A "NEARLY SEMI-QUANTITATIVE" EXPLANATION OF ELECTRICAL BREAKDOWN EFFECTS REPORTED BY JULIUS CAESAR AND PLINY THE ELDER

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MeVARC 2019, Padova, Sept. 2019

Introduction

Julius Caesar – "De Bello Africo", 47

47. About this time a most incredible accident befell Caesar's army; for the Pleiades being set, about the second watch of the night, a terrible storm arose, attended by hail of an uncommon size. But what contributed to render this misfortune the greater was, that Caesar had not, like other generals, put his troops into winter quarters, but was every three or four days changing his camp, to gain ground on the enemy; which keeping the soldiers continually employed they were utterly unprovided with any conveniences to protect them from the inclemency of the weather.

Besides, he had brought over his army from Sicily with such strictness, that neither officer nor soldier had been permitted to take their equipages or utensils with them, nor so much as a vessel or a single slave; and so far had they been from acquiring or providing themselves with any thing in Africa, that, on account of the great scarcity of provisions, they had even consumed their former stores. ….

Julius Caesar – "De Bello Africo", 47

Impoverished by these accidents, very few of them had tents; the rest had made themselves a kind of covering, either by spreading their clothes, or with mats and rushes. But these being soon penetrated by the storm and hail, the soldiers had no resource left, but wandered up and down the camp, covering their heads with their bucklers to shelter them from the violence of the weather. In a short time the whole camp was under water, the fires extinguished, and all their provisions washed away or spoiled. The same night the shafts of the javelins belonging to the fifth legion, of their own accord, took fire.

In short it was:

- **Stormy – presumably high wind**
- **Wet**
- **Dark**

Pliny the Elder – "De Naturalis" – Chap. XXXVII

Chapter XXXVII.

Of the Stars called Castor and Pollux[1].

I have seen myself, in the Camp, from the Sentinels in the Night-watch, the Resemblance of Lightning to fix on the Spears set before the Rampart. They settle also upon the Yards, and other Parts of the Ship, at Sea: making a Kind of vocal Sound, and shifting their Places as Birds do which fly from Bough to Bough. They are dangerous when they come singly, for they sink those Ships on which they alight;

Pliny the Elder – "De Naturalis" – Chap. XXXVII

or they set them on Fire if they fall upon the Bottom of the Keel. But if the Pair appear, they are salutary, and foretel a prosperous Voyage; for by their coming, it is supposed that the dreadful and threatening Meteor called *Helena*, is driven away. And therefore it is, that Men assign this mighty Power to *Castor* and *Pollux*, and invocate them as Gods at Sea. Men's Heads, also, in the Evening are seen to shine round about; which presageth some great Matter. Of all these Things there is no certain Reason to be given; but they are hidden in the Majesty of Nature.

This phenomenon is commonly called St Elmo's fire.

St Elmo's fire

St. Elmo's fire on a ship at sea

St Elmo's fire is a weather phenomenon in which a luminous "ball" or "flame" is created at the tip of a tall pointed object, such as a ship's mast.

It has been reported by many reliable witnesses including:

- Julius Caesar & Pliny the elder;
- 15th century Chinese admiral Zheng He;
- Charles Darwin, when HMS Beagle was anchored in the estuary of the Rio de la Plata.

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St Elmo's fire has also been reported (formally or informally) at the tips of many other long, pointed objects, including: spear tips; church spires; vertical pipes; trees; fingers; cattle horns; and the ears of a horse. [See Wikipedia for more details.]

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St. Elmo's fire on a ship at sea

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St. Elmo's fire on a ship at sea

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it is now known that it is a form of Corona Discharge.

Current explanations are largely qualitative

Aims of talk

Aim of talk is to provide better physical understanding – where possible "nearly semi-quantitative" understanding – of the occurrence of various corona discharge phenomena, in particular St Elmo's fire.

This understanding draws on:

(1) Ideas and formulae developed in the context of the electrostatics of field electron emitter arrays – particularly carbon nanotube (CNT) arrays

(2) Basic ideas of "electrical thermodynamics".

Ideas in (1) were largely developed in a collaboration between myself, Prof. Thiago de Assis of the Federal University of Bahia, Brazil and Prof. Fernando Dall'Agnol of the Federal University of Santa Catarina, Brazil.

Structure of Talk

1. Introduction
2. Elements of field emitter electrostatics
3. Elements of electrical thermodynamics
4. Electric-field requirements for St Elmo's fire
5. Ships' masts & Roman Javelins
6. The case of Surgeon Braid's horse
7. The possible role of Gilbert-Gray cone-jets ("Taylor cones")
8. Possible larger-scale applications
9. Summary

Elements of field emitter electrostatics

2a: The single-emitter case

Single emitters – the paradigm situation

We need to understand single-emitter electrostatics.

The paradigm model situation (taken as a starting point) is a:
single, perfectly conducting, post-like emitter standing on one of a pair of infinitely large (laterally) parallel plane plates, with separation very much greater than the post height. The post itself is modelled as a cylinder, capped by a hemisphere.

This model situation is termed the hemisphere-on-cylindrical-post (HCP) model.

Terminology: "plate field"

voltage between plates V_p ("plate voltage") (+ve)

separation of plane-parallel electrodes d_{sep}

Plate field (–ve) $E_P = -V_p/d_{sep}$

Terminology: "field enhancement factor"

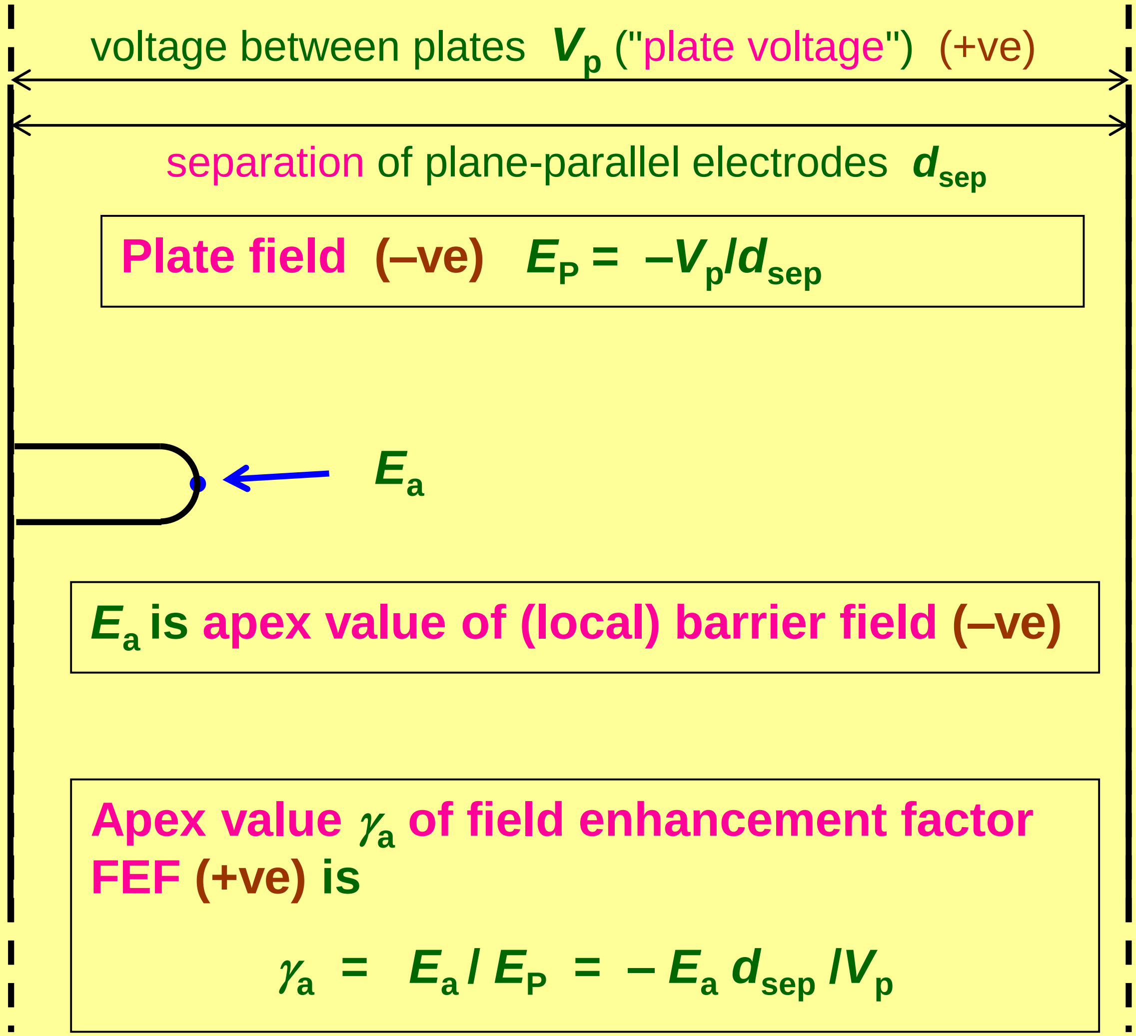

Plate field (–ve) $E_P = -V_p/d_{sep}$

E_a **is apex value of (local) barrier field (–ve)**

Apex value γ_a of field enhancement factor FEF (+ve) is

$$\gamma_a = E_a / E_P = -E_a d_{sep} / V_p$$

FE electrostatics as electron thermodynamics

FE electrostatics is primarily a branch of electron thermodynamics, NOT primarily a branch of Coulomb/Maxwell-type classical electrostatics (though this also is involved).

The underlying physical principle is that:

The condition for an electron conductor to be in thermodynamic equilibrium is that the Fermi Level be the same everywhere in the conductor.

This condition was established by Fowler in the 1930s.

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This condition was established by Fowler in the 1930s.

For simplicity (at this stage of the subject's development), the assumption is nearly always made that the local work function is the same at all points on the emitter surface. This generates the modelling assumption that:

The classical electrostatic potential has the same value (normally taken as zero) at all points immediately outside the surface of an emitter *in internal thermodynamic equilibrium.*

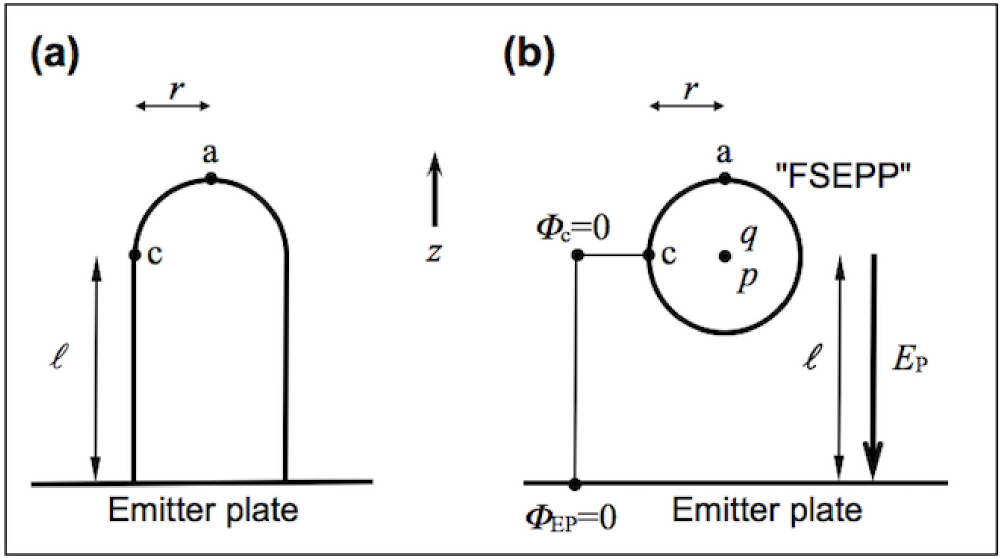
"Floating Sphere at Emitter Plate Potential" model

For the single emitter case, there is a particularly simple model. This is a basic version of the so-called Floating Sphere at Emitter Plate Potential (FSEPP) model. It involves only a single charge and a single fitting point for potential, taken on the sphere equatorial plane (as shown overleaf).

"Floating Sphere at Emitter Plate Potential" model

In the lowest approximation, the usual "Floating Sphere at Emitter Plate Potential (FSEPP)" model works as follows:

0) The electrostatic potential of the emitter plate is taken as zero, i.e. $\Phi_{EP} = 0$.



1) Sphere is in uniform (–ve) field E_P; this contributes a (+ve) amount $-E_P l$ [$=|E_P|l$] to the electrostatic potential Φ_c at point "c".

2) To get Φ_c to zero, place (–ve) charge q at sphere centre, of size such that

$$q/4\pi\varepsilon_0 r \; - \; E_P l = 0 \, .$$

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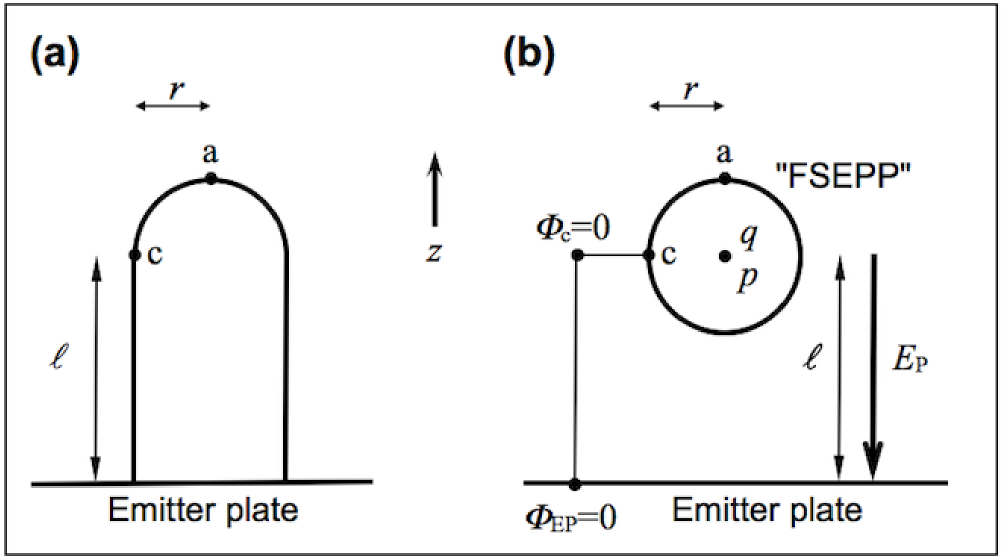

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$$q/4\pi\varepsilon_0 r \; - \; E_P l = 0 \; .$$

3) The (–ve) apex field E_a is: $\quad E_a = q/4\pi\varepsilon_0 r^2 = E_P(l/r)$.

4) Hence the (+ve) zero-current apex FEF γ_a^{zc} is given (in this model) by:

$$\gamma_a^{zc} = E_a/E_P = l/r \; .$$

The conducting-post & floating-rod formulae

For practical emitters the ratio l/r >>1, and it is usual to put the total post height h [= $l+r$] into the formula.

Also, more exact treatments yield a correction factor α , typically around 0.7 for the HCP model. Thus, the final result is:

$$\gamma_a^{zc} = \alpha \cdot (h/r) \approx h/r \, .$$

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Because of the symmetry of the situation, the same field enhancement is achieved by a "floating rod" of total length $L=2h$, aligned parallel to a macroscopic field. Hence we get the floating-rod formula

$$\gamma_a = \alpha\ (L/2r) \approx L/2r .$$

Schottky's conjecture

Situations can arise where emitter shapes can be modelled as "a little post (or protrusion), with apex FEF γ_1" on top of "a large post (or protrusion), with apex FEF γ_2" .

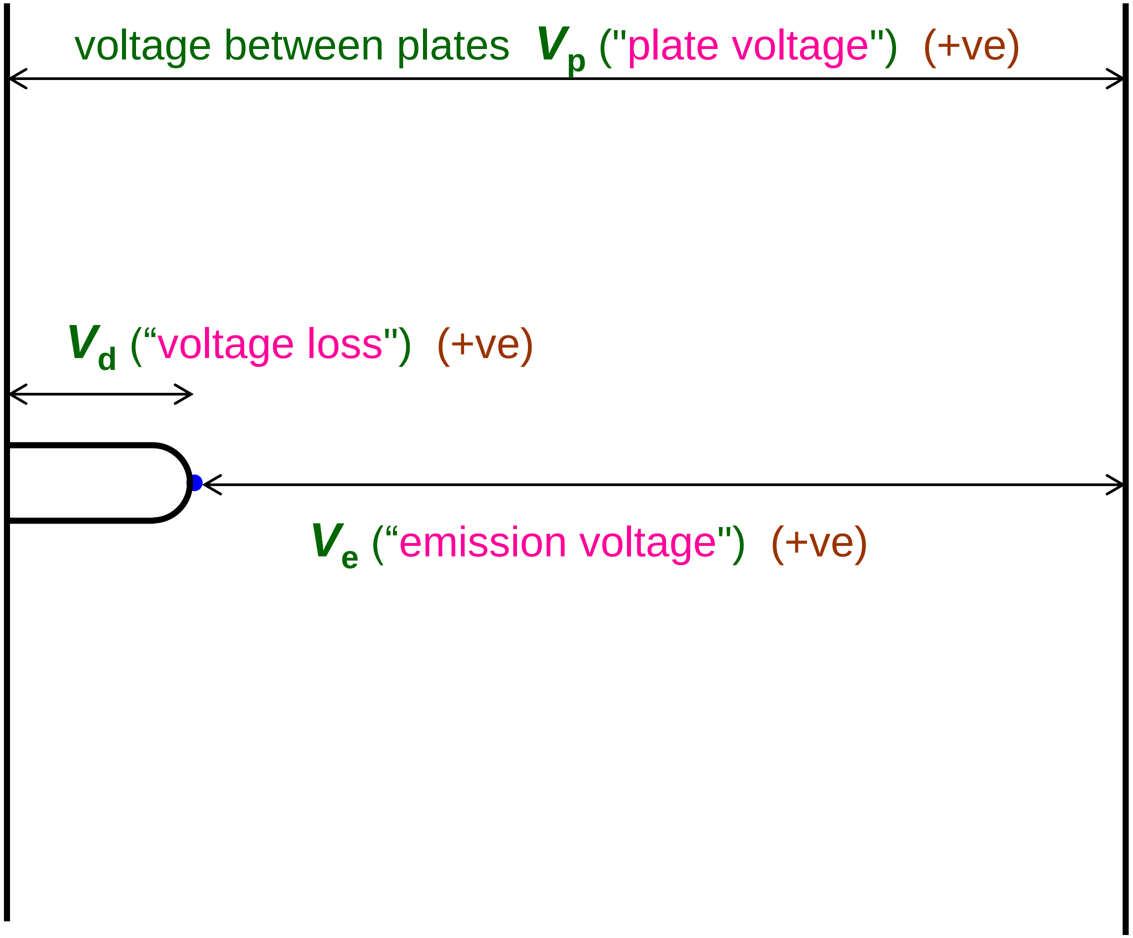
Schottky's conjecture is that the total apex FEF γ_{tot} is given by:

$$\gamma_{tot} = \gamma_1 \gamma_2 \, .$$

It turns out that this is often a better approximation than one might anticipate.

2b: The link between voltage loss and FEF reduction

More voltage definitions

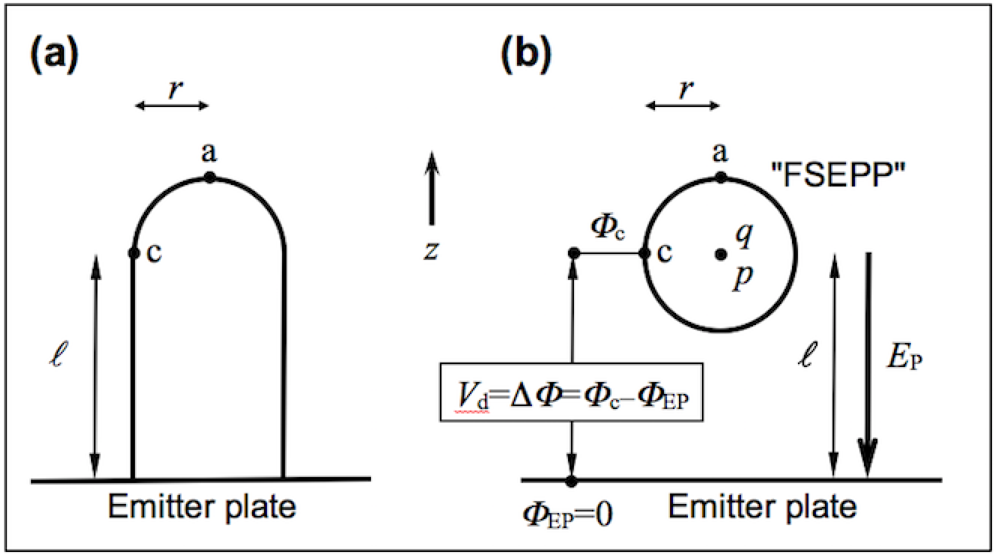


For ideal devices, and for non-ideal devices without additional series resistance, we have V_p equal to the measured voltage V_m.

Floating Sphere at Positive Potential V_d

To allow (+ve) voltage loss V_d along post, a simple "Floating Sphere" model is used.

Taking the local work-function as constant means that the (+ve) potential difference $\Delta\Phi$ between "c" and emitter plate needs to equal V_d .



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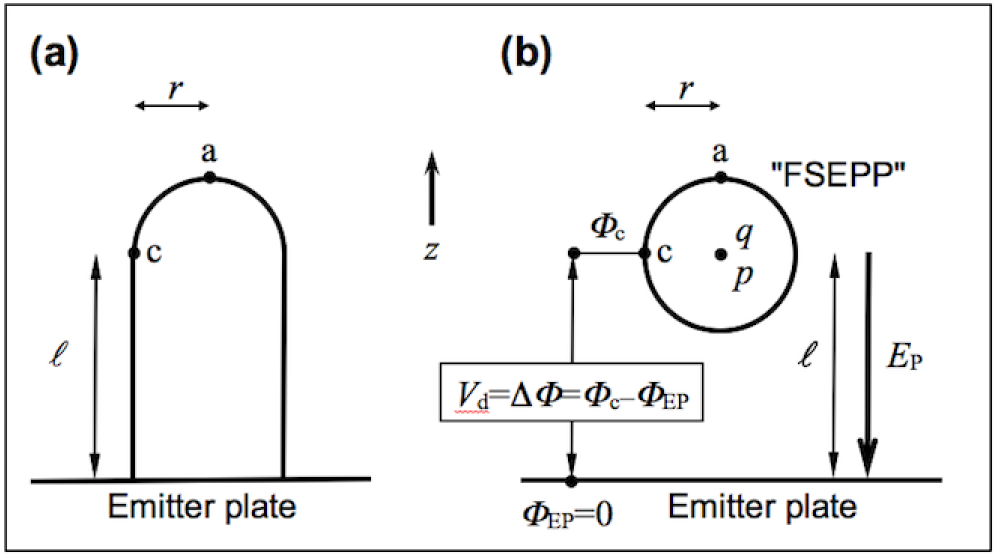

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3) The (–ve) apex field E_a is: $E_a = q/4\pi\varepsilon_0 r^2 = E_P(l/r) + (V_d/l)(l/r)$

4) Hence (+ve) operative apex FEF γ_a^{op} is given by:

$$\gamma_a^{op} = E_a/E_P = (l/r) + (V_d/E_P l)(l/r) = [1 - (V_d / |E_P|l)]\, \gamma_a^{zc} .$$

Generality of the argument

Although a specific formula has just been deduced, the underlying principles are more general, and apply to any "pointy electron emitter".

