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The BIRD model
Insights and Future Developments

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16/09/19

OUTLINE

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- **SEMI-CLASSICA**L Approach to the BIRD Model
  - The BIRD model
  - The Electron Cold Emission from a Dielectric Layer

- **EXPERIMENTAL** Evidence
  - Old Data (Furuta et al. 2005)
  - New Data HVSGTF (Padova 2019)

- **THEORETICAL** development
  - Infinite well + Triangular barrier
  - Calculation of λ *(probability/time)*

- **SUMMARY**

Breakdown Induced by Rupture of Dielectric layer

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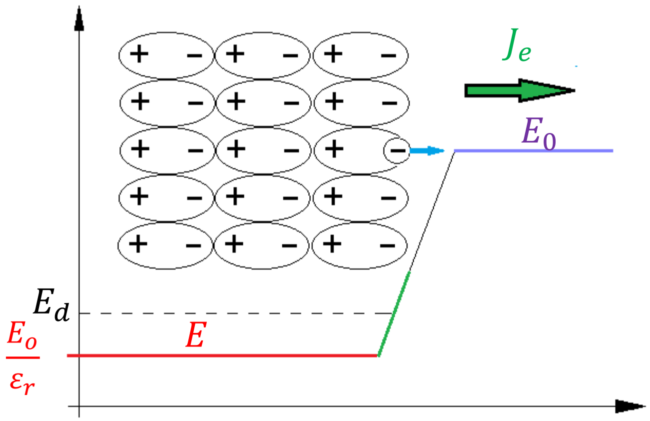
The semi-classical Approach (2019)

IEEE TRANSACTIONS ON PLASMA SCIENCE, VOL. 47, NO. 5, MAY 2019

$$J_e = env \qquad E > E_d$$



$$J_e = k(E_0 - E)(E - E_d)$$

$$J_e \approx k(E_0 - E)E\, exp\left(-\frac{E_d}{E}\right)$$

$$J_e(t=0) \approx k_1 E_0^2 exp\left(-\frac{k_2}{E_0}\right)$$

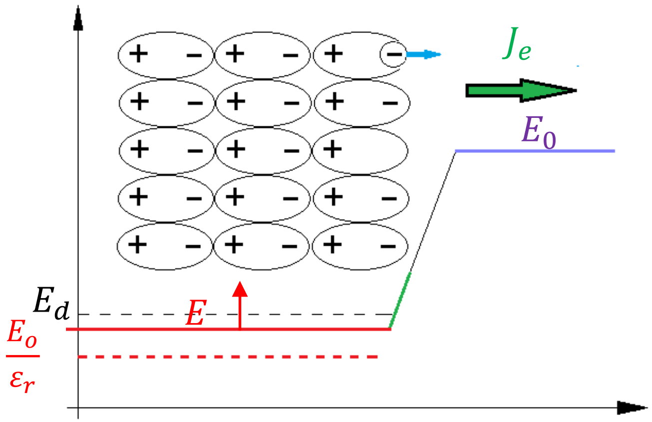
- The same law as F.N.
- Different constants k_1 and k_2
- Higher CURRENT DENSITY
- Lower β needed

$$k_1 = \frac{\varepsilon_r - 1}{\varepsilon_r^2} k \qquad k_2 = \varepsilon_r E_d$$

$$k = \frac{e\varepsilon_0 b}{\hbar} \approx 3.8 \cdot 10^{-11} \frac{A}{V^2}$$

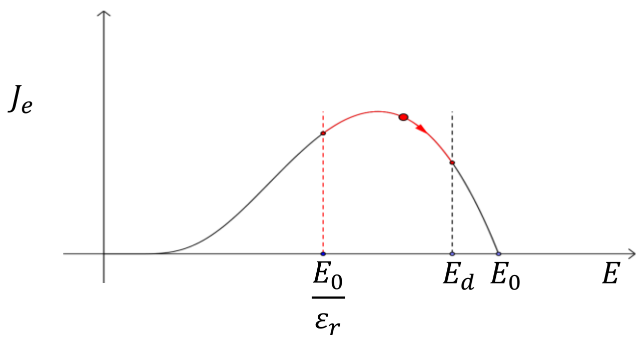
Breakdown Induced by Rupture of Dielectric layer

The semi-classical Approach (2019)



…at constant VOLTAGE…

$$J_e \approx k(E_0 - E)E\ exp\left(-\frac{E_d}{E}\right)$$



- The Electric Current initially increases and then decreases
- Eventually it reaches the dielectric strength E_d

Breakdown Induced by Rupture of Dielectric layer

The semi-classical Approach (2019)

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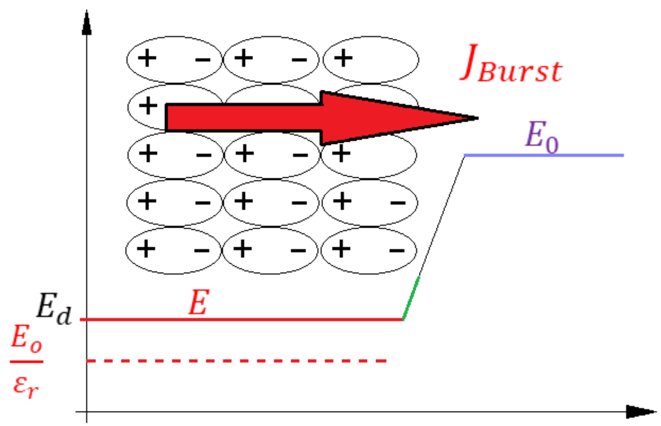

