# Review of Space Charge Effects for Transverse Collective Instabilities of Bunched Beams in Circular Machines

Alexey Burov

Fermilab

Many thanks to
J. Eldred, V. Kornilov, V. Lebedev, E. Metral, M. Migliorati, A. Oeftiger.

MCBI ICFA Workshop, Sep 23, 2019
Zermatt, Switzerland

# Merle-Mohl²-Schonauer Equation (1969,1974)

THE STABILIZING INFLUENCE OF NON-LINEAR SPACE-CHARGE ON TRANSVERSE COHERENT OSCILLATIONS

G. Merle*) and D. Möhl

October 1969

$$Z_k''(s) + Q_o^2 Z_k - 2Q_o\Delta Q_c^{(k)}\overline{Z} - 2\,Q_o\Delta Q_{ic}^{(k)}(Z_k-\overline{Z}) - W^{(k)}\overline{Z} = 0$$

LANDAU DAMPING BY NON-LINEAR SPACE-CHARGE FORCES AND OCTUPOLES

D. Möhl and H. Schönauer,

CERN, Geneva, Switzerland

1974

3.1 No external non-linearities (i.e. $\nu_a = 0$, $\nu_b = 0$)

In this case incoherent space-charge has no effect, as was found already in Ref. 4. In fact it is easily

PHYSICAL REVIEW SPECIAL TOPICS - ACCELERATORS AND BEAMS **12**, 034201 (2009)

**Transverse instabilities of coasting beams with space charge**

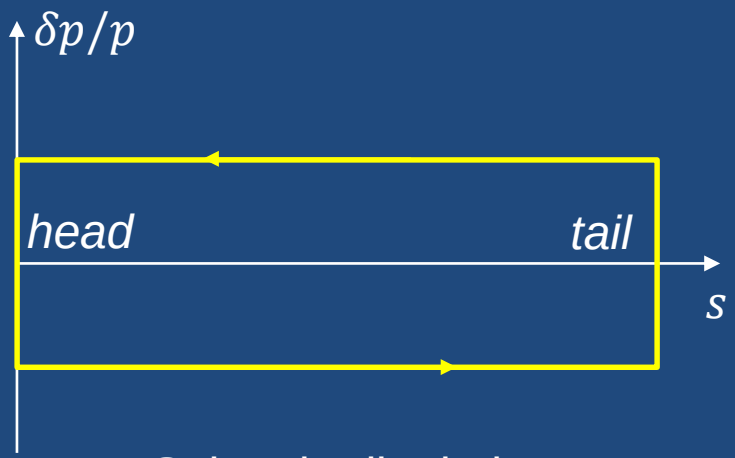
A. Burov and V. Lebedev

MMS Eq = rigid-slice approximation, valid for strong space charge only.

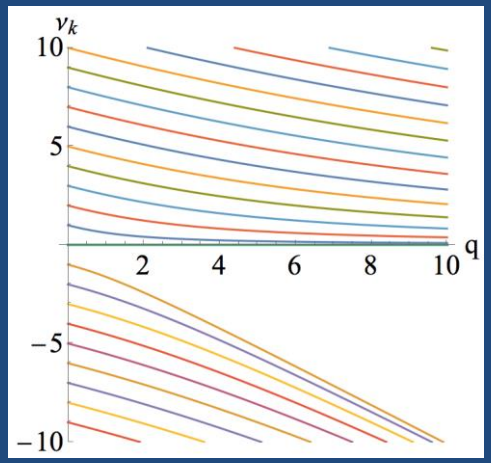
PHYSICAL REVIEW SPECIAL TOPICS - ACCELERATORS AND BEAMS, VOLUME 1, 044201 (1998)

# Fast head-tail instability with space charge

M. Blaskiewicz




ABS: longitudinal plane



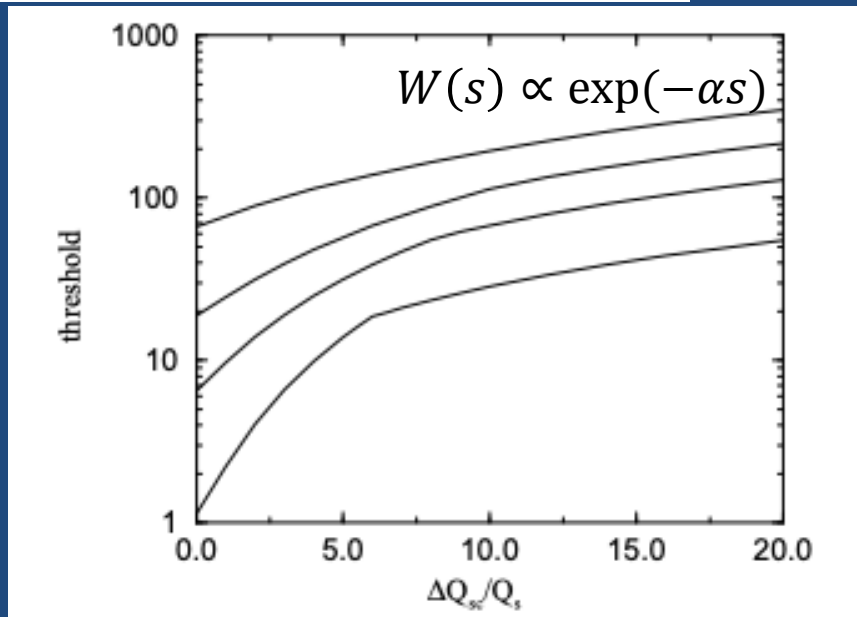



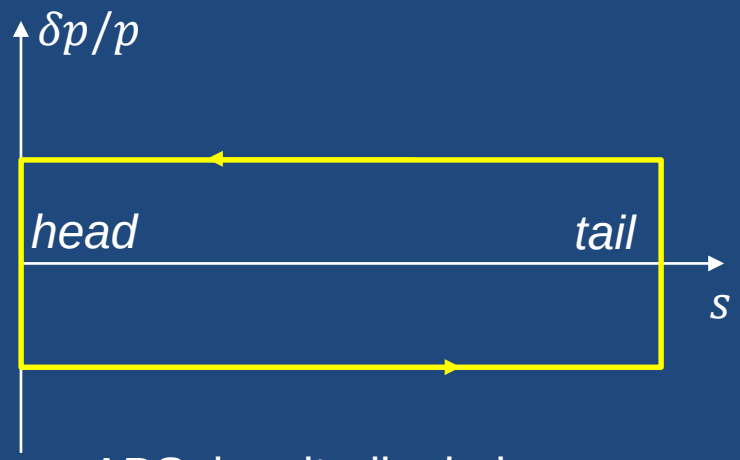

FIG. 14. Threshold value of $\kappa W_0 \tau_b / Q_s$ vs $\Delta Q_{sc}/Q_s$ for the analytic square well with $\chi = 0$ and $\alpha \tau_b = 0, 5, 10, 20$, from bottom to top.

$$\left\{\frac{\kappa W_0 \tau_b}{Q_s}\right\}_{\text{thresh}} = a(\alpha\tau_b) + b(\alpha\tau_b)\frac{\Delta Q_{sc}}{Q_s}.$$

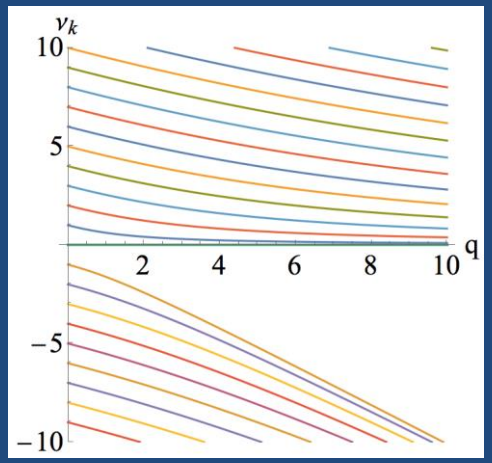
PHYSICAL REVIEW SPECIAL TOPICS - ACCELERATORS AND BEAMS, VOLUME 1, 044201 (1998)

