

# Free-Electron Lasers

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LCLS, EU-XFEL, FERMI@Elettra, ...

## **Summary**

## Introduction

Free-Electron Lasers (FELs) are tuneable sources of monochromatic synchrotron radiation. Compared to typical storage ring beamlines, the pulses have much higher peak power, higher brightness and better coherence.

In simple terms, they consist of:

- A **relativistic beam of electrons**
- Travelling through a **periodic magnetic array** (undulator)
- Co-propagating with a **radiation field**, with a wavelength in resonance with the transverse oscillations

Many different types of FELs have been constructed or proposed, each of which can enhance certain properties (pulse length, peak power, repetition rate, wavelength, ...)

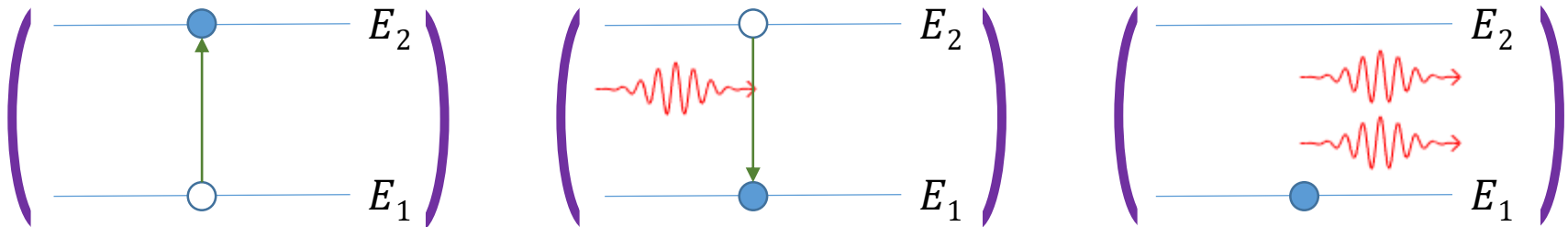
In this lecture, we will:

- Establish the difference between a classical laser and a free-electron laser
- Review the different types of FEL
- Establish the basic 1D equations governing the FEL process in low and high-gain regimes
- Look at some present day FEL user facilities

## Introduction

### *Classical Laser:*

- Emits high-power, monochromatic light via optical amplification
- Quantum source, requiring a lasing medium, an energy pump and an optical cavity
- Stimulated emission leads to well-defined phase relation, i.e. coherence



1) Gain medium is put into an excited state by the energy pump (population inversion)

2) Incident photon at the energy  $\hbar\omega = E_2 - E_1$  increases the likelihood of electron transition to the lower energy state

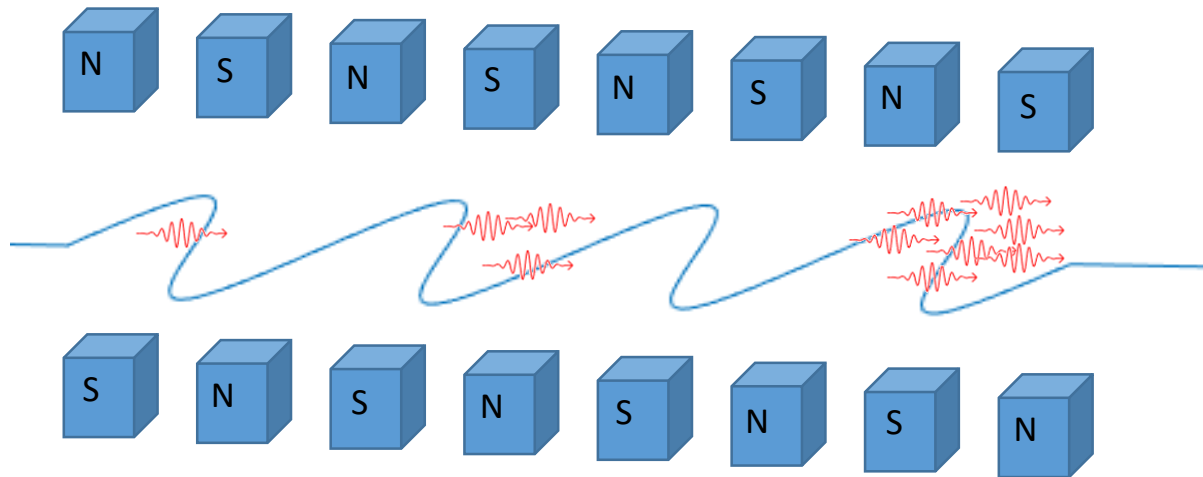
3) As the atom changes state it emits a photon of energy  $\hbar\omega$

An optical cavity surrounds the lasing medium, containing the photons. The first photon is emitted spontaneously, and this photon stimulates the emission of another. The number of photons stored within the cavity increases exponentially over time. One of the mirrors is permeable, allowing some photons to escape.

# Introduction

## *Free-Electron Laser:*

- Emits high-power, **tuneable**, monochromatic light
- Classical source; an electron beam travelling through an undulator provides the lasing medium
- Transverse motion of the electrons couples to the horizontal component of the radiation field, leading to energy transfer between the electrons to the radiation field

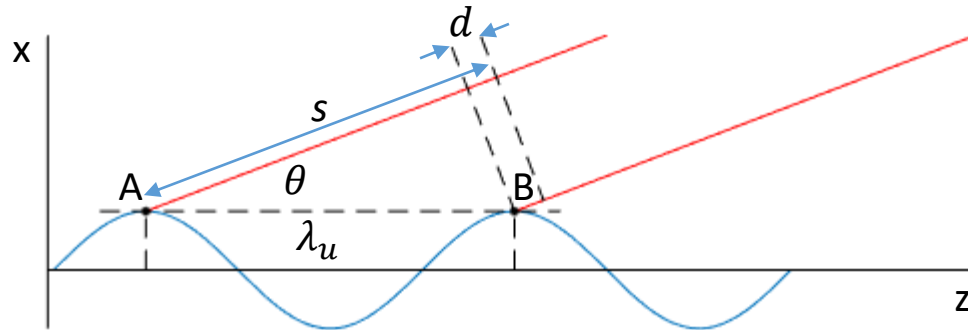


Energy exchange:  $\Delta W = -e \int \mathbf{v} \cdot \mathbf{E} dt$  (no energy exchange without the undulator!)

## The Importance of Coherence

The radiated power emitted by FELs is much higher than the spontaneous emission in typical undulator beamlines due to the principle of **coherent emission**.

From the lectures on synchrotron radiation and insertion devices, we used the condition for constructive interference to calculate the resonant wavelength  $\lambda_r$  for the photons



This resulted in the equation:

$$\lambda_r = \frac{\lambda_u}{2n\gamma^2} \left( 1 + \frac{K^2}{2} + \theta^2 \gamma^2 \right), \quad K = \frac{eB_0\lambda_u}{2\pi m_e c}$$

As an example, a 3 GeV electron beam passing through an undulator of period  $\lambda_u = 20$  mm will emit radiation with a typical scale length of  $\sim 0.3$  nm. This is orders of magnitude shorter than the electron bunch itself. As such, the spontaneous radiation emitted by the electrons will have arbitrary phase relationships.

## Incoherent emission

If the phase of each photon emitted in the undulator is random, then there will be partial cancelation of all the electric fields.

As an example, if all the individual photons have wavelength  $\lambda$  and amplitude  $E_0$ , then the total electric field will be the sum over all photons:

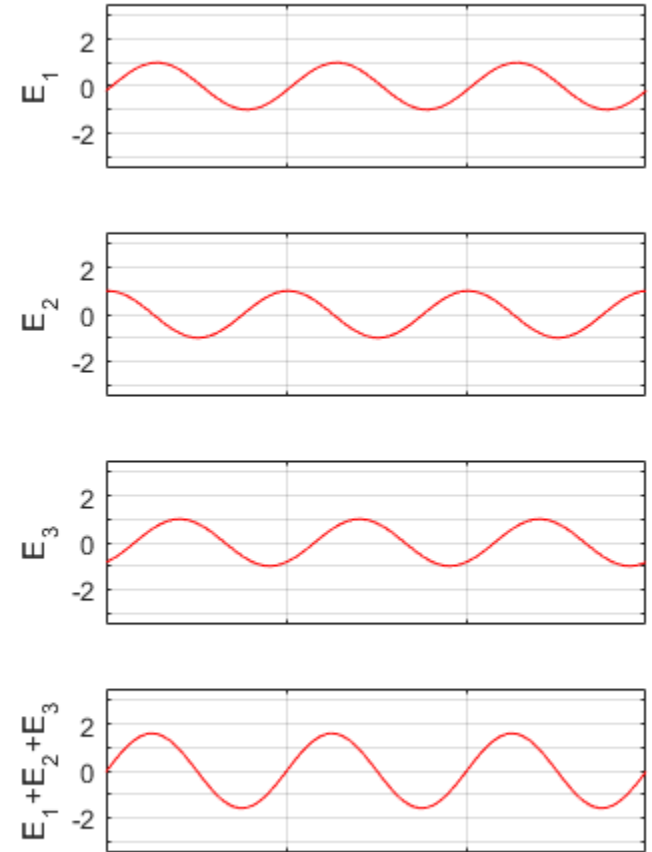
$$E_{tot}(\lambda) = \sum_{n=1}^N E_0(\lambda) e^{i\phi_n}$$

where  $\phi_n$  is the phase of the  $n^{\text{th}}$  photon.

