FIELD ELECTRON EMISSION IN AN EXTERNAL MAGNETIC FIELD PARALLEL TO THE SURFACE



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Introduction

Theory of the field electron emission based on a quantum-mechanical tunneling of electrons through a surface potential barrier was developed by R. H. Fowler and L. W. Nordheim in 1928 [1] but it still remains the main theory used to calculate field emission current density for the time being. Electron motion becomes relativistic as voltage and interelectrode gaps increase. The relativistic effects should be considered to develop a general approach describing tunneling of the electrons through the potential barrier and to refine an expression for a transmission coefficient for tasks with high fields and work function (for example, electron emission from the polar region of strongly magnetized neutron stars).

This work derives a relativistic expression for a transmission coefficient for the electron tunneling through the potential barrier, with influence of a parallel magnetic field considered. Wave functions of an electron motion at relativistic speed to be found, the Klein-Gordon equation instead of the Schrödinger equation was used.

The Klein-Gordon equation allows consideration of the external uniform magnetic field parallel to the metal surface but neglects the electron spin. The case when the charged particle just deflects under the influence of the magnetic field and moves to infinity under the action of an electric field (that is when $E^2 - (cB)^2 > 0$) is considered.

Relativistic generalization of the transmission coefficient

The wave function of the electron in vacuum in the presence of an electric field satisfies the Klein-Gordon equation:

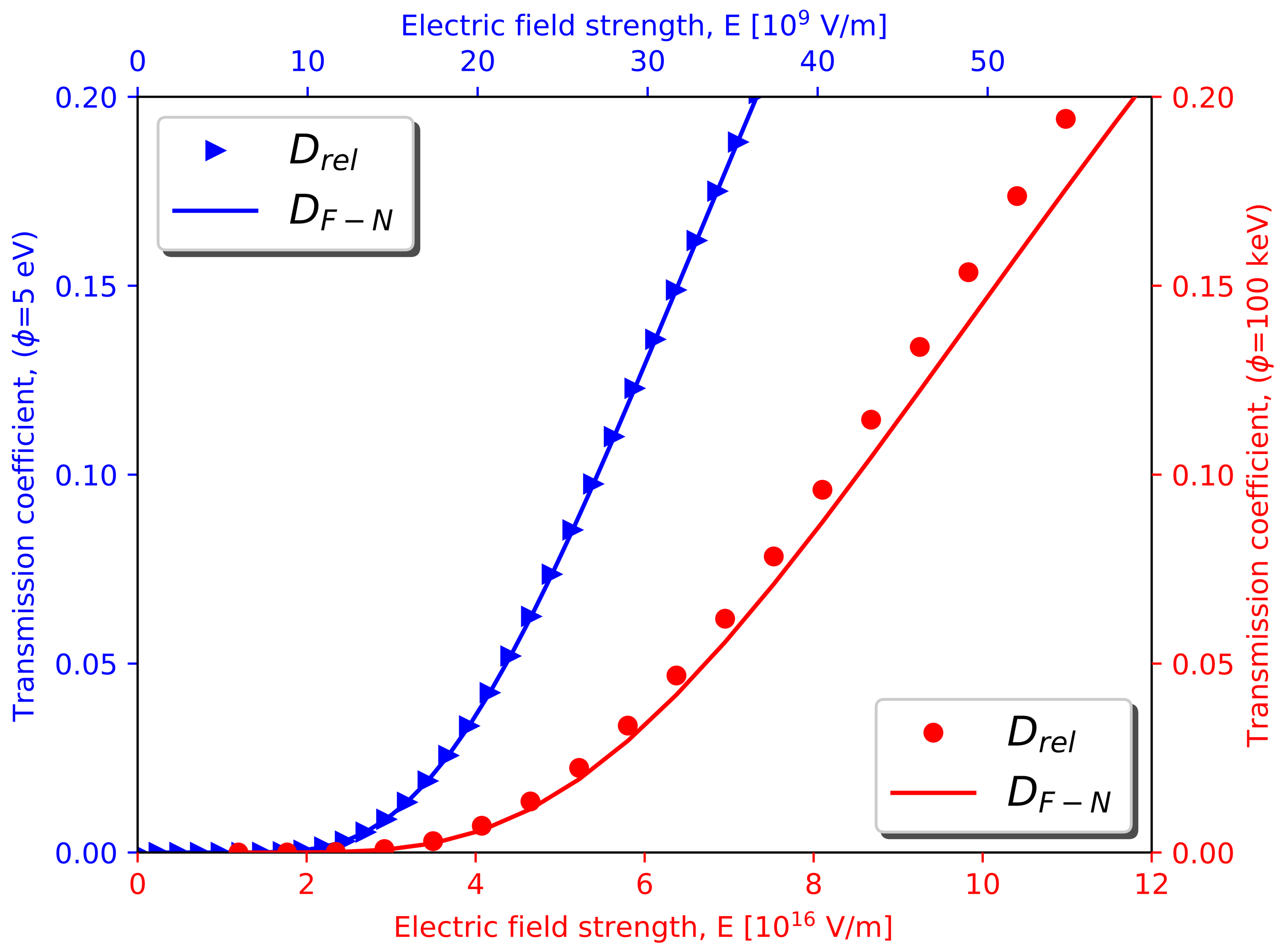
$$\left[\frac{\partial^2}{\partial \xi^2} + \frac{\xi^2}{4} - a\right]\psi(\xi) = 0, \tag{1}$$