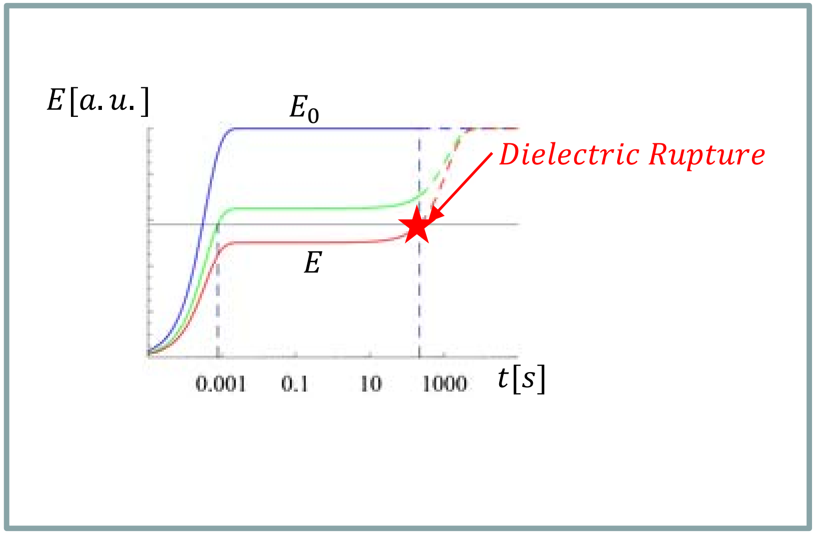

When the internal Field E reaches the Dielectric Strenght E_d we have a

- Local rupture of the dielectric layer
- Burst of Current

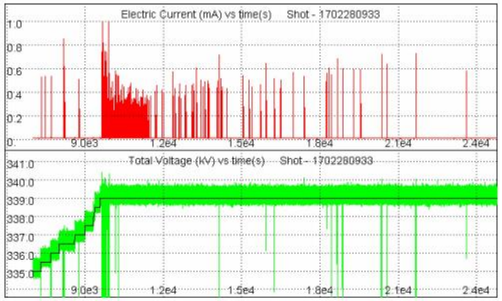

Fig. 5. Experimental current bursts (red line), reference (black line), and measured (green line) voltage in HVPTF during conditioning. A constant voltage $V = 339$ kV has been applied at time $t = 9588$ s.

BIRD Model: EXPERIMENTAL EVIDENCE

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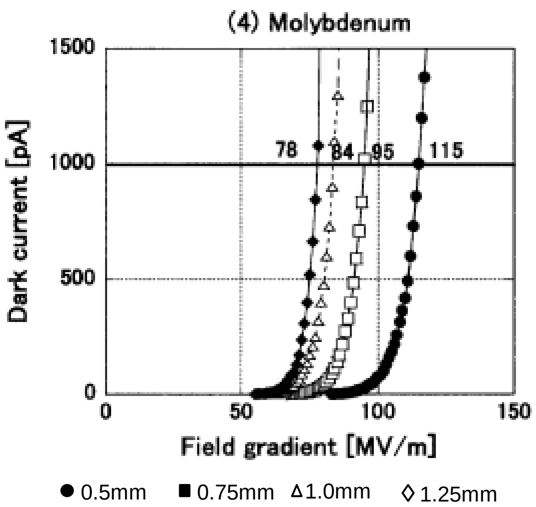
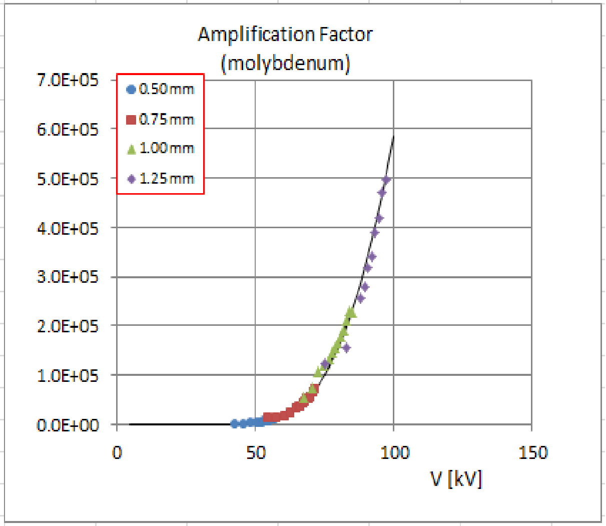
FURUTA et al. data (2005)