Since all the phases are random, we have

$$E_{tot} \approx \sqrt{N} E_0$$

The total field scales with the square root of the number of electrons, and the **power scales linearly with  $N$** . (Power  $\propto E^2$ )



## Coherent emission

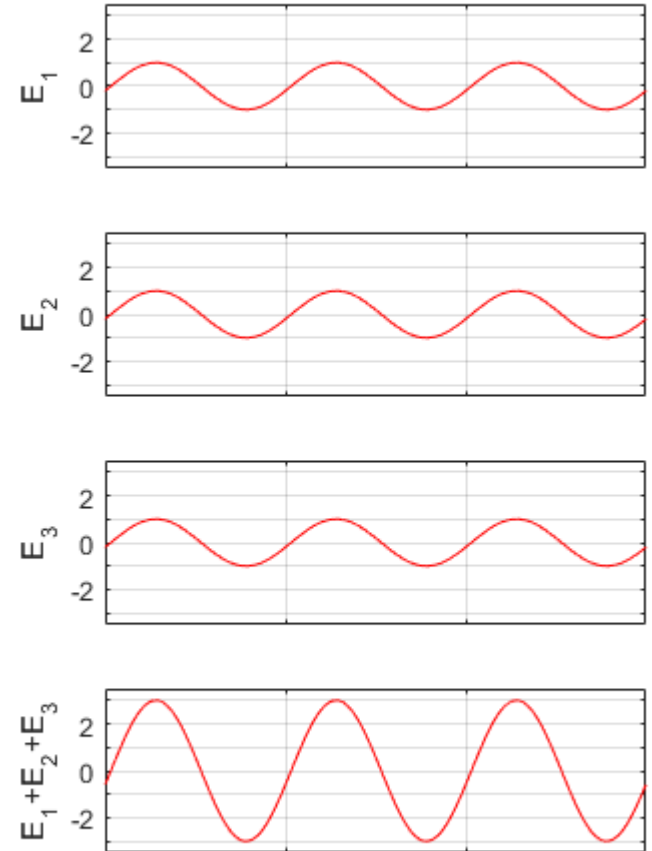
If, however, all of the photons are emitted with the same phase at a given point in space, then all of the fields will add linearly.

In this case, we have

$$E_{tot}(\lambda) = \sum_{n=1}^N E_0(\lambda) e^{i\phi_0} = NE_0$$

The total field scales linearly with the number of electrons, and the **power scales with  $N^2$** .

The radiation in this case is temporally coherent, and since there are a very high number of electrons within the bunch, it can be many orders of magnitude greater than the standard, incoherent undulator radiation emission.





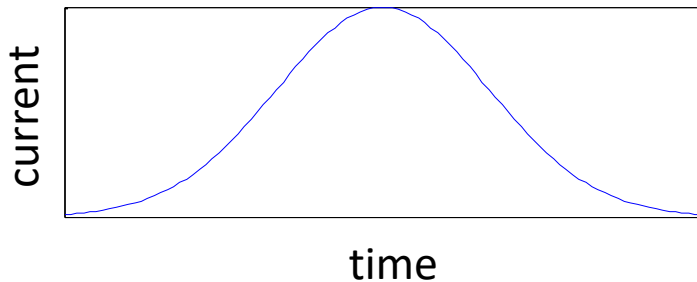
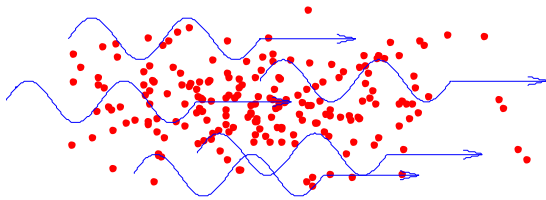
## Coherent emission

If all of the electrons were at a single point in space, then the radiation emitted in an undulator would be fully coherent. This is of course not achievable in practice.

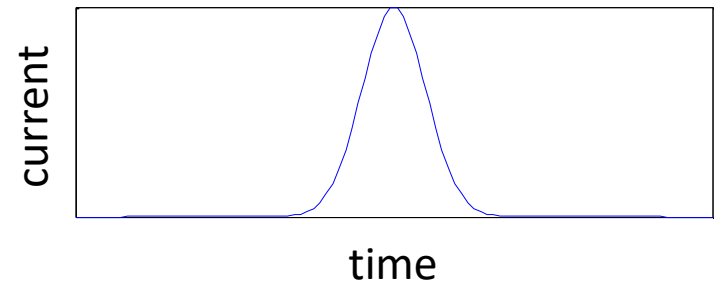
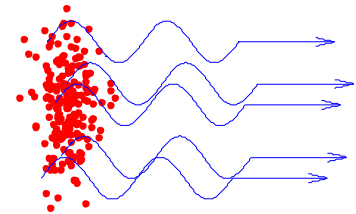
However, assuming the radiation wavelength ( $\lambda_r$ ) is long in comparison to the electron bunch length, there will still be a significant component of coherent emission at  $\lambda_r$ . Alternatively, if micro-bunches can be formed within the electron bunch, each shorter than and separated by  $\lambda_r$ , then each of these micro-bunches will emit coherently.

This is the fundamental principle behind the free-electron laser.

$$P(\lambda) \propto N_{elec}$$

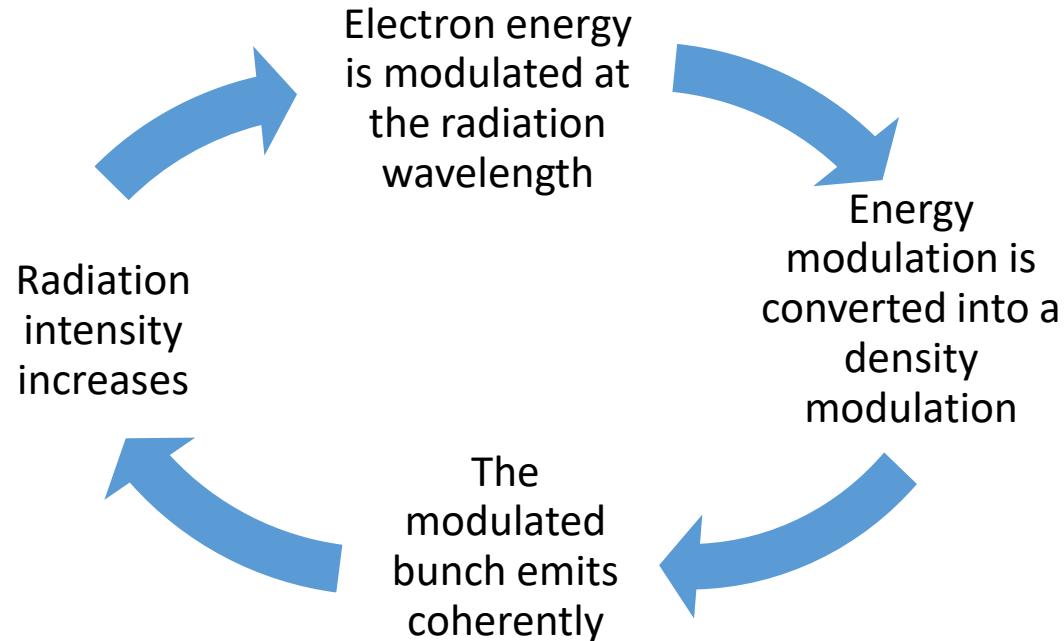


$$P(\lambda) \propto N_{elec}^2$$



## Free-Electron Lasers

In a free-electron laser, a positive feedback mechanism occurs between the electrons within the bunch and the emitted radiation field:



There is an energy-transfer between the electrons and the radiation field. Internal micro-bunches appear in the electron bunch at the radiation wavelength. Each micro-bunch effectively acts as a point-source, emitting coherent radiation proportional to the square of the number of electrons within that segment.

