



# Simulations of Vacuum Arcs and Gas Discharges with Liquid Cathodes

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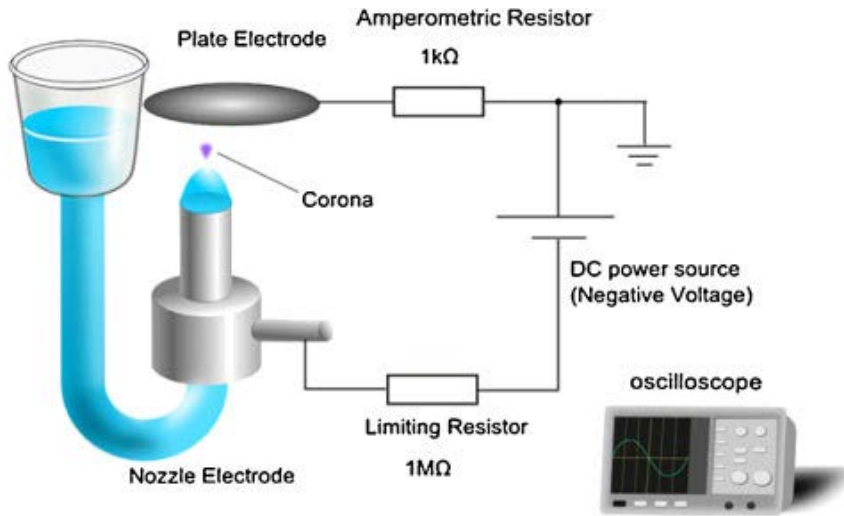
8th International Workshop on Mechanisms of Vacuum Arcs  
(MeVArc 2019)  
15-19 September 2019  
Orto Botanico – Padova, Italy

# Background and Motivation

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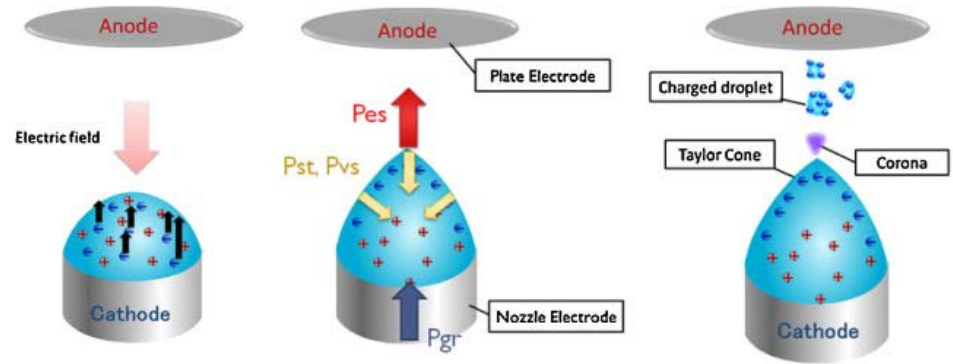
- Simulations of vacuum arcs and gas discharges with liquid electrodes require coupling plasma physics with multi-phase flow science
- Specifics of gas breakdown and vacuum breakdown (volume processes vs surface processes)
- Computational challenges:
  - Different time scales for liquid dynamics and plasma dynamics (ms and ns, respectively)
  - Disparity of electron and ion time scales
  - Explosive Electron Emission: continuous phase transition (solid → liquid → gas → plasma)
  - Plasma expansion into vacuum or into a background gas vs ionization wave development

# Negative Corona on Liquid Water Cathode

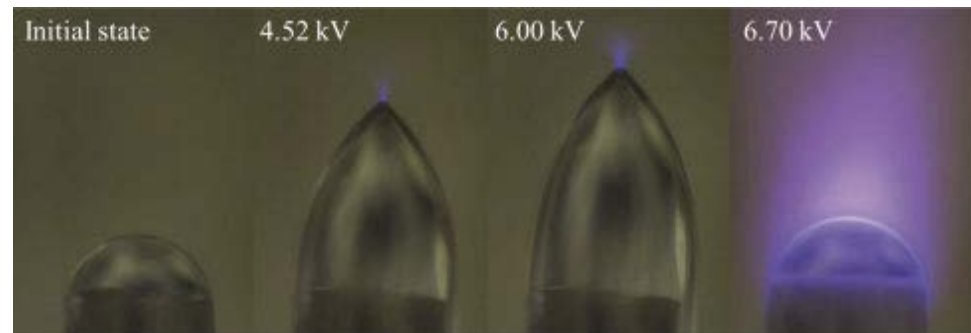


*Experimental setup for corona discharge using Taylor cone*

**Atmospheric-pressure air**  
**Cathode-anode gap is 5-20 mm**



*Principle of Taylor cone formation. (a) Initial state, (b) Taylor cone formation, and (c) Corona discharge at the tip of Taylor cone*



*Corona discharge using Taylor cone and shape of the liquid surface*

# Computational Model for Liquid Dynamics

Navier-Stokes equation for two-phase flows:

$$\nabla \cdot \mathbf{u} = 0$$

$$\rho \frac{\partial \mathbf{u}}{\partial t} + \rho (\mathbf{u} \cdot \nabla) \mathbf{u} = \nabla \cdot [-p\mathbf{I} + \mu(\nabla \mathbf{u} + (\nabla \mathbf{u})^T)] + \frac{p}{l} \mathbf{n}$$

Gas-liquid interface tracked using Volume- of- Fluid method, <http://basilisk.fr>

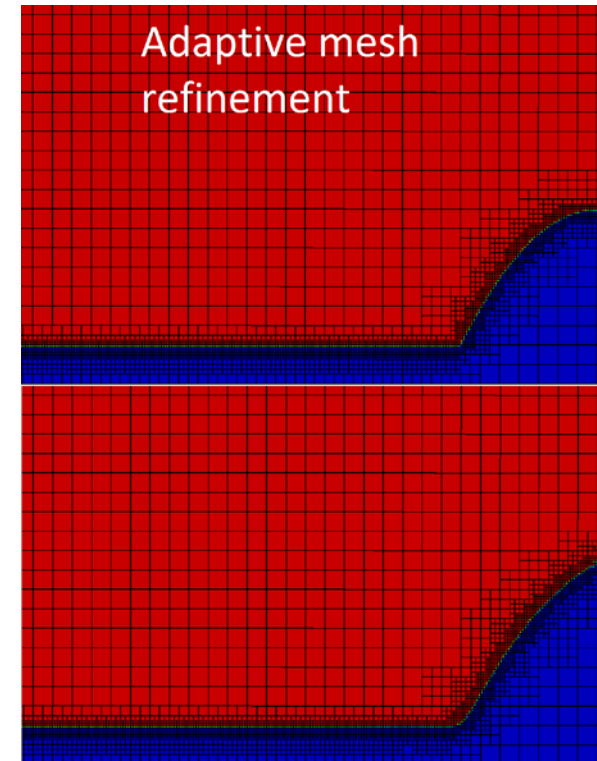
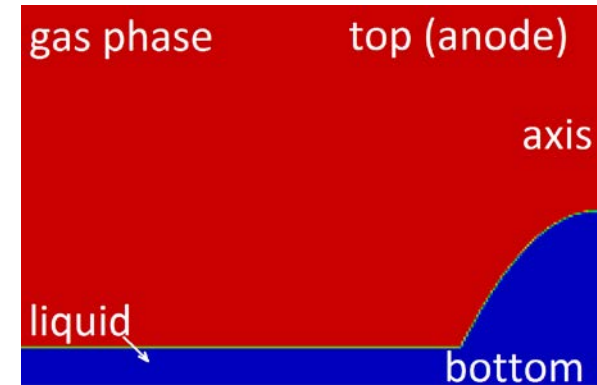
Two competitive forces at interface: surface tension and electrostatic

