Theory of the field emission electron based on a quantum-mechanical tunneling of electrons through a surface potential barrier was developed by R. H. Fowler and L. W. Nordheim in 1928 [1] but it still remains the main theory used to calculate field emission current density for the time being. Electron motion becomes relativistic at high fields and interelectrode gaps increase. The relativistic effects should be considered to develop a general approach describing tunneling of the electrons through the potential barrier and to refine an expression for a transmission coefficient for tasks with high fields and work function (for example, electron emission from the polar region of strongly magnetized neutron stars).

This work derives a relativistic expression for a transmission coefficient for the electron tunneling through the potential barrier, with influence of a parallel magnetic field considered. Functions of an electron motion at relativistic speed to be found, the Klein-Gordon equation instead of the Schrödinger equation was used. The Klein-Gordon equation allows consideration of the external uniform magnetic field parallel to the metal surface but neglects the electron spin. The case when the charged particle just deflects under the influence of the magnetic field and moves to infinity under the action of an electric field (that is when \( \vec{E}^2 - \alpha \vec{B}^2 > 0 \)) is considered.

### Relativistic generalization of the transmission coefficient

The wave function of the electron in vacuum in the presence of an electric field satisfies the Klein-Gordon equation:

\[
\frac{\partial^2 \psi}{\partial x^2} - \frac{\partial^2 \psi}{\partial t^2} - \frac{m^2}{c^2} \frac{\partial^2 \psi}{\partial \vec{B}^2} = 0, \tag{1}
\]

where \( \frac{m}{c^2} = \sqrt{\frac{m^2 c^4}{\hbar^2} + \frac{\vec{E}^2}{\vec{B}^2}} \) is the electron’s energy. The transmission coefficient of the potential barrier can be found only numerically in the general case. We use the following parameter values hereafter:

\( E \approx 100 \text{ keV} \), \( \vec{E} \approx 5 \text{ eV} \) are the typical values for laboratory conditions and \( E \approx 10^2 \text{ eV} \), \( \phi \approx 100 \text{ keV} \) are typical values for field emission from neutron stars surface [2, 3].

### Conclusions

- The Fowler-Nordheim equation for field emission current density has been generalized to the relativistic case used to calculate field emission current density for the time being. Electron motion becomes relativistic at high fields and work function (for example, electron emission from the polar region of strongly magnetized neutron stars).

### References