SAPIENZA
UNIVERSITÀ DI ROMA

DIPARTIMENTO DI SCIENZE DI BASE
E APPLICATE PER L'INGEGNERIA

INFN

ARIES

Review of impedance-induced instabilities and their possible mitigation techniques

M. Migliorati, E. Métral, M. Zobov

Acknowledgements:

N. Biancacci, D. Quartullo, B. Salvant

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Research and Innovation programme under Grant Agreement No 730871.

Agenda

- Acknowledgements
- What do we mean by impedance-induced instabilities and some useful definitions
- Tools for studying impedance-induced instabilities
  - Vlasov (and Fokker Planck) equation
  - Few-particles models, simulation codes
- Instabilities in circular machines (and mitigations techniques)
  - mode coupling, microwave instability, transverse head tail, coupled bunch
- Instabilities in LINACS (and mitigation techniques):
  - Beam break-up
- What I had no time to mention and conclusions

Acknowledgements

- First studies of impedance-induced instabilities were developed in the early 70s, and even before, with the initial concepts of dispersion relations and coupling impedance

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ARCHIVES CERN

ISR-RF/66-35
November 18, 1966

LONGITUDINAL INSTABILITY OF A COASTING BEAM ABOVE TRANSITION, DUE TO THE ACTION OF LUMPED DISCONTINUITIES.

by V.G. Vaccaro

1. Generalities

We assume that the electrical action on an ion beam, of a discontinuity in a tank is that of an impedance. We still consider the case in which this discontinuity is sufficiently small compared with the wavelength of the perturbation, to be considered as concentrated.

where a is the beam radius. The passage of an ion beam induces a field in the discontinuity, which is given by:

$$E_d = -Z\ I/d, \qquad (4)$$

where d is the magnitude of the discontinuity, and Z is the impedance of the discontinuity.

CERN 67-2
ISR-Division
February 6, 1967

LONGITUDINAL INSTABILITIES OF AZIMUTHALLY UNIFORM BEAMS IN CIRCULAR VACUUM CHAMBERS WITH WALLS OF ARBITRARY ELECTRICAL PROPERTIES

by

A. Sessler and V. Vaccaro

The Xie Jialin Prize for outstanding work in the accelerator field, with no age limit.

Prof. Vittorio Giorgio VACCARO

'For his pioneering studies of instabilities in particle beam physics, the introduction of the impedance concept in storage rings and, in the course of his academic career, for disseminating knowledge in accelerator physics throughout many generations of young scientists."

IPAC 2019

Acknowledgements

- First studies of impedance-induced instabilities were developed in the early 70s, and even before, with the initial concepts of dispersion relations and coupling impedance

- Some influential people who made the story of this important, intriguing and always in fashion topic of particle accelerators are:
  - A. Chao, C. Pellegrini, A. M. Sessler, V. Vaccaro, F. Sacherer, J. L. Laclare, B. Zotter, K. Yokoya, Y. Chin, J. Haissinski, A. Hoffmann, V. K. Neil, L. J. Laslett, M. Sands, E. D. Courant, …

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… and, of course, also many colleagues participating to this workshop!

Acknowledgements

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  - …

A short phrase to summarize the work that has been done could be:

"Particle Accelerators Work and are Successful"

Are we just lucky? After 50 years it couldn't be only a coincidence …

What are impedance-induced instabilities and some useful definitions

When a beam of charged particles traverses a device which

- is not a perfect conductor
- or is not smooth

it produces electromagnetic fields that perturb the following particles

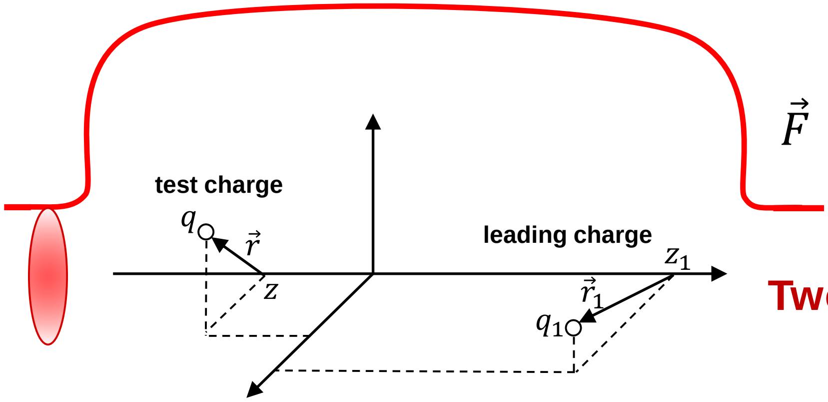
Are these fields an issue?

Differently from those of magnets and RF cavities, these fields depend on beam intensity and their amplitude cannot be easily changed.

Under some conditions they can induce instabilities

These fields are described in time domain → wakefields, or in frequency domain → impedance

What are impedance-induced instabilities and some useful definitions



$$\vec{F} = q\left[E_z\hat{z} + \left(E_x - vB_y\right)\hat{x} + \left(E_y + vB_x\right)\hat{y}\right] \equiv \vec{F}_\parallel + \vec{F}_\perp$$

Two approximations:
- rigid beam
- impulse approximation

Longitudinally: **(energy change)**

$$U(\Delta z) = \int_0^L F_\parallel ds \Rightarrow w_\parallel(\Delta z) = -\frac{U(\Delta z)}{qq_1}$$

Longitudinal wake function

Transversally: **(momentum kick)**

$$\vec{M}(\vec{r}_1, \Delta z) = \int_0^L \vec{F}_\perp ds \Rightarrow \vec{w}_\perp(\Delta z) = \frac{1}{r_1}\frac{\vec{M}(\vec{r}_1, \Delta z)}{qq_1}$$

transverse dipole wake function

$$Z_\parallel(\omega) = \frac{1}{v}\int_{-\infty}^{\infty} w_\parallel(z) e^{i\omega\frac{z}{v}} dz$$

Coupling impedance

$$Z_\perp(\omega) = \frac{i}{\beta v}\int_{-\infty}^{\infty} w_\perp(z) e^{i\omega\frac{z}{v}} dz$$

Tools for studying impedance-induced instabilities

- The basic idea is quite simple: take the motion of a single particle and include the force (wakefield) due to all the others
- Instead of considering $10^{10} - 10^{12}$ equations of motion we generally

NB:

- Go to the extreme by considering a continuous function (distribution function) describing the motion of the beam as superposition of modes. In this treatment the force is also described in terms of the distribution function → Vlasov or Fokker Planck equation

- Simplify the problem by reducing the number of equations: instead of $10^{10} - 10^{12}$ particles, we use macroparticles and simulation codes which track, turn after turn, about $10^6 - 10^7$ macroparticles, taking into account their em interactions by using the concept of wakefields.

- Simple models, as two-particles. These few-particles models allow to understand many physical aspects with quite manageable expressions.

Vlasov (and Fokker Planck) equation

A. Chao

- The **Vlasov** equation describes the collective behaviour of a multiparticle system under the influence of electromagnetic forces.
- It is valid if we can ignore diffusion or external damping effects, for example for proton beams.
- For electron beams, synchrotron radiation cannot be neglected and we have another equation called **Fokker-Planck** equation

Vlasov
(constant local particle density in phase space)

$$0=\boxed{\boxed{\frac{\partial\psi}{\partial t}+\frac{\partial\psi}{\partial q}\frac{\partial H}{\partial p}-\frac{\partial\psi}{\partial p}\frac{\partial H}{\partial q}}= A\frac{\partial}{\partial p}(\psi p)+\frac{D}{2}\frac{\partial^2\psi}{\partial p^2}}$$

Fokker-Planck

q and p: set of canonical variables (e.g. time and energy offset)