Laplace equation for electrostatic potential

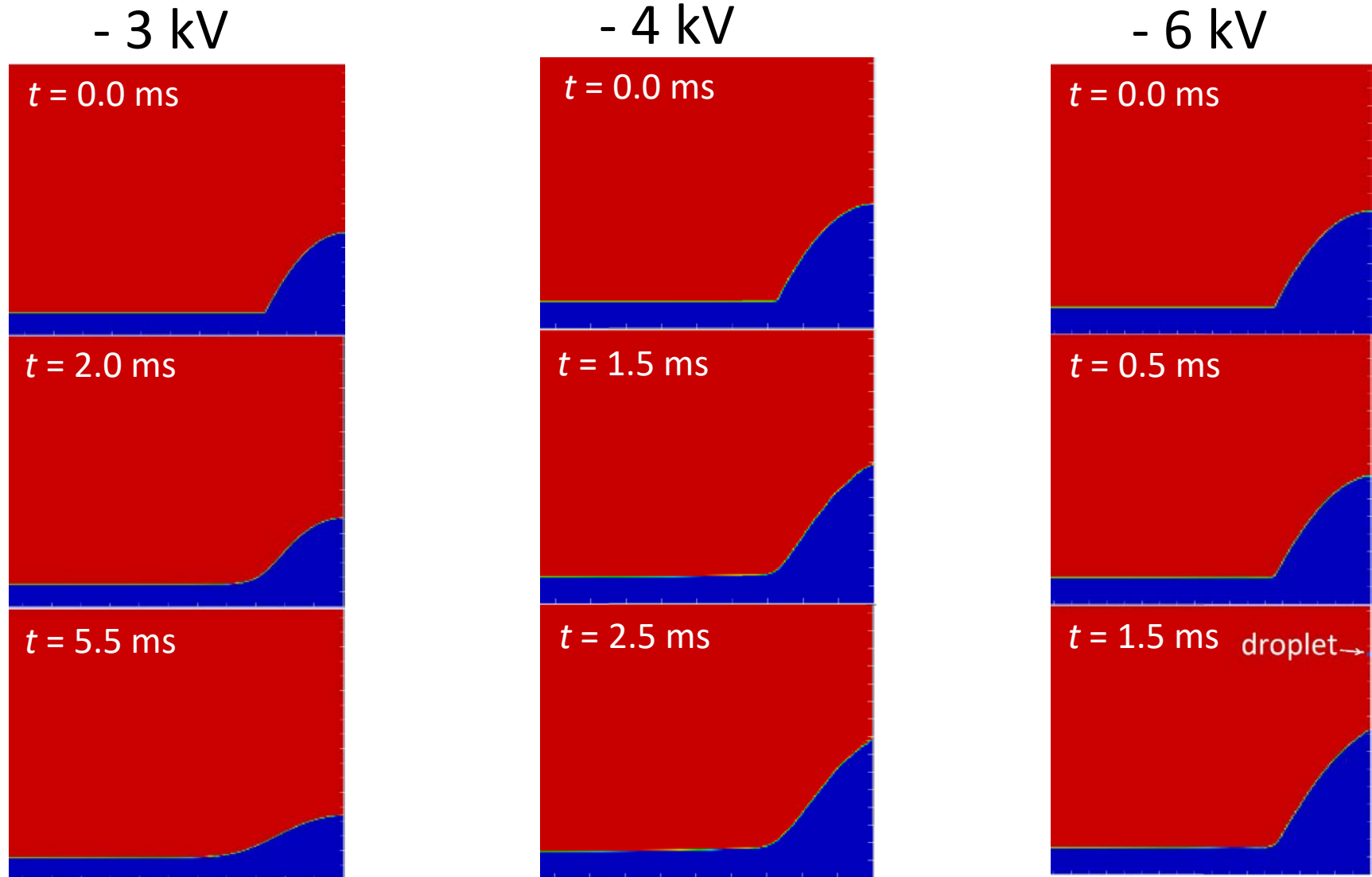
$$\Delta \varphi = 0$$

$$\varphi_{top} = 0, \varphi_{bottom} = \varphi_0$$

Time scale is milliseconds  $\rightarrow \Delta t \sim 10^{-7} - 10^{-6}$  s



# Dynamics of Taylor Cone Formation in Electric Field



- Interplay between surface tension and electrostatic forces leads to Taylor cone formation
- The cone development terminates with droplet ejection from the tip (electrospray)
- Voltage influences the cone shape and height, and the droplet ejection rate

# Gas Breakdown: Electron Kinetics

$$\frac{\partial f_e}{\partial t} + v \frac{\partial f_e}{\partial x} - \frac{eE - F(v)}{m_e} \frac{\partial f_e}{\partial v} = S_{ion}(v)$$

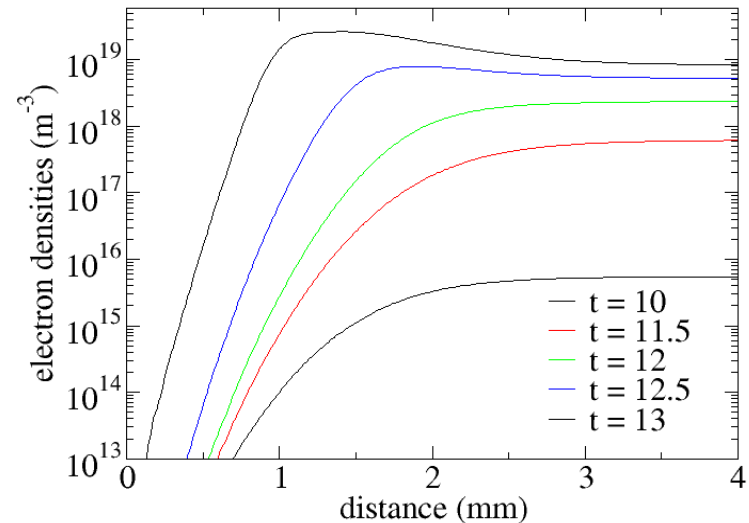
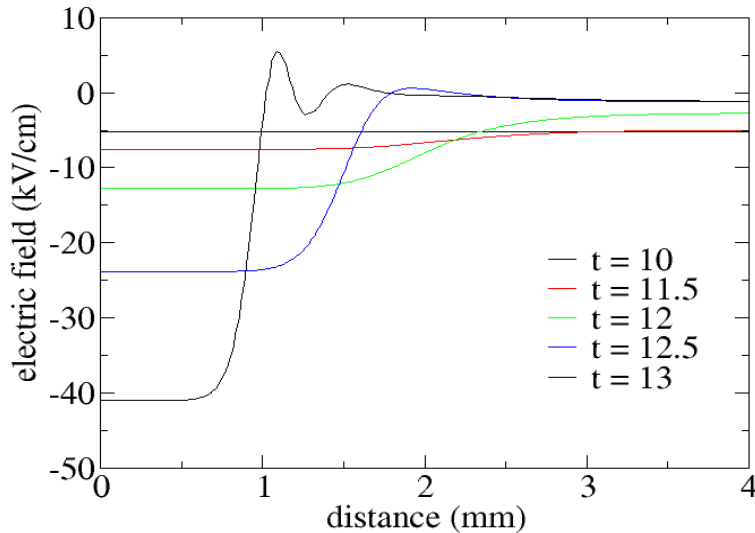
$$n_e = \int_{-\infty}^{+\infty} f_e dv$$

$$\frac{dn_i}{dt} = S_{ion} \quad \frac{d^2\phi}{dx^2} = \frac{e}{\epsilon_0} (n_e - n_i)$$

$$S_{ion}(v) = S(v) \int_{-\infty}^{+\infty} n_g \sigma_{ion}(v') f_e(x, v', t) dv'$$

$$F(\epsilon_e) = \frac{e^4 Z n_g}{8\pi \epsilon_0^2 \epsilon_e} \cdot \ln\left(\frac{\epsilon_e}{I}\right), \text{ where } \epsilon_e = \frac{m_e v^2}{2}$$

400 Torr, initial plasma density  $2 \times 10^{11} \text{ m}^{-3}$ ,  
 gap  $d = 6 \text{ mm}$ ,  $dU/dt = 8.4 \times 10^{10} \text{ V/s}$ ,  
 $\sigma_{ion} = 2 \times 10^{-20} \text{ m}^{-2}$



# Fluid Model: Gas Breakdown and Plasma Formation

Drift-diffusion approximation for both electrons and ions:

$$\frac{\partial n_k}{\partial t} + \nabla \cdot \Gamma_k = S_k$$

$$\Gamma_k = -\mu_k n_k \nabla \varphi - D_k \nabla n_k$$

Electron energy balance:

$$\frac{\partial \varepsilon_e}{\partial t} + \nabla \cdot \Gamma_\varepsilon = S_\varepsilon$$

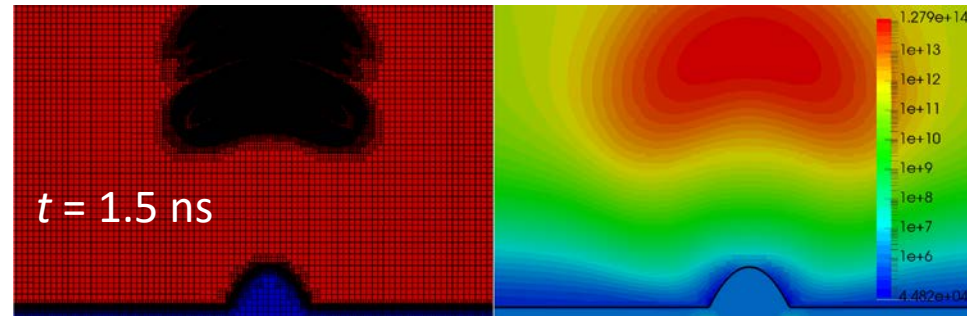
Poisson equation for the electrostatic potential:

$$\Delta \varphi = \frac{q_e}{\varepsilon_0} (n_e - n_i)$$

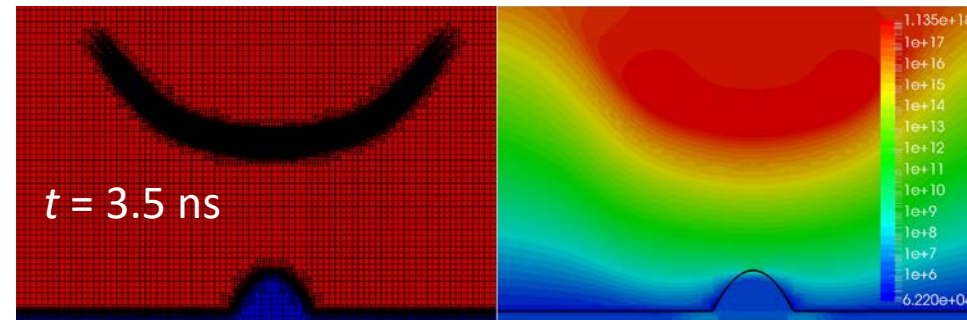
Time scale is nanoseconds  $\rightarrow \Delta t \sim 10^{-14} - 10^{-12}$  s