Nuclear Instruments and Methods in Physics Research A 538 (2005) 33–44



$$MoO_3 \begin{cases} k_1 \approx \dfrac{\varepsilon_r - 1}{\varepsilon_r^2} k = 0.91 \cdot 10^{-11} \dfrac{A}{V^2} \\ k_2 \approx \varepsilon_r E_d = 800\ kV/mm \end{cases}$$

$$\frac{I_{Furuta}}{J_e}$$



BIRD Model: EXPERIMENTAL EVIDENCE

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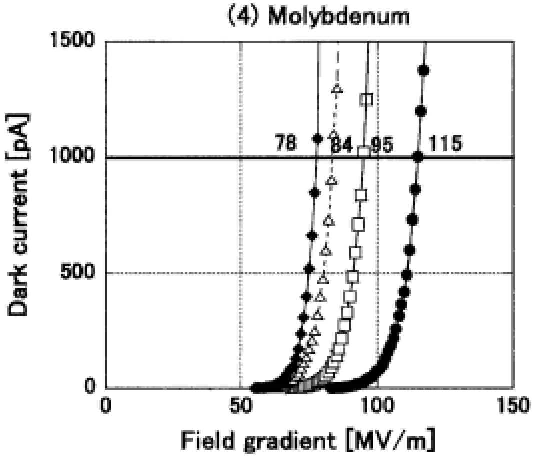

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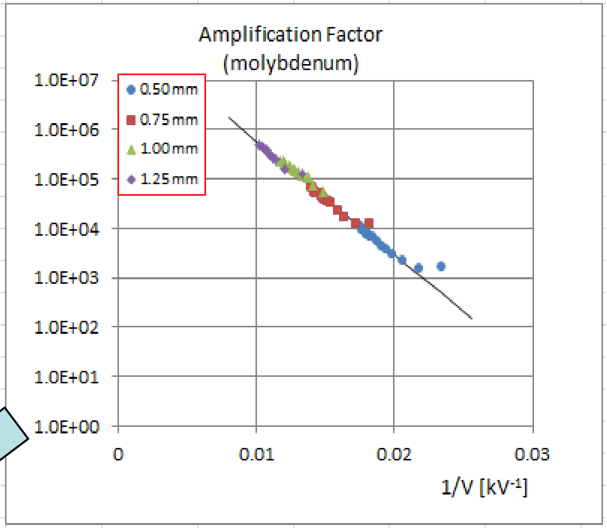
$$\frac{I_{Furuta}}{J_e}$$

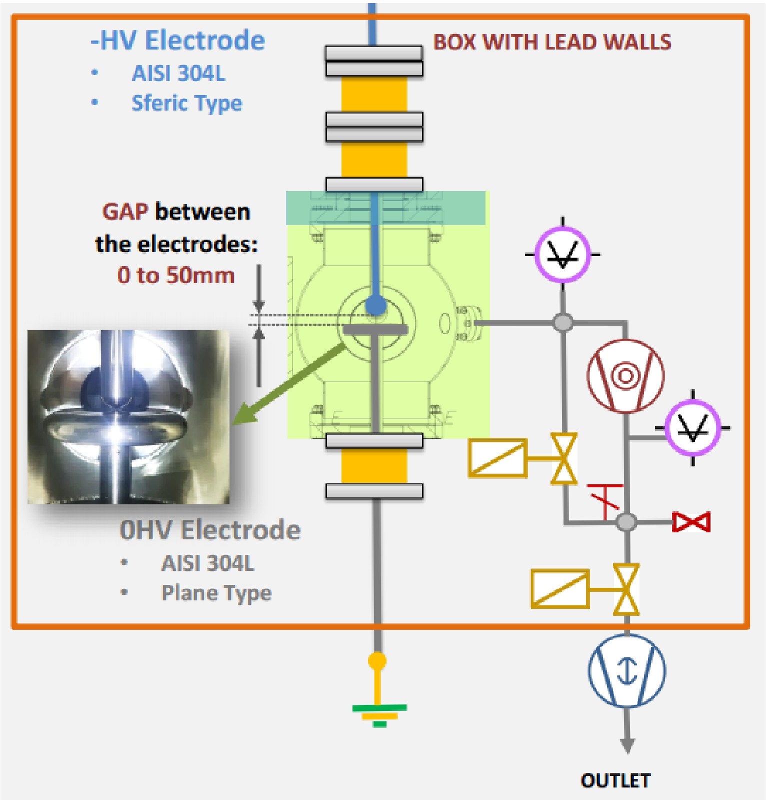


$$A(V) = \frac{I_{Furuta}}{J_e} \sim exp\left(-\frac{k_3}{V}\right)$$

BIRD Model: EXPERIMENTAL EVIDENCE

HVSGTF (2019)

High Voltage Short Gap Test Facility



A new Facility has been recentely developed in Padova,
to study dark-current at short gap.