A and D are related, respectively, to the damping and diffusion coefficients

H is the Hamiltonian of the system

NB: for transverse plane we need to consider 4D phase space

Simulation codes

- Single particle equations of motion:

$$\Delta\varepsilon_i = \frac{qV_{RF}(\sin\varphi_i - \sin\varphi_s) - qV_{wf}(\varphi_i) + R(T_0)}{E_s} - 2\frac{T_0}{\tau_s}\varepsilon_i$$

$$\Delta(\varphi_i - \varphi_s) = -\frac{2\pi h\eta}{\beta^2}\varepsilon_i$$

Simulation codes

- Single particle equations of motion:

induced wakefield voltage

synchronous phase

normalized energy difference with respect to the synchronous particle

one turn variation

$$\Delta\varepsilon_i = \frac{qV_{RF}(\sin\varphi_i - \sin\varphi_s) - qV_{wf}(\varphi_i) + R(T_0)}{E_s} - 2\frac{T_0}{\tau_s}\varepsilon_i$$

longitudinal damping time

RF peak voltage

phase with respect to RF

$$\Delta(\varphi_i - \varphi_s) = -\frac{2\pi h\eta}{\beta^2}\varepsilon_i$$

stochastic variable changing each turn and taking into account the fact that the electromagnetic radiation occurs in quanta of discrete energy

harmonic number

slippage factor

Simulation codes

- Single particle equations of motion:

$$\Delta\varepsilon_i = \frac{qV_{RF}(\sin\varphi_i - \sin\varphi_s) - qV_{wf}(\varphi_i) + R(T_0)}{E_s} - 2\frac{T_0}{\tau_s}\varepsilon_i$$

$$\Delta(\varphi_i - \varphi_s) = -\frac{2\pi h\eta}{\beta^2}\varepsilon_i$$

$V_{wf}(\varphi_i)$ is the term coupling the equations of all the particles:

$$V_{wf}(\varphi_i) = \frac{Q_{tot}}{N_m}\sum_{j=1}^{N_m} w_{\parallel}(\varphi_i - \varphi_j)$$

to be evaluated at each turn for each of N_m macroparticles → $(N_m - 1)N_m/2$ operations for each turn

In order to reduce the computing time and only in the evaluation of the wakefield effects, the bunch is generally divided into N_s slices:

number of macroparticles of the slice j

$$V_{wf}(\varphi_{i\Delta}) = \frac{Q_{tot}}{N_m}\sum_{j=1}^{N_s} n_j(\Delta) w_{\parallel}(\varphi_{i\Delta} - \varphi_{j\Delta})$$

centre of the slice i

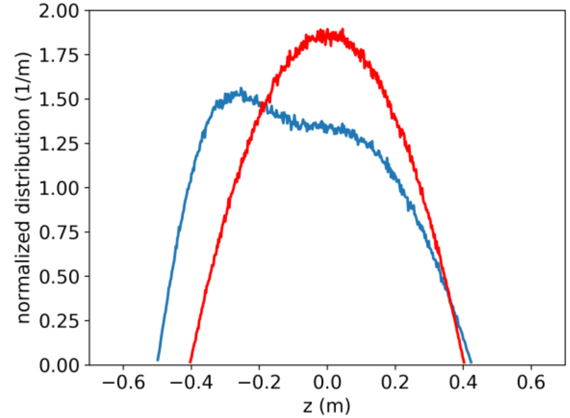
width of the slice i

Stationary solution: $\frac{\partial \psi_0}{\partial t} = 0$

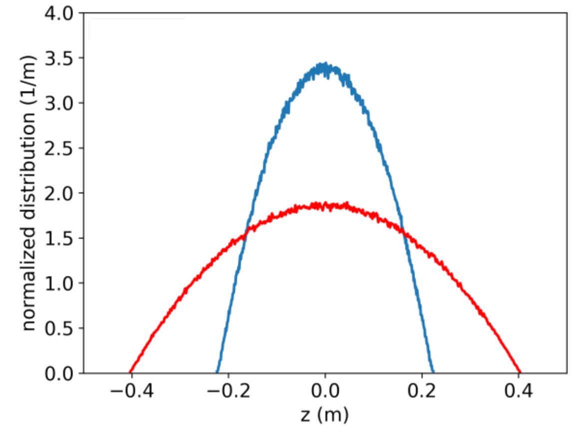
proton beams: $$\frac{\partial \psi_0}{\partial q}\frac{\partial H_0}{\partial p} - \frac{\partial \psi_0}{\partial p}\frac{\partial H_0}{\partial q} = 0 \rightarrow$$
$$\psi_0(q,p) = \text{any function of } H_0(q,p)$$

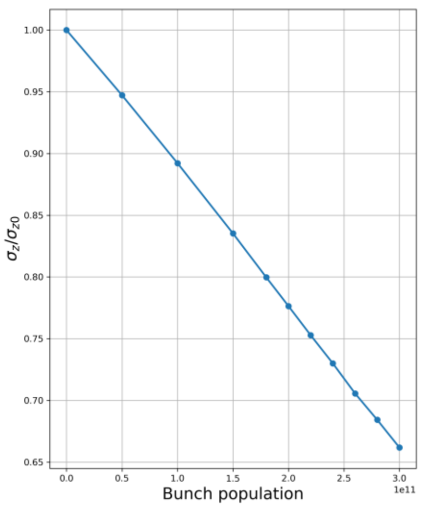
The Hamiltonian H_0 contains the wakefield → the distribution function depends on wakes (potential well distortion)

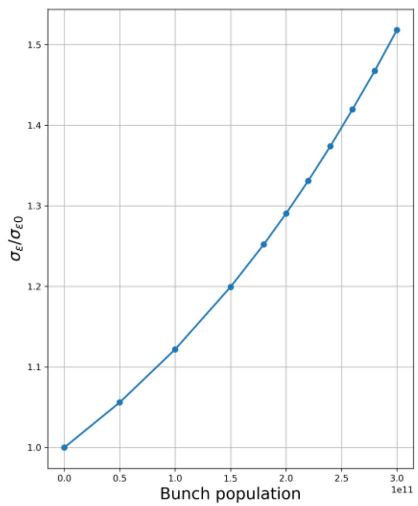
BBR impedance affect the symmetry of the parabolic potential well

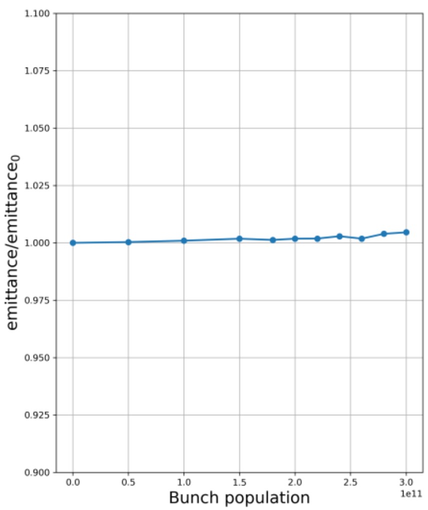


pure reactive impedance maintains the symmetry of the parabolic potential well









Stationary solution: $\frac{\partial \psi_0}{\partial t} = 0$

electron beams: $\frac{\partial \psi_0}{\partial q}\frac{\partial H_0}{\partial p} - \frac{\partial \psi_0}{\partial p}\frac{\partial H_0}{\partial q} = A\frac{\partial}{\partial p}(\psi_0 p) + \frac{D}{2}\frac{\partial^2 \psi_0}{\partial p^2} \rightarrow$ Haissinski equation

'potential'

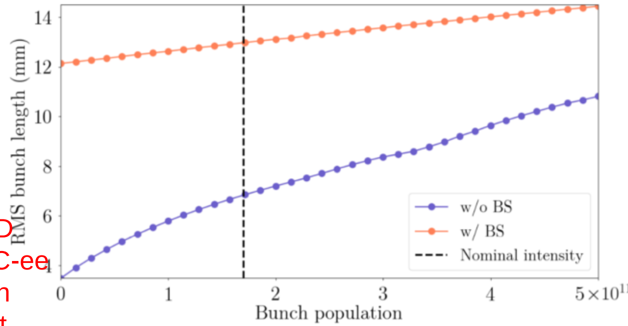
wakefields

$$\varphi(z) = \frac{1}{L_0}\int_0^z [eV_{RF}(z') - U_0]dz' - \frac{eN^2}{L_0}\int_0^z dz' \int_{-\infty}^{\infty} \lambda_0(z'')\, w_{\parallel}(z''-z')dz''$$