## Dynamics of Gas Breakdown

Mesh is adapted on  $(n_e - n_i)$

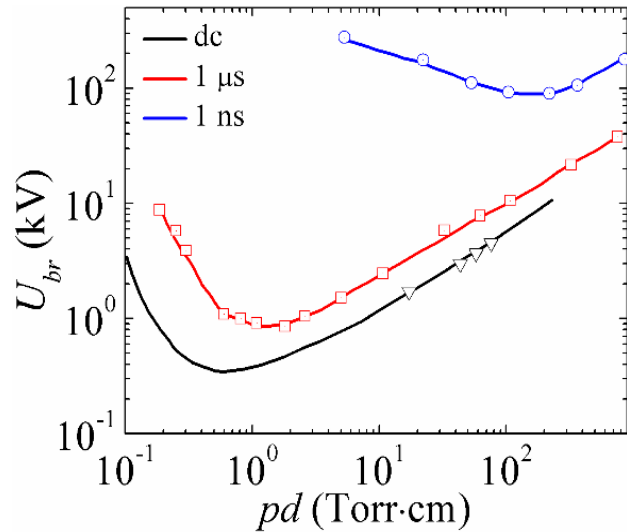


Mesh is adapted on field

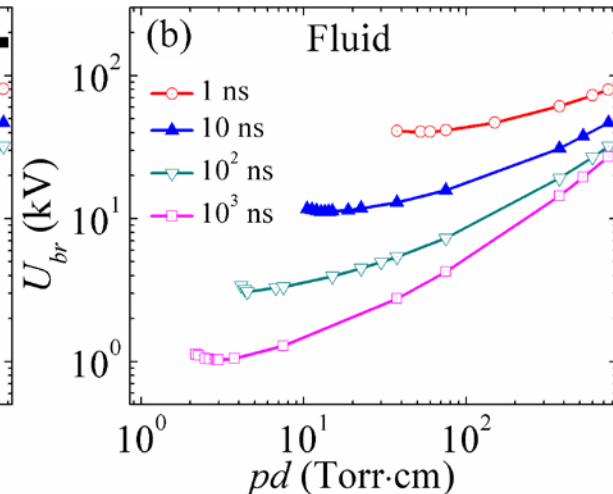
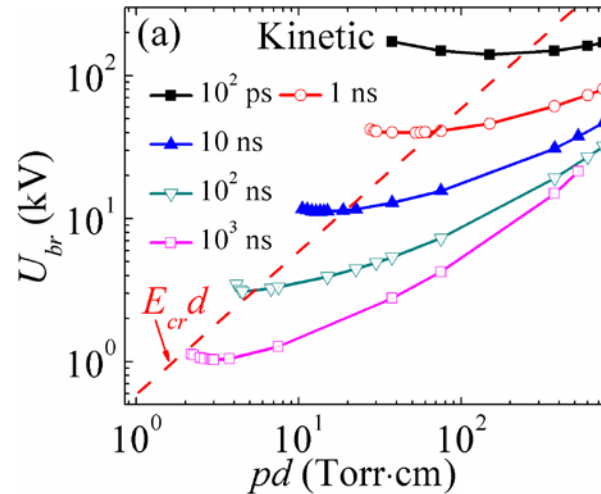
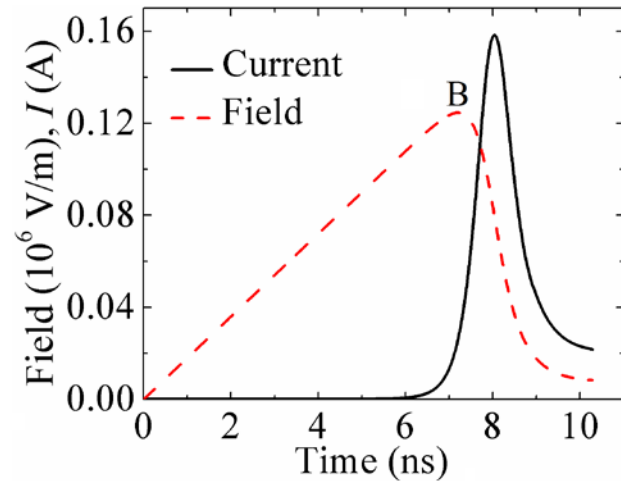


*Quasi-neutral plasma near the anode*

# Paschen Curves for Pulsed Gas Breakdown



- Voltage rise time  $\tau$  affects the Paschen curves for planar gaps.
- Both kinetic and fluid models predict the experimentally observed shift of the curves toward higher voltages and the shift of Paschen minima toward higher  $pd$  with decreasing  $\tau$ .
- On the right branch, the agreement between both models is obtained for all  $\tau$ .
- The fluid model could not describe the left branch of the Paschen curves.



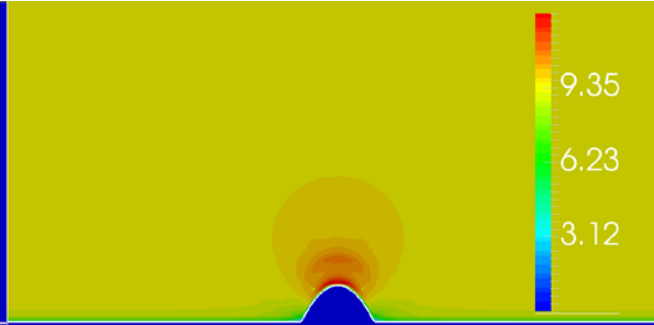
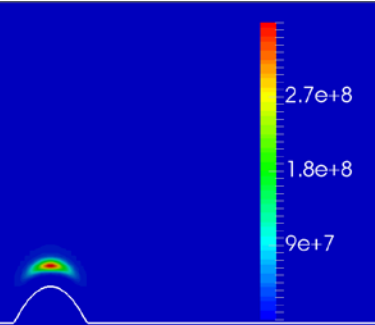


# Dynamics of Corona Discharge Development

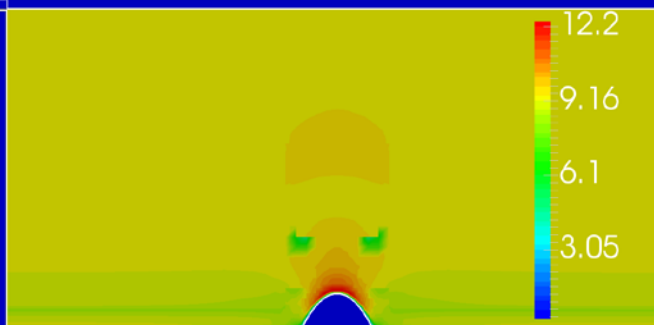
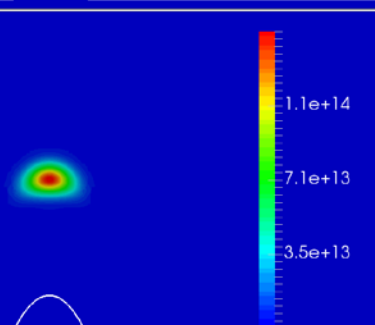
$n_e$  in  $[m^{-3}]$

$T_e$  in [eV]

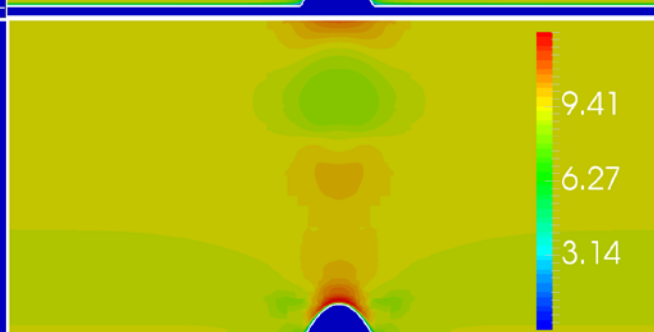
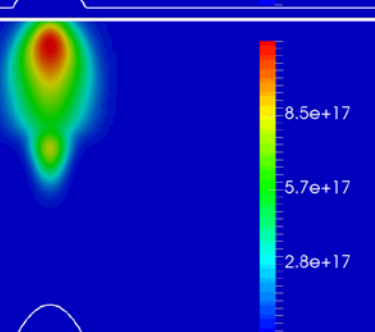
$t = 1.0$  ns



