# 3D Modeling of field electron emission from micro-structured surfaces

## Exploration of thermal interaction between intensively emitting close tips

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# Motivations

CONTEXT :

Intense electron emission from **many asperities** in vacuum

→

Dark current leakage and plasma discharge in high voltage devices

Need for understanding the physical phenomena at stake

PART OF THE ANSWER : SIMULATIONS

2D axi-symmetric simulation :

- small computation time
- easier to process data
- isolated asperity

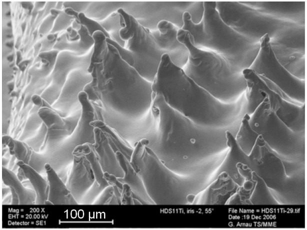
**3D simulation :**

- long computation time
- asperities proximity
- **closer to reality**

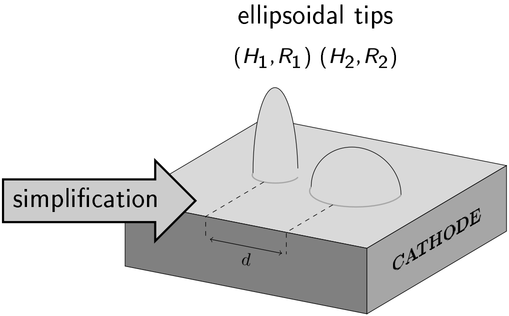
# Simulation scope

We want to study tips interaction

- Reduce interaction to only two tips.
- Use simplified tip profile : ellipsoid.



Ti electrode surface aspect after having arced [Antoine,2012]



Scheme of a typical configuration

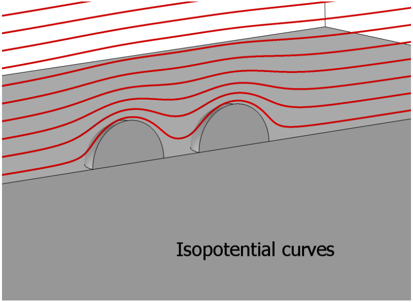
Highly simplified but can't use 2D axial symmetry anymore : need for 3D.

# Asperities interactions

Tips proximity leads to two interaction types :

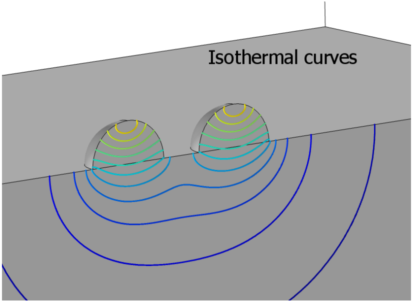
**Electrostatic screening**

- has a major effect on electron emission
- is well studied



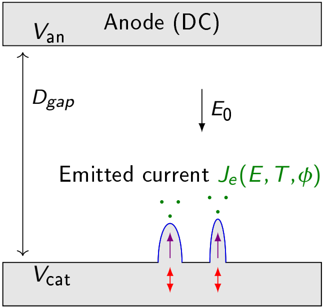
**Thermal coupling**

- hasn't been much explored
- what magnitude ?



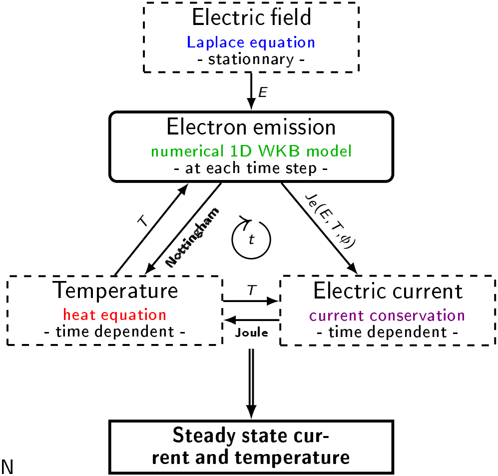
1. **Modeling background**
2. Procedure
3. Results
4. Conclusions and outlooks

