Statistical Analysis of Dark Currents

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Introduction

- Presently, current analysis is done only on mean values of the current, specifically with β estimation on the FN plot.
- We present two novel methods of β estimation. These methods can provide a more accurate estimation in a shorter time and do not require a measurement of multiple field values.
- Additionally, we present an improvement to the current method of β estimation[a].

Improving Current Estimations

- The current method of β analysis[a] is based on the FN plot. The original equations for this method are:

$$\begin{aligned} I &= \frac{ASE^2}{\phi t(s)^2} exp\left(-\frac{B\phi^{3/2}v(s)}{E}\right) \\ v(s) &= 2^{-1/2}\left[(1+1-s^2)^{1/2}/2\right]^{1/2}\left(E(k^2)-\frac{s^2K(k^2)}{1+(1-y^2)^{1/2}}\right) \\ s &= C\cdot E^{1/2}/\phi \\ k^2 &= \frac{2(1-y^2)^{1/2}}{1+(1-y^2)^{1/2}} \\ t(s) &= v(s)-\frac{2}{3}s\frac{dv}{ds} \end{aligned}$$

With S being the emitter surface area, ϕ the work function, E the emitter field, $K(k^2)$ and $E(k^2)$ the complete elliptic integrals of the first and second kind respectively and $A = 1.54\cdot 10^{-6}\left(\frac{m^{-2}\cdot eV}{(V/m)^2}\right)$, $B = 6.83\cdot 10^{9}\left(\frac{V/m}{eV^{3/2}}\right)$, $C = 3.79\cdot 10^{-5}\left(\frac{eV}{(V/m)^{1/2}}\right)$.

- This equation was then approximated by $v(s)\cong 0.956-1.062s^2$ and $t(s)\cong 1$.
- Using a more accurate approximation[b]