$t = 2.5$  ns



$t = 3.5$  ns

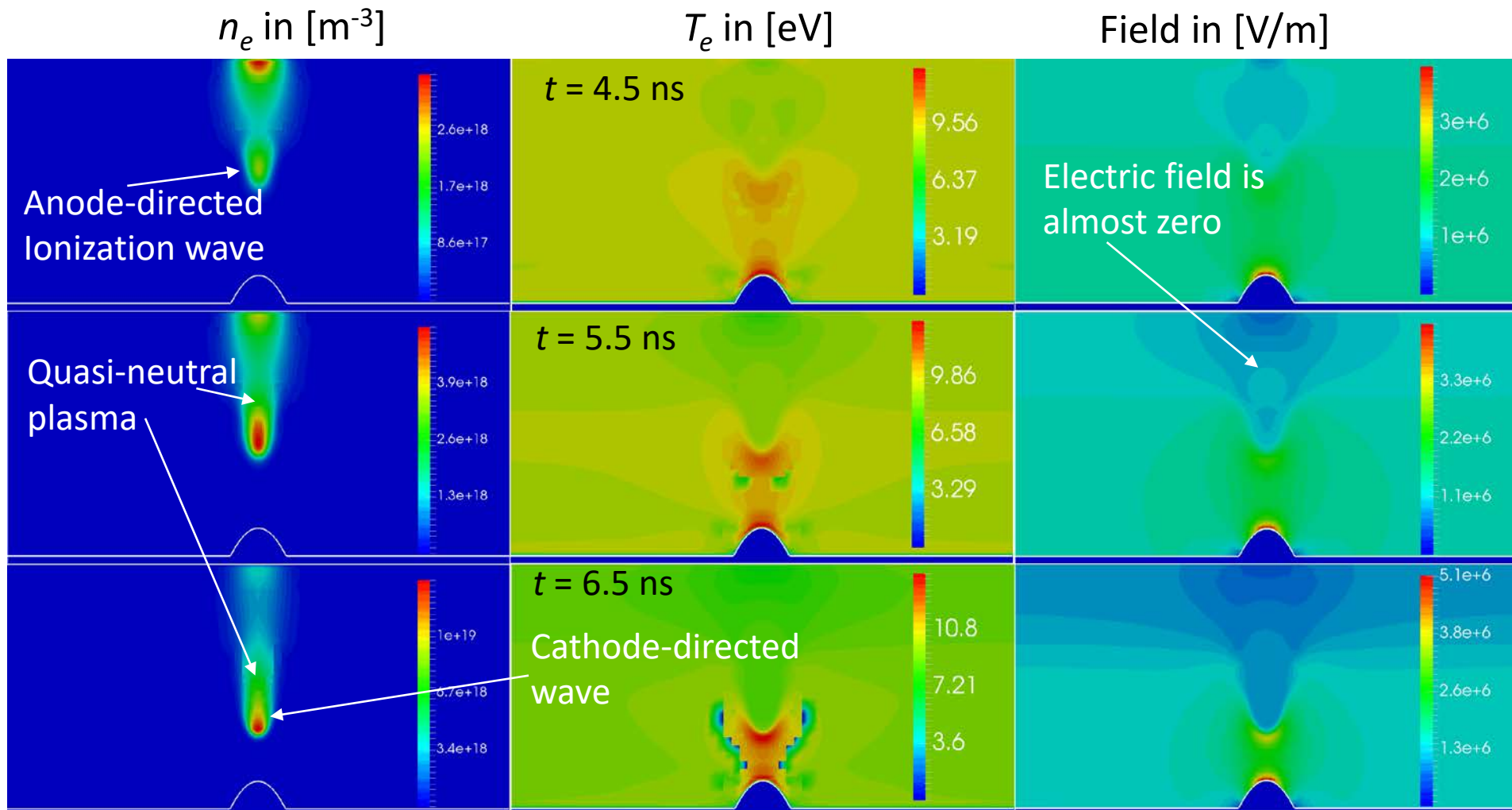


Negative corona discharge is ignited at low voltages; ionization wave propagates toward the anode

Plasma density does not exceed  $10^{15} m^{-3}$ ; Electric field remains undistorted; Highest  $T_e$  is obtained at the cone tip

Primary avalanche leaves behind uncompensated positive space charge; ions propagate to the cathode on much longer time scale

# Dynamics of Glow Discharge Development



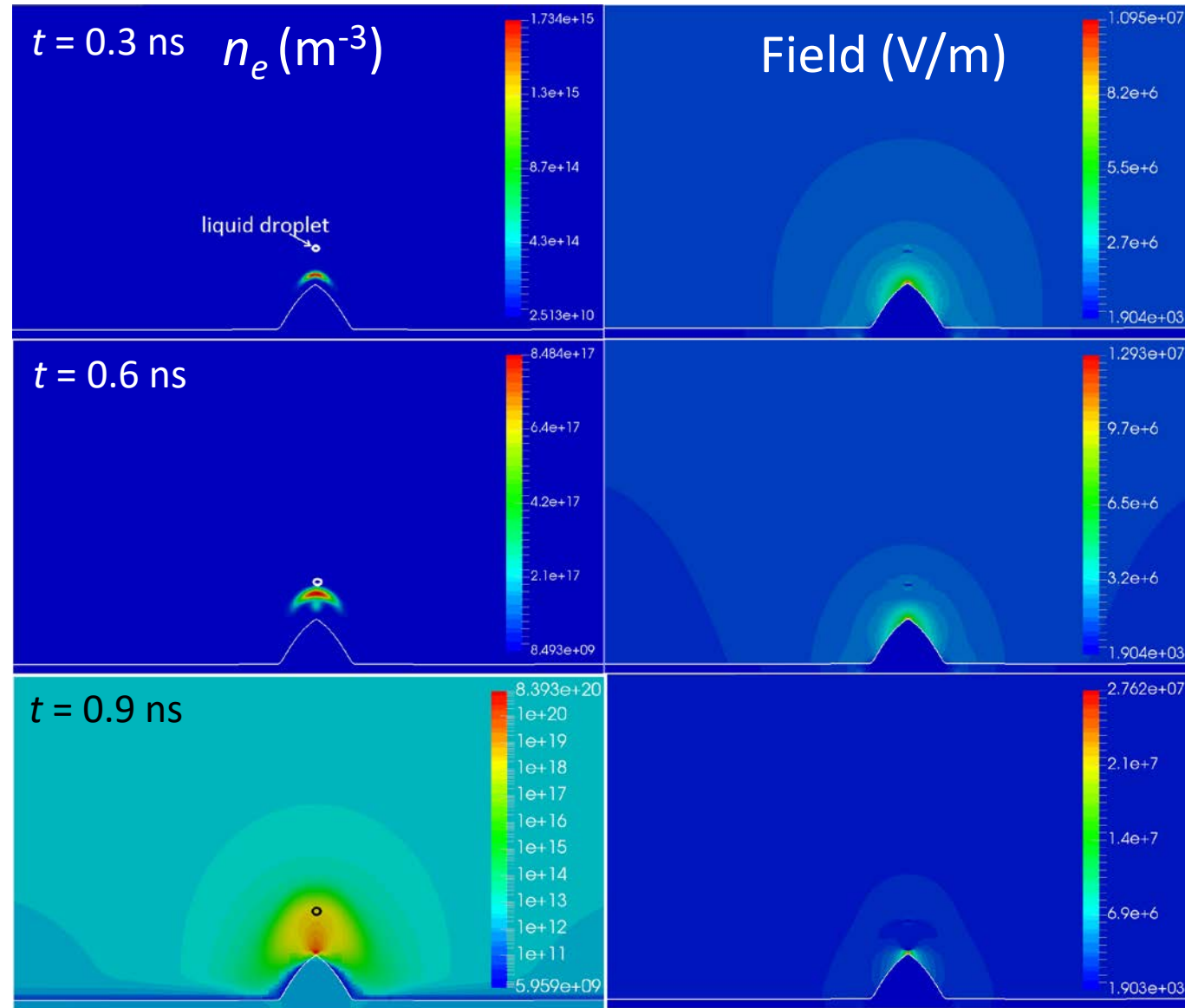
The anode-directed ionization wave stops and follows the fast-axial wave propagating towards the cathode: the cathode sheath is formed. Further development of the discharge occurs at the (slow) ion time scale.

