Beam Beam Effects

JAI Graduate Course

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This lecture is based on previous lectures from Ted Wilson and Werner Herr

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Introduction

Beam Beam

When two beams collide, they interact.

1. By collisions between particles (elastic, inelastic etc), which is our desired goal
2. By other electromagnetic interactions (that we would rather avoid).

Typically in a collider:

- $< 0.01\%$ or less of the particles collide
- BUT $> 99.999\%$ or more feel the force of the other bunch

i.e. in the LHC there are 100,000 protons per bunch but only 20 collisions per crossing at nominal luminosity.

Luminosity Considerations

Beam-beam effects are the main limit in past, present and future colliders. Consideration of these effects is important for high density beams (high intensity and/or small). There are many different effects involved, in this lecture we will look at some of the key ones.

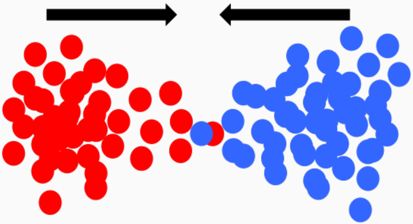
Recall that luminosity (which is what we want from a collider) goes as:

$$L = \frac{N_1 N_2 F n_B}{4\pi\sigma_x\sigma_y} \tag{1}$$

Where N is the number of particles in each bunch, F is the revolution frequency, n_B the number of bunches and σ is the rms beam size.

Luminosity Considerations

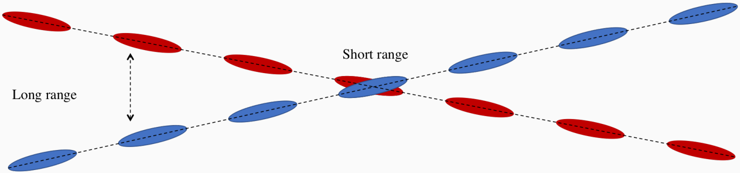
The beam parameters can change due to beam-beam interaction. The particles in one bunch act like an electromagnetic lens on the other bunch.



- the forces from one beam on another are non-linear
- we cannot avoid the beams exerting forces on each other
- this can have a serious detrimental effect, and can lower the luminosity

Beam Beam

There are two types of interactions that we must consider, long range and short-range interactions.



We will start by considering what happens in the short-range picture, around the collision point.

Forces from Beam Beam

To calculate the forces, we first take the charge distribution and fields, transformed into the moving frame, and see what the force is on a test particle. Note that the force on a test particle can be defocusing or focusing depending on the beams, i.e. and e+e- collider or pp collider.

For a gaussian beam, the charge density (for x, similar for y) is:

$$\rho_x = \frac{1}{\sigma_x \sqrt{2\pi}} \exp\left(\frac{-x^2}{2\sigma_x^2}\right) \tag{2}$$

We can derive the potential $U(x, y, z)$ from the Poisson equation: $\Delta U = -\frac{1}{\epsilon_0}\rho(x, y, z)$

Potential of Beam

This gives (see W. Herr, Advanced CAS notes, Appendix 1):

$$U(x, y, \sigma_x, \sigma y) = \frac{\lambda e}{4\pi\epsilon_0} \int_0^\infty \frac{\exp(-\frac{x^2}{2\sigma_x^2+q} - \frac{y^2}{2\sigma_y^2+q})}{\sqrt{(2\sigma_x^2 + q)(2\sigma_y^2 + q)}} dq \tag{3}$$

where λ is the line density of particles in the beam.