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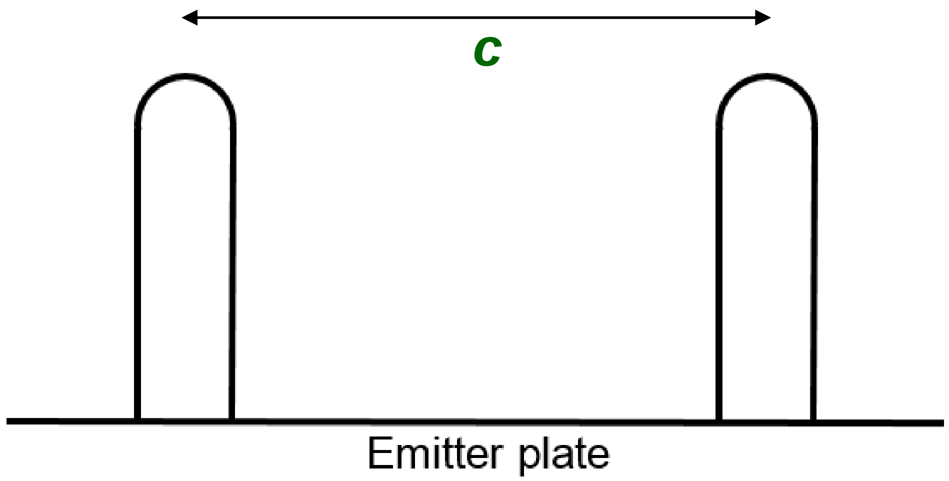
My view is that this link between current flow and "electrostatic" effects (i.e. effects related to surface charge distributions) is not widely understood. Perhaps one might call this topic current-related electrostatics, as opposed to "zero-current electrostatics".

2c: The physics of two identical HCP emitters

Electrostatic depolarization

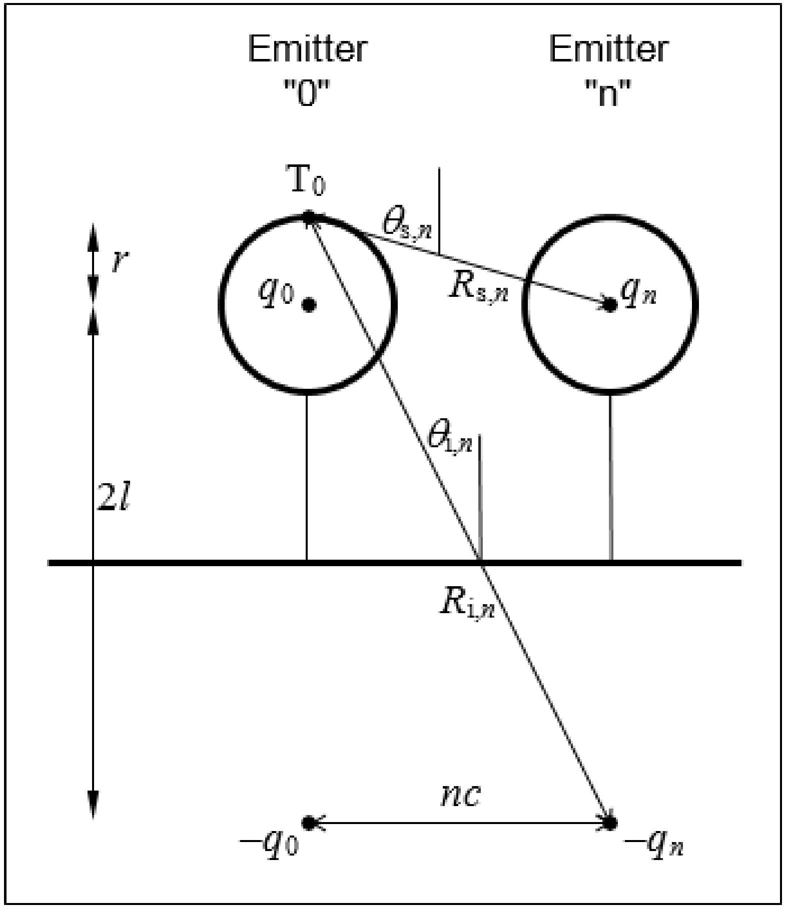


When two (or more) emitters stand on an earthed plate, there is an electrostatic interaction between them.

This has been called by various names, including "shielding", "screening", "charge blunting" and the "shadowing effect", but I now prefer the term electrostatic depolarization.

This is a depolarization of finite dipole-like charge distributions, rather than mutual depolarization of electrostatic point dipoles (which is better known).

Electrostatic depolarization



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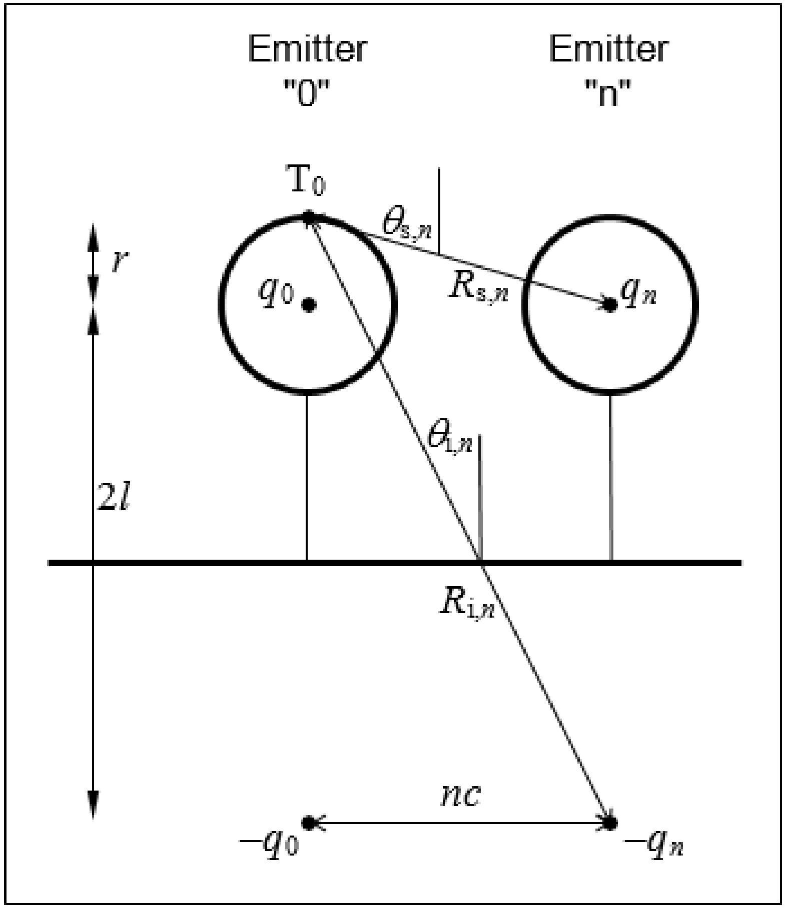

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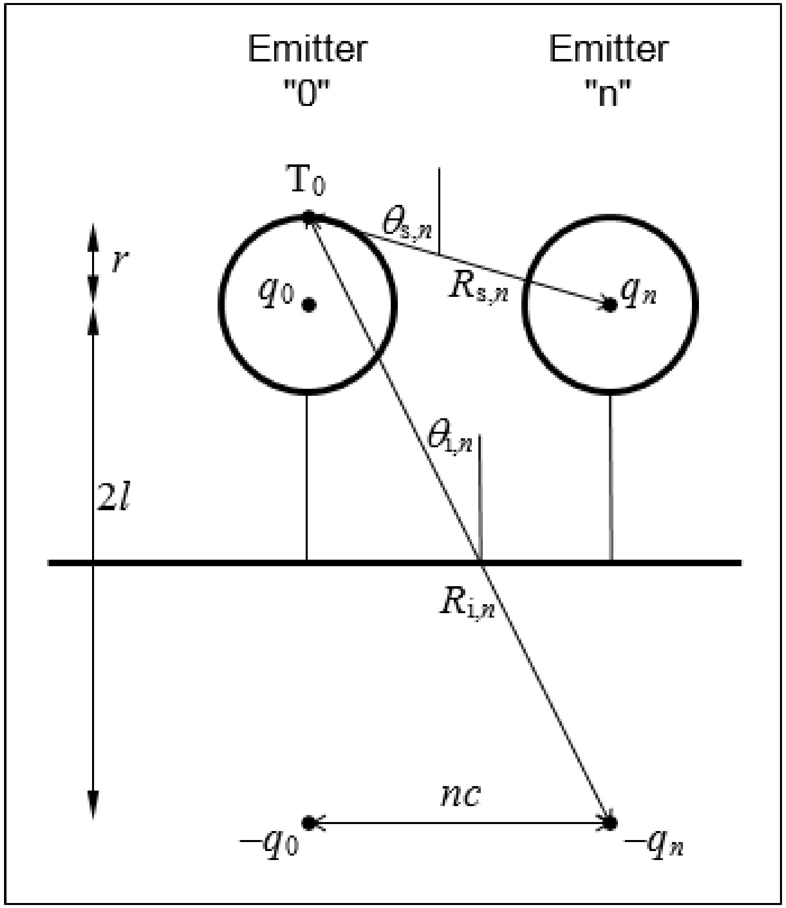
The way the model works is as follows.

(1) Before the effects of emitter “n” are taken into account, the classical electrostatic potential at T_0 is zero.

Electrostatic depolarization



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The way the model works is as follows.

(1) Before the effects of emitter "n" are taken into account, the classical electrostatic potential at T_0 is zero.

(2) The charges q_n and $-q_n$ would cause the potential at T_0 to change away from zero.

(3) The "thermodynamic" requirement that the potential at T_0 remain at zero means that the magnitude of charge q_0 must reduce, in order to keep the potential at T_0 equal to zero.

(4) Hence the magnitude of the apex field (at T_0) is decreased, and the related apex FEF is also decreased.

Fractional reduction in apex FEF

Let γ_1 denote the apex FEF for a single isolated HCP emitter.

Let γ_2 denote the reduced apex FEF for two interacting HCP emitters.

Define a fractional increase δ, and a fractional reduction ρ, in apex FEF by

$$\rho \;=\; (-\,\delta) \;=\; (\gamma_2 - \gamma_1)\,/\,\gamma_1\,.$$

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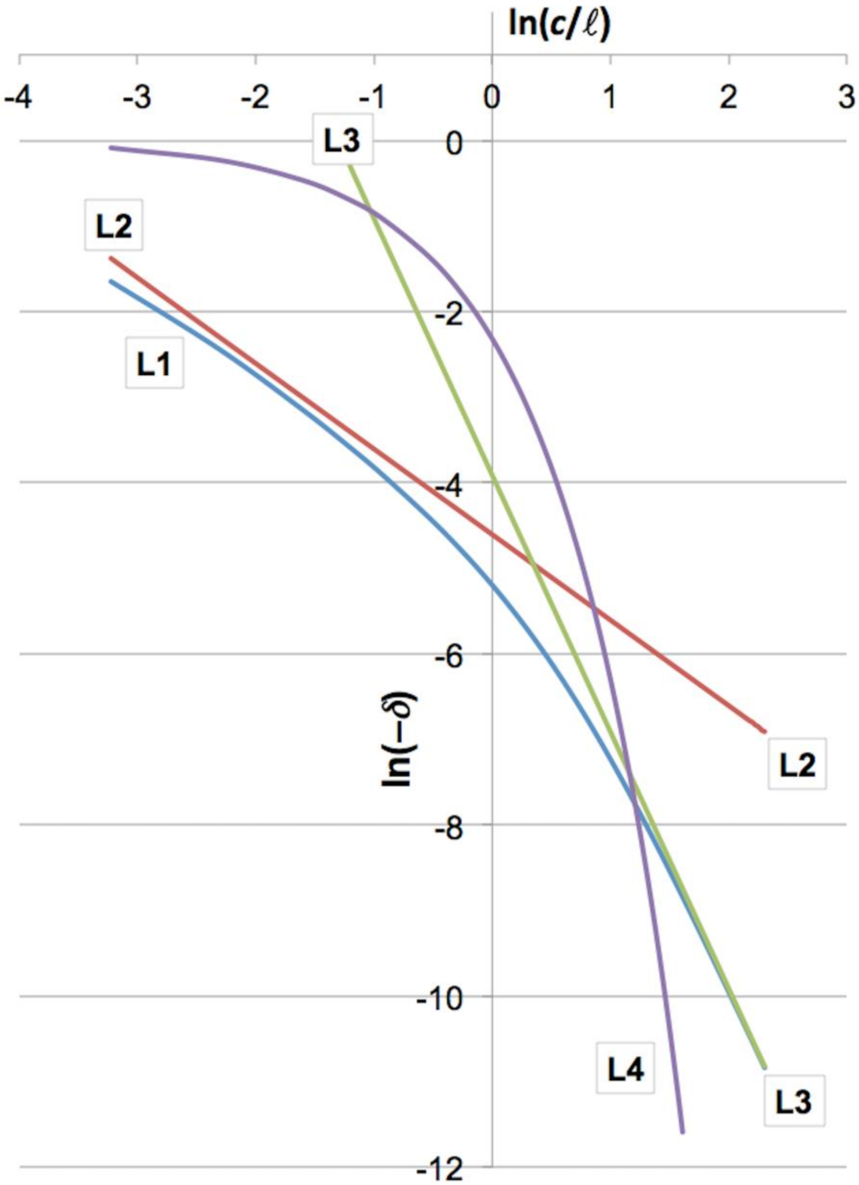
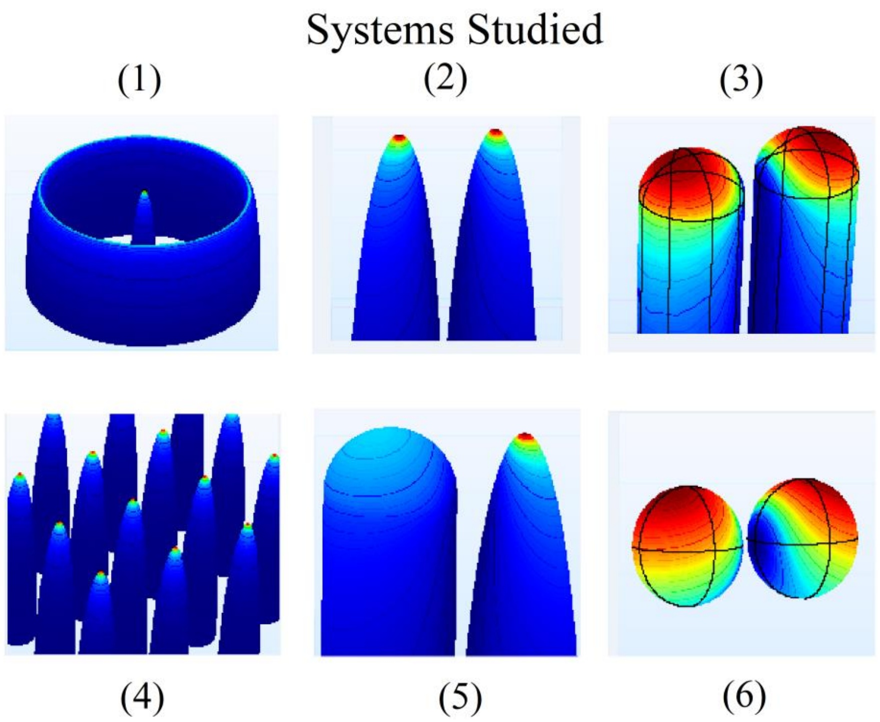
Calculations within the FSEPP model yielded an analytical formula plotted as **line L1** alongside.

For large separations *c*, this becomes the **power law**: $(-\delta) \sim (c/l)^{-3}$, shown as **line L3**.



Universal nature of the c^{-3} law



Because there was a discrepancy between the existing exponential fitting law, and the c^{-3} result, de Assis and Dall'Agnol investigated this situation extensively, using numerical simulations, and various structures (as above).

Their numerical results suggested that the c^{-3} dependence was a universal law, applicable to a protrusion of any shape.

Physical origin of the c^{-3} law

I was then able to show that this large-separation result is expected from general electrostatic principles, and is obtained by modelling the distant protrusions as a point dipole.

It turns out that (at sufficiently large distances) one expects the fractional reduction to scale in accordance with the parameter

$$r\, l^{2} / c^{3}\ .$$

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A more important conclusion was that all these nanoscale physics formulae are in fact general formulae of conductor electrostatics, and apply on any scale.

Main lessons from electron-conductor theory

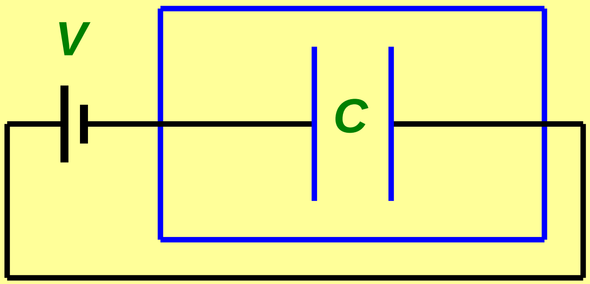
1. **The conducting post and floating rod formulae.**
2. **The link between voltage loss and “reduction in field enhancement”**
3. **The idea of mutual electrostatic depolarization**
4. **The generality of the effects**

Elements of electrical thermodynamics

3a: Basic theory

The electrical Gibbs function

Consider a complex system that has internal capacitance *C*, and suppose that an ideal external battery (or ideal high-voltage generator) that supplies a voltage *V* is attached to the system.



If a change happens within the system that causes the capacitance to increase by an amount δC then the external voltage source will "charge" the increased capacitance, by passing charge $\delta q = V\delta C$ around the electrical circuit. The external source thus does electrical work w^{el} on the system, with $w^{el} = V^2\delta C$.

The electrical Gibbs function

Normally, as in the applications of interest to us, the change in capacitance is caused by a change in electrode shape.

Normally, the work done on the system in such a change is purely electrical: no external mechanical work is done.

To describe the thermodynamics of such changes, it is necessary to introduce electrical terms into Helmholtz free energies, and to introduce the electrical Gibbs function.

The electrical Gibbs function

A general definition of a Gibbs function *G* is

$$G = ☞ - w \,,$$

where ☞ is Helmholtz free energy, and *w* is the work done ON the system by an external agency.

The electrical Gibbs function G^{el} is

$$G^{el} = ☞ - Vq \,,$$

Since the voltage applied by a battery can be taken as constant:

$$\delta G^{el} = \delta ☞ - V\delta q \,.$$

Basic predictions

The Helmholtz free-energy ☞ has to contain:

- a bulk term;
- a zero-field surface-energy term, which involves the (zero-field) surface free energy per unit area γ^0 (also called "surface tension");
- field-dependent surface terms;
- an electrostatic-capacitance (*C*) term, which can alternatively be interpreted as an (electrostatic) field-energy term.

Theory can be constructed at three levels.

(1) A classical macroscopic level.

(2) An atomic level, representing atoms by point charges and dipoles.

(3) An atomic level, using quantum mechanics

If you use the simplest possible theory, you reach the formula

$$\delta G^{el} = \gamma^0 \delta A - \tfrac{1}{2} V^2 \delta C$$

where δA is change in surface area *A.*

Basic predictions

$$\delta G^{el} = \gamma^0 \delta A - \tfrac{1}{2} V^2 \delta C$$

A general rule of thermodynamics is that **a system tends to change in such a direction that its Gibbs function becomes more negative.**

This rule predicts

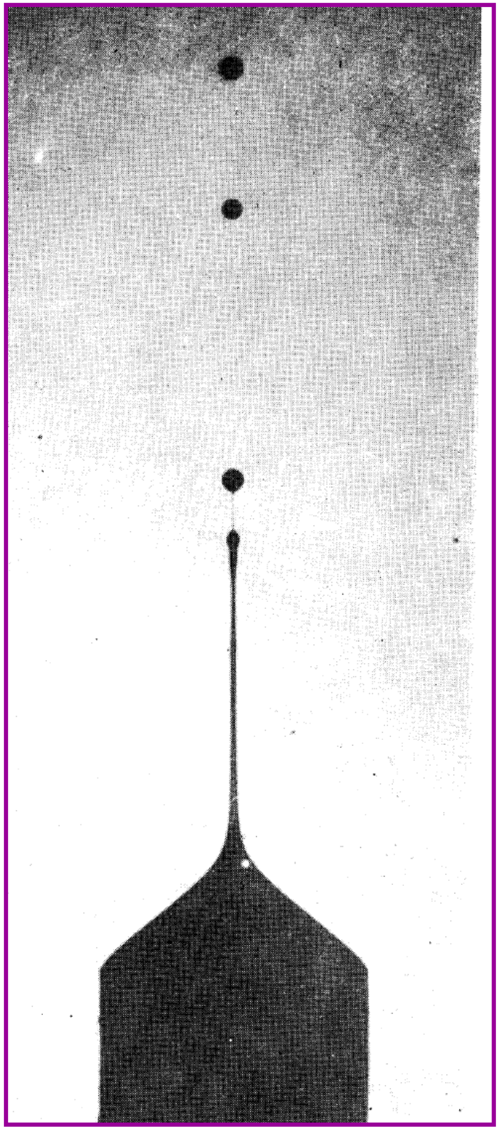
1) **When the applied voltage is very small, then the system-shape changes in such a way as to minimize the surface area, that is the system tends to "ball up".**

2) **When the applied voltage is sufficiently large, then the system-shape changes is such a way as to maximize the capacitance between the "active" (i.e. shape-changing) electrode and the counter-electrode, i.e. the system tends to "grow spikes".**

3) **There is a change-over condition, as voltage increases, from surface-area-driven behaviour to capacitance-driven behaviour.**

3b: Illustrations of the "spike-growth" regime

Experimentally observed cone-jets



Field induced Tellis-oil jet

Diagram courtesy: G. Taylor,
Proc. Roy. Soc. Lond, A313, 453 (1969), Fig. 8(b).