HVSGTF is designed to carry out experiments up to 100 kVdc with metal and non metal electrodes

BIRD Model: EXPERIMENTAL EVIDENCE

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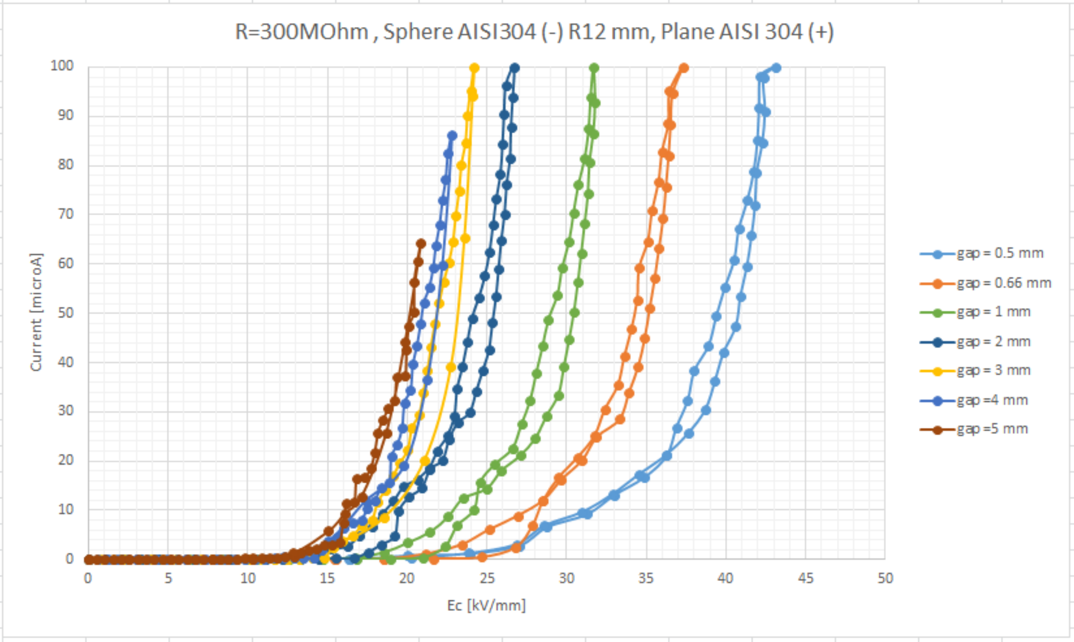
HVSGTF (2019)

First Result



BIRD Model: EXPERIMENTAL EVIDENCE

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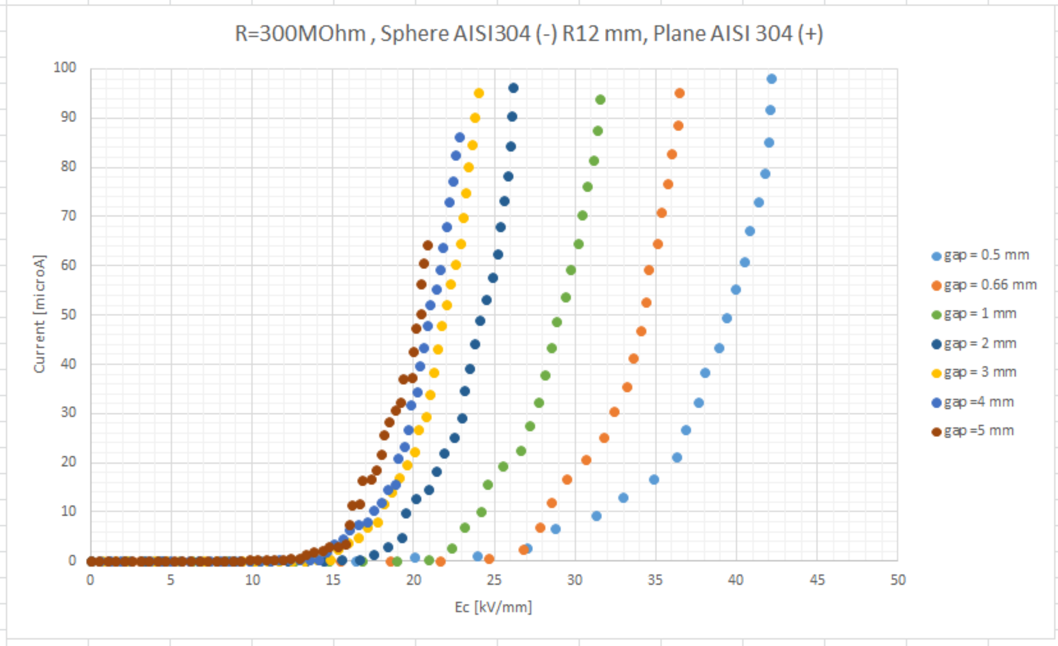
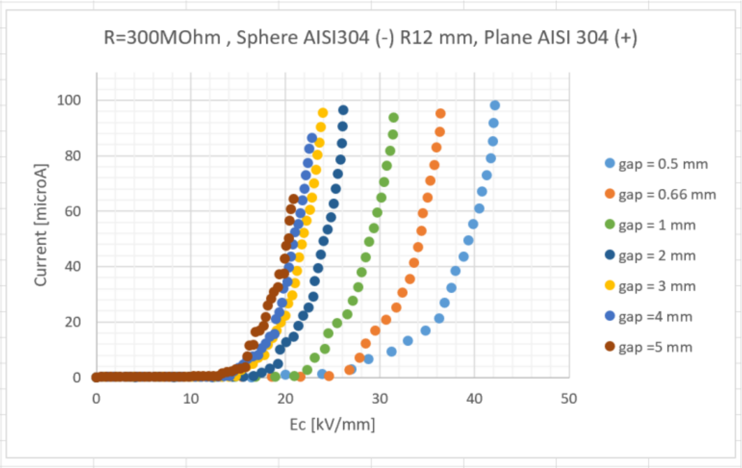
HVSGTF (2019)

