SPECIAL AND GENERAL RELATIVITY AND COSMOLOGY FOR TEACHERS AND HIGH-SCHOOL STUDENTS

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INTRODUCTION: A PLEA FOR MODERN (ASTRO-) PHYSICS AT SCHOOL

The increasing distance between physics as taught at school and modern science has become a central problem of physics teaching. While our knowledge about physics increases every 8 or 10 years by a factor of about two, physics teaching very often suffers from obsolete contents and an overcharge with facts. Instead of emphasizing the concepts and intellectual contents with an eye for the cultural merits of physics, we very often become aware of switching over to still more technical applications.

The grand old man of child psychology, Bruno Bettelheim, exemplifies the situation strikingly as follows (Bettelheim, 1982; re-translation from German by the author):

> Mastering the fundamental rules of arithmetic is undoubtedly useful, and all children are taught them. But even though mathematics is that useful, most children don't pursue it further on as soon as they got the minimum knowledge without which there is nothing to bring about. The reason is that because all emphasis is put on the usefulness of rudimentary arithmetic, in their mathematics lessons children learn nothing about how fascinating the world of numbers is and that mathematics provides the key to a deeper understanding of the world. Only the few who were captured by mathematics for some particular reason do realize what it is really about beyond its practical usefulness. I don't know whether this higher and more apt idea of mathematics is potentially accessible to everybody, but undoubtedly it could be made intelligible to a much greater number of pupils if it wasn't emphasized again and again that the main value of mathematics rests upon its practical applicability.

Even the "theoretical" objection, modern physics cannot be taught at school because it removed too far from mankind and everyday phenomena and can, as far as teaching is concerned, only be what is called "Kreidephysik" anyway, fails of many students' discontent with such a self-restriction of (physics) teaching. Consequently, they resort to other sources of information which frequently present modern physics incorrectly and sometimes promise that the big bang could have happened in any classroom and that black holes could solve the

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energy crisis. It seems that the teachers have left the initiative to the media and that the gap between school physics and modern science is also one between school physics and "popular" science. But for the public the teachers should be the best representatives of science, and students should not depend on external sources of information. Instead they should learn to critically evaluate them.

Relativity, quantum physics, chaotic dynamics etc. will be comprehensible more easily for a generation that grows up with them. But *how* can modern reasoning in physics enter physics classes? Certainly not by simply expanding the curriculum while, on the other hand, the number of classes remains unchanged or even gets reduced. Not at all by dispensing with classical physics and substituting the very latest developements for it. This is because a great deal of the intellectual merits of modern physics rests on its foundations. A transparent connection with these foundations and, if need be, their re-explanation from a modern point of view is, together with the confirmation by experiment, the main source of confidence in modern physics and, consequently, its teachability.

Even the big bang and the black holes, being so remarkable in themselves that they don't need popular-scientific exaggeration, must become subjects of research in physics teaching.

With regard to that it is intended to present a few suggestions based upon positive teaching experiences. In doing so we emphasize the conceptual ideas in order to gain a modern understanding of the physical universe as was made possible by twentieth-century physics, in particular Einstein's relativity theory.

DESCRIPTION OF THE COURSE: DIDACTIC CONCEPTION AND TEACHING SUGGESTIONS

Special Relativity

Even though special relativity as part of *physics* can already be considered classic, it is still young as part of *physics teaching*. Its paradoxes (among them the twin paradox and the length contraction paradox) are well-known (Rindler, 1993), not yet obscured by habituation, and give rise to mental conflicts quasi automatically. In order that the student does not repeat the sometimes contra-intuitive lessons of special relativity merely as new "physics vocabulary" together with already existing (pre-) knowledge, it is not sufficient to simply teach them in a correct manner. Let alone the lack of skill of many teachers, this would be difficult enough. However, the student should *experience the changes* in his mind. Therefore he has to think over these paradoxes and to reflect the genesis of the notions of physics. Thus we arrive at the necessity to combine modern physics with traditional subject-matters without which the changes of rigid ways of thinking could not be experienced.

To begin with, we compare the propagation of light with that of sound thus reminding that already classical mechanics had its principle of relativity. With this bearing in mind and in view of the difficulties with the mechanistic ether concept it becomes all the more obvious what the difference between Einstein's principle of relativity and that of classical mechanics is and that Einstein's achievement lies in the simultaneous recognition of the apparently contradictory principles of relativity and constancy of the speed of light. The presentation of physics as an intellectual process in a historical context is *one* aspect of the human dimension of physics so often missed at school.

The clock synchronization is discussed together with the relativity of simultaneity, the latter being illustrated with two spaceships of "equal psychological power" (Sexl, 1980) instead of the Einstein train (fig. 1).

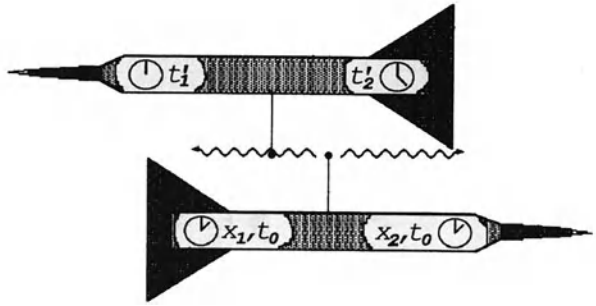


Figure 1. The relativity of simultaneity. The light flashes are emitted halfway between the clocks when the two spaceships face each other. From the point of view of the spaceship shown in the lower half of the figure, the light flashes switch on the clock at the tail of the other spaceship first, then the two own clocks simultaneously, and finally the one at the bow of the other spaceship.

The time dilation is being introduced making use of light clocks which, from the point of view of didactics, represent the highest possible reduction of a clock to the two principles of special relativity. As far as length contraction is concerned, our main interest lies in the visual *appearance* of objects moving at relativistic velocities.

All that is dealt with still before the Lorentz transformation is introduced. Thus, the approach to a quantitative reasoning takes place step by step; gedankenexperiments and model making are of primary importance for the time being. Without physical reasoning which is to be expressed by the student in an everyday language, mathematical formalism alone makes no sense. Comprehension of the problem comes first, then that of the theory!

For a derivation of the Lorentz transformation the method of Bondi's k factor turned out to be the most suitable one (d'Inverno, 1992), for it combines a maximum of physical intuition with a minimum of required mathematics. Moreover, the relativistic Doppler effect gives the k factor a clear physical meaning, and the composition law of velocities comes out immediately.

Finally, when the Lorentz transformation is at hand, the handling of the mathematical formalism (which is particularly simple in this case) is aimed at qualitative problems. It turns out that the way back from the formulas into everyday language is more difficult than vice versa. First of all, we verify all the well-known effects once more. We also demonstrate that the relativity of equal places and that of simultaneity are completely equivalent. While the former seems to be obvious, the latter is a true difficulty in understanding relativity (fig. 2).

It is of decisive importance for the verification of the theory by the μ-meson experiment that it can be understood from the point of view of a stationary observer at the surface of the earth as well as from the point of view of a co-moving one. The explanations given by the two observers may be different, the result, namely the detectability of μ mesons from the upper atmosphere at the surface of the earth, is, however, the same. After all, relativity does *not* mean "everything is relative".

In order to retrospectively compare special relativity with Galilei-Newtonian mechanics, it is necessary to carry out approximations for velocities which are small in comparison with that of light. A mathematical addendum is devoted to such approximations (Table 1).

Even though the students know calculus sufficiently well such that Taylor's theorem could be dealt with, the more elementary binomial formulas are sufficient for special relativity (and even more, fig. 3). Inequalities like $u/c \ll 1$ are important connecting links

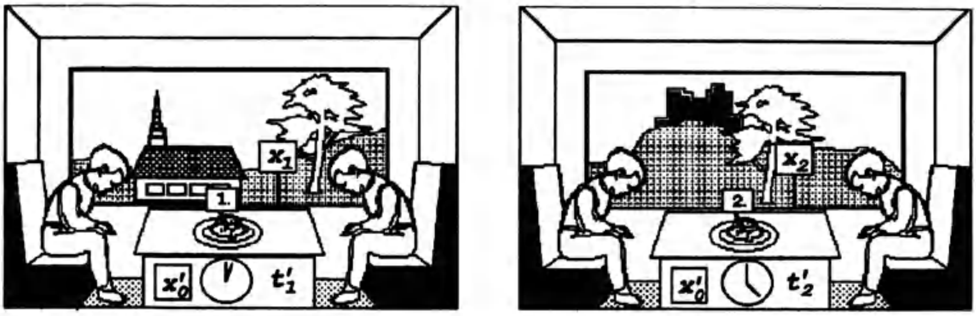


Figure 2. Relativity of equal places. A traveller who takes two meals at one and the same seat in a moving dining-car sees, when looking out of the window, different landscapes.

which recombine physics from its different parts. Moreover, they yield another argument why modern science must not be taught at the expense of classical physics.

Approximations are inherent to the method of physics and even to the most fundamental of its notions. A good example is the notion of inertial frames which lies at the heart of special relativity (Lotze, 1993). At school we pay too less attention to that or do not emphasize it at all thus allowing for wrong ideas about how true and exact science can be. However, there is no reason to be afraid of rattling the student by dealing with approximations. To accept them is not a weakness but a prerequisite for the strength of physics.

Table 1. Calculations involving small quantities by making use of the binomial formulas only, and two applications (fig. 3).

1. $x = \sqrt{1-\varepsilon}$: $x^2 = 1-\varepsilon$, add the quadratic completion ($\varepsilon << 1$)

$$x^2 \approx 1-\varepsilon+\frac{\varepsilon^2}{4} = (1-\frac{\varepsilon}{2})^2$$

$$\boxed{\sqrt{1-\varepsilon} \approx 1-\frac{\varepsilon}{2}}$$

2. $x = \frac{1}{1-\varepsilon}$: Division of polynomials $1:(1-\varepsilon) = 1+\varepsilon+\varepsilon^2+\ldots \approx 1+\varepsilon$

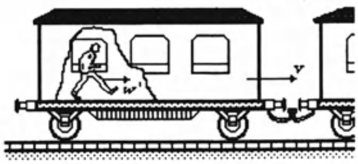
$$\boxed{\frac{1}{1-\varepsilon} \approx 1+\varepsilon}$$

3. $x = \frac{1}{(1+\varepsilon)^2}$: $x \approx [1+(-\varepsilon)]^2 = 1-2\varepsilon+\varepsilon^2$, omit the quadratic completion

$$\boxed{\frac{1}{(1+\varepsilon)^2} \approx 1-2\varepsilon}$$

4. $x = \frac{1}{\sqrt{1-\varepsilon}}$: $x \approx \frac{1}{1-\frac{\varepsilon}{2}} \approx 1+\frac{\varepsilon}{2}$, $\boxed{\frac{1}{\sqrt{1-\varepsilon}} \approx 1+\frac{\varepsilon}{2}}$

The composition law of velocities



$$w = \frac{v+w'}{1+\frac{vw'}{c^2}} \approx (v+w')(1-\frac{vw'}{c^2})$$

For $\frac{v}{c} << 1$, $\frac{w'}{c} << 1$: $\quad \varepsilon = \frac{vw'}{c^2}$

What has a Galilean parabola to do with Kepler's ellipse?

- On the earth:
  The orbit of projectile motion is a *parabola* (bound motion)
- In the sky:
  Bound motion is an *ellipse*
  (A Galilean parabola is not Kepler's parabola)
- Same physics in the sky and on earth?

$$ma = -G\frac{Mm}{(R+h)^2} = -m\cdot\frac{GM}{R^2}\cdot\frac{1}{(1+\frac{h}{R})^2}$$

$$\xrightarrow{\varepsilon=h/R} \quad -mg\cdot(1-2\frac{h}{R})$$

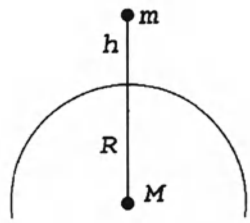


Figure 3

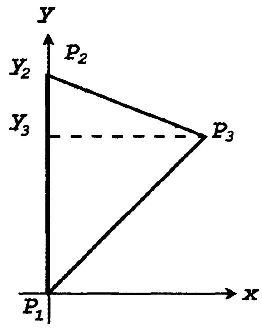
Finally, this tool is being applied constructively to obtain the mass-energy equivalence by means of a generalization of kinetic energy from classical mechanics.

In order to grasp the essence of general relativity and its applications the training of a geometrical view of space and time is of particular importance. Therefore we demonstrate, using the Lorentz transformation, that the distance $(\Delta s)^2 = (\Delta x)^2 - c^2(\Delta t)^2$ between two events in spacetime has the same invariant meaning for a transformation from one inertial frame to another as the spatial distance $(\Delta r)^2 = (\Delta x)^2 + (\Delta y)^2$ of two points has for rotations in a two-dimensional plane. This "Pythagorean theorem" in spacetime with a minus instead of a plus sign already represents the most elementary case of non-Euclidean geometry. Graphically it manifests itself in a skewing of the x and (ct) coordinate axes through the same angle towards the invariant *light cone.*

At first we make use of this geometrical way of thinking to solve the twin paradox: Because of the motion of the travelling twin his world line comes closer to the light cone than that of the twin at home and is therefore - from the spacetime point of view - the shorter one (fig. 4).

Every spacetime event has its own light cone which divides spacetime into an absolute past and an absolute future with respect to this event. This is one of the central messages which we save for general relativity in order to make transparent, for instance, the physics of black holes for which a rigorous mathematical discussion is out of question at this level.

Already at this early stage of the course the concept of the light cone and the finite and constant speed of light inherent in it gives rise to a conclusion which is fascinating to



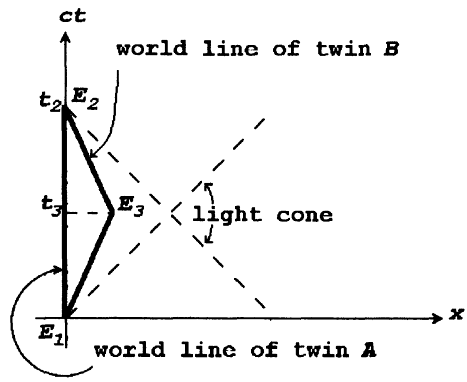


Figure 4. While in Euclidean space (left) the straight line is the shortest connection between the points P_1 and P_2, in Minkowski space (right) the straight line connecting the events E_1 and E_2 is the world line with the longest proper-time interval.

everybody who meditates on cosmology for the first time: Whenever we look into the depths of space we also look into the history of the Universe. This is because we don't see the stars and galaxies as they are now. We see them as they were when they emitted the light we receive today.

General Relativity

The time has come to demythologize general relativity and to make it accessible to secondary schools. In order to embody it in classical physics we start with a critical retrospective on Newton's theory of gravity. What is to be taught about it from a modern point of view?

In the same way as we subject the notion of inertial frame to a particularly critical analysis in special relativity, we now examine the concept of *mass*. With regard to physics teaching one lesson of special relativity is that the teacher should provoke mental conflicts even when they do no more or not at all arise of their own. Thus we explain that mass appears as active and passive gravitational as well as inertial mass. That one finally need not distinguish between active and passive gravitational mass can easily be proved using Newton's third law. That, however, gravitational and inertial mass are equal is an independend law characteristic of gravity, which cannot be understood in the framework of Newton's theory. Here a comparison with electromagnetism is always helpful: The idea of a mass spectrograph rests on the fact that for different particles the relation between *electric* charge and inertial mass is different. A gravitational field, however, does not separate the particles because for all of them the ratio between "gravitational charge" (gravitational mass) and inertial mass is the same.

There is a direct way from this critical analysis of the concept of mass to Einstein's elevator gedankenexperiments and to the equivalence principle according to which physics in *small* freely falling laboratories is the same as in inertial frames. Taking this principle as the starting-point of the theory one can *derive* that different bodies dropped simultaneously from the same height hit the ground at the same time.

While special relativity postulated the existence of global inertial frames but could not find them, inertial frames do exist in general relativity as freely falling laboratories, but these are allowed to be small (local) ones only. But what is meant by "small"?

Now we come to a key point of our chapter on general relativity. Let us consider two test particles (pin heads) falling freely in the gravitational field of the earth. Each of them is attracted by the earth's gravity; on the other hand, their mutual attraction may be neglected completely. With increasing initial distance between the particles it becomes more and more obvious that each of them falls towards the centre of the earth individually and that they suffer an acceleration towards each other even though their mutual attraction was neglected. Thus the gravitational field of the earth reveals itself, like all "real" gravitational fields, by means of a *tidal* action which is proportional to distance. (Of course this is the appropriate place to discuss the ocean tides.)

By means of the elevator gedankenexperiments we draw, at least qualitatively, two conclusions from the equivalence principle which belong to the classical tests of general relativity. On one hand, this is gravitational redshift, in other words, the fact that clocks are running the more slowly the deeper they are in a gravitational potential. On the other hand it is the bending of light by a gravitational field. In quantitative respects we encounter a geometrical measure of the strength of a gravitational field for the first time here, namely the Schwarzschild radius $R_S = 2GM/c^2$, sometimes called (without the factor 2) "compactness". In its relation to the geometrical radius of a celestial body of mass M it determines the order of magnitude of gravitational effects produced by this body.

In view of the curvilinear light rays we, however, need not contemptuously abandon the dogma of our very first physics lessons, namely that light always propagates along straight lines, if only we formulate correctly: The worldlines of light are the straightest possible lightlike trajectories (null geodesics) in a curved spacetime.

In the same way general relativity replaces the interpretation, planetary orbits are brought about by a force which distracts the planets from a straight line, by the geometric one according to which the planet moves on the straightest possible path through a spacetime curved by the sun. For the purpose of illustration a quadratic rubber membrane is well suited which is stretched in a frame and gets deformed by a weight at its centre. If one causes, with some training, tiny balls representing the planets to move in nearly elliptical orbits on this curved surface one can even demonstrate rather clearly the third of the classical effects of general relativity, namely the advance of the perihelion of planetary orbits.

Nevertheless, this model suffers from one drawback which should not be concealed if one wants to exhaust all the educational possibilities inherent to models and analogies. It does not answer the question how the weak gravitational field of the sun (as compared e.g. with those of neutron stars and black holes) can give rise to planetary orbits which are curved as much as ellipses are. The rubber membrane demonstrates the curvature of space only, not that of *spacetime* what relativity is really about. If one represents the planetary motions in a spacetime diagram (as astronomical almanacs do e.g. with the Jovian moons) and turns the time axis of this diagram into a spatial one by multiplication with the speed of light (as was done in special relativity from the very beginning), a planetary orbit appears as an extremely stretched and therefore, in accordance with the weak gravitational field of the sun, very weakly curved helix. Its projection from spacetime into space may very well be an ellipse (fig. 5).

After having discussed the most fundamental ideas of special and general relativity we now arrived at a branch point as far as applications are concerned. On one hand there is relativistic astrophysics with focus on late stages of stellar evolution, pulsars and black holes, gravitational lenses and gravitational waves (Lotze, 1994), on the other hand there is cosmology.

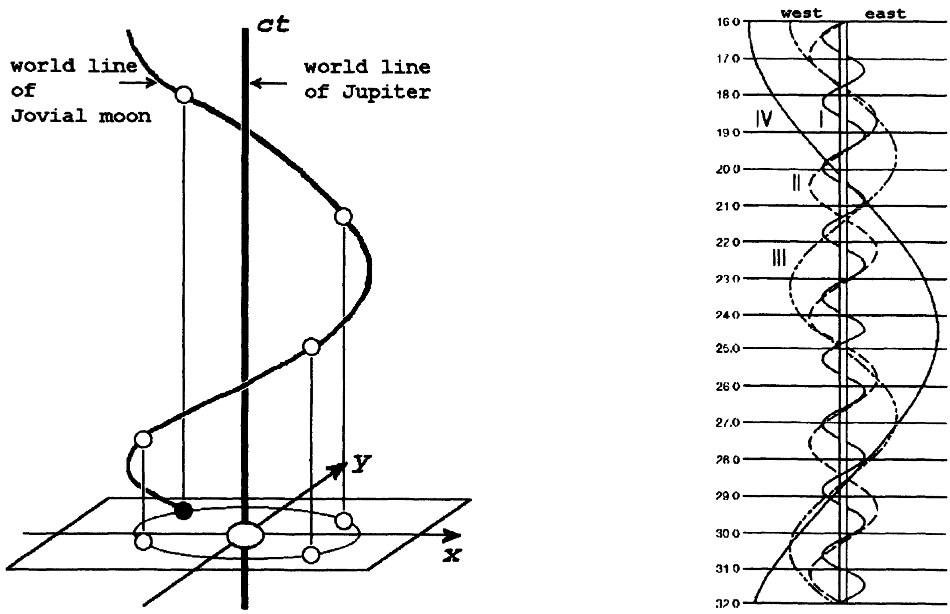


Figure 5. Worldlines of the Jovian moons are weakly curved helixes in space and time. Their projections into space are elliptical orbits (left). On the right the worldlines of the four Galilean moons of Jupiter are shown with an arbitrary scale of the time axis as they are in September 1994. With an appropriate time scale the helix-shaped worldlines must be stretched by a factor of about 35000.

Relativistic Astrophysics

Here we use the notion "relativistic astrophysics" in order to discriminate between applications of relativity to isolated objects like collapsing stars, galaxies as gravitational lenses, and binary stars as sources of gravitational radiation on one hand, and cosmology, the large-scale structure of the Universe, its past, present, and future on the other hand. However, we begin with a few remarks on traditional astronomy and astrophysics.

Classical Astronomy and Astrophysics. For the sake of self-consistency of the course and in order to account for the differences in the (pre-) knowledge of the students, an introduction into astronomy is indispensable which is aimed at the main topics of the course, including cosmology even though cosmology does not care about many astronomical details. Of course, at secondary schools astronomy has its own significance beyond the context discussed in this paper.

We discuss in detail

- astronomical methods to determine cosmic distances,
- properties of stars, their energy-production mechanisms and their finite luminous lifetime,
- the Hertzsprung-Russell diagram and possible final states of stellar evolution (white dwarfs, neutron stars, black holes),
- the evolution of binary stars and the limitations of classical celestial mechanics to determine the orbital elements of spectroscopic binaries.

The intellectual merit of the Hertzsprung-Russell diagram is no less than that of the evolutionary ideas of biology. Different objects which can already be observed with amateur telescopes like e.g. the Great Nebula in the constellation Orion, the ring-shaped nebula in Lyra and the Crab Nebula in Taurus become parts of a whole story, and the teacher can really be a guide through the complex. The mere fact that heavy elements occur in our environment and in ourselves already means that we did not emerge from the first generation of stars. Supernova explosions must have taken place during which these elements only could have been produced. Insights of this kind demonstrate that not only gazing at the starry night sky but also the theoretical conclusions of astronomy are particularly suitable to combine science with adventure. Even though the awareness of the *physical* position of man in the Universe cannot completely satisfy the students' need for orientation in this world, such insights are being absorbed by the students, according to the experiences of the author, not only by reason but also with imagination and emotion.

Furthermore, we discuss also

- the structure and size of the Milky Way galaxy and the position of the solar system in it and
- different types of galaxies and the reasons why dark matter should occur in them.

The successes of astronomy very much depend on the use of advanced technology and sophisticated methods. Among these, the pedagogical importance of the Hubble Space Telescope can hardly be overestimated. It makes *visible* objects (like the moon Charon in clear separation from its planet Pluto; planetary systems in statu nascendi around young stars; the impact of intergalactic environment on the shape of galaxies; dust rings around black holes which are supposed to exist in the centres of galaxies; images of quasars formed by gravitational lenses etc.) which, to an extent that a summary of the entire course can be based on it, confirm what up to now existed in the minds of theoreticians only.

Gravitational Lenses and Wine Glasses as Tools of Relativity. That the prediction of general relativity, light rays should be deflected in the vicinity of the sun, was confirmed for the first time during a solar eclipse in 1919, is well known today. Sixty years later a related discovery was made, indeed much more silently but of no less importance: The double image of the quasar QSO 0957+561 was identified as a gravitational-lens phenomenon.

Gravitational lenses offer everything one can expect from astronomy - new observations together with new theoretical interpretations which, in this case, do not only confirm general relativity but reveal a new approach to important problems of cosmology like e.g. the existence of dark matter.

Unlike many discoveries in astronomy, in the case of gravitational lenses the theory was known long before observation. It has common roots with the theory of light deflection in the vincinity of the sun. Here we take up the latter again after having argued *qualitatively* on the basis of the very foundations of general relativity that light must be deflected in a gravitational field. Here we do not fail to point out that the heuristic analogies and qualitative arguments we frequently make use of are not yet the physical theory. For instance, according to the principle of equivalence the light deflection comes out wrong by a factor of two. In order to obtain the correct value (of measurement) one needs, however, the mathematical formulation of the theory.

With regard to teaching general relativity gravitational lenses are lucky chance in two respects. Firstly, they meet the request for vividness - it just deals with gravitational *optics*. Secondly, they even provide an opportunity to carry out experiments in the classroom. It is important that students carry out experiments, and in the classroom they usually can do that with classical physics only. We, however, may ask what shape a lens made of *glass* must have in order to produce the same images as a static gravitational lens. The answer is that it

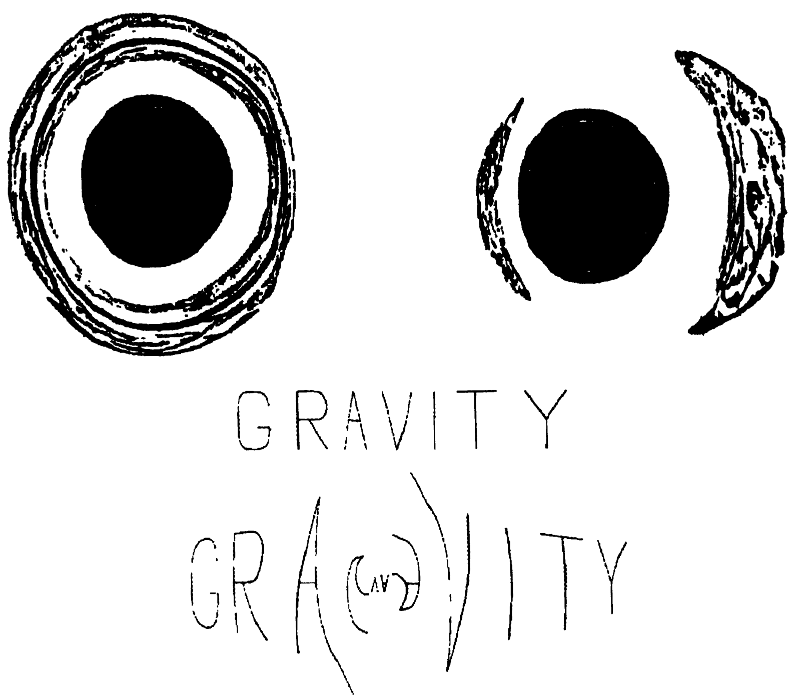


Figure 6. A portrait of Einstein and the word GRAVITY on a billboard would be distorted as shown when a black hole passed by acting as a gravitational lens between these objects and the observer (adapted resp. from Zahn, 1992 and Gott, 1983). The same images can be seen through a glass lens shaped like the base of a wine glass.

must be shaped like the base of a wine glass. Light rays entering the flat bottom parallel to and near the axis get deflected more than distant ones. At least the profile of the rim of the lens can even be derived from Snell's law making use of high-school mathematics only.

Various authors generated gravitational-lens images using a computer thus demonstrating what, for instance, a portrait of Einstein or a billboard with the word "GRAVITY" on it, which are located in a typical quasar distance, would look like when imaged by a black hole or a galaxy halfway between us and these sources.

To perform a gravitational-lens experiment one has to look at the Einstein portrait resp. at the billboard through the glass lens in a way such that the flat bottom through which the light rays enter is kept parallel to the object plane. The position and number of images depend crucially on the arrangement of the lens with respect to source (object) and observer. When they are perfectly aligned the Einstein portrait degenerates to a ring which decays into two crescent-shaped portraits on both sides of the lens when it is placed off the axis source - observer (fig. 6). If the size of the lens is appropriate one can see three images of two letters (A and V, say) of the word GRAVITY, two of them inverted. Thus the student can gain some ideas about the optical properties of gravitational lenses at least qualitatively.

Black Holes. There is no other astrophysical object to stimulate imagination more than black holes. No other object is subject to such fantastic and unrealistic ideas like that of black monsters of extreme density which have such strong a gravitational field that all the matter in the Universe can be swallowed by them. Therefore, when discussing black holes it is particularly important to combine simplicity with correctness.

Black holes are defined by the event horizon, i.e. that surface at which the escape velocity is equal to that of light. Here we once more encounter the Schwarzschild radius $R_S = 2GM/c^2$ and become acquainted with black holes as the most compact objects in the sense that their geometrical radius equals (per definitionem) the Schwarzschild radius. However, this does not mean that the average mass density μ is always extremely high (Matzner, 1980). For a sphere with the Schwarzschild radius we get

$$\mu = \frac{3}{8\pi G}\left(\frac{c}{R_S}\right)^2 = 1.85 \cdot 10^{16}\left(\frac{M_\odot}{M}\right)^2 \frac{\text{g}}{\text{cm}^3}.$$

Accordingly, the mass density is extremely high for a one-solar-mass ($M_\odot$) black hole indeed. For a black hole of 10 solar masses we already get the density of standard nuclear matter, and it is by no means hard to imagine a black hole so massive that its density becomes that of water.

The strength of the gravitational field also varies with mass. At the event horizon the gravitational acceleration is

$$g = \frac{c^2}{2R_S} = 1.53 \cdot 10^{13}\frac{M_\odot}{M}\frac{\text{m}}{\text{s}^2}.$$

How strong it may ever be: The equivalence principle still applies, and in free fall towards or around a black hole one locally does not experience the force of gravity at all. The situation is quite the same as in the vicinity of the earth. It is, therefore, not the gravitational acceleration which is important but its change over short distances. At the event horizon the ratio between the tidal forces which tend to rip apart an observer of height h who is in free fall head-on towards the black hole, and his weight at the surface of the *earth* is equal to

$$8.1 \cdot 10^{15}\,\text{m} \cdot \frac{h}{R_S^2}.$$

Thus for $h = 2$ m and a one-solar-mass black hole the tidal forces at the event horizon are $1.8 \cdot 10^9$ times the weight on earth!

Imagine an observer indestructible even by such strong tidal forces who follows the collapse of a star to a black hole at the surface of that star. To him everything appears like in an inertial frame except for the tidal forces.

Suppose this observer emits light signals at a constant rate to another one who rests at a large distance from this black hole in the making. If the exaggerated ideas of black holes would be correct such an observer should not exist at all for black holes are said to swallow all the matter in the Universe. In reality, however, the motion of any planets at large distances from a black hole (about 10 Schwarzschild radii will do) is independent of whether the central body is a black hole or a normal star of equal mass (Mtzner, 1980). Thus the remote observer may well exist.

Nevertheless, the signals sent out so regularly by the freely falling observer arrive at the distant one with progressive time delay and redshift. For the distant observer it would take an infinite amount of time that the co-moving observer reaches the event horizon if he did not disappear from view much earlier because of the reddening of light. The message is that when discussing relativistic problems one always has to specify the observer one is talking about.

In order to make this instructive example more transparent we take advantage of the light cone that was introduced in great detail in our chapter on special relativity. We emphasized already there that every spacetime event has its own light cone. In a spacetime diagram of gravitational collapse with the angular coordinates suppressed (Eddington-Finkelstein diagram), the *local* light cones, to which light rays are bound, demonstrate how complicated the spacetime geometry is that was responsible for the time delay of light signals.

Gravitational Waves. The award of the 1993 Nobel prize in physics to R. Hulse and J. Taylor for their discovery of the binary pulsar PSR 1913+16 makes a discussion of gravitational waves at secondary school timely.

For the first time general relativity has been turned from a theory yet to be confirmed by experiment into a tool of celestial mechanics in order to achieve what was not feasible on the basis of classical celestial mechanics: In consideration of the advance of the periastron and the special-relativistic as well as gravitational slow down of the pulsar "clock" near the periastron, Hulse and Taylor succeeded in determining the masses of the two components of the binary with so high an accuracy that this was the most precise mass determination of celestial bodies beyond the solar system.

What is even more, they could find out, relying on the high accuracy of the methods of radio astronomy, that the two compact celestial bodies, which are about 23 000 light years away from us, approach each other 3.5 meters per year! This is in good agreement with the theoretical prediction that the binary should lose energy of orbital motion by emission of gravitational waves the existence of which has been proved in this way, although indirectly.

What gravitational waves are and why it is so hard to detect them can be best explained at school by comparing and contrasting them with electromagnetic waves (Davies, 1980). With the intent to genetically combine gravitational waves with the concepts of classical physics the teacher is, however, forced to emphasize properties of electromagnetic waves which otherwise would hardly be mentioned at all. Frequently the very fact is taken for granted too carelessly that there are positive and negative electric charges such that the total electric charge of an aggregation of particles may be zero in contrast to its "gravitational charge" (gravitational mass), because there are no negative masses in Nature. Consequently, an oscillating doublet of positive and negative electric charges has an electric dipole moment but an oscillating doublet of masses does not have a mass dipole moment.

It is rarely mentioned that the electric field surrounding a charge is itself uncharged, but, on the other hand, there are no "uncharged" gravitational fields. They, too, are "charged" with mass (energy). Therefore, if an electric charge is being accelerated electrically, charge and field separate from each other. If, however, a mass is being accelerated gravitationally, at first sight the field falls together with the mass according to the equivalence principle. Does this mean that the equivalence principle totally prevents gravitational waves from being produced? No, for it is a local principle whereas the gravitational field extends to large distances. In the far-field regions the external, accelerating gravitational field is weaker than near the mass. Therefore the far field will fall more slowly than the near field, and it is again the tidal forces that try to pull the gravitational field from the mass.

This series of topics which should - in this case - be taught on classical electrodynamics could be continued. The forces exerted by electromagnetic waves on simple arrangements of test particles also belong to it.

Cosmology

Cosmology is particularly attractive from the point of view of natural philosophy. Therefore, it is useful not only for specific educational aims of physics. It interacts with almost all branches of physics and astronomy, but also with philosophy, intellectual history, and even religion. All that forces us to a selection of topics. Preferably we taught cosmology as part of relativity and astronomy. The interplay between cosmology and elementary-particle physics and thus the very early Universe played a comparatively modest part. Contemporary ideas on elementary particles and their interactions could not be acquired by the way, of course.

We begin with a detailed discussion about how cosmology is feasible at all and what is meant by the "Universe".

If we define (Harrison, 1981) that the *physical* Universe contains everything (including space and time!) which can be described by theories with predictive power and investigated by controlled observations and experiments and nothing else, we are well prepared to answer questions which the students have since the days of their childhood when they, long before any instruction, tried to develop their first ideas about the infinity of the Universe. Frequently these unanswered questions fell into oblivion. They must be laid open and confronted with the concepts of science, e.g. the above definition of "Universe". Pointing out the genesis of ideas can be helpful, for the questions of an individual like e.g. "Is there a centre of the Universe?" or "Is there an edge of the Universe and if so, what comes beyond that edge?" played an important part also in the history of scientific thought. It seems that the evolution from a child to an adult repeats the phylogenetic one. First of all, ahistoric answers to such questions would be much too abstract, for *"the most important and fundamental ideas of our science are being discussed in more detail only during their process of developement, afterwards they are more or less believed or handled as self-evident and without hesitation"* (d'Inverno, 1992; translated by the author).

Our definition of the Universe also implies ist uniqueness. Therefore, we cannot deal with it like with other physical objects which we investigate by experiment subjecting them to various conditions which depend on the questions posed. All we can do about cosmology is to *freely invent models* of the Universe and confront their predictions with astronomical observations. Once more observations and experiments can be the starting points of our instructions just as little as they are the only sources of knowledge.

The most simple cosmological models result from the cosmological principle according to which the Universe is, at a sufficiently large scale, homogeneous and isotropic in all its measurable quantities. Whereas isotropy can be verified by observation, homogeneity cannot. This is because - as we know from special relativity - gazing at remote galaxies does not provide information about what the Universe looks like from there *now*. Instead, we invoke another, rather plausible, principle in order to infer homogeneity, namely the Copernican idea that we do not live at a privileged place in the Universe.

In a discussion of these cosmological principles conducted as detailed as possible we can directly link up with the questions of the students. The most important and, from the point of view of teaching, most prolific among these questions is: "Why the sky is dark at night?" The darkness of the starry night sky (which has to be brought home to many people of this time) is the most elementary and most important observation that links us with the Universe at a cosmic scale. Moreover, this topic convincingly demonstrates how much history of science can help to comprehend modern physics (Lotze, in press). The student can understand still without relativity why the sky is dark at night if only he bears in mind how sparsely the stars are distributed and that they have a finite luminous lifetime. Among other topics the material for introductory astronomy was selected in order to prepare the student for just that.

Only now the *relativistic* cosmological models are being discussed. To begin with, we make any effort toward a correct explanation of the nature of the Hubble effect (Harrison, 1981; Lotze, 1994) which sometimes is erroneously interpreted as Doppler effect in the literature. However, one has to discriminate carefully among Doppler, gravitational, and expansion redshifts (fig. 7)! Otherwise an attempt is made to make the expansion redshift vivid at the price of its misrepresentation. But vividness is not sufficient for comprehension,

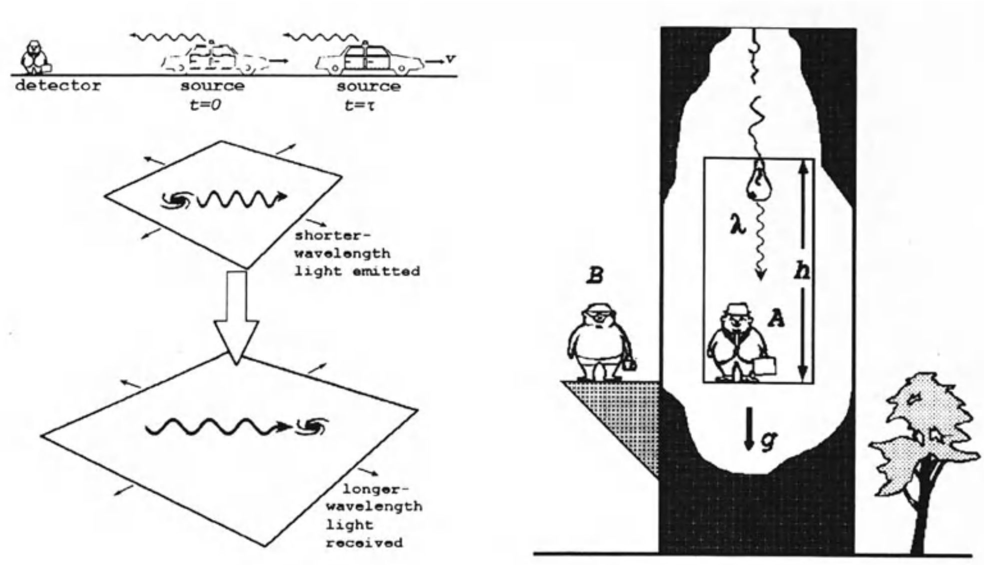


Figure 7. The three redshifts. For the (relativistic) Doppler effect of light (above) the shift of the wavelength depends on the relative velocity emitter and detector have just in that moment when light is being emitted. The expansion redshift of light (below) depends on what happens to the light waves when they are on their way between the galaxies in an expanding universe. Right: Gravitational shift of the wavelength of light (shown here as a blueshift as seen by the non-inertial observer B).

and the teacher should not force it. In our case, however, we are lucky: Standing waves in a "rubber-balloon universe" make clear the difference between expansion and Doppler redshift.

The rubber-balloon universe is the favourite analogy in teaching cosmology, with the aid of which the expanding universe is visualized by the two-dimensional surface of a rubber balloon. The use of this model cannot be criticized as long as one tries to live as a "flatlander" completely within the surface of the balloon thereby forgetting that a rubber balloon, when inflated, expands into the surrounding space. Because our Universe contains space and time and not vice versa we cannot, however, look at it from outside like at the balloon.

Now the Gaussian *theorema egregium* implies that this higher dimension is not necessary at all. Instead, local measurements *within* the surface space are completely sufficient to determine its geometry.

This makes possible a simplification of the mathematical formalism and gives rise to a more thorough treatment of the properties of triangles and circles at various curved two-dimensional surfaces. Even though the students had at hand the definition of a sphere as the place where all points have the same *distance* from a fixed point and even though they, of course, knew that one needs only *two* geographic coordinates in order to fix a point on a globe, the recognition of the sphere as a two-dimensional geometrical object was a problem from time to time!

From the plenty of relativistic cosmological models we select the Friedman models, of course. The cosmological principle they are based upon implies among other things that global properties can be represented locally (homogeneity and isotropy). Therefore, we can refer to Newtonian mechanics and study the collapse or expansion of a sphere of constant matter density as a substitute for the Universe as a whole. Instead of solving complicated differential equations we can refer to the qualitative methods of Newtonian dynamics (motion in an effective potential), and the three Friedman models correspond in perfect analogy to the ellipse, parabola, and hyperbola of classical celestial mechanics.

On the basis of data from astronomical observations we discuss how well suited the Friedman models are in order to describe the Universe we live in. Thereby unsolved problems like e.g. our present inability to discriminate among the three models and to make predictions about the future evolution of the Universe must not be concealed. The resolution of very distant galaxies into single stars by the Hubble Space Telescope, among them also "standard candles" for a more accurate determination of cosmic distances, is very likely to bring us closer to a solution of these problems.

On the other hand, the decision about which of the Friedman models describes our Universe best, is not at all a prerequisite to study its radiation-determined initial state ("big bang") which was essentially different from its present-day situation. Of the early history of the Universe we discuss merely the 2.7 K background radiation together with the synthesis of light elements during the first three minutes of cosmic evolution.

REPORT ON SUCCESSFUL COURSES

The table of contents of the course is given in the Appendix.

The course was offered first for student teachers at the University of Jena, Germany, as a two-hour, three-semester seminar. The first semester covered special relativity (A) and general relativity (B), the second one covered relativistic astrophysics (C) with emphasis on black holes, gravitational lenses and gravitational waves, and the third semester focussed on cosmology (D). Thus the time volume of the entire course was 90 hours.

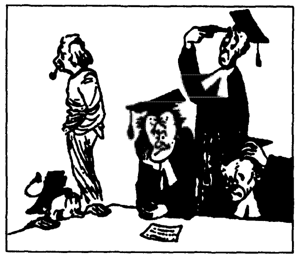
Special relativity has already become part of the physics curriculum at secondary schools, general relativity not yet. Fifteen of altogether twenty five hours were taught in accordance with part (A) of our course at two secondary schools in Thuringia and in numerous in-service training classes of teachers. The remaining ten hours were devoted to repetition of the material and discussion of homework exercises. We required each student to solve three problems per week and at the same time to answer various questions verbally because knowledge which is expressed in everyday language can be kept in mind for longer a time than formulas. The same applies to the one-hour written exam at the end of the course (fig. 8).

The chapters on general relativity, astrophysics, and cosmology have been taught to high ability pre-college students who were particularly motivated and above average. It is just these students who need an adequate intellectual demand that, as a rule, cannot be offered at school. Therefore, summer academies have been launched for high-school students in order just to meet this demand. Students who are considered qualified like e.g. successful participants at competitions in mathematics or foreign languages work together for two weeks during their summer holidays under the supervision of instructors from schools and universities on various subjects which complete and transcend the curriculum. The intellectual communication among participants of various courses as well as attractive recreational activities are favourable to the developement of the students. Summer school classes attract students from all over the country who are interested in academic challenges.

Our first summer academy was conducted in 1993 at St. Peter-Ording, North Sea. Together with fifteen students we discussed parts A, B, and D (cosmology) of our course within 45 hours. The second academy was held in 1994 at Schloß Spetzgart, Lake Constance. There we taught parts A, B, and C (relativistic astrophysics). It was accompanied by astronomical observations using amateur telescopes (another 15 hours), which had only loose contact with the main topics, of course.

EXERCISES ON SPECIAL RELATIVITY

(Short test)

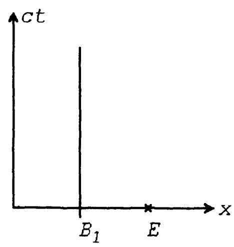


Problems:

1. An astronaut at the age of 25 sets out for a space trip at a speed $v = \frac{12}{13}c$. Upon his return his twin is 69 years old. What's the age of the astronaut?

2. On a train moving with the speed u, a tramp turns on a flashlight and shines in the direction *opposite* to the direction of motion. Using the composition law of velocities, find the speed of light relative to the railway platform.

3. Starting with the event E, draw the world line of an observer B_2 who moves first in the positive x direction with constant speed, then slows down, and comes to rest at a certain distance from B_1!



4. The radiation output of the sun is $4 \cdot 10^{26}$ W. Find out how much mass the sun loses each second due to radiation.

Fragen:

1. Under what circumstances two events are simultaneous at different locations?

2. Why the time delation is not a particular feature of light clocks, even though it has been derived for them only?

3. What does the equivalence of mass and energy mean?

Figure 8

The student's ability to grasp the matters immediately was convincingly demonstrated by their numerous far-sighted questions which anticipated the next step. Such questions very much strengthened the self-confidence of the students who had an opportunity to be truly active learners.

We required each student to write a paper on a special topic chosen from the table of contents of the course, which was published as part of a documentation of the academy edited by the Verein Bildung und Begabung, Bonn. The basis of this paper was, as a rule, one or two articles from Scientific American which fortunately have not all been available in german translation, or other material at this level. The articles should be selected with particular care, for many students are not experienced with sources of scientific information which go beyond popular science.

In order that the written papers be more than mere excerpts of the articles studied it is of particular importance that each student gives an oral presentation of his paper. This requires *to speak about physics* in such a way that the fellow students can follow, to overcome initial shyness and to deepen the understanding of the subject.

To conclude, one can say that our course could be taught to these students without loss of content at the freshman-level of the first two university semesters. The experience of taking similar courses at the university was anticipated as closely as possible. More extensive experiences as with special relativity have, however, yet to be made.

It is hoped that the suggestions made in this paper will stimulate further attempts and efforts which are aimed at modernization of physics teaching.

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APPENDIX: COURSE CONTENTS

Relativity, Relativistic Astrophysics and Cosmology

A. SPECIAL RELATIVITY

1. Classical mechanics and its Galilean principle of relativity

1.1. Inertial frames
1.2. Propagation of sound and propagation of light

2. The fundamental principles of special relativity and its kinematic consequences

2.1. The two fundamental principles of special relativity
2.2. Relativity of simultaneity
2.3. Spacetime diagrams: events and world lines
2.4. Time dilation
2.5. The relativistic Doppler effect and Bondi's k factor. The composition law of velocities
2.6. The Lorentz transformation

3. Minkowski's four-dimensional spacetime

3.1. A graphic representation of the Lorentz transformation: light cone and particle horizon
3.2. The proper-time interval
3.3. Geodesics in Minkowski space: The twin paradox

4. Relativistic dynamics: The inertia of energy

B. GENERAL RELATIVITY

1. The principle of equivalence

1.1. The concept of mass in Newtonian mechanics
1.2. Relativity of free fall. The principle of equivalence
1.3. Gravitational redshift
1.4. Light deflection by a gravitational field and the need for non-Euclidean geometry
1.5. Inhomogeneous gravitational fields and local inertial frames
1.6. Tides

2. Gravity and curvature of space

2.1. Curvilinear coordinates but flat spaces
2.2. Gauss' "theorema egregium"
2.3. Curvature of space near a point mass
2.4. Geodesics. Advance of the perihelon of Mercury

C. RELATIVISTIC ASTROPHYSICS

1. Gravitational lenses

1.1. Bending of light by the sun: historical and theoretical aspects
1.2. The gravitational-lens effect: Representative examples and simulation by a glass lens
1.3. The night sky as seen from a black hole
1.4. The gravitational-lens equation

2. Black holes

2.1. Stars: their characteristics and evolution
2.2. A journey through the Hertzsprung-Russell diagram
2.3. The nature of pulsars and how they were discovered
2.4. Schwarzschild black holes and gravitational collapse. The Eddington-Finkelstein diagram
2.5. Rotating black holes. The Penrose process
2.6. Black holes in the centres of galaxies

3. The binary pulsar PSR 1913+16 and gravitational waves

3.1. Binary stars: their masses and radial velocities
3.2. Nobel prize for astronomers: A binary star emits gravitational waves
3.3. Electromagnetic waves and gravitational waves compared and contrasted. Consequences of the equivalence principle

D. COSMOLOGY

1. On the history of cosmological ideas

1.1. What cosmology is about. The cosmological principle
1.2. Is there a centre of the Universe?
1.3. Is there an edge of the Universe?

2. Astronomical observations relevant to cosmology

2.1. Why the sky is dark at night? - An old paradox re-examined by modern science
2.2. The Andromeda galaxy and the Milky Way galaxy - cosmic twins?
2.3. The cosmic distance scale and our cosmic frame of reference
2.4. Dark matter: Are galaxies rotating too fast?
2.5. Searching for dark matter with gravitational lenses

3. The expansion redshift and the nature of the Hubble effect

4. The Friedman models of the Universe

4.1. The Friedman equation: 1. How good is the cosmological model of Newtonian dynamics?
4.2. The Friedman equation: 2. Motion in an effective potential
4.3. Universes made of "dust" and radiation
4.4. Will the Universe expand forever? The age of the Universe and the event horizon

5. The standard model of the hot big bang

5.1. The 2.7 K background radiation
5.2. The synthesis of light elements during the first three minutes

6. Special topics

6.1. Dirac's large numbers - nothing more than a caprice of the Universe?
6.2. The Anthropic principle - does the Universe need man?

E. SUMMARY: Astronomical results of the Hubble Space Telescope