# Discharge formation between cone tip and droplet

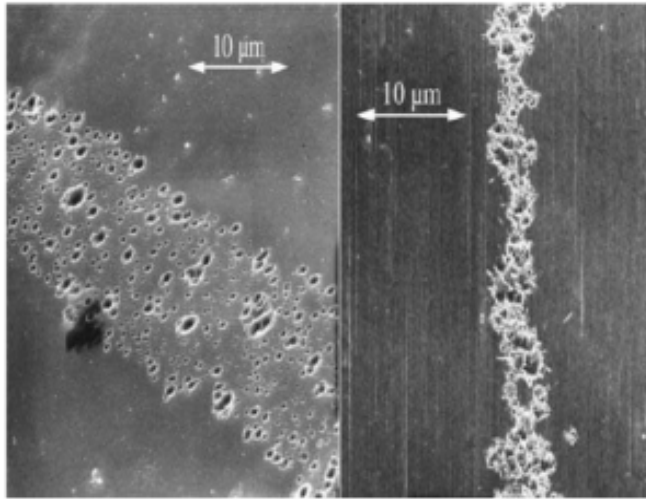
The cone growth stops once droplets are ejected from the tip; they shield the electric field

Electric field sufficient for gas breakdown is obtained between the cone tip and the droplet

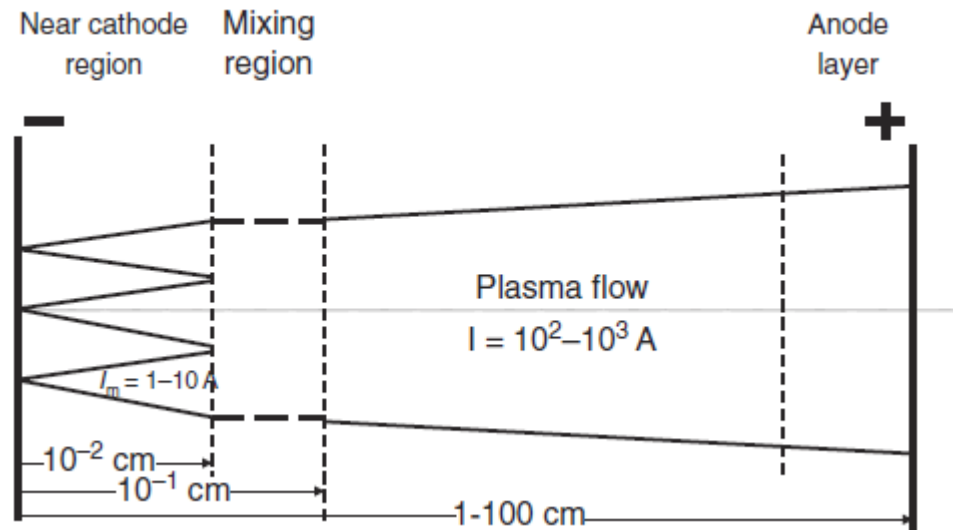
Plasma density in the vicinity of the cathode is  $\sim 10^{21} \text{ m}^{-3}$  and keeps growing; sheath thickness is  $< 0.1 \mu\text{m}$



# Explosive Electron Emission in Vacuum Arcs



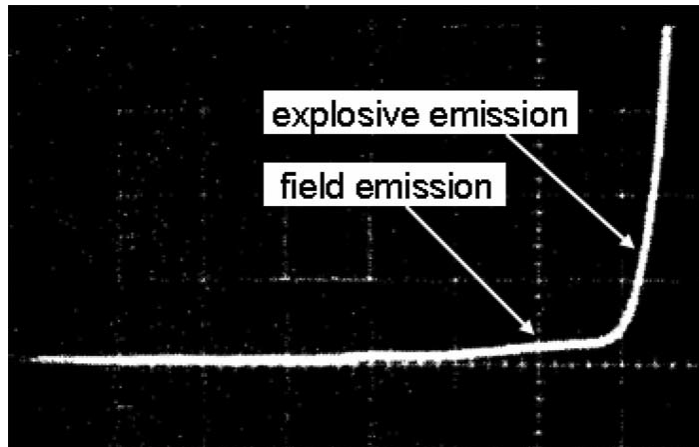
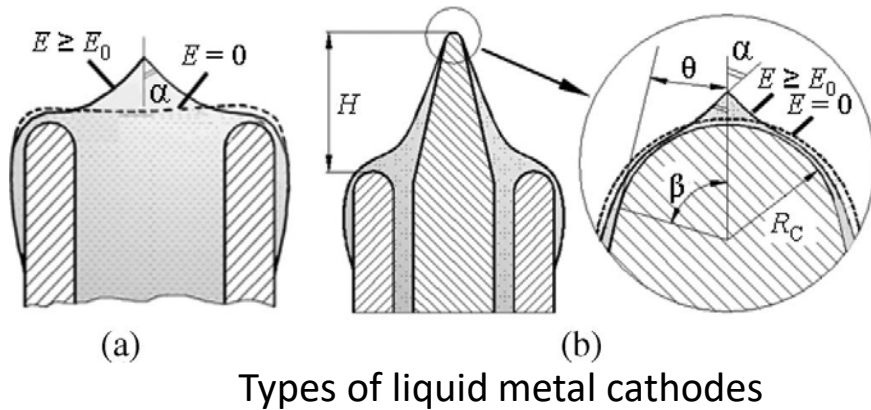
*Arc traces on cathode left by vacuum arcs*



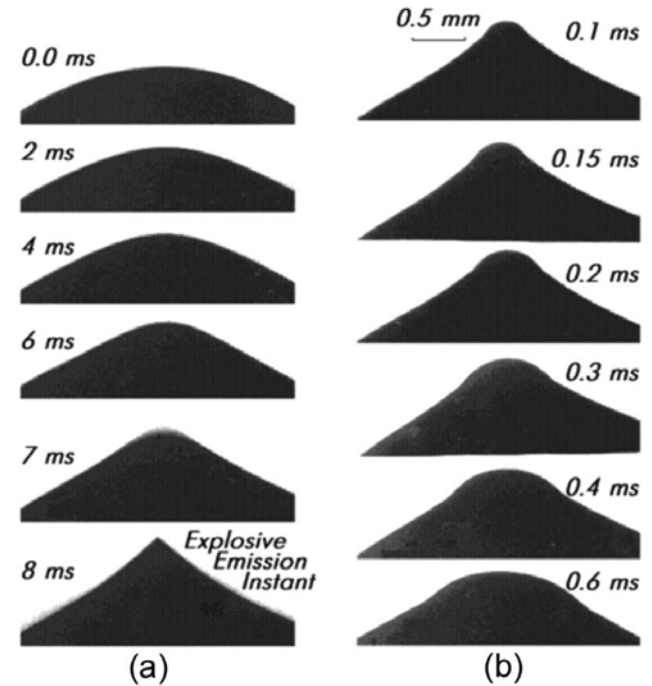
Schematic diagram of vacuum arc discharge

- A distinctive feature of electric discharges in a vacuum is rapid phase transitions of the cathode material from a solid state to liquid, gaseous, and plasma states
- The plasma state changes from a dense non-ideal to a moderately rarefied, and to a collisionless.
- The cathode material is emitted from micro-explosions (ectons) and progressively accelerated to velocities of about  $10^6$  cm/s, the particle density decreases from  $10^{23}$  to  $10^{10}$  cm $^{-3}$

# Vacuum Discharge Ignition from Liquid-Metal Cathodes



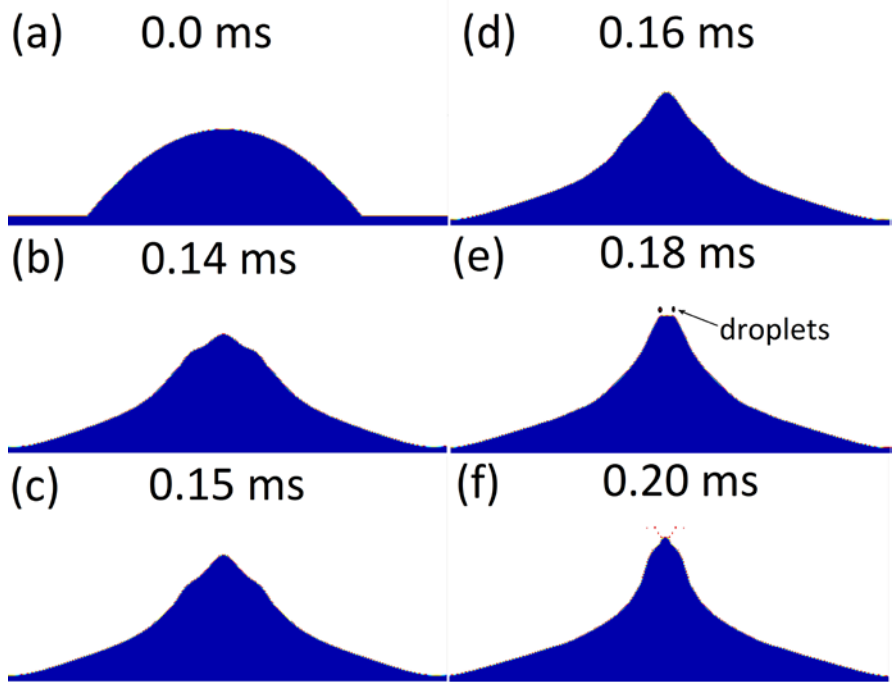
Electron emission current oscillogram  
L. W. Swanson and G. A. Schwind, J. Appl. Phys. 49, 5655 (1978)