Then, we can obtain the fields by $\vec{E} = -\nabla U(x, y, \sigma_x, \sigma_y)$

E and B Fields of Beam

We take the case where the beam is round, so $\sigma_x = \sigma_y = \sigma$
The Lorentz force, $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$ can be re-written in cylindrical co-ordinates:

$$\vec{F} = q(E_r + \beta c B_\phi) \times \vec{r} \tag{4}$$

And then re-write Eqn. 4 in terms of $r = \sqrt{x^2 + y^2}$, to give:

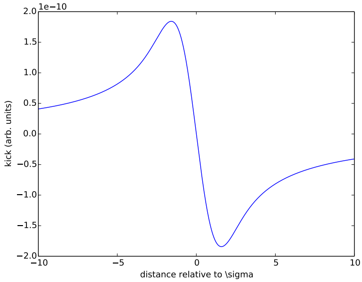
$$E_r = -\frac{\lambda e}{4\pi\epsilon_0} \frac{\partial}{\partial r} \int_0^\infty \frac{\exp(-\frac{r^2}{2\sigma^2 + q})}{(2\sigma^2 + q)} dq \tag{5}$$

$$B_\phi = -\frac{\lambda e \beta c \mu_0}{4\pi} \frac{\partial}{\partial r} \int_0^\infty \frac{\exp(-\frac{r^2}{2\sigma^2 + q})}{(2\sigma^2 + q)} dq \tag{6}$$

Beam Beam Force

Taking the results, note that the force from Eqn. 4 is radial only. We evaluate the integrals in Eqns. 5 and 6, and using $c = \frac{1}{\epsilon_0 \mu_0}$ we get:

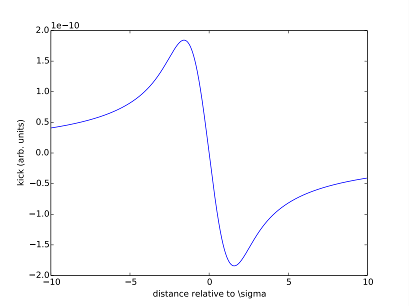
$$F_r(r) = -\frac{\lambda e^2(1+\beta^2)}{2\pi\epsilon_0}\frac{1}{r}\left[1 - \exp(-\frac{r^2}{2\sigma^2})\right] \quad (7)$$



Beam Beam Force

Note that this force is:

- Approximately linear for small amplitudes ($< 1\sigma$). So the small amplitude particle will experience a change in tune, as if the focusing were different.
- Very non-linear at larger amplitudes, but here the tune change is amplitude-dependent

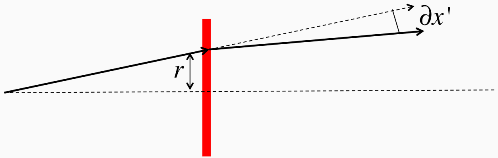


Beam Beam Parameter

Beam Beam Tune Shift

So what is the tune shift that this ‘kick’ produces? If we recall the tune shift from a quadrupole error is:

$$\Delta Q = \frac{\beta}{4\pi f} = \frac{1}{4\pi}\beta\frac{B'L}{B\rho} \tag{8}$$



But in this case f, the error in focal strength, is due to the angular kick:

$$\partial x' = \frac{r}{f} \tag{9}$$

Beam Beam Tune Shift

So from Eqn. 8 and 9:

$$\Delta Q = \frac{1}{4\pi}\beta^* \frac{\partial x'}{r} \tag{10}$$

We can describe the radial impulse as:

$$\partial x' = \frac{\partial p_r}{p_z} = \frac{F(r)dt}{p_z} \tag{11}$$

And therefore the tune shift becomes:

$$\Delta Q = \frac{1}{4\pi}\beta^* \frac{F(r)dt}{p_z} \tag{12}$$

Beam Beam Tune Shift

$$\Delta Q = \frac{1}{4\pi}\beta^* \frac{F(r)ds}{2\beta c p_z} \tag{13}$$

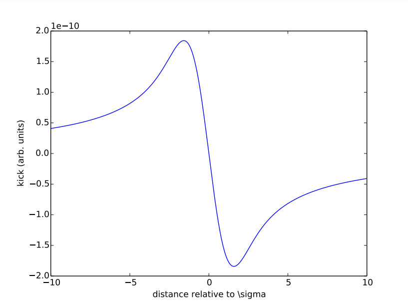
$$= \frac{1}{4\pi}\beta^* \frac{F(r)ds}{2\beta^2 \gamma m_0 c^2} \tag{14}$$

$$= \beta^* \frac{\lambda(1+\beta^2)ds}{\gamma(2\beta^2)\sigma^2}\left(\frac{q^2}{4\pi\epsilon_0 m_0 c^2}\right) \tag{15}$$

$$\therefore \Delta Q = \frac{\beta^* N r_0}{\gamma \sigma^2} \tag{16}$$

Beam Beam Tune Shift

Tune shift (linear) vs tune spread (non-linear). Recall the beam-beam 'kick':