## Free-Electron Lasers

Two different regimes exist.

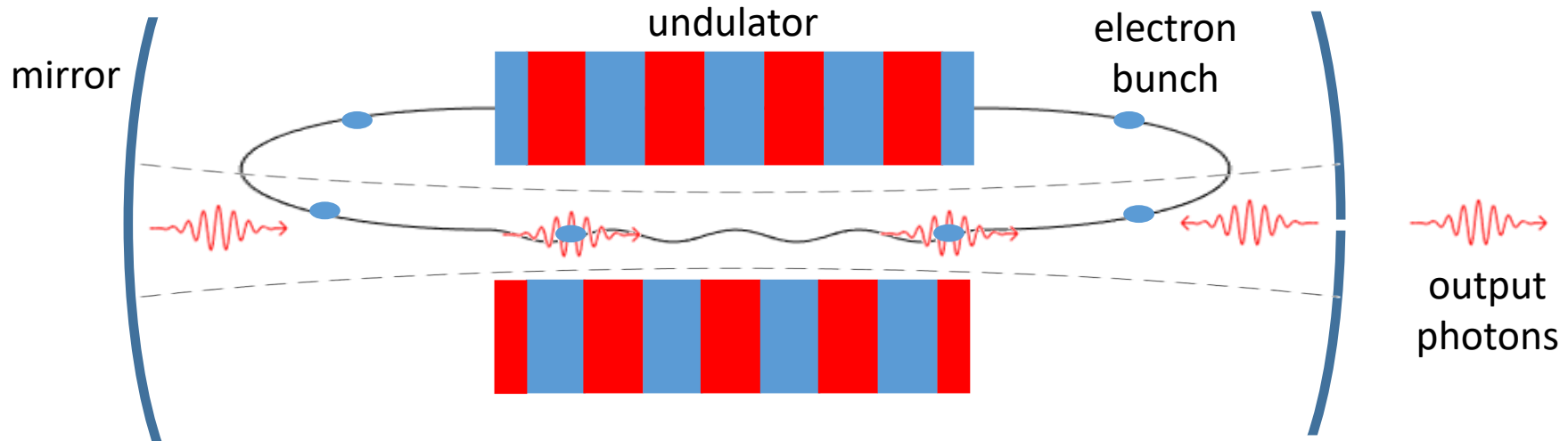
### *Low-gain regime:*

- Electrons make repeated passes through the undulator
- Requires an optical cavity to store the radiation field
- The amplitude of the radiation field grows slowly over time, and is approximately constant during each passage of the electron bunch
- During each passage, energy is transferred from the electrons to the light wave

### *High-gain regime:*

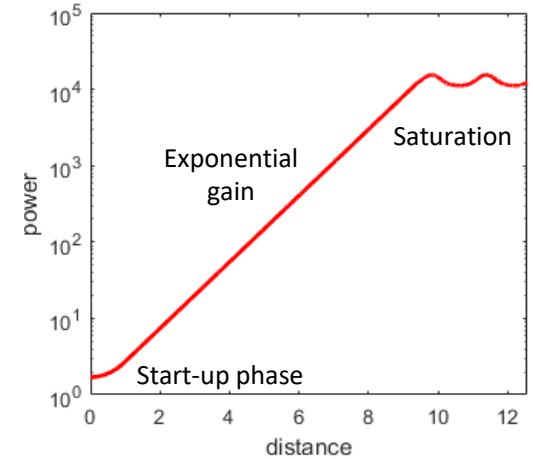
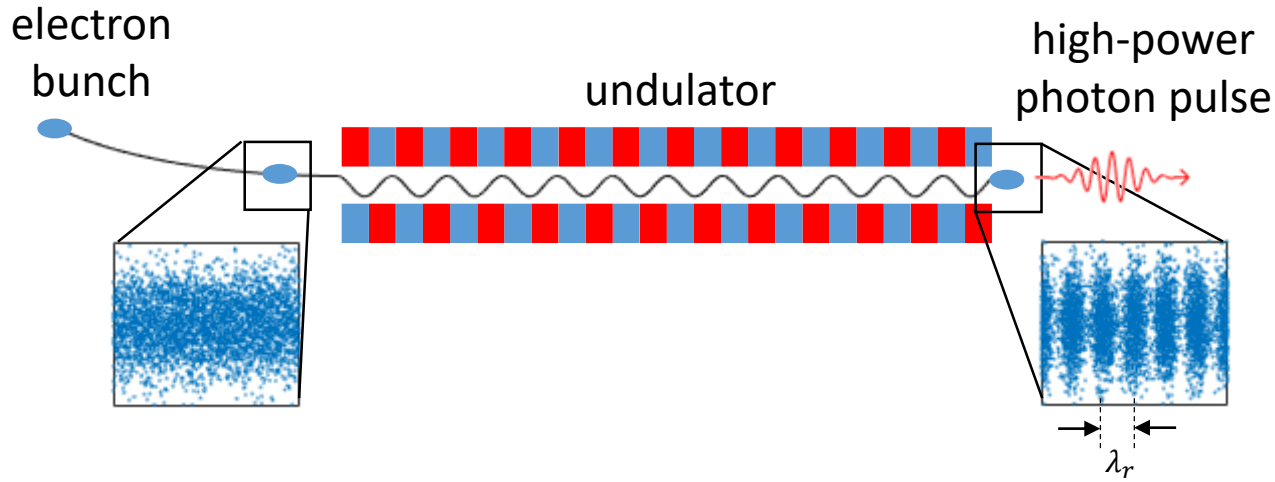
- Usually consist of a long undulator at the end of a linear accelerator
- The feedback mechanism is strong enough that significant micro-bunching occurs within a single passage
- Coherent emission from the micro-bunching gives a rapid increase in the field intensity
- Field grows exponentially with distance into the undulator

## Types of FEL: FEL Oscillator



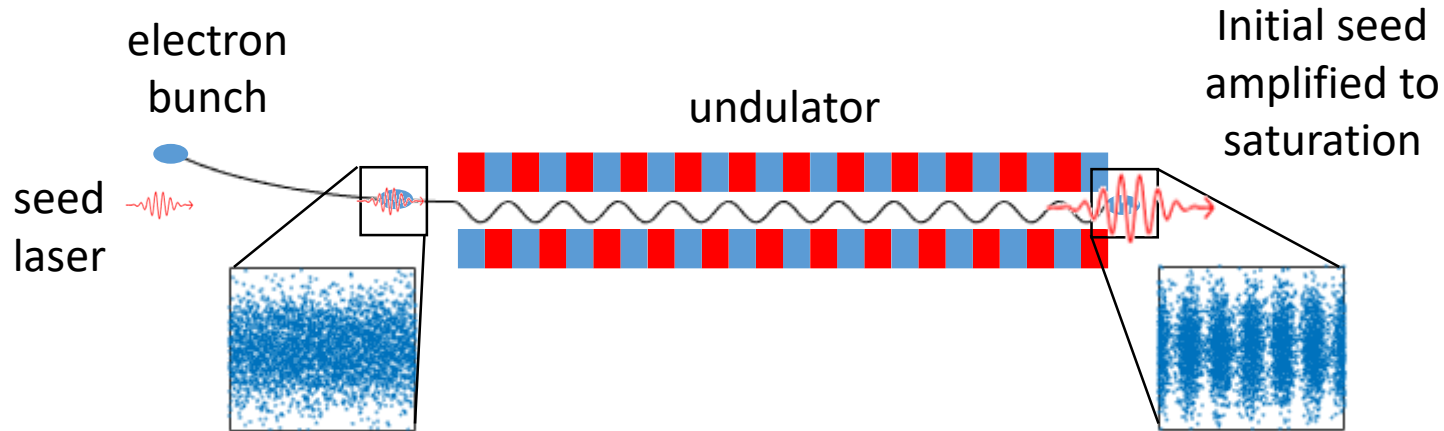
- Low-gain FEL
- Short undulator in a storage ring
- FEL starts up from spontaneous emission from the electrons
- Radiation is trapped within the optical cavity
- Energy exchange between the electrons and radiation causes the electrons to bunch at the radiation wavelength as they pass through the undulator
- Intensity of radiation increases over many passes

## Types of FEL: SASE



- High-gain FEL (no optical cavity)
- Single Pass
- Long undulator at the end of a linear accelerator
- FEL starts up fresh each time from spontaneous emission from the electrons
- Micro-bunches form in the electron bunch
- Radiation field grows exponentially as the bunch travels along the undulator
- Field eventually saturates after  $\sim 18$ - $20$  gain lengths
- Relatively long electron bunch means radiation consists of many temporally independent spikes (relatively low longitudinal coherence)

## Types of FEL: Amplifier FEL



- Similar to the SASE FEL
- Single Pass
- High-gain (no optical cavity)
- Initial (quantum) laser seed initiates the FEL process
- Micro-bunches form around the seed laser field
- Initial laser seed gets amplified, minimising the statistical noise inherent in the SASE FEL and increasing the longitudinal coherence
- Suitable, high-power seed lasers difficult to produce at short wavelengths

## FEL Equations

In order to describe the complete FEL process, we need to:

- 1) Establish the equations of motion for the electrons travelling in the combined fields of the undulator and the synchrotron radiation
- 2) Work out the energy exchange between electrons and radiation field
- 3) Investigate how the energy exchange impacts both the electron motion and the radiation field

These processes are described by Maxwell's equations and the Lorentz force.

The equations must be solved self-consistently, as the motion of the particles depends upon the radiation field, and the radiation field depends upon the particle distribution.

We will begin by examining the low-gain regime, for which the radiation intensity can be assumed to be roughly constant during a single passage of the electron beam. We will follow closely the treatment given in [1].

## Electron motion in an undulator

To start, we will assume an idealised, planar undulator, for which the magnetic field follows a simple sine wave:

$$B_y = B_0 \sin(k_u z), \quad k_u = \frac{2\pi}{\lambda_u}$$

Considering the Lorentz force  $\mathbf{F} = e(\mathbf{E} + \mathbf{v} \times \mathbf{B})$  and relativistic energies ( $v_z \sim c$ ), the equation of motion in the horizontal plane as a function of  $z$  is:

$$\ddot{x}(z) = \frac{d^2 x}{dz^2} = -\frac{eB_0}{\gamma m_e c} \sin(k_u z)$$

From this, we can integrate once to get the transverse velocity:

$$\dot{x}(z) = \frac{eB_0}{\gamma m_e c} \frac{\cos(k_u z)}{k_u} = \frac{K}{\gamma} \cos(k_u z)$$

and a second time to get the position:

$$x(z) = \frac{K}{\gamma k_u} \sin(k_u z)$$



## Electron motion in an undulator

To get the longitudinal velocity, we use the fact that the total velocity is constant, i.e.

$$\beta^2 = \beta_x^2 + \beta_z^2$$
$$v_z \approx c \left( 1 - \frac{1}{2\gamma^2} \left( 1 + \frac{\gamma^2 v_x^2}{c^2} \right) \right)$$

Substituting in  $v_x = \frac{Kc}{\gamma} \cos(k_u z)$  and using the identity  $2 \cos^2 a = 1 + \cos 2a$ ,

$$v_z \approx c \left( 1 - \frac{1}{2\gamma^2} \left( 1 + \frac{K^2}{2} \right) \right) - \frac{cK^2}{4\gamma^2} \cos(2k_u z)$$

The average velocity in the longitudinal plane is therefore

$$\bar{v}_z \approx c \left( 1 - \frac{1}{2\gamma^2} \left( 1 + \frac{K^2}{2} \right) \right)$$

After integration and moving to the time domain we have finally ( $\omega_u = ck_u$ ):

$$x(t) = \frac{K}{\gamma k_u} \sin(\omega_u t) \quad z(t) = \bar{v}_z t - \frac{K^2}{8\gamma^2 k_u} \sin(2\omega_u t)$$

## Energy Exchange

Now we consider how the electrons interact with the co-propagating radiation. We assume that there exists a field (either an external laser or the initial, spontaneous radiation), given by

$$E_x(z, t) = E_0 \cos(k_r z - \omega_r t + \psi_0)$$

where  $E_0$  is the field amplitude,  $\psi_0$  is an initial phase offset and  $\omega_r = ck_r$ .

This electric field will exert a force on the electrons, causing them to change energy. The rate of change of energy is given by

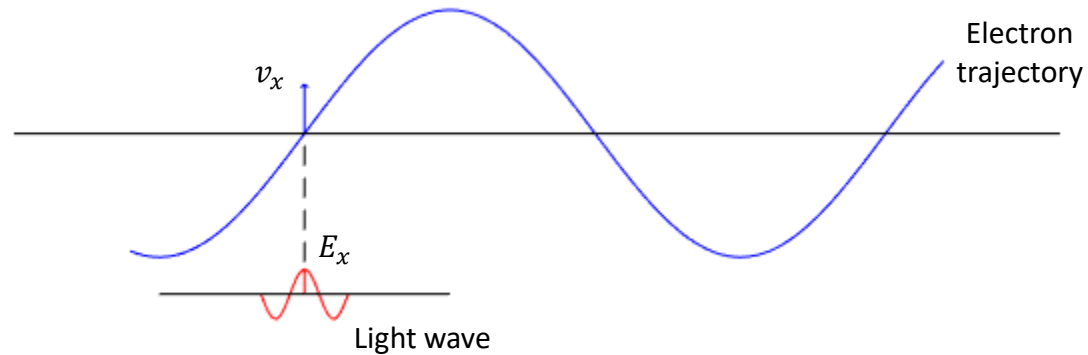
$$\frac{dW}{dt} = v \cdot F = -ev_x(t)E_x(t)$$

The sign of  $dW/dt$  determines whether the electrons gain or lose energy to the light wave.

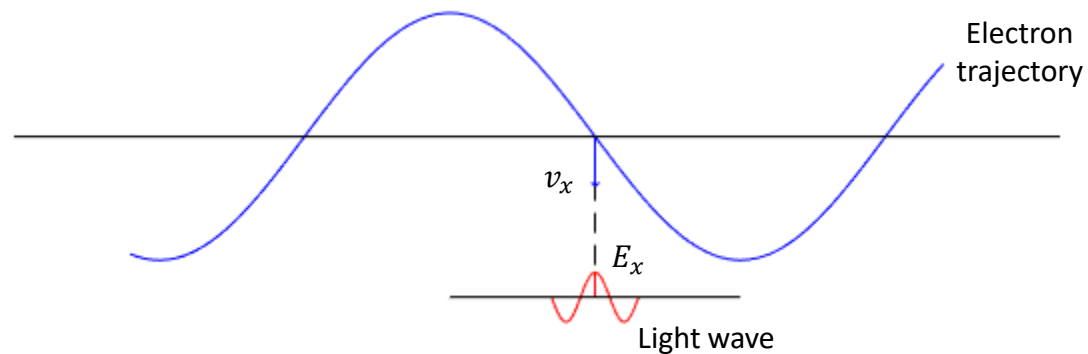
## Energy Exchange

Let us consider two cases:

1) Electron velocity and electric field in same direction => electron loses energy

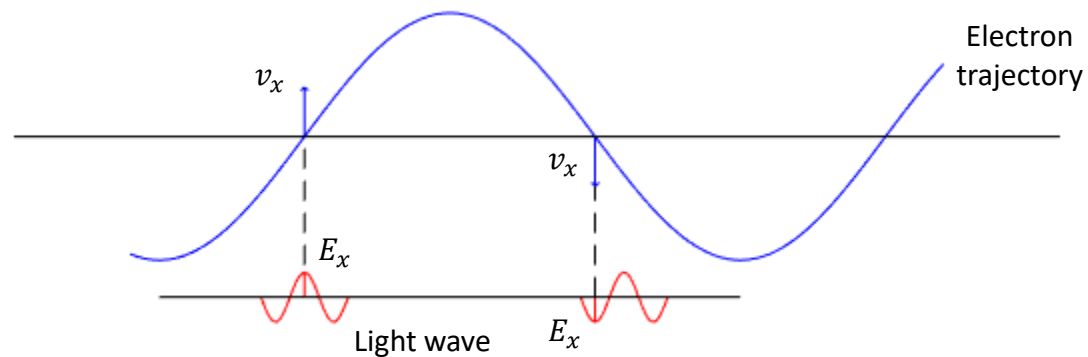


2) Electron velocity and electric field in opposite direction => electron gains energy



## Energy Exchange

As the electrons travel along the undulator, the radiation slips forward ahead of the electrons. In order for there to be sustained energy transfer from the electron beam to the light wave, the light wave must advance by  $\lambda_r/2$  when the electrons have travelled a distance  $\lambda_u/2$ .



This is the same condition as for constructive interference in an undulator, i.e. the condition for continuous energy transfer from the electron beam to the light wave is:

$$\lambda_r = \frac{\lambda_u}{2\gamma^2} \left( 1 + \frac{K^2}{2} \right)$$

## Energy Exchange

Let us now calculate the amount by which the electron energy changes. We have:

$$\frac{dW}{dt} = -ev_x(t)E_x(t) = -e \frac{cK}{\gamma} \cos(k_u z) E_0 \cos(k_r z - \omega_r t + \psi_0)$$

This can be re-written as the sum of two terms:

$$\frac{dW}{dt} = -\frac{ecKE_0}{2\gamma} [\cos((k_r + k_u)z - \omega_r t + \psi_0) + \cos((k_r - k_u)z - \omega_r t + \psi_0)]$$

Or more simply as

$$\frac{dW}{dt} = -\frac{ecKE_0}{2\gamma} [\cos(\psi(t)) + \cos(\chi(t))]$$

Where we have defined the *ponderomotive* phase as  $\psi(t) = (k_r + k_u)z(t) - \omega_r t + \psi_0$ , and defined  $\chi(t) = (k_r - k_u)z(t) - \omega_r t + \psi_0$ .

The (nonlinear) ponderomotive force in this case is provided by the combined undulator-radiation electromagnetic field.

## Energy Exchange

We now have two terms describing the energy exchange. As discussed previously, for there to be continuous energy transfer from the electron beam to the light wave, we need to ensure there is a constant phase relationship between the two, i.e.  $\psi(t) = \text{const}$ . This can only be fulfilled for a constant wavelength. To find this, we differentiate the first term to get

$$\frac{d\psi}{dt} = (k_r + k_u)\bar{v}_z - k_r c = 0$$

Substituting in for the average longitudinal velocity, we again arrive at the condition for continuous energy transfer:

$$\lambda_r = \frac{\lambda_u}{2\gamma^2} \left( 1 + \frac{K^2}{2} \right)$$

If we now use the result  $\chi(z) = \psi(z) - 2k_u z$ , we see that if the above condition is satisfied and  $\psi(t)$  is constant, then  $\chi(t)$  must make two complete oscillations per undulator period and will cancel out. We are then left with the result:

$$\frac{dW}{dt} = -\frac{ecKE_0}{2\gamma} \cos(\psi(t))$$

This is known as the slowly varying envelope approximation.

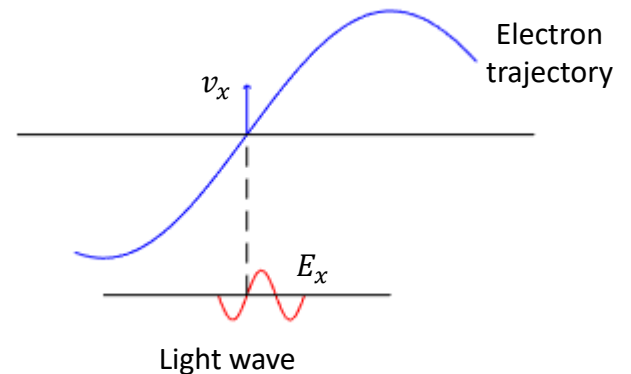
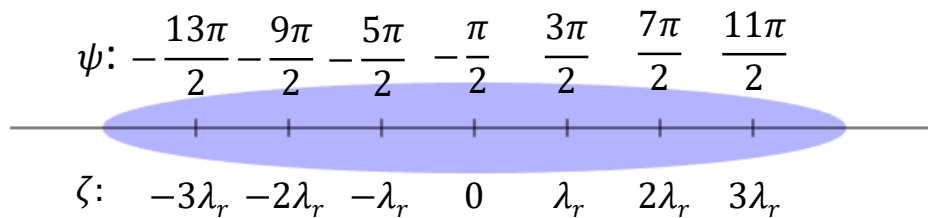
## Electron Coordinates

At this stage, it is convenient to move a coordinate system that moves with the average speed of the bunch. Using the ponderomotive phase, we can define the position of an arbitrary electron as:

$$\zeta(t) = \frac{\psi(t) + \pi/2}{k_r + k_u} \approx \lambda_r \frac{\psi(t) + \pi/2}{2\pi}$$

The reference electron in this case will be at  $\zeta = 0$ , with phase  $\psi = -\pi/2$

For the reference electron, there is no energy exchange. Particles  $\zeta$  in the range  $-\lambda_r/2$  to  $0$  will gain energy, and those in the range  $0$  to  $+\lambda_r/2$  will lose energy.



The position of an arbitrary electron along the undulator is therefore:

$$z(t) = \overline{v_x}t + \zeta(t) = z_r(t) + \zeta(t)$$

## Electron Coordinates

We can also define a reference energy for the particle as  $W_r = \gamma_r m_e c^2$  which satisfies the equation

$$\lambda_r = \frac{\lambda_u}{2\gamma_r^2} \left( 1 + \frac{K^2}{2} \right)$$

From this, we can then define the relative energy deviation of the electron as

$$\eta = \frac{W - W_r}{W_r}$$

Due to the interaction with the light wave, both the energy and ponderomotive phase of the electron will change by an amount that depends upon their position within the electron bunch.



## The Pendulum Equations

For particles with a finite energy deviation, the ponderomotive phase is not constant and so will have a non-zero time derivative. We have:

$$\frac{d\psi}{dt} = (k_l + k_u)\bar{v}_z - k_r c$$

Inserting the expression for  $\bar{v}_z$  and assuming  $k_l - k_u \approx k_l$  this can be written as

$$\frac{d\psi}{dt} = k_u c - \frac{k_l c}{2\gamma^2} \left(1 + \frac{K^2}{2}\right)$$

From the definition of  $\lambda_r$  for an undulator we have  $k_u = \frac{k_r}{2\gamma_r^2} \left(1 + \frac{K^2}{2}\right)$ , so

$$\frac{d\psi}{dt} = \frac{k_l c}{2} \left(1 + \frac{K^2}{2}\right) \left(\frac{1}{\gamma_r^2} - \frac{1}{\gamma^2}\right)$$

and we have to a good approximation

$$\frac{d\psi}{dt} = 2k_u c \eta$$

## The Pendulum Equations

To get the change in energy deviation with time we use the earlier result

$$\frac{dW}{dt} = -\frac{ecKE_0}{2\gamma} \cos(\psi(t))$$

and re-express in terms of  $\eta$  to get

$$\frac{d\eta}{dt} = -\frac{eKE_0}{2\gamma_r^2 m_e c} \cos(\psi(t))$$

We now have the coupled pendulum equations that describe the electron motion in the combined undulator – radiation field:

$$\frac{d\psi}{dt} = 2k_u c\eta$$

$$\frac{d\eta}{dt} = -\frac{eKE_0}{2\gamma_r^2 m_e c} \cos(\psi(t))$$

## The Pendulum Equations

To stress the equivalence with the equations describing the motion of a pendulum with redefine the phase as  $\phi(t) = \psi(t) + \pi/2$ , such that

$$\frac{d\phi}{dt} = 2k_u c \eta$$

$$\frac{d\eta}{dt} = -\frac{eKE_0}{2\gamma_r^2 m_e c} \sin(\phi(t))$$

and we can write

$$\frac{d^2\phi}{dt^2} - \Omega^2 \sin(\phi(t)) = 0, \quad \Omega^2 = \frac{ek_u KE_0}{m_e \gamma_r^2}$$

With this analogy, we can say the electrons are moving in an FEL bucket, where the separatrix between bounded and unbounded motion has an energy acceptance of

$$\eta_{sep}(\phi) = \pm \sqrt{\frac{eE_0 K}{k_u m_e c^2 \gamma_r^2}} \cos\left(\frac{\phi}{2}\right)$$

Note: so far we have assumed the electric field amplitude  $E_0$  is constant. This is OK for the low-gain FEL analysis, but not the high gain FEL.

## FEL Bucket: Electron Phase Space Motion

The phase-space trajectories for 13 particles have been plotted below for a given time interval. The initial phases have been evenly distributed in the range  $-\pi$  to  $\pi$ , with all particles at the resonant energy ( $\eta=0$ ). There are two cases:

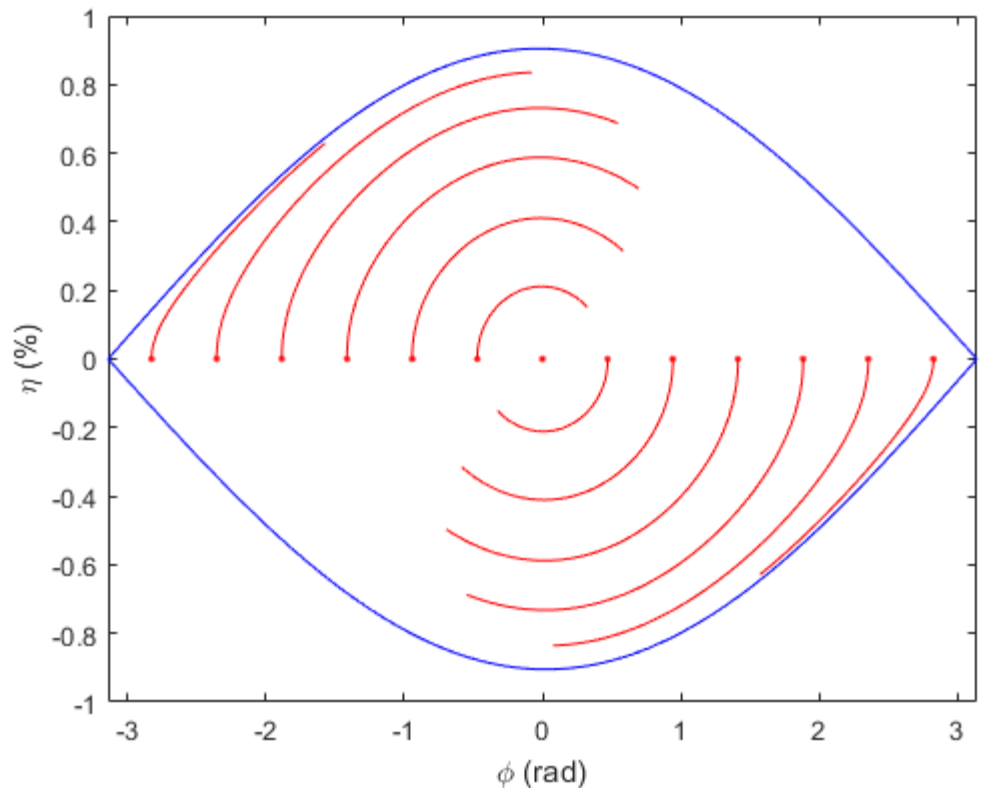
$\phi$  in range  $[-\pi, 0]$ : electrons gain energy

$\phi$  in range  $[0, \pi]$ : electrons loses energy

In this example, an equal number of particles are losing energy as gaining energy; there is no net transfer between the electron bunch and radiation.

In addition, we see the point  $(\phi, \eta) = (0, 0)$  is a stable stationary fixed point; the particle neither gains nor loses energy.

The points  $(\phi, \eta) = (\pm\pi, 0)$  are a unstable fixed points.



## FEL Bucket: Electron Phase Space Motion

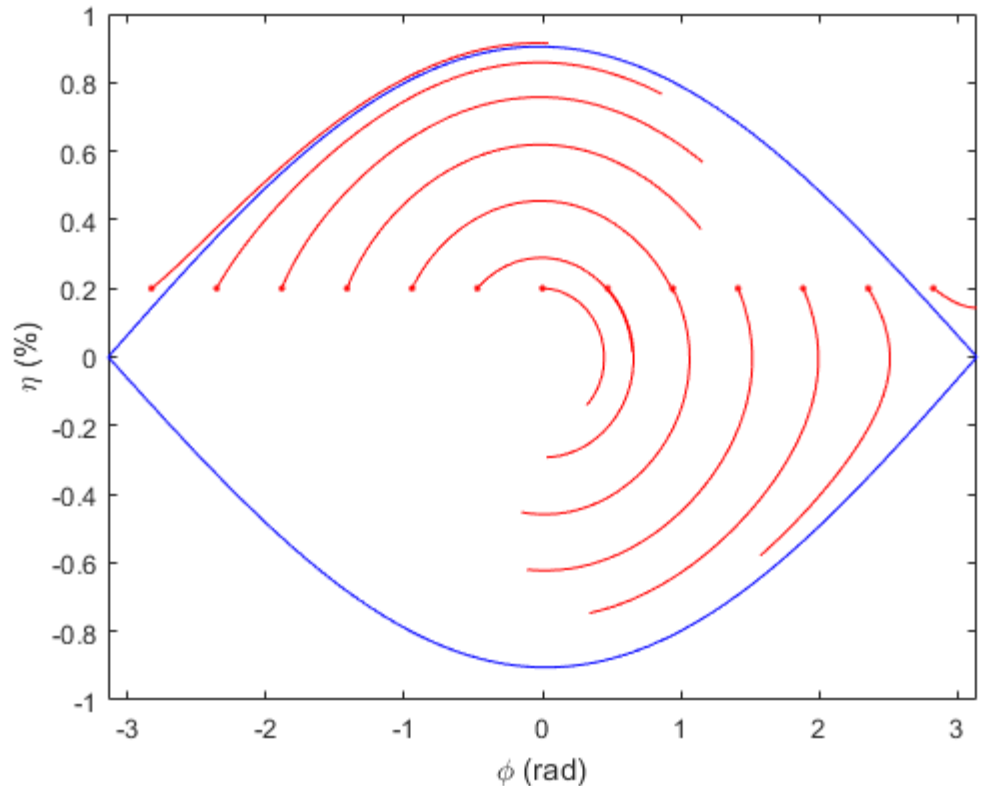
If, however, the energy of the electron beam is slightly offset (detuned) from the resonant wavelength energy, the situation is slightly different.

This time we see some of the particles are outside of the FEL bucket and are lost.

Of the particles that remain, only 3 particles have gained energy, but 8 have lost energy.

There has been a net transfer of energy from the electron beam to the radiation field.

The particle at phase  $\phi = 0$  no longer occupies the stable fixed point.



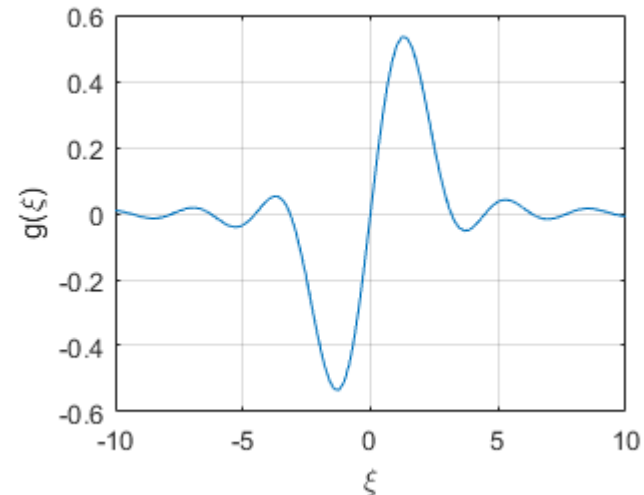
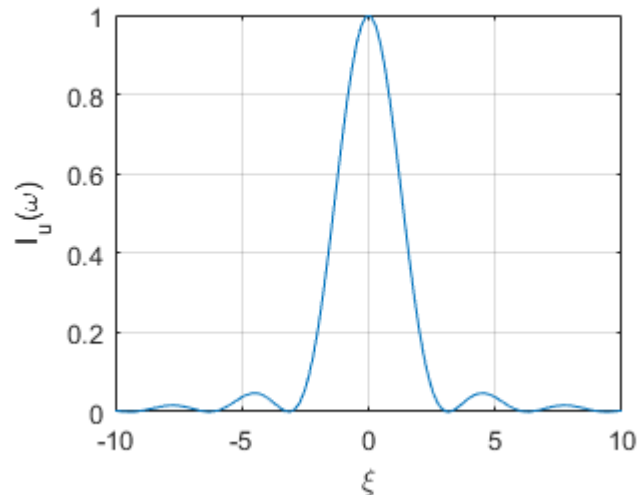
## FEL Gain and Madey's Theorem

As we have seen, there is a net gain in energy for the light wave if  $\eta > 0$ , and net loss if  $\eta < 0$ . This can be quantified by the gain function for the light wave as [1]:

$$G(\xi) = \frac{\Delta W_r}{W_r} = -\frac{\pi e^2 K^2 N_u^2 \lambda_u^2 n_e}{4 \epsilon_0 m_e c^2 \gamma_r^3} \left[ J_0 \left( \frac{K^2}{4 + 2K^2} \right) - J_1 \left( \frac{K^2}{4 + 2K^2} \right) \right]^2 \frac{d}{d\xi} \left( \frac{\sin^2 \xi}{\xi^2} \right)$$

where  $J_n$  are the Bessel functions and  $\xi = \frac{\pi N_u (\omega_1 - \omega)}{\omega_1}$ . This result stems from Madey's theorem, which states:

*The FEL gain curve is proportional to the derivative of the line-shape curve of undulator radiation,  $I_u(\omega)$*



## High-Gain FELs

Up to now we have been considering the case for a low-gain FEL, that is, an optical cavity surrounding a short undulator, for which the radiation field amplitude can be considered constant for each passage of the electron bunch.