**Fast head-tail instability with space charge**

M. Blaskiewicz




ABS: longitudinal plane



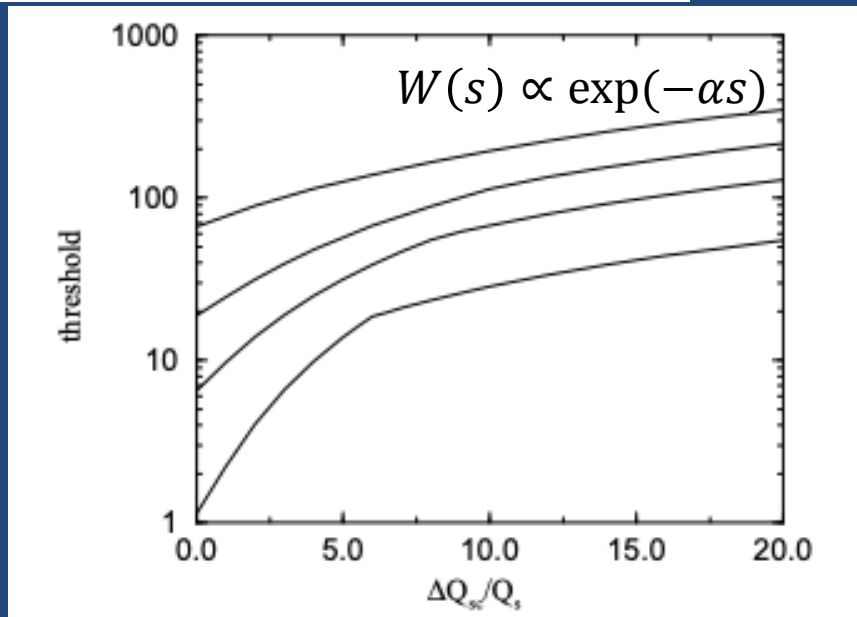




FIG. 14. Threshold value of $\kappa W_0 \tau_b / Q_s$ vs $\Delta Q_{sc}/Q_s$ for the analytic square well with $\chi = 0$ and $\alpha \tau_b = 0, 5, 10, 20$, from bottom to top.

$$\left\{\frac{\kappa W_0 \tau_b}{Q_s}\right\}_{\text{thresh}} = a(\alpha\tau_b) + b(\alpha\tau_b)\frac{\Delta Q_{sc}}{Q_s}.$$

If emittance is small enough, there is no threshold!

PHYSICAL REVIEW SPECIAL TOPICS - ACCELERATORS AND BEAMS **12**, 044202 (2009)

# Head-tail modes for strong space charge

A. Burov

$$\nu\bar{y}(\tau) + \frac{1}{Q_{\rm eff}(\tau)}\frac{d}{d\tau}\left(u^2\frac{d\bar{y}}{d\tau}\right) = \kappa(\hat{\mathbf{W}}\,\bar{y} + \hat{\mathbf{D}}\,\bar{y}) \qquad \hat{\mathbf{W}}\,\bar{y} \equiv \int_\tau^\infty W(\tau - s)\exp[i\zeta(\tau - s)]\rho(s)\bar{y}(s)ds;$$

$$\hat{\mathbf{D}}\,\bar{y} \equiv \bar{y}(\tau)\int_\tau^\infty D(\tau - s)\rho(s)ds.$$

$$\bar{y}\,'(\pm\tau_*) = 0.$$

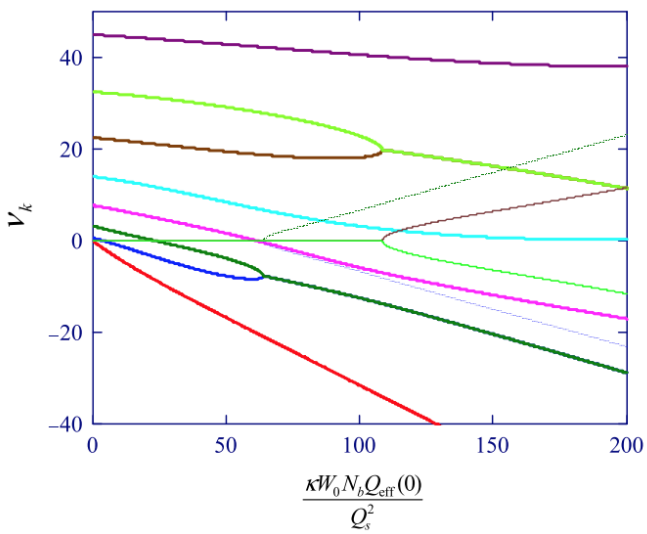



FIG. 5. (Color) Same as Fig. 4, but for the resistive wake function $W(\tau) = -W_0/\sqrt{\tau}$.

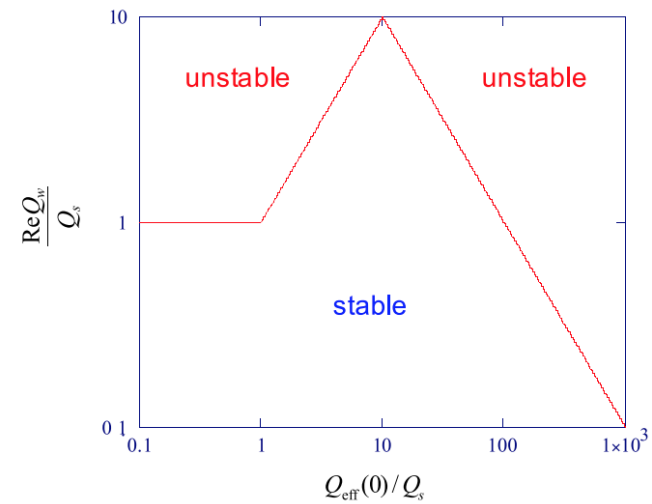



FIG. 7. (Color) A schematic behavior of the TMCI threshold for the coherent tune shift versus the space charge tune shift. Both tune shifts are in units of the synchrotron tune.

PHYSICAL REVIEW SPECIAL TOPICS—ACCELERATORS AND BEAMS **18,** 074401 (2015)

# Simulation of transverse modes with their intrinsic Landau damping for bunched beams in the presence of space charge

Alexandru Macridin, Alexey Burov, Eric Stern, James Amundson, and Panagiotis Spentzouris

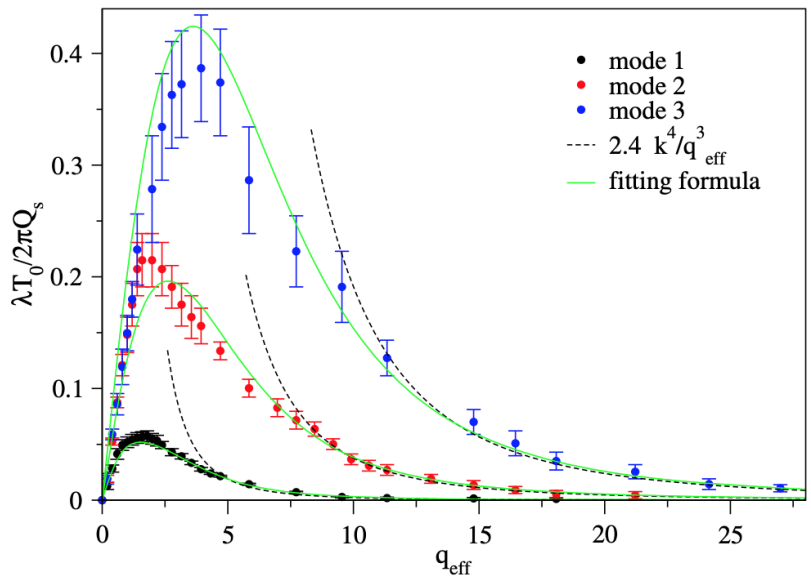



FIG. 6. 3D-G bunch. The Landau damping for modes 1, 2, and 3 versus the space charge parameter $q_{\mathrm{eff}}$. At small $q_{\mathrm{eff}}$ the damping increases quickly with increasing $q_{\mathrm{eff}}$. In the strong space charge regime, $q > \approx 4k$, we find that $\lambda T_0 \approx 2.4 \frac{k^4 2\pi Q_s}{q_{\mathrm{eff}}^3}$, where $k$ is the mode number (dashed lines). This behavior is in agreement with the theoretical predictions [6]. The proportionality factor of 2.4 is characteristic of transverse Gaussian beams. The damping rates of all three modes can be fitted reasonably well for the entire range of the space charge strength by employing Eq. (33) (green lines).