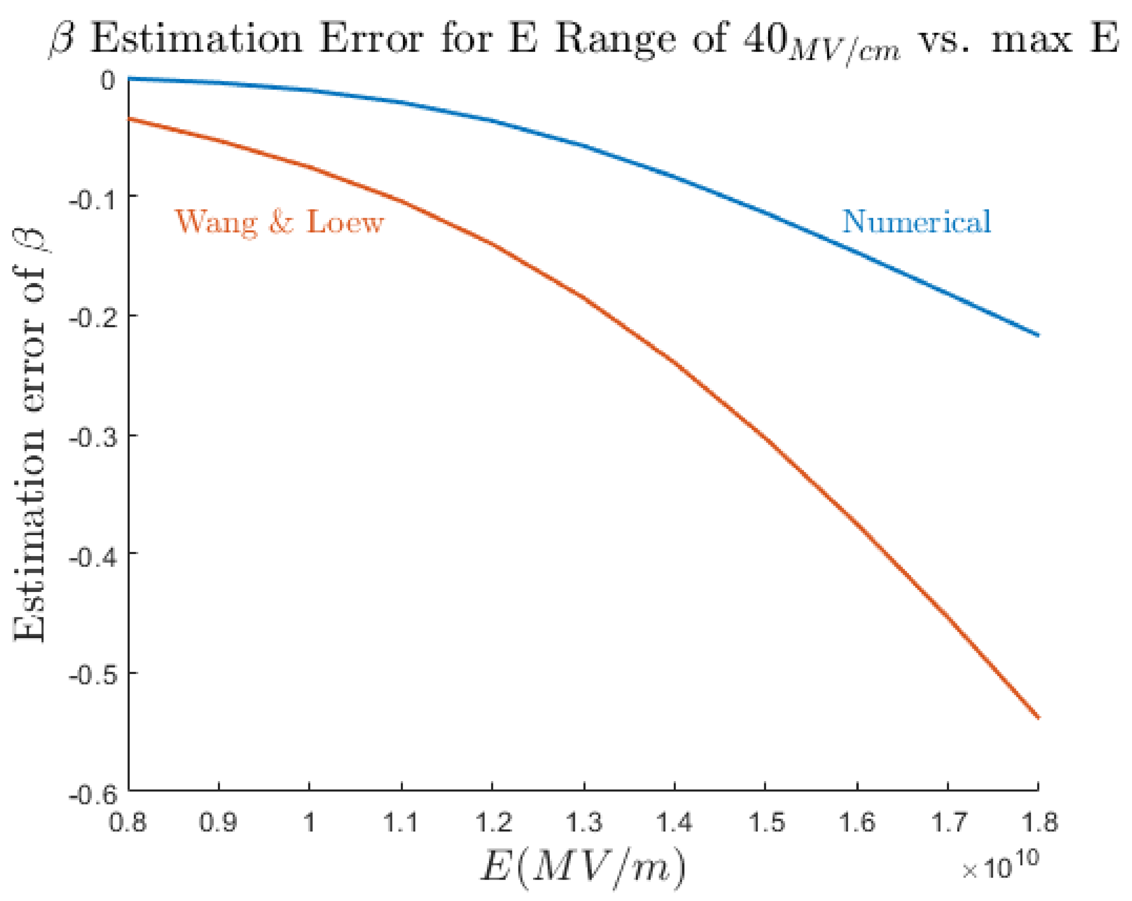
$$v(s) = 1-s^2+\frac{1}{3}s^2 ln(s)$$

a better estimation can be achieved as can be seen in figure (1).

- The observed error is reduced from about 7.6% to about 1.1% at lower fields.

The error was calculated as $1-\frac{\beta_{est}}{\beta_{act}}$ where β_{est} is the estimted value and β_{act} is the actual simulated value. In this fashion, a consistent negative bias of the error (as observed here) indicates an estimation bias of larger β values.

Figure 1: β estimation error for FN plot method

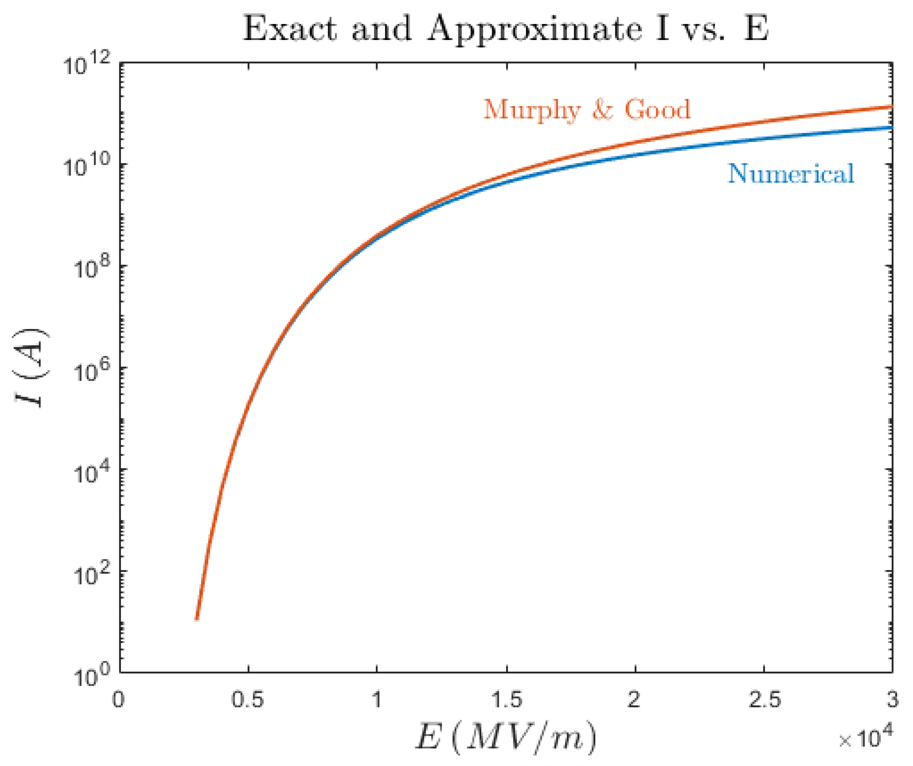


Numerical Calculation of Current

This work relies on the theoretical work of Murphy and Good[c] who solved the 1D dark current problem for the SN barrier. In their work, the equations for the electronic supply function $N(W)$ and tunneling probability $D(W)$ were found.