Shadow photographs showing (a) growth and (b) decay of a protrusion on the liquid metal cathode with diameter 4 mm

**InGa is liquid at room temperature**

# Taylor cone formation on Liquid Metal Cathode



- Electric field at the cathode tip increases with time
- Unstable Taylor cone is formed during 0.2 ms
- Liquid droplets are ejected from the cone tip
- Droplet ejection precedes the field emission onset

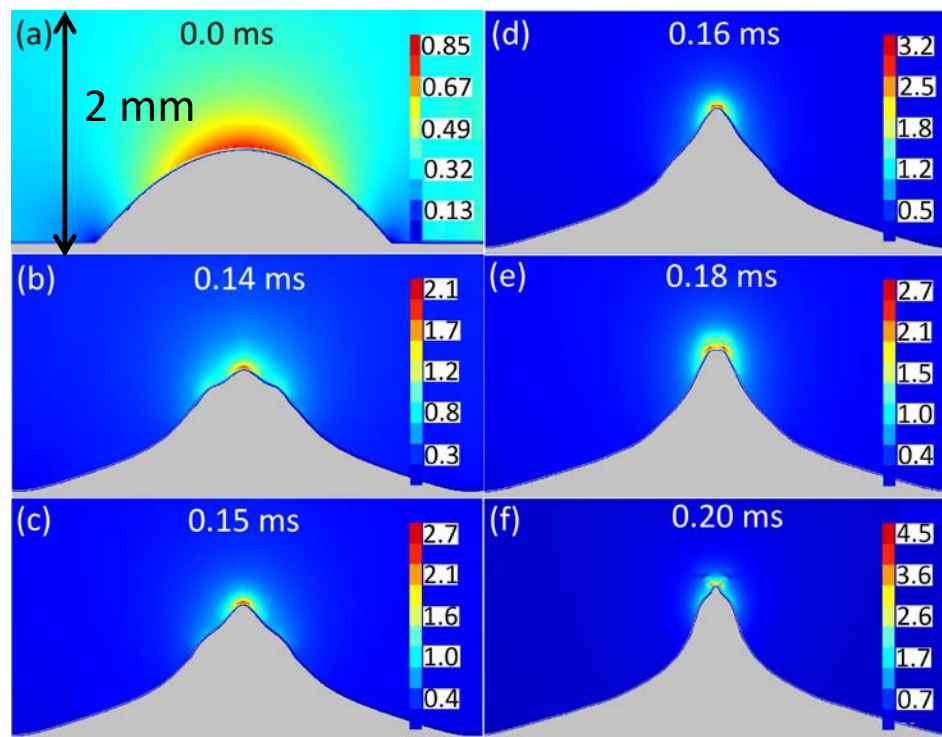
## Conditions:

InGa liquid cathode

Cathode potential -40 kV

Cathode-anode gap is 2 mm

Electric field distribution ( $10^8$  V/m)



# Liquid Metal Protrusion Heating

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Axisymmetric model of micro-protrusion heating:

$$\rho C_p \frac{\partial T(\mathbf{r}, t)}{\partial t} = \nabla(\lambda(T)\nabla T(\mathbf{r}, t)) + F(\mathbf{r}, t)$$

Heating is due to electric current and Thomson effect:

$$F(\mathbf{r}, t) = \mathbf{j}(\mathbf{r}, t)\mathbf{E}(\mathbf{r}, t) + g(T)(\mathbf{j}(\mathbf{r}, t)\nabla T(\mathbf{r}, t))$$

Solve the Poisson's equation to define the current density:

$$\nabla[\sigma(T)\nabla\varphi(\mathbf{r}, t)] = 0 \rightarrow \mathbf{j}(\mathbf{r}, t) = \sigma(T)\nabla\varphi(\mathbf{r}, t)$$

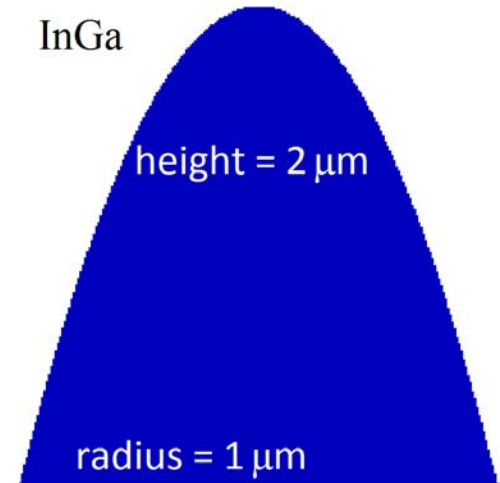
Boundary conditions:

$$\lambda(T) \frac{\partial T(\mathbf{r}, t)}{\partial \mathbf{n}} = \frac{j_{FE}(E_{surf}, T_{surf})}{e} \Delta E(E_{surf}, T_{surf}) - \sigma_{SB} T_{surf}^4$$

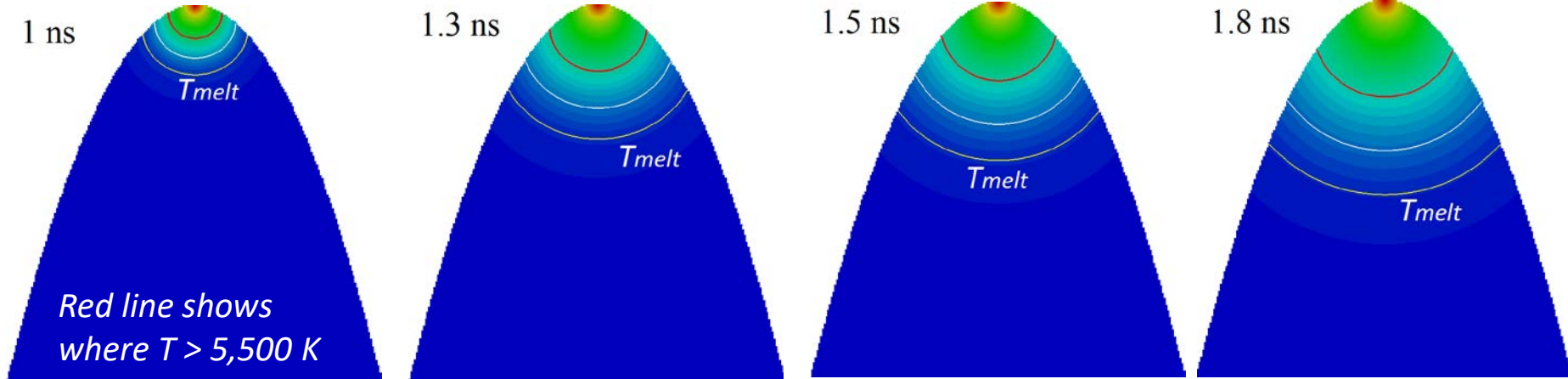
$$\sigma(T) \frac{\partial \varphi(\mathbf{r}, t)}{\partial \mathbf{n}} = j_{FE}(E_{surf}, T_{surf})$$

$g(T)$  is the Thomson constant,  $\sigma(T)$  is the conductivity,  $\Delta E$  energy carried out by emitted electrons; takes into account the Nottingham effect,  $\sigma_{SB}$  is the Stefan-Boltzmann constant, and  $j_{FE}(E_{surf}, T_{surf})$  is the known function of the emitter shape

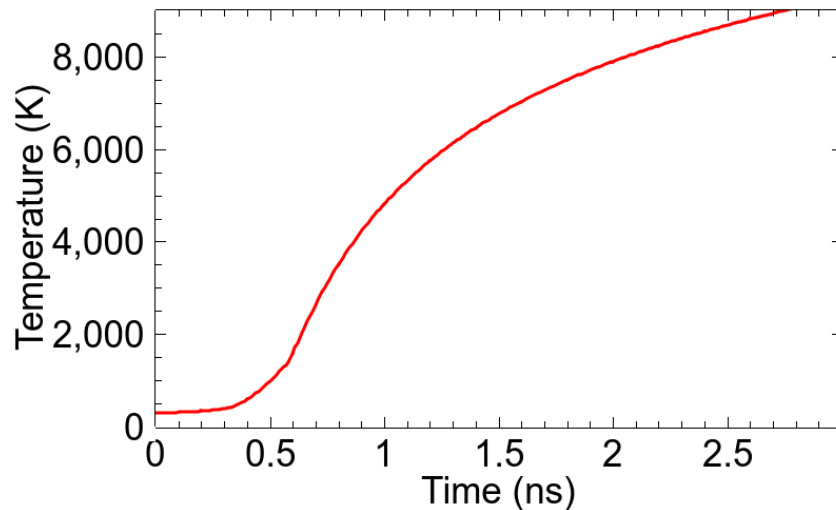
from Glazanov, Baskin and Fursey, Journal of Technical Physics **59**, 60 (1989)