First Result



BIRD Model: EXPERIMENTAL EVIDENCE

HVSGTF (2019)



$$I = SJ_e(E_0)\mathrm{A}(V)$$

$$Sk_1\beta^2E_0^2 exp\left(-\frac{k_2}{\beta E_0}\right)\cdot\left[1+\xi exp\left(-\frac{k_3}{V}\right)\right]$$

BIRD Model: EXPERIMENTAL EVIDENCE

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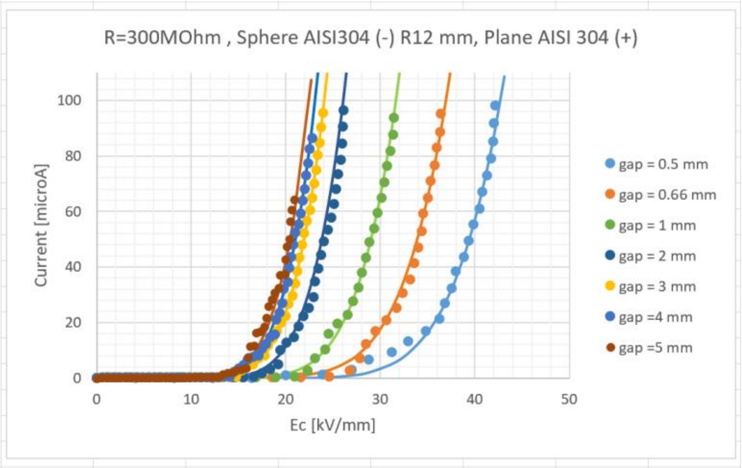
HVSGTF (2019)



$$I = SJ_e(E_0)A(V)$$

$$Sk_1\beta^2 E_0^2 exp\left(-\frac{k_2}{\beta E_0}\right) \cdot \left[1 + \xi exp\left(-\frac{k_3}{V}\right)\right]$$

Const	Value	
$Sk_1\beta^2$	5.20	$\cdot 10^{-20} \frac{Am^2}{V^2}$
$\frac{k_2}{\beta}$	82.7	kV/mm
ξ	400	
k_3	87.9	kV

Bisquare Method – Matlab Tool

BIRD Model: EXPERIMENTAL EVIDENCE

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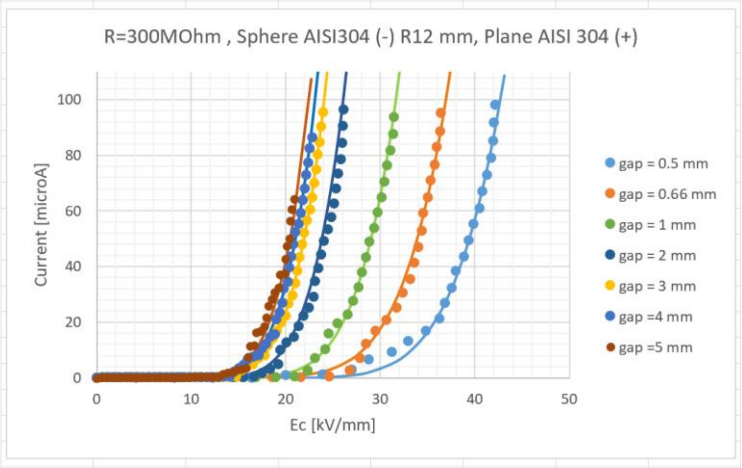
HVSGTF (2019)



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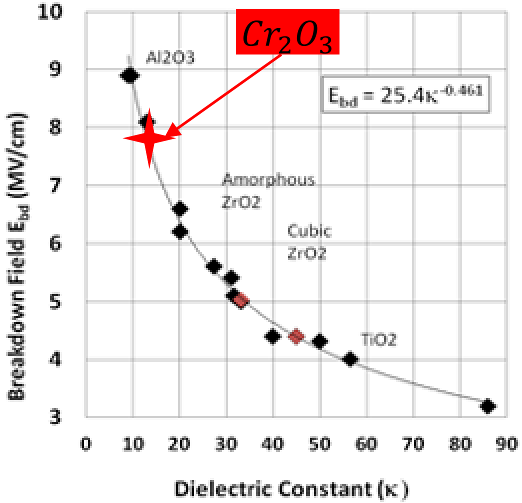
Bisquare Method – Matlab Tool