In the XUV and X-Ray part of the electromagnetic spectrum however, the lack of suitable mirrors means such a set-up is difficult to achieve. As such, the radiation must be amplified to saturation within a single passage of the undulator. This in turn implies that the undulator must be much longer, and that the radiation field intensity can no longer be considered to be constant during the analysis.

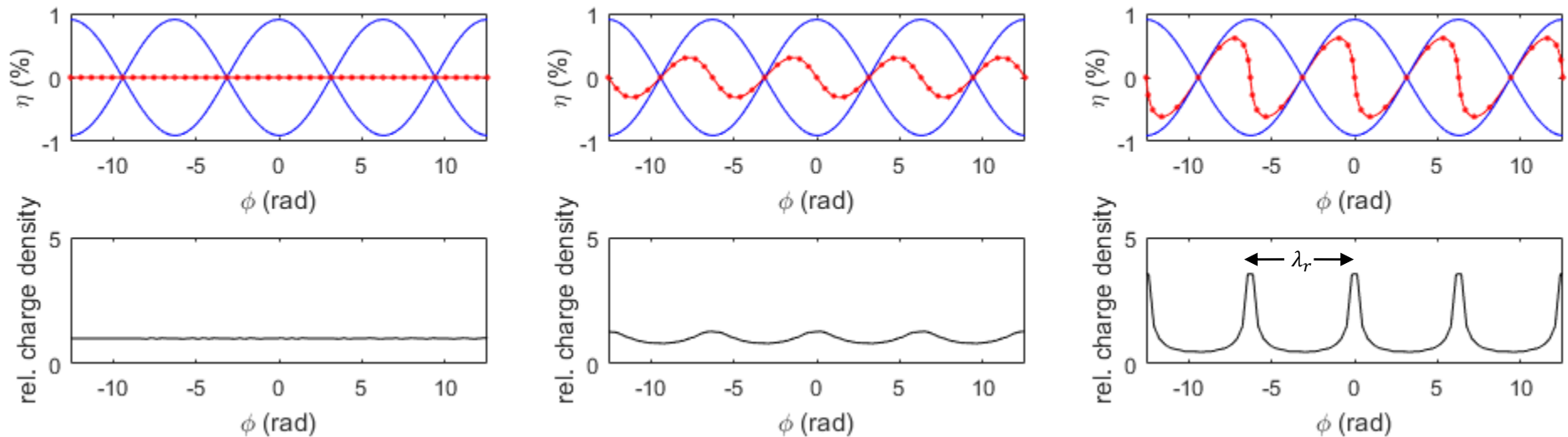
The main differences between the low and high-gain FEL theory are:

- Self-consistent calculation of the field amplitude in parallel with the evolution of the pendulum equations
- The formation of micro-bunching within the FEL bucket at the radiation wavelength, leading to coherent emission and rapid growth of the field amplitude

We will present only the main results of the theory, with the relevant derivations available elsewhere [see for example 1-4].

## Micro-bunching

To illustrate the concept of micro-bunching, to begin with we will continue to ignore the growth in field amplitude and investigate only what happens to the particle density if they interact with the radiation for a sustained period.



As the electrons start to change energy and rotate in the FEL bucket, the electrons that gain energy move forward in phase, and those that lose energy move backwards. The electrons will bunch together at the synchronous phase, and micro-bunches will start to form at the radiation wavelength and begin to emit coherently.

The rotation in phase space stems from the fact low-energy particles will be deflected by a larger amount by the undulator poles, and will thus travel a longer distance and slip in phase.



## Electron charge density

The formation of micro-bunches is incorporated into the high-gain FEL theory through a complex charge density function:

$$\tilde{\rho}(\psi, z) = \rho_0 + \tilde{\rho}_1(z)e^{i\psi}$$

The charge density is assumed to be uniform at the entrance to the undulator with amplitude  $\rho_0$ , and becomes modulated according to the ponderomotive phase of the particle and the growth of  $\tilde{\rho}_1(z)$ .

Correspondingly, the longitudinal current density ( $j_z = v_z\rho$ ) is described by:

$$\tilde{j}_z(\psi, z) = j_0 + \tilde{j}_1(z)e^{i\psi}$$

The modulation in the longitudinal particle velocity is neglected in this analysis, and the current modulation moves along the axis of the undulator with the approximately the average velocity of the electron bunch ( $\bar{v}_z$ ). In this high-gain theory, there is a slight slippage between the FEL bucket and the electron bunch.

## Radiation Field

In the low-gain analysis the electric field was assumed to be purely horizontal with constant amplitude,  $E_x = E_0 \cos(k_r z - \omega_r t + \psi_0)$ . We now take into account the growth of the field amplitude with  $z$  and use a complex notation:

$$\tilde{E}_x(z, t) = \tilde{E}_x(z) \exp[i(k_r z - \omega_r t)]$$

The 1D wave equation for the electric field is given by Maxwell's Equations:

$$\left[ \frac{\partial^2}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right] \tilde{E}_x(z, t) = \mu_0 \frac{\partial \tilde{j}_x}{\partial t}$$

After inserting  $\tilde{E}_x(z, t)$  into the wave equation, making the slowly varying envelope approximation and re-expressing the horizontal current density in terms of the longitudinal, we arrive at:

$$\frac{d\tilde{E}_x}{dz} = -\frac{i\mu_0 K}{2k_r \gamma} \frac{\partial \tilde{j}_z}{\partial t} \exp[-i(k_r z - \omega_r t)] \cos(k_u z)$$

We can then insert  $\tilde{j}_z = j_0 + \tilde{j}_1(z)e^{i\psi}$  and take the time derivative to get:

$$\frac{d\tilde{E}_x}{dz} = -\frac{\mu_0 c K}{4\gamma} \tilde{j}_1$$

## FEL Equations

The 1D coupled equations describing the evolution of both the electrons in phase space and the radiation field as a function of  $z$  are:

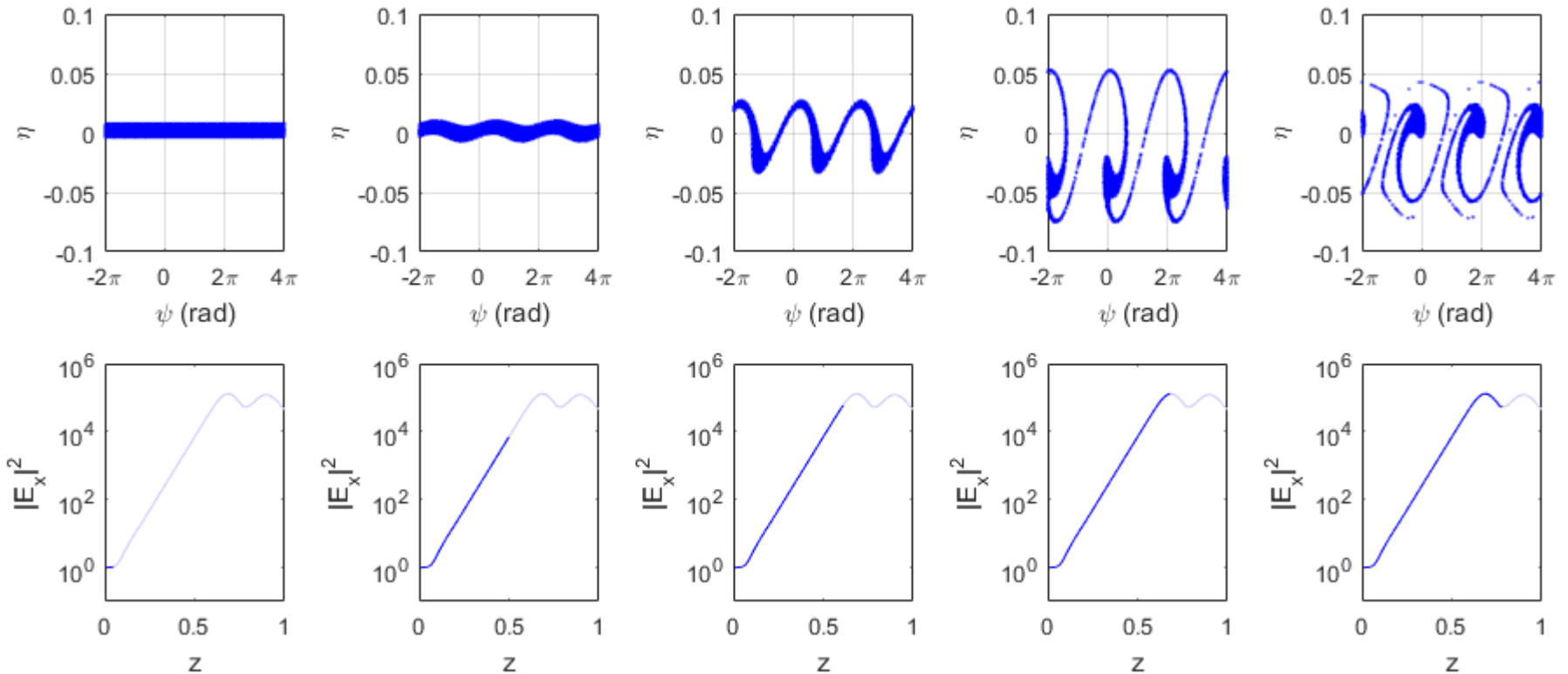
$$\frac{d\psi}{dz} = 2k_u\eta$$

$$\frac{d\eta}{dz} = -\frac{e}{\gamma_r m_e c^2} \operatorname{Re} \left[ \left( \frac{K\tilde{E}_x}{2\gamma_r} - \frac{i\mu_0 c^2}{\omega_r} \tilde{j}_1 \right) \exp(i\psi) \right]$$