Electron micrograph of emitting liquid-metal ion source (LMIS) at very high emission current

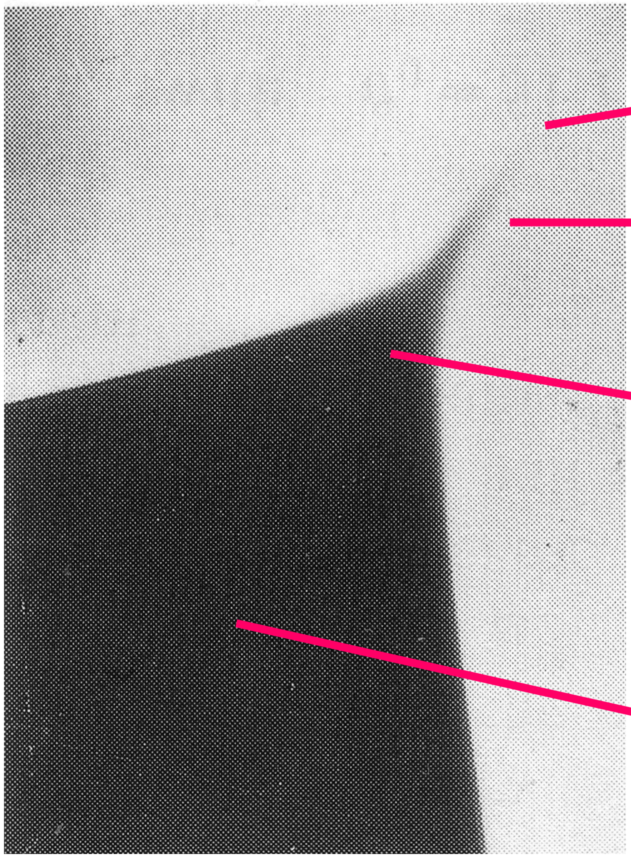
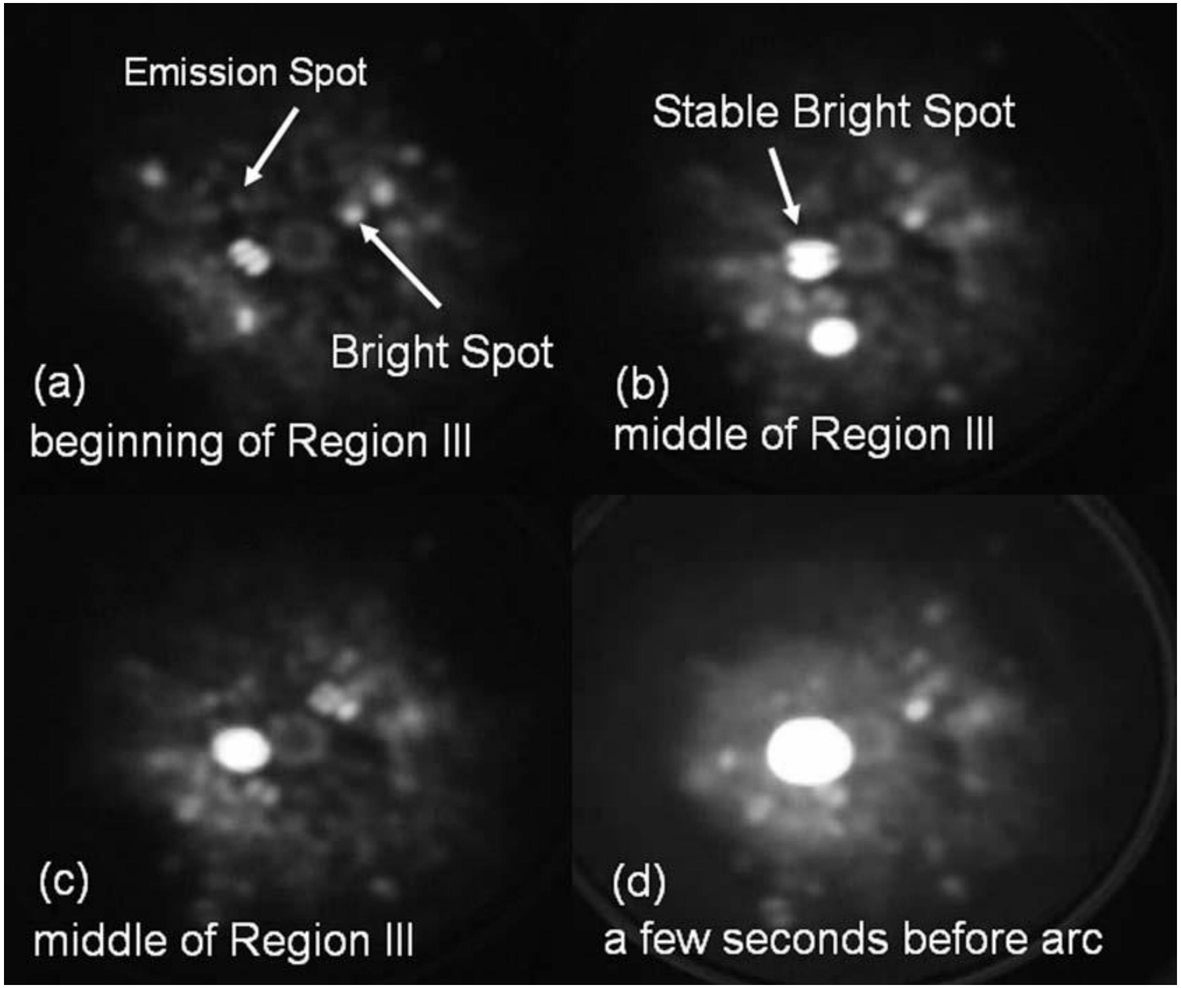


Diagram courtesy:

G. Benassayag, 3rd cycle Thesis, Toulouse, 1984.

Life cycle of cold W field electron emitter



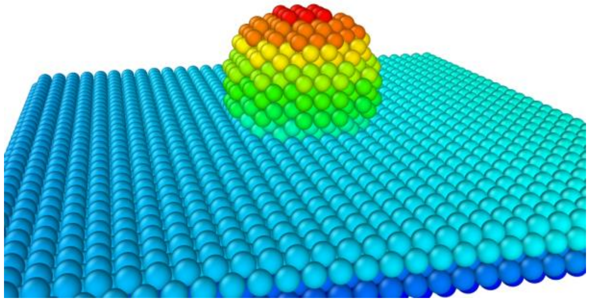
This sequence shows unambiguously that, with cold metal field electron emitters, arc initiation is preceded by nanoprotrusion growth on top of the field electron emitter.

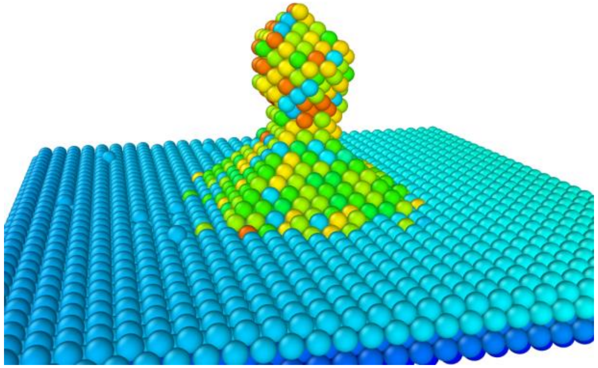
Diagram courtesy: K.S. Yeong & J.T.L. Thong, J. Appl. Phys, 99, 104903 (2006), Fig. 8.

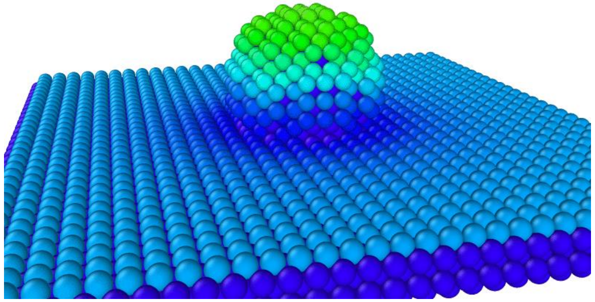
Growth of W nanotips

W nanotip at 3000 K in 1 GV/m applied field

Radius = 1.4 nm
Height = 2.0 nm







Colours according to the initial atom positions to show the diffusion

Colours: field

The bias diffusion in fields causes formation of nanotips

Slide provided by V. Janssen

V. Jansson et al. To be submitted.

3d: Atomic-level theoretical terms

The polarization & partial ionization terms

The Helmholtz free-energy ☞ has to contain:

- **a bulk term;**
- **a zero-field surface-energy term, which involves the (zero-field) surface free energy per unit area γ^0 (also called "surface tension");**
- **field-dependent surface terms;**
- **an electrostatic-capacitance (*C*) term, which can alternatively be interpreted as an (electrostatic) field-energy term.**

The contribution due to field-dependent surface terms is sometimes described as due to polarization and partial ionization (PPI), and can be denoted by $☞_{PPI}$. For a single atom, it is assume that we can write:

$$☞_{PPI} = \mu_{PPI} F + \tfrac{1}{2} c_{PPI} F^2$$

where F is an appropriate field, μ_{PPI} is the linear PPI coefficient (my name), and c_{PPI} is usually described as an "effective polarizability".

The polarization & partial ionization terms

The values of μ_{PPI} and c_{PPI} are expected to vary with position and with the (very local) electric field at that position.

Progress has been made, in particular by the Helsinki group, in extracting values or "effective average values" from density functional theory (DFT) calculations.

Thus, this fuller atomic-level theory has TWO F^2 terms

- **a PPI term, and**
- **a field-energy term.**

Questions arise as to which term is dominant is any particular situation, and are not resolved in all cases.

Electric field requirements for St Elmo's Fire

St Elmo's fire

St. Elmo's fire on a ship at sea

St Elmo's fire is a weather phenomenon in which a luminous "ball" or "flame" is created at the tip of a tall pointed object, such as a ship's mast.

It has been reported by many reliable witnesses including:

- Julius Caesar & Pliny the elder;
- 15th century Chinese admiral Zheng He;
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St Elmo's fire has also been reported (formally or informally) at the tips of many other long pointed objects, including: spear tips; church spires; vertical pipes; trees; fingers; cattle horns; and the ears of a horse. [See Wikipedia for more details.]

St Elmo’s fire as corona discharge

A feature of St Elmo’s fire is that typically it appears in (very) bad weather, when it is or has been raining, and when there are thunderclouds present (and probably high wind). In these circumstances there can be strong electrostatic fields between the clouds and the ground, found to be as much as 4×10^3 V/m (though often less).

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It is now known that St Elmo's fire is an electrical discharge phenomenon called corona discharge. It is NOT a fire, and the tall pointed object is NOT consumed. The luminosity is due to formation of a luminous plasma. The electrostatic field need to sustain a corona discharge is typically 3×10^6 V/m .

In current thinking, the reason why tall pointed objects are relevant is the so-called (electrostatic) lightning rod effect. A rod-shaped conductor, buried in the ground (or linked to the sea) at one end, enhances the field at its upper end, so the rod apex field is significantly greater than the ambient field.

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$$\gamma_a = \alpha(h/r) \approx h/r\,.$$

Clearly this formula provides a qualitative physical explanation of why both height and apex radius are important in the lightning rod effect and in the physical explanation of the occurrence of St. Elmo’s fire.

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Ships' Masts & Roman Javelins

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Perhaps the second-stage field enhancement is "shorted out" when the ear gets wet.

Also note he observed "sparks".

The possible role of Gilbert-Gray cone-jets

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Life cycle of cold W field electron emitter

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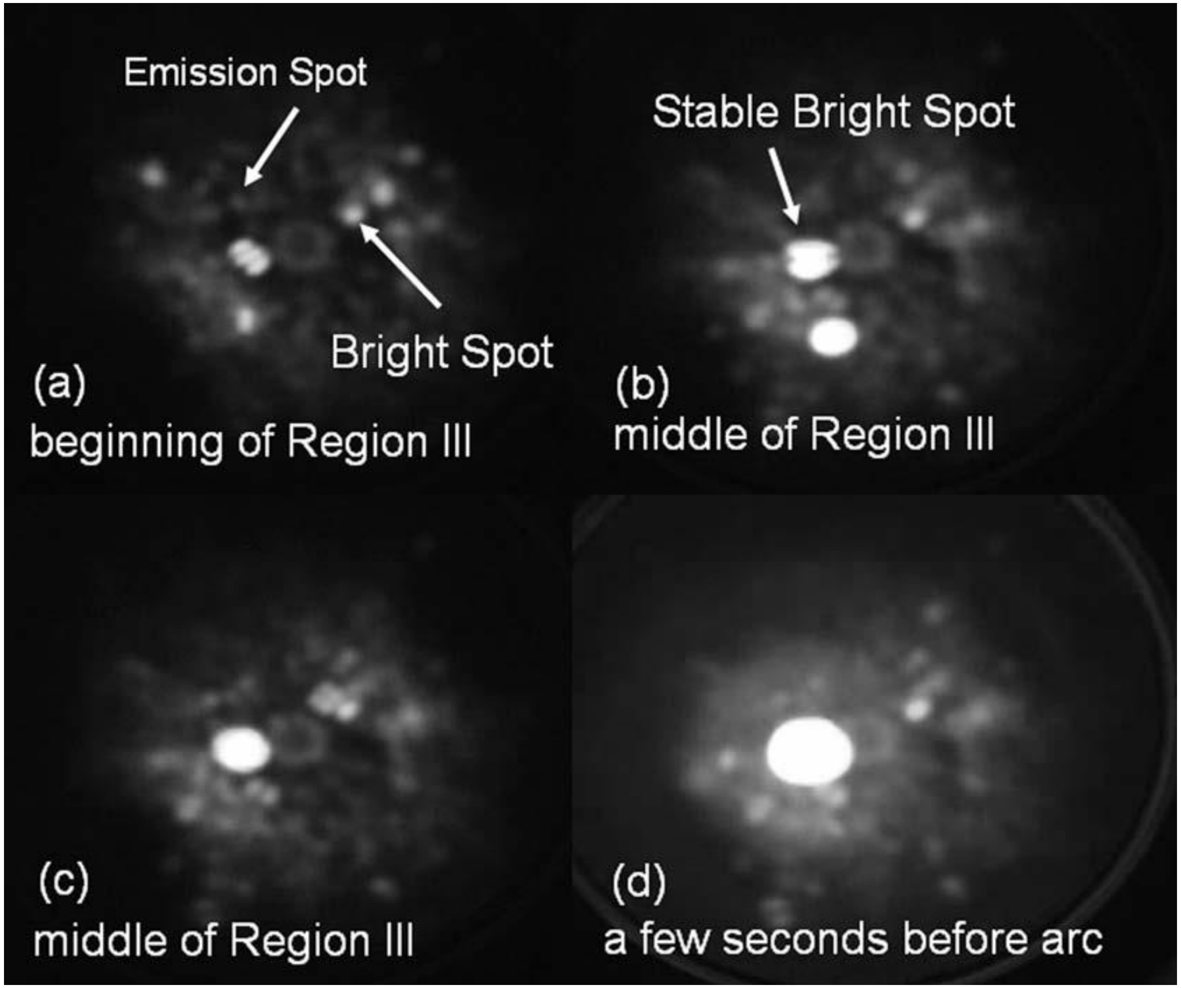

This sequence shows unambiguously that, with cold metal field electron emitters, arc initiation is preceded by nanoprotrusion growth on top of the field electron emitter.

Diagram courtesy: K.S. Yeong & J.T.L. Thong, J. Appl. Phys, 99, 104903 (2006), Fig. 8.

Electron micrograph of emitting liquid-metal ion source (LMIS) at very high emission current

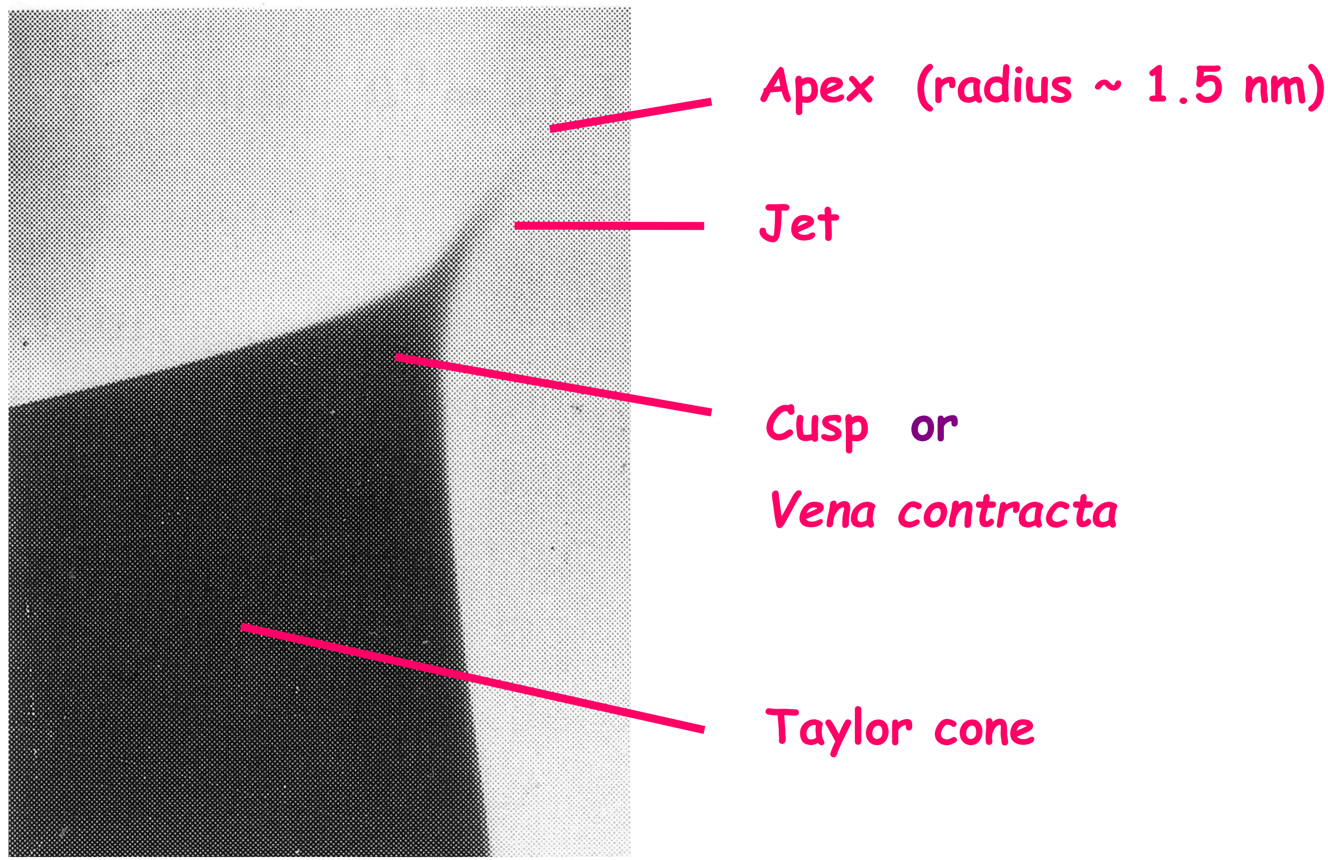

Diagram courtesy:

G. Benassayag, 3rd cycle Thesis, Toulouse, 1984.

Gray's 1733 Report of Discharge Phenomenon

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A modern report

IOP Publishing Journal of Physics D: Applied Physics

J. Phys. D: Appl. Phys. 50 (2017) 085201 (8pp) doi:10.1088/1361-6463/aa5760

Dynamic corona characteristics of water droplets on charged conductor surface

Pengfei Xu, Bo Zhang, Zezhong Wang, Shuiming Chen and Jinliang He

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Received 16 October 2016, revised 26 December 2016
Accepted for publication 6 January 2017
Published 30 January 2017

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A modern report

Abstract

The formation of the Taylor cone of a water droplet on the surface of the conductor in a line-ground electrode system is captured using a high-speed camera, while the corona current is synchronously measured using a current measurement system. Repeated Taylor cone deformation is observed, yielding regular groupings of corona current pulses. The underlying mechanism of this deformation is studied and the correlation between corona discharge characteristics and cone deformation is investigated. Depending on the applied voltage and rate of water supply, the Taylor cone may be stable or unstable and has a significant influence on the characteristics of the corona currents. If the rate of water supply is large enough, the Taylor cone tends to be unstable and generates corona-current pulses of numerous induced current pulses with low amplitudes. In consequence, this difference suggests that large rainfall results in simultaneously lower radio interference and higher corona loss.

Keywords: corona, rainfall, water droplet, Taylor cone

Masts and Spears

Perhaps (in at least some cases) the primary role of the "long pointed object" is just to enhance the "environmental field" to the level where a Gilbert-Gray cone-jet ("Taylor cone") can form.