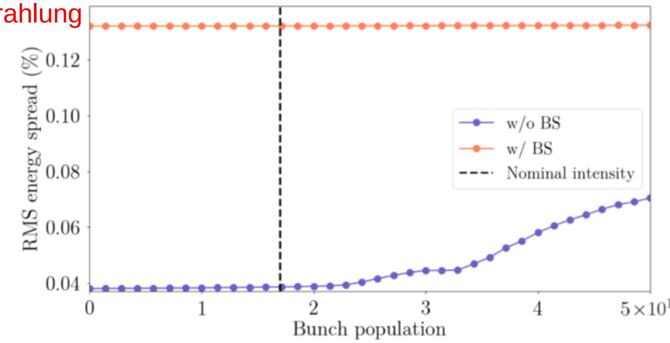
RF voltage

$$\lambda_0(z) = \int_{-\infty}^{\infty} \psi_0(\varepsilon, z)d\varepsilon = \bar{\lambda}\exp[-C\varphi(z)]$$

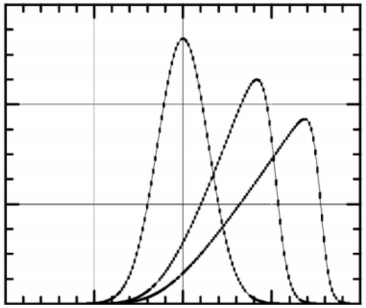
Due to random quantum radiation and fluctuations the stationary distribution is always Gaussian in ε



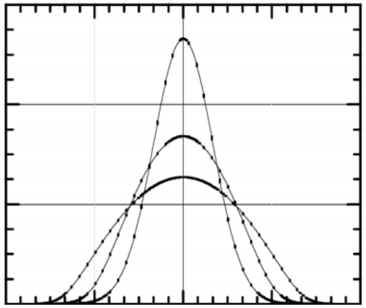
E. Belli PhD thesis: FCC-ee collider with and without beamstrahlung



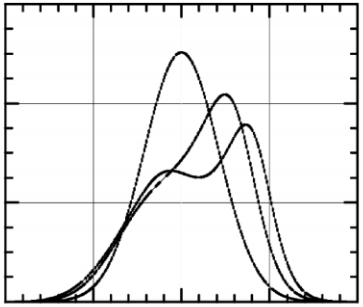
Pure resistive impedance



Pure inductive impedance

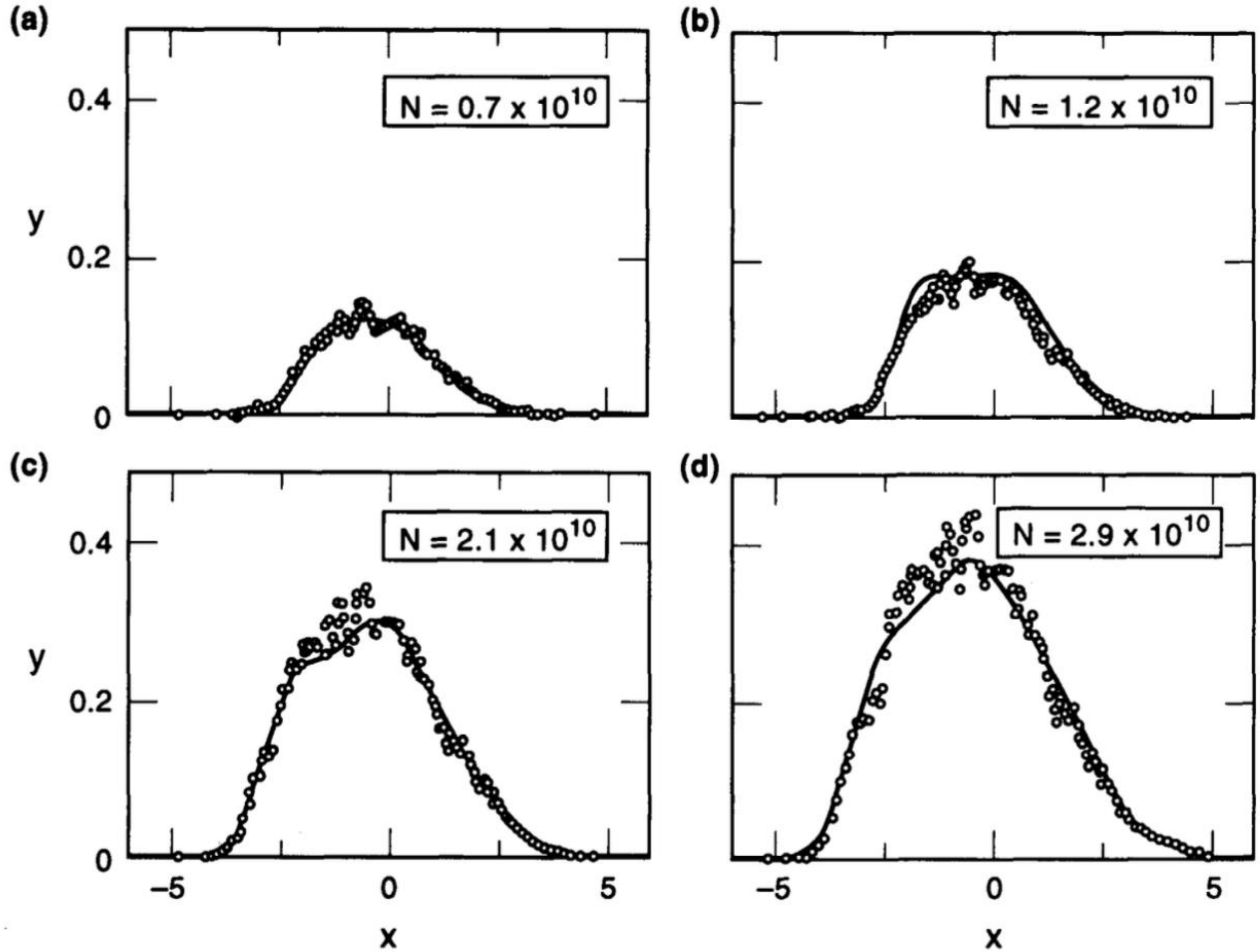


Broad band resonator



Stationary solution: $\frac{\partial \psi_0}{\partial t} = 0$

Famous picture: potential-well distortion of bunch shape for various beam intensities for the SLC damping ring. The open circles are the measured results. The horizontal axis is $x = -z/\sigma_{z0}$ where σ_{z0} is the unperturbed rms bunch length. (Courtesy Karl Bane, 1992)



Linearization of Vlasov equation and perturbation theory

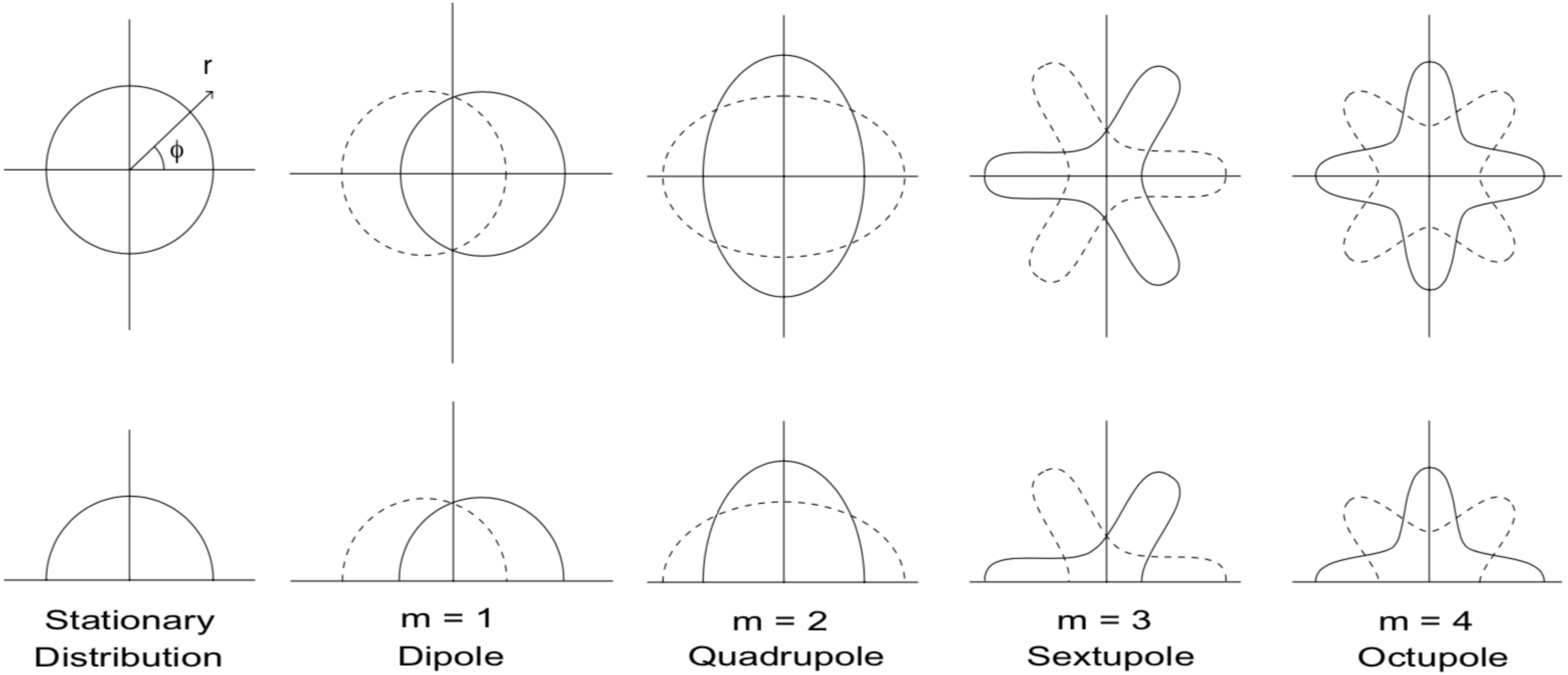
Very interesting is the case when the beam is unstable and phase space distribution depends on time. The steps to study the Vlasov equation in case of instability are generally the following:

1. Use a perturbation method: $\psi(q,p;t) = \psi_0(q,p) + \Delta\psi\,(q,p;t)$
2. Use action-angle coordinates (I,ϕ), and consider the disturbance as sum of azimuthal (m) coherent modes oscillating with a coherent frequency Ω to be found:

$$\Delta\psi(I,\phi;t) = \sum_{m=-\infty}^{\infty} R_m(I) e^{im\phi} e^{-i\Omega t}$$

3. The instability is produced by the wakefields excited by the perturbation and not by the stationary distribution
4. From the Valsov equation, the so-called Sacherer integral equation is obtained. Multi-bunch case can be treated in a similar way.

Linearization of Vlasov equation and perturbation theory



PHYSICS OF INTENSITY DEPENDENT BEAM INSTABILITIES

K. Y. Ng

Linearization of Vlasov equation and perturbation theory

5. There are several methods to get the solution (J. L. Laclare, bunched beam coherent instabilities):
   - Sacherer's approach, G. Besnier expansion in orthogonal polynomials, J. L. Laclare's eigenvalue problem, J. Whang and C. Pellegrini approach.
   - Each azimuthal mode $R_m(I)$ is expanded in terms of a set of orthonormal functions with a proper weight function which depends on (the derivative of) the stationary distribution:

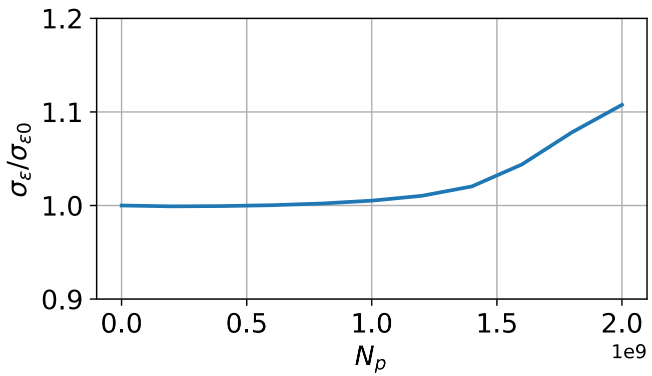
$$R_m(I) = W(I) \sum_{k=0}^{\infty} \alpha_{mk}\, g_{mk}(I)$$

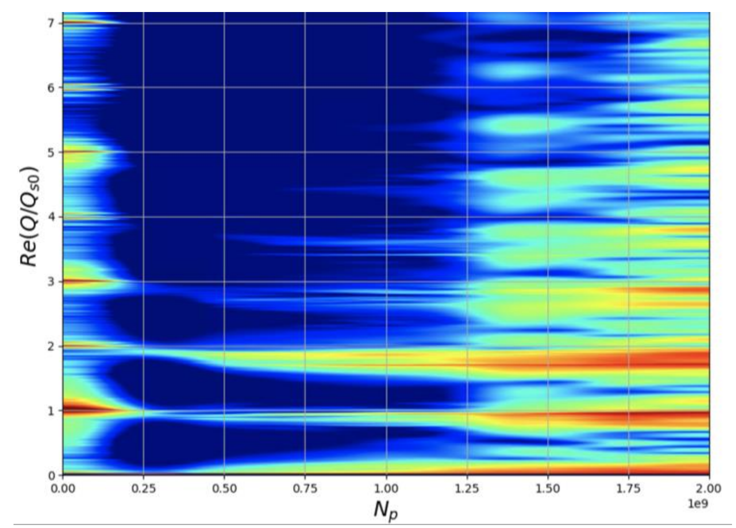
6. An infinite set of linear equations is finally obtained. The eigenvalues represent the coherent frequencies and the eigenvectors the corresponding modes

$$(\Omega - m\omega_s)\alpha_{mk} = \sum_{m'=-\infty}^{\infty} \sum_{k'=0}^{\infty} M_{kk'}^{mm'}\, \alpha_{m'k'}$$

Low intensity coherent modes of oscillations

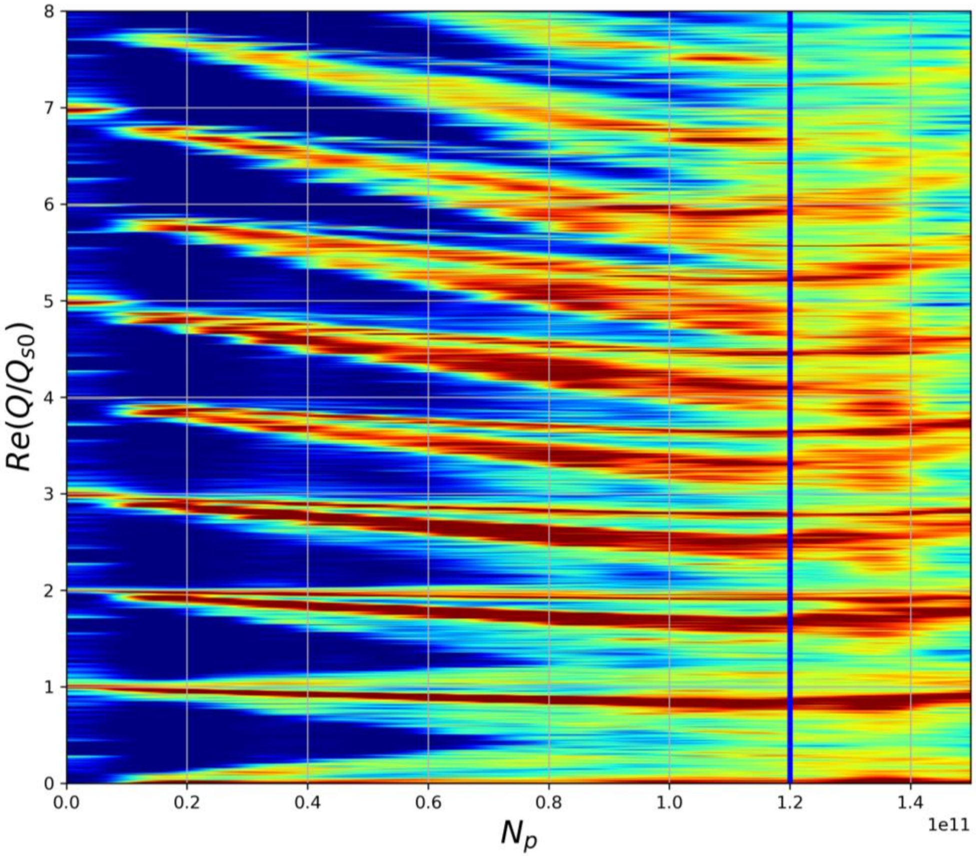
- For low intensity we ignore the coupling of radial modes that belong to different azimuthal families $(m = m')$, the matrix of the eigenvalue system is Hermitian, the eigenvalues are always real and no instability occurs (only in longitudinal plane)
- Only coupled bunch instabilities (interaction with high Q resonators) can occur if we consider single azimuthal modes
- Can a longitudinal instability occur due to coupling of radial modes of the same azimuthal family if we include the potential well distortion in the weight function of the radial modes?

