The Harvey M. Krueger Family Center for Nanoscience and Nanotechnology

האוניברסיטה העברית בירושלים
THE HEBREW UNIVERSITY OF JERUSALEM

Statistical Analysis of Dark Currents and β Estimation

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Background

- Prevailing method of dark current analysis account for the mean behavior of the measurement.
- However, the measurement has inherent randomness:
  - System generated noise.
  - The experimental systems have fluctuations in supplied voltage.
  - The physical process is quantum and therefore stochastic in nature.
  - Stochastic evolution of field emitters
- In this talk I will present two novel methods of β estimation as well as a more accurate numerical estimation of β through FN plot analysis.

Numerical Simulation

- The basis of this work is in a numerical simulation of dark currents.
- The simulation is based on the theory of Murphy and Good[1] on the SN barrier.
- Contrary to the current analytical formulas, a current emitter was split into many single incident energy electron bands.
- These currents were simulated independently to generate a current for each band and was then summed to arrive at a total current of the emitter.

[1]E.L. Murphy and R.H. Good. Thermionic emission, field emission, and the transition region. Phys. Rev., 102(6) 1464, 1956.

Numerical Simulation

$$N(W) = \frac{4\pi mkT}{h^3} L\left(\frac{W-\mu}{kT}\right)$$

$$\begin{aligned} D &= \left\{1 + exp\left(\frac{4\sqrt{2}}{3}\left(\frac{E\hbar^4}{m^2 e^5}\right)^{-\frac{1}{4}} y^{-\frac{3}{2}} v(y)\right)\right\}^{-1} \\ &\cong \left\{1 + exp\left(\tilde{e}\Delta v(y)\frac{1}{E}\right)\right\}^{-1} \end{aligned}$$

Analytical Estimation

$$I = \int D(W)\, N(W)\, dW$$

Numerical Gaussian Distribution

$$\mu_W = SD(W)N(W)$$

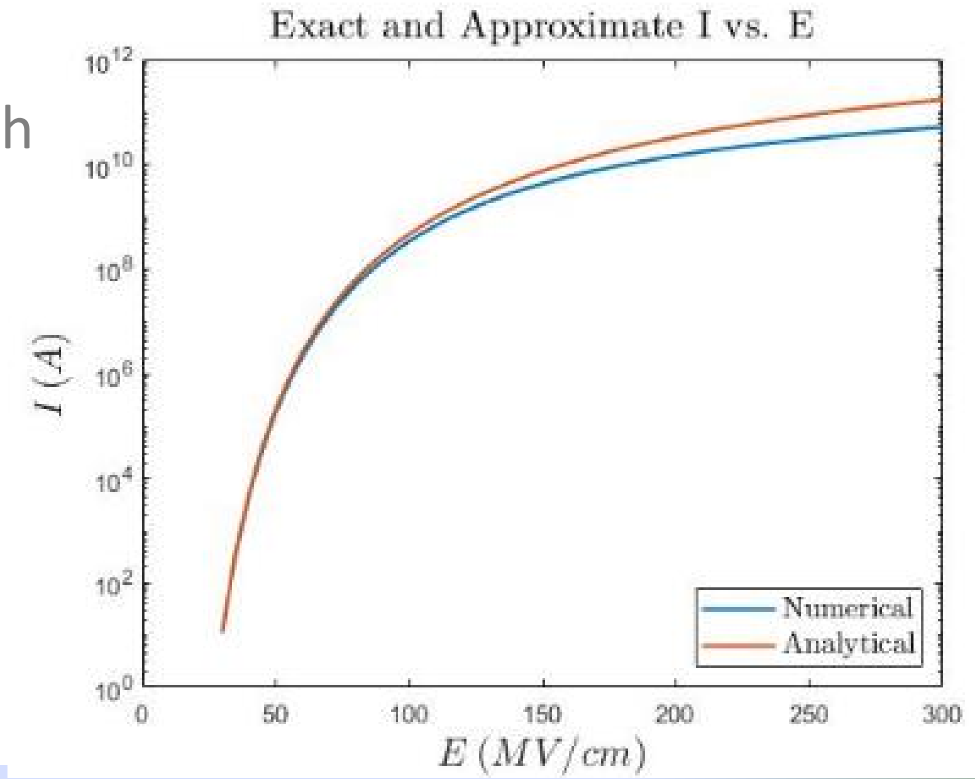
$$\sigma_W = S\sqrt{\big(1 - D(W)\big)D(W)N(W)}$$

Numerical Simulation

- The resulting current agrees with the analytical formula[2] at fields up to about 40MV/cm with an error of up to 10%.

$$I = \frac{ASE^2}{\phi t^2(s)} \exp\left(-\frac{B\phi^{3/2}v(s)}{E}\right); s = C \cdot E^{0.5}/\phi$$

- While this field (4,000MV/m) seems big, the CLIC FG system exhibits an applied field of 35MV/m with a measured $\beta = 300 - 400$ so that the "emitter" field is on the order of 10,000MV/m (100MV/cm).



[2]J.W. Wang and G. A. Loew. Field emission and rf breakdown in high-gradient room-temperature linac structures. Technical report, .S. Department of Energy, 1997.

Impact on FN Plot Analysis

- The most commonly used analysis method of dark currents and β estimation is that of the FN plot.
- This method uses an approximated formula of the current[2]

$$v(s) \cong 0.956 - 1.062 s^2 \qquad t \approx 1$$

- This approximation only holds for values of $s \approx 0.5$ ($E = 37MV/cm$) and adds additional errors to the already approximated analytical equation.

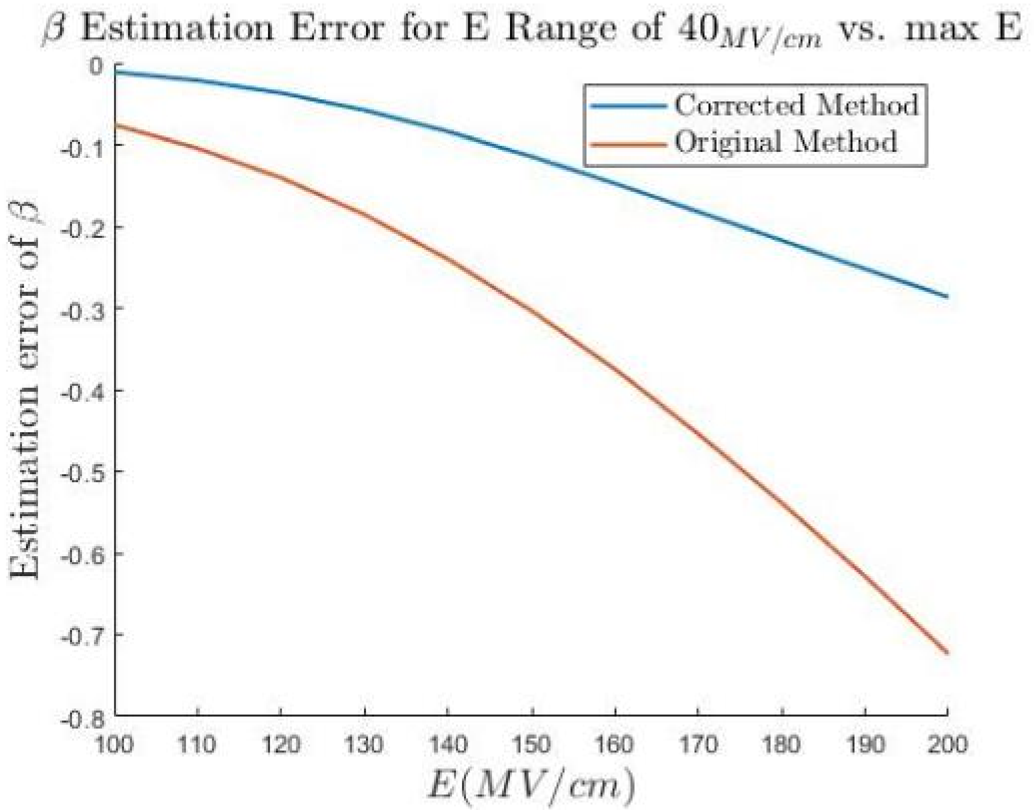
$$I = \frac{ASE^2}{\phi t^2(s)} \exp\left(-\frac{B\phi^{3/2}v(s)}{E}\right) \longrightarrow I = \frac{AS10^{4.52\phi^{-0.5}}E^2}{\phi} \exp\left(-\frac{0.956B\phi^{3/2}}{E}\right)$$

[2] J.W. Wang and G. A. Loew. Field emission and rf breakdown in high-gradient room-temperature linac structures. Technical report, .S. Department of Energy, 1997.

Application of Forbes Approximation

- By taking a better approximation of $v(s)$ and $t(s)^3$ we can arrive at a much better approximation with little effort.

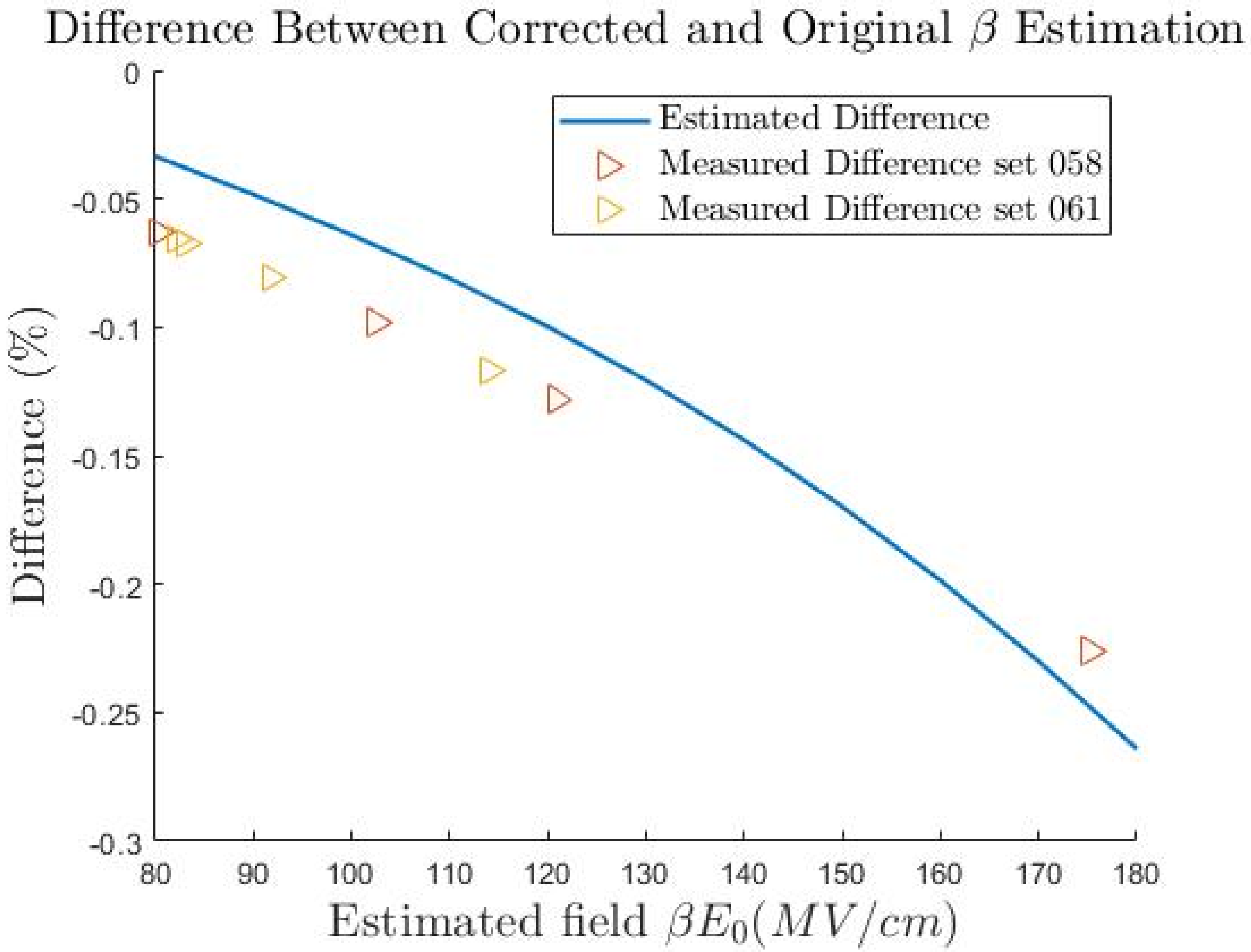
$$v(s) = 1 - s^2 + \frac{1}{3}s^2 ln(s) \qquad t = v(s) - \frac{2}{3}s\frac{dv}{ds}$$



[3] R. G. Forbes. Simple good approximations for the special elliptic functions in standard fowler-nordheim tunneling theory for a schottky-nordheim barrier. Appl. Phys. Lett., 89(113122), 2006.

Impact on FN Plot Analysis – Measurements

- The corrected algorithm was applied on measurements from the CLIC FG system.
- This plot shows the difference between the estimation of the field from the current and corrected methods, along with the estimated difference from the simulation.
- As can be seen, the measured difference is close to the estimated values.



Measurements by Iaroslava Profatilova

Issues with the FN Plot

- Two issues with the current method of β estimations are:
  1. This method assumes a constant β over different field values and therefore cannot detect changes in β as a function of field.
  2. The time required to get a complete scan of the electric field required to get an estimation is quite long (might be as high as 20 minutes).
- Can we switch to instantaneous measurements?
- Such a measurement will hopefully allow a much shorter estimation time (a few seconds at most).
- The shorter estimation time means that β evolution can be observed at much shorter time scales and for each field independently.

Field Variation

- Here we try to put to use the fact that the applied voltage on the structure isn't constant – whether by fluctuations in the power supply or from RF interference.
- The resulting fluctuations in the applied field practically scan of a small field range.
- From the fluctuation theory (using σ_I and σ_E the STD of current and applied field):

$$\frac{\sigma_I}{\sigma_E} = \frac{\partial I}{\partial E_0}$$

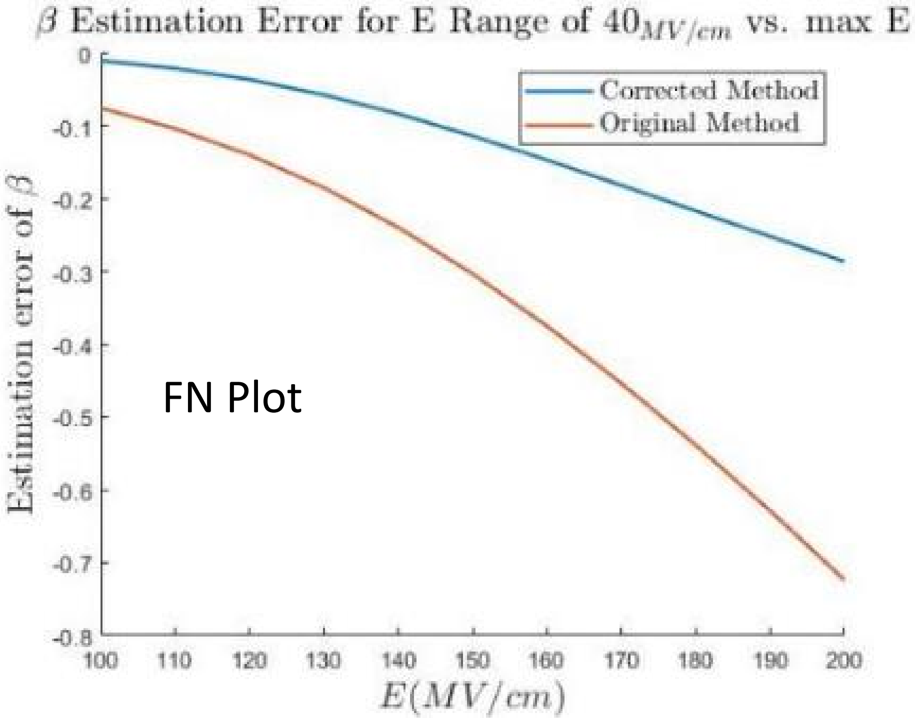
- Removing dependency on the unknown emitter surface area

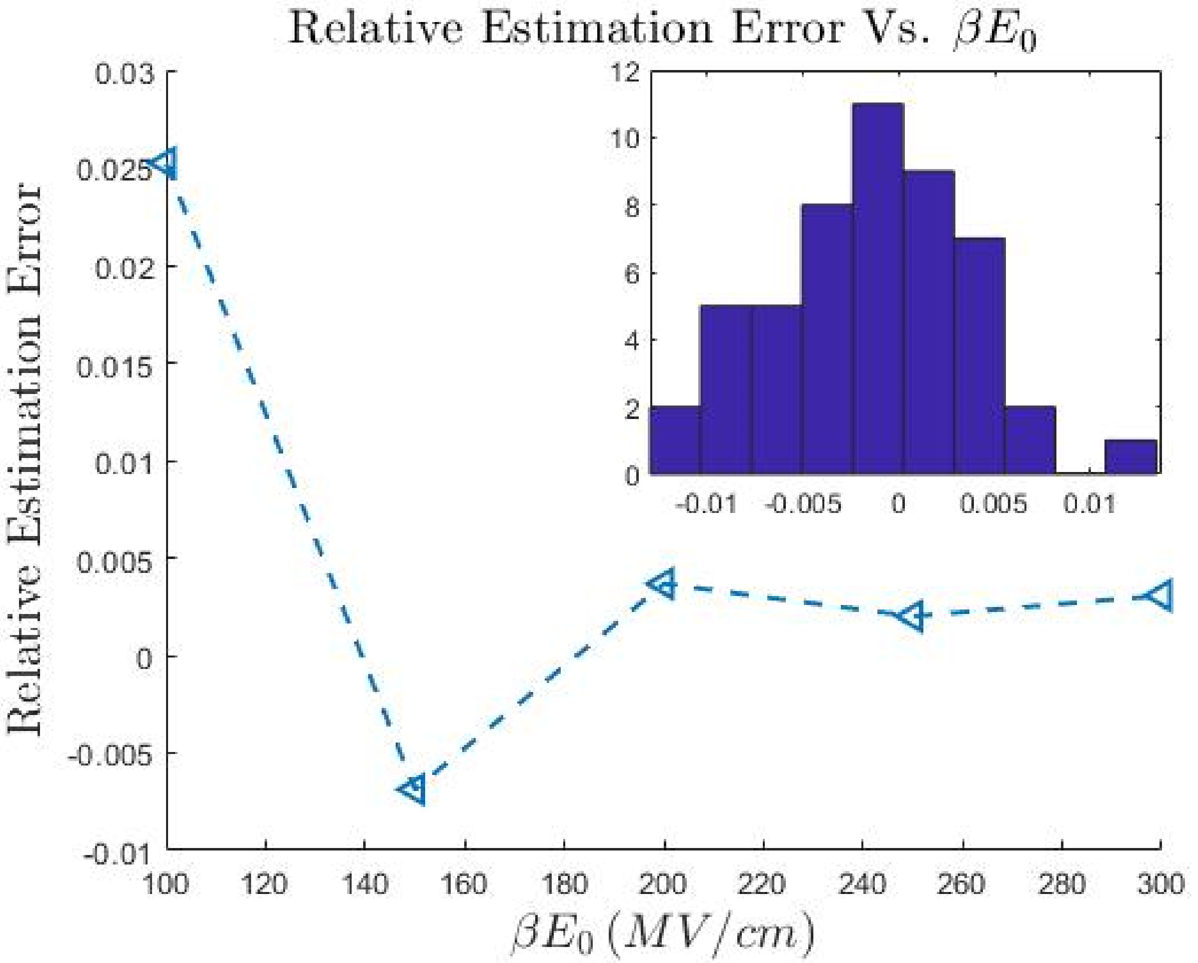
$$\frac{1}{I_0}\frac{\sigma_{I_0}}{\sigma_E} = \frac{\beta}{I}\frac{\partial I}{\partial E}$$

With σ_{I_0} the *measured* current STD, I_0 the *measured* mean current and $\frac{1}{I}\frac{\partial I}{\partial E}$ the derivative of the current by the emitter field normalized by the current.

Field Variation – Estimation Error

- This method improves the FN plot analysis errors.
- And can be applied by actively varying the field (possibly allowing for even shorter measurements).





Shot Noise - Theory

- Shot noise (In the context of FN currents) is due to quantum fluctuations of the measured current around the theoretical mean value.
- Modeling the statistical nature of the system as a Gaussian, we get that for a given incident energy of electrons W:

$$\mu_W = SD(W,E)N(W)$$
$$\sigma_W = S\sqrt{D(W,E)\big(1 - D(W,E)\big)N(W)}$$

- The ratio of these two is:

$$1 - \frac{\sigma_W}{\mu_W} = 1 - \sqrt{\frac{\big(1 - D(W,E)\big)}{D(W,E)N(W)}}$$

- This result is independent of the tunneling surface S.

Shot Noise - Theory

- For a current that is the sum of many energies:

$$\mu = \int \mu_W dW = S\int D(W,E)N(W)dW$$

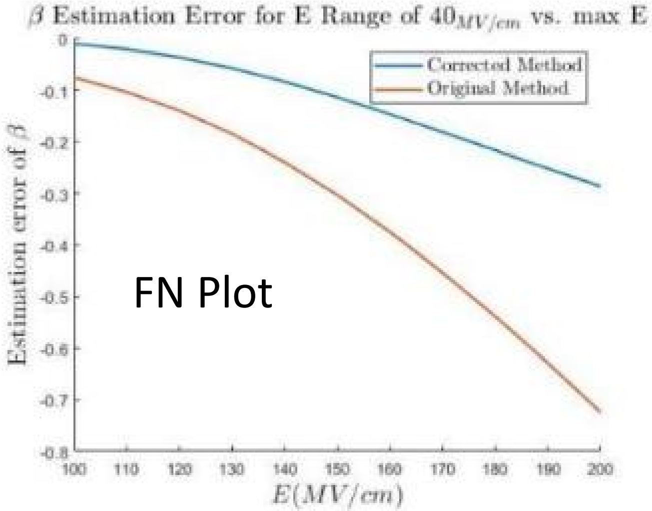
$$\sigma = \sqrt{\int \sigma_W^2 dW} = S\sqrt{\int D(W,E)\big(1 - D(W,E)\big)N(W)dW}$$

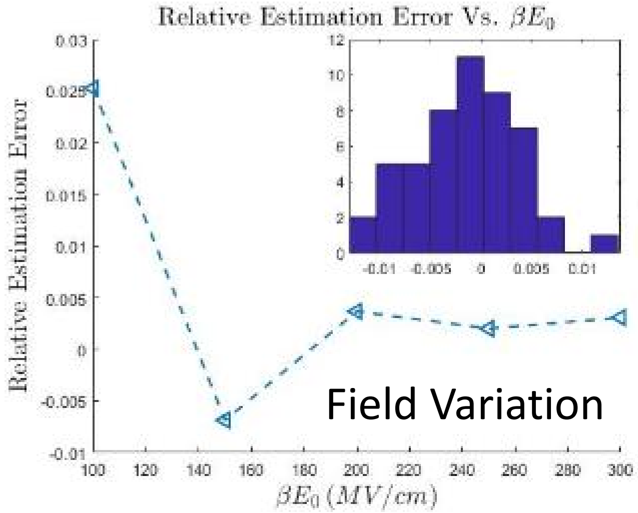
- The ratio of these two, while more complicated and not easily written analytically, is still vitally independent of S.
- Evaluating the ratio of the shot noise relative to the mean current we get an expression independent of S that depends only on the emitter field:

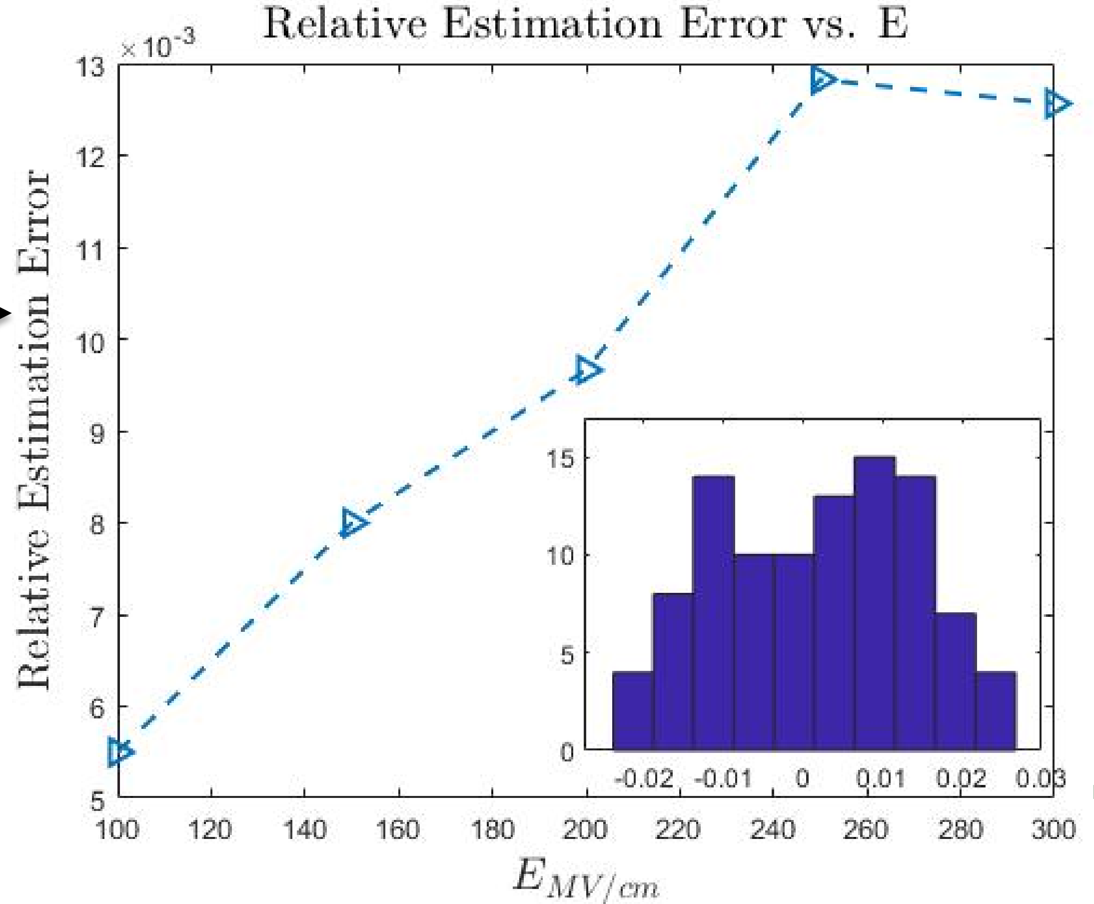
$$1 - \frac{\sigma}{\mu} = 1 - \frac{\int D(W,E)N(W)dW}{\sqrt{\int D(W,E)\big(1 - D(W,E)\big)N(W)dW}} \equiv D(E)$$

Shot Noise - Simulation

- Calculating the variable D numerically as a function of the field E, a reference data set was generated.
- Using a Monte Carlo simulation of currents, the variable D was used to estimate the emitter field.
- If measurable, this may lead to improved accuracy.

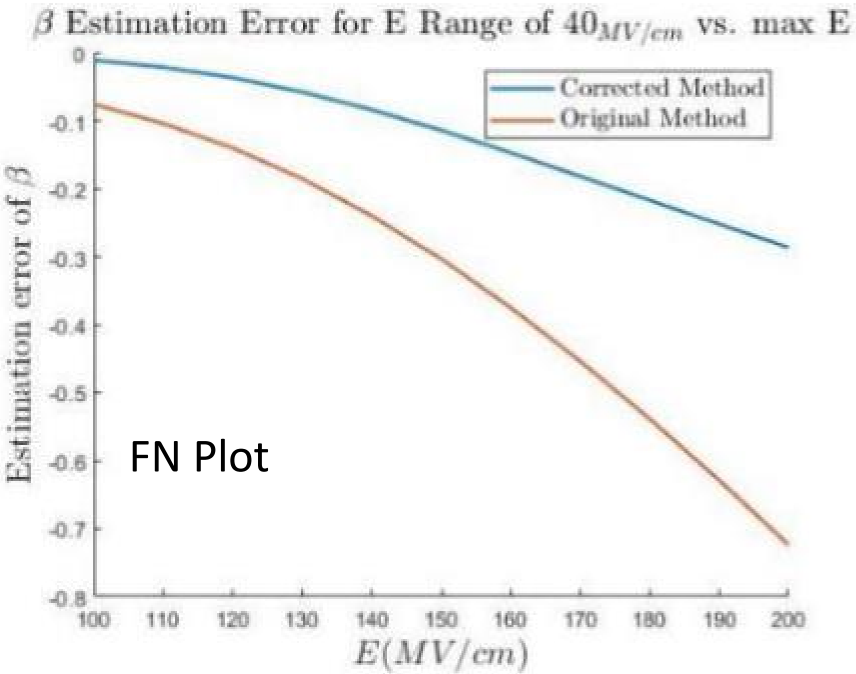


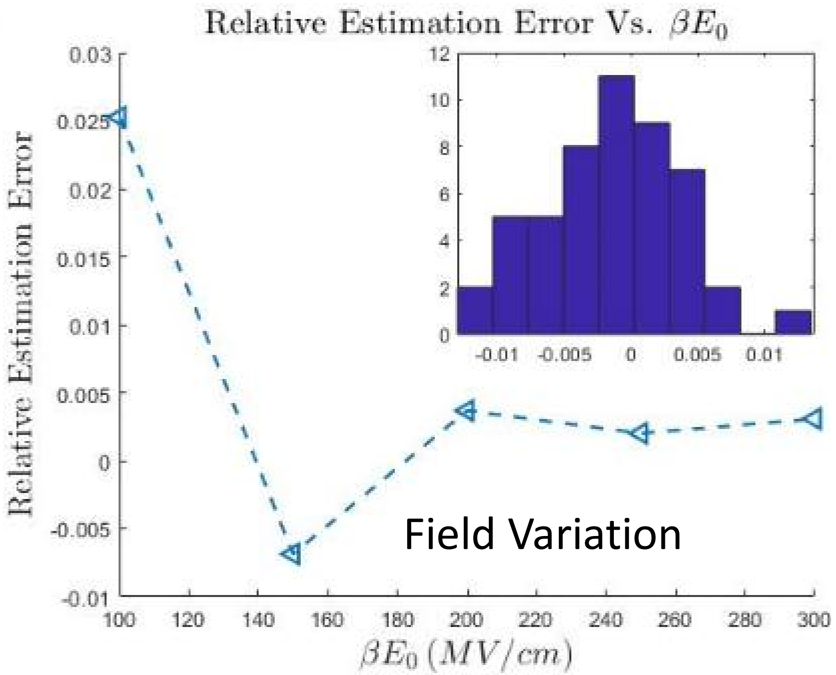


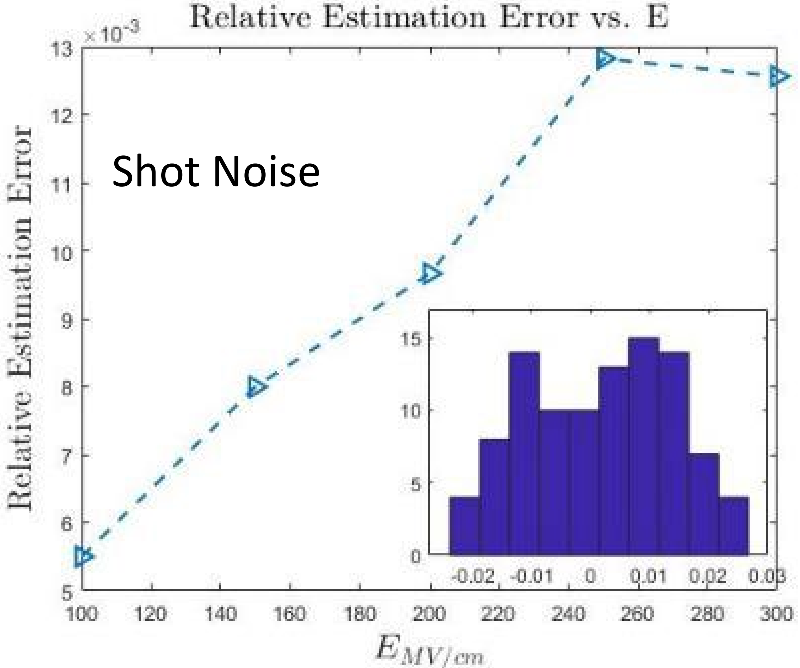


Conclusions

- We applied the Forbes correction to the current FN plot analysis.
- This leads to improved β estimation.
- We've demonstrated two optional new methods for β estimation.
- These methods may allow for real time monitoring of high field systems, as well as characterization of the systems as a function of field.







THANK YOU!