Review of Space Charge Effects for Transverse Collective Instabilities of Bunched Beams in Circular Machines

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Many thanks to J. Eldred, V. Kornilov, V. Lebedev, E. Metral, M. Migliorati, A. Oeftiger.

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Merle-Mohl\textsuperscript{2}-Schonauer Equation (1969, 1974)

\begin{equation}
Z_k''(\lambda) + Q_o^2 Z_k - 2Q_o\Delta Q(k)\frac{1}{Z} - 2Q_o\Delta Q(k)\frac{1}{\zeta_{Zk}}(Z_k-\bar{Z}) - W(k)\frac{1}{Z} = 0
\end{equation}

Landau damping by non-linear space-charge forces and octupoles

D. Möhl and H. Schönauer,
CERN, Geneva, Switzerland 1974

3.1 No external non-linearities (i.e. $\nu_a = 0$, $\nu_b = 0$)

In this case incoherent space-charge has no effect, as was found already in Ref. 4. In fact it is easily verified for any order of incoherence, for this case.

Physical Review Special Topics - Accelerators and Beams 12, 034201 (2009)

Transverse instabilities of coasting beams with space charge

A. Burov and V. Lebedev

MMS Eq = rigid-slice approximation, valid for strong space charge only.
Fast head-tail instability with space charge

M. Blaskiewicz

ABS: longitudinal plane

$\delta p/p$

head  tail

$s$

$W(s) \propto \exp(-\alpha s)$

FIG. 14. Threshold value of $\kappa W_0 \tau_b/Q_s$ vs $\Delta Q_{sc}/Q_s$ for the analytic square well with $\chi = 0$ and $\alpha \tau_b = 0, 5, 10, 20$, from bottom to top.

$$\left\{ \frac{\kappa W_0 \tau_b}{Q_s} \right\}_{\text{thresh}} = a(\alpha \tau_b) + b(\alpha \tau_b) \frac{\Delta Q_{sc}}{Q_s}.$$
Fast head-tail instability with space charge

M. Blaskiewicz

If emittance is small enough, there is no threshold!
Head-tail modes for strong space charge

A. Burov

\[ \nu \ddot{y}(\tau) + \frac{1}{Q_{\text{eff}}(\tau)} \frac{d}{d\tau} \left( u^2 \frac{d\ddot{y}}{d\tau} \right) = \kappa (\hat{W} \ddot{y} + \hat{D} \dot{y}) \quad \hat{W} \ddot{y} = \int_{\tau}^{\infty} W(\tau - s) \exp[i \zeta (\tau - s)] \rho(s) \dot{y}(s) ds; \]

\[ \hat{D} \dot{y} = \ddot{y}(\tau) \int_{\tau}^{\infty} D(\tau - s) \rho(s) ds. \]

\[ \ddot{y}'(\pm \tau_*) = 0. \]

FIG. 5. (Color) Same as Fig. 4, but for the resistive wake function \( W(\tau) = -W_0/\sqrt{\tau}. \)

FIG. 7. (Color) A schematic behavior of the TMCI threshold for the coherent tune shift versus the space charge tune shift. Both tune shifts are in units of the synchrotron tune.
Simulation of transverse modes with their intrinsic Landau damping for bunched beams in the presence of space charge

Alexandru Macridin, Alexey Burov, Eric Stern, James Amundson, and Panagiotis Spentzouris

FIG. 6. 3D-G bunch. The Landau damping for modes 1, 2, and 3 versus the space charge parameter $q_{\text{eff}}$. At small $q_{\text{eff}}$ the damping increases quickly with increasing $q_{\text{eff}}$. In the strong space charge regime, $q > \approx 4k$, we find that $\lambda T_0 \approx 2.4 \frac{k^3}{q_{\text{eff}}^3}$, where $k$ is the mode number (dashed lines). This behavior is in agreement with the theoretical predictions [6]. The proportionality factor of 2.4 is characteristic of transverse Gaussian beams. The damping rates of all three modes can be fitted reasonably well for the entire range of the space charge strength by employing Eq. (33) (green lines).
Transverse modes of a bunched beam with space charge dominated impedance