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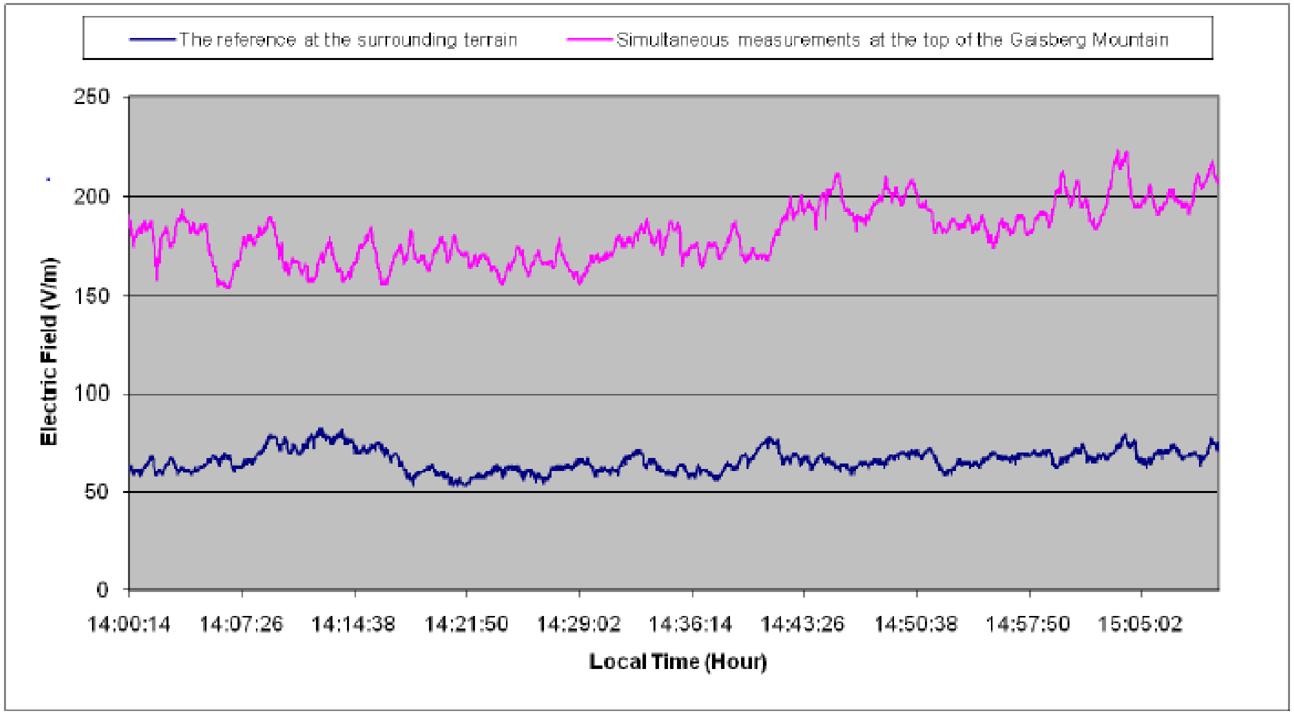
8. Possible larger-scale applications

Electrostatic fields near mountains



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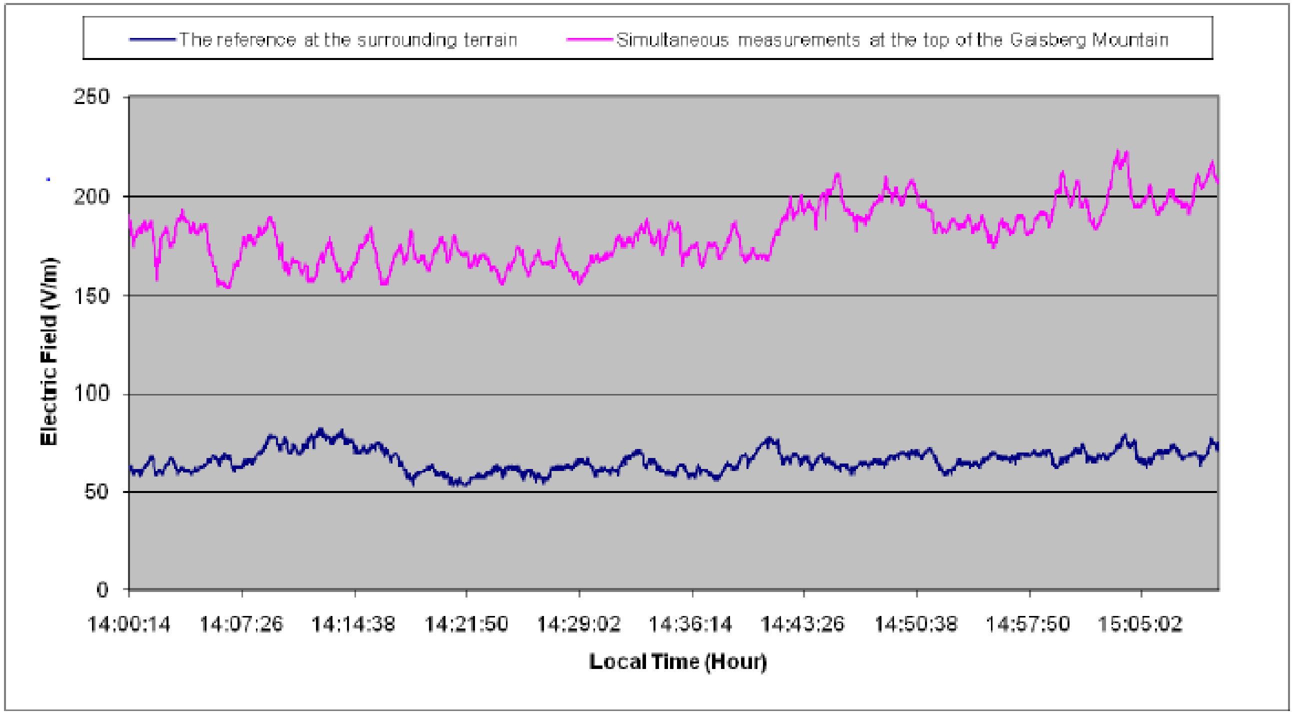

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If we modelled the mountain as "approximately hemispherical", we could explain the experimental observation as an electrostatic effect.

Masts and Spears

Perhaps we should think of the Earth as "a great big electrical (electron) conductor surrounded by a system of electrostatic fields". [And these fields are also influenced by atmospheric phenomena.]

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Summary

Main take-home suggestions

3. Elements of electrical thermodynamics – and have shown that spike-type growth is a consequence at sufficiently high voltages and fields
4. Electric-field requirements for St Elmo's fire
5. Ships' masts & Roman Javelins – and have offered two forms of explanation.
6. The case of Surgeon Braid's horse
7. The possible role of Gilbert-Gray cone-jets (“Taylor cones”) – as the second stage of field enhancement, possibly/probaby giving enough to allow field electron emission
8. Possible larger-scale applications.

Main-take-home suggestions

1. Perhaps we need (or need to recognize) a more comprehensive theory of electricity.

2. Maxwell's theory is not enough – we also need:
   a. Electron thermodynamics
   b. Electrical thermodynamics and the "electrical Gibbs function"

3. Perhaps we should think of the whole Earth as an electrical (electron) conductor.

Thanks for your attention

The role of Gilbert-Gray cone-jets ("Taylor cones")

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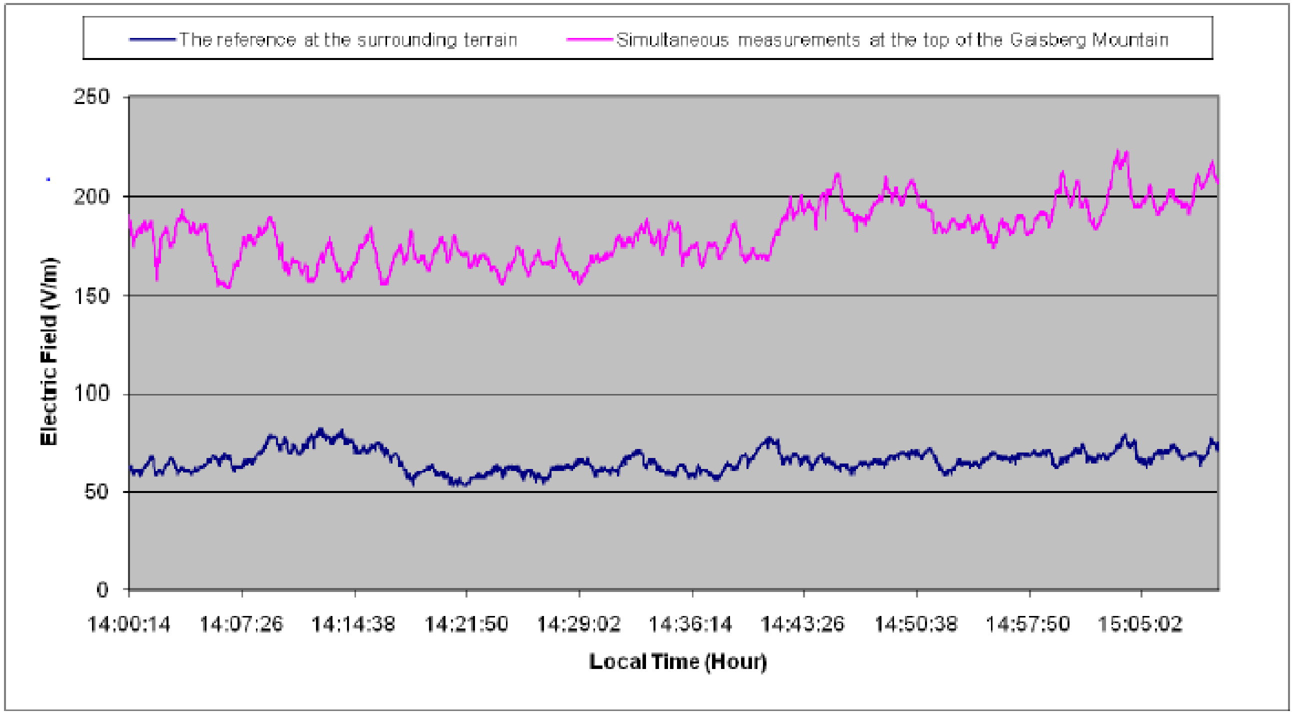

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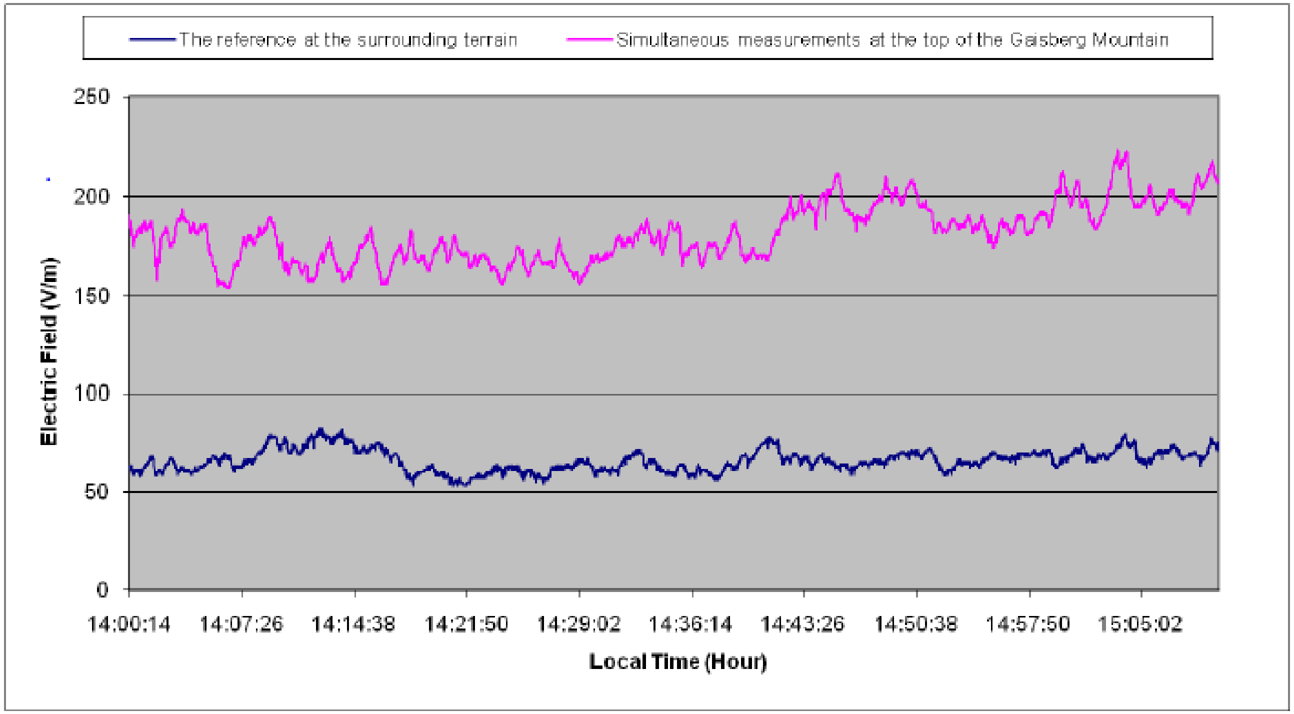

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9: Conclusions on high-field nanoscience

We have made considerable progress.

Very much remains to be done.

There are many “elephants” still out there !

Thanks for your attention

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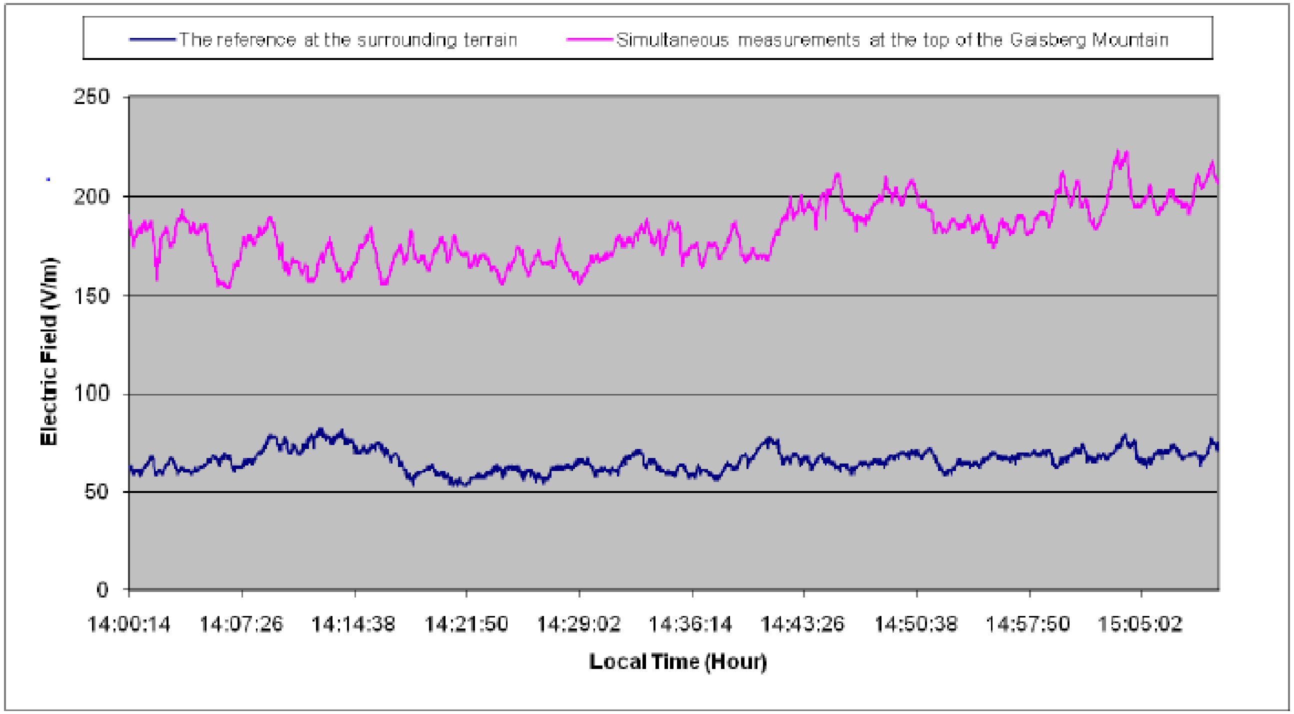

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Conclusions

We have made considerable progress in clarifying and quantifying various aspects of nanoscale electrostatics.

It has become clear that this nanoscale electrostatics (which is based ultimately on electron thermodynamics) is in fact a theory of more general applicability, in particular to “ordinary scale” electrostatic phenomena.

It provides an element that has hitherto been missing in the theory of electricity, and seems able to provide "nearly quantitative" explanations of historically described phenomena which at present have no satisfactory quantitative explanation.

In particular, it should be part of the background theory associated with detailed theories of lightning strikes and the operation of lightning rods.

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Book of Abstracts, p.12

The link between voltage loss, reduction in field enhancement factor, and saturation effects in Fowler-Nordheim plots

Richard G. Forbes

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University of Surrey, Guildford, Surrey GU2 7XH, UK
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With a large-area field electron emitter (LAFE), when an individual post-like emitter is sufficiently resistive, and current through it sufficiently large, then voltage loss occurs along it. This presentation begins by providing a simple analytical and conceptual demonstration that this voltage loss V_d is directly and inextricably linked to a reduction in the field enhancement factor (FEF) γ_a at the post apex. A formula relating apex-FEF reduction to this voltage loss was obtained by Minoux et al. [1], by fitting to numerical results from a Laplace solver. In its simplest form, this formula has the form

$$\gamma_a \approx \gamma_a^{sc} \{1 - V_d/|E_M \ell)|\} ,$$

where γ_a^{sc} is the small-current FEF, E_M is the true macroscopic field, and ℓ is the post length.

This presentation derives the same formula analytically by using a "floating sphere" model. The derivation has already been published [2]. The analytical proof brings out the underlying physics more clearly, and shows that the effect is a general phenomenon, related to reduction in the magnitude of the surface charge in the most protruding parts of an emitter.

A Fowler-Nordheim (FN) plot [3] is a plot of measured current i_m as a function of measured voltage V_m, plotted in the form $\ln\{i_m/V_m^2\}$ vs $1/V_m$. Voltage-dependent FEF-reduction is one cause of "saturation" (i.e. flattening, and maybe turn-over) in experimental FN plots. Another is a voltage-divider effect, due to measurement-circuit resistance. An integrated theory of both effects will be presented, and shows that both together, or either by itself, can cause saturation. In order to distinguish between them, it can be predicted that, if FEF-reduction is the cause of saturation then one would expect the emitted electrons to exhibit relative small measured energy deficits, but that if series resistance is the cause then one might expect the energy deficits to be relatively large and that a plot of emitted current versus measured voltage would be approximately linear.

With point emitters in the classical field electron microscope (FEM) situation, one would expect similar experimental effects on energy distributions and on current-voltage plots, although detailed theory is much more difficult to formulate. For ICCNA, it may be of interest that, for both these limiting cases, in the classical FEM situation, clear experimental examples were first found in Jordan, at Mu'tah University [4,5]. Further, now that we know more clearly what the characteristic shape of a FN plot looks like in the high-series-resistance limiting case, we can look for (and have found) other examples of this type of plot in the literature [6], including in work [7] (on carbon nanotube fibres) aimed at developing fast-switching LAFEs as possible electron sources for microwave generators, in the context of US military requirements and potential civilian applications.

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THE LINK BETWEEN VOLTAGE LOSS, REDUCTION IN FIELD ENHANCEMENT FACTOR, AND SATURATION EFFECTS IN FOWLER-NORDHEIM PLOTS

METADATA

This oral presentation was made at the First International Conference on Current Nanotechnology and its Applications (ICCNA – 2018), held at the Jordan University of Science and Technology, Irbid, Jordan, 10-12 April 2018. It may be cited as:

R.G. Forbes, "The link between voltage loss, reduction in field enhancement factor, and saturation effects in Fowler-Nordheim plots", First International Conf. on Current Nanotechnology and its Applications, Irbid, Jordan, April 2018. doi:10.13140/RG.2.2.18804.24964/1 .

The abstract as recorded in the Book of Abstracts (p.12) is shown at the end of this file.

This presentation is a shortened and slightly modified version of an oral presentation made at the 30th International Vacuum Nanoelectronics Conference, Regensburg, July 2017, which itself was based on a letter published as Appl. Phys. Lett. 110, 133109 (2017).

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Background

Sign-convention issues

This talk uses the classical electrostatics sign convention, in which the field at a field electron emitter is taken as negative.

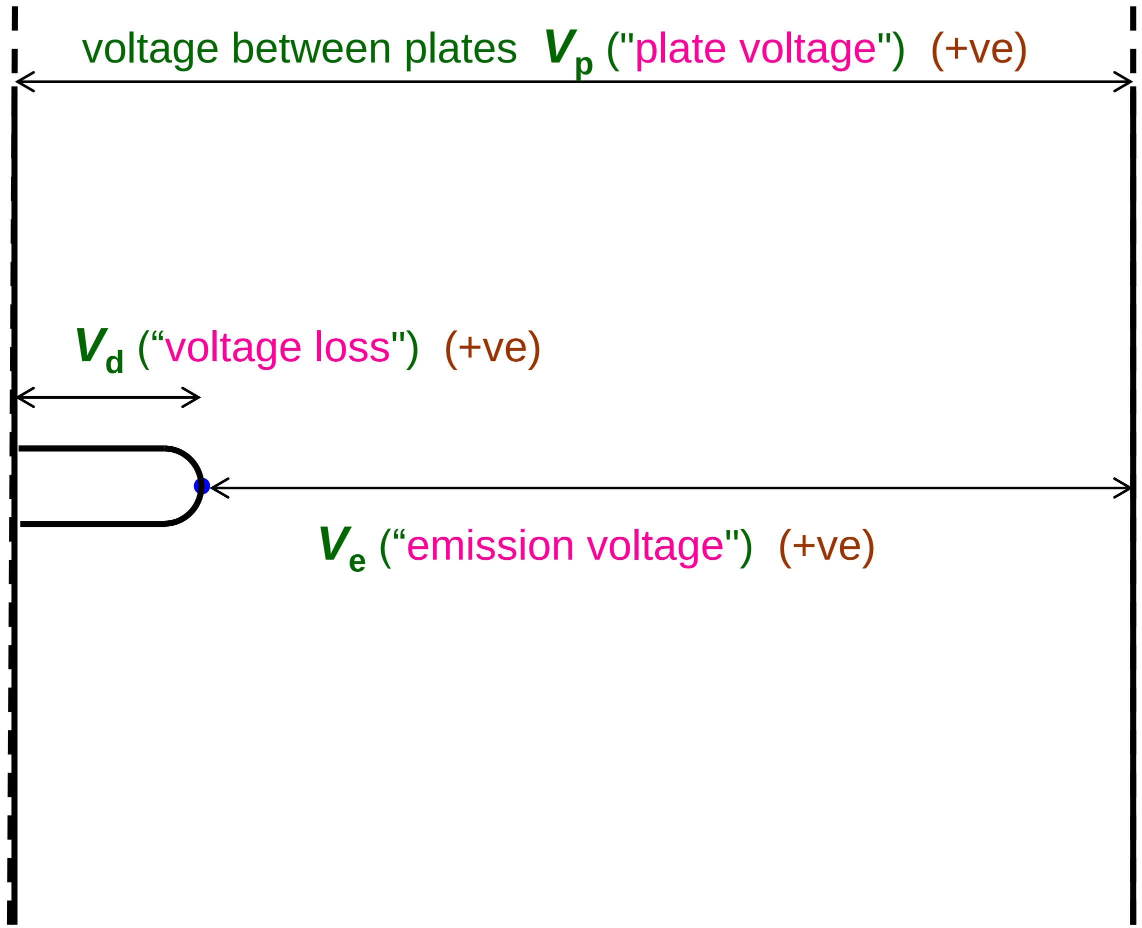
Classical electrostatic field is denoted by the conventional symbol *E*, as is usually done in mainstream science outside field electron emission (FE).

Hence, my symbol *E* denotes a quantity that is negative in value, in FE.

This convention differs from the usual modern FE experimentalists' convention, which is to use *E* to denote a parameter that is positive in value.

To help avoid confusion, I often indicate whether my FE parameters are normally positive or normally negative.

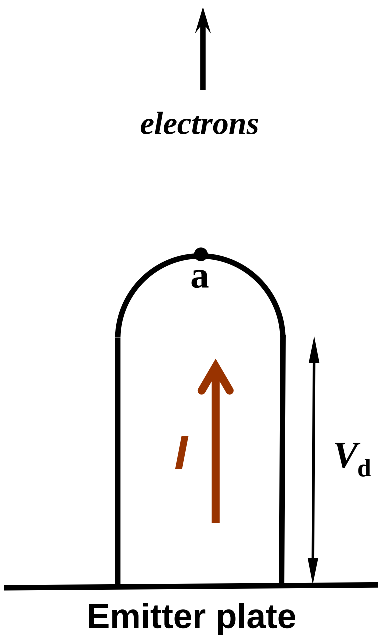
Terminology: voltages



For ideal devices/systems: $V_d = 0$,

$V_p = V_e = V_m$

The effects of voltage loss

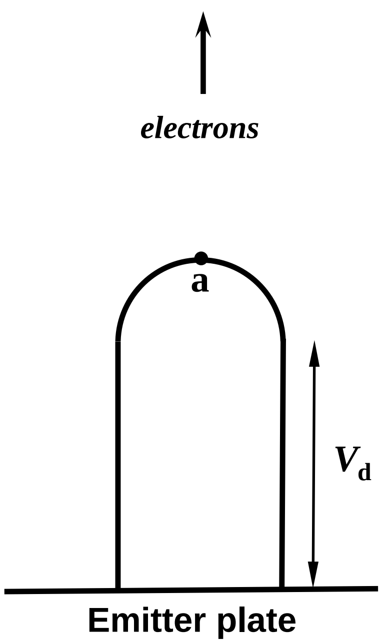


Cause:

voltage loss along post-like emitter, due to current flow

The effects of voltage loss

Effect 1:
Peak shift in total energy distribution



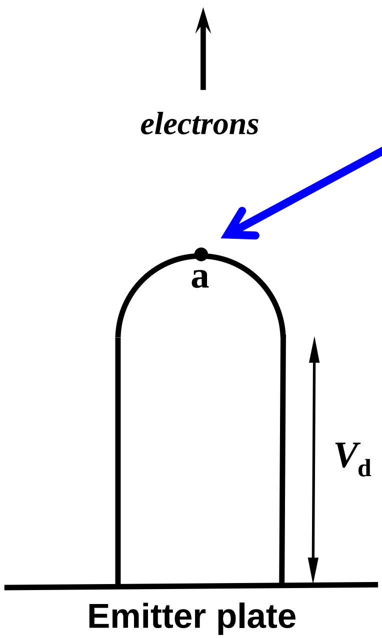
Cause:
voltage loss along post-like emitter, due to current flow

When the electric current through a resistive post-like emitter (or along its surface) is sufficiently high, the resulting (+ve) voltage loss V_d along the post causes two effects:

(1) Peak shift in the total energy distribution (TED).

