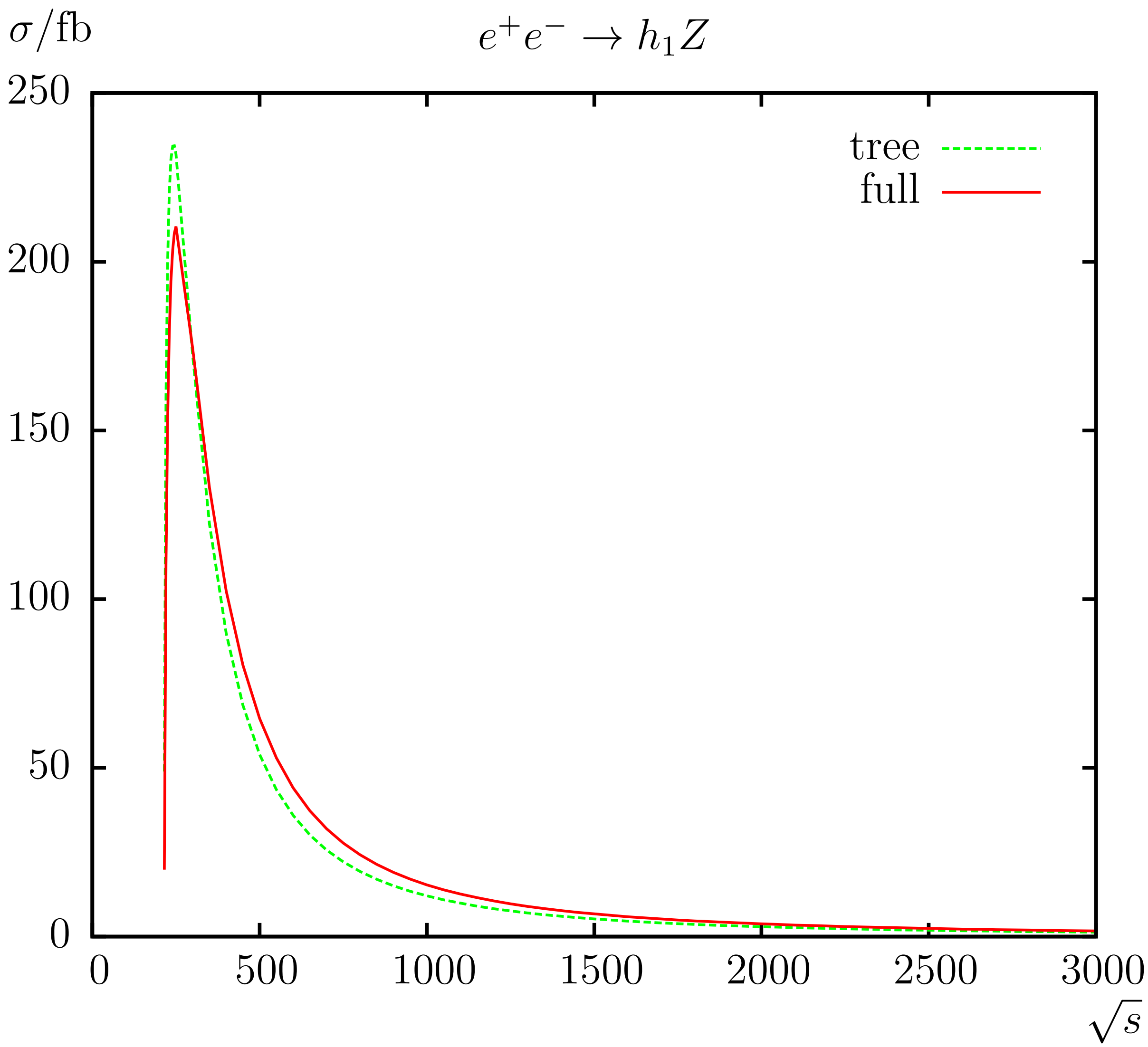
$e^+e^- \to h_i Z$:





$e^+e^- \to h_1 Z$ (CEPC $\to$ CLIC):