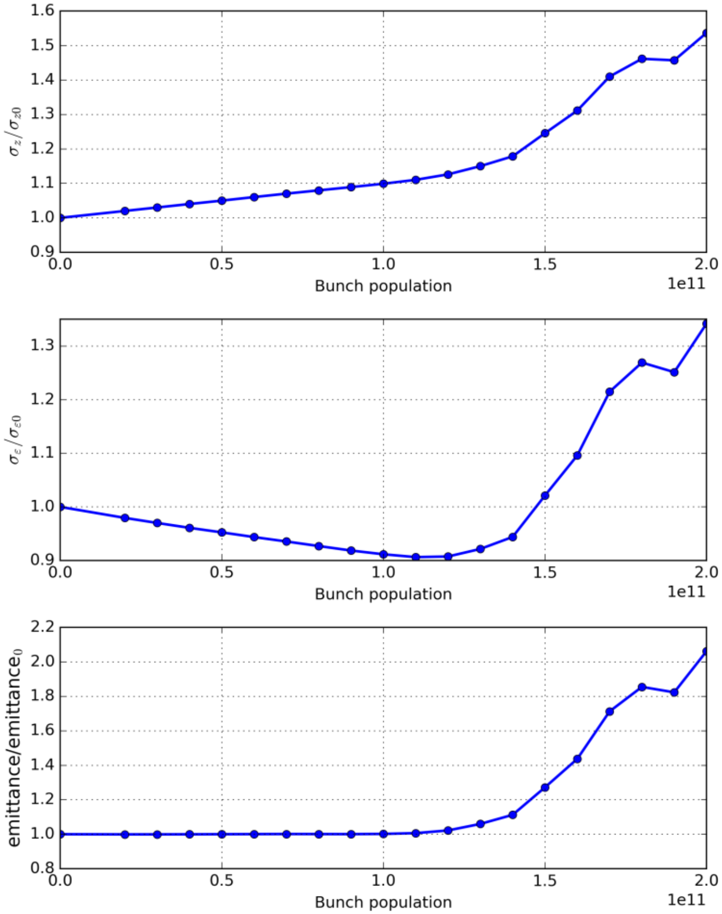




High intensity and mode coupling

- Mode coupling can occur at high intensity by taking into account different azimuthal modes.





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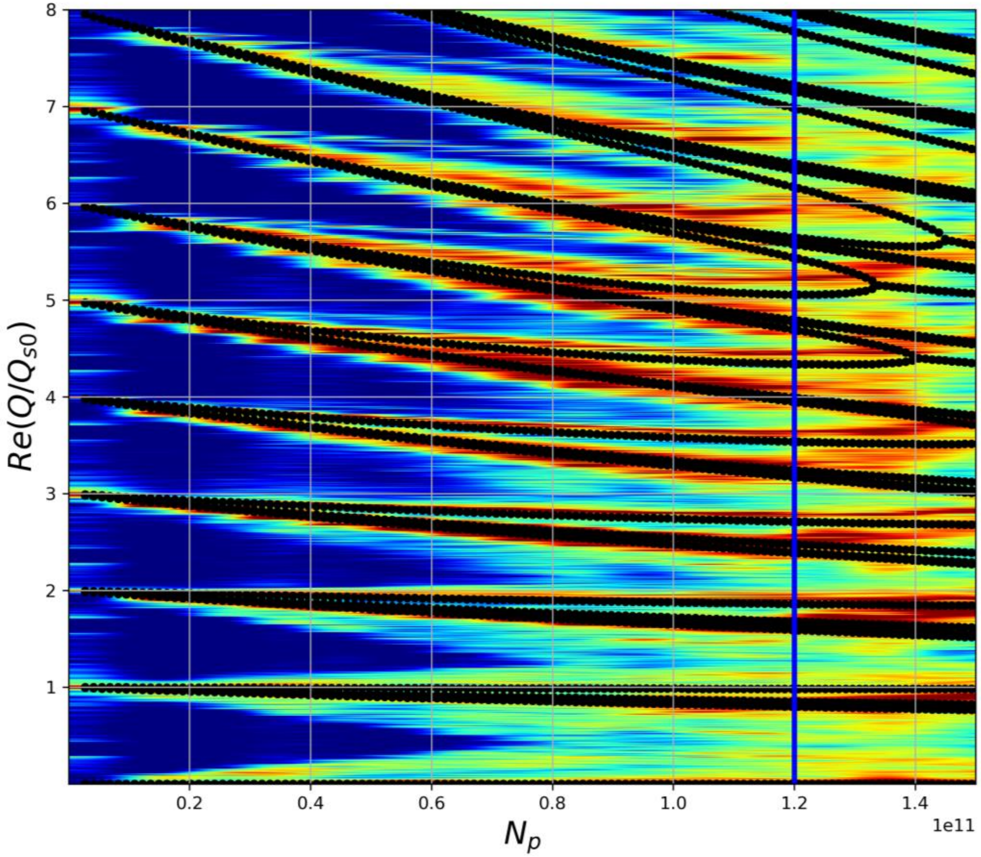


Black curves: E. Métral, GALACLIC Vlasov solver (with simplest model of PWD, i.e. neglecting the -small- asymmetry due to real part of impedance)

Some considerations on the longitudinal microwave instability for protons

- Synchrotron tunes in proton machines are in general much smaller than those in electron machines
- When considering collective instabilities, in some cases the synchrotron period of protons can be neglected because much longer then the instability growth times
- The wavelength of the perturbation producing the instability is often of the size of the radius of the vacuum chamber, which is usually much shorter than the length of the proton bunch

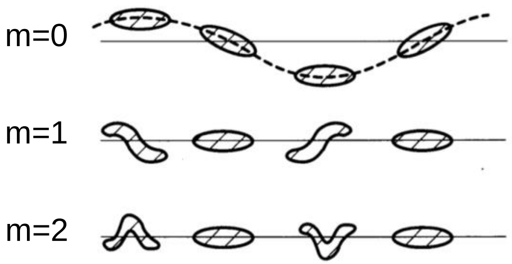
→ proton bunches, in some cases can be viewed locally as coasting beams in many instabilities considerations. Boussard suggested to apply the same criterion of coasting beams (Keil-Schnell) to bunched beams

$$I_{th} = \frac{\sqrt{2\pi}|\eta|(E_0/e)\sigma_{\varepsilon 0}^2 \sigma_z}{R|Z/n|}$$

Transverse case at low intensity

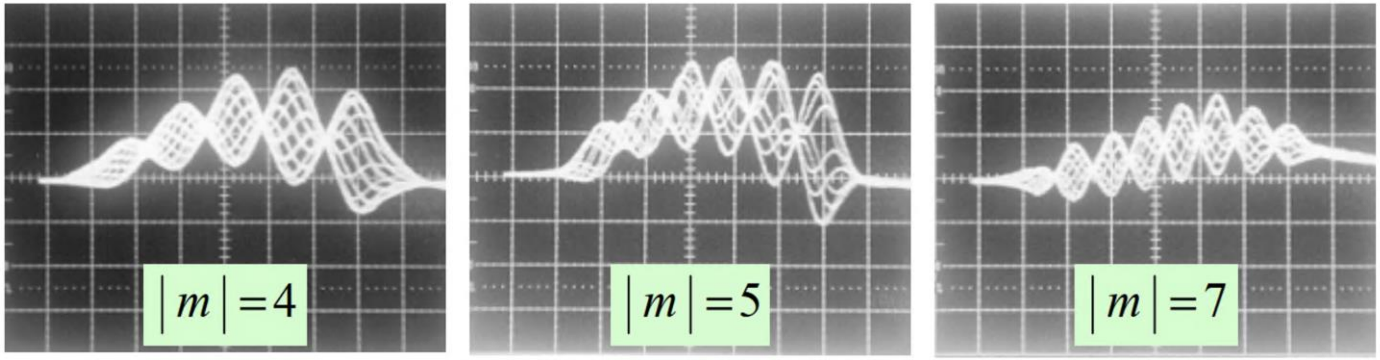
- The transverse case is similar to the longitudinal one with few differences:
  - the bunch is supposed to have only a dipole moment in the transverse plane
  - This dipole moment is not constant longitudinally. Depending on the longitudinal mode number m, its longitudinal structure may be simple or complicated

From A. Chao book



  - The modes are called transverse modes, but the transverse structure is a pure dipole and the main task is to find their longitudinal structure
  - the Vlasov equation needs to take into account both the transverse and the longitudinal phase spaces. Fortunately, however, the transverse structure of the beam is simple.