# Liquid Metal Protrusion Heating



Temperature in the protrusion bulk  
(1.7  $\mu\text{m}$  from the bottom)



- Thermal instability develops only when the current density at the tip exceed  $10^8$  A/cm<sup>2</sup> in agreement with the Mesyats' theory
- Then, the micro-protrusion temperature at the tip starts exceeding the boiling temperature on the ns time scale
- This clearly indicates that the field-to-explosive electron emission occurs after 1 ns
- Now we know the metal mass which experiences phase transition from solid to non-ideal plasma: can use it for plasma expansion modeling;



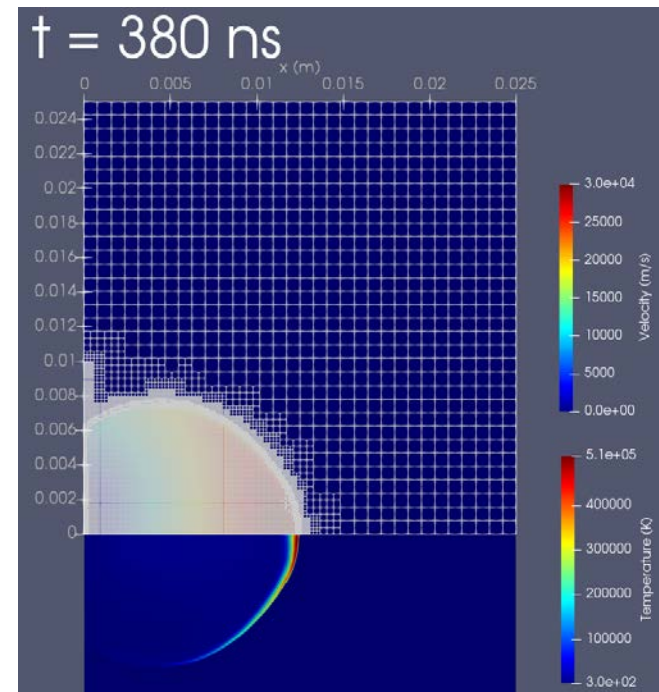
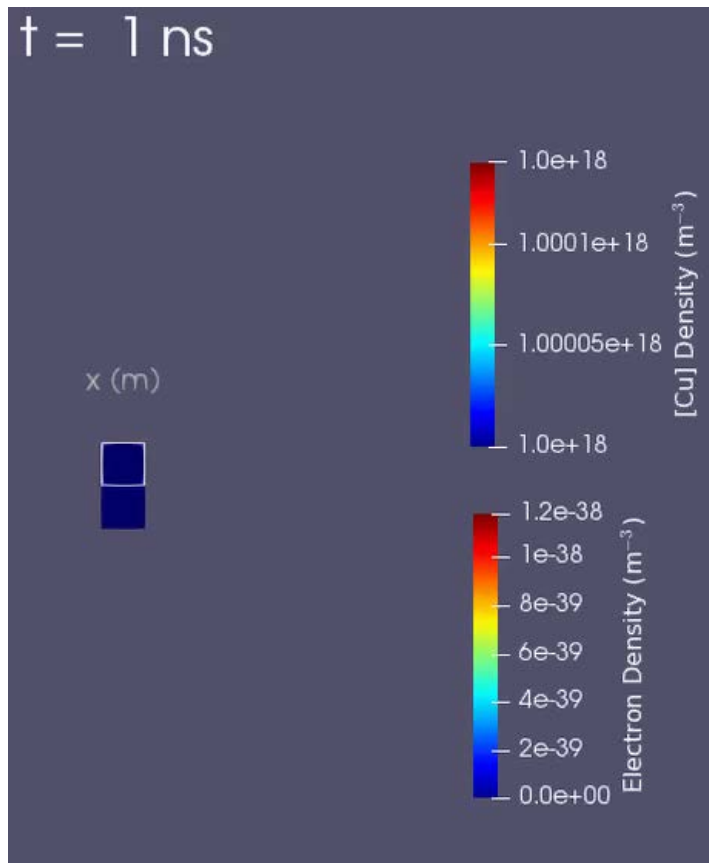
# Plasma Expansion: Gas-Dynamic LTE Model

LTE model is widely used for pulsed laser ablation (Saha equation)

## Adaptive Computation Domain

- Implemented 2D planar and axisymmetric solvers with ACD
- Generic EOS to describe continuous phase transition

## Dynamically Adaptive Cartesian Mesh



# Fluid Model for Plasma Expansion into Vacuum

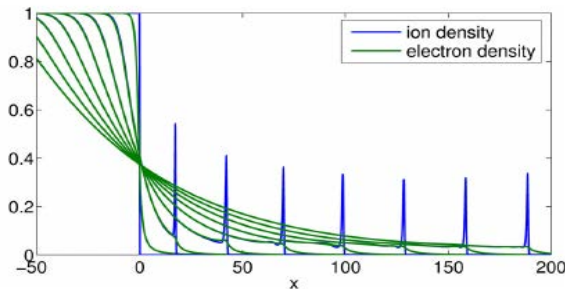
Collisionless, cold ions,  
Boltzmann-electron model:

$$\frac{\partial n_i}{\partial t} + \nabla \cdot (n_i \vec{u}_i) = 0,$$

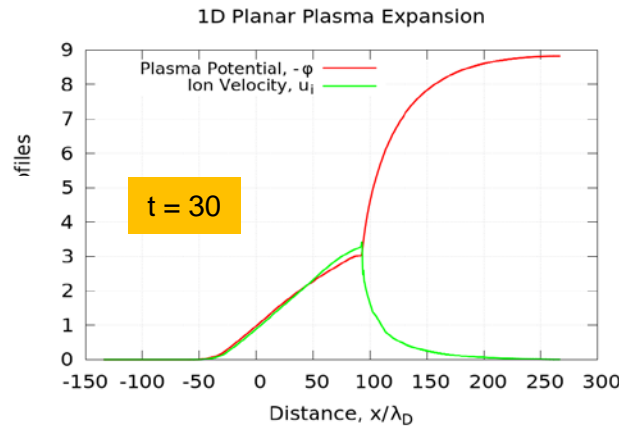
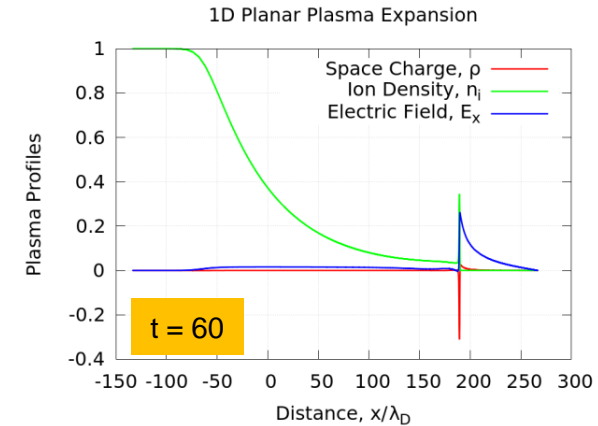
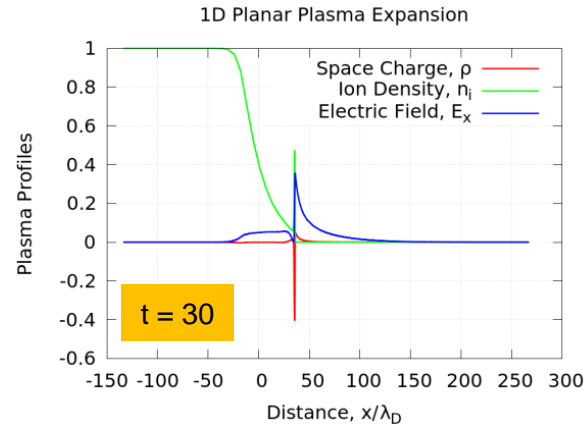
$$m_i \frac{\partial \vec{u}_i}{\partial t} + m_i (\vec{u}_i \cdot \nabla) \vec{u}_i = -e \nabla \phi,$$