where $\frac{\xi}{\sqrt{2}} = \sqrt{\frac{|e|E}{\hbar c}}\left(x + \frac{\varepsilon - U_0}{|e|E}\right)$, $\varepsilon = mc^2 + W_e$, $a = \frac{m^2c^3}{2\hbar eE}$, U_0 is the potential barrier height, E is electric field strength, W_e is the electron's kinetic energy.

The transmission coefficient of the potential barrier can be found only numerically in the general case. We use the following parameter values hereafter:

$E \cong 10^9 \frac{V}{m}, \phi \cong 5eV$ are the typical values for laboratory conditions and

$E \cong 10^{16} \frac{V}{m}, \phi \cong 100keV$ are typical values for field emission from neutron stars surface [2, 3]



Numerical calculation of the obtained by Fowler-Nordheim and relativistic generalized transmission coefficients for different work function values [4].

Figure shows the dependence of the potential barrier transmission coefficient on the electric field strength. It can be concluded that in the laboratory conditions the relativistic correction makes an extremely small contribution and is not experimentally observable. But in the case of field emission from neutron stars surface, the contribution is substantial and should be taken into account. Also let's note an increasing of the transmission coefficient due to relativistic corrections.

With respect to conditions $\frac{\phi}{mc^2} \ll 1$ and $\frac{\phi^2}{eE\hbar c} \sim 1$,the transmission coefficient of a potential barrier at the metal–vacuum boundary is accurately up to second-order terms can be written as

$$D_{rel} = D_{F-N}\left(1 + \frac{\sqrt{2}}{5}\frac{(U_0 - W_e)^{\frac{5}{2}}}{\sqrt{m}c^2 eE\hbar} + \frac{\sqrt{2}}{48}\frac{(7U_0 - 12W_e)\, eE\hbar}{U_0\sqrt{m}(U_0 - W_e)^{3/2}} + \frac{1}{120}\frac{37U_0 - 79W_e + 12\frac{{W_e}^2}{U_0}}{mc^2} + \frac{1}{1536}\frac{\left(49{U_0}^2 - 216U_0W_e + 192{W_e}^2\right)e^2E^2\hbar^2}{{U_0}^2 m (U_0 - W_e)^3}\right), \tag{2}$$

$$D_{F-N} = 4\sqrt{U_0 - W_e}\sqrt{W_e}{U_0}^{-1} e^{-\frac{4}{3}\frac{(U_0 - W_e)^{\frac{3}{2}}\sqrt{2m}}{\hbar eE}}. \tag{3}$$

Equation (3) is the transmission coefficient obtained by Fowler and Nordheim [1]. At the same time, we note that the second and the last terms of the expression (which do not contain the speed of light c) are the correction to the expression for the transmission coefficient obtained by Fowler and Nordheim and can be obtained from their calculations. The first and the third terms are purely relativistic and cannot be obtained in the framework of the Fowler and Nordheim approach.