PHYSICAL REVIEW SPECIAL TOPICS - ACCELERATORS AND BEAMS **12**, 124402 (2009)

**Transverse modes of a bunched beam with space charge dominated impedance**

V. Balbekov*

PHYSICAL REVIEW SPECIAL TOPICS - ACCELERATORS AND BEAMS **14**, 094401 (2011)

**Transverse instability of a bunched beam with space charge and wakefield**

V. Balbekov*

$$U^2\bar{Y}'' - \left(\theta + \frac{U^2\rho'/\rho(0)}{\nu + \rho/\rho(0)}\right)\bar{Y}' + \frac{\nu[\nu + \rho/\rho(0)]}{\mu^2}\bar{Y} = 0.$$

Phys. Rev. ST Accel. Beams **14,** 094401 (2011)

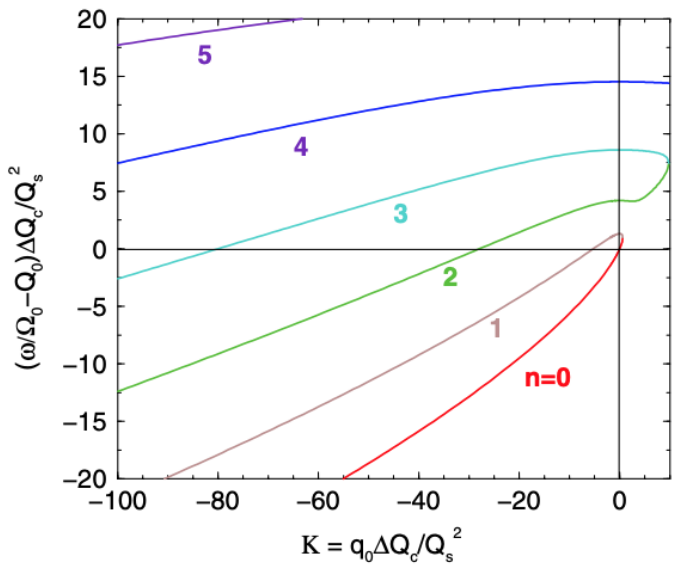



FIG. 8. Eigentunes of several modes against wake strength (Gaussian bunch, rectangular wake).

PHYSICAL REVIEW ACCELERATORS AND BEAMS **20,** 034401 (2017)

# Transverse mode coupling instability threshold with space charge and different wakefields

V. Balbekov*

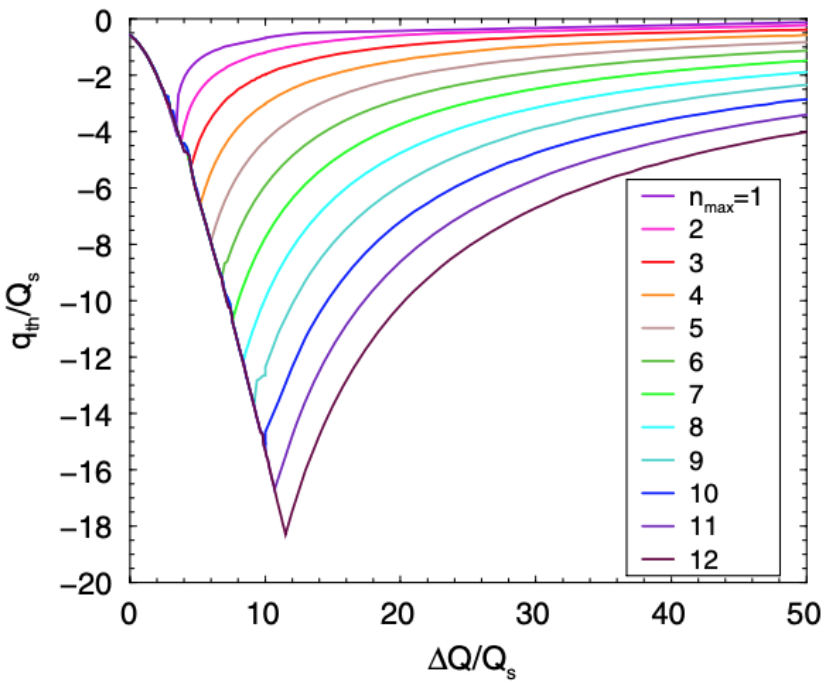



FIG. 5. Threshold curve of the boxcar bunch in different approximations. The index $n_{max}$ means maximal power of the Legendre polynomial in the expansion. The left-hand line shows the TMCI threshold, and the right-hand rising lines are unphysical because of an absence of the convergence.

The case of very high space charge has been considered in Refs. [6,7]. It was confirmed in both papers that the space charge heightens the TMCI threshold until the ratio of the SC tune shift to the synchrotron tune reaches the border in several tens or a hundred units. However, the authors have expressed different opinions about the further behavior of the threshold. As it follows from Ref. [7], the threshold growth should continue at higher SC as well. On the contrary, it was suggested in Ref. [6] that the threshold growth can cease and turn back over the mentioned border.

The last statement has been supported recently in Ref. [8]. I have used the known eigenfunctions of the boxcar bunch [9] to get a convenient basis for investigation of the TMCI problem in depth. However, a disclosure of some errors at the numerical solutions of the obtained equations forces me to revise the conclusions. The equations are recomputed in the present paper at any value of the SC tune shift and different wakes including the resistive wall, square, and oscillating ones. The increase of the TMCI threshold by the SC is observed in all the cases.

[6] A. Burov, Head-tail modes for strong space charge, Phys. Rev. ST Accel. Beams **12**, 044202 (2009).
[7] V. Balbekov, Transverse instability of a bunched beam with space charge and wakefield, Phys. Rev. ST Accel. Beams **14**, 094401 (2011).
[8] V. Balbekov, Report No. FERMILAB-PUB-16-079-APC, 2016.

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PHYSICAL REVIEW ACCELERATORS AND BEAMS **21,** 104201 (2018)

**Transverse mode-coupling instability and space charge**

T. Zolkin* and A. Burov
*Fermilab, P.O. Box 500, Batavia, Illinois 60510-5011, USA*

B. Pandey
*The University of Chicago, 5801 S Ellis Avenue, Chicago, Illinois 60637, USA*

Blaskiewicz' linear rule for TMCI threshold $W_{th} \propto \Delta Q_{sc}$ is essentially universal.