The effects of voltage loss

Effect 1:
Peak shift in total energy distribution



Effect 2:
Reduction in apex FEF γ_a

Cause:
voltage loss along post-like emitter, due to current flow

When the electric current through a resistive post-like emitter (or along its surface) is sufficiently high, the resulting (+ve) voltage loss V_d along the post causes two effects:

(1) Peak shift in the total energy distribution (TED).

(2) Reduction in the apex FEF γ_a .

[FEF= field enhancement factor]

FEF reduction at carbon tips – Minoux 2005

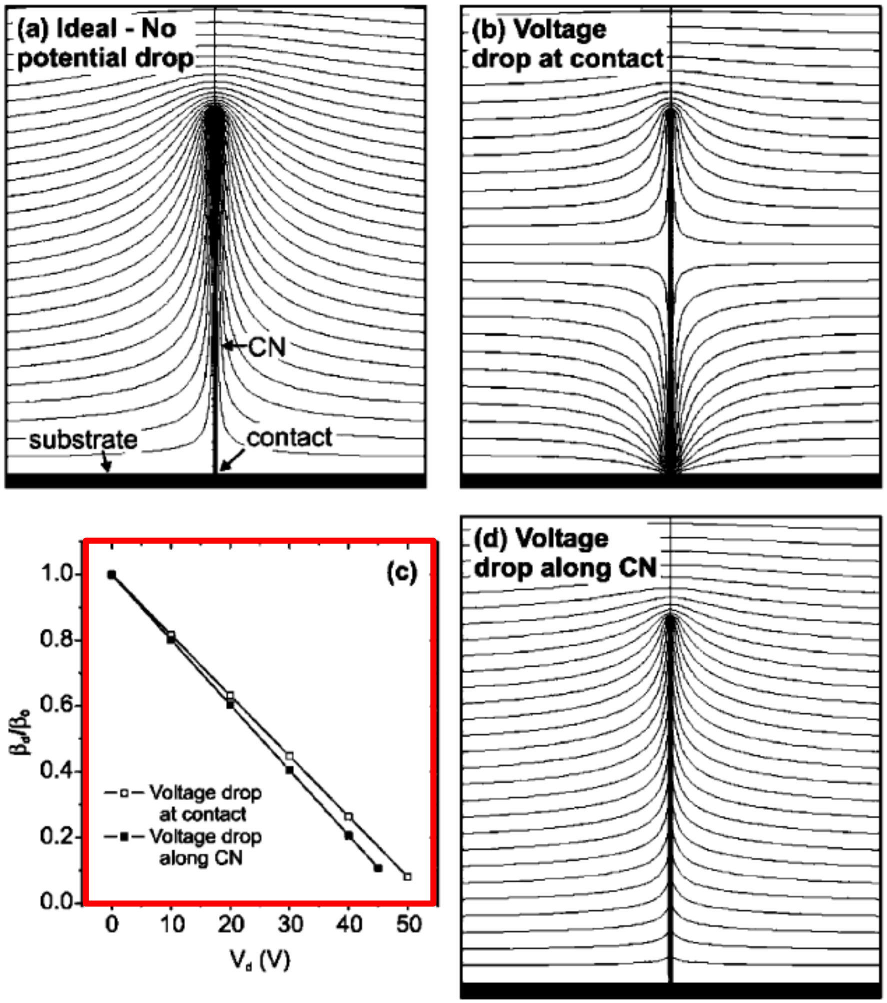


Figure 3. Simulations show the distribution of equipotential lines between the CN and the anode: (a) depicts the case of a perfect emitter where there is no potential drop along it, (b) an emitter with a voltage drop at the emitter/substrate interface, and (d) an emitter with a voltage drop along its length. The graph in (c) plots the reduction of field enhancement factor as a function of the voltage drop. In this particular case, the emitter height was 5 μm, its radius was 25 nm, and the applied electric field was 10 V μm^{-1}.

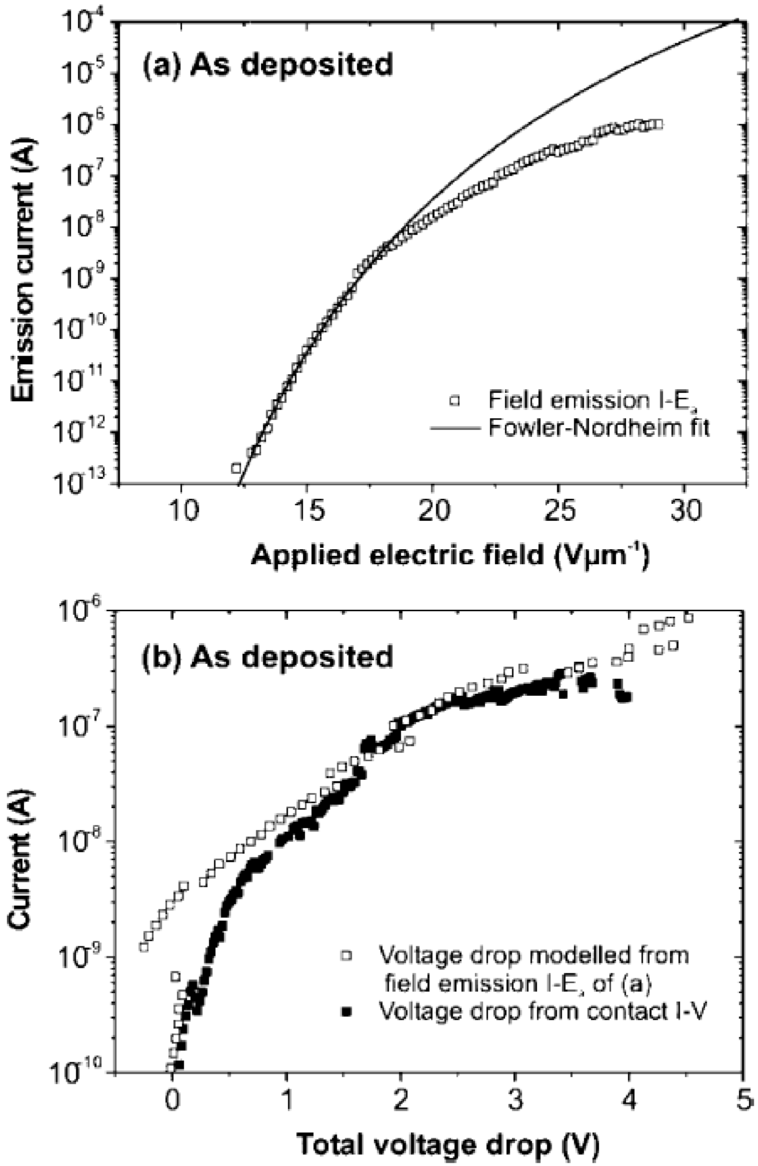
For a long thin carbon post, with an assumed voltage drop, Minoux et al. (2005) used finite-element methods to simulate the potential and field distributions in two cases:

(a) voltage drop at the substrate-to-post contact;

(b) voltage drop along carbon post.

They found a relationship between apex FEF and voltage drop, as illustrated in their Fig. 3(c).

FEF reduction at carbon tips – Minoux 2005



Minoux et al. also:

(1) showed that saturation effects in the current-voltage characteristics were associated with voltage drop along a carbon emitter [see their Fig. 2(a) alongside] ;

(2) measured how the voltage drop depended on current, using a direct-contact probe [see their Fig. 2(b) alongside] ;

FEF reduction at carbon tips – Minoux 2005

By fitting to their numerical results, Minoux et al. deduced a formula that (in my notation) is written:

$$\gamma_a^{op}(V_d) = (1 - \alpha V_d / |E_M| l)\, \gamma_a^{sc} \, ,$$

where: l is the length of the carbon post;

E_M is the (–ve) applied macroscopic classical electrostatic field;

V_d is the (+ve) voltage drop (or "voltage loss") between the post apex and the substrate;

α is a constant, discussed below, related to the physical situation;

γ_a^{sc} is the small-current apex FEF;

γ_a^{op} is the operative apex FEF.

Minoux et al. found that:

- if the voltage drop occurred at the substrate/post contact, then $\alpha = 0.92$;
- if the voltage drop occurred along the post, then $\alpha = 1$.

FEF reduction due to voltage loss along post

In summary, by fitting to their numerical results, Minoux et al. deduced that, when voltage loss is due to resistive ("*iR*") voltage loss along the emitter post (i.e., is "post voltage loss" V_d), the formula for operative FEF is:

$$\gamma_a^{op}(V_d) = (1 - V_d/|E_M|l)\ \gamma_a^{sc} \ .$$

My role has been to:

(1) provide a simple analytical derivation of this formula:
this brings out the underlying physics clearly, and shows that this link between FEF reduction and voltage loss is a general electrostatic effect;

(2) build a more general (formal) treatment of FN plot-saturation that incorporates the effect of apex-FEF reduction.

Analytical proof of the formula for operative FEF

Terminology: "Barrier strength"

In the simple-JWKB formalism, the transmission probability D_F for an electron tunnelling "at the Fermi level" can be written formally as:

$$D_F = \exp[-G_F] .$$

I call the mathematical parameter G the barrier strength. (It is also called the "Gamow factor").

G_F is its value for a Fermi-level electron (which sees a barrier of zero-field height equal to the local work function).

The origin of non-ideal behaviour

Until recently, it has been assumed that the main physical reason for non-ideality (i.e., non-orthodox behaviour) in FN plots was series resistance in the measurement circuit.

Whilst this is still the reason in some cases, the results just discussed suggest that in many cases the physical reason for non-ideality in large area field emitters may be current-dependence in field enhancement factors.

Hence we need a theory of FN plots that takes both factors into account.

The correction factors Θ_{fr} and Θ_{sr}

For convenience, define a correction factor Θ_{fr} [$\equiv (1 - V_d I / |E_M| I)$] "due to FEF reduction", and write the previous equation as: $\gamma_a^{op} = \Theta_{fr} \gamma_a^{sc}$.

If, in addition, there is other series resistance in the measurement circuit (for example, across a poorly conducting silicon substrate) then the plate voltage V_p will be related to the measured voltage V_m by a current-dependent correction factor Θ_{sr} "due to series resistance", with $V_p = \Theta_{sr} V_m$.

Links to FN plot saturation

When applying a FN-type equation to a parallel-plane-plate-type LAFE, and examining the form of the expression for barrier strength G_F at the emitter apex, the following equations are nearly always adequately valid:

$$G_F = \nu_F b \phi^{3/2} / |E_a| ; \qquad E_a = \gamma_a E_M ; \qquad E_M = -V_p / d_{sep} ,$$

where the symbols have their usual meanings [ν_F is the barrier form correction factor for the chosen barrier of interest].

However, there are also equations that depend on whether series resistance ("sr") and/or FEF reduction ("fr") effects are taking place, as set out below:

NO "sr" or "fr"	BOTH "sr" and "fr"
$\gamma_a = \gamma_a^{sc}$	$\gamma_a = \gamma_a^{op} = \Theta_{fr} \gamma_a^{sc}$
$V_p = V_m$	$V_p = \Theta_{sr} V_m$
$G_F = \nu_F b \phi^{3/2} d_{sep} / \gamma_a^{sc} V_m$	$G_F = \nu_F b \phi^{3/2} d_{sep} / \gamma_a^{sc} \Theta_{fr} \Theta_{sr} V_m$

Links to FN plot saturation

Thus, if both effects are taking place, the barrier strength will be given by

$$G_F = \nu_F b \phi^{3/2} d_{sep} / \gamma_a^{sc} \Theta_{fr} \Theta_{sr} V_m \,,$$

where Θ_{fr} and Θ_{sr} (and also ν_F) will be functions of V_m.

When finding a theoretical expression for FN-plot slope (for plots involving measured variables), one needs the derivative $dG_F/d(V_m^{-1})$. The resulting expression will contain voltage-dependent terms relating to Θ_{fr} and Θ_{sr}, and to their derivatives with respect to V_m^{-1}. At sufficiently high voltages V_m, these terms will/may generate curvature in the FN plot and the plot will/may "saturate". Applying elementary or orthodox data-analysis theory to the "saturated" part of the plot will lead to spurious results.

Clearly, either series resistance or FEF reduction or a combination of both can generate saturation. In any particular case where saturation occurs, there is an issue of "What has caused saturation?"

The following argument may provide an useful indication …..

The more likely cause of saturation

Consider an emitting post that has radius 10 nm, length 1 μm, and γ_a^{sc}~100. When this emits at a classical local barrier field of –4 V/nm, it is sitting in a classical macroscopic field E_M of ~ –40 V/μm, and the quantity $|E_M l|$~ 40 V . So a value Θ_{fr}~ 0.5 occurs when the voltage loss V_d along the post is about 20 V.

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On the other hand, if d_{sep}~ 25 μm, the plate voltage V_p needed to generate E_M= –40 V/μm is 1000 V, and a value Θ_{sr}~ 0.5 implies a measured voltage of 2000 V and a voltage loss (across the substrate, say) of 1000 V .

Hence, expectation is that saturation by FEF reduction will be associated with much smaller (+ve) voltage loss than saturation related to series resistance.

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Hence, expectation is that saturation by FEF reduction will be associated with much smaller (+ve) voltage loss than saturation related to series resistance.

Voltage losses of this kind show up as (+ve) voltage deficits as measured by direct-contact probe techniques or as (+ve) energy deficits as found when measuring total electron energy distributions.

If, therefore, saturation in the $i_m(V_m)$ characteristics is observed simultaneously with relatively small measured voltage deficits or energy deficits, then the more likely cause of the saturation is FEF reduction.

Carbon nanofibre behaviour

However, with carbon fibres, in the classical field electron microscope configuration where the same effects occur but with slightly different theory, BOTH causes of saturation have been detected, in different experimental circumstances,

[Linearity in a current-voltage plot implies large series resistance in the measuring circuit.]

Carbon nanofibre behaviour

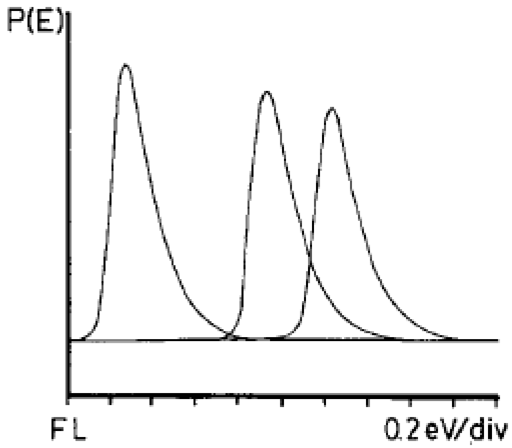
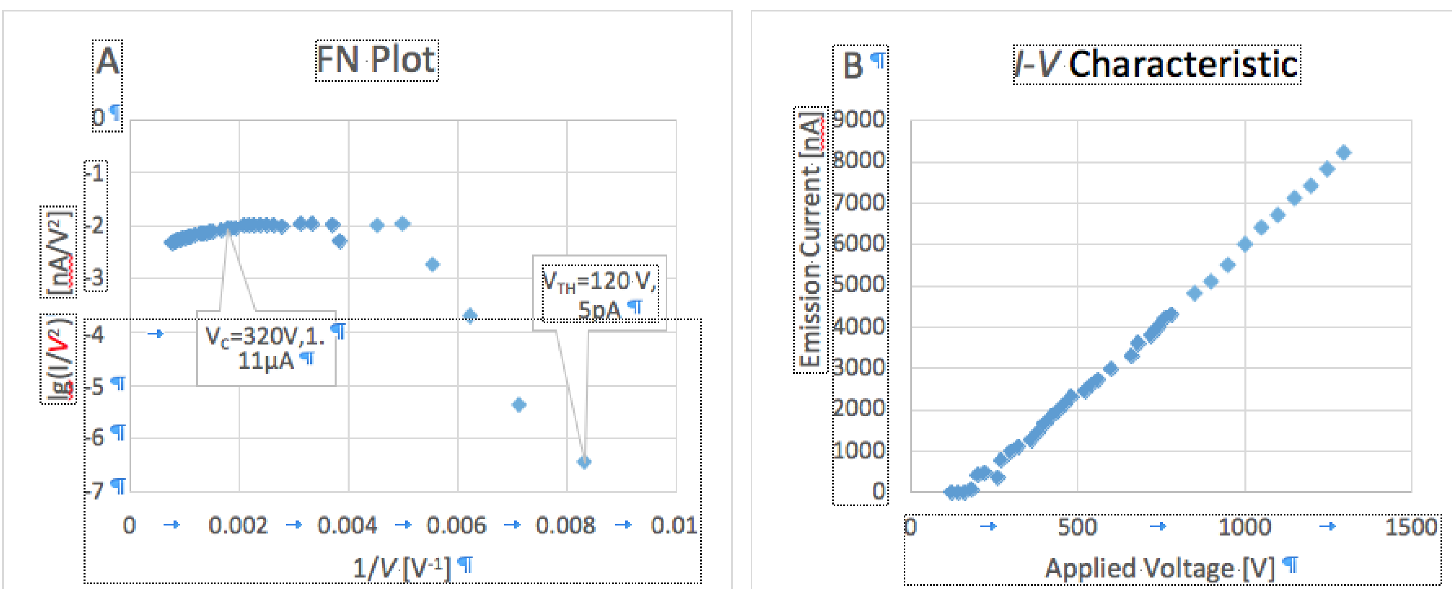


Fig. 5. Sequence of energy spectra that illustrates how the shift of the peak from the Fermi level varies with the emission current.

Alongside: M.S. Mousa, Appl. Surf. Sci. 94/95 (1996) 129.

Below: M.S. Mousa, M. H. AL-Akhras, S. I. Daradkeh, Jordan J. Phys. (in press)

In both cases, these effects have been discovered first in Jordan.



Some outcomes

(1) It has been demonstrated clearly that current flow affects the electrostatics of conductors – this effect may not be as widely known as it perhaps should be.

(2) It has been shown clearly that, even for a single class of material, more than one factor may make field emitter devices non-ideal in practice. In different circumstances, different factors may dominate.

Thanks for your attention

Sign-convention issues

This talk uses the classical electrostatics sign convention, in which the field at a field electron emitter is taken as negative.

Classical electrostatic field is denoted by the conventional symbol *E*, as is usually done in mainstream science outside field electron emission (FE).

Hence, my symbol *E* denotes a quantity that is negative in value, in FE.

In practice, in field emission literature, the equations are written in terms of a field-like quantity that is positive in value. I denote this positive quantity by *F* and define it by: $F = -E$.

To help avoid confusion, I often indicate whether my parameters are normally positive or normally negative.

Arrow-convention issues

For reference, the classical electrostatic arrow conventions being used by the author in this talk are as follows:

Coordinates: The direction of the arrow shows the direction in which the coordinate increases positively.

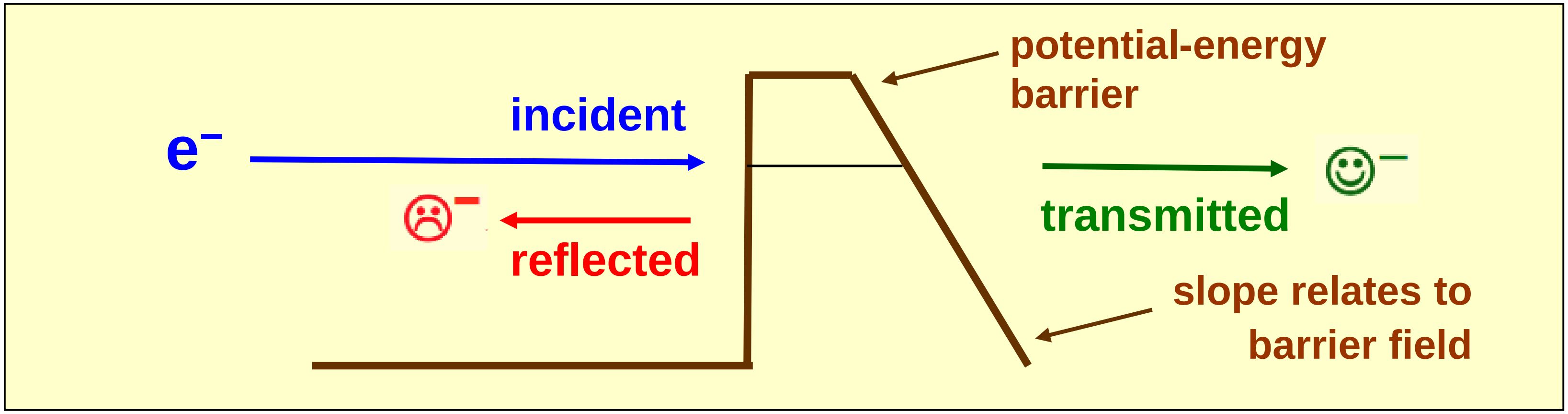
Fields: The direction of the arrow shows the direction in which a positive test charge would move when the electrostatic field is positive in value.

Currents: The direction of the arrow shows the direction of flow of conventional positive charge, when the field is positive in value. [Since FE fields are negative, the actual direction of conventional current flow in FE is opposite to that of the arrow, but the arrow does show the direction of electron flow in FE.]

Basic field electron emission process

Electrons escape from field electron emitters by wave-mechanical tunnelling through a field-lowered potential-energy barrier.

Tunnelling is not mysterious: it is a property of a wave-theory; tunnelling occurs with light, and also with waves on strings.



The new-standard FN-type equation

Field electron emission (FE) theory has many different formulations and approximations. Core theory provides expressions for the local emission current density (LECD) J , in terms of the local work function ϕ and the local barrier field F .