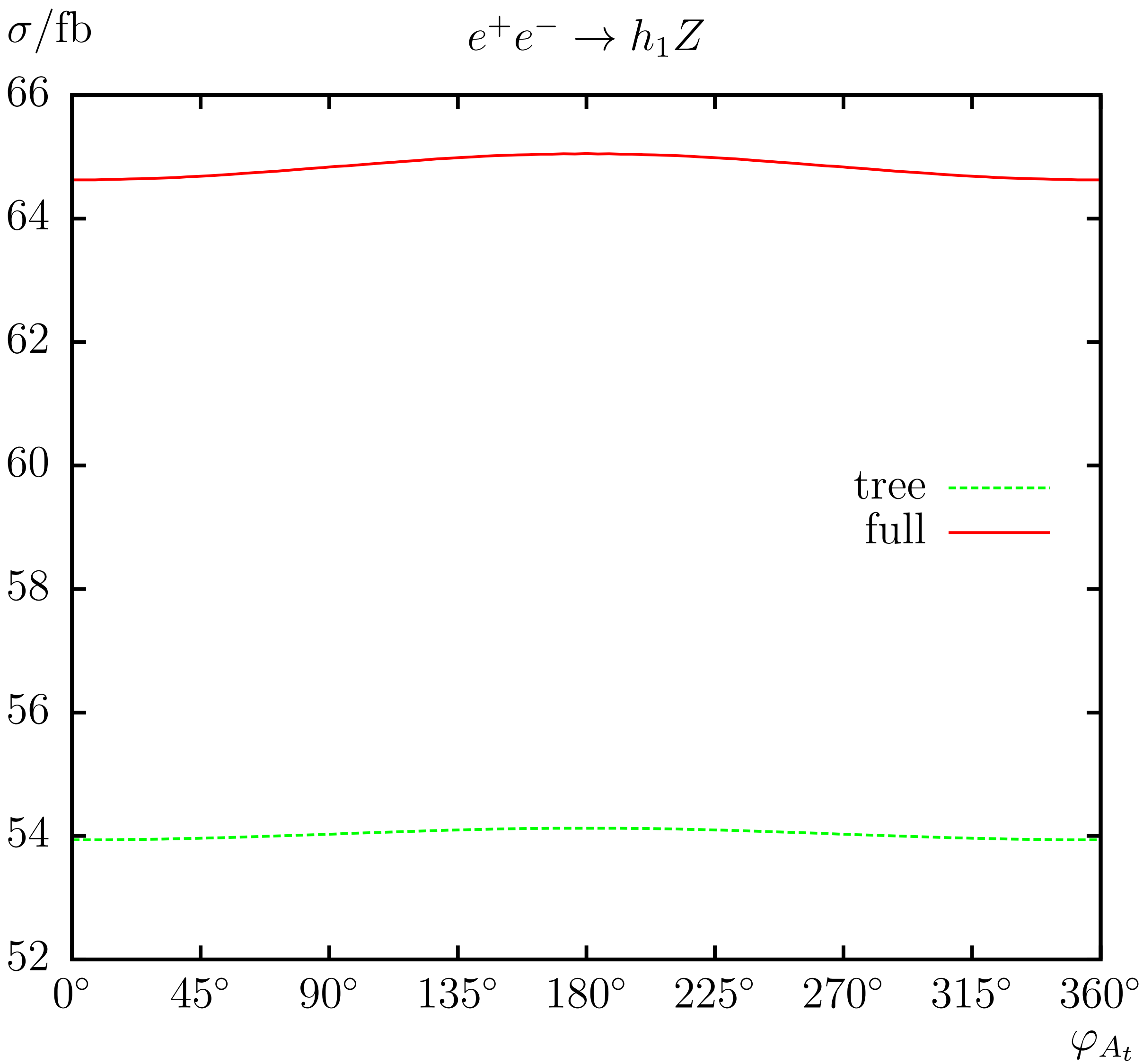
[S.H., C. Schappacher '16]



$\Rightarrow$ loop corrections crucial $\Rightarrow$ two-loop required

$e^+e^- \to h_1 Z$: effects of complex parameter

[S.H., C. Schappacher '16]



⇒ complex parameters have per-cent effects, has to be included!