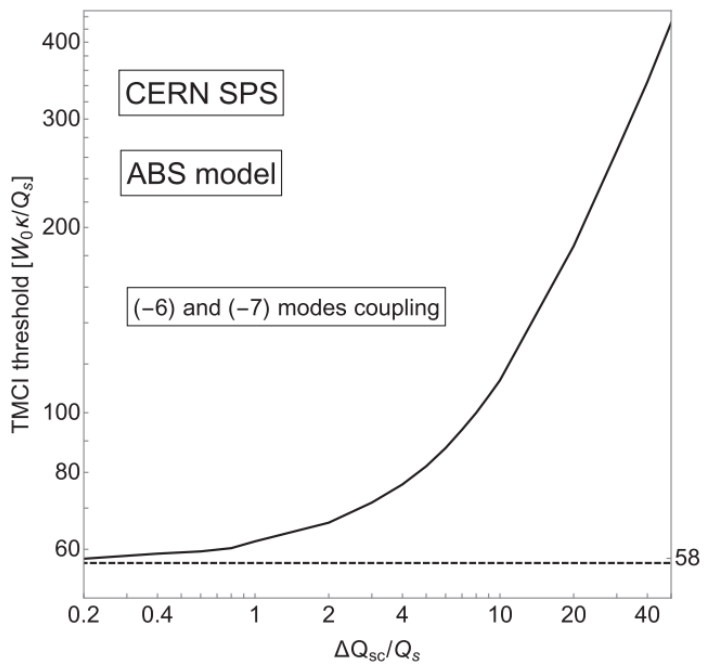
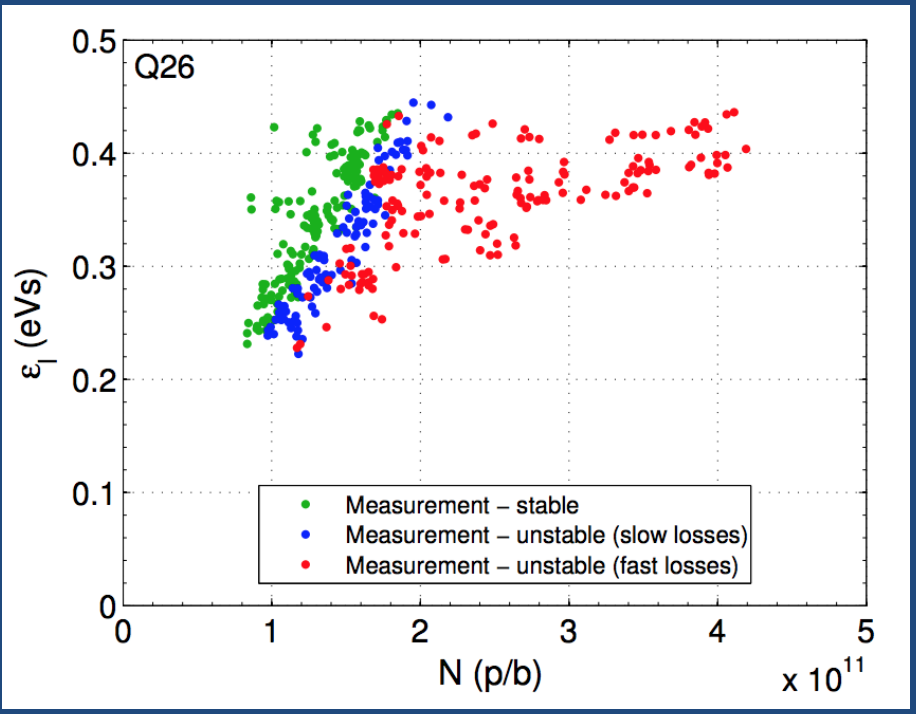



FIG. 18. The lowest TMCI threshold as a function of space charge for CERN SPS ring (ABS model). Dashed line shows the value of threshold at zero $\Delta Q_{sc}$.

# SPS Instability

H. Bartosik, *Beam dynamics and optics studies for the LHC injectors upgrade*, Ph.D. thesis, TU Vienna (2013-10-23).



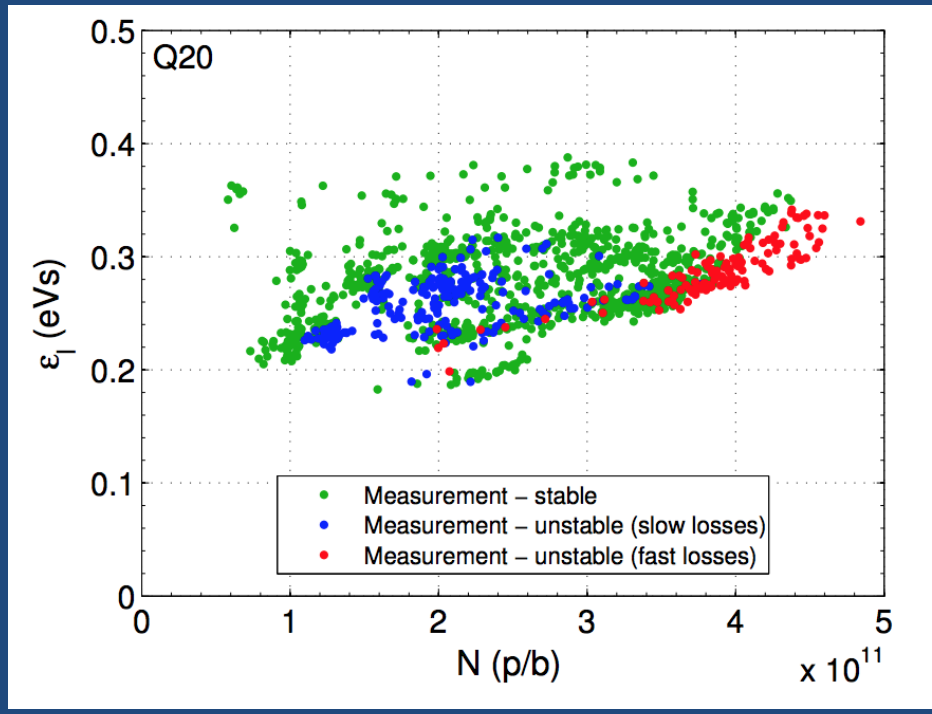




**Figure 4.17**: Experimental study of beam stability at injection with the Q26 optics (top) and Q20 optics (bottom). Each data point shows the longitudinal emittance measured in the PS before bunch rotation as a function of the beam intensity at the PS extraction, where the color code indicates if the beam was stable in the SPS. Green points correspond to stable cases. Unstable cases are marked by red dots if losses occurred within the first 1000 turns after injection ("fast losses") and blue dots if losses occurred later in the cycle.

These measurements reasonably agree with no SC TMCI threshold,
but SC was really huge at Q26, q > 20.
These measurements apparently contradict to everything in the TMCI @ SSC theory!

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# Resolution of the contradictions

PHYSICAL REVIEW ACCELERATORS AND BEAMS **22,** 034202 (2019)

**Convective instabilities of bunched beams with space charge**

A. Burov*

Beam stability requires more than $\Im\,\nu \leq 0$ !

# Centroid oscillations, strong-strong case

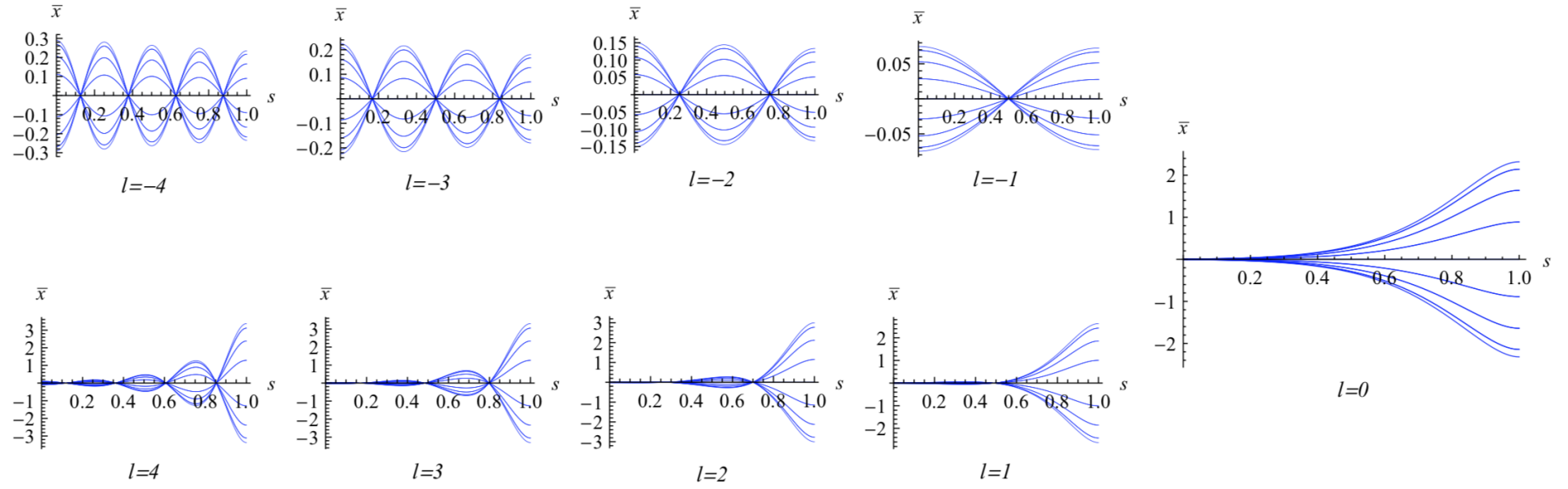



FIG. 6. Stroboscopic images of the centroid oscillations for the same parameters and modes as in Fig. 5. Number of nodes for each mode is identical to the modulus of its number.