Cr_2O_3

$$\frac{k_2}{\beta} = 82.7\,\frac{kV}{mm} = \frac{\varepsilon_r E_d}{\beta}$$

$$\varepsilon_r = 12.5 \quad E_d = 7.9\,\frac{kV}{mm}$$

https://intermolecular.com/materials/metal-oxides/



→ $\beta = 1.2$

BIRD Model: EXPERIMENTAL EVIDENCE

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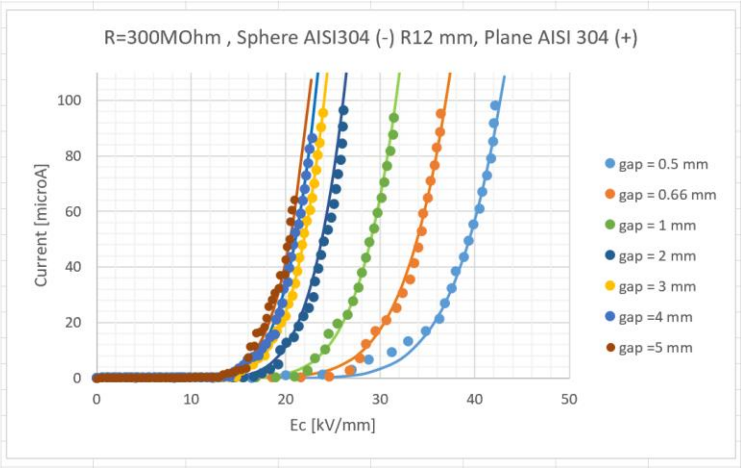
HVSGTF (2019)



Cr_2O_3

$$Sk_1\beta^2 = 5.20 \cdot 10^{-20}\,\frac{Am^2}{V^2}$$

$$k_1 = \frac{\varepsilon_r - 1}{\varepsilon_r^2}\,k \sim 2.8 \cdot 10^{-12}\,\frac{A}{m^2}$$

Const	Value	
$Sk_1\beta^2$	5.20	$\cdot\, 10^{-20}\,\frac{Am^2}{V^2}$
$\frac{k_2}{\beta}$	82.7	kV/mm
ξ	400	
k_3	87.9	kV

Bisquare Method – Matlab Tool

→ $S = 1.3 \cdot 10^{-2} mm^2$

BIRD Model: EXPERIMENTAL EVIDENCE

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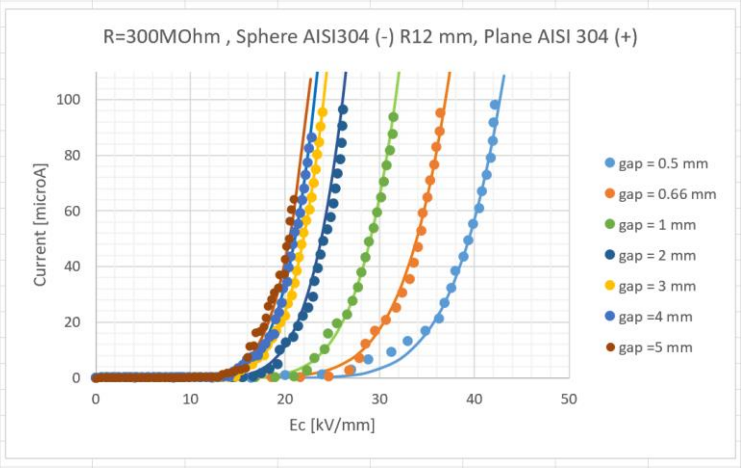
HVSGTF (2019)



Const	Value	
$Sk_1\beta^2$	5.20	$\cdot 10^{-20} \frac{Am^2}{V^2}$
$\frac{k_2}{\beta}$	82.7	kV/mm
ξ	400	
k_3	87.9	kV

Bisquare Method – Matlab Tool

Cr_2O_3

$\xi = 400$

?

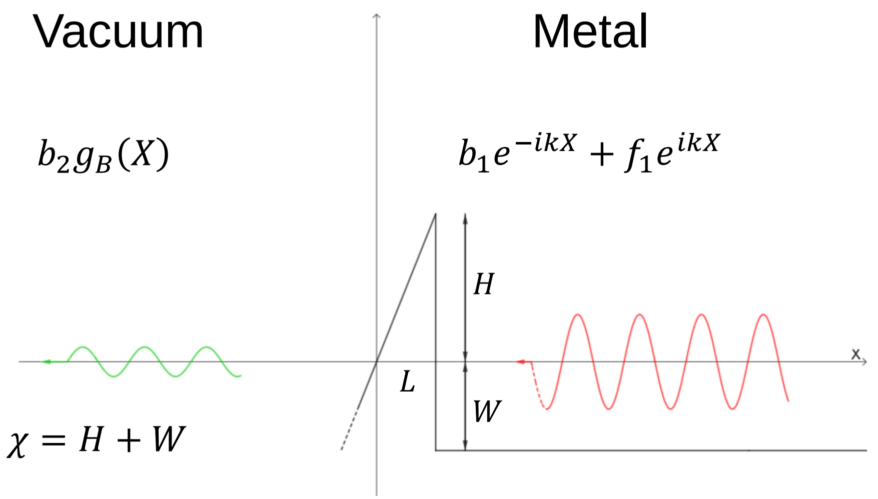
$k_3 = 87.9\ kV$

….model for A(V) is needed…

BIRD Model: THEORETICAL DEVELOPMENT

ECE from Metal

«G. Forbes and J. Deane» Proc. R. Soc. A (2011) 467, 2927-2947



$$g_B(X) = [Ai(k_A X) - iBi(k_A X)]$$

$$T = \frac{k_A}{\pi k}\left|\frac{b_2}{b_1}\right|^2$$

$$f_1 = \frac{1}{2}\left(g_B(x_L) - i\frac{k_A}{k}g'_B(x_L)\right)e^{-ikL}b_2$$

$$b_1 = \frac{1}{2}\left(g_B(x_L) + i\frac{k_A}{k}g'_B(x_L)\right)e^{ikL}b_2$$

$$x_L = k_A L = \left(\frac{2m}{\hbar^2}eE\right)^{\frac{1}{3}}\frac{H}{eE} = \left(\frac{2m}{\hbar^2}\right)^{\frac{1}{3}}\frac{H}{(eE)^{\frac{2}{3}}}$$

$$x_L{}^{\frac{3}{2}} = \left(\frac{2m}{\hbar^2}\right)^{\frac{1}{2}}\frac{H^{\frac{3}{2}}}{eE} = \frac{E_d}{E}$$

$x_L \to \infty$ ⟺ $E \to 0$

$$T = \frac{4}{\left(\frac{k_A\sqrt{x_L}}{k}\right) + \left(\frac{k_A\sqrt{x_L}}{k}\right)^{-1}}exp\left(-\frac{4}{3}x_l^{\frac{3}{2}}\right)$$

$$J = \int D(W)T(W)dW$$

$$J = k_1 E^2 exp\left(-\frac{k_2}{E}\right)$$

BIRD Model: THEORETICAL DEVELOPMENT

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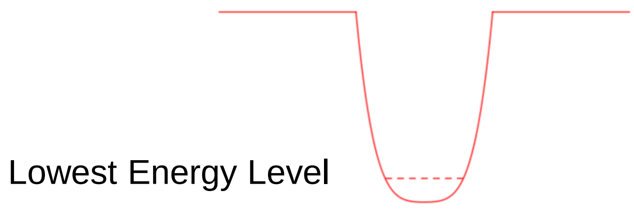
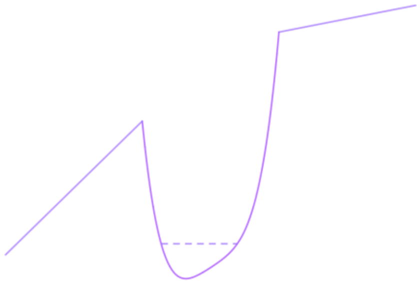
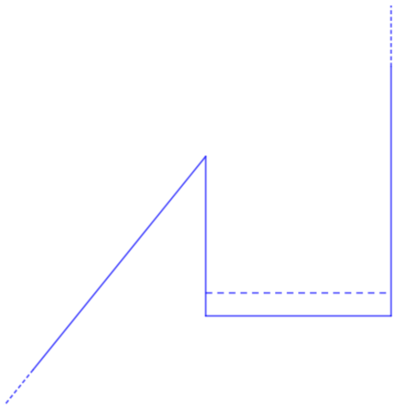
ECE from Dielectrics

Electrons are confined in polarization structures….





…modified in presence of an Electric Field



The simplest possible model

BIRD Model: THEORETICAL DEVELOPMENT

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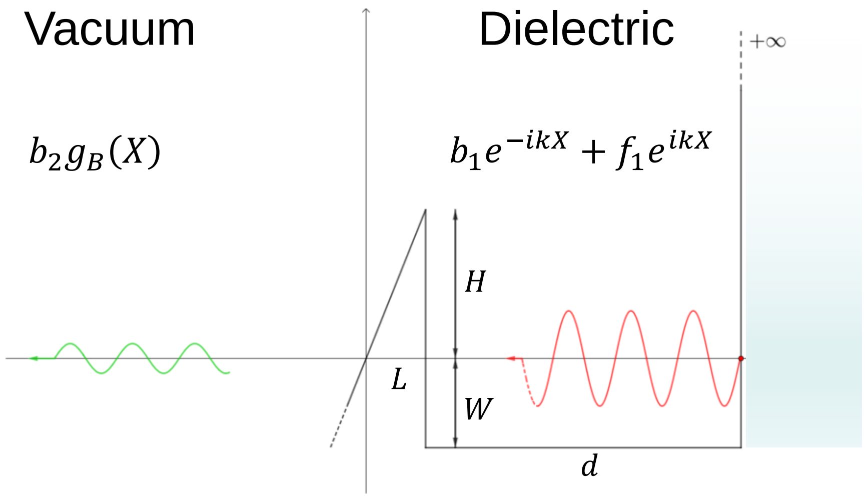
ECE from Dielectrics



$$g_B(X) = [Ai(k_A X) - iBi(k_A X)]$$

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$$b_1 = \frac{1}{2}\left(g_B(x_L) + i\frac{k_A}{k}g'_B(x_L)\right)e^{ikL}b_2$$

$$b_1 e^{-ik(L+d)} + f_1 e^{ik(L+d)} = 0 \quad \Rightarrow \quad \text{tg}(kd) = -\frac{k}{k_A}\frac{g_B(x_L)}{g'_B(x_L)}$$

$$W \to W + i\frac{\hbar}{2}\lambda \qquad e^{\frac{i}{\hbar}W} \to e^{\frac{i}{\hbar}W} \cdot e^{-\frac{\lambda}{2}t} \qquad |\psi|^2 \to |\psi|^2 e^{-\lambda t}$$