V. Balbekov*

Transverse instability of a bunched beam with space charge and wakefield

V. Balbekov*

\[ U^2 \dddot{Y} - \left( \theta + \frac{U^2 \rho'/\rho(0)}{\nu + \rho/\rho(0)} \right) \dddot{Y} + \frac{\nu[\nu + \rho/\rho(0)]}{\mu^2} \ddot{Y} = 0. \]


FIG. 8. Eigentunes of several modes against wake strength (Gaussian bunch, rectangular wake).
Transverse mode coupling instability threshold with space charge and different wakefields

V. Balbekov

The case of very high space charge has been considered in Refs. [6,7]. It was confirmed in both papers that the space charge heightens the TMCI threshold until the ratio of the SC tune shift to the synchrotron tune reaches the border in several tens or a hundred units. However, the authors have expressed different opinions about the further behavior of the threshold. As it follows from Ref. [7], the threshold growth should continue at higher SC as well. On the contrary, it was suggested in Ref. [6] that the threshold growth can cease and turn back over the mentioned border.

The last statement has been supported recently in Ref. [8]. I have used the known eigenfunctions of the boxcar bunch [9] to get a convenient basis for investigation of the TMCI problem in depth. However, a disclosure of some errors at the numerical solutions of the obtained equations forces me to revise the conclusions. The equations are recomputed in the present paper at any value of the SC tune shift and different wakes including the resistive wall, square, and oscillating ones. The increase of the TMCI threshold by the SC is observed in all the cases.

FIG. 5. Threshold curve of the boxcar bunch in different approximations. The index $n_{\text{max}}$ means maximal power of the Legendre polynomial in the expansion. The left-hand line shows the TMCI threshold, and the right-hand rising lines are unphysical because of an absence of the convergence.

Blaskiewicz’ linear rule for TMCI threshold \( W_{th} \propto \Delta Q_{sc} \) is essentially universal.

**FIG. 18.** The lowest TMCI threshold as a function of space charge for CERN SPS ring (ABS model). Dashed line shows the value of threshold at zero \( \Delta Q_{sc} \).
These measurements reasonably agree with no SC TMCI threshold, but SC was really huge at Q26, $q > 20$. These measurements apparently contradict to everything in the TMCI @ SSC theory!
Resolution of the contradictions

Beam stability requires more than $\Im \nu \leq 0$!
Centroid oscillations, strong-strong case

![Graph showing centroid oscillations for different values of l with q = 20 and w = 13.](image)

**FIG. 6.** Stroboscopic images of the centroid oscillations for the same parameters and modes as in Fig. [5]. Number of nodes for each mode is identical to the modulus of its number.

\[ q = 20 \quad w = 13 \]

Threshold \( w = 115 \) at this \( q \)
\( \bar{x} = (x^+ + x^-)/2 \)

FIG. 12. Time evolution of the local centroids \( \bar{x}(\theta, s) = [x^+(\theta, s) + x^-(\theta, s)]/2 \) for the same case, i.e. for \( q = 20 \), \( w = 13 \) and constant initial conditions, \( x^\pm = 1 \). The amplification is saturated within \( \sim 1 \) synchrotron period.
The main intensity limitation for single bunches in the SPS has been identified as transverse mode coupling instability (TMCI) at injection [67], which is mainly caused by the vertical beam coupling impedance of the kicker magnets [68]. The threshold
Convective instabilities of bunched beams with space charge

A. Burov

FIG. 21. Amplification for SC $q = 20$. The black dashed line of no-SC TMCI threshold is close to the contour line $K \approx 300 - 1000$ for large interval of the phase advances. For the entire area of the parameters, the system is absolutely stable, $\Im \nu = 0$. 

$q = 20$
Resolution of the Conundrum

\[ w \approx a + bq \]

Blaskiewicz' asymptote

Convective Instabilities

TMCI
Figure 4: Fast instability observed in the CERN PS near transition (~6 GeV total energy) in 2000. Single-turn signals from a wide-band pick-up. From top to bottom: $\Sigma$, $\Delta x$, and $\Delta y$. Time scale: 10 ns/div. The head of the bunch is stable and only the tail is unstable in the vertical plane. The particles lost at the tail of the bunch can be seen from the hollow in the bunch profile.