# Simulation steps



⬚ solved with COMSOL

▢ home-made model solved in FORTRAN



Electric field
Laplace equation
- stationnary -

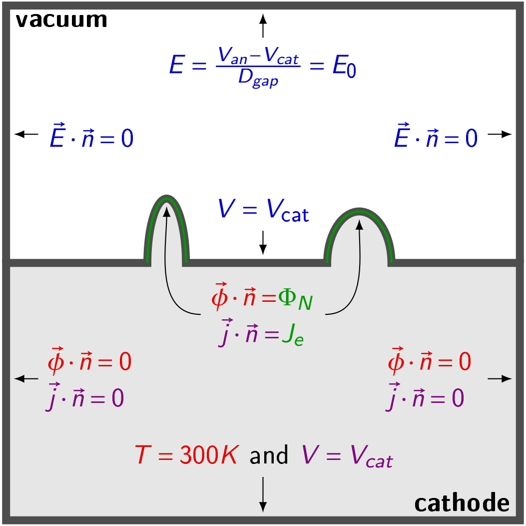
Electron emission
numerical 1D WKB model
- at each time step -

Temperature
heat equation
- time dependent -

Electric current
current conservation
- time dependent -

**Steady state current and temperature**

# Boundary conditions



Simulation domain

The electric field equation is solved in vacuum.

Our electron emission model is solved at the tips surfaces.

Heat and current equations are solved together in the cathode.

# Parameters range and example

Ellipsoidal tips, $f = H/R \equiv$ aspect ratio, $\beta_a \equiv$ apex field enhancement factor

$f = 1,\ \beta_a = 3$ $\quad$ $f = 2,\ \beta_a \sim 6$ $\quad$ $f = 5,\ \beta_a \sim 17$ $\quad$ $f = 10,\ \beta_a \sim 50$

$H$ and $R$ both range from 1 to 10 $\mu m$

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**Example for two identical tips :**

$H = 10\mu m,\ R = 1\mu m$ and $d = 3\mu m$

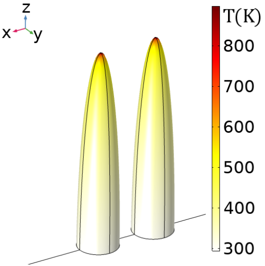
- **titanium tips** :

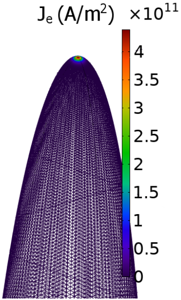
$\Phi = 4.3eV$ (homogenous)

- **applied voltage** :

$\Delta V = 32kV$ on $D_{gap} = 200\mu m$

$\Rightarrow E_0 = 1.6 \times 10^8\,V/m$





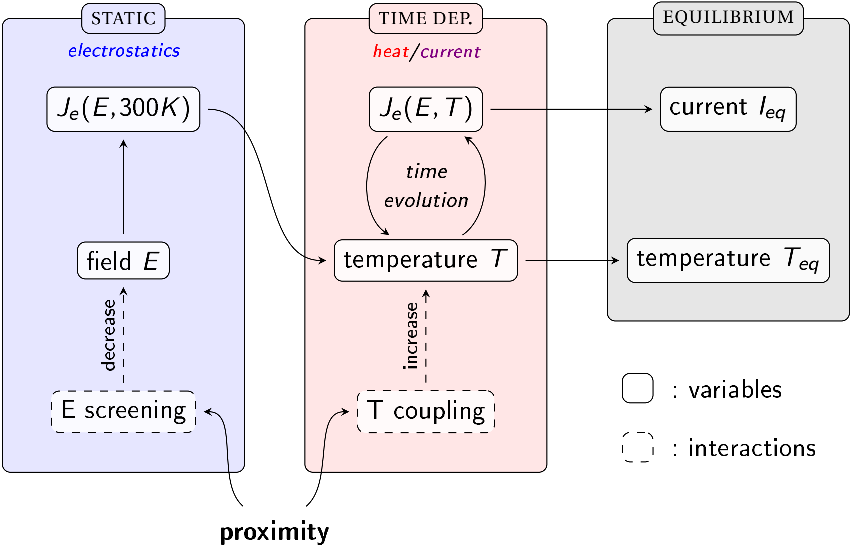
1. Modeling background

2. **Procedure**

3. Results

4. Conclusions and outlooks

# Study method



STATIC

*electrostatics*

$J_e(E, 300K)$

field $E$

decrease

E screening

TIME DEP.