In this work a Monte-Carlo numerical integration of electronic energy bands (W) was performed to generate a current value.

Figure 2: Analytical and numerical current plot



- The statistical model used for the generation was of a Gaussian PDF for each energy band W with the parameters

$$\begin{aligned} \mu_W &= S\cdot N(W)D(W) \\ \sigma_W &= S\cdot N(W)D(W)(1-D(W)) \end{aligned}$$

- The same parameters after integration are:

$$\begin{aligned} \mu &= \int \mu_W dW &&= S\int N(W)D(W)dW \\ \sigma &= \sqrt{\int \sigma_W^2 dW} &&= S\sqrt{\int (N(W)D(W)(1-D(W)))^2 dW} \end{aligned}$$

- This method of current calculation gives a current that's smaller than the analytical approximation (figure (2)).

Novel β Estimation Methods

1 The first method (and currently the more practical) is using calculus of variation and the inherent fluctuations of the experimental system to get a short "scan" of fields. Specifically, using the equation

$$\frac{1}{I_m}\frac{\sigma_I}{\sigma_{E_0}} = \beta\frac{1}{I_r}\frac{\partial I}{\partial E}$$

- Here, on the l.h.s. I_m, σ_I and σ_{E_0} are the measured mean current, current deviation and applied field deviation respectively and on the r.h.s. I_r (the mean field) and the derivative $\frac{\partial I}{\partial E}$ are calculated on a reference numerical data set with an arbitrarily chosen surface area.
- The division by I is important to remove the linear dependence on the surface area and measurement time.

2 The second method is with the shot-noise of the system. For this method, the parameter D is defined as:

$$D \equiv 1-\frac{\sigma}{\mu} = 1-\frac{\sqrt{\int (N(W)D(W)(1-D(W)))^2 dW}}{\int N(W)D(W)dW} = D(E,\phi)$$

and is independent of the surface area.

- By comparing the measured D to a numerically calculated reference set of D, the tunneling field can be estimated.
- An important point to note about these methods is that they should take significantly less time than the current method of FN plot analysis and are measured in a single field strength. In addition to better real time monitoring of the system, this allows for a better characterization of the system as a function of field.

Simulating experimental Scenario

- To test these methods compared with the current analysis method, a Monte-Carlo simulation of currents was performed with a set β value.
- The emitter field was set to be in the range of about 10,000-30,000MV/m while currently high field structures at CERN's CLIC operate at an estimated emitter field of about $\beta E\sim 10,000 MV/m$ ($E_0\sim 35MV/m$ with $\beta\cong 300-400$ at the fixed gap system).
- Figures (3) and (4) show the relative estimation error of the field variation and shot noise estimations respectively. As can be seen, both methods present low errors - especially compared with the current analysis method (red line in figure (1)).
- The reason for the increase in error of the shot noise as a function of field is that $\frac{dE}{dD}$ diverges as D (and E) increases and so the same variation in measurement of D is more impactful on accuracy at higher fields.

