The Les Houches benchmarks on Heavy Quarks in Deep-Inelastic Scattering

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In collaboration with S. Forte, J. Huston, P. Nadolsky, P. Nason, F. Olness, R. Thorne, G. Watt

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HEAVY QUARKS SCHEMES IN DIS

Heavy quark schemes in perturbative QCD

Heavy quark schemes in DIS have received a lot of attention in the recent years, specially due to its impact in the determination of PDFs and correspondingly in LHC physics.

Two very well-understood schemes:

1. Assume heavy quarks effectively massless above $Q^2 = m_H^2 \to$ Massless scheme, valid for $Q^2 \gg m_H^2$
2. Heavy quarks retain their mass for all $Q^2 \to$ Massive scheme, valid for $Q^2 \sim m_H^2$

However, their matching in a unique GM-VFN scheme valid at all Q^2 is much more challenging

Matched calculations

Matched calculations aim to combined mass effects in the massive scheme with log resummation in the massless scheme

Several proposals in the literature, including:

1. ACOT: Used in CTEQ family of PDF analysis since 2006 (Collins and Tung 86, Aivazis et al 93, Collins 98). Several variants: Simplified ACOT (S-ACOT), S-ACOT-χ
2. Thorne-Roberts: Used in MSTW family of PDF analysis (Thorne and Roberts 98, Thorne 06)
3. FONLL: originally formulated for hadronic collisions (Cacciari, Greco and Nason 98), recently applied to DIS (Forte, Laenen, Nason, Rojo 10). Currently being implemented in the NNPDF family of PDF analysis

Detailed discussions on the theoretical framework underlying the various GM-VFN schemes presented at previous PDF4LHC meetings: P. Nason, CERN 08 and CERN 09, R. Thorne DESY 09

Now focus on numerical quantitative comparisons between these schemes

THE FONLL APPROACH

Heavy quarks in deep-inelastic scattering,
Stefano Forte, Eric Laenen, Paolo Nason, Juan Rojo
arXiv:1001.2312 [hep-ph]

FONLL in a nutshell

- Express the massive result $F^{(n_l)}$ in terms of the massless PDFs and α_s (non trivial from $\mathcal{O}\left(\alpha_s^2\right)$)

$$F^{(n_l)}(x,Q^2) = x\int_x^1 \frac{dy}{y}\sum_{i=q,\bar{q},g} B_i\left(\frac{x}{y},\frac{Q^2}{m^2},\alpha_s^{(n_l+1)}(Q^2)\right) f_i^{(n_l+1)}(y,Q^2),$$

- Define massless limit of the massive computation as

$$F^{(n_l,0)}(x,Q^2) \equiv x\int_x^1 \frac{dy}{y}\sum_{i=q,\bar{q},g} B_i^{(0)}\left(\frac{x}{y},\frac{Q^2}{m^2},\alpha_s^{(n_l+1)}(Q^2)\right) f_i^{(n_l+1)}(y,Q^2),$$

$$\lim_{m\to 0}\left[B_i\left(x,\frac{Q^2}{m^2}\right) - B_i^{(0)}\left(x,\frac{Q^2}{m^2}\right)\right] = 0$$

- The FONLL approximation is then

$$F^{\mathrm{FONLL}}(x,Q^2) \equiv F^{(d)}(x,Q^2) + F^{(n_l)}(x,Q^2),$$
$$F^{(d)}(x,Q^2) \equiv \left[F^{(n_l+1)}(x,Q^2) - F^{(n_l,0)}(x,Q^2)\right]$$

All technical details in arXiv:1001.2312

FONLL in a nutshell

- Far from threshold, $Q^2 \gg m^2$ $F^{(n_l,0)}(x,Q^2) \sim F^{(n_l)}(x,Q^2) \to$ the massless computation recovered

$$F^{\rm FONLL}(x,Q^2) \sim F^{(n_l+1)}(x,Q^2)$$

- Near threshold the "difference term" is formally higher order but unreliable, so one can correct it by mass suppressed terms, using for example a damping factor (FONLL default)

$$F^{(d,\,th)}(x,Q^2) \equiv f_{\rm thr}(x,Q^2)F^{(d)}(x,Q^2), \quad f_{\rm thr}(x,Q^2) = \Theta(Q^2-m^2)\left(1-\frac{Q^2}{m^2}\right)^2 ,$$

or some form of $\chi-$scaling,

$$F^{(d,\,\chi)}(x,Q^2) \equiv F^{(d)}(x,Q^2) = x\int_{\chi(x,Q^2)} \frac{dy}{y} C\left(\frac{\chi(x,Q^2)}{y},\alpha(Q^2)\right) f(y,Q^2),$$

$$F^{(d,\,\chi,v2)}(x,Q^2) \equiv F^{(d)}(\chi(x,Q^2),Q^2), \quad \chi = x\left(1+\frac{4m^2}{Q^2}\right) .$$

The choice of threshold prescription represent an intrinsic ambiguity of the matching procedure. Can it be eliminated?