*heat*/*current*

$J_e(E,T)$

*time evolution*

temperature $T$

increase

T coupling

EQUILIBRIUM

current $I_{eq}$

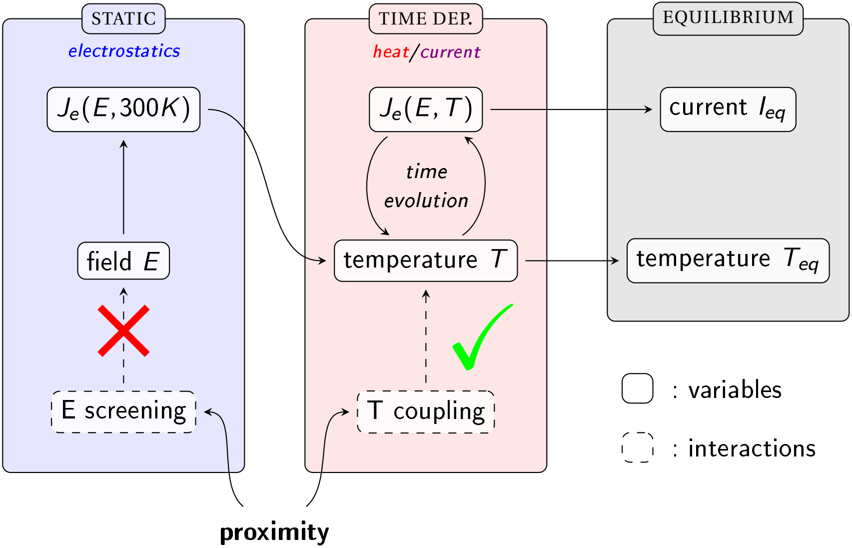
temperature $T_{eq}$

☐ : variables

⬚ : interactions

**proximity**

# Study method



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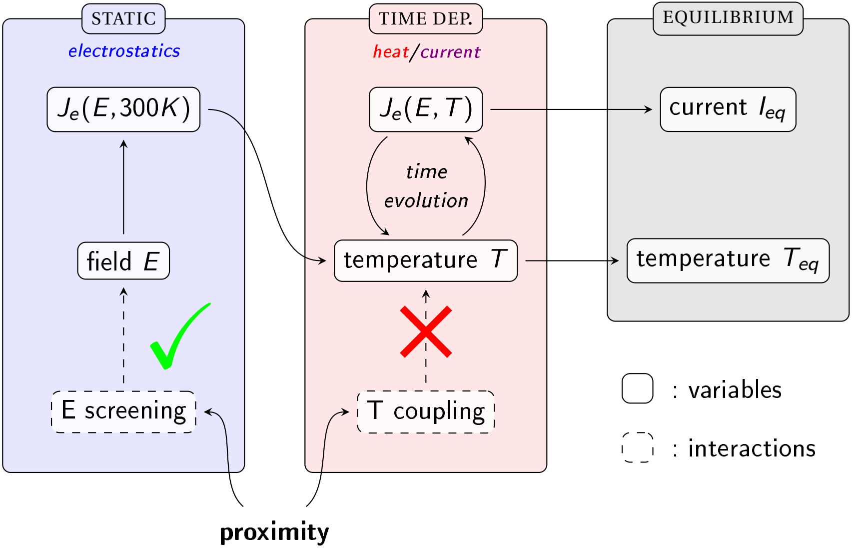

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✓

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*time evolution*

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T coupling

✗

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**proximity**

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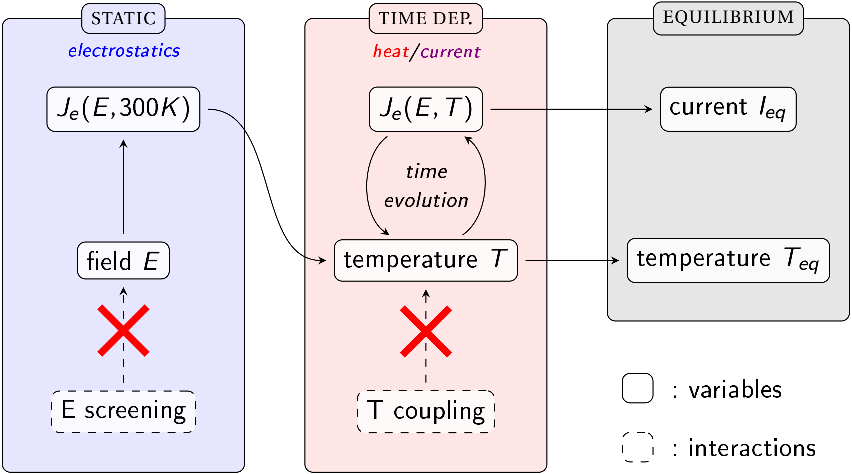