E. Métral, G. Rumolo, R. Steerenberg and B. Salvant, proceedings of PAC07, Albuquerque, New Mexico, USA, pp. 4210 - 4212



Transverse case at low intensity

- The eigenvalue system in this case is of the kind

$$(\Omega - \omega_\beta - m\omega_s)\alpha_{mk} = \sum_{m'=-\infty}^{\infty} \sum_{k'=0}^{\infty} M_{kk'}^{mm'} \alpha_{m'k'}$$

- The matrix elements, in this case, depend also on chromaticity. When this is zero, similarly to the longitudinal plane, the only instability for low intensity beams is due to high Q resonators

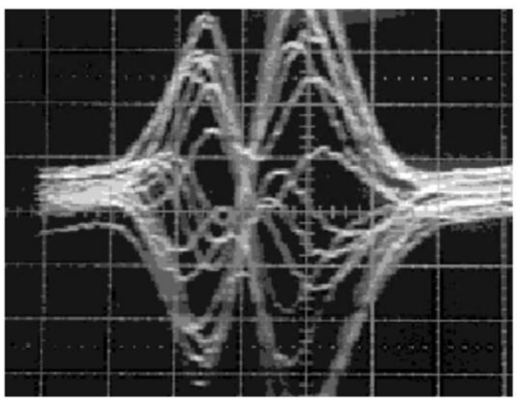
- If the chromaticity is different from zero, differently from the longitudinal plane, single azimuthal modes can be unstable. This instability is called head-tail instability

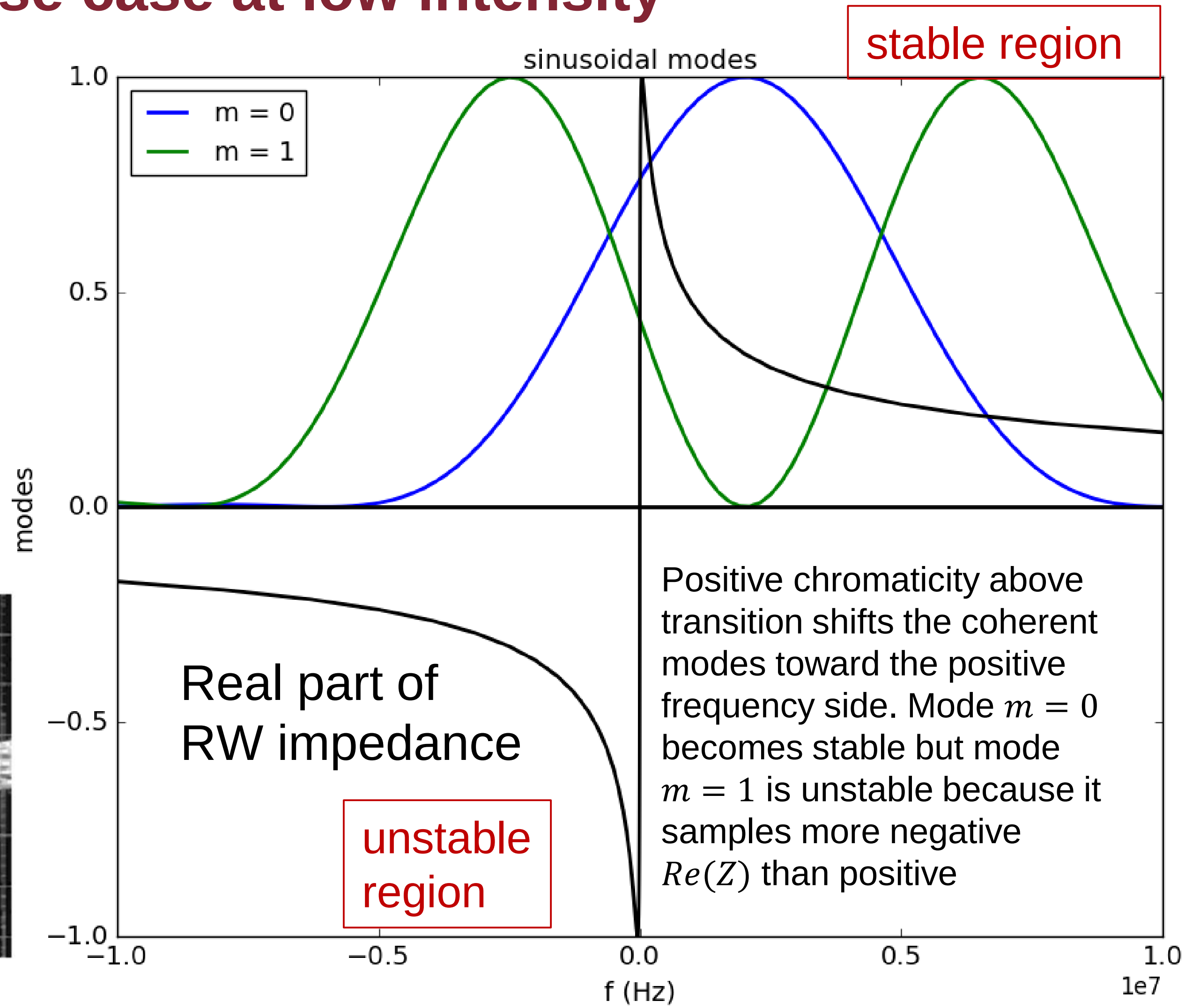
- Head-tail instability is not an intensity threshold mechanism.