The new-standard FN-type equation

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In the "Modinos-Forbes" formulation, the LECD is given by the so-called new-standard Fowler-Nordheim-type equation:

$$J \;=\; \lambda a \phi^{-1} F^2 \exp[-v_F b \phi^{3/2}/F] \;,$$

where: a and b are universal constants called the Fowler-Nordheim (FN) constants;

v_F is an appropriate particular value of a special mathematical function v called the principal Schottky-Nordheim barrier function, or alternatively the field emission v-function;

λ is a general-purpose pre-exponential correction factor whose value is not well known, but is thought (in late 2018) to lie in the range $0.005 < \lambda < 14$.

Local (measured-)voltage conversion length

As just stated, the the local emission current density (LECDO) is given by

$$J \;=\; \lambda\, a\phi^{-1}F^{2}\,\mathrm{exp}[-\mathrm{v}_{F}b\phi^{3/2}/F]\ .$$

For electron tunnelling to be significant, the local barrier field *F* must be of high magnitude, typically in the range 1-10 V/nm .

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In a practical application, *F* can be related to the measured voltage V_m by the auxiliary formula

$$F \;= V_{m}\,/\,\zeta\,,$$

where is the local (measured-) voltage conversion length (VCL).

The VCL is a mathematical parameter that characterises the emitting device/system, NOT a physical length. Its value is determined by the (small-current) electrostatics of the device/system, and−−in the case of non-Ideal devices/systems−−by various other factors.

Today's talk is mostly about ideal devices/systems.

The need for low voltages

In a practical application, the measured voltage V_m is related to F and E by

$$V_m = F\zeta = -E\zeta .$$

In practical applications, one needs to achieve operating F-values of around 5 V/nm, but at measured voltages that are as low as reasonably possible.

With ideal devices, we need ζ to be as low as reasonably possible.

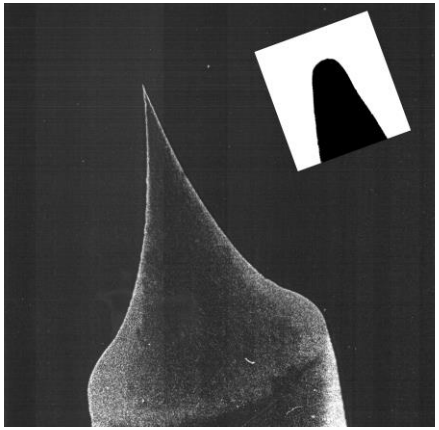
Hence, there is interest in the electrostatics of how to achieve this.

In practice, we are most interested in values (E_a and ζ_a) at the post apex.

Two types of field electron emitter

Single-tip field electron emitter (STFE)

[Tungsten wire emitter shown]

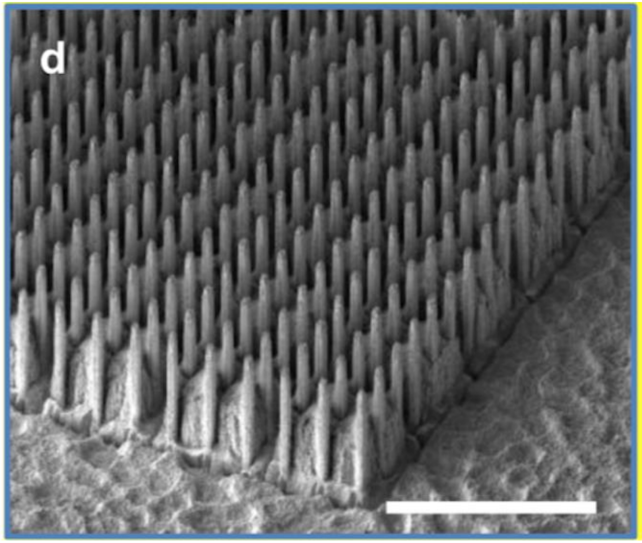


Large-area field electron emitter (LAFE)

[Has many/very-many individual emitters/emission sites.]

[Silicon carbide pillar array shown]

[Diagram courtesy: M.-G. Kang, H.J. Lezec & F. Sharifi, Nanotechnology 24, 065201 (2013), Fig. 1.]



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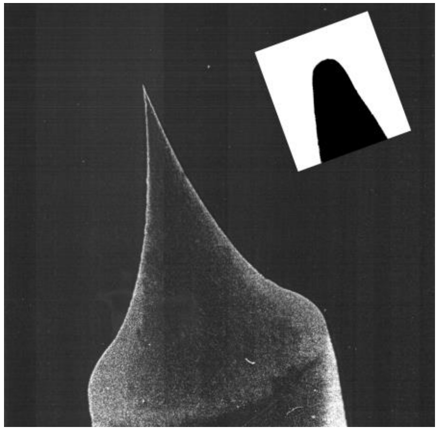
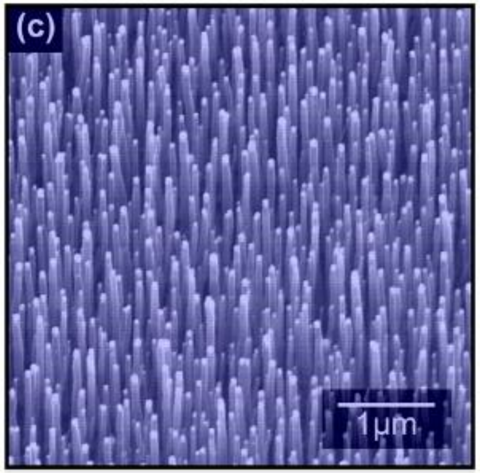


Large-area field electron emitter

(LAFE)

[Has many/very-many individual emitters/emission sites.]

[Carbon nanotube field emitter array shown]



Main applications and contexts

(A) The main contemporary applications of field electron emitters are:

- **STFEs:** **as electron sources for electron microscopes and related machines, including pulsed-laser machines**

  as the electron emitter in a field electron microscope or a related energy-analysis machine
- **LAFEs:** **as large-area electron sources for X-ray machines and microwave generators**

  possibly as spacecraft neutralisers

(B) Another important context is:

- **understanding the causes and mechanisms of electrical breakdown in vacuum and in low-pressure gases, and how to prevent breakdown**

The link between apex FEF and apex VCL

The apex field enhancement factor (apex FEF) γ_a (+ve) is

$$\gamma_a = -E_a d_{sep}/V_p ,$$

And the measured voltage V_m is related to E_a by

$$V_m = F\zeta = -E_a \zeta_a .$$

For an *ideal* device/system, we have

$$V_p = V_m .$$

Hence:

$$\boxed{\gamma_a = d_{sep}/\zeta_a .}$$

[This result also applies to some classes of non-ideal device.]

The process of "getting the VCL ζ_a as low as reasonably possible" means that, in the paradigm model situation (and in other plane-parallel-plate models), the aim is to "get the FEF γ_a as high as reasonably possible".

A focus of the Newton-Fund collaboration has been to develop improved methods of calculating FEFs, particularly apex FEFs.

General principles relating to emitter electrostatics

The operation of electrostatic models

1. In plane-parallel plate geometry, one emitter model with an analytical solution has long been known, namely the hemi-ellipsoidal-post model.

2. With the charge-distribution models, the modus operandi is as follows:

(a) choose the shape of the equipotential surface you wish to model;

(b) choose a distribution of charges so that the electrostatic potential has the correct value at one or many points on the chosen shape (depending on the sophistication of the model);

(c) use this charge distribution to calculate field and FEF values at any desired point – in our case at the emitter apex;

(d) where appropriate, deduce a fitting formula that describes how the relevant FEF (often the apex FEF) varies as a function of model parameters. Any such formula is necessarily approximate.

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3. For the operation of numerical models based on Laplace analysis, see next slide.

4. There are also special models based on other approaches, but these are outside the scope of this talk.

The operation of electrostatic models

With numerical models based on Laplace analysis, the main objectives are:

(a) derive accurate numerical results for (e.g, for apex FEF, but also for charge distributions), using (in our case) the COMSOL software package.

(b) assess the precision of the numerical Laplace analysis;

(c) derive accurate fitting formulae;

(d) compare the Laplace-based results with the results from other methods.

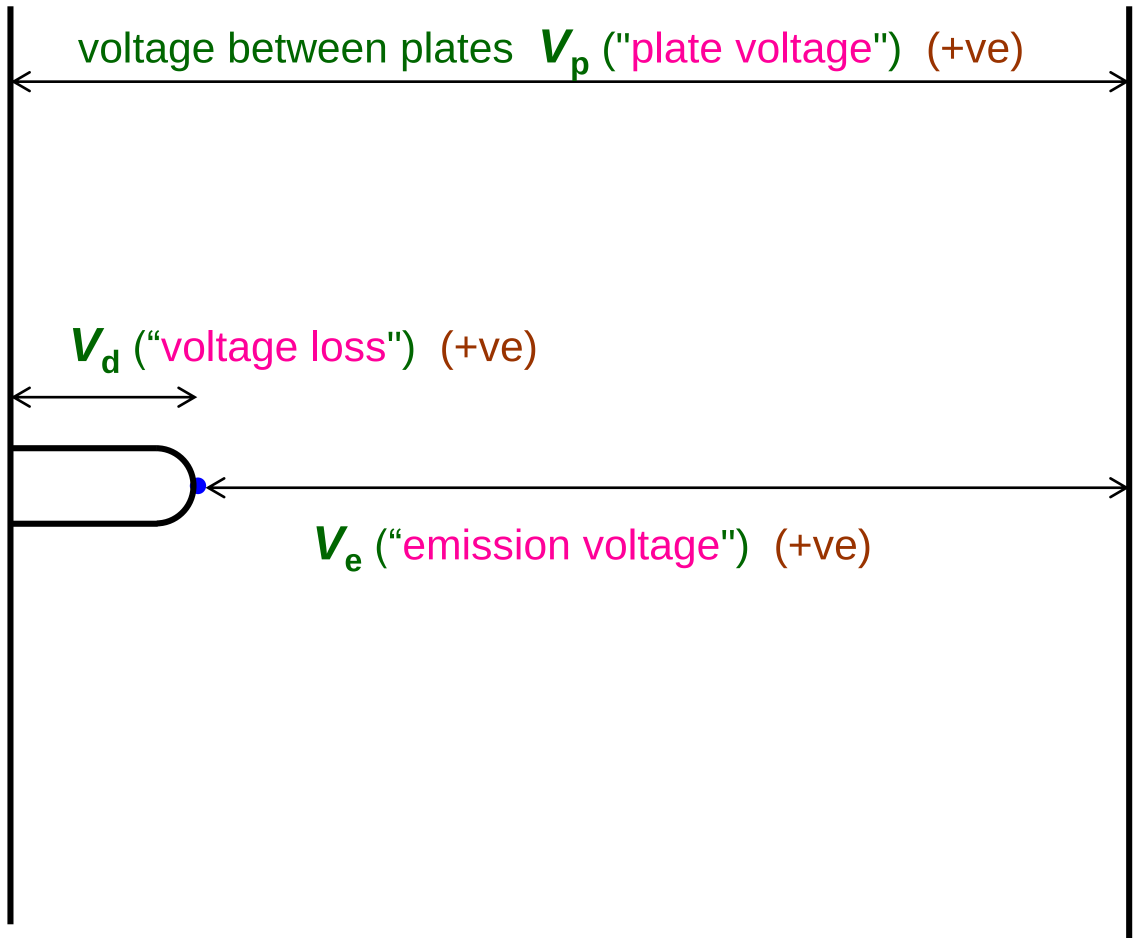
The single isolated HCP emitter

3a: Basic behaviour

The single isolated HCP emitter

3b: The link between voltage loss and FEF reduction

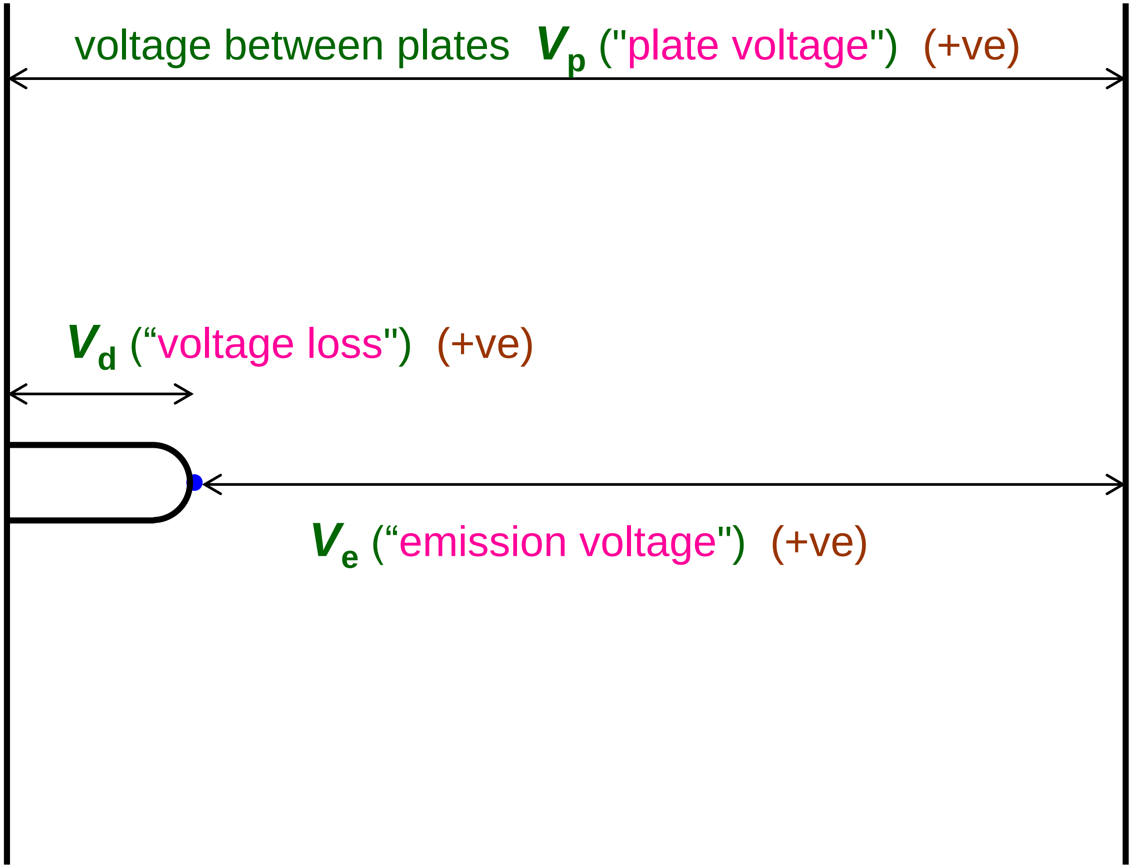
The measurement circuit



For ideal devices, and for non-ideal devices without additional series resistance R_{s1}, we have

$$V_p = V_m$$

Terminology: voltages ***



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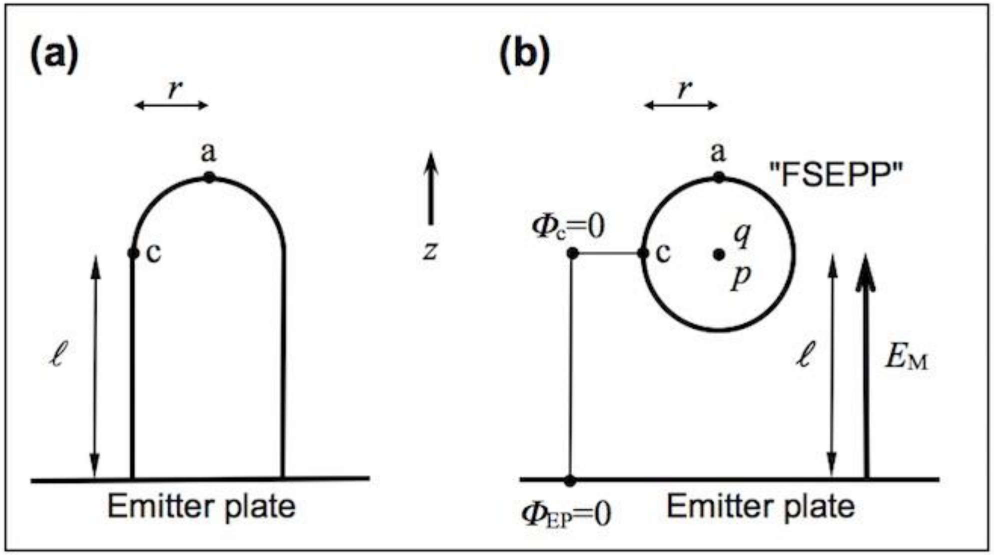
"Floating Sphere at Emitter Plate Potential" model

In the lowest approximation, the usual "Floating Sphere at Emitter Plate Potential (FSEPP)" model works as follows:

0) The electrostatic potential of the emitter plate is taken as zero, i.e. $\Phi_{EP} = 0$.



1) Sphere is in uniform (–ve) field E_M; this contributes a (+ve) amount $-E_M l$ [$=|E_M|l$] to the electrostatic potential Φ_c at point "c".

2) To get Φ_c to zero, place (–ve) charge q at sphere centre, of size such that

$$q/4\pi\varepsilon_0 r \; - \; E_M l = 0 \, .$$

3) The (–ve) apex field E_a is: $E_a = q/4\pi\varepsilon_0 r^2 = E_M(l/r)$.

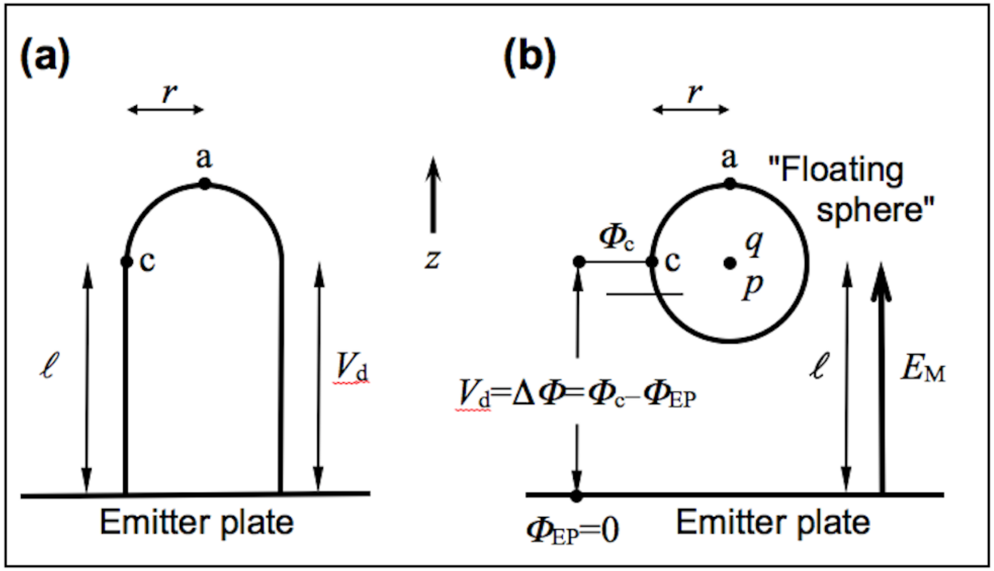
4) Hence the (+ve) small-current apex FEF γ_a^{sc} is given (in this model) by:

$$\gamma_a^{sc} = E_a/E_M = l/r \, .$$

Floating Sphere at Positive Potential V_d

To allow (+ve) voltage loss V_d along post, a simple "Floating Sphere" model is used.

Taking the local work-function as constant means that the (+ve) potential difference $\Delta\Phi$ between "c" and emitter plate needs to equal V_d .



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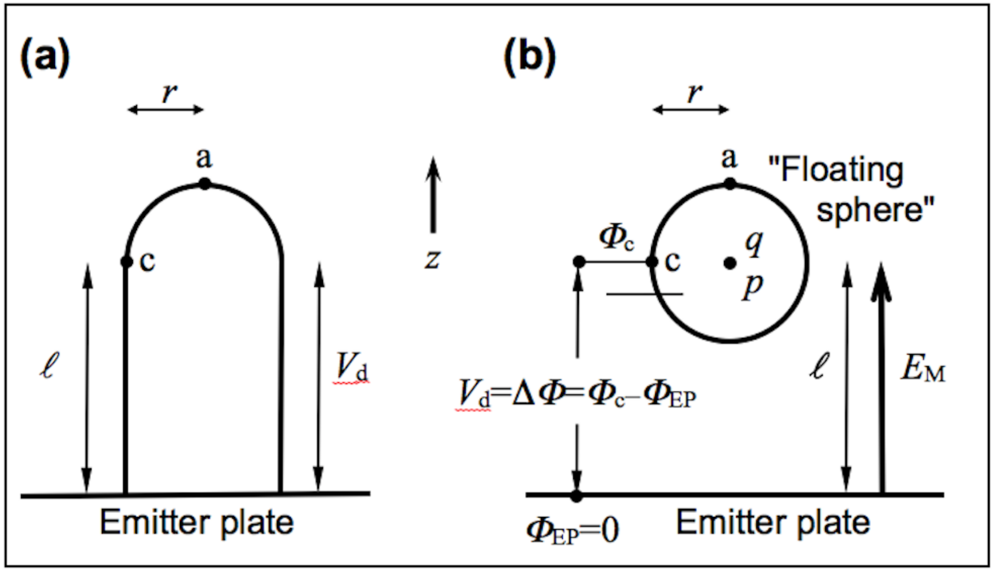

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2) To get $\Delta\Phi$ equal to V_d, place (–ve) charge q at sphere centre, of size such that $\Delta\Phi = q/4\pi\varepsilon_0 r - E_M l = V_d$.