STATIC

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$J_e(E, 300K)$

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▢ : variables

⬚ : interactions

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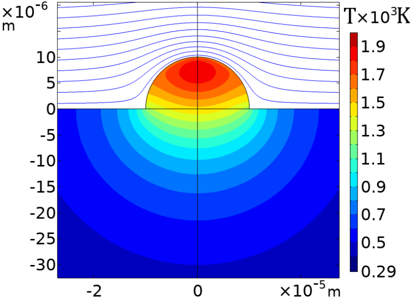
# Reference with only one tip



2D sectional view showing isopotentials in vacuum and isothermals inside the cathode

✗ E screening

✗ T coupling

Tip parameters :

$$\boxed{\begin{array}{l} H = 10\mu m,\ R = 10\mu m \\ \phi = 4.3eV \\ E_0 = 1.87V/nm,\ \beta = 3 \end{array}}$$

Reference current at equilibrium :

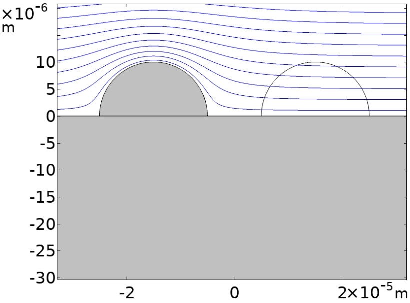
$$I_{eq} = 3.49\ A$$

Reference maximum temperature at equilibrium :

$$T_{eq}^{max} = 1897\ K$$

# Two tips with thermal coupling only

two tips at d=3R (3D)



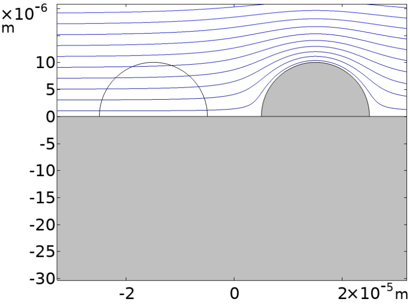
2D sectional view showing isopotentials in vacuum

✗ E screening

$d = 3R$, identical tips
$\phi = 4.3 eV$
$E_0 = 1.87 V/nm$, $\beta = 3$

# Two tips with thermal coupling only

two tips at d=3R (3D)



2D sectional view showing isopotentials in vacuum

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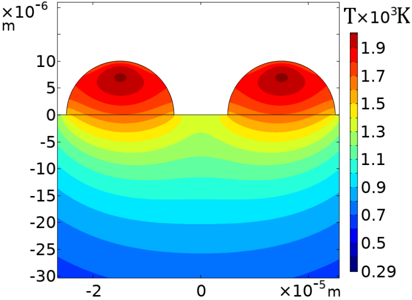

2D sectional view showing isothermals inside the cathode

$d = 3R$, identical tips
$\phi = 4.3eV$
$E_0 = 1.87V/nm,\ \beta = 3$

✗ E screening

✓ T coupling

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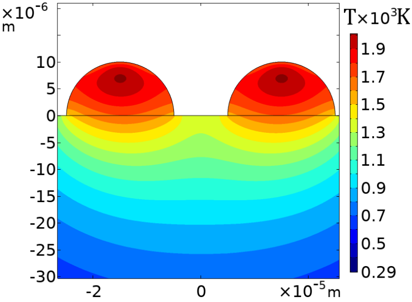


2D sectional view showing isothermals inside the cathode

✗ E screening

✓ T coupling

$d = 3R$, identical tips
$\phi = 4.3eV$
$E_0 = 1.87V/nm,\ \beta = 3$

**Effect of thermal coupling only**

Current *per tip* at equilibrium :

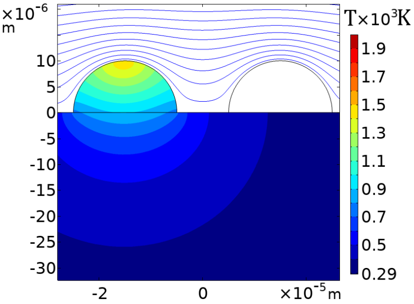
$$I_{eq} = 3.72\ A$$

Maximum temperature at equilibrium :

$$T_{eq}^{max} = 2011\ K$$

⇒ **variation of** +6.6% **in current and** +6.0% **in temperature with respect to reference.**