- The particles with small amplitudes have a large tune shift.
- Particles at larger amplitudes have a smaller tune shift.
- As the amplitude increases further the tune shift eventually becomes zero.

Beam Beam Parameter

We can characterise this effect in terms of the 'beam beam parameter'.

$$\xi = \frac{N r_0 \beta^*}{4\pi\gamma\sigma^2} \tag{17}$$

	LEP (e+e-)	LHC (pp)
Beam size	$160 - 200\mu m$, $2 - 4\mu m$	$16.6\mu m$, $16.6\mu m$
Intensity N	4×10^{11}ppb	1.15×10^{11}ppb
Energy	100 GeV	7000 GeV
$\epsilon_{x,y}$	20 nm, 0.2 nm	0.5 nm, 0.5 nm
$\beta^*_{x,y}$	1.25 m, 0.05 m	0.55 m, 0.55 m
Crossing angle	0.0	285 μrad
Beam-beam parameter ξ	0.0700	0.0037

Beam Beam Parameter Comparsion

It's tempting to use the beam-beam parameter to quantify the strength of the beam-beam interaction, but recall it only reflects the linear part of the behaviour. But we can use it for comparison.

In general for non-round beams $\beta_x^* \neq \beta_y^*$:

$$\xi_{x,y} = \frac{N r_0 \beta_{x,y}^*}{2\pi\gamma\sigma_{x,y}(\sigma_x + \sigma_y)} \tag{18}$$

Comparison of Machines

An unusual effect is observed in lepton colliders, but not hadron colliders.

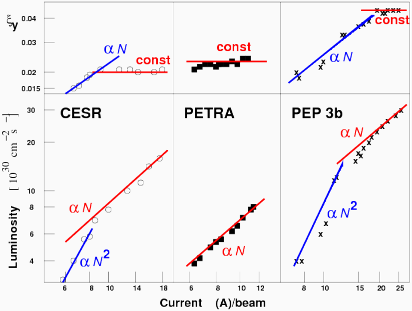


Image credit: Werner Herr

Comparison of Machines

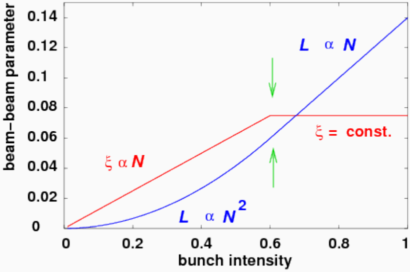


Image credit: Werner Herr

- Above a certain current, $L \propto I_{beam}$ rather than $L \propto I_{beam}^2$, which we expect from the luminosity definition in Eqn. 1.
- The beam-beam parameter ξ seems to saturate at the same value of the intensity.

Beam Beam Limit

This 'beam beam limit' can be understood as an effect with the vertical emittance:

- The horizontal emittance is larger in e+e- colliders and doesn't change much
- So for the luminosity to increase, the vertical beam size must increase in proportion to the intensity above the beam-beam limit ($\frac{N}{\sigma} \propto$ constant)
- As the vertical beam size is very small, it can keep increasing until any beam loss is observed

Note however, that the beam beam 'limit' is NOT a constant and cannot really be predicted. Simulations are needed to provide an idea for each machine.