$q = 20 \qquad w = 13$

Threshold w=115 at this q

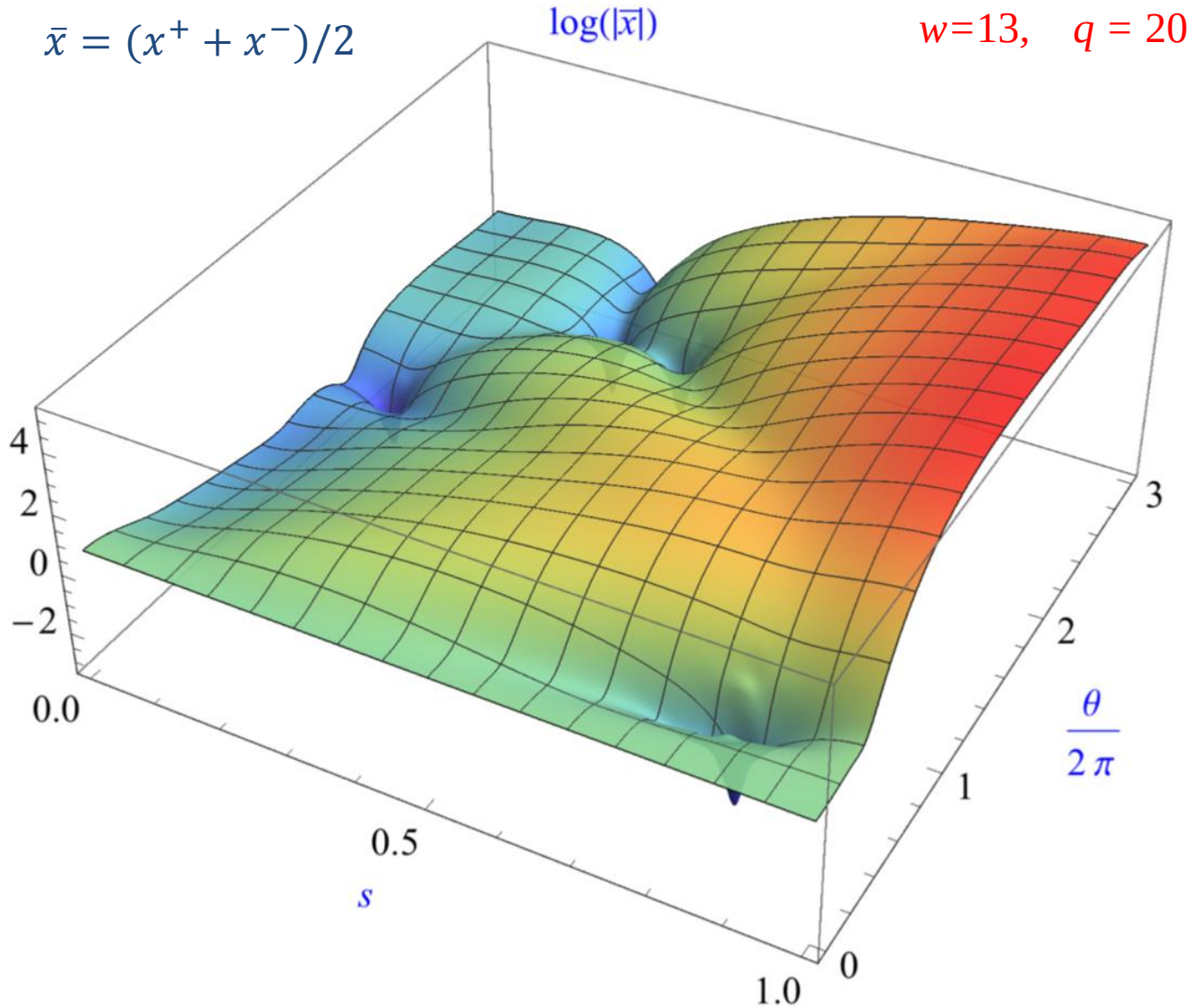
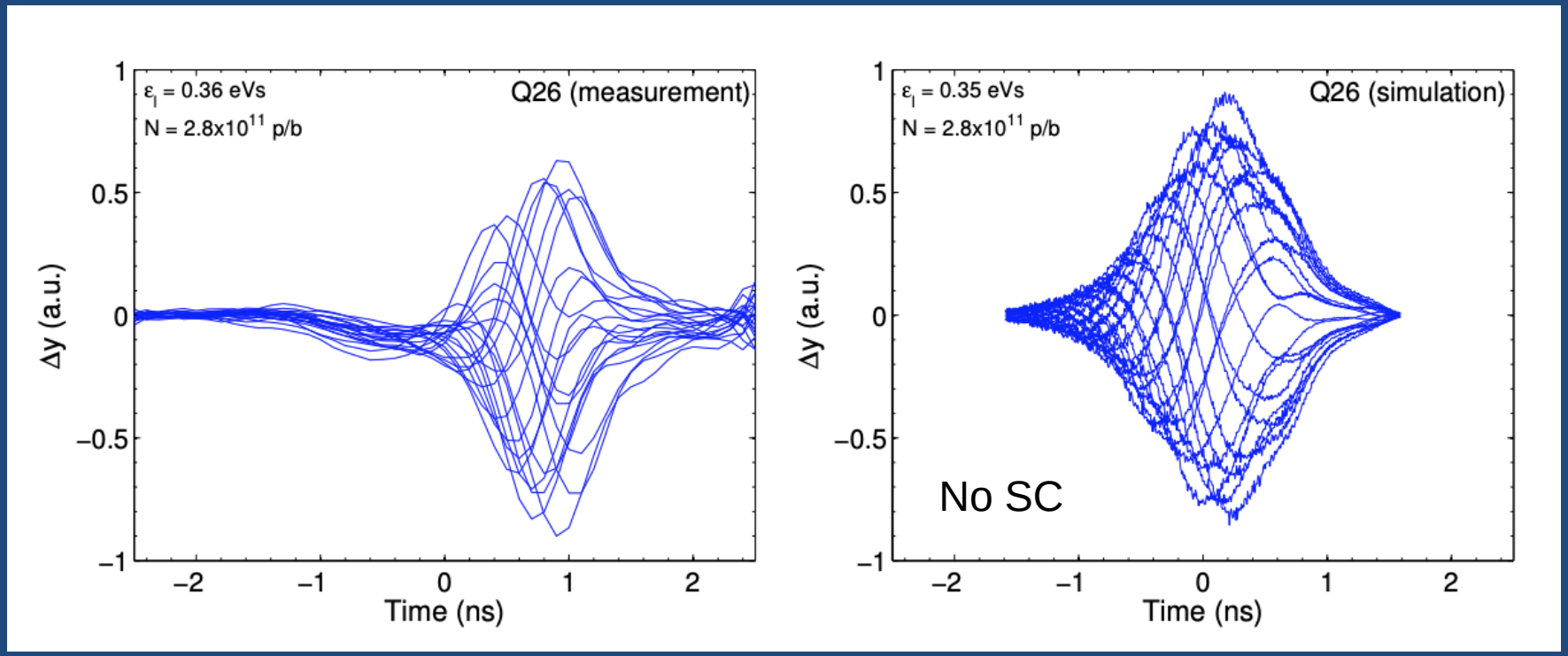



FIG. 12. Time evolution of the local centroids $\bar{x}(\theta, s) = [x^+(\theta, s) + x^-(\theta, s)]/2$ for the same case, i.e. for $q = 20$, $w = 13$ and constant initial conditions, $x^{\pm} = 1$. The amplification is saturated within ~1 synchrotron period.