3) The (–ve) apex field E_a is: $E_a = q/4\pi\varepsilon_0 r^2 = E_M(l/r) + (V_d/l)(l/r)$

4) Hence (+ve) operative apex FEF γ_a^{op} is given by:

$$\gamma_a^{op} = E_a/E_M = (l/r) + (V_d/E_M l)(l/r) = [1 - (V_d / |E_M|l)]\, \gamma_a^{sc} .$$

Generality of the argument

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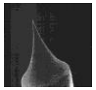

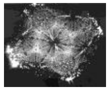

Thanks for your attention

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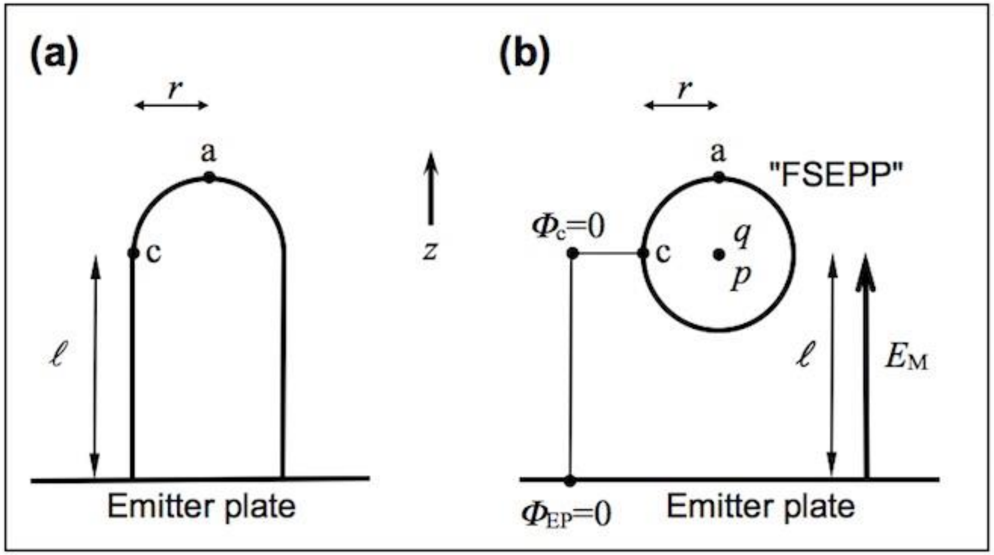
"Floating Sphere at Emitter Plate Potential" model

In the lowest approximation, the usual "Floating Sphere at Emitter Plate Potential (FSEPP)" model works as follows:

0) **The electrostatic potential of the emitter plate is taken as zero, i.e. $\Phi_{EP} = 0$.**



1) **Sphere is in uniform (–ve) field E_M; this contributes a (+ve) amount $-E_M l$ [$=|E_M|l$] to the electrostatic potential Φ_c at point "c".**

2) **To get Φ_c to zero, place (–ve) charge q at sphere centre, of size such that**

$$q/4\pi\varepsilon_0 r - E_M l = 0 \, .$$

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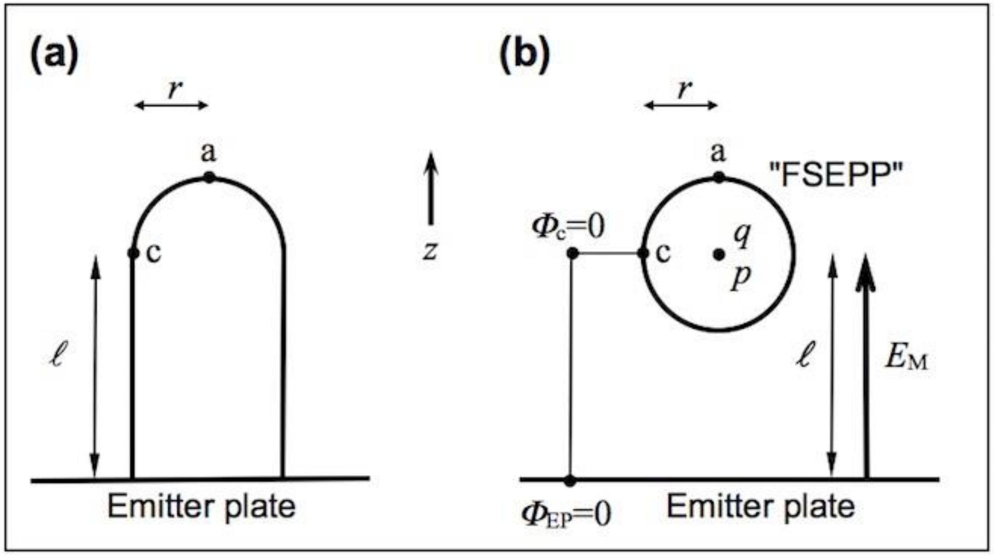

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$$q/4\pi\varepsilon_0 r \; - \; E_M l = 0 \, .$$

3) The (–ve) apex field E_a is: $\quad E_a = q/4\pi\varepsilon_0 r^2 = E_M(l/r)$.

4) Hence the (+ve) apex FEF γ_a is given (in this model) by:

$$\gamma_a = E_a/E_M = l/r \, .$$

The conducting-post & floating-rod formulae

For practical emitters the ratio l/r >>1, and it is usual to put the total post height h [= $l+r$] into the formula.

Also, more exact treatments yield a correction factor α, typically around 0.7 for the HCP model. Thus, the final result is:

$$\gamma_a^{zc} = \alpha \cdot (h/r) \approx h/r .$$

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Because of the symmetry of the situation, the same field enhancement is achieved by a "floating rod" of total length $L=2h$, aligned parallel to a macroscopic field. Hence we get the floating-rod formula

$$\gamma_a = \alpha \, (L/2r) \approx L/2r \,.$$

Schottky's conjecture

Situations can arise where emitter shapes can be modelled as "a little post (or protrusion), with apex FEF γ_1" on top of "a large post (or protrusion), with apex FEF γ_2" .

Schottky's conjecture is that the total apex FEF γ_{tot} is given by:

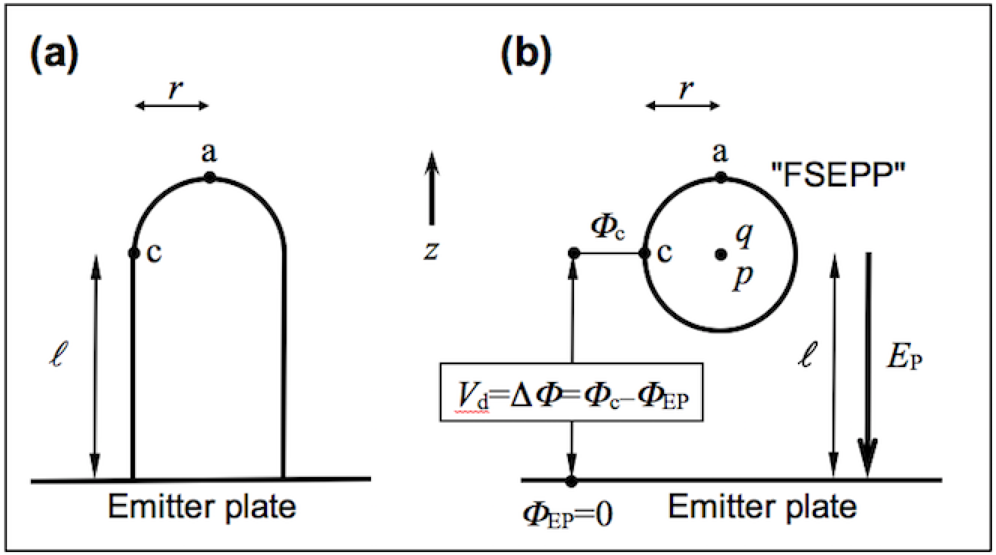
$$\gamma_{tot} = \gamma_1 \gamma_2 .$$

Recent work has investigated the situations in which the conjecture is adequately valid. It turns out that it is often a better approximation than one might anticipate.

Floating Sphere at Positive Potential V_d

To allow (+ve) voltage loss V_d along post, a simple "Floating Sphere" model is used.

Taking the local work-function as constant means that the (+ve) potential difference $\Delta\Phi$ between "c" and emitter plate needs to equal V_d .



1) Sphere is in uniform (–ve) field E_P; this contributes a (+ve) amount $-E_P l$ [$=|E_P|l$] to the electrostatic potential Φ_c at point "c", as before.

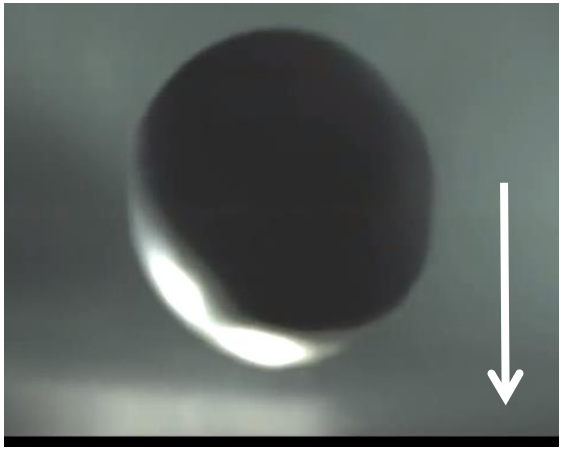
2) To get $\Delta\Phi$ equal to V_d, place (–ve) charge q at sphere centre, of size such that $\Delta\Phi = q/4\pi\varepsilon_0 r - E_P l = V_d$.

3) The (–ve) apex field E_a is: $E_a = q/4\pi\varepsilon_0 r^2 = E_P(l/r) + (V_d/l)(l/r)$

4) Hence (+ve) operative apex FEF γ_a^{op} is given by:

$$\gamma_a^{op} = E_a/E_P = (l/r) + (V_d/E_P l)(l/r) = [1 - (V_d / |E_P|l)]\, \gamma_a^{zc} .$$

Reaction of sodium with water



Experiment: A small piece of sodium is dropped towards a water surface and is photographed from underneath.

Hypothesis: initial heating drives off electrons, leaving positively charged body, and then we get:

Diagrams courtesy: P.F. Mason et al., Nature Chemistry, 7, 250 (2015).

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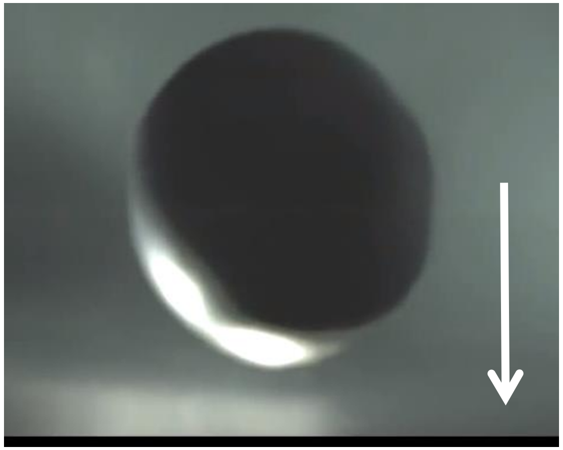

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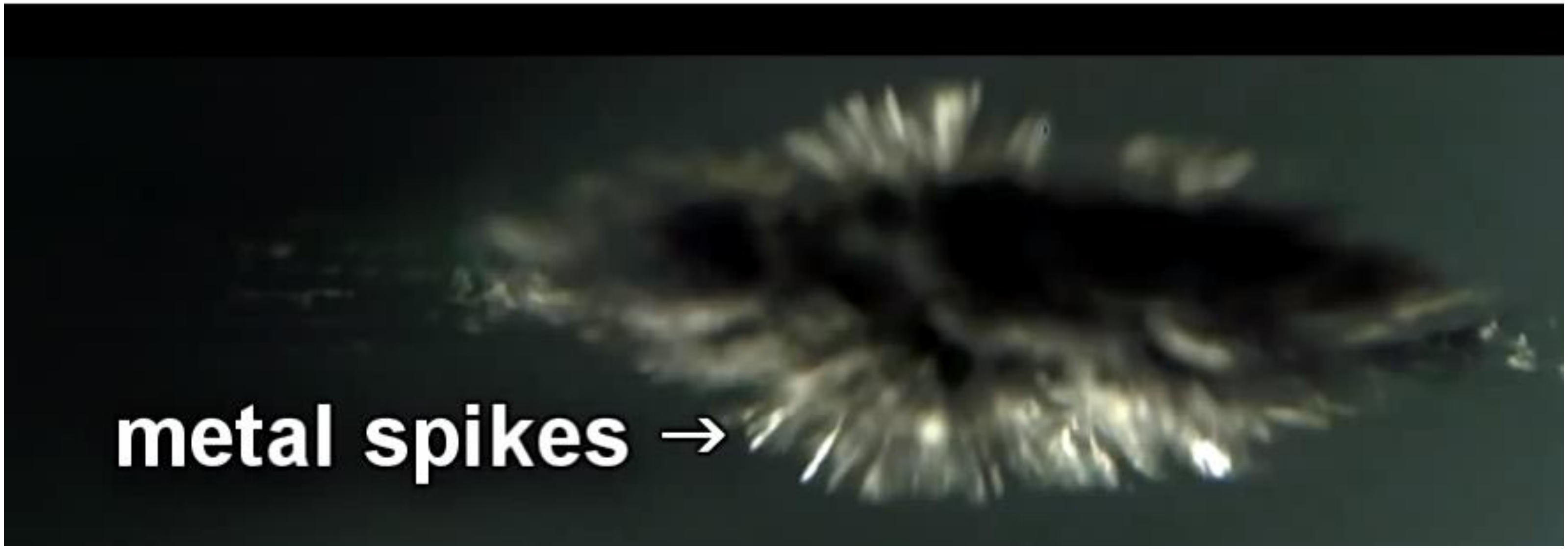

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ATI Advanced Technology Institute | Electrical & Electronic Engineering

LMIS Dynamic effects

UNIVERSITY OF SURREY

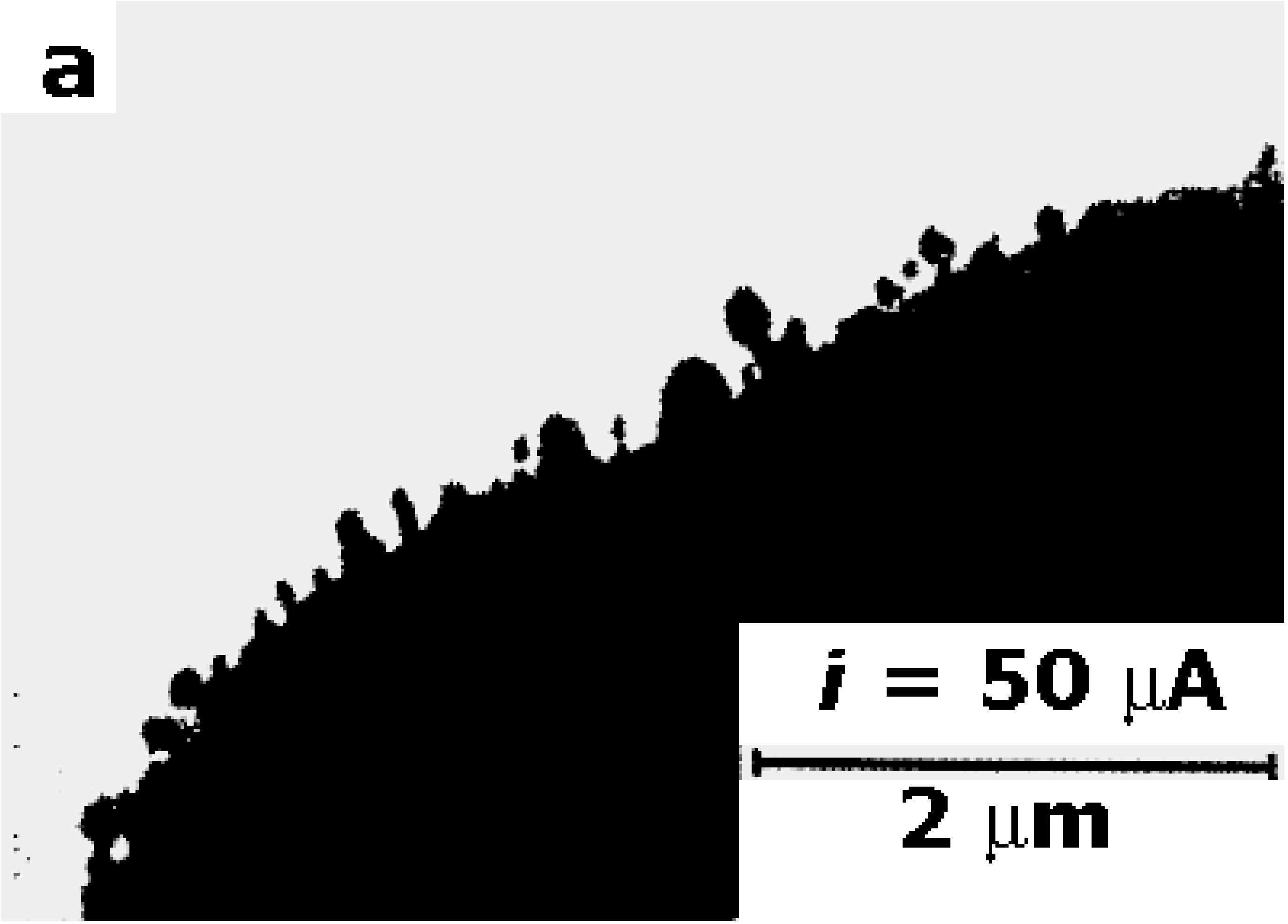
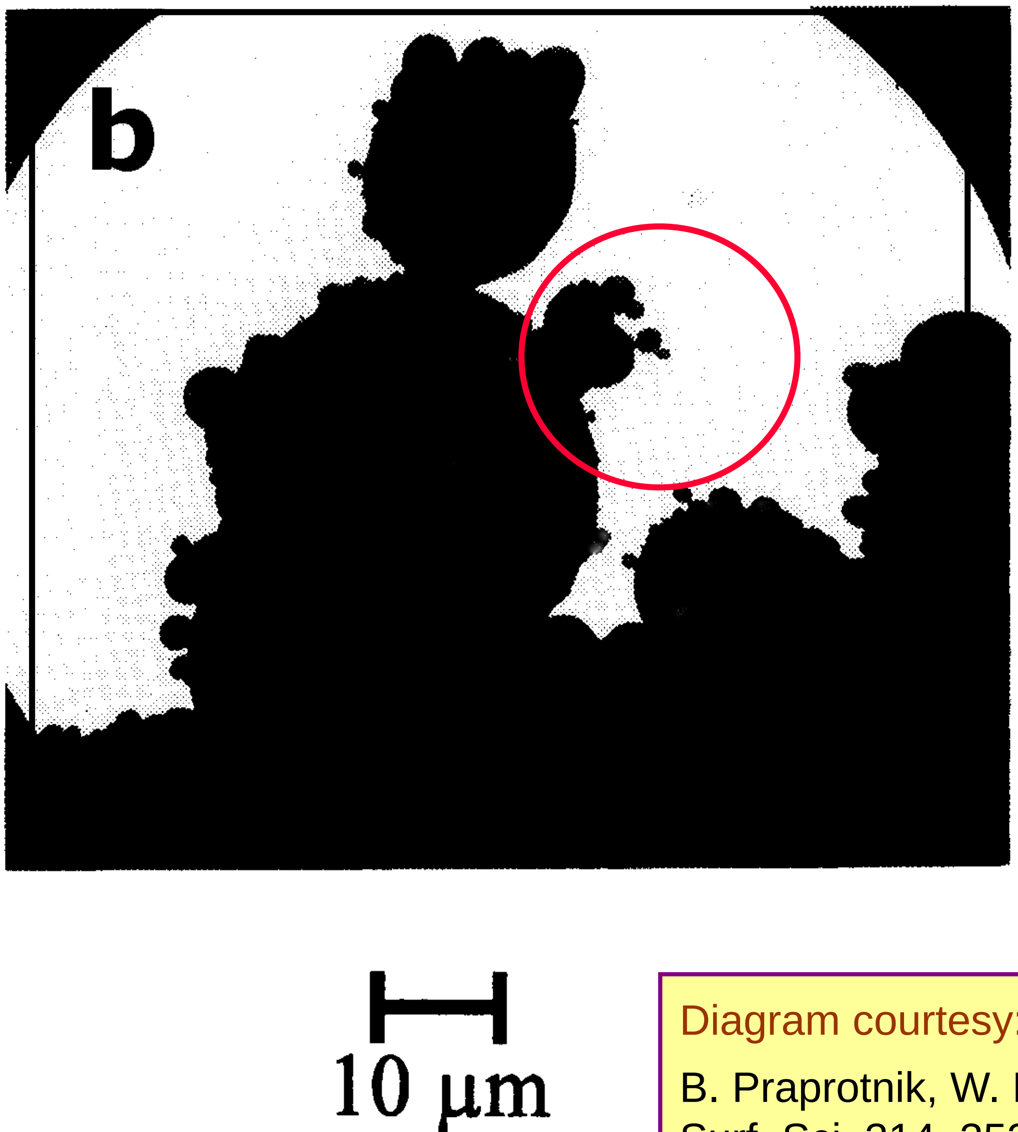


Diagram courtesy: H. Niedrig, W. Driesel & B. Praprotnik.

Extracted from video recording provided privately by H. Niedrig.

LMIS Dynamic effects



One logical explanation of the circled effect is that the thermodynamic drive changes several times as the protrusion is growing.

Perhaps a disruptive event of some kind causes a change in growth mode.

Diagram courtesy:

B. Praprotnik, W. Driesel, Ch. Dietsch & H. Niedrig, Surf. Sci. 314, 353 (1994), Fig. 7 (e).

3c: More theory and discussion

Basic voltage-based change-over condition

In the special ideal case where γ^0 is assumed uniform and the system is a sphere of radius r, surrounded by a spherical counter-electrode of very large radius, then: $A = 4\pi r^2$, $C \approx 4\pi\varepsilon_0 r$. This yields change-over voltage

$$(V_{co})^2 = (4/\varepsilon_0)\,\gamma^0 r\ .$$

In this ideal spherical case, put $V_{co} = rF_{co}$, where F_{co} is the change-over field. This this gives the stress-based change-over condition:

$$\tfrac{1}{2}\varepsilon_0(F_{co})^2 = 2\gamma^0/r\ .$$

This may be read as:

"Maxwell-stress outwards equals surface-tension stress inwards" .

Change-over fields

This formula can be re-arranged into the forms

$$r^{1/2}F_{co} \;=\; (4\gamma^0/\varepsilon_0)^{1/2} \;=\; c_{TF}\,.$$

$$F_{co} \;=\; c_{TF}\,/\,r^{1/2}$$

Some selected vales of c_{TF} are:

	γ0	cTF		γ0	cTF
	eV/nm2	V nm^–1/2		eV/nm2	V nm^–1/2
Al	5.7	20.3	Ag	6.0	20.9
Si	5.4	19.8	Ta	13.4	31.2
Ti	10.3	27.3	W	15.6	33.6
Fe	11.7	29.1	Pt	11.2	28.5
Cu	8.1	24.3	Au	7.3	23.0
Nb	11.9	29.3	Hg	3.1	15.0
Mo	14.0	31.9			

For example, for tungsten:

a 1 nm radius tip has $F_{co} \approx 34$ V/nm ;

a 100 nm radius tip has $F_{co} \approx 3.4$ V/nm .