$$\frac{d\tilde{E}_x}{dz} = -\frac{\mu_0 c K}{4\gamma} \tilde{j}_1$$

Note that the second equation also includes a term accounting for the space-charge force acting on the electrons which grows with the charge density modulation.

## Evolution in the High-Gain Regime



The evolution consists of several stages; an initial start-up (lethargy) phase, exponential growth, followed by saturation. The height of the FEL bucket can be seen to grow along the length of the undulator.

In contrast to the low-gain FEL, the electron beam in a high-gain FEL starts at the resonant energy. There is an overall exchange of energy from the electrons to the radiation due to a slippage of the FEL bucket with respect to the electrons.

## Effects which can degrade the FEL Process

In order to achieve effective coupling between the electron beam and the radiation field and for the high-gain FEL to reach saturation, a number of conditions must be satisfied:

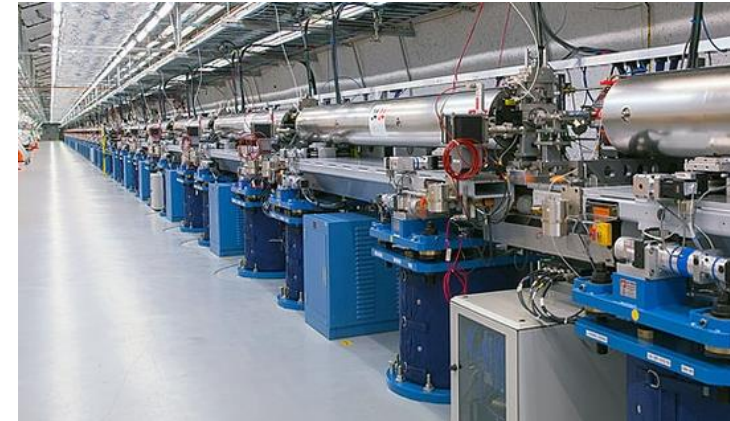
- **High peak current** in order to benefit from the increase in radiation amplitude from coherent emission
- **Long undulator** compared to the gain length for the FEL interaction ( $P(z) = P_0 e^{z/Lg}$ ); the FEL will typically saturate in 18-20 gain lengths
- **Low emittance electron beam** to minimise path-length variation amongst the electrons which would otherwise degrade the FEL gain
- **Low energy spread electron beam** to ensure the spread in electron energies is less than the bandwidth of the FEL, related to line width of the undulator radiation
- **Excellent overlap between electrons and light wave** to ensure sustained interaction between the two beams
- **Gain length less than Rayleigh Length** to counter diffraction of the light wave

## LCLS and LCLS-II



- World's first hard x-ray free electron laser
- Uses SASE principle to generate light in range 0.15 – 1.5 nm
- Pulse durations few fs to >100 fs
- 13.6 GeV electron beam, using final 1/3 of original SLAC linac
- Normal conducting linac, with 120 Hz repetition rate
- Many different modes to enhance the output:
  - Tapering
  - Self-seeding
  - Twin pulses (slotted foil + others)

- Planned upgrade to the LCLS-II: new superconducting linac to increase rep rate to 1 MHz
- 4 GeV electron beam and new undulators, in parallel with existing capabilities
- Increases photon energy range





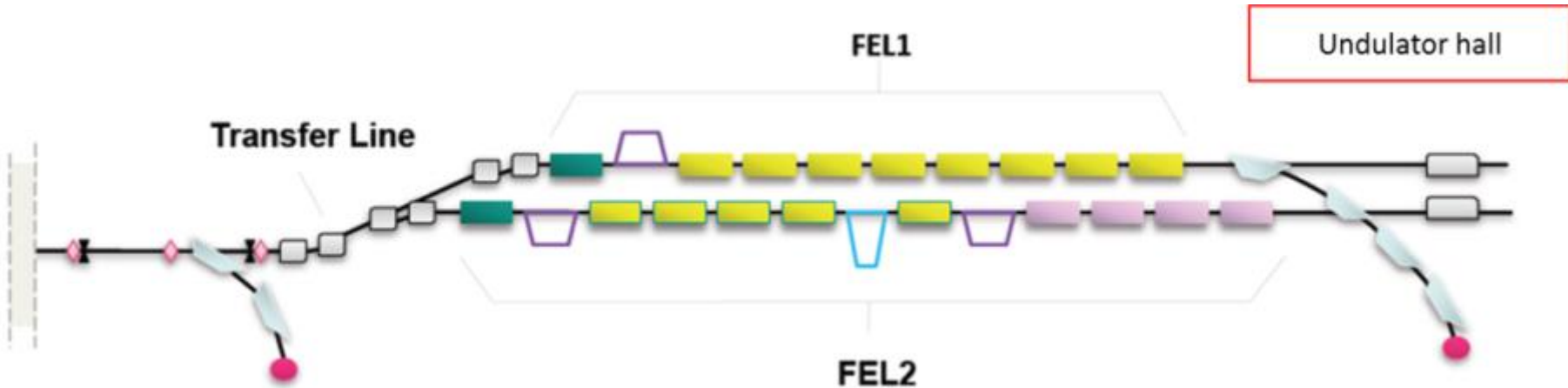
## European X-FEL

- European project between 12 participating countries
- Located in Hamburg, Germany
- 2.1 km long linac, 17.5 GeV beam
- Produces light in range 0.4 nm to 4.8 nm
- Commissioned in 2016/2017
- Superconducting linac
- 2,700 individual pulses at 10 Hz (1.3 GHz linac RF)



## FERMI

- Soft x-ray/UV FEL, spanning 4-80 nm
- Located in Trieste, Italy
- Normal conducting / 50 Hz, 1.2 GeV beam
- Uses external seed laser to initiate FEL process
- 'High-Gain Harmonic Generation' FEL
- Seed laser at 266 nm gets up-converted
- Initial modulator undulator creates an energy modulation
- Chicane converts energy modulation into density modulation
- Radiator undulator generates FEL light at harmonic of bunching
- FEL-2 is two-stage cascade





## SACLA / SwissFEL / PAL X-FEL

### **SACLA** is a Japanese SASE-FEL (2012)

- Compact design; short undulator period (18 mm) and gap (3.5 mm)
- Wavelength range 0.06-0.3 nm
- C-band linac for high gradient (5.7 GHz)
- 8 GeV electron beam (60 Hz)



### **SwissFEL** under construction at PSI (SASE)

- Wavelength range 0.1-7 nm
- Compact design (15 mm period, 3.2 mm gap!)
- Mix of S and C-band linac
- 6 GeV electron beam (100 Hz)
- User operations to begin in 2019

### **PAL-XFEL** in Pohang, S. Korea (SASE)

- Wavelength range 0.1-6 nm
- Undulators 26 mm period, 8.3 mm gap
- S-band linac
- 10 GeV electron beam (60 Hz)
- User operations from 2017



## Summary

FELs are a source of high-power, tuneable, coherent radiation

Can produce peak powers orders of magnitude greater than is available from typical storage-ring based facilities, with pulse durations in the range 1-100 fs

Many different types of FEL:

- low gain oscillator/amplifier FEL

- high gain SASE/amplifier FEL

- regenerative amplifier FEL

- laser-plasma wakefield accelerator-based FEL

- ...

Different technologies used to enhance different aspects:

- seeded vs. unseeded (longitudinal coherence)

- normal conducting linac vs. super-conducting linac (rep. rate)

- harmonic generation (wavelength)

Efficient operation requires high-quality electron beams (low emittance, low energy spread, high peak current)

Highly-active, fast developing field of research

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