Weak-Strong and Strong-Strong Interactions

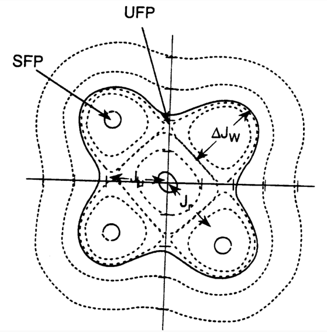
Sometimes beam-beam effects are classified into different categories depending on the two beams colliding:

- Both beams are very strong (strong-strong), both are affected, very challenging (LHC, LEP, RHIC, ...)
- One beam is much stronger than the other (weak-strong), only weak beam is affected (SPS, Tevatron, ...)

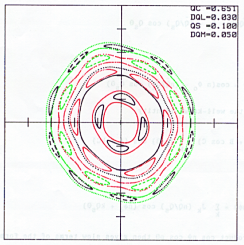
Non-linear Behaviour

What is the influence of nonlinear detuning on the phase space trajectories? Since the tune depends on amplitude - amplitude growth of particles on resonance leads to a tune change which tends to move the particles off resonance. So this can stabilise the resonance! In the phase space, stable and unstable fixed points appear.

For example, the phase space from octupole contribution looks like:



Non-linear Behaviour



L. Evans, Hadron collider luminosity limitations, in: 4th US-CERN School on Particle Accelerators, 1990.

- Below the beam-beam limit see the many archipelagos of islands due to different multipole components in the beam-beam potential.
- Increasing the beam-beam interaction enlarges the islands so that they overlap in stochastic regions where particle may diffuse out in 4 dimensional phase space.
- Increasing amplitude tune slope will merge the islands

Long Range Interactions

LHC Interaction Region

In the LHC, the beams are brought into the same beam pipe for around 120m in each interaction region.

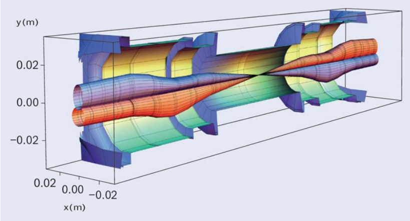


Image credit: CERN Courier

With a small crossing angle, separation of the two beams is only $6 - 12\sigma$, so interactions can occur. We call these long-range interactions.

Long range interactions

The tune spread from long range interactions follows a kind of scaling law:

$$\Delta Q_{LR} \propto -\frac{N}{d^2} \tag{19}$$

Where N is the bunch intensity and d the separation. Note that now the large amplitude particles are more perturbed than the small amplitude ones!

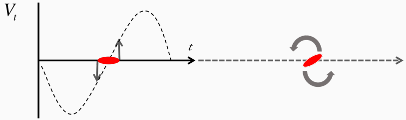
- The beam-beam effect can also affect the orbit.
- It's possible to also create coherent modes by exciting dipole oscillations. (Like a series of coupled oscillators).

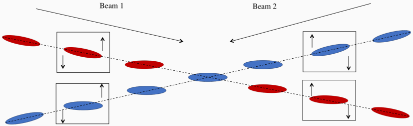
Question: what would happen if a bunch were 'missing'?

Mitigating Beam Beam

Crab Cavities

If we increase the crossing angle to mitigate beam-beam, there is a loss in luminosity. 'Crab' cavities introduce a rotation of the bunch before the collision point, which is then re-rotated after. Using RF transverse deflection:

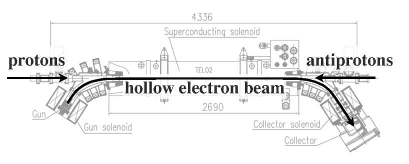




Active Compensation

There have been tests and developments in techniques of active compensation for beam-beam effects:

- Electron lenses: linear or non-linear (tested at Tevatron and RHIC)



- Current carrying wires (force is 1/r at large distances, same as the force from a current carrying wire!)

Summary

Summary

- The beam beam effect is one of the main limiting effects in modern collider experiments
- The linear effect can be summarised in terms of the *beam beam parameter*, ξ
- The full non-linear beam beam effect does not have a fully consistent theory
- New ideas to mitigate beam beam issues are being implemented

Questions?

References i

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