$$\lambda = \text{decay constant} \quad = \frac{Probability}{time}$$

BIRD Model: THEORETICAL DEVELOPMENT

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ECE from Dielectrics

for $E \to 0$ $(x_L \to \infty)$

$$T = \frac{k_A}{\pi k}\left|\frac{b_2}{b_1}\right|^2$$

$$T = \frac{4k}{k_A\sqrt{x_L}} exp\left(-\frac{4}{3}x_L^{\frac{3}{2}}\right)$$

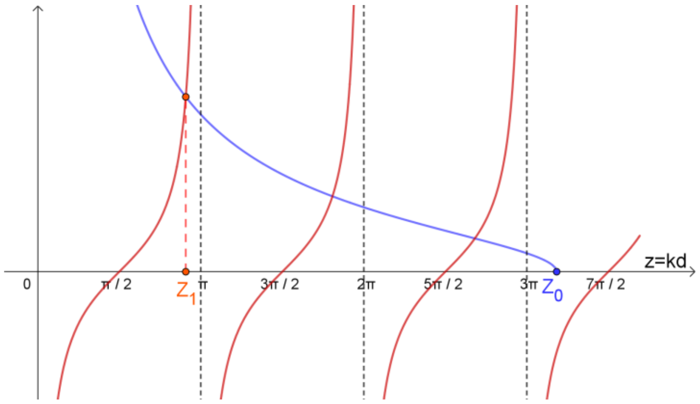
$$\mathrm{tg}(kd) = -\frac{k}{k_A}\frac{g_B(x_L)}{g'_B(x_L)}$$

$$\mathrm{tg}(kd) = -\frac{k}{\sqrt{k_0^2 - k^2}}$$

$$\mathrm{tg}\left(z - \frac{\pi}{2}\right) = \sqrt{\frac{z_0^2}{z^2} - 1}$$

$$z_0 = k_0 d = \frac{\sqrt{2m\chi}}{\hbar} d$$

$$z = kd$$



$$\frac{\pi}{2} < z_1 < \pi \qquad k_1 \sim \frac{1}{d} \qquad W_1 \sim \frac{\hbar^2}{md^2}$$

BIRD Model: THEORETICAL DEVELOPMENT

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ECE from Dielectrics

How can we calculate λ ?

Gamow theory of alpha decay - 1928

G. Gamow, ZP, **51,** 204

We can now solve the problem for two symmetrical potential barriers [Fig. 3]. We shall seek two solution.

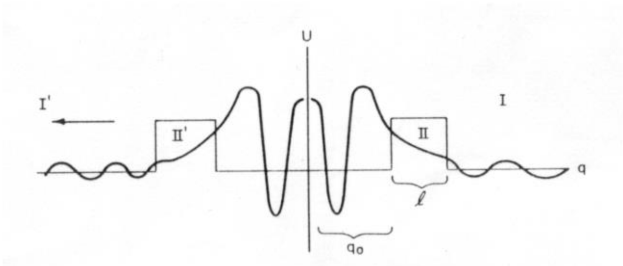

Figure 3:

The conservation principle then reads

$$\frac{\partial}{\partial t} e^{-\lambda t} \int_{-(q_0+l)}^{+(q_0+l)} \Psi^{(q)}_{II,III} \cdot \Psi^{(q)}_{II,\ III}\, dq = -2\, \frac{A^2 h}{4\pi \cdot i \cdot m} \cdot 2ik \cdot e^{-\lambda t},$$

from which we obtain

$$\lambda = \frac{4hk \sin^2\theta}{\pi m \left[1 + \left(\frac{k'}{k^0}\right)^2\right] 2(l+q_0)_k} \cdot e^{-\frac{4\pi l\sqrt{2m}}{h}\sqrt{U_0-E}}, \qquad (5)$$

where k is a number of order of magnitude one.

$$\lambda \sim \frac{\hbar k}{m(L+d)} T$$

BIRD Model: THEORETICAL DEVELOPMENT

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ECE from Dielectrics

$$\lambda = \frac{\hbar k}{mL} T = \frac{\hbar k}{mL} \cdot \frac{k_A}{\pi k} \left| \frac{b_2}{b_1} \right|^2 = \frac{8W_1}{\hbar \sqrt{x_L^3}} \exp\left(-\frac{4}{3}\sqrt{x_L^3}\right) \propto E \cdot exp\left(-\frac{4}{3}\frac{E_d}{E}\right)$$

$$\sigma = \varepsilon_0 (E_0 - E) \qquad J_e = \sigma\lambda$$

$$\boldsymbol{J_e = \sigma\lambda = k(E_0 - E)E\, exp\left(-\frac{4}{3}\frac{E_d}{E}\right)}$$

…the same functional shape for J_e has been recovered !

BIRD Model - CONCLUSION

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BIRD model

- Explain, at least qualitatively, some features (ECE – Bursts)
- The semi-classical model is consistent with the first experimental results
- An amplification factor A(V) is needed
- A simple quantum model gives the expected shape of ECE

Future investigations require:

- More experimental work (Current evolution – Different electrodes -..)
- The development of a much more solid quantum model
- To understand the physical origin of A(V)

THANK YOU FOR THE ATTENTION