Perturbative ordering in FONLL

Three FONLL schemes for different ordering of the perturbative expansion can be defined:

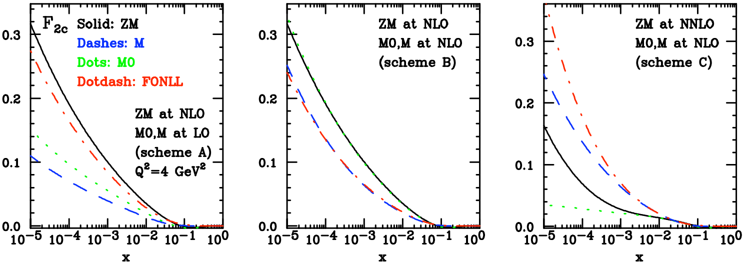
1. Scheme A $\rightarrow \mathcal{O}(\alpha_s)$ in massless and in massive
2. Scheme B $\rightarrow \mathcal{O}(\alpha_s)$ in massless and $\mathcal{O}(\alpha_s^2)$ in massive
3. Scheme C $\rightarrow \mathcal{O}(\alpha_s^2)$ in massless and in massive

In any of the three schemes, any threshold prescription can be implemented
These schemes can be related to existing approaches

1. Scheme A is identical to S-ACOT
2. Scheme B was formulated with similar scope as TR (use the information from the $\mathcal{O}(\alpha_s^2)$ massive computation in a NLO GM-VFN scheme), but they turn to be completely different
3. Scheme C should be S-ACOT at NNLO?

$F_{2c}(x, Q^2)$ in FONLL

The different contributions to FONLL for $F_{2c}(x, Q^2)$:



In FONLL scheme B ZM$\sim$M0 even at $Q^2 \sim 20$ GeV^2 , so FONLL$\sim$Massive
Greatly reduced sensitivity to choice of (arbitrary) threshold prescription present in scheme A

In all schemes mass-suppressed corrections are important even at moderate Q^2

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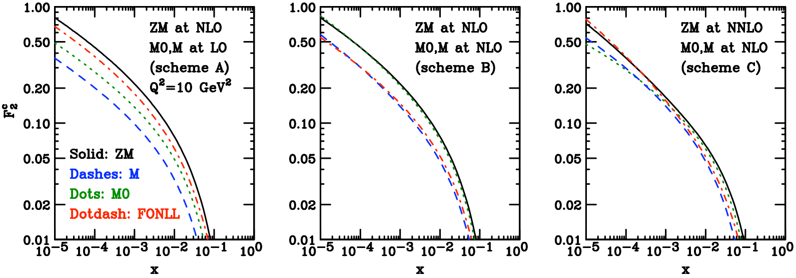

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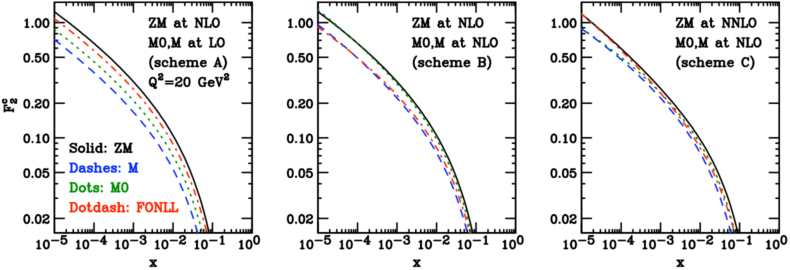
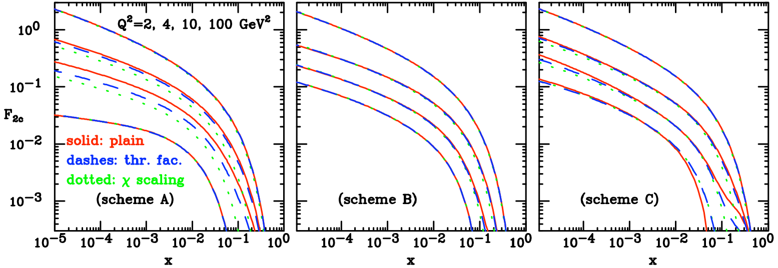


In FONLL scheme B ZM∼M0 even at $Q^2 \sim 20$ GeV^2 , so FONLL∼Massive
Greatly reduced sensitivity to choice of (arbitrary) threshold prescription present in scheme A

In all schemes mass-suppressed corrections are important even at moderate Q^2

$F_{2c}(x, Q^2)$ in FONLL - threshold prescriptions

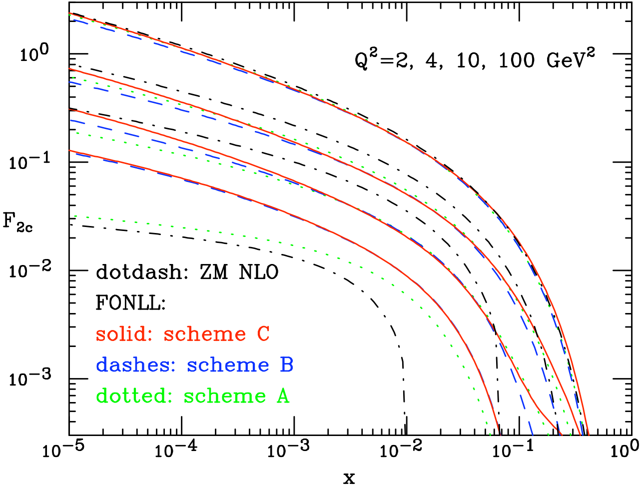
The FONLL result for $F_{2c}(x, Q^2)$ with different threshold prescriptions



In FONLL the ambiguity due to choice of (arbitrary) threshold prescription present in scheme A dissapears in scheme B

This threshold ambiguity can be as large as the resummation itself

$F_{2c}(x, Q^2)$ in FONLL - Summary



FONLL - Summary

- The FONLL approach provides a simple, flexible and practically viable implementation of heavy quark effects in DIS
- FONLL allows for the combination of fixed order heavy quark emission terms with the all–order resummation of collinear logs which appear at scales much larger than the heavy quark mass.
- A significant feature is that the perturbative order at which the fixed–order and resummed results are obtained can be chosen independently of each other in the most suitable way
- By exploiting this flexibility, we have defined scheme B, which thanks to the use of the α_S^2 massive coefficients cures the threshold ambiguities which affect scheme A $\rightarrow$ FONLL-B is A NLO GM-VFN scheme without (almost) threshold ambiguity
- FONLL can be easily generalized to arbitrary perturbative orders, as shown by scheme C, a relatively simple NNLO GM-VFN scheme