# SPS Observations

H. Bartosik, CERN, Thesis, 2013




The main intensity limitation for single bunches in the SPS has been identified as transverse mode coupling instability (TMCI) at injection [67], which is mainly caused by the vertical beam coupling impedance of the kicker magnets [68]. The threshold

PHYSICAL REVIEW ACCELERATORS AND BEAMS **22,** 034202 (2019)

**Convective instabilities of bunched beams with space charge**

A. Burov*

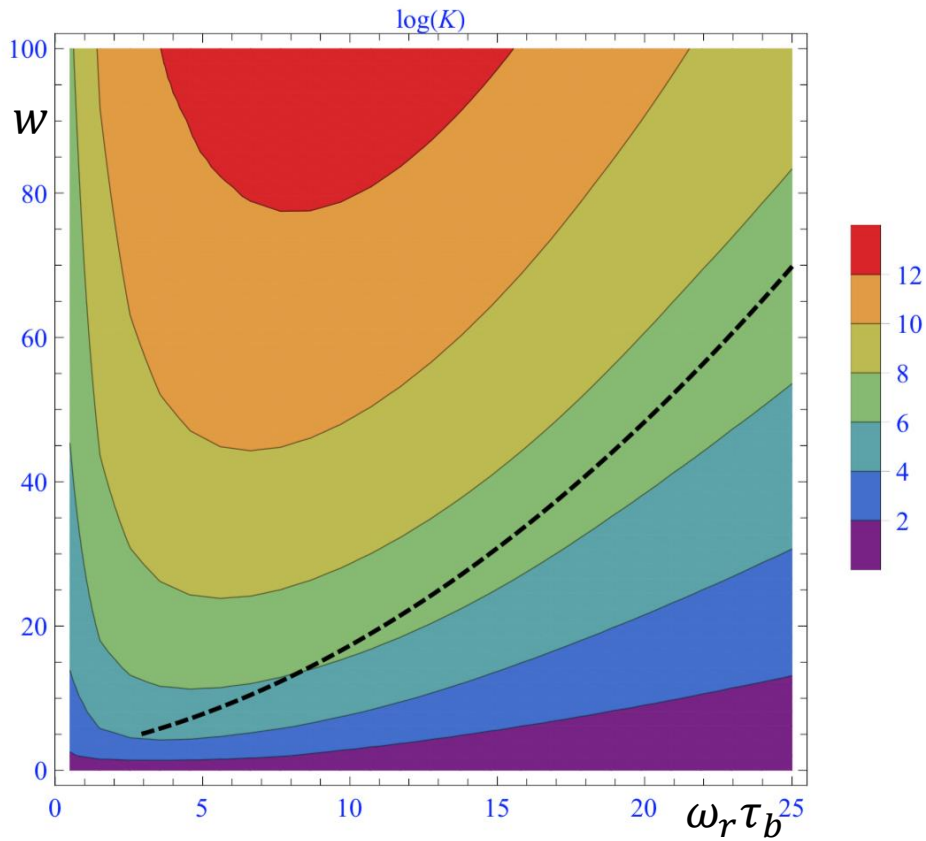
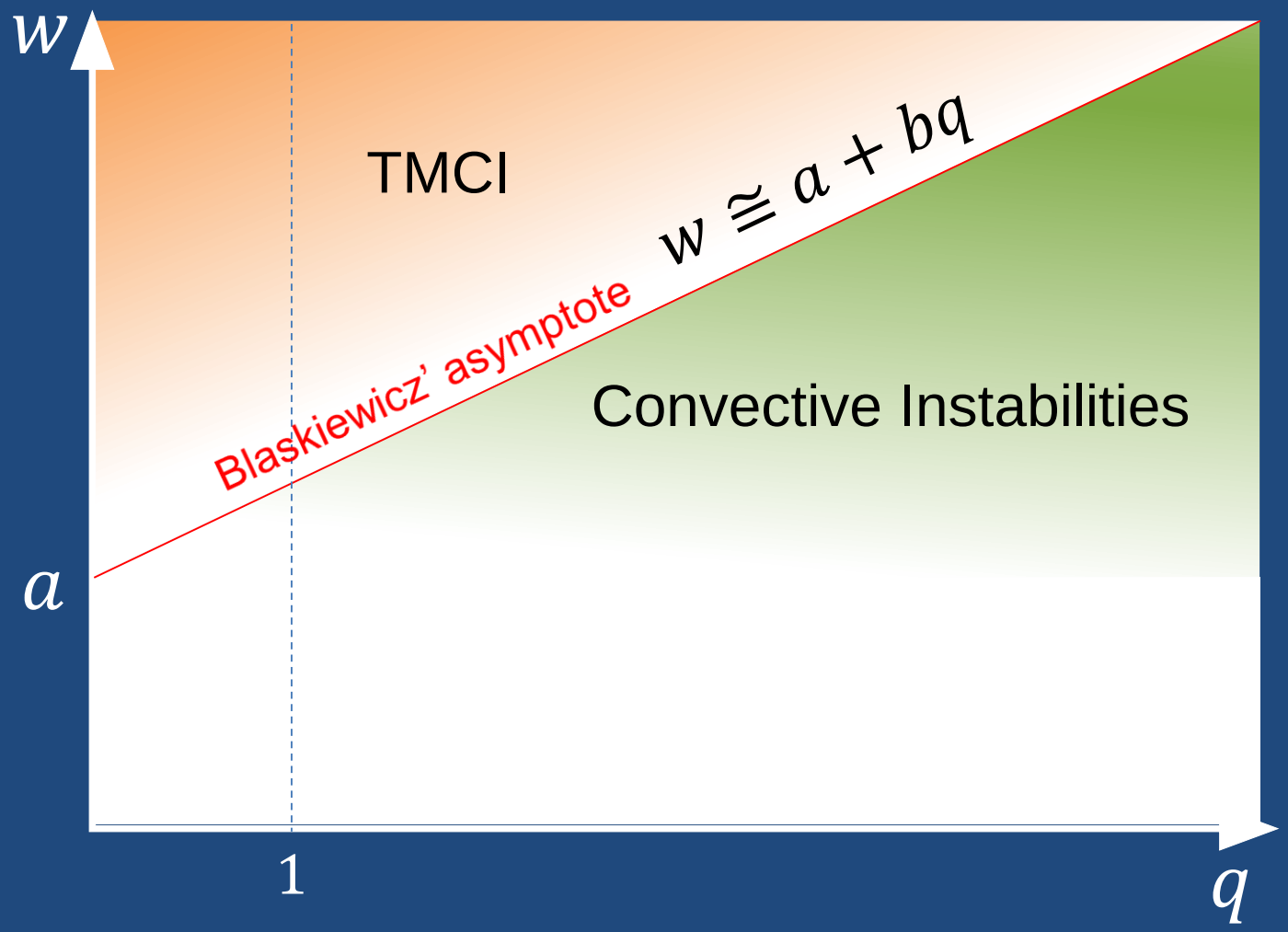



FIG. 21. Amplification for SC $q = 20$. The black dashed line of no-SC TMCI threshold is close to the contour line $K \simeq 300 - 1000$ for large interval of the phase advances. For the entire area of the parameters, the system is absolutely stable, $\Im\nu = 0$.

$q = 20$

# Resolution of the Conundrum



# PS Observations

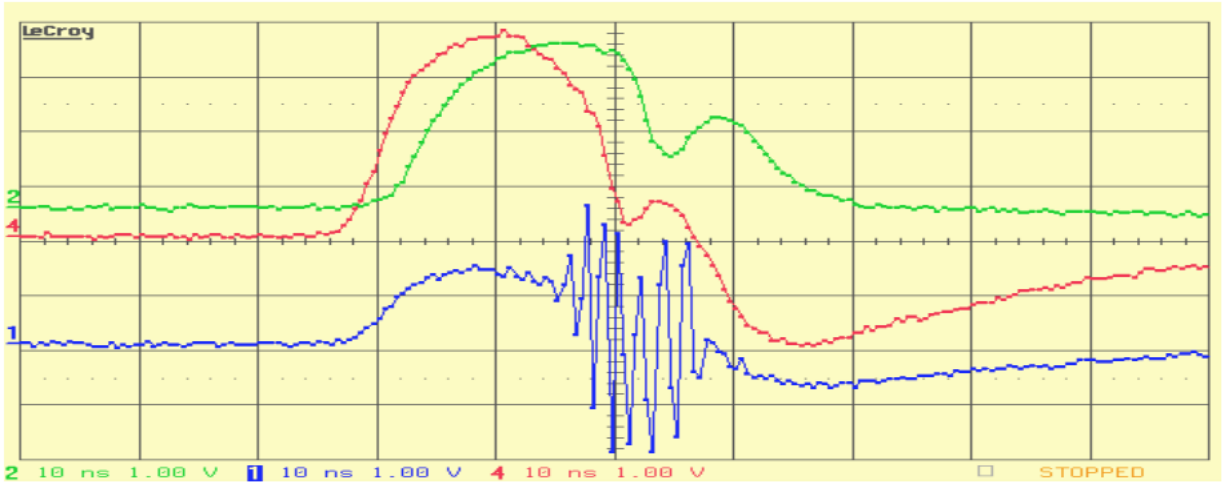



Figure 4: Fast instability observed in the CERN PS near transition (~6 GeV total energy) in 2000. Single-turn signals from a wide-band pick-up. From top to bottom: Σ, Δx, and Δy. Time scale: 10 ns/div. The head of the bunch is stable and only the tail is unstable in the vertical plane. The particles lost at the tail of the bunch can be seen from the hollow in the bunch profile.