# Two tips with electrostatic screening only


two tips at d=3R (3D)

2D sectional view showing isopotentials in vacuum and isothermals inside the cathode

✓ E screening

✗ T coupling

$d = 3R$,identical tips
$\phi = 4.3eV$
$E_0 = 1.87V/nm,\ \beta = 3$

**Effect of electrostatic screening only**

Current *per tip* at equilibrium :

$$I_{eq} = 1.93\ A$$

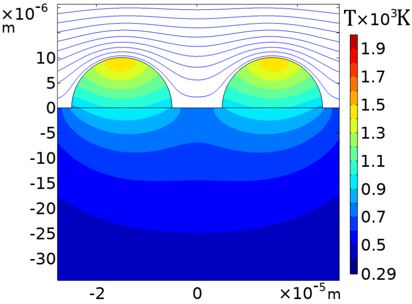
Maximum temperature at equilibrium :

$$T_{eq}^{max} = 1429\ K$$

**⇒ variation of $-50\%$ in current and $-25\%$ in temperature with respect to reference.**

# Results with two close tips

two tips at d=3R (3D)



2D sectional view showing isopotentials in vacuum and isothermals inside the cathode

✓ E screening

✓ T coupling

$d = 3R$, identical tips
$\phi = 4.3eV$
$E_0 = 1.87V/nm,\ \beta = 3$

**Effect with both interactions**

Current *per tip* at equilibrium :

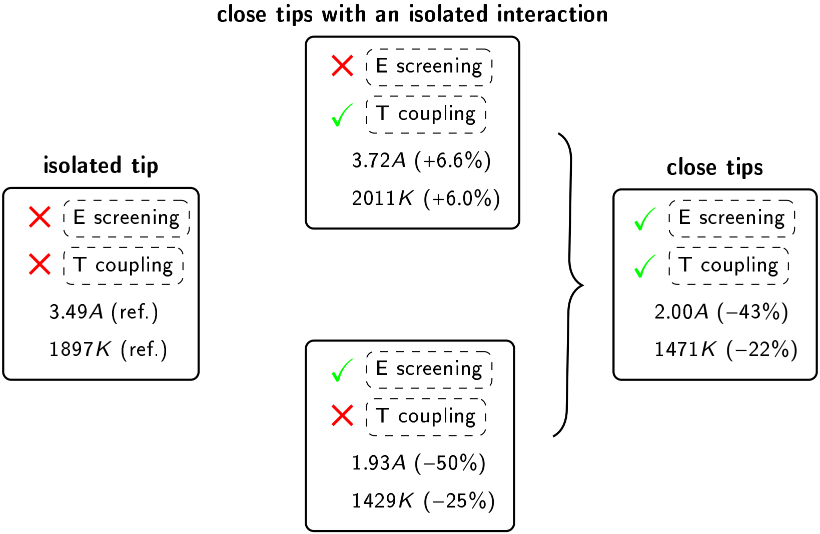
$$I_{eq} = 2.00\ A$$

Maximum temperature at equilibrium :

$$T_{eq}^{max} = 1471\ K$$

**⇒ variation of $-43\%$ in current and $-22\%$ in temperature with respect to reference**

# Recap



**isolated tip**

- ✗ E screening
- ✗ T coupling

3.49$A$ (ref.)

1897$K$ (ref.)

**close tips with an isolated interaction**

- ✗ E screening
- ✓ T coupling

3.72$A$ (+6.6%)

2011$K$ (+6.0%)

- ✓ E screening
- ✗ T coupling

1.93$A$ (−50%)

1429$K$ (−25%)

**close tips**

- ✓ E screening
- ✓ T coupling

2.00$A$ (−43%)

1471$K$ (−22%)

1. Modeling background

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# Conclusions and outlooks

To sum up :