Transverse case at low intensity

Example of head tail instability due to the RW impedance

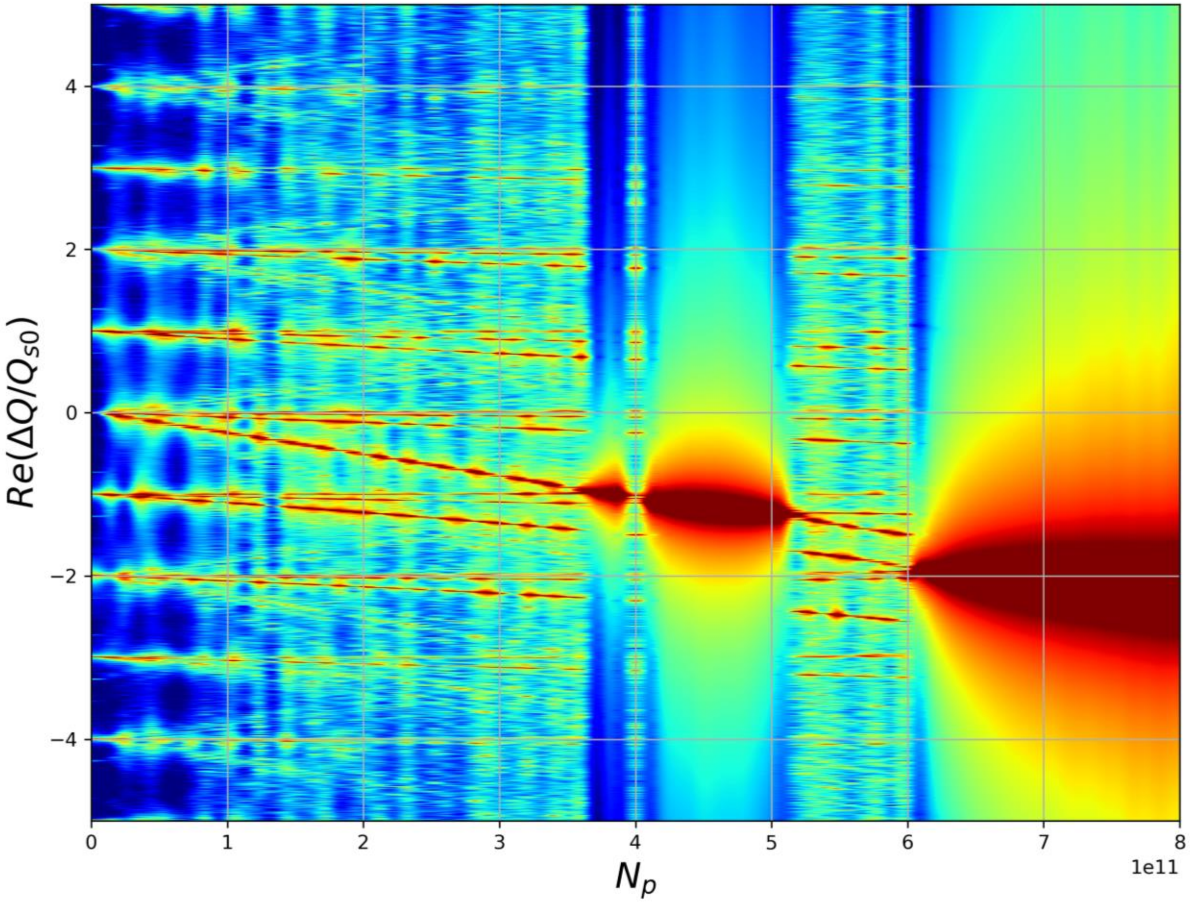
Courtesy: E. Métral





Transverse case at high intensity

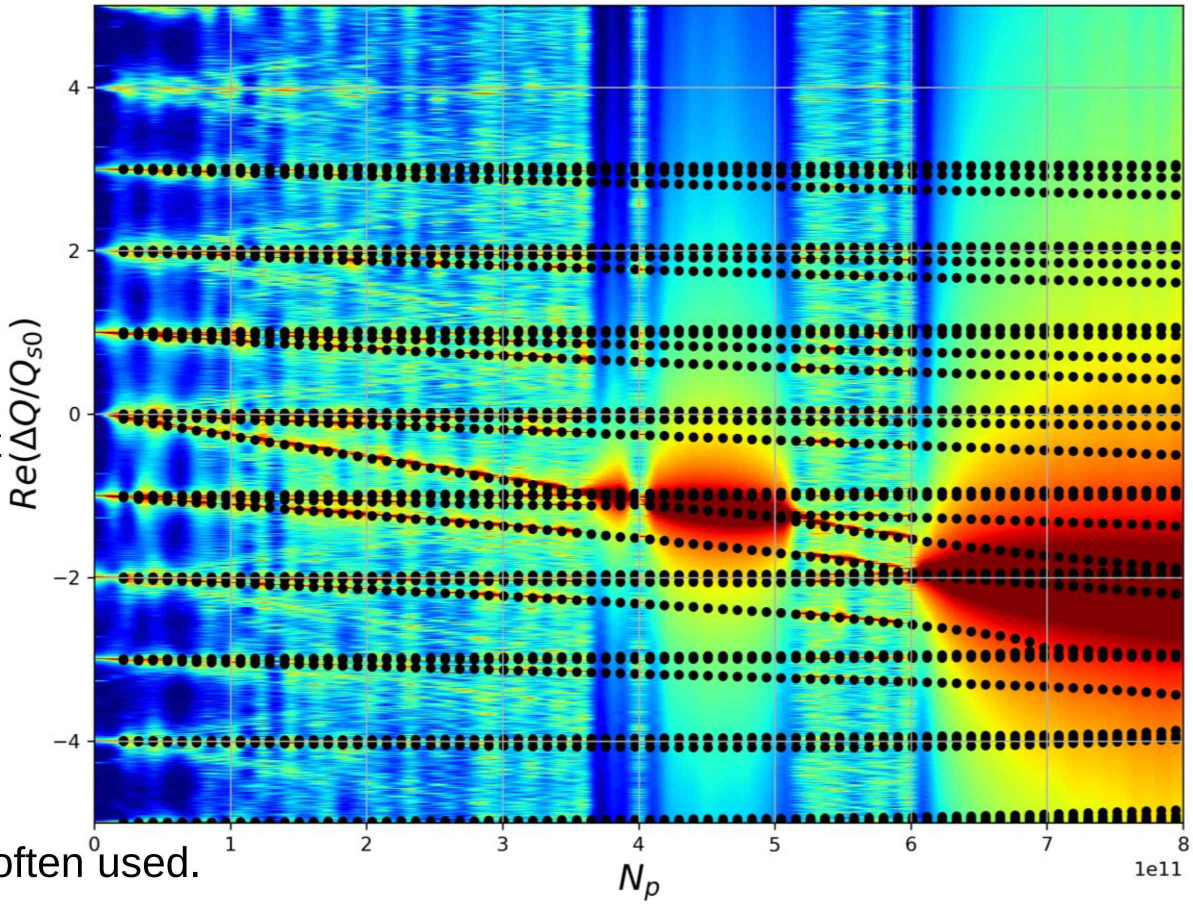
- As for longitudinal case, mode coupling can occur at high intensity by taking into account different azimuthal modes.



Transverse case at high intensity

- As for longitudinal case, mode coupling can occur at high intensity by taking into account different azimuthal modes.

Black curves: E. Métral, GALACTIC Vlasov solver (NB: in the past MOSES code from Y. H. Chin and, more recently, the NHT from A. Burov and DELPHI from N. Mounet are often used.



Transverse case at high intensity

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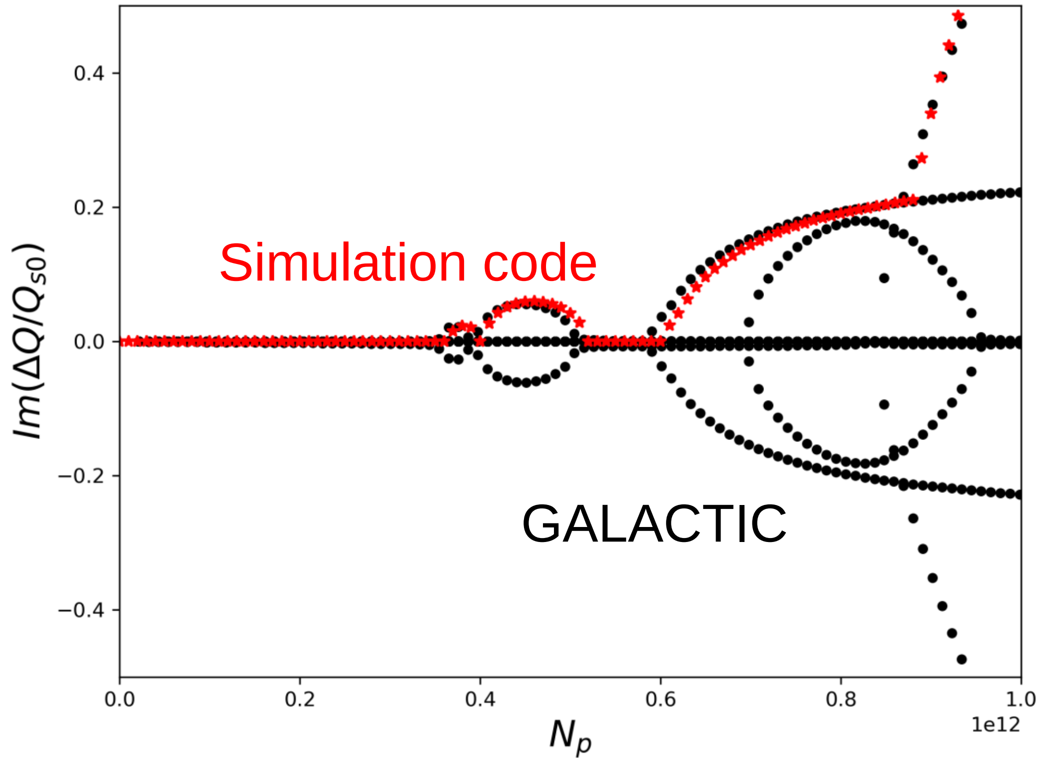
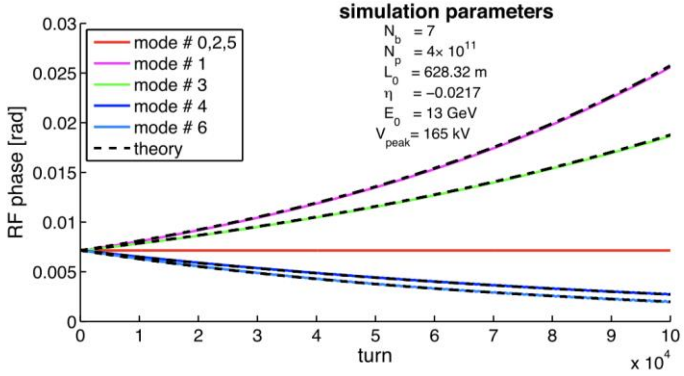