$$\nabla^2 \phi = \frac{e}{\epsilon_0} (n_e - n_i)$$

$$n_e = n_0 \exp\left(\frac{e\phi}{k_B T_e}\right)$$



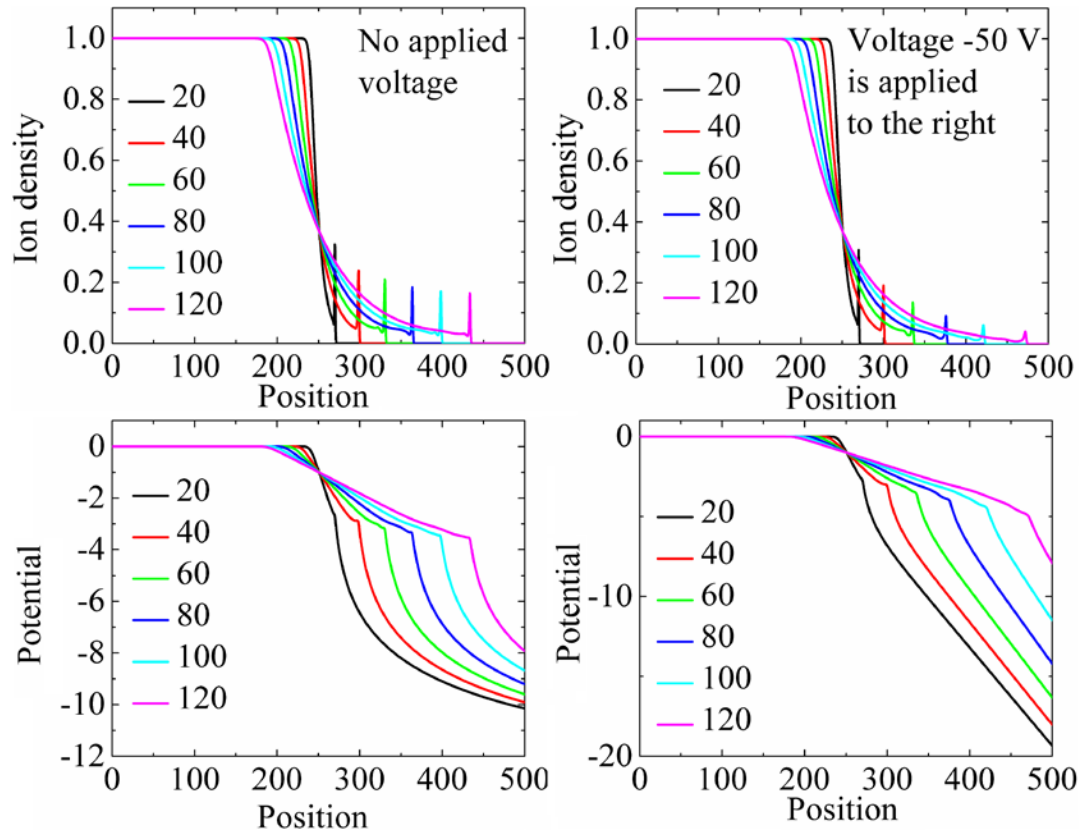
*J. E. Allen and M. Perego, On the ion front of a plasma expanding into a vacuum, Physics of Plasmas 21, 034504 (2014);*



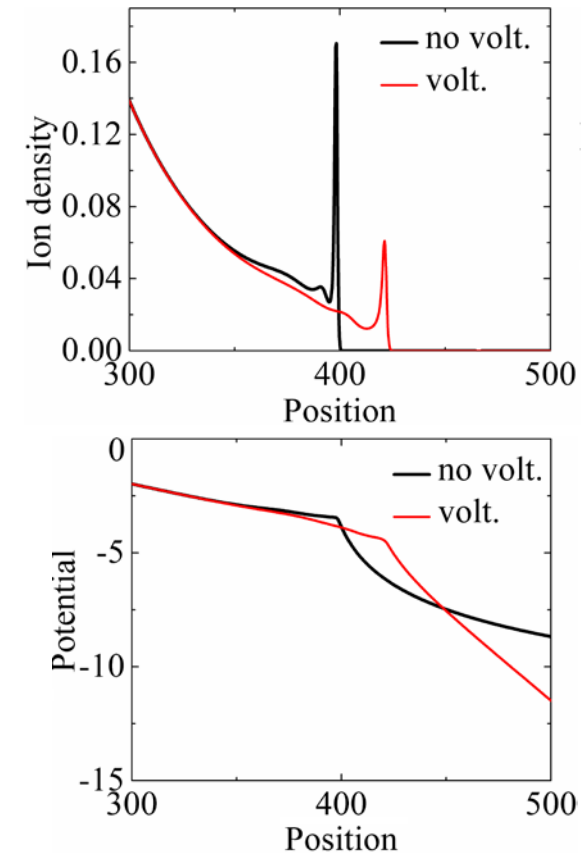
Very sharp ion density peaks predicted (peak widths a few  $\lambda_D$ )

- Good agreement with other 1D computations using both Eulerian and Lagrangian approaches. Both spatial and temporal profiles are well reproduced
- The code can be used to study collisionless plasma expansion dynamics in multi-dimensions

# Plasma Expansion: Influence of External Field



## Ion front in both cases

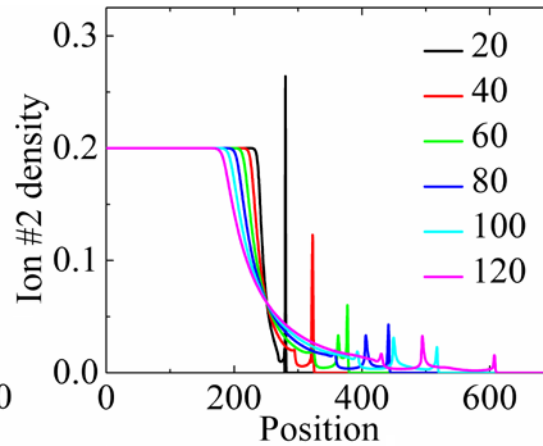
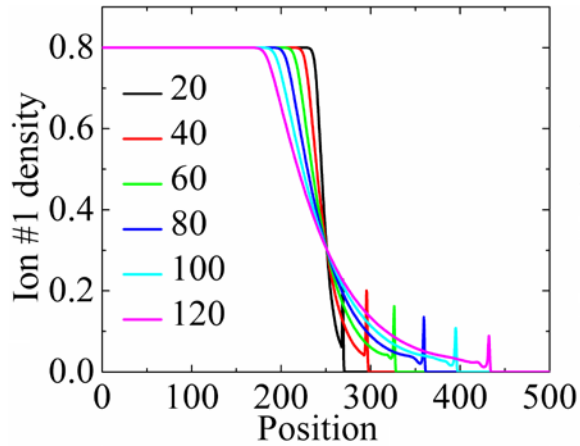


Gap  
 $1000\lambda_D$

Voltage  
 $\sim 50T_e$

- Applied voltage starts influencing the plasma front propagation only when it is far from the “core” dense plasma: the plasma density decreases  $\rightarrow$  Debye length decreases  $\rightarrow$  external electric field starts penetrating the plasma
- Front velocity increases with time because the electric field between the front and the right boundary increases with time

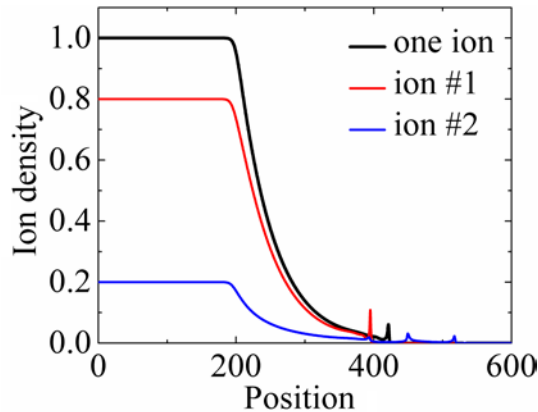
# Two Types of Positive Ions



$$\frac{Z_1}{Z_2} = \frac{1}{2}$$

$$\frac{n_1}{n_2} = \frac{8}{2}$$

Comparison with 1 ion plasma expansion



- Ions with larger charge propagate faster
- Density of Ion #2 has multippeak structure: the 1<sup>st</sup> peak coincides with the position of the Ion #1 peak, and the 3<sup>rd</sup> peak is the ion front
- All the peaks are formed at different stages: the 3<sup>rd</sup> peak is formed at the earliest stage of the front formation
- Velocity of the Ion #1 only slightly depends on the applied voltage
- Velocity of the Ion #2 increases with time
- Ion #1 moves slower than in the case of one ion species plasma

# Conclusions

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- **We have developed computational models for multi-phase plasmas with adaptive Cartesian mesh**
- **Simulated dynamics of Taylor cone formation on liquid cathodes using Volume-of-Fluid model**
- **Illustrated development of Corona and Glow Discharges with liquid cathodes**
- **Plasma expansion into vacuum has been simulated with fluid models**
- **Adaptive Computation Domain methodology has been introduced**
- **The differences between gas breakdown and vacuum breakdown have been identified and illustrated**

# Outlook

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- **Combine fluid and kinetic models for expanding plasma**
- **Simulate dynamics of vacuum breakdown and clarify effects of anode processes**
- **Quantify effects of background gas pressure on electron emission and volume ionization processes during breakdown and plasma formation**
- **Identify mechanisms of nano-particle production during adiabatic expansion of dense plasma into vacuum and background gas**
- **Identify similarities and differences between pulsed laser ablation (from fs to ns scales) and dynamics of spark and arc discharges**

# Acknowledgements

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- **DoE SBIR Phase II Contract DE- DE-SC0015746  
“Simulations of Explosive Electron Emission in  
Cathodic Arcs”**
- **NSF EPSCoR project OIA-1655280 “Connecting the  
Plasma Universe to Plasma Technology in AL: The  
Science and Technology of Low-Temperature Plasma”**