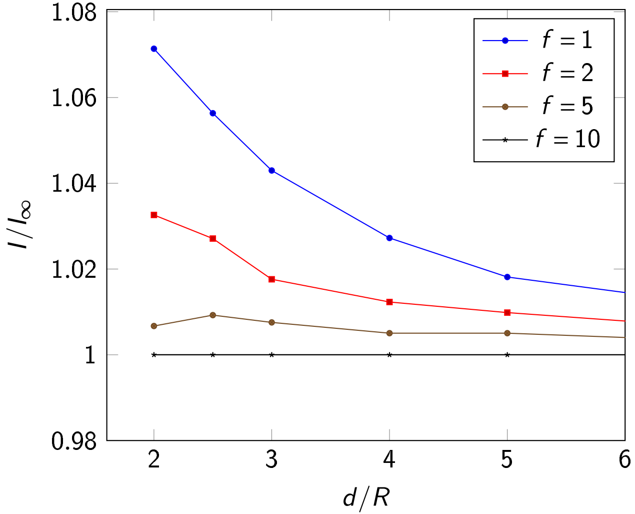
- Tips proximity increases both thermal and electrostatic interactions.
- Thermal coupling can have a noticeable effect in specific configurations (large asperities, high temperature).
- Electrostatic screening reduces thermal coupling effect and clearly makes it a second order phenomenon in terms of magnitude compared to screening itself.

What's next :

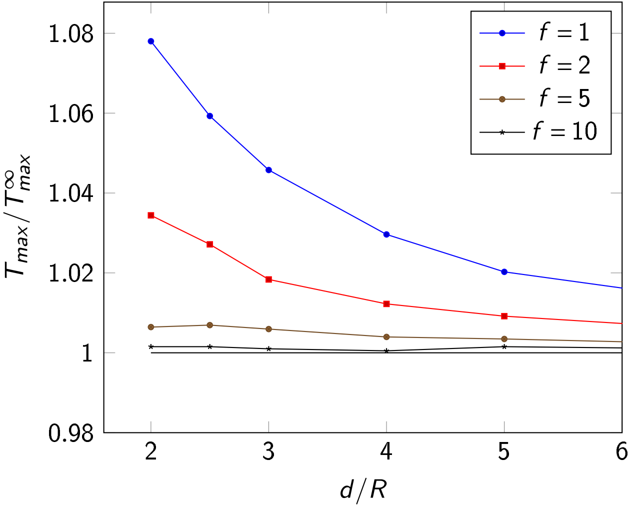
- Explore with refractory metals and other geometries.
- Explore with more tips (ex : tips array).
- Use these results to propose a simplified approach to simulate complex 3D configurations (paper to come).

# 5. Back up slides

# Parameters exploration



# Parameters exploration

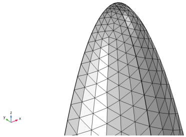


Visualisation of isolated thermal coupling effect on temperature versus distance at breaking potential for different aspect ratios

# Gross evaluation of mesh related error

For a tip alone ($H = 10\mu m$, $R = 1\mu m$, $E_0 = 1.6 \times 10^8 V/m$ with titanium), we compare the results of a 2D axi-symetric ultra finely meshed result to an equivalent in 3D with different meshes.

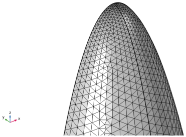
The reference current is : $I_{2D}(300K) = 11.177\ mA$



mesh1 : gross
($3.5 \times 10^5$ elements)

$\sigma_E^* = 2.44\%$

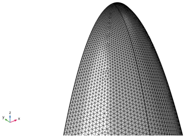
$I_{3D}(300K) =$
$9.3577\ mA$
variation of -16.3%



mesh2 : acceptable
($4.6 \times 10^5$ éléments)

$\sigma_E^* = 0.641\%$

$I_{3D}(300K) =$
$10.879\ mA$
variation of -2.67%



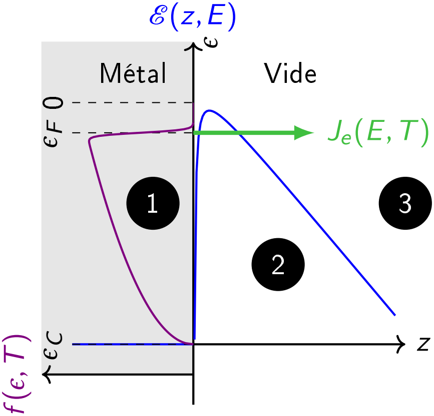
mesh3 : usual
($1.1 \times 10^6$ elements)

$\sigma_E^* = 0.167\%$

$I_{3D}(300K) =$
$10.999\ mA$
variation of -1.59%

# Field electron emission model

Code input : initial temperature $T$ and linear Efield $E$



1. from $T \rightarrow$ Sommerfeld Energy Distribution Function of electrons : $f(\epsilon, T)$
2. from $E \rightarrow$ Linear electric field + image charge correction : $\mathcal{E}(z,E)$
3. Numerical computation of current density† : $J_e(E,T)$

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†. where **the transparency coefficient** $D$ is obtained through numerical solving of 1D Schrödinger equation with WKB approximation and numerical compuation of the elliptic integral functions.