E. Metral, HB 2005
$\epsilon_s = 0.3 \text{ eVs}$

$\epsilon_\perp = 1 \mu m$

Figure 8: Intensity threshold of the Beam Break-Up Instability in the PS during transition (without $\gamma_t$ jump), as a function of the rf voltage for $h = 8$. For the intensities above the presented values (protons per bunch) strong, fast losses have been observed.
Instability studies at the CERN Proton Synchrotron during transition crossing

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Comment on “Instability studies at the CERN Proton Synchrotron during transition crossing”

A. Burov

in many publications for the last twenty years. Do the observations and modeling of the Article refute the long-standing idea that the PS instability is the TMCI? Do they rather confirm it? Do they neither refute, nor confirm it? Apparently, we deal here with new physics, so the question is important.

Reply to “Comment on “Instability Studies at the CERN Proton Synchrotron during transition crossing””

M. Migliorati,1,2 S. Aumon,2 E. Koukovini-Platia,2 A. Huschauer,2 E. Métral,2 G. Sterbini,2 and N. Wang3

As suggested by the Author in the conclusion, if, apparently, we deal here with new physics, it could be very useful to check if the theory developed in [2] can reproduce the thresholds, as those reported in Fig. 14 of [1], under the different conditions of the machine, determining also the role of the space change in this instability at different longitudinal emittances.


Transverse Microwave Convective Instability at Transition Crossing

Alexey Burov
(Submitted on 8 Aug 2019)

\( \epsilon \perp = 5 \, \mu m \)

\[
\frac{w}{3\alpha^2 b} \sqrt{\frac{w \omega_{sc}}{\kappa}} = \Lambda_{th}.
\] (23)

\[ \Lambda_{th} = \ln(a/x_0) = 18 \]

\[ x_0 = \sigma_x/\sqrt{N_{eff}} \]

\[ W(s) = W_0 \exp(-\alpha s) \sin(\kappa s) \]

\[ b = 2 \frac{\dot{\gamma}}{\gamma^3} \frac{\delta p}{p} \]

\[ N_{th} \propto \epsilon_s^{3/4} \epsilon_{\perp}^{1/4} \]

FIG. 1. Threshold number of protons per bunch (ppb) versus longitudinal rms emittance for the PS bunch with the reported transverse rms emittance 5\( \mu \)m [5]. The line corresponds to our formula (23), the dots copy the data of Fig. 14 of Ref. [5].

Red dot \( \bullet \) is my theoretical result for these emittances,
\[
\epsilon_s = 0.3 \text{ eVs} \\
\epsilon_\perp = 1 \mu m
\]
Core-Halo Collective Instabilities

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(Dated: August 28, 2018)

At strong space charge, transverse modes of the bunch core may effectively couple with those of the halo, leading to instabilities well below the core-only transverse mode-coupling threshold.

FIG. 2. Centroid stroboscopic images of the core and halo components of the most unstable core-halo mode for the same $q$ and $w$ as in Fig. 1, at the most unstable $\tilde{q} = 0.29$. Waists instead of nodes in the halo image tell about an absolute instability.

FIG. 4. Growth rates of the most unstable modes versus wake parameter for three different SC parameters. Note the conventional TMCI threshold for $q_c = 5$ at $w \approx 15$. 

Transverse Instabilities of a Bunch with Space Charge, Wake and Feedback

Alexey Burov

(Submitted on 18 Sep 2018)

FIG. 3. With sufficient gain and wake, SC becomes a mode coupling factor, as in here, with +1st and +2nd modes coupling. The wake is same, $w = 2$; the gain $g = 5$. MUM is +1st. Mode 0 (green dots) is insensitive to SC, being just shifted down by $w/2 = 1$. 
Convective instability at the FNAL Booster,
July 2\textsuperscript{nd}, 2019
Mitigations are left totally out of my talk:

Chromaticity (limited by incoherent SC)

Octupoles (hardly useful at SSC)

Landau e-lenses (Alexahin’s talk)

Feedbacks (Lebedev’s talk)

Landau cavities (may be a bit helpful at SSC)

RFQ (should be studied for SSC)
Many thanks!
FIG. 17. Time evolution of the ACI for the same parameters as Fig. 16. An exponential growth is clearly seen.