THE LES HOUCHES HEAVY QUARK BENCHMARKS STUDY

Les Houches HQ Benchmark settings

- A unique set of PDFs is used → We adopt the Les Houches toy PDF set, evolved in Q^2 to NLO and NNLO using the HOPPET program (G. P. Salam and J. Rojo, Comput. Phys. Commun. 180 (2009) 120)
- $\alpha_s\left(Q^2\right)$ obtained from exact solution of the RG equations from $\alpha_s\left(Q^2 = m_c^2 = 2\,\mathrm{GeV}^2\right) = 0.35$
- The charm quark is the only heavy quark ($m_b \to \infty$)
- F_{2c} (F_{Lc}) defined as the contribution to F_2 (F_L) when light quark charges set to zero
- Compare F_{2c} and F_{Lc} for the different GM-VFN schemes for different values of Q^2, from 4 GeV^2 to 100 GeV^2
  Here concentrate on F_{2c} - see extras for F_{Lc}
- Comparisons available between ACOT, TR and FONLL

Results: F_{2c} in FONLL vs. S-ACOT

The $\chi-$scaling threshold prescription used in S-ACOT-χ can be implemented in two alternative ways (with the difference being subleading)

- $x \to \chi$ replacement only inside convolutions

$$F^{(\chi)}(x,Q^2) = x\int_{\chi(x,Q^2)} \frac{dy}{y} C\left(\frac{\chi(x,Q^2)}{y},\alpha(Q^2)\right) f(y,Q^2),$$

- $x \to \chi$ replacement in the structure function argument

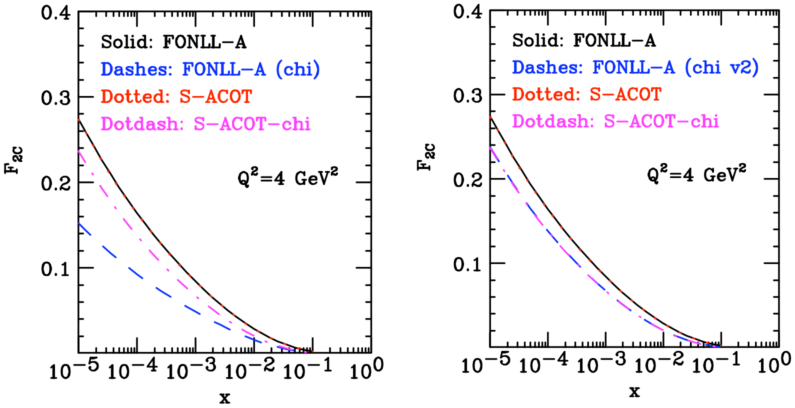
$$F^{(\chi,v2)}(x,Q^2) = \chi(x,Q^2)\int_{\chi(x,Q^2)} \frac{dy}{y} C\left(\frac{\chi(x,Q^2)}{y},\alpha(Q^2)\right) f(y,Q^2),$$

$$\chi(x,Q^2) = x\left(1+\frac{4m^2}{Q^2}\right).$$

$F^{(\chi)}(x,Q^2)$ used in CTEQ6.6, while $F^{(\chi,v2)}(x,Q^2)$ implemented in MSTW2008

Results: F_{2c} in FONLL vs. S-ACOT

S-ACOT is identical to FONLL scheme A
S-ACOT-χ is identical to FONLL scheme A with χ scaling (v2)



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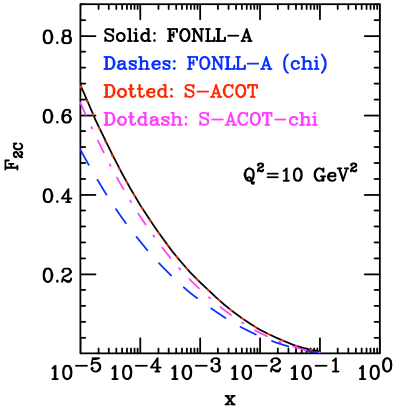
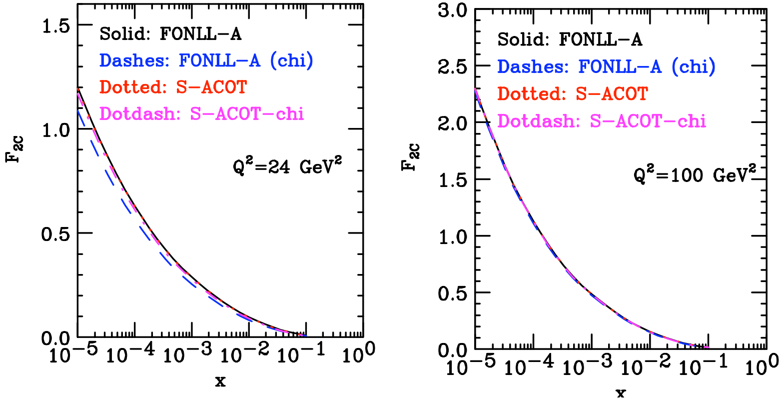


Results: F_{2c} in FONLL vs. S-ACOT

As Q^2 increases all schemes are identical (threshold effects negligible)