E. Metral, HB 2005

CERN-ATS-Note-2013-038 MD
January 31, 2013
v.kornilov@gsi.de

## MD at the PS: Beam Break-Up Instability Near Transition

V. Kornilov, GSI Helmholtzzentrum, Darmstadt, Germany
S. Aumon, S. Gilardoni, P. Freyermuth, CERN, Geneva

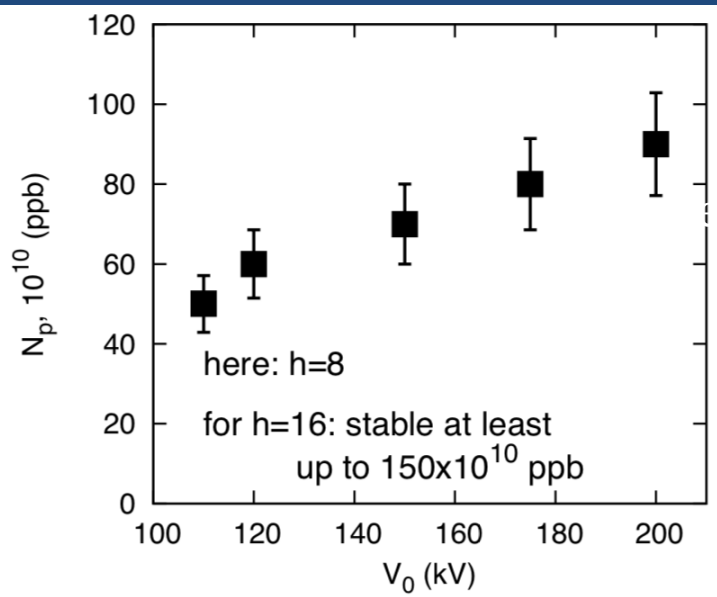



Figure 8: Intensity threshold of the Beam Break-Up Instability in the PS during transition (without $\gamma_t$ jump), as a function of the rf voltage for $h = 8$. For the intensities above the presented values (protons per bunch) strong, fast losses have been observed.

$\epsilon_s = 0.3$ eVs

$\epsilon_\perp = 1\ \mu m$

PHYSICAL REVIEW ACCELERATORS AND BEAMS **21,** 120101 (2018)

# Instability studies at the CERN Proton Synchrotron during transition crossing

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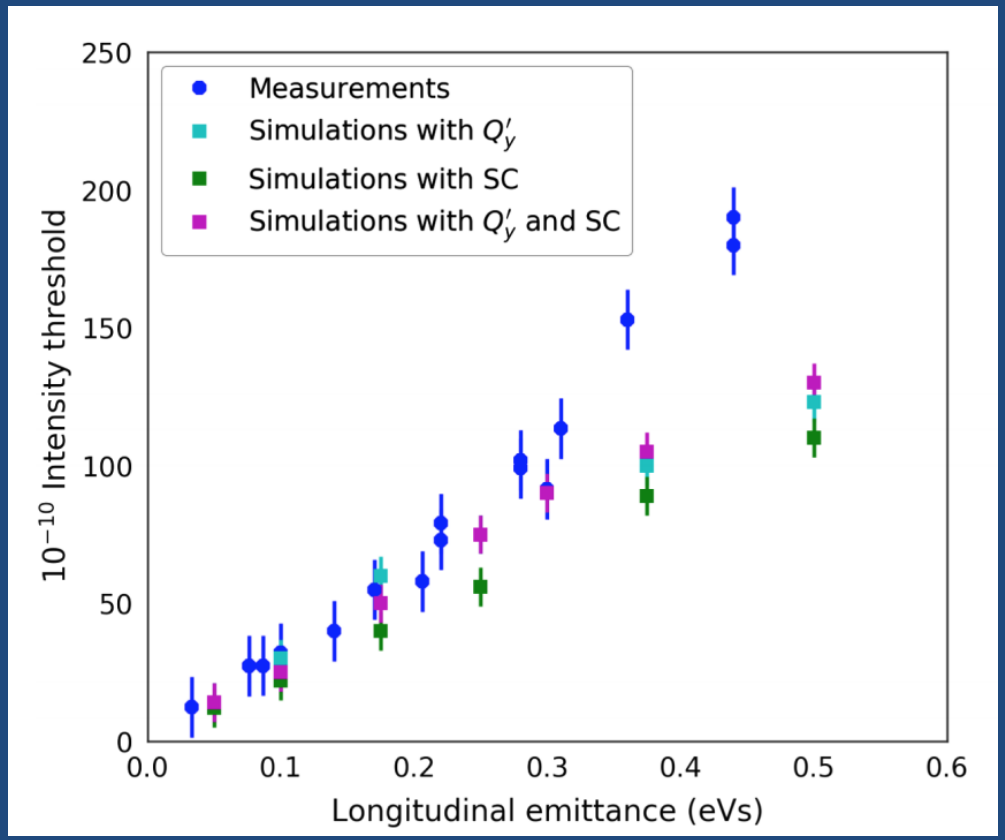
*University of Rome 'La Sapienza' and INFN Sez. Roma1—00185 Roma—Italy and CERN, 1211 Geneva, Switzerland*

S. Aumon, E. Koukovini-Platia, A. Huschauer, E. Métral, and G. Sterbini
*CERN, 1211 Geneva, Switzerland*

N. Wang
*Institute of High Energy Physics, 100049 Beijing, China*



PHYSICAL REVIEW ACCELERATORS AND BEAMS **22,** 068001 (2019)

**Comment on "Instability studies at the CERN Proton Synchrotron during transition crossing"**

A. Burov*

in many publications for the last twenty years. Do the observations and modeling of the Article refute the long-standing idea that the PS instability is the TMCI? Do they rather confirm it? Do they neither refute, nor confirm it? Apparently, we deal here with new physics, so the question is important.

PHYSICAL REVIEW ACCELERATORS AND BEAMS **22,** 068002 (2019)

**Reply to "Comment on "Instability Studies at the CERN Proton Synchrotron during transition crossing""**

M. Migliorati,[1,2] S. Aumon,[2] E. Koukovini-Platia,[2] A. Huschauer,[2]
E. Métral,[2] G. Sterbini,[2] and N. Wang[3]

tion of the paper. Therefore, the answer to the question of the Author of the Comment is that we neither refute, nor confirm that the PS instability is the TMCI because we did not investigate that.

As suggested by the Author in the conclusion, if, apparently, we deal here with new physics, it could be very useful to check if the theory developed in [2] can reproduce the thresholds, as those reported in Fig. 14 of [1], under the different conditions of the machine, determining also the role of the space change in this instability at different longitudinal emittances.

[1] M. Migliorati, S. Aumon, E. Koukovini-Platia, A. Huschauer, E. Métral, G. Sterbini, and N. Wang, preceding Comment, Instability studies at the CERN Proton Synchrotron during transition crossing, Phys. Rev. Accel. Beams **21**, 120101 (2018).

[2] A. Burov, Convective instabilities of bunched beams with space charge, Phys. Rev. Accel. Beams **22**, 034202 (2019).