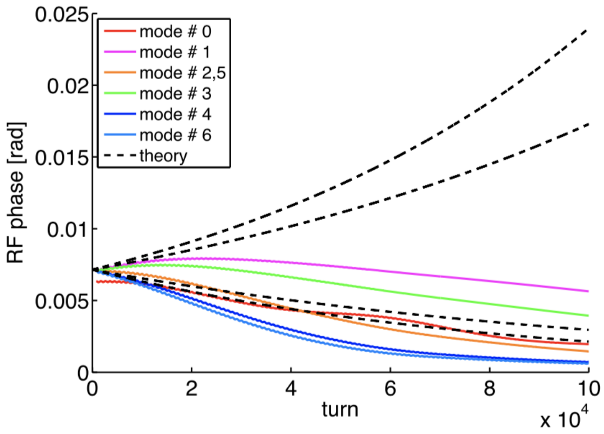
Single bunch instabilities: mitigations

- A very effective way to mitigate all instabilities is to reduce the machine coupling impedance
- Longitudinal plane:
  - No feedback systems can be used to suppress the microwave instability
  - The Boussard criterion can be a good indicator on how to cope with the longitudinal instability. For example: increase of momentum compaction (strong factor), energy spread (heating the bunch, e.g. with wigglers (laser?) in electron machines)
  - Some machines work in the microwave instability regime!
- Transverse plane:
  - In general the lattice choice is quite a strong factor to mitigate the instabilities: tunes, linear and nonlinear chromaticity, coupling, tune dependence on the oscillation amplitude etc.
  - Feedback systems with proton beams
- **Others?**

Coupled bunch instability

- Coupled bunch instability: similar approach as single bunch with the addition of another index taking into account the coupled bunch modes
- The instability is due to the interaction of the beam with high Q resonators





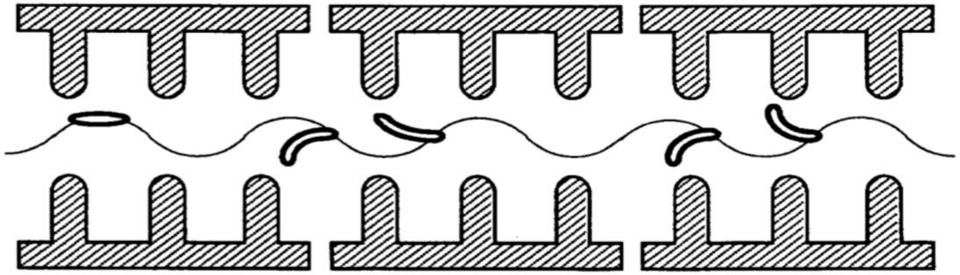
- Mitigations: HOMs damping, feedback system, higher harmonic (Landau) cavity, RF voltage modulation, uneven fill
- Others?

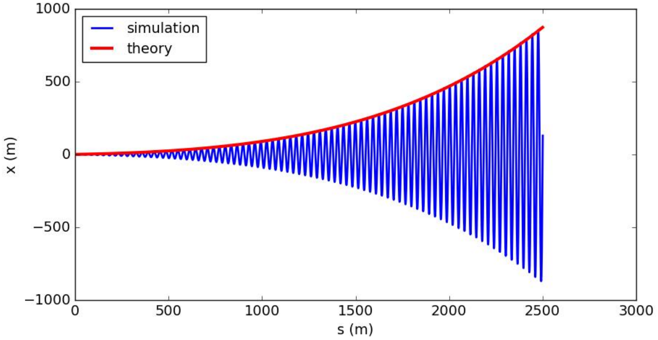
LINACS: BBU and mitigation

- The Beam BreakUp (BBU) instability can be viewed as a mode coupling instability in the limit of $\omega_s \to 0$ (longitudinal frozen motion)
- It can be driven either by coherent oscillations due to injection errors or by misalignments of accelerating structures
- It can be described in its simplest form as an harmonic oscillation of the bunch tail driven on resonance by the bunch head:

$$y_2'' + k_\beta^2 y_2 = \frac{Ne^2 w_\perp(\Delta z)}{2\beta^2 E_0 L_w} \hat{y}_1 \cos k_\beta s$$

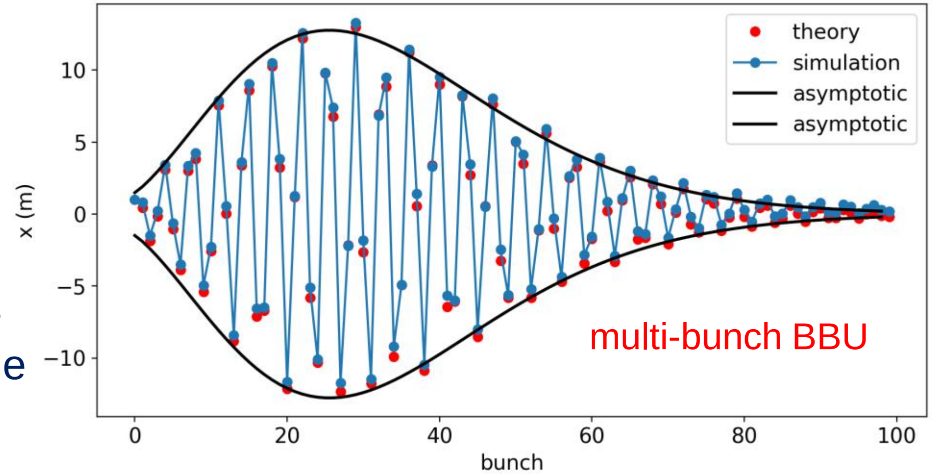
- In addition to A. Chao book, a comprehensive paper is: A. Mosnier, Instabilities in Linacs, CERN Report No. 95-06, 2005, p. 481





LINACS: BBU and mitigation

- There are several approaches to the BBU: simple two-particle model, single bunch general distribution, multi-bunch BBU, with and without acceleration.
- Mitigations:
  - BNS damping (after Balakin-Novokhatski-Smirnov, 1983). By a proper difference in betatron frequencies (stronger focusing of the bunch tail), the resonant growth of the tail oscillation can be suppressed
  - Adiabatic damping (acceleration)
  - Damping of the resonant modes
  - Adding a modest energy spread
  - Introducing a distribution of mode frequencies from section to section or from cell to cell so that the deflecting mode will no longer be excited coherently by the beam (detuning technique).



What I had no time to mention

- Coasting beam instabilities (e.g. negative mass instability)
- Not relativistic beam ($\beta < 1$)
- Space charge effects
- Landau damping and dispersion integrals
- Sawtooth instabilities for electrons
- Robinson's instability
- Transition crossing
- Microbunching instability in RF and magnetic compressors
- Other impedance induced effects which are not real instabilities but can influence the machine performances (e.g. detuning impedance, beam energy spread in LINACS …).

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- … for sure I'm forgetting something …

Conclusions

- The subject of impedance-induced instability is one of the main topics facing modern high performance accelerators
- Even if the roots of this subject are more than 50 years old, it is still a cutting-edge in the beam physics
- Many researchers have been working over the years on this subject and very elegant and well-established theories have been proposed explaining many experimental observations
- We still need to study in more detail the interplays among different mechanisms (e.g. with optics) and in particular we need to better understand the mitigation techniques, which is the subject of this workshop!
- There are still “dark sides” that have to be illuminated by the young generation, which, we hope, will continue the work with the passion that has marked so far the protagonists of this fascinating subject

Thank you very much for your patience (and attention)