Balling-up of field emitter in zero-field case

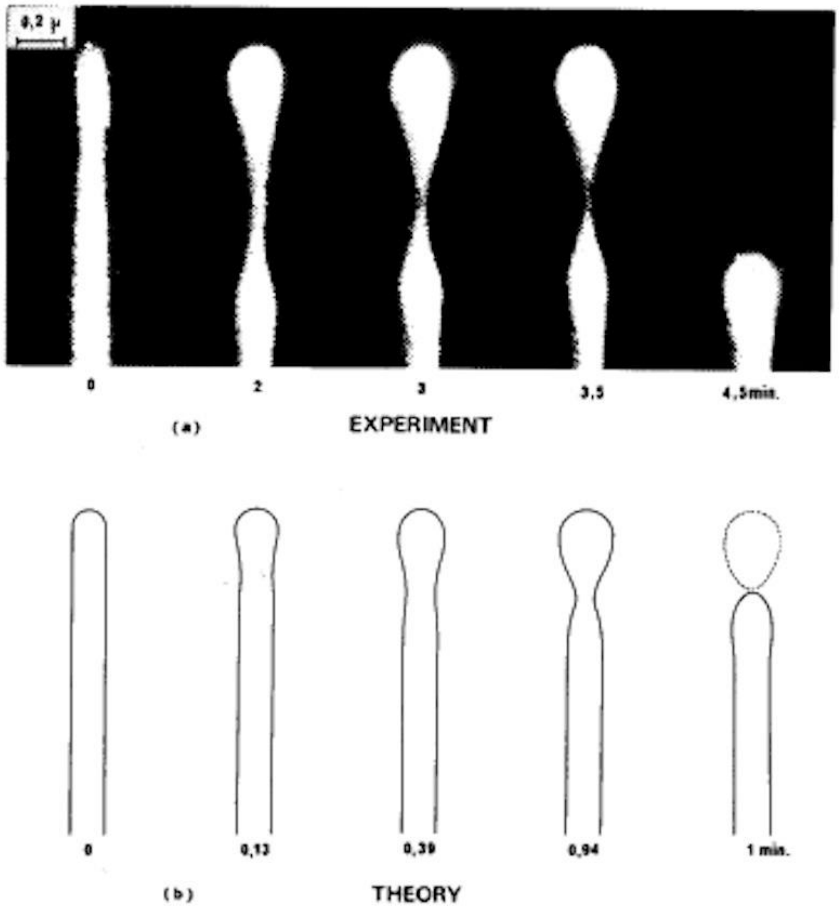
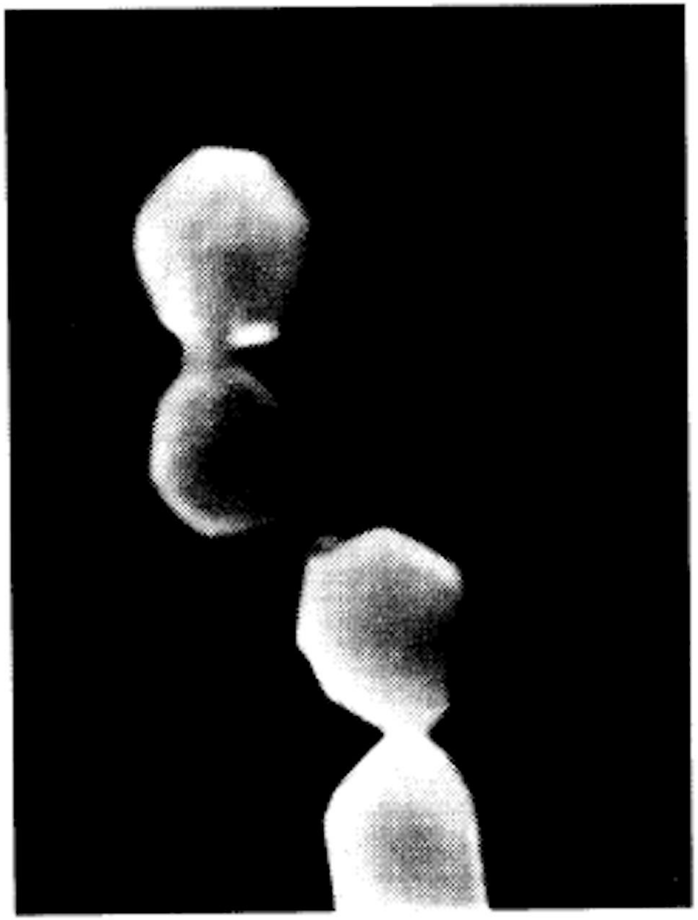


Fig. 3. Evolution of a solid tungsten drop. Experimental (a) and theoretical (b) result.[19)]

Balling-up effects can also occur with hot solid emitters, as a result of surface-atom migration (in this case in the absence of any applied field).

Diagram courtesy:
M. Drechsler, Proc. 2nd Intern. Conf. on Solid Surfaces, 1974; in Japan J. Appl. Phys. Suppl. 2, pt 2, p. 25.

The effect of surface-energy anisotropy, in zero-field case



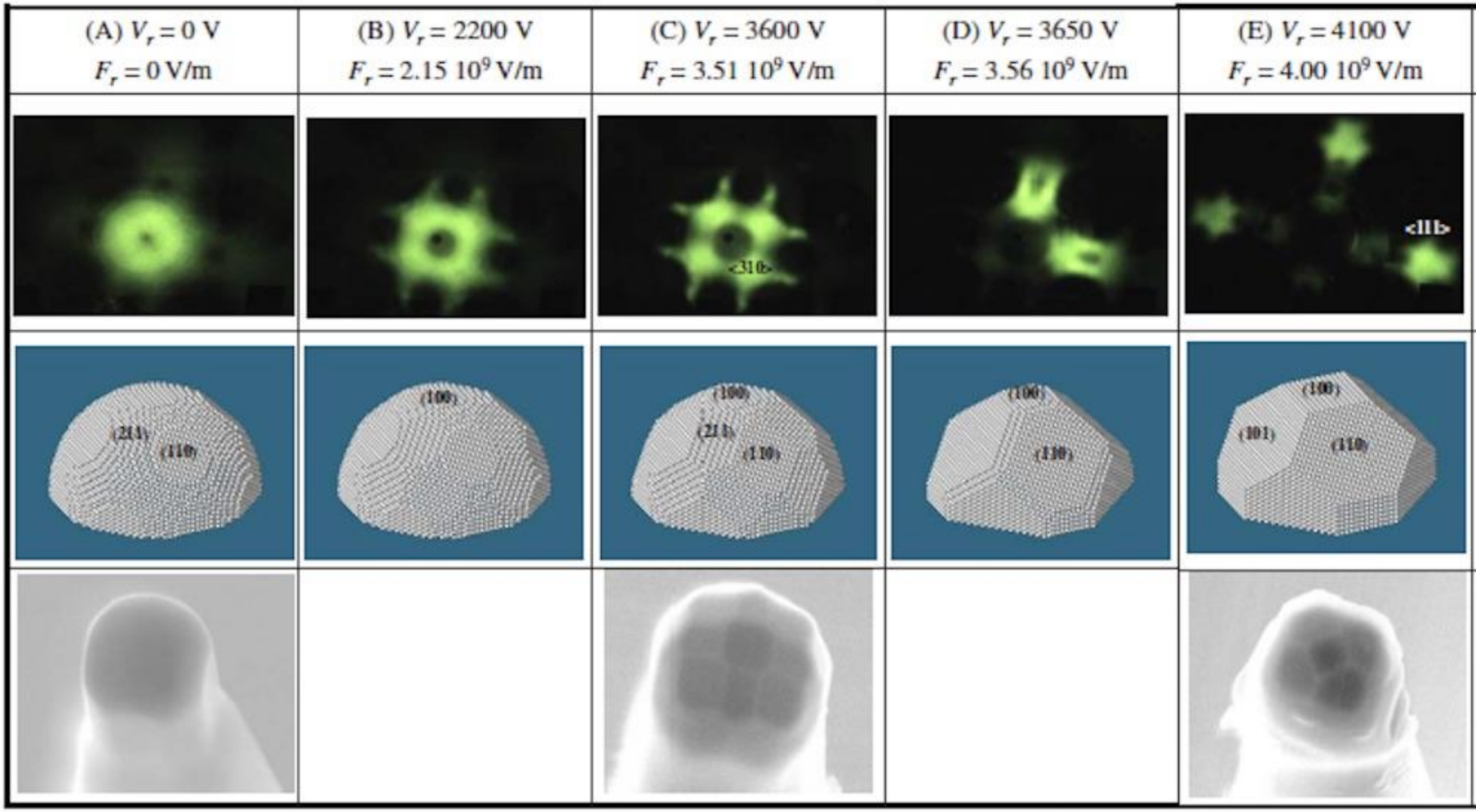
Surface self-diffusion shape of an initially conical Ni tip, surface not clean.

In the zero-field case, when there are strong differences in surface free energy per unit area, as between different crystallographic faces, then facetting can occur.

Results can be strongly affected by the presence of adsorbates.

Diagram courtesy:
M. Drechsler. Proc. 2nd Intern. Conf. on Solid Surfaces, 1974; in Japan J. Appl. Phys. Suppl. 2, pt 2, p. 25, Fig.16.

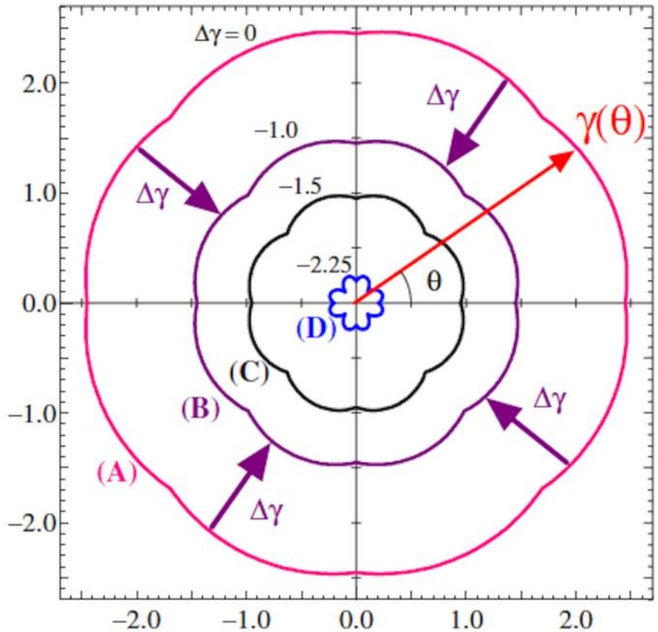
Equilibrium emitter shape depends on applied field



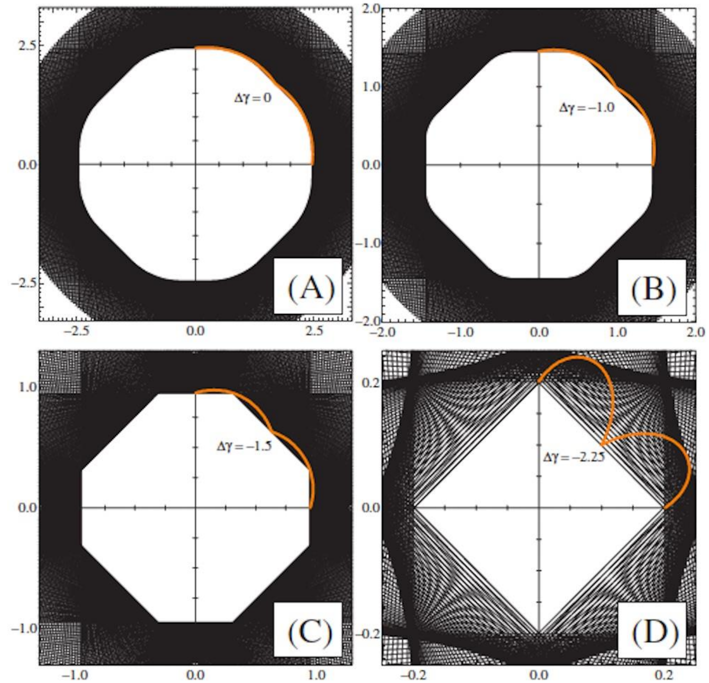
More generally, the equilibrium shape of a field emitter tip depends on the value of the applied field (and, obviously, affects the field electron emission images).

Diagram courtesy: S. Fujiota & H. Shimoyama, Phys. Rev. B 75, 235431 (2007), Fig. 5.

– and may be explained by Wulff-type methods



(a) Surface tension reduction by electrostatic field



(b) Evolution of Equilibrium Crystal Shape

Effects of this kind can be explained by a Wulff-type methods, with the local "electrical Gibbs function per unit area" taking the place of the local surface free energy per unit area ("local surface tension"). Alternatively, the Gibbs function can be treated as a (reduced) "effective surface tension", as in the diagrams above.

Diagram courtesy: S. Fujiota & H. Shimoyama, Phys. Rev. B 75, 235431 (2007), Fig. 10.

The field dependence of activation energy for surface migration

At a "bridge site" on a charged surface, the local field is higher than at the adjacent bonding sites. This "pulls down" the bonding potential energy at the bridge site, more so than at the adjacent bonding sites.

Consequently, when the entity migrating on a surface is a significantly charged atom (i.e., a partial ion), the field terms reduce the activation energy for thermal-field surface migration.

This situation defies semi-classical analysis, because everything changes – fields, coefficients, PPI terms and field-energy terms.

My original inclination has been to think that this would probably be another field-energy effect. However, the work of Jansson et al*., presented at conferences last summer, challenges this. It now seems much more likely that this is a PPI effect.

*Now summarised at: V. Jansson et al., arXiv1709.04694.

Surface migration in high electric fields

and its possible role in electrical breakdown

Nanoprotrusions and electrical breakdown

One possible cause of electrical breakdown is field electron emission from a nanoprotrusion.

A short nanoprotrusion will grow in the presence of a high electric field, if it is hot enough.

This thermodynamic trend is predicted qualitatively BOTH by classical field energy-arguments AND by arguments based on PPI effects.

The Helsinki group, in particular Ville Jannson, have explored the kinetics of nanoprotrusion growth, using molecular dynamics calculations. A brief indication of Jansson's work follows.

Adatom diffusion in high electric fields

V. Jansson, E. Baibuz, M. Veske, A. Kyritsakis, V. Zadin, F. Djurabekova

Helsinki Institute of Physics
Department of Physics, University of Helsinki

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Surface diffusion in electric fields

- Adatoms in electric fields become polarized

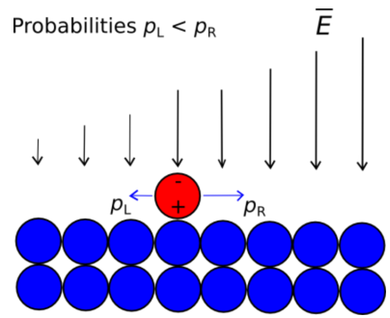
- This introduces a dipole force, perpendicular to the field, that will bias the adatom's migration towards stronger fields

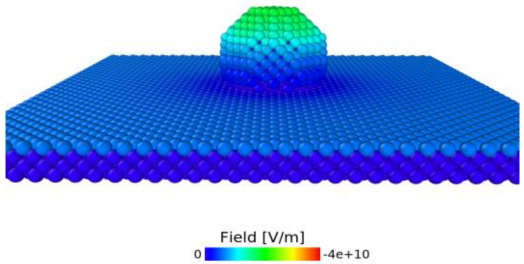
- The bias of the adatom migration is achieved by using migration barriers modified by the field at the lattice point (0) and the saddle point (s):

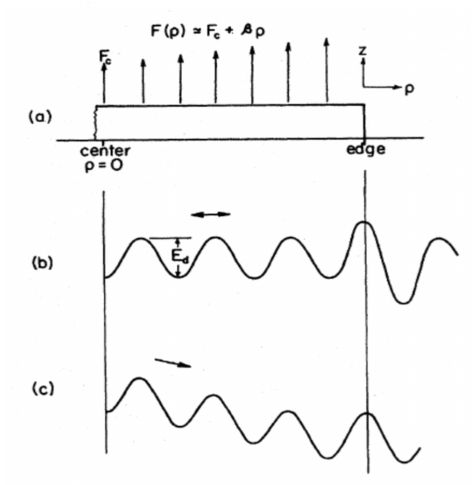
$$E_m = E - (\mu_s F_s - \mu_0 F_0) - \frac{1}{2}(\alpha_s F_s^2 - \alpha_0 F_0^2)$$

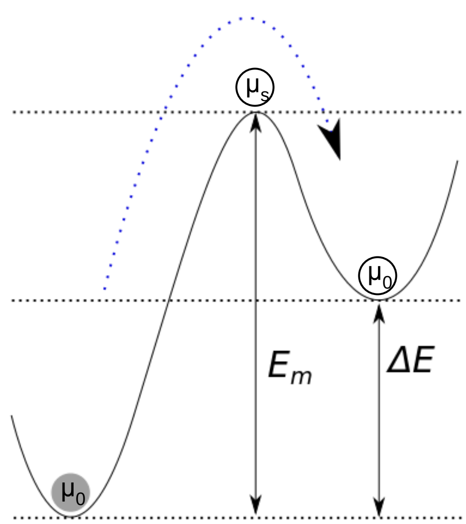
- At lattice positions, the dipole moment μ_0 and polarizability α_0 are calculated with DFT

- μ_s is fitted to experiment and $\alpha_s = \alpha_0$









Tsong & Kellogg PRB 1975

Slide provided by V. Jansson

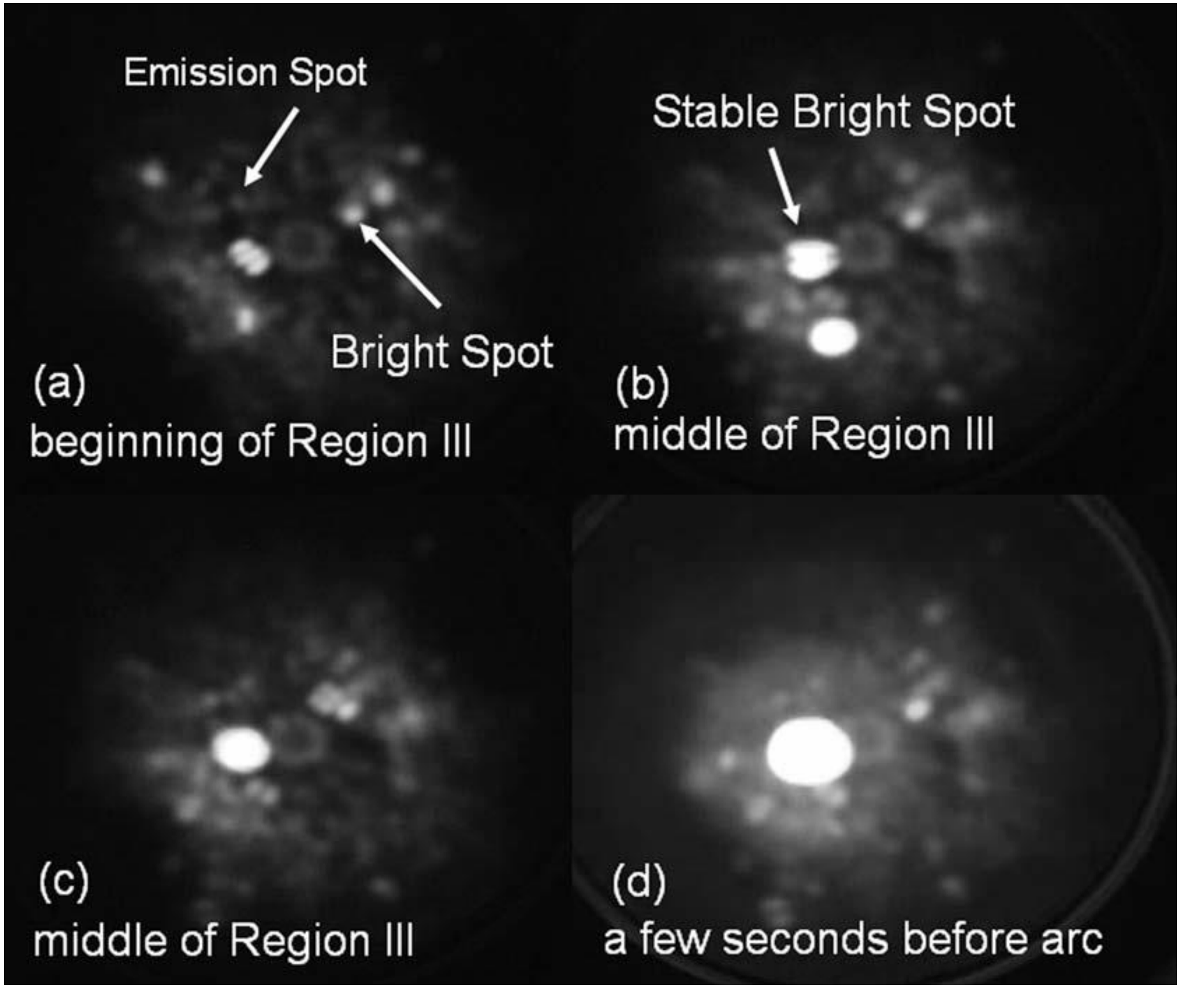
What initiates metal nanotip formation?

It remains my provisional view that flat solid surfaces are not likely to spontaneous develop nanotips, even when hot. So what initiates nanotip formation ? Currently, I can see at least four options worth thinking about:

- **Very small pre-existing protrusion.**
- **Nanotip growth at edge of crystal facet.**
- **Metallurgical process in the substrate that causes surface irregularity [suggested by Helsinki group].**
- **Impact of heavy particle onto surface.**

Presumably more than one mechanism may operate.

Life cycle of cold W field electron emitter



This sequence shows unambiguously that, with cold metal field electron emitters, arc initiation is preceded by nanoprotrusion growth on top of the field electron emitter.

Diagram courtesy: K.S. Yeong & J.T.L. Thong, J. Appl. Phys., 99, 104903 (2006), Fig. 8.

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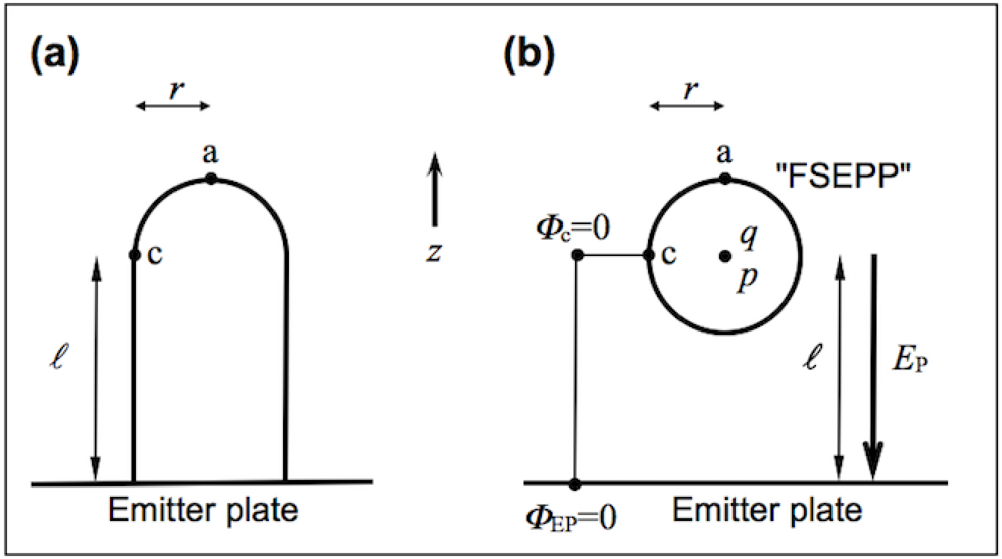

Note on arrow convention for fields [as used here – which differs from earlier work]

As per the standard convention for fields in electrostatics, the arrows used here indicate the direction in which a positive test charge would travel.

Since a positive test charge moves "downhill", it follows that the electrostatic potential at "c", due to a field as shown by the arrow in Fig. (b), is positive relative to the plate. More generally (here) we can write: $\Phi = \beta z$, where β is positive. From the usual relation, it follows that the related classical electrostatic field E is given by

$$E = -\mathrm{d}\Phi/\mathrm{d}z = -\beta .$$

Hence the classical electrostatic field E_P as shown on the diagram is negative.