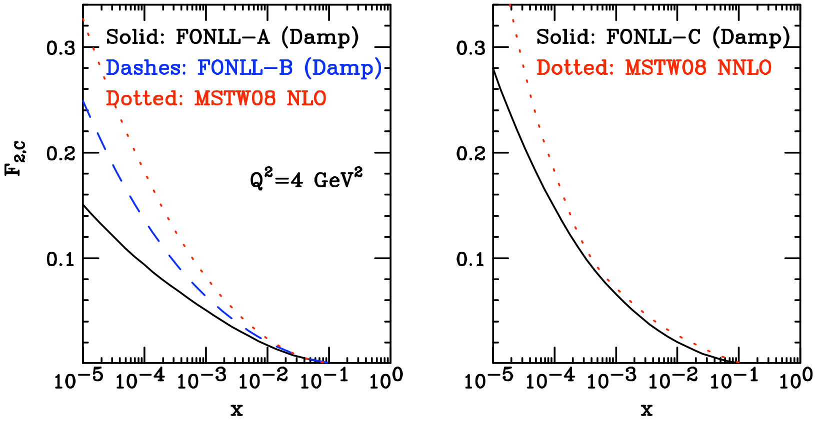


Results: F_{2c} in FONLL vs. S-ACOT

- FONLL-A (plain) is identical to S-ACOT (both for F_{2c} and for F_{Lc})
- FONLL-A is identical to S-ACOT-χ once the proper threshold prescription is adopted
- The S-ACOT-χ numbers provided by F. Olness use a different $\chi-$scaling than the ones used in the CTEQ6.6 fit (P. Nadolsky)
- It is crucial to carefully state the threshold prescription used in each case $\rightarrow$ In FONLL scheme A (and in S-ACOT) the effect of the threshold prescription can be as large as the resummation itself
- The default threshold prescription used in FONLL (damping factor) falls between the two implementations of $\chi-$scaling

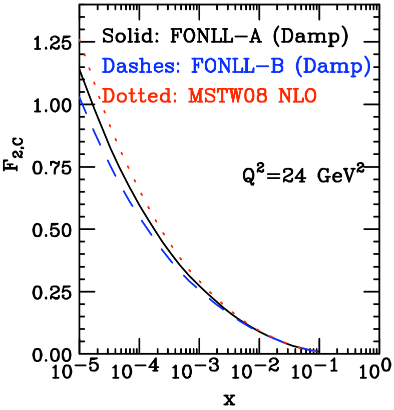
Results: F_{2c} in FONLL vs. MSTW08

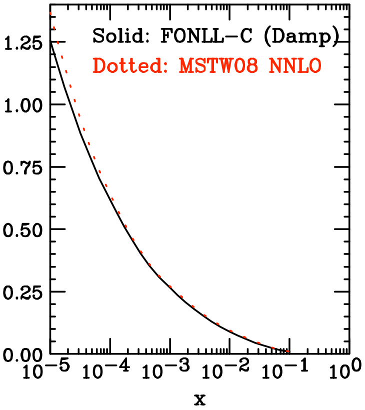
With default threshold prescriptions:



Results: F_{2c} in FONLL vs. MSTW08

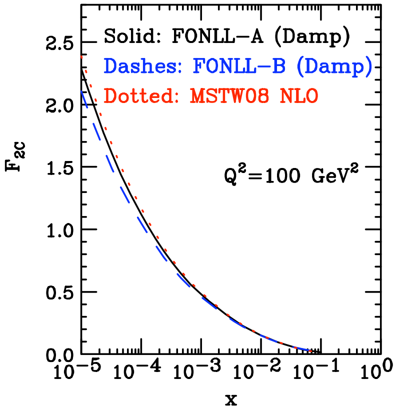
With default threshold prescriptions:





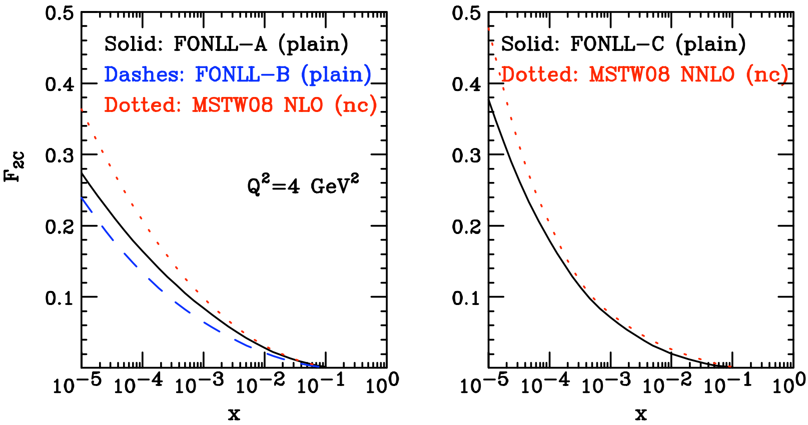
Results: F_{2c} in FONLL vs. MSTW08

With default threshold prescriptions:



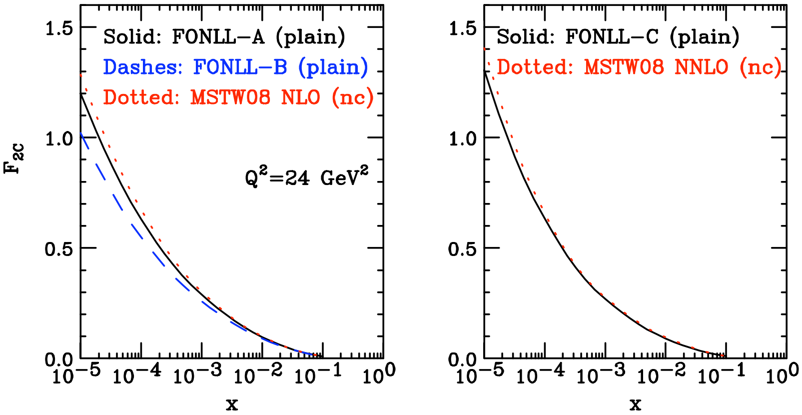
Results: F_{2c} in FONLL vs. MSTW08

With threshold prescriptions switched off:



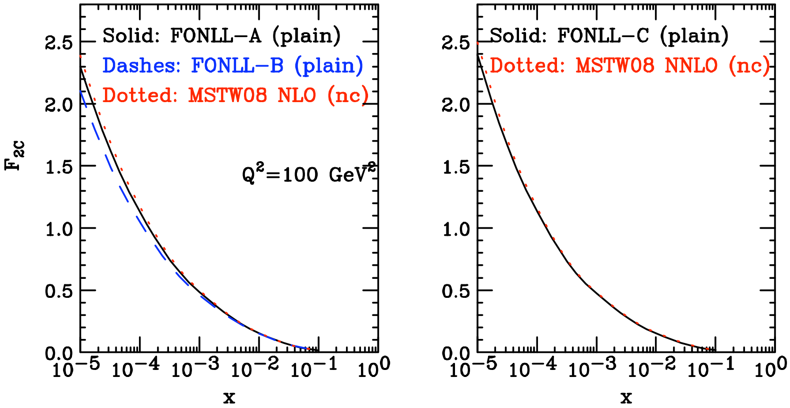
Results: F_{2c} in FONLL vs. MSTW08

With threshold prescriptions switched off:



Results: F_{2c} in FONLL vs. MSTW08

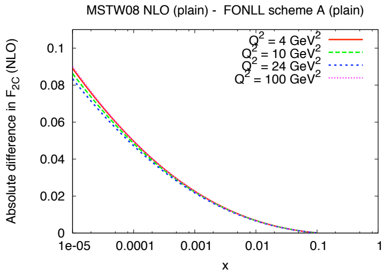
With threshold prescriptions switched off:

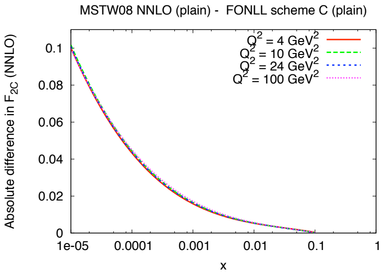


Results: F_{2c} in FONLL vs. MSTW08

The only difference for $F_{2c}(x, Q^2)$ between FONLL scheme A (and scheme C) and MSTW08 NLO (and NNLO) (without threshold prescriptions) is a Q^2-independent matching term f in MSTW08:

$$\text{FONLL-A - MSTW08-NLO} = f\left(x, \alpha_s\left(m_c^2\right)\right)$$





The same conclusions holds for S-ACOT vs. MSTW08 NLO

Results: F_{2c} in FONLL vs. MSTW08

Summary of the TR/MSTW08 vs. FONLL comparison

- FONLL scheme B was formulated with a similar motivation than TR $\to$ Use all information from the $\mathcal{O}\left(\alpha_s^2\right)$ massive computation in the NLO GM-VFN scheme
- In practice, since TR freeze their $\mathcal{O}\left(\alpha_s^2\right)$ term at $Q^2 = m_c^2$, for F_{2c} TR and FONLL-B turn out to be completely different schemes
- TR NLO is S-ACOT/FONLL-A plus the constant (subleading) term, and shares with these schemes the large dependence on the choice of (arbitrary) threshold prescription (unlike FONLL-B which is unaffected by this choice of prescription)
- Similar conclusions for TR NNLO and FONLL-C: identical up to a Q^2-independent subleading term
- For F_{Lc} instead the TR ordering leads to similar results between FONLL-B and MSTW08.

SUMMARY AND FUTURE WORK

Summary

The Les Houches benchmark study on heavy quarks GM-VFN schemes has allowed to identify and quantify similarities and differences between schemes:

1. FONLL-A (plain) is identical to S-ACOT, and FONLL-A (χ,v2) is identical to S-ACOT-χ (Olness), both for F_{2c} and F_{Lc}
2. The only difference between FONLL-A (plain) (and S-ACOT) and MSTW08 NLO for F_{2c} is a Q^2-independent matching term present in MSTW08
3. The only difference between FONLL-C (plain) and MSTW08 NNLO for F_{2c} is a Q^2-independent matching term present in MSTW08
4. FONLL scheme B is a completely different scheme from MSTW08 NLO for F_{2c}. In particular, unlike S-ACOT or MSTW08, it is independent of the threshold prescription
5. On the other hand, due to the TR ordering, FONLL scheme B(C) is very close to MSTW08 NLO(NNLO) for F_{Lc}

Outlook

More work is still required:
From the theoretical point of view:

- CTEQ is planning NNLO PDF fits → would S-ACOT at NNLO be identical to FONLL scheme C?
- MSTW has studied variations of his GM-VFN scheme with respect his default values (R. Thorne, PDF4LHC DESY 10/09) → How these new settings affect the benchmark comparisons?