Effect of the Lorentz contraction of the potential step

To explain the obtained effect of increase of the transmission coefficient due to relativistic effects we consider a simple problem of the electron tunneling through a rectangular barrier. Then the potential barrier width in relativistic case can be written as

$$h_{rel} = h\sqrt{1 - V^2/c^2}, \tag{4}$$

where h is the potential barrier width and $V^2 = \frac{U_0 - W_e}{2m}$.

Thus, the relativistic transmission coefficient will describes by the same equation as in non-relativistic case but with decreased barrier width, $h_{rel} < h$. This effect can be compared to Lorentz contraction of the potential barrier width. As a result, the transmission coefficient accordingly increases, which explains the effect obtained in the previous chapter.

The influence of a magnetic field on the transmission coefficient

The Klein-Gordon equation for external mutually perpendicular electric $\vec{E}(-E, 0, 0)$ and magnetic $\vec{B}(0, B, 0)$ fields has form (1) with following symbols:

$$\frac{\xi}{\sqrt{2}} = \left(\frac{e^2\left(E^2 - c^2B^2\right)}{c^2\hbar^2}\right)^{\frac{1}{4}}(x - x_c), \quad x_c = -\frac{(\varepsilon - U_0)\, E}{e\left(E^2 - c^2B^2\right)},$$

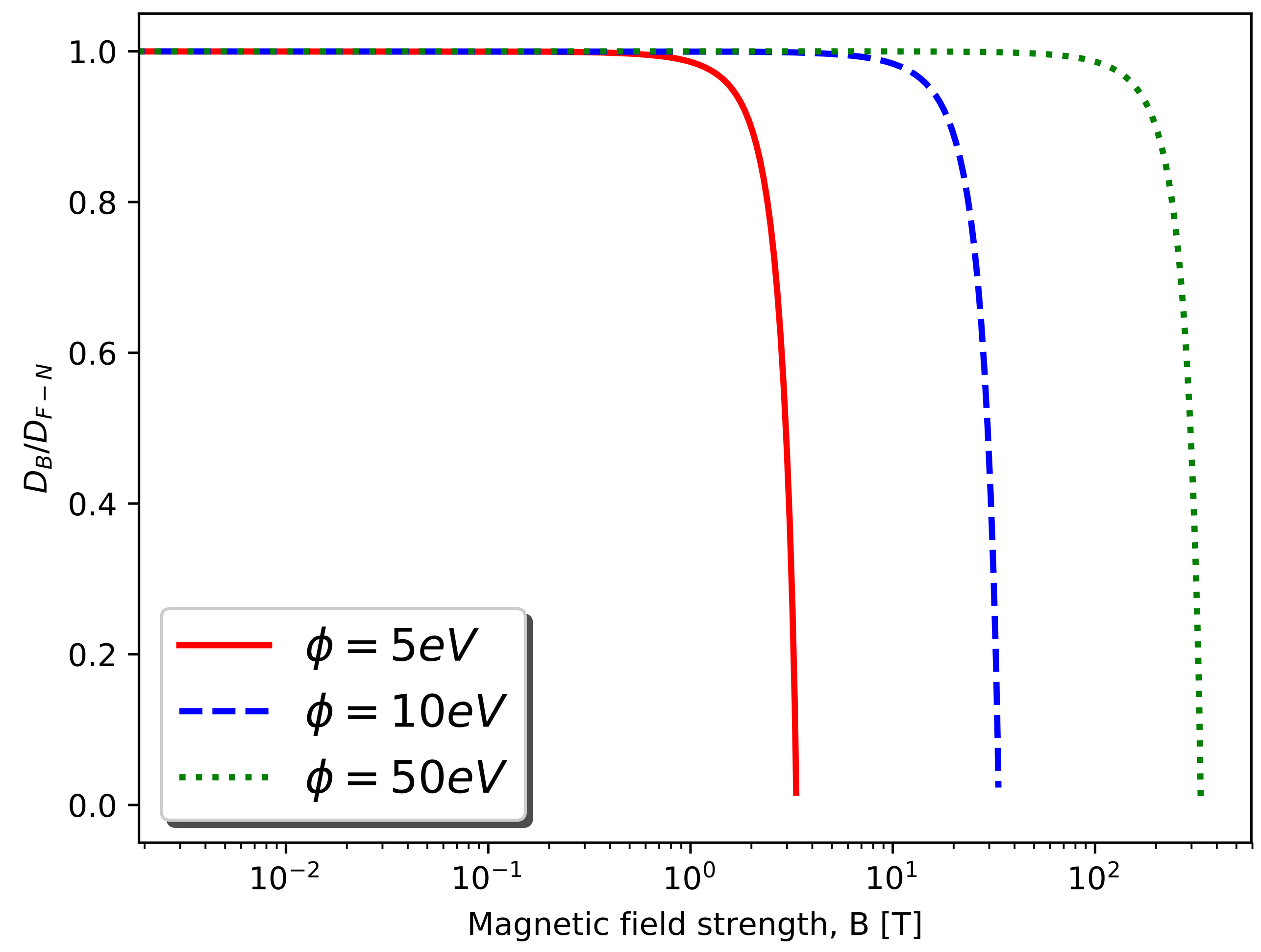
$$a = \frac{\left((\varepsilon - U_0)\, E + B^2p_2\right)^2}{2ec\hbar\left(E^2 - c^2B^2\right)^{\frac{3}{2}}} - \frac{(\varepsilon - U_0)^2}{2ec\hbar\sqrt{E^2 - c^2B^2}} + \frac{m^2c^3 + p_2^2c}{2e\hbar\sqrt{E^2 - c^2B^2}}$$