Figure 3: Estimation error of field variation

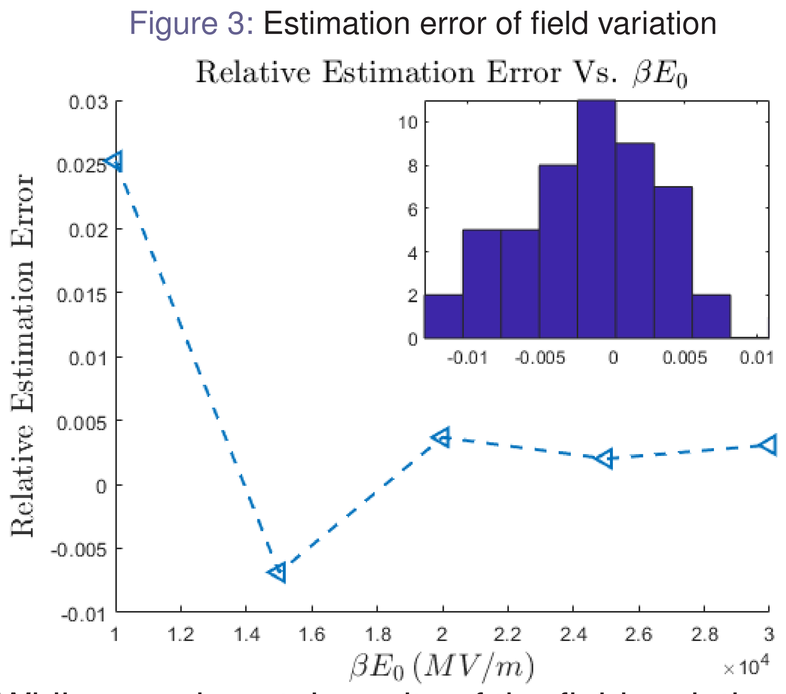
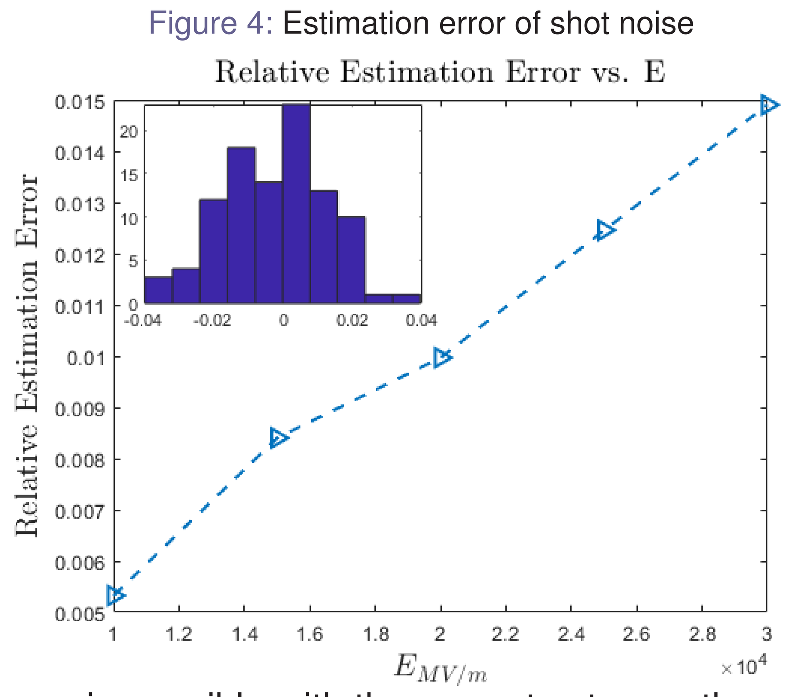


Figure 4: Estimation error of shot noise



- While experimental results of the field variation method were impossible with the current setup as there were too many noise sources and the variation was controlled by noise, we were able to compare the corrected FN plot analysis with the current method.
- The estimated values were mostly around 6% smaller than the original estimates, as was expected from the numerical estimates (figure (1)).

Conclusions

- We have shown a simple numerical correction to decrease the estimation error of the FN plot analysis.
- We have shown two novel methods of β estimation. These methods are faster than the current method and also require only a single field point for the estimation.
- In particular, the field variation method can be easily implemented in any high field system and can help with the characterization and monitoring of high field systems.

[a] J.W. Wang and G. A. Loew 1997
[b] R. G. Forbes Appl. Phys. Lett., 89(113122), 2006
[c] E.L. Murphy and R.H. Good Phys. Rev.,102(6):1464, 1956

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