What is naturalness?

the fairly biased view of a particle physicist

based on

P Athron, C Balázs, B Farmer, A Fowlie, D Harries, D Kim	JHEP 1710 (2017) 160
A Fowlie, C Balázs, G White, L Marzola, M Raidal	JHEP 1608 (2016) 100
D Kim, P Athron, C Balázs, B Farmer, E Hutchison	PRD90 (2014) 5 055008
C Balázs, A Buckley, D Carter, B Farmer, M White	EPJC73 (2013) 2563

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Outline

What is naturalness?

How to quantify fine tuning?

What is the meaning of a fine tuning measure?

"Everything is natural: if it weren't, it wouldn't be."

Mary Catherine Bateson

M.C. Bateson On the Naturalness of Things
"How Things Are: A Science Toolkit for the Mind"
ed. J. Brockman and K. Matson

“Everything is natural: if it weren’t, it wouldn’t be.”

Mary Catherine Bateson

Consequently, if a phenomenon appears unnatural
then there’s a problem with our understanding
of the phenomenon *or* naturalness.

M.C. Bateson On the Naturalness of Things
“How Things Are: A Science Toolkit for the Mind”
ed. J. Brockman and K. Matson

Tenish years ago my notion of naturalness was…

Naturalness is a theoretical prejudice.

Naturalness is a subjective principle.
(Compared to e.g. the gauge principle.)

Naturalness is qualitative and unquantifiable.

Consequently, naturalness shouldn’t be used in physics.

In particular, I didn't understand the Barbieri-Ellis-Giudice measure:

$$\frac{\partial(electroweak\ observable)}{\partial(theory\ parameter)}$$

* For the observable people use m_Z, m_h, v, etc. Why?
* For the parameter people use μ, M_0, $M_{1/2}$, etc. Why?
* The form changes: $\frac{\partial m_Z}{\partial \mu}$, $\frac{\partial m_Z^2}{\partial \mu^2}$, $\frac{\partial \log(m_Z)}{\partial \log(\mu)}$, etc. Why?
* For more parameters: Max? Quadrature? Another combination? Why?
* What does $\frac{\partial m_Z}{\partial \mu}$ have to do with the other fine tuning measures?
* Beyond MSSM, NMSSM, SUSY and EW fine tuning?
* How much fine tuning is too much?

Since then, I learned a few things...

Physics is naturally divided across length scales

universe	$10^{+30}m$	cosmology
galaxy	$10^{+20}m$	astrophysics
star	$10^{+10}m$	solar physics
rock	$10^{0}\ m$	solid state physics
atom	$10^{-10}m$	atomic physics
top quark	$10^{-20}m$	particle physics
Planck cell	$10^{-30}m$	quantum gravity

Naturalness

Physical phenomena characterized by
disparate (energy or length) scales are separated:

Their governing laws can be understood
largely independently from each other.

They obey the naturalness principle.

Natural sciences exists as we know them because of naturalness.

Example 1

The orbit of the moon is not affected by sub-nuclear physics.

Newton’s gravity is natural.

Example 2

A doctor may cure a patient without being an expert in cosmology.

Medical science (we may say) is natural.

Example 3

In the effective description of the Standard Model of particles
the Higgs boson mass acquires a dependence on the Planck scale.

The Higgs and Planck masses are many orders of magnitudes apart.

The Standard Model violates naturalness.

Example 3

In the effective description of the Standard Model

the Higgs boson mass is given by

$$m_h^2 = -\mu^2 + \Lambda^2$$

If Λ is the Planck scale then Λ^2 and μ^2 have to be very finely tuned to obtain the observed Higgs mass.

Naturalness can be quantified in terms of fine tuning.

But how can we quantify fine tuning?

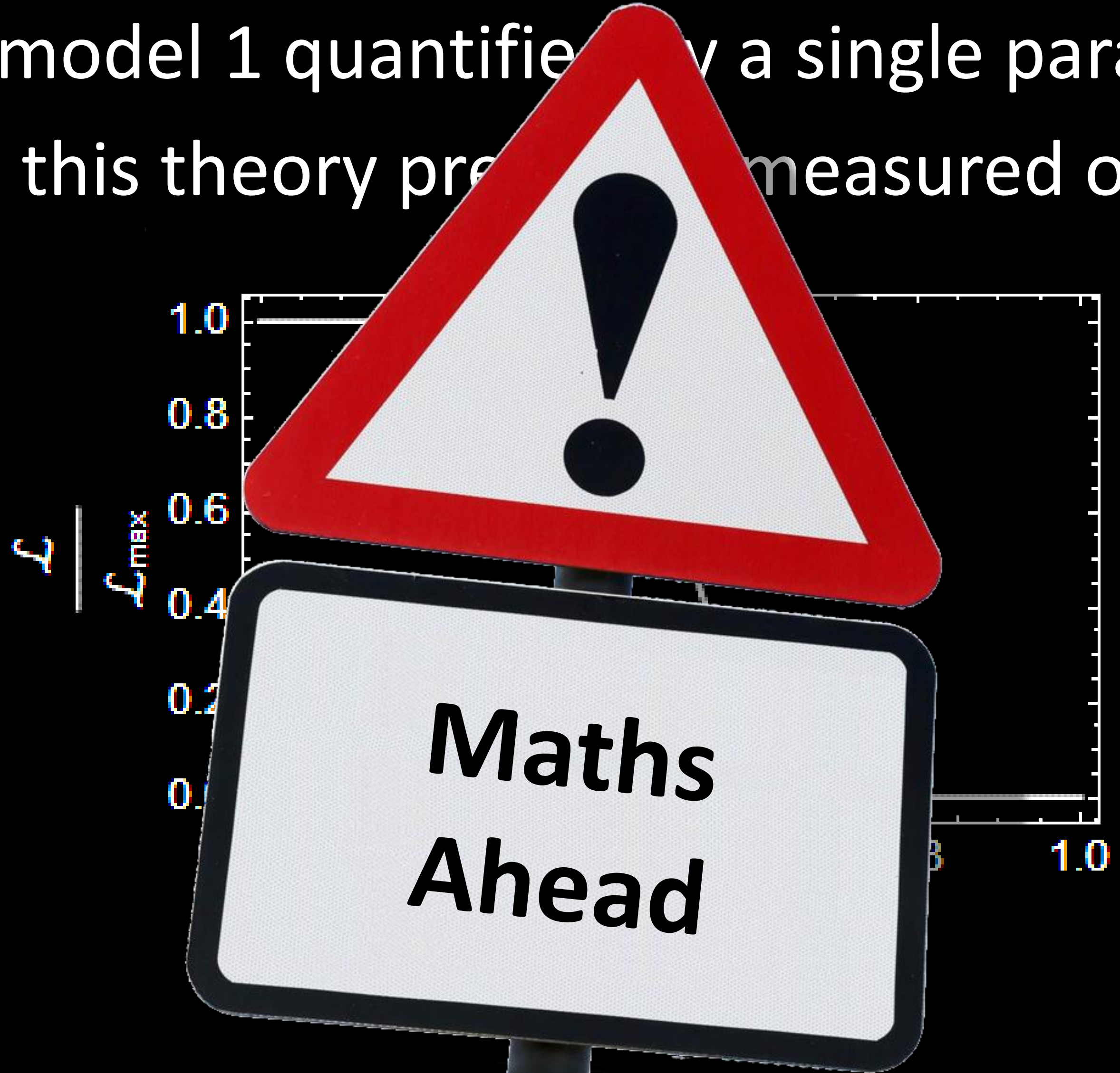
Many different means exists for quantifying fine tuning.

I only concentrate on one of them.

I’m convinced that many of them are essentially equivalent.

Consider model 1 quantifie y a single parameter p.

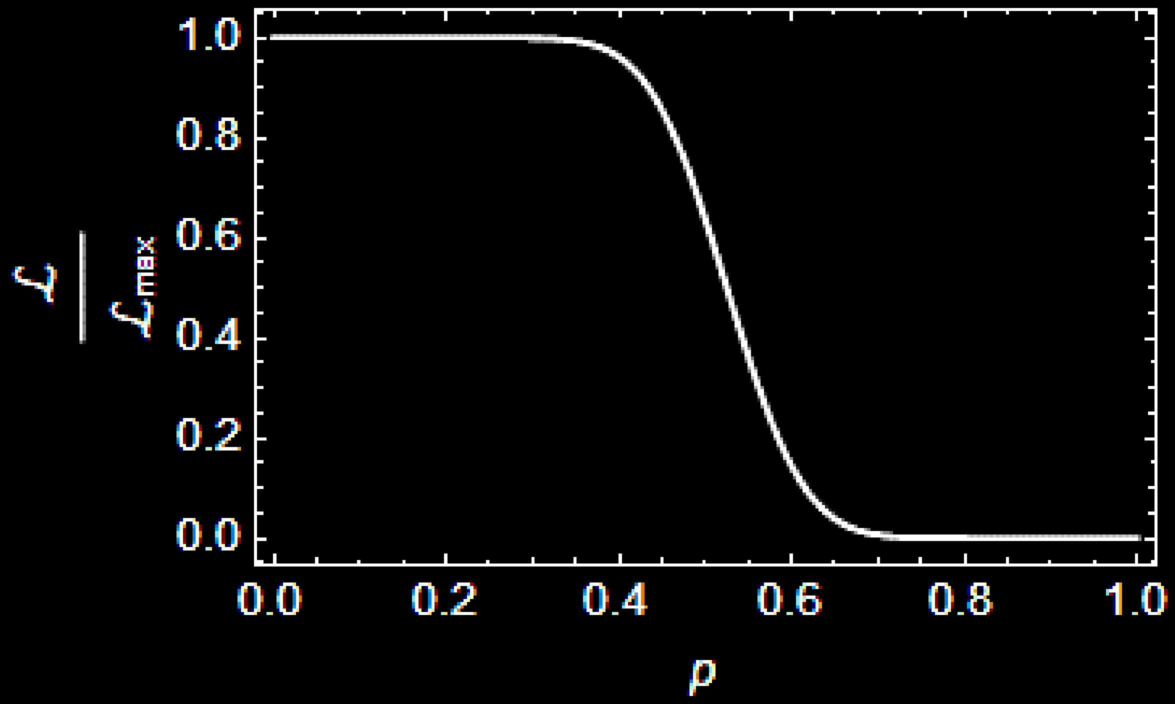
Assume that this theory pre measured observable o.



$\mathcal{L}(o, p)$ is the likelihood of the odel predicting the observable.

Consider model 1 quantified by a single parameter p.

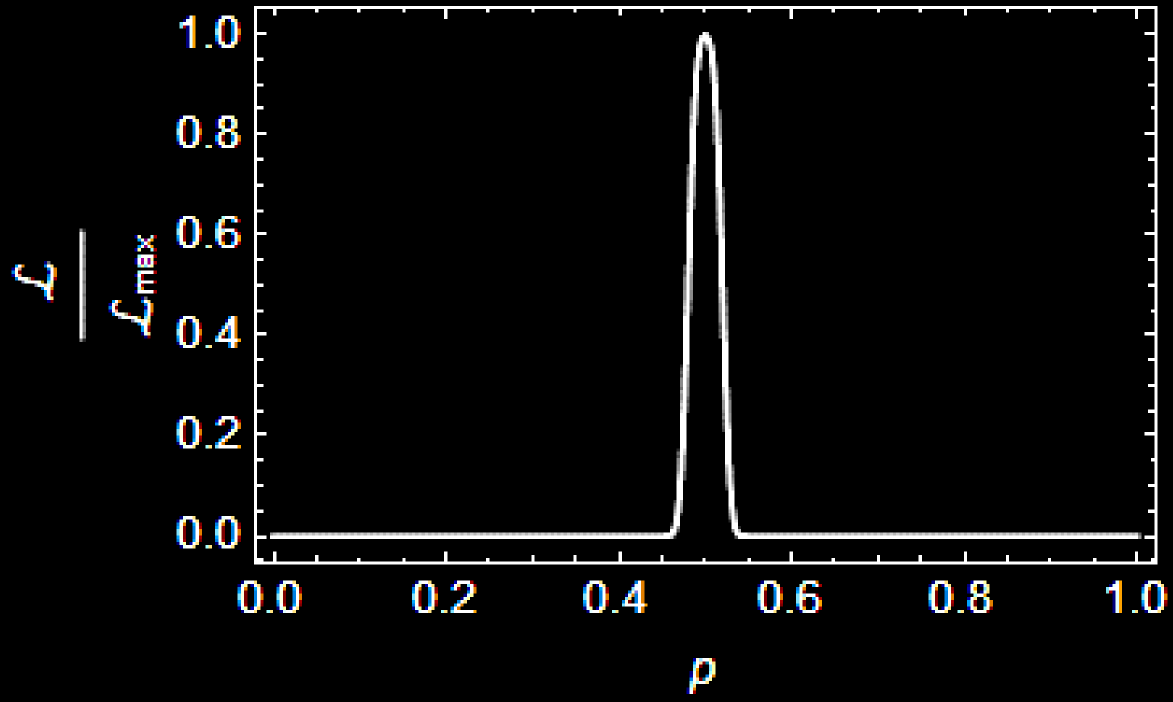
Assume that this theory predicts a measured observable o.



$\mathcal{L}(o, p)$ is the likelihood of the model predicting the observable.

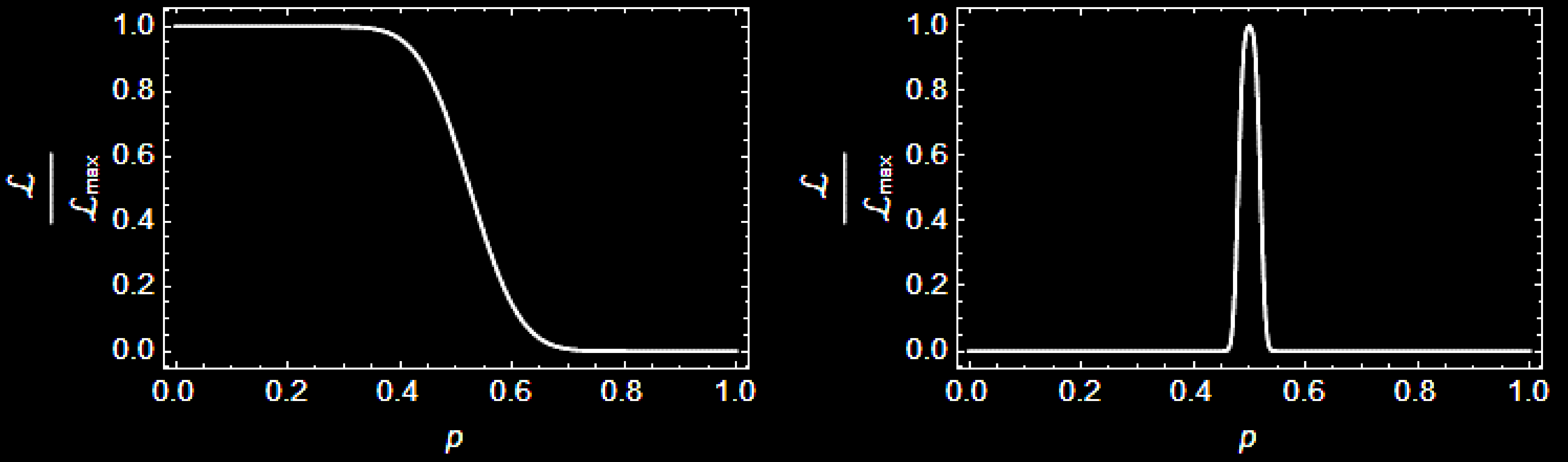
Consider model 2 quantified by the same parameter p.

Assume that this theory also predicts the observable o.



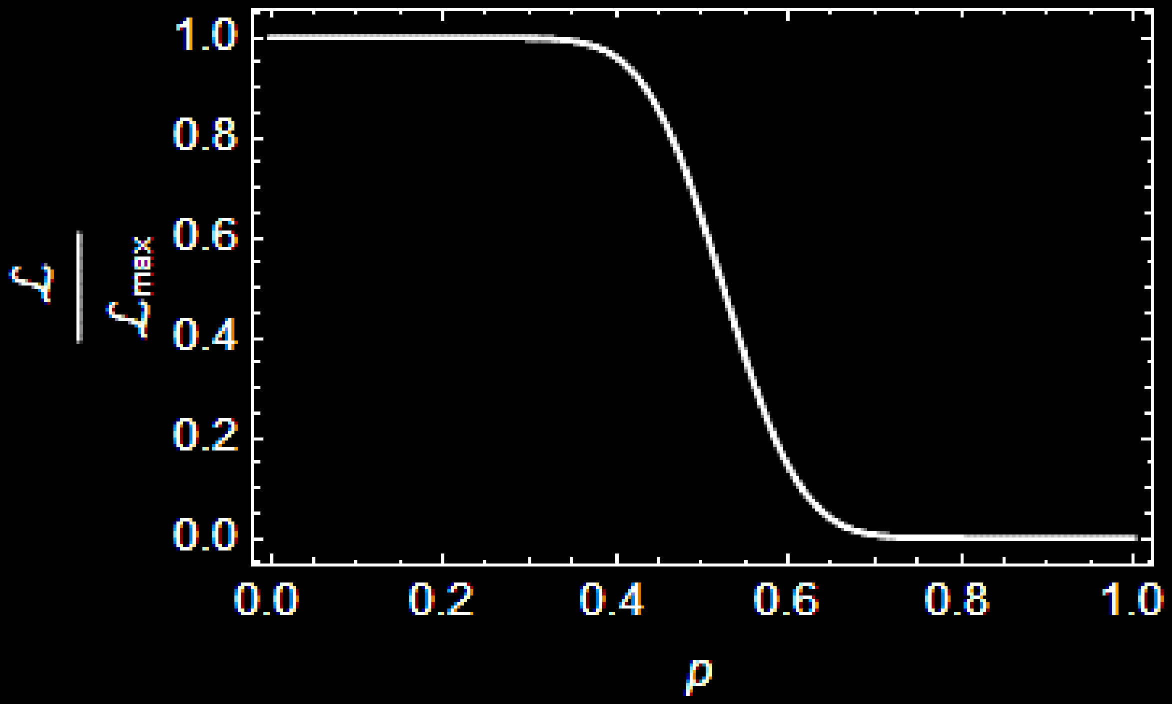
$\mathcal{L}(o, p)$ is the likelihood of the model predicting the observable.

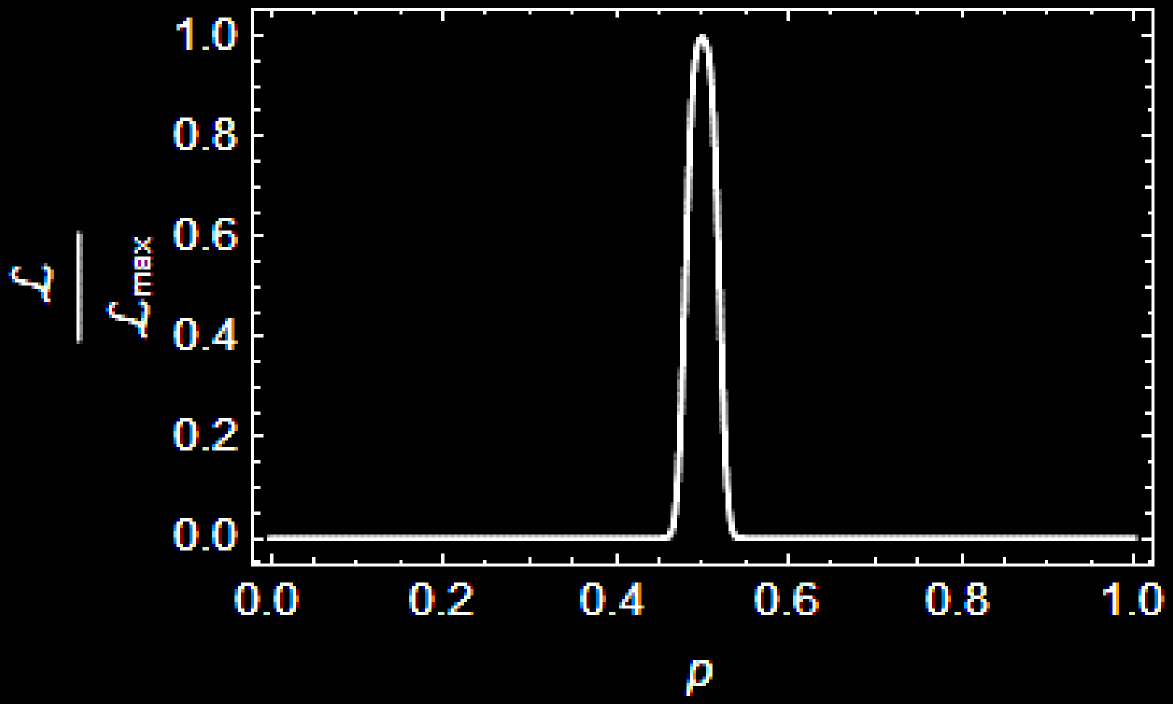
Which model looks less fine-tuned?



If you said 'the first one', why?

Why does the first model look less fine tuned?

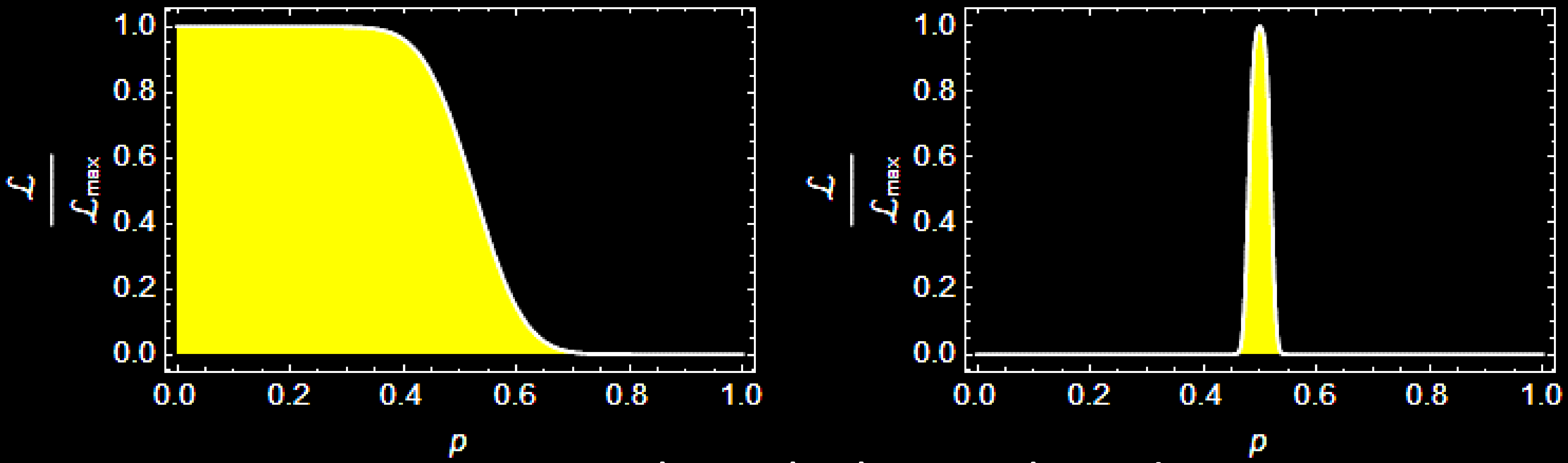




Particle physicist's answer: because $\mathcal{L}(o, p)$ varies milder with p.

But here I'll go with the astrophysicist's answer.

Why does the first model look less fine tuned?



Because it has a higher evidence!

(Bayesians: assume flat prior, or plot the posterior :)

What does the evidence have to do with fine tuning?

Baye ce:

on p before data d

data d upo

Bayesian evidence:

$$\mathcal{E} = \int \mathcal{L}(o,p)\,\pi(p)\,dp$$

probability distribution of p
before observation o

Bayesian evidence:

$$\mathcal{E} = \int \mathcal{L}(o, p)\, \pi(p)\, dp$$

observation o

updating prior info π

the Bayesian evidence is …

$$\mathcal{E} = \int \mathcal{L}(o,p)\,\pi(p)\,dp$$

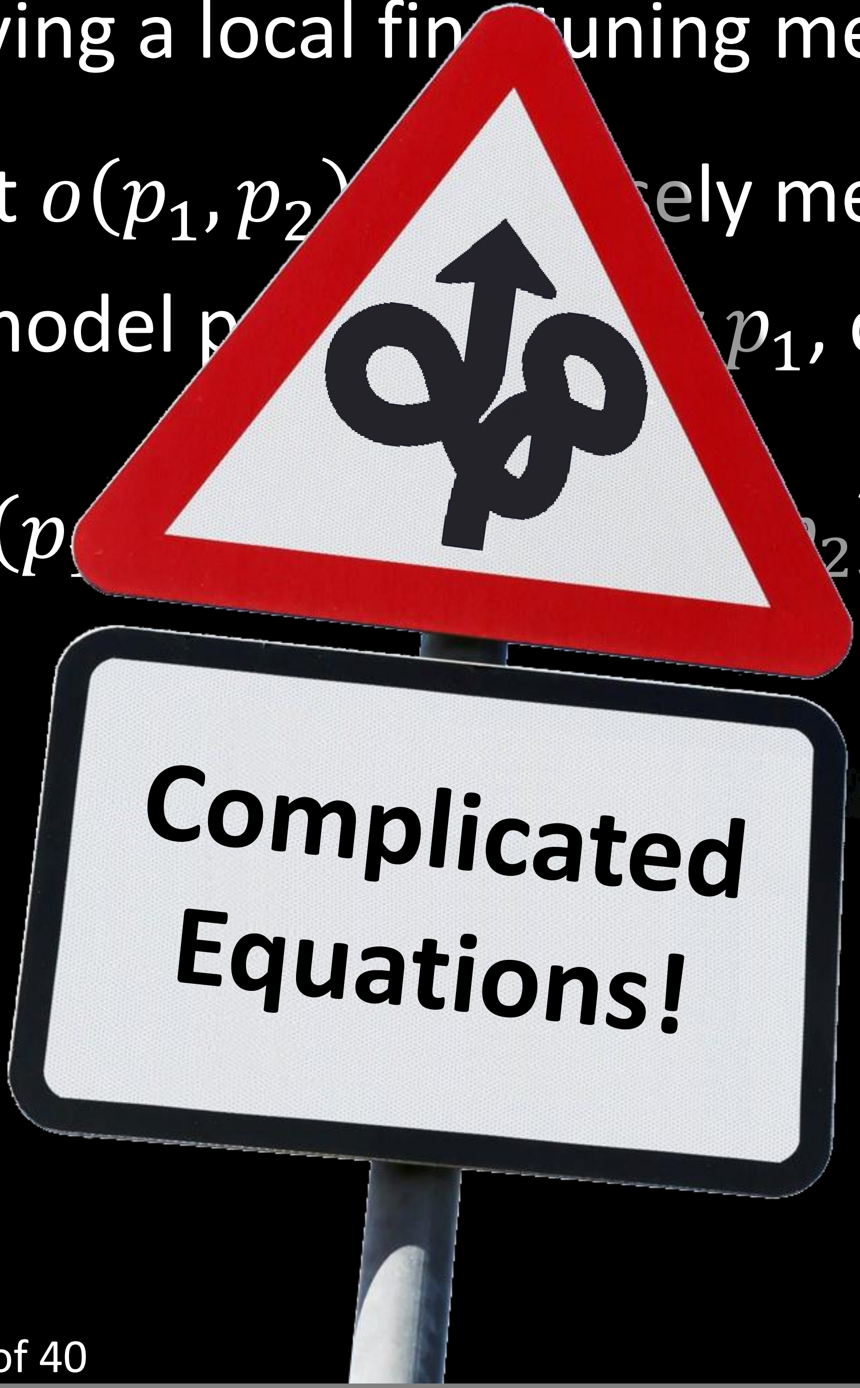
… the plausibility that a hypothesis reproduces an observation

… proportional to global fine tuning (will see this quantitatively in a moment) deriving a local fin … uning measure

assume that $o(p_1, p_2$ … ely measured: o_{exp}

then one model p … p_1, can be fixed:

$\mathcal{E} = \int \delta\big(o(p$ … $_2)\,\frac{\partial p_1}{\partial o}\, do\, dp_2$



deriving a local fine tuning measure

assume that $o(p_1, p_2)$ is precisely measured: o_{exp}

then one model parameter, say p_1, can be fixed:

$$\mathcal{E} = \int \delta\big(o(p_1, p_2) - o_{exp}\big)\, \pi(p_1, p_2) \frac{\partial p_1}{\partial o} do\, dp_2$$

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$$\mathcal{E} = \int \pi(p_{1,exp}, p_2) \left.\frac{\partial p_1}{\partial o}\right|_{o_{exp}} dp_2$$

the prior of p_2 got modified by $\frac{\partial p_1}{\partial o}$

hence the name naturalness prior

deriving a local fine tuning measure

assume that $o(p_1, p_2)$ is precisely measured: o_{exp}

then one model parameter, say p_1, can be fixed:

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$\frac{\partial p_1}{\partial o}$ is the inverse of Barbieri-Ellis-Giudice fine tuning measure !

deriving a local fine tuning measure

assume that $o(p_1, p_2)$ is precisely measured: o_{exp}

then one model parameter, say p_1, can be fixed:

$$\mathcal{E} = \int \delta\big(o(p_1, p_2) - o_{exp}\big)\, \pi(p_1, p_2) \frac{\partial p_1}{\partial o}\, do\, dp_2$$

$$\mathcal{E} = \int \pi(p_{1,exp}, p_2) \left.\frac{\partial p_1}{\partial o}\right|_{o_{exp}} dp_2$$

$\frac{\partial p_1}{\partial o}$ measures fine tuning at every value of p_2

(o depends on p_2, and so does $\frac{\partial p_1}{\partial o}$)

properties of the fine tuning measure

in the context of the MSSM: $p_1 = \mu$, $o = m_Z$, $p_2 =$ rest of para.

$$\mathcal{E} = \int \pi(\mu_{exp}, p_2) \left. \frac{\partial \mu}{\partial m_Z} \right|_{m_{Z,exp}} dp_2$$

* when the fine tuning is large, $\frac{\partial \mu}{\partial m_Z}$ is small, $\mathcal{E}$ is suppressed, so

$\frac{\partial \mu}{\partial m_Z}$ measures the local fine tuning, and

$\mathcal{E}$ measures the global fine tuning

properties of the fine tuning measure

in the context of the MSSM: $p_1 = \mu$, $o = m_Z$, $p_2 =$ rest of para.

$$\mathcal{E} = \int \pi(\mu_{exp}, p_2) \left.\frac{\partial \mu}{\partial m_Z}\right|_{m_{Z,exp}} dp_2$$

* $\frac{\partial \mu}{\partial m_Z}$ measures fine tuning because μ was traded for m_Z

consequently, $\frac{\partial m_Z}{\partial M_0}, \frac{\partial m_Z}{\partial M_{1/2}}$, etc. are not qunatifying fine tuning

* $\frac{\partial \mu}{\partial m_Z}$ depends on $M_0, M_{1/2}, \ldots$ so it measures local fine tuning

Constrained Minimal Supersymmetric Model

the set of parameters $p = \{\mu, y_t, B_0\}$

are traded for observables $o = \{m_Z, m_t, tan\beta\}$

invoking the determinant

$$\frac{\partial o}{\partial p} = \begin{vmatrix} \frac{\partial m_Z}{\partial \mu} & \frac{\partial m_t}{\partial \mu} & \frac{\partial tan\beta}{\partial \mu} \\ \frac{\partial m_Z}{\partial y_t} & \frac{\partial m_t}{\partial y_t} & \frac{\partial tan\beta}{\partial y_t} \\ \frac{\partial m_Z}{\partial B_0} & \frac{\partial m_t}{\partial B_0} & \frac{\partial tan\beta}{\partial B_0} \end{vmatrix}$$

What did I learn about the Barbieri-Ellis-Giudice measure?

$$\frac{\partial(electroweak\ observable)}{\partial(theory\ parameter)}$$

* Observable: m_Z, m_h, v, etc. A subjective choice. Changes $\frac{\partial o}{\partial p}$ but not $\mathcal{E}$!
* Parameter: μ, M_0, $M_{1/2}$, etc. A subjective choice. Only eliminated para measure FT!
* The form: $\frac{\partial m_Z}{\partial \mu}$, $\frac{\partial m_Z^2}{\partial \mu^2}$, $\frac{\partial \log(m_Z)}{\partial \log(\mu)}$, etc. Depends on assumed prior, a bit subjective!
* Max? Quadrature? Another combination? Determinant of the Jacobian!
* What does $\frac{\partial m_Z}{\partial \mu}$ have to do with the other measures? Unexplained in this talk.
* Beyond MSSM, SUSY, EW? Easy: Bayesian methods apply to any models & FTs.

- How much fine tuning is too much? $\frac{\partial o}{\partial p}$ is a probability density, $\mathcal{E}$ is relative!

  Consequently, one should only use them comparing *relative* fine tunings.

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The gross amount of fine tuning is encoded in the model
via the dependence of the observables on the parameters.

The amount of this fine tuning is fixed by the above relations
and quantified by the Bayesian evidence.

This is true for all kinds of fine tunings:
electroweak, dark matter, cosmological, etc.

What did I learn about the Barbieri-Ellis-Giudice measure?

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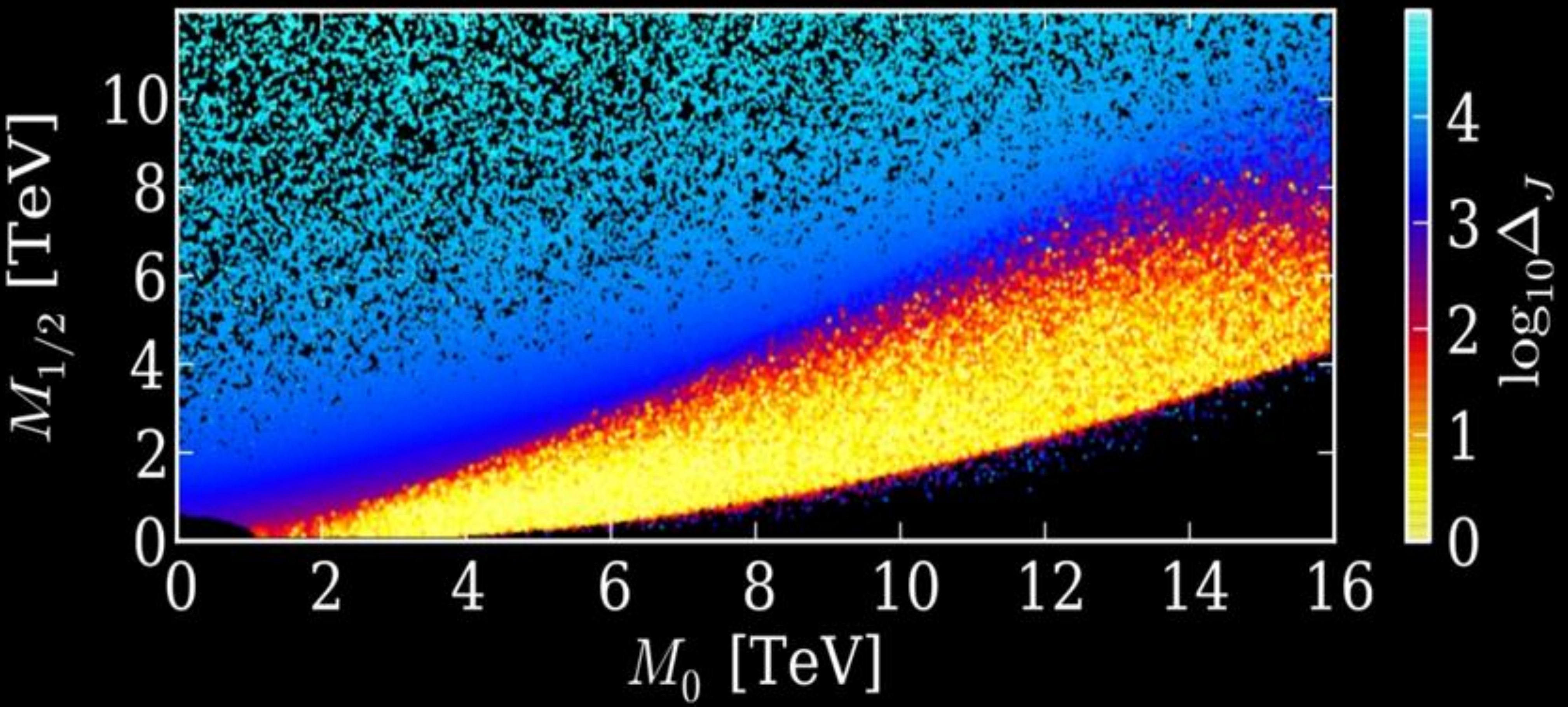
What did I learn about the Barbieri-Ellis-Giudice measure?

It’s a fine tuning measure that is deeply rooted in observation.

It measures the plausibility of a model recovering a given observation locally in the parameter space.

It’s closely related to Bayesian inference and as such it’s a crucial ingredient of model selection.

Fine tuning in the CNMSSM

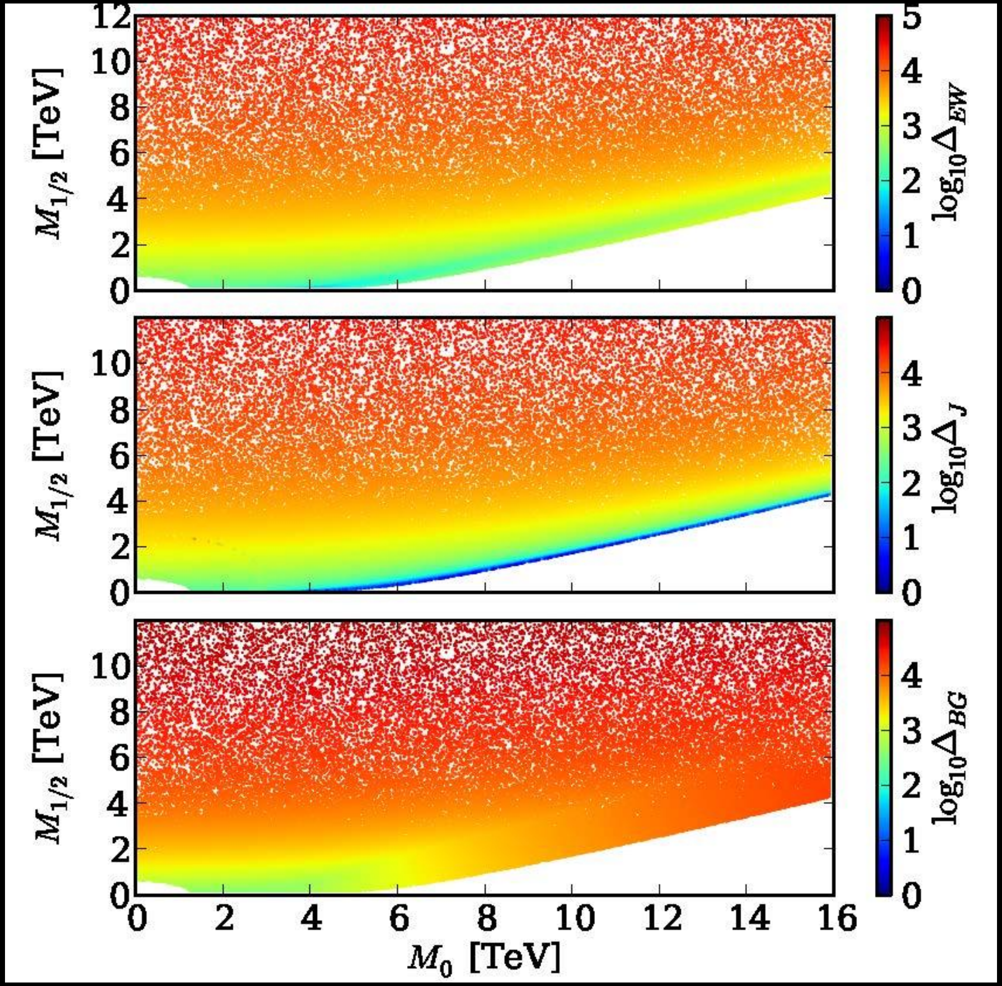


$A_0 = -2.5$ TeV, $tan\beta = 10$

Ahron,Balazs,Farmer,Kim PRD90 5 055008 (2014)

naturalness prior against other FT measures

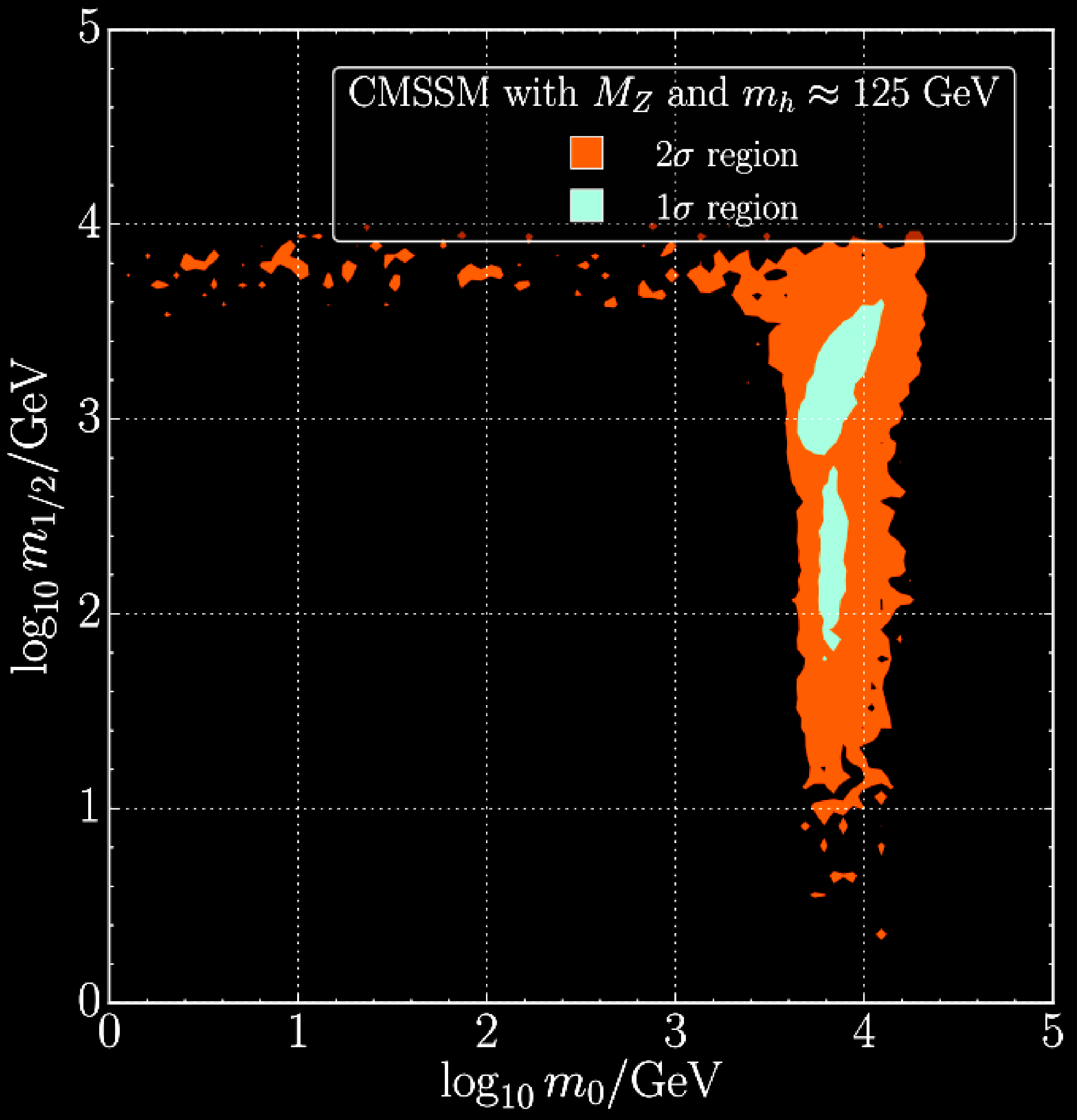
$A_0 = -2.5 \text{ TeV}, tan\beta$

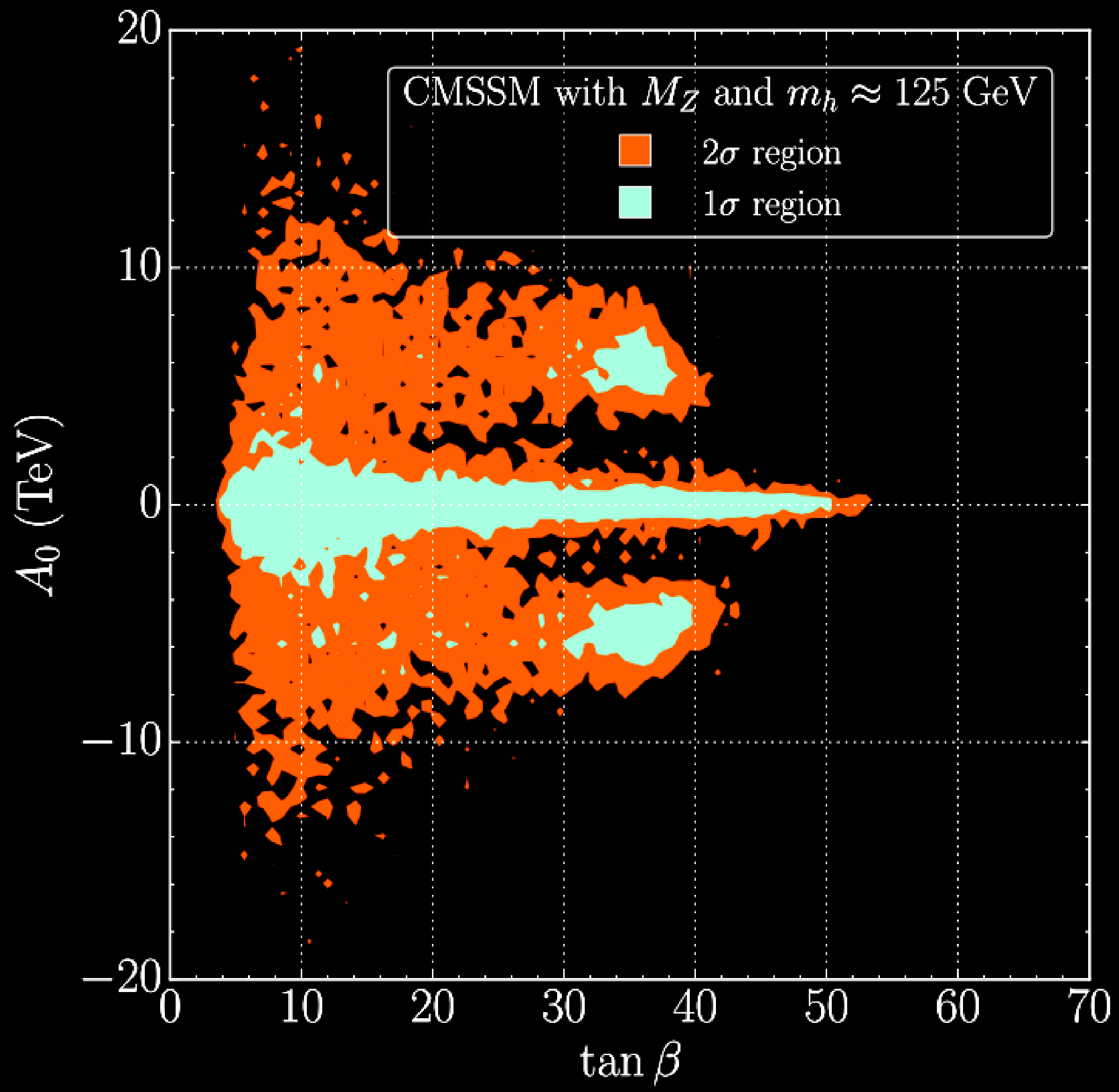


since Δ_{BG} is a quadratic sum or max and Δ_J is a determinant: $\Delta_{BG} > \Delta_J$

Ahron,Balazs,Farmer,Kim PRD90 5 055008 (2014)

lowest fine tuning in the CMSSM





Conclusions

Naturalness is a robust fundamental organizing principle that can, and should, guide model building.

Naturalness can be quantified by fine tuning.

Fine tuning is measured by the Bayesian evidence: the plausibility that a theory reproduces observations.

Bayesian naturalness, surprisingly, tells us that the least fine tuned regions of the (N)MSSM haven’t been experimentally explored yet.