and p_2 is the component of the electron momentum along a magnetic field.

The transmission coefficient in the first nonvanishing approximation can be written as

$$D_B = \mathrm{e}^{-\frac{4}{3}\frac{\sqrt{2}\left(E^2 - 2\left(\frac{U_0 - W_e}{mc^2}\right)c^2B^2\right)\sqrt{m}(U_0 - W_e)^{3/2}}{E^3eh}} \times \frac{4\sqrt{U_0 - W_e}\left(E^2 - B^2c^2\right)^{3/4}E^{3/2}\sqrt{W_e}}{W_eE^3 + \left(E^2 - B^2c^2\right)^{3/2}(U_0 - W_e)}. \tag{5}$$

In the case of $B = 0$ equation (5) coincides with the transmission coefficient obtained by Fowler and Nordheim [1].



Dependence of the transmission coefficient vs magnetic field strength for the different work function values [4].

Figure shows the dependence of the transmission coefficient at the metal–vacuum interface on the induction of an external magnetic field parallel to the metal surface. We can see an insignificant change in transmission coefficient at small values of magnetic field strength. However, the transmission coefficient decreases when induction increases to the value $B = E/c$.

The experiments on suppression of vacuum breakdowns by a magnetic field have been carried out at CERN and IAP NASU with field values of $E = 144\ MV/m$, $B = 0.5\ T$ and $E = 100\ MV/m$, $B = 0.33\ T$ respectively [5]. Then we can find from (5) that in the case of laboratory parameters the transmission coefficient reduction is less than 0.015%. The transmission coefficient decreases by 0.1%, 5% and 50% when cB is equal to $0.1\ E$, $0.5\ E$ and $0.9\ E$ respectively. To obtain experimentally noticeable reduction of the transmission coefficient of about 10%, it is necessary to apply magnetic field of order of $10\ T$.

Conclusions

- The Fowler-Nordheim equation for field emission current density has been generalized to the relativistic case. An approximate formula for transmission coefficient of a potential barrier at the metal-vacuum boundary was presented.
- The effect of Lorentz contraction of a potential barrier at the metal-vacuum interface was found. This effect results in increasing of the transmission coefficient by 0.015% for field emission in laboratory conditions and by 15% for emission from neutron stars surface.
- An expression for the transmission coefficient was found when the condition $cB < E$ is satisfied. For typical experimental values of E and B [5], the effect of the magnetic field on the transmission coefficient was found.

References

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