From the phenomenological point of view:

- Impact of different GM-VFN schemes in the determination of PDF with benchmark-like settings
- Impact of different GM-VFN schemes in relevant LHC observables

EXTRA MATERIAL

Scheme accuracy (Nason, CERN TH inst 09)

Accuracy: (k and l stand for ANY integer from 0 to ∞)

$$\sigma = \sum_{j=1}^{n_l} f_j^{(n_l)}(x,\mu)\,\hat{\sigma}_j^{n_l}(px,\mu,m_h,....)\ \text{(Massive scheme)}$$

Born	NLO	NNLO	...
$\alpha_s^b \times (\alpha_s \log \mu/\Lambda)^k$	$\alpha_s^{b+1} \times (\alpha_s \log \mu/\Lambda)^k$	$\alpha_s^{b+2} \times (\alpha_s \log \mu/\Lambda)^k$	

$$\sigma = \sum_{j=1}^{n_f} f_j^{(n_f)}(x,\mu)\,\hat{\sigma}_j^{(n_f)}(px,\mu,...)\ \text{(Massless scheme)}$$

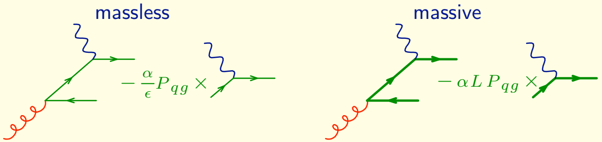
Born	NLO	NNLO	...
$\alpha_s^b \times (\alpha_s \log \mu/\Lambda)^k \times (\alpha_s \log \mu/m_h)^l + \mathcal{O}(m_h/\mu)$	$\alpha_s^{b+1} \times (\alpha_s \log \mu/\Lambda)^k \times (\alpha_s \log \mu/m_h)^l + \mathcal{O}(m_h/\mu)$	$\alpha_s^{b+2} \times (\alpha_s \log \mu/\Lambda)^k \times (\alpha_s \log \mu/m_h)^l + \mathcal{O}(m_h/\mu)$	

Matched calculations (Nason, CERN TH inst 09)

ACOT (Aivazis, Collins, Olness, Tung, 1988,1994)

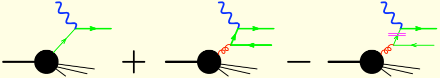
Use the $\overline{\text{MS}}$ scheme above m_h without setting m_h to zero.

If $m_h = 0$: $1/\epsilon$ poles to subtract;   if $m_h > 0$, $L = \log Q/m_h$ terms to subtract



Formal basis: factorization with massive quarks (Collins,1998)

ACOT At NLO: PDF subtraction (3rd graph) depends upon 1st graph.



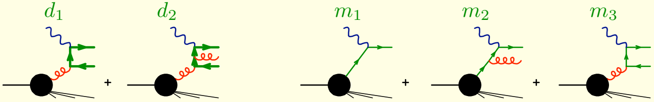
How to include mass in the 1st graph is not fully specified ...

3rd graph takes away from 2nd graph what was already included in 1st.

Matched calculations (Nason, CERN TH inst 09)

TR SCHEME (Thorne and Roberts, 1998)

Basic idea: a structure function computed in the decoupling scheme does not match a structure function computed in the massless scheme when $Q \approx m_h$.
Correct the massless scheme so that they match.



$Q \approx m_h$: $\mathcal{O}(\alpha_s)$ $\quad$ $\mathcal{O}(\alpha_s^2)$ $\quad$ $\mathcal{O}(\alpha_s)$ $\quad$ $\mathcal{O}(\alpha_s^2)$ $\quad$ $\mathcal{O}(\alpha_s)$

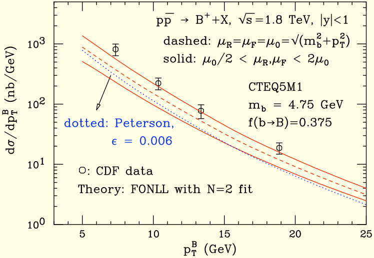
(When counting the order for $Q \approx m_h$, remember that $f_h \approx \alpha_s$)
Since F_2 is $\mathcal{O}(1)$, matching at NLO can be interpreted as: $\mathcal{O}(\alpha_s)$ terms only.
This approach essentially recovers ACOT

TR try to match at NLO, including $\mathcal{O}(\alpha_s^2)$ terms by imposing continuity at $Q = M$ at $\mathcal{O}(\alpha_s^2)$, up to the derivative with respect to Q.
Since m_1 and m_2 vanish at $Q = m$, this implies that d_2 is added to their result. They add $d_2(Q = m_h)$, to avoid $(\alpha_s L)^2$ terms arising for $Q \gg m_h$.

Matched calculations (Nason, CERN TH inst 09)

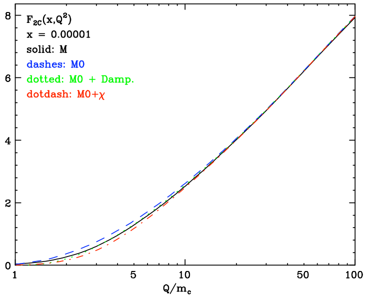
FONLL (Cacciari, Greco, P.N., 1998)

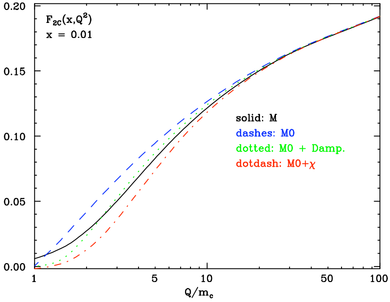
Totally independent approach introduced in the context of heavy flavour production at high p_T, in order to address the discrepancy between theory and Tevatron data in b production.
Besides the pdf, it also deals with b fragmentation.