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arXiv.org > physics > arXiv:1908.02916

Physics > Accelerator Physics

# Transverse Microwave Convective Instability at Transition Crossing

Alexey Burov

*(Submitted on 8 Aug 2019)*

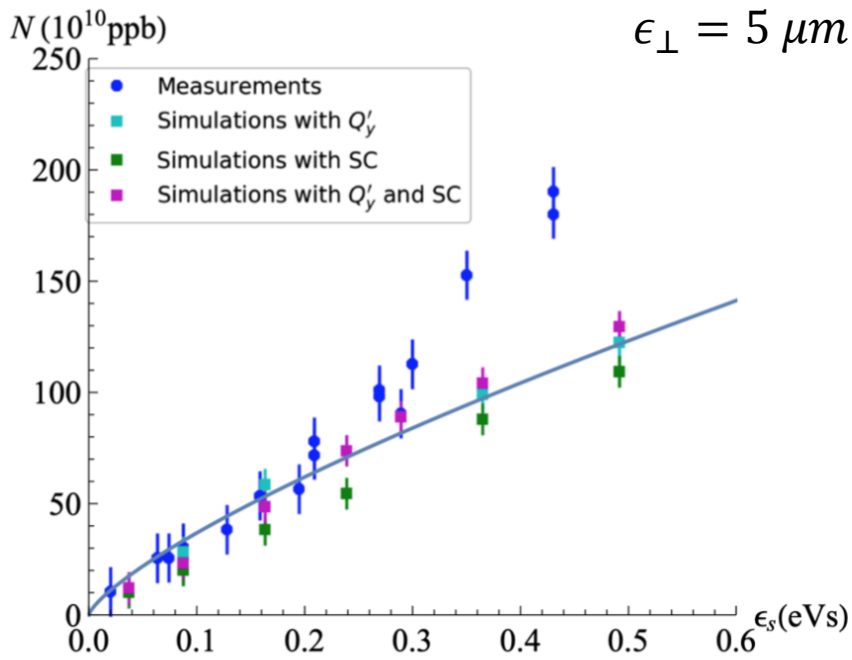


FIG. 1. Threshold number of protons per bunch (ppb) versus longitudinal rms emittance for the PS bunch with the reported transverse rms emittance $5\mu$m [5]. The line corresponds to our formula (23), the dots copy the data of Fig. 14 of Ref. [5]

$$\frac{w}{3\alpha^2 b}\sqrt{\frac{w\,\omega_{\rm sc}}{\kappa}} = \Lambda_{\rm th}\,. \qquad (23)$$

$$\Lambda_{th} = \ln(a/x_0) = 18$$

$$x_0 = \sigma_x/\sqrt{N_{eff}}$$

$$W(s) = W_0 \exp(-\alpha s)\,\sin(\kappa s)$$

$$b = 2\frac{\dot{\gamma}}{\gamma^3}\frac{\delta p}{p}$$

$$N_{\rm th} \propto \epsilon_s^{3/4}\epsilon_\perp^{1/4}$$

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CERN-ATS-Note-2013-038 MD
January 31, 2013
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## MD at the PS: Beam Break-Up Instability Near Transition

V. Kornilov, GSI Helmholtzzentrum, Darmstadt, Germany
S. Aumon, S. Gilardoni, P. Freyermuth, CERN, Geneva

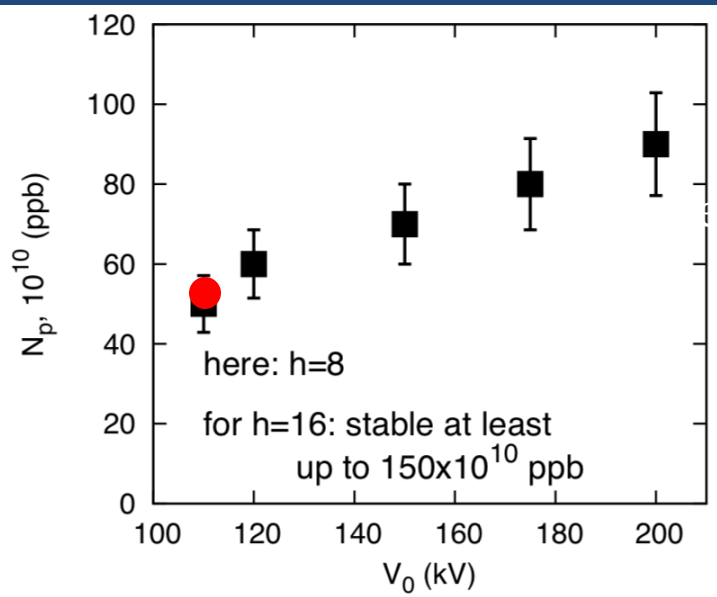


Figure 8: Intensity threshold of the Beam Break-Up Instability in the PS during transition (without $\gamma_t$ jump), as a function of the rf voltage for $h = 8$. For the intensities above the presented values (protons per bunch) strong, fast losses have been observed.

Red dot ● is my

theoretical result
for these emittances,

$\epsilon_s = 0.3$ eVs

$\epsilon_\perp = 1\ \mu m$

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# Core-Halo Collective Instabilities

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*Fermilab, PO Box 500, Batavia, IL 60510-5011*
(Dated: August 28, 2018) https://arxiv.org/pdf/1808.08498.pdf

At strong space charge, transverse modes of the bunch core may effectively couple with those of the halo, leading to instabilities well below the core-only transverse mode-coupling threshold.

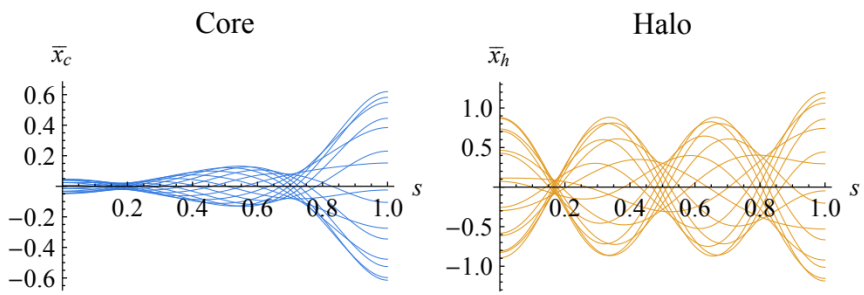



FIG. 2. Centroid stroboscopic images of the core and halo components of the most unstable core-halo mode for the same $q$ and $w$ as in Fig. 1, at the most unstable $\tilde{q} = 0.29$. Waists instead of nodes in the halo image tell about an absolute instability.

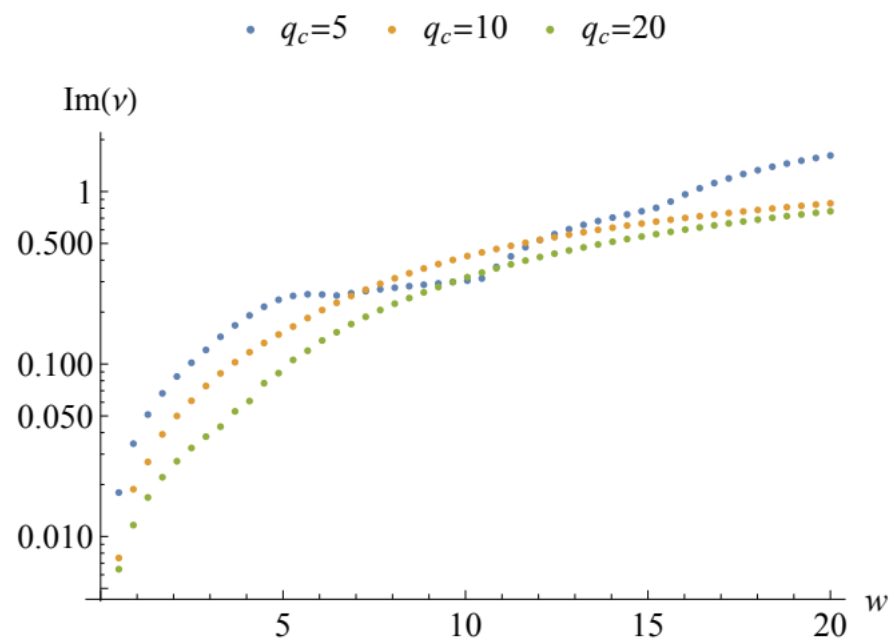



FIG. 4. Growth rates of the most unstable modes versus wake parameter for three different SC parameters. Note the conventional TMCI threshold for $q_c = 5$ at $w \approx 15$.

arXiv.org > physics > arXiv:1809.06927

Physics > Accelerator Physics

# Transverse Instabilities of a Bunch with Space Charge, Wake and Feedback

Alexey Burov

*(Submitted on 18 Sep 2018)*