Most complete implementation in cMSSM: FeynHiggs

Evaluation of all MSSM Higgs boson masses and mixing angles

- $M_{h_1}, M_{h_2}, M_{h_3}, M_{H^\pm}$, α_{eff}, $\mathbf{Z}_{ij}$, $\mathbf{U}_{ij}$, . . .

Evaluation of all neutral MSSM Higgs boson decay channels (so far)

- total decay width Γ_{tot}
- BR$(h_i \to f\bar{f})$: decay to SM fermions: full 1L, running m_q at 3L, $\mathbf{Z}_{ij}$
- BR$(h_i \to Z^{(*)}Z^{(*)}, W^{(*)}W^{(*)})$: decay to massive SM gauge bosons: Prophecy4f $\oplus$ coupling factors, $\mathbf{U}_{ij}$
- BR$(h_i \to \gamma\gamma, gg)$: decay to massless SM gauge bosons: NLO QCD, gg: NNLO, NNLL from SM, $\mathbf{U}_{ij}$
- BR$(h_i \to h_j Z^{(*)}, h_j h_k)$: decay to gauge and Higgs bosons: $h_j Z^{(*)}$: $\mathbf{U}_{ij}$, $h_j h_k$: full 1L, log-resum, $\mathbf{Z}_{ij}$
- BR$(h_i \to \tilde{f}_i \tilde{f}_j)$: decay to sfermions: $\mathbf{U}_{ij}$
- BR$(h_i \to \tilde{\chi}^\pm_i \tilde{\chi}^\mp_j, \tilde{\chi}^0_i \tilde{\chi}^0_j)$: decay to charginos, neutralinos: $\mathbf{U}_{ij}$

Overall (N)MSSM Higgs decay intrinsic uncertainty estimates

[*F. Domingo, S.H., S. Passehr, G. Weiglein '18*]

- $h_i \to q\bar{q}$: SM-like: SM NNLO QCD, EW NNLO, SUSY 2L: $\sim 5\%$
  heavy: as SM-like, Sudakov logs: $\sim 5 - 10\%$
- $h_i \to \ell\bar{\ell}$: SM-like: $\lesssim 1\%$
  heavy: Sudakov logs for very heavy Higgses $\lesssim 10\%$
- $h_i \to WW^{(*)}, ZZ^{(*)}$: SM-like: $\lesssim 1\%$
  heavy: missing 2L (very small width): $\lesssim 50\%$
- $h_i \to \gamma\gamma, gg, \gamma Z$: $\gamma\gamma$: NNLO QCD, EW: $\lesssim 4\%$
  gg: NNLO QCD, EW: $\lesssim 4\%$
  γZ: NLO: $\sim 5\%$
- $h_i \to$ SUSY SUSY: [*S.H., C. Schappacher '14-'16*]
  1L effects $10 - 20\%$, 2L?
- all decays: $\mathbf{U}_{ij}$, $\mathbf{Z}_{ij}$: few %, effects close to threshold?

$\Rightarrow$ approaching e^+e^- precision for SM-like Higgs
(not for heavy Higgses yet)

7. Conclusions

- High anticipated experimental precision for Higgs/EWPO at future e^+e^- colliders
- Crucial: theory uncertainties: intrinsic and parametric

$$\text{total} = \sqrt{\text{experimental}^2 + \text{parametric}^2} + \text{intrinsic}$$

- We give (realistic/optimistic) estimates for future intrinsic and parametric uncertainties
- EWPO: intrinsic unc. larger than anticipated experimental unc.
  parametric unc. often larger than experimental uncertainties
  $\Rightarrow$ particularly true for M_W and $\sin^2\theta_{\text{eff}}$
- SM Higgs: cross section can be under control with full $2 \to 2$ calc.
  intrinsic unc. can be relevant for $H \to WW/ZZ \to$ 4f
  parametric unc. can be relevant, in particular for $H \to WW/ZZ \to$ 4f
- BSM Higgs: deviations in per-cent range expected
  $\Rightarrow$ MSSM is "best case" of "difficult case"!
  cross sections can be under control with full $2 \to 2$ calc.
  intrinsic unc. approaching e^+e^- precision for SM-like Higgs
  parametric unc. at least as large as in SM
- Uncertainties should be taken into account by experimental analyses!


Further Questions?