### Improving Current Estimations

The current method of $\beta$ analysis is based on the FN plot. The original equations for this method are:

\[
\begin{align*}
I &= \frac{A eV}{\hbar \nu_f} \exp \left( \frac{\hbar\nu_f}{kT} \right) \\
\nu_f &= 2 \left( 1 - [1 - e^{-\beta}]^{1/2} \right) \left( E(\beta) - \frac{e\phi}{\hbar\nu_f} \right)
\end{align*}
\]

$\beta$ indicates an estimation value. In this fashion, an estimated value and calculated as $1 - \beta$.

Numerical Calculation of Current

This work relies on the theoretical work of Murphy and Good who solved the 1D dark current problem for the SN barrier. In their work, the equations for the electronic supply function $N(W)$ and tunneling probability $D(W)$ were found. In this work a Monte-Carlo numerical integral of electronic energy bands ($W$) was performed to generate a current value.

The statistical model used for the generation was of a Gaussian PDF for each energy band $W$ with the parameters

\[
\begin{align*}
\mu &= \frac{1}{\sqrt{2\pi} \sigma} \exp \left( - \frac{(W - \mu)^2}{2\sigma^2} \right) \\
\sigma &= \frac{1}{\sqrt{2\pi} \sigma} \int \exp \left( - \frac{(W - \mu)^2}{2\sigma^2} \right) dW
\end{align*}
\]

The same parameters after integration are:

\[
\begin{align*}
\mu &= \int \frac{1}{\sqrt{2\pi} \sigma} \exp \left( - \frac{(W - \mu)^2}{2\sigma^2} \right) dW \\
\sigma &= \int \frac{1}{\sqrt{2\pi} \sigma} \exp \left( - \frac{(W - \mu)^2}{2\sigma^2} \right) dW
\end{align*}
\]

This method of current calculation gives a current that’s smaller than the analytical approximation (figure 2).

### Novel $\beta$ Estimation Methods

1. The first method (and currently the more practical) is using calculus of variation and the inherent fluctuations of the experimental system to get a short “scan” of fields. Specifically, using the equation

\[
\frac{\partial}{\partial \mu} \int \left[ N(W)D(W)(1 - D(W)) \right]^2 dW = 0
\]

2. Here, on the l.h.s. $\mu$, $\eta$ and $\sigma$ are the measured mean current, current deviation and applied field deviation respectively and on the r.h.s. $\mu$ (the mean field) and the derivative $\sigma$ are calculated on a reference numerical data set with an arbitrarily chosen surface area.

3. The division by $\mu$ is important to remove the linear dependence on the surface area and measurement time.

4. An important point to note about these methods is that they should take significantly less time than the current method of FN plot analysis and are measured in a single field strength. In addition to better real time monitoring of the system, this allows for a better characterization of the system as a function of field.

### Simulating experimental Scenario

To test these methods compared with the current analysis method, a Monte-Carlo simulation of currents was performed with a set $\beta$ value.

The emitter field was set to be in the range of about 10,000-30,000MV/m while currently high field structures at CERN’s CLIC operate at an estimated emitter field of about $\sim 35$ MV/m with $\beta \geq 300 - 400$ at the fixed gap system.

Figures (3) and (4) show the relative estimation error of the field variation and shot noise estimations respectively. As can be seen, both methods present low errors - especially compared with the current analysis method (red line in figure (1)).

The reason for the increase in error of the shot noise as a function of field is that $\phi$ diverges as $D$ (and $E$) increases and so the same variation in measurement of $D$ is more impactful on accuracy at higher fields.

### Conclusions

We have shown a simple numerical correction to decrease the estimation error of the FN plot analysis.

We have shown two novel methods of $\beta$ estimation. These methods are faster than the current method and also require only a single field point for the estimation.

In particular, the field variation method can be easily implemented in any high field system and can help with the characterization and monitoring of high field systems.

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**References**