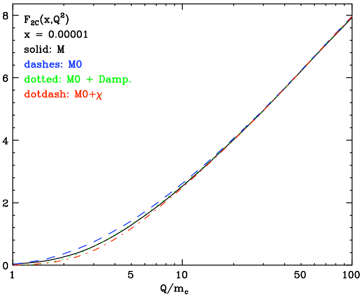
It was used to match the NLO heavy flavour production calculation of Ellis, Dawson and P.N. (massive scheme) with that of Cacciari and Greco (massless scheme). The method is totally general. It has been applied to heavy flavour production in e^+e^- annihilation, but never to DIS.

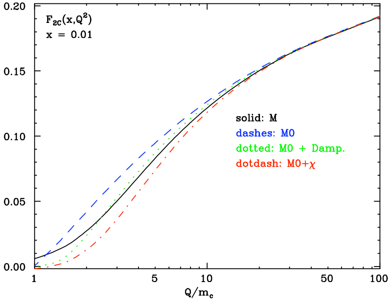
Threshold prescriptions





Threshold prescriptions





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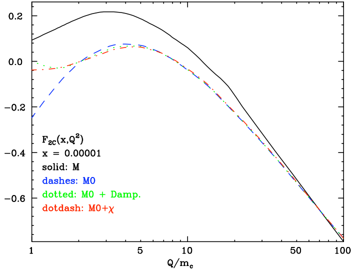
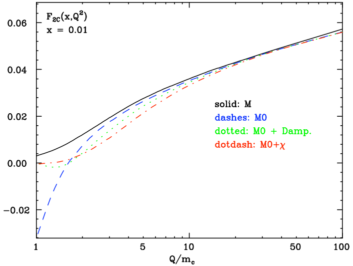
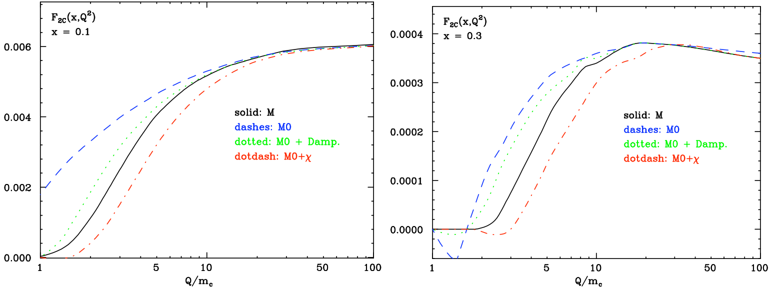


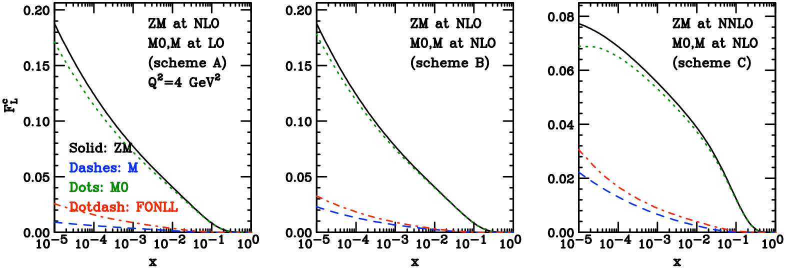


Threshold prescriptions



$F_{Lc}(x, Q^2)$ in FONLL

The different contributions to FONLL for $F_{Lc}(x, Q^2)$



In FONLL scheme B ZM$\sim$M0 even at $Q^2 \sim 20$ GeV^2 , so FONLL$\sim$Massive

Reduced sensitivity to choice of (arbitrary) threshold prescription present in scheme A

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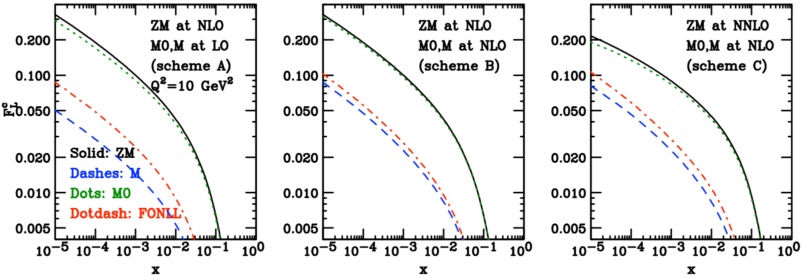

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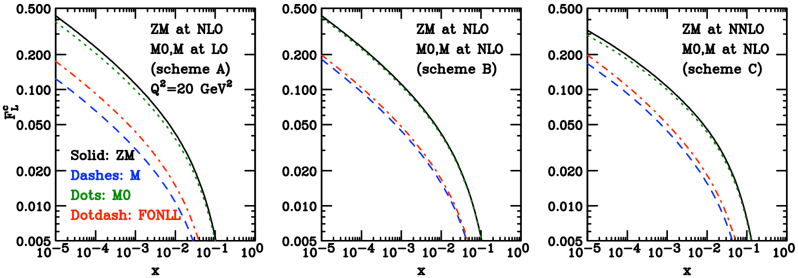
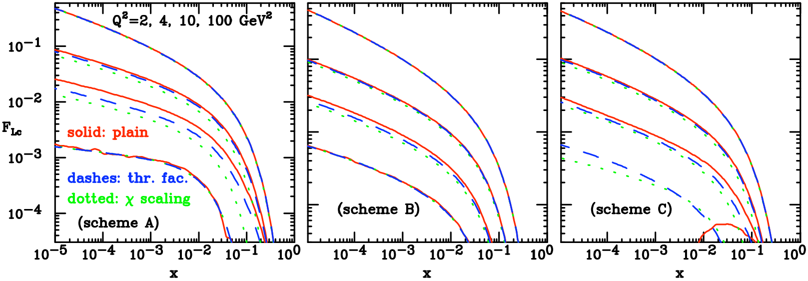


In FONLL scheme B ZM∼M0 even at $Q^2 \sim 20$ GeV^2 , so FONLL∼Massive

Reduced sensitivity to choice of (arbitrary) threshold prescription present in scheme A

$F_{Lc}(x, Q^2)$ in FONLL - threshold prescriptions

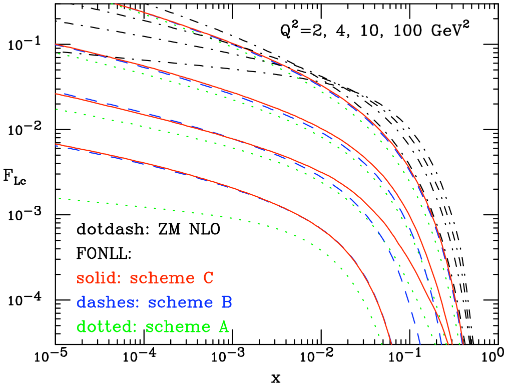
The FONLL result for $F_{Lc}(x, Q^2)$ with different threshold prescriptions



In FONLL the ambiguity due to choice of (arbitrary) threshold prescription present in scheme A dissapears in scheme B

This threshold ambiguity can be as large as the resummation itself

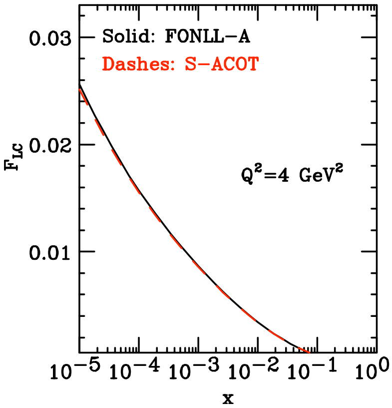
$F_{Lc}(x, Q^2)$ in FONLL - Summary

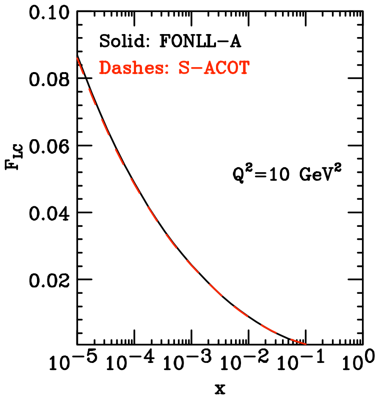


The massless is very far from FONLL even at large Q^2 for $F_{Lc}(x, Q^2)$

Results: F_{Lc} in FONLL vs. S-ACOT

S-ACOT is identical to FONLL scheme A also for F_{Lc}





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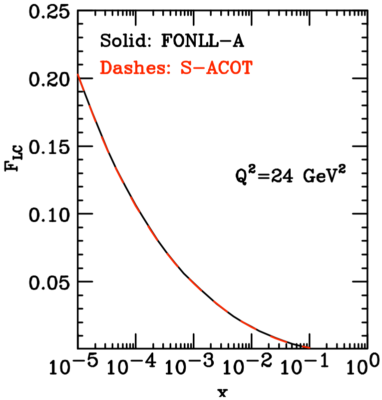
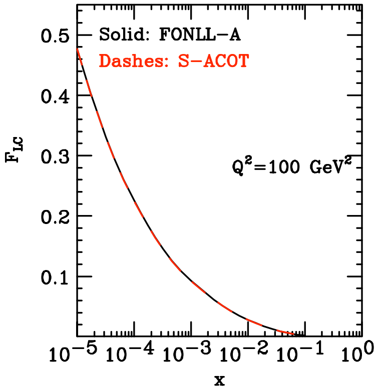
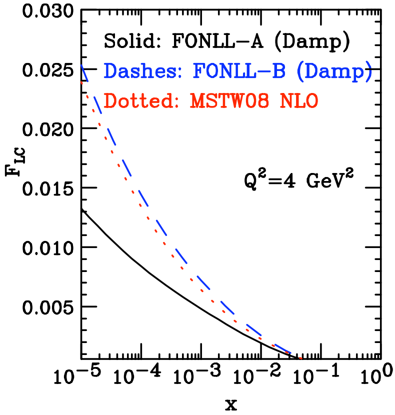




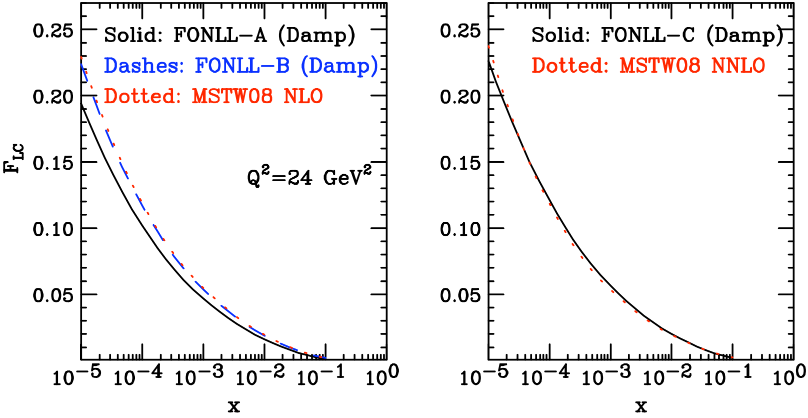
Results: F_{Lc} in FONLL vs. MSTW08

With default threshold prescriptions:



Results: F_{Lc} in FONLL vs. MSTW08

With default threshold prescriptions:



Results: F_{Lc} in FONLL vs. MSTW08

With default threshold prescriptions: