Finite-size scaling, intermittency and the QCD critical point

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Summary of previous presentation: Basic ideas

Critical region (point) $\Rightarrow$ **scaling** in **order parameter fluctuations**

$\swarrow$ $\searrow$

σ-condensate net baryon density

Experimental access $\Rightarrow$ **Critical opalescence**

$\Downarrow$

fractality in real space $\longrightarrow$ **power-law correlations** in momentum space

Transfer to QCD CEP case:

Finite size scaling $\Rightarrow$ power-law correlations in momentum space (small momentum differences)

$\Downarrow$

Intermittency in proton transverse momentum space $\Rightarrow$ Measurement of **isothermal critical exponent** δ: $F_2(M) \sim M^{2\phi_2}$; $\phi_2 = \frac{\delta}{\delta+1} \approx \frac{5}{6}$

Summary of previous presentation: Basic ideas

- Size of the critical region $\Rightarrow$ Use Ising-QCD partition function constructed based on 3d-Ising effective action:

$$\mathcal{Z}_{IQCD} = \sum_{N=0}^{\Lambda} \zeta^N \exp\left[-\frac{1}{2}\hat{m}^2 \frac{N^2}{\Lambda} - g_4 \hat{m} \frac{N^4}{\Lambda^3} - g_6 \frac{N^6}{\Lambda^5}\right]$$

with $\zeta = \exp[(\mu_B - \mu_c)\beta_c]$, $\hat{m} = \beta_c \xi^{-1}$, $\xi \sim |t|^{-2/3}$, $t = \frac{T-T_c}{T_c}$, $N =$ proton number and $\Lambda = \dfrac{V}{V_0}$ ($V_0 =$ proton volume).

- Calculate proton multiplicity moments $\langle N^k \rangle$!
- Obtain FSS laws:

$$\langle N^k \rangle \sim \Lambda^{kq}, \qquad q = \frac{5}{6}, \qquad k = 1, 2, ..$$

- **Deviations** can be used to estimate the **size** of **critical region**

Summary of previous presentation: Main questions

1. Q: Why and how scaling in configuration space is transformed in power-law correlations in momentum space?

   A: The FSS property leads to a density-density correlation function (diagonal element of the two particle density matrix averaged over the ensemble) in **r**-space which possess a power-law decay for large distances. The Fourier transform allows us to represent the two particle density matrix in momentum space. The power-law decay in **r**-space leads to power-law singularity of the density-density correlation function in **p**-space. The decay exponent in **r**-space and the singularity exponent in **p**-space are related to each other. In QCD case the Fourier transform refers to transverse space (free streaming era $\Rightarrow$ rapidity is the same in both spaces) leading to:

$\tilde{d}_{F,\perp} = 2 - d_{F,\perp} \approx \frac{1}{3}$ (3d-Ising); N. Antoniou, *et al.*, Phys. Rev. C 93, 014908 (2016)

Summary of previous presentation: Main questions

2. Q: What is the role of non-measurable neutrons?

   A: Due to isospin symmetry the correlations of neutrons are identical to those of protons. However, as mentioned in the question, they are not measurable.

   See Y. Hatta and M. A. Stephanov, PRL 91, 102003 (2003) for more details.

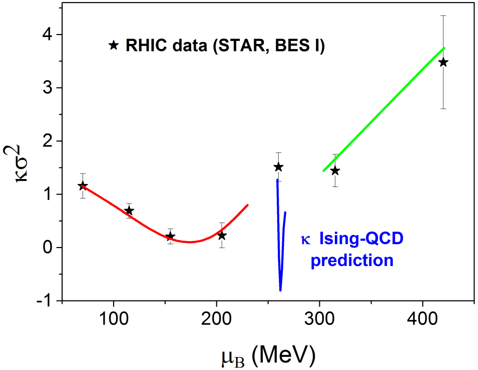
3. Q: Why can the correlation analysis be restricted only to protons?

   A: We are interested of the singular part of the density-density correlation function in momentum space. This part is the same for protons, antiprotons and neutrons (see reference above). We use protons because they are measurable and have higher statistics.

Summary of previous presentation: Main questions

4. Q: How do compare predictions for the behaviour of kurtosis in Ising-QCD with RHIC-BES I results?

   A: The Ising-QCD critical region is very narrow! Furthermore, using the Ising-QCD partition function the kurtosis turns out to attain a sharp negative minimum (See N. G. Antoniou, *et al.*, arXiv:1711.10315 [nucl-th]):



Summary of previous presentation: Main questions

5. Q: What is the correspondence with the work by Gorenstein *et al.* where they see CP in van der Waals model?

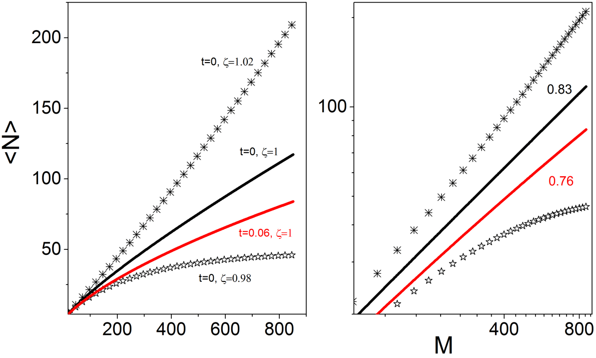
   A: Van der Waals model possesses a critical point in the mean field universality class (not 3d Ising). The results of RHIC can be described by a ϕ^4 Landau free-energy (mean field description) away from the critical point (see N. G. Antoniou, F. K. D., N. Kalntis and A. Kanargias, arXiv:1711.10315 [nucl-th]).

6. Q: Critical fluctuations in other conserved quantities (charge, strangeness, ..)?

   A: Connection to order parameter (σ-field) needed!

Size of the critical region

Departing from the critical point $\Rightarrow$ Gradual destruction of the FSS law:
$(\zeta = 1, t = 0)$ $\langle N \rangle \sim \Lambda^{\frac{5}{6}}$



Notation: $\zeta = \exp[(\mu_B - \mu_c)\beta_c]$, $t = \frac{T - T_c}{T_c}$ $(\alpha = 0)$

Size of the critical region (continued)

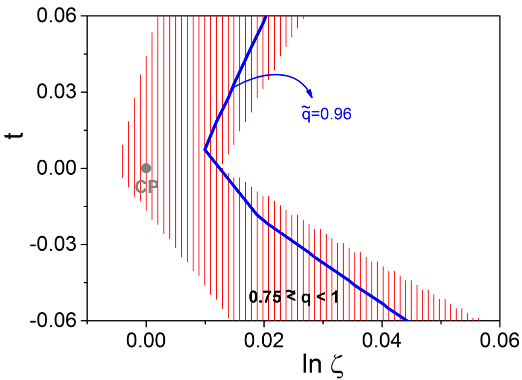
In a region around $\zeta = 1$, $t = 0$ (CP) it holds:

$$\langle N \rangle \sim \Lambda^{\tilde{q}}$$

- $\tilde{q} = \frac{3}{4} \Rightarrow$ scaling (q) in mean field theory
- $\tilde{q} = 1 \Rightarrow$ trivial scaling

Critical region: region in $(\ln \zeta, t)$-plane for which $\frac{3}{4} < \tilde{q} < 1$

Size of the critical region (first result)



Critical region $\Delta\mu_B \approx 5\ MeV$

(for $T_c \approx 160\ MeV$)

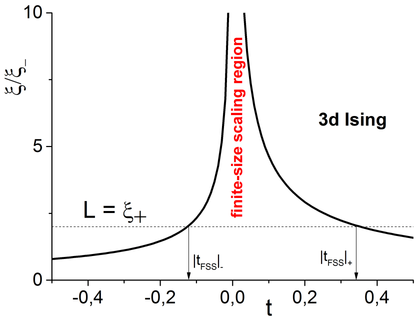
$\Downarrow$

Very **narrow** along μ_B-axis

N.G. Antoniou, F.K. D., X.N. Maintas, C.E. Tsagkarakis, PRD 97, 034015 (2018)

Finite size scaling region

FSS condition: $\xi_\infty > V^{1/3}$



Bounds along the t axis!

System dependent!

For **medium** $(20 < A < 50)$ size nucleii,

FSS region:

$3\ MeV < \Delta T < 5\ MeV$

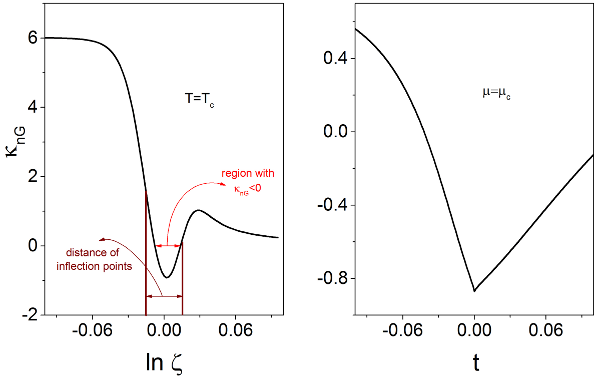
(for $T_c \approx 160\ MeV$)

$\Downarrow$

Narrowness along T-axis too

N.G. Antoniou, F.K. D., arXiv:1802.05857 [hep-ph]

Kurtosis within the critical region



Negative sharp minimum

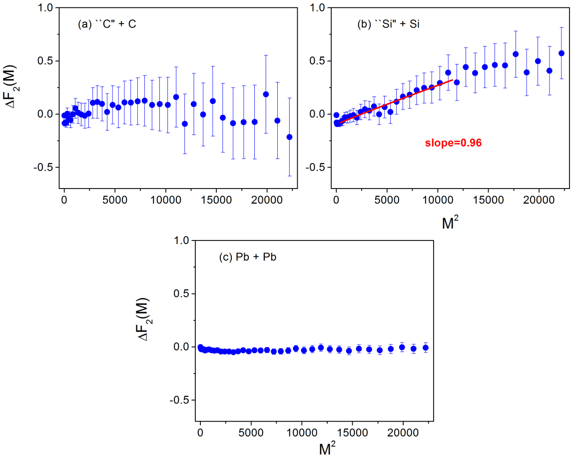
of κ_{nG} at the CP

N.G. Antoniou, F.K. D., N. Kalntis, A. Kanargias

arXiv:1711.10315 [nucl-th]

Alternative(s) for the critical region size

N.G. Antoniou, F.K. D., arXiv:1802.05857 [hep-ph]



Si + Si central collisions at $\sqrt{s} = 17.2$ GeV create a final state **within the critical region:** $\tilde{q} \approx 0.96$!

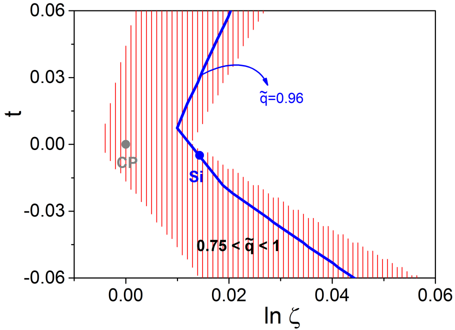
$\Downarrow$

Use this result to locate CEP!

Ignore **large** statistical errors...

Typical range for spatial correlations $[1, 8]$ *fm*
$\Rightarrow$ tr. mom. space $M^2 \in [400, 11000]$

Locating Si + Si final state within the critical region



Line $\tilde{q} = 0.96$ determines μ_c for known T_c (Lattice QCD)

recent result: $T_c = 163\ MeV$

S. Datta et al, PRD 95, 054512 (2017)

Freeze-out of Si*:

$$(\mu_{Si}, T_{Si}) = (260, 162.2)\ MeV$$

$$T_c = 163\ MeV$$
$$\Downarrow$$

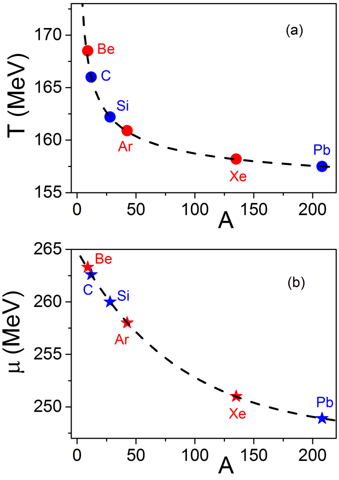
$$\ln \zeta_{Si} = 0.0143 \Rightarrow$$

$$\mu_c = 257.7\ MeV$$

*: *F. Becattini, J. Manninen and M. Gazdzicki, PRC 73, 044905 (2006)*

Predictions for NA61/SHINE freeze-out states

Freeze-out conditions for Ar+Sc and Xe+La $\Rightarrow$ use NA49 results



$$\sqrt{s} = 17.2\ GeV$$

Freeze-out of central Ar+Sc:

$$(\mu_{ArSc}, T_{ArSc}) = (258, 160.9)\ MeV$$

Freeze-out of central Xe+La:

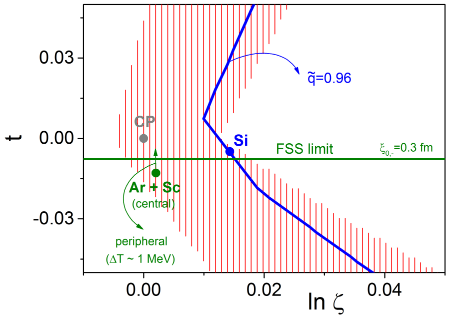
$$(\mu_{XeLa}, T_{XeLa}) = (251, 158.2)\ MeV$$

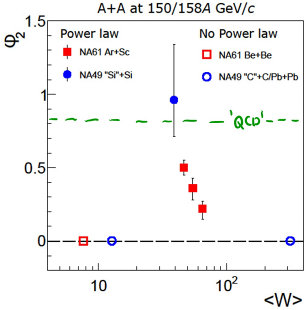
N.G. Antoniou, F.K. D., arXiv:1802.05857 [hep-ph]

Enriched sketch of the critical region

M. Gazdzicki, indico.cern.ch;

N. Davis, CPOD 2018 talk





F. Becattini, et al, PRC 90, 054907 (2014);

N.G. Antoniou, F.K. D., arXiv:1802.05857

[hep-ph]

Conclusions

- **Critical (FSS) region** is **very narrow** $O(5\ MeV)$ along the μ_B and the T axis.
- Beam energy scan program at RHIC with $\Delta\mu_B \approx 50\ MeV$ is very unlike to approach the critical region.
- **Important NA49 result: freeze-out state of central Si+Si collisions at $\sqrt{s} = 17.2\ GeV$ lies within the critical (FSS) region!** (needs accurate measurements to reduce statistical errors)

$$\Downarrow$$

Can be used as a **guide** for detecting the QCD CEP.

- **Basic strategy: Accurate measurements of FSS exponent $\tilde{q}$ (intermittency analysis) and corresponding freeze-out parameters (μ_B, T) in A+A collisions with $25 < \mathrm{A} < 50$.**

Conclusions (continued)

- $\sqrt{s} \approx 17\ GeV$ seems to be the **appropriate beam energy** for approaching μ_c. **Peripheral collisions** can be used for **fine changes in** T allowing the entrance into the FSS region.

  For A+A collisions at $\sqrt{s} = 17.2\ GeV$ we propose:

  Accurate measurements of $(\tilde{q}, \mu_B, T)$ in central collisions for $25 < \mathrm{A} < 32$.

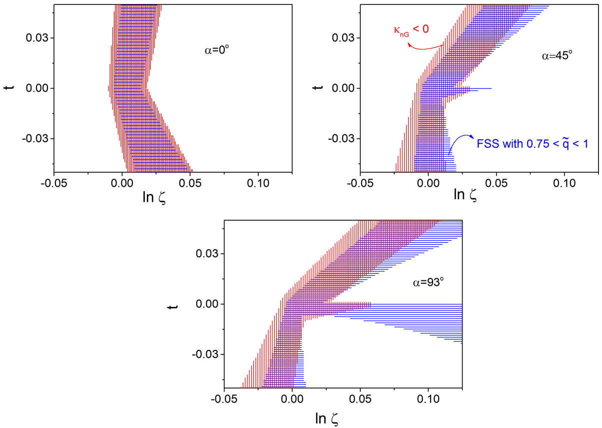
  Accurate measurements of $(\tilde{q}, \mu_B, T)$ in peripheral collisions for $32 < \mathrm{A} < 50$.

  Prediction: Strong intermittency effect in peripheral Ar+Sc collisions at $\sqrt{s} \approx 17\ GeV$ (NA61/SHINE experiment).

  (See *N.G. Antoniou, F.K. D., arXiv:1802.05857 [hep-ph]*)

Thank you!

Critical region size: κ_{nG} vs. FSS varying α



Critical region size along μ_B is 3 $MeV \leq \Delta\mu_B \leq 11 MeV$ for all α!

N.G. Antoniou, F.K. D., arXiv:1802.05857 [hep-ph]

Enriched sketch of the critical region for $\alpha \neq 0$