# Heat sources : Joule

Current in the tip volume

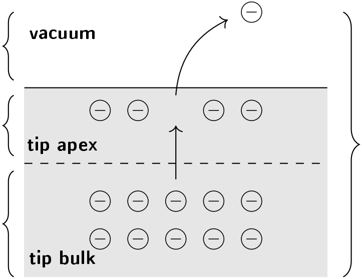
→

**Joule heating :**
$p_J = \rho j^2$

- can only heat
- heat is dissipated in volume : $[p_J] = W.m^{-3}$

# Heat sources : Nottingham

Electron emission at the tip apex → **Nottingham flux**



*emitted electrons :*
*mean energy* $= \langle \epsilon \rangle_{em}$

*surface electrons*

*replacement electrons :*
$\langle \epsilon \rangle_{rep} = \mu(T) \sim \epsilon_F$

*Average energy change for each emitted electron :*

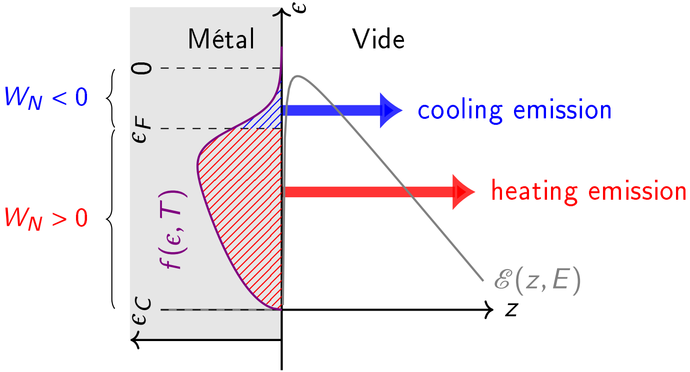
$$W_N(T) = \langle \epsilon \rangle_{rep} - \langle \epsilon \rangle_{em}(T)$$

$$\Rightarrow \text{ Heat flux : } \Phi_N(E,T) = \frac{J_e(E,T)}{-e} W_N(T)$$

- can heat or cool ($W_N$ can be positive or negative)
- heat flux from the emission surface : $[\Phi_N] = W.m^{-2}$

# Heat sources : Nottingham

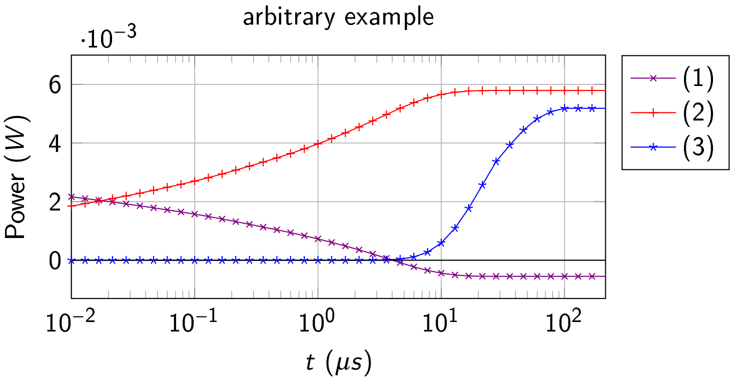
Theoretical view of Nottingham cooling or heating



$$\boxed{W_N = \epsilon_F - \epsilon}$$

- heat when emitting cold electrons
- cool when emitting hot electrons

# heat evolution toward equilibrium



at final time $t_f$ :

$$\overbrace{\iint_{S_{ext}} \Phi_N(t_f) \cdot \mathbf{dS}}^{(1)\,:\,\text{Nottingham heating}} + \overbrace{\iiint_{V_{\mathbf{tot}}} p_J(t_f) dV}^{(2)\,:\,\text{Joule heating}} = \overbrace{\iint_{S_{int}} \mathbf{q}(t_f) \cdot \mathbf{dS}}^{(3)\,:\,\text{output heat flux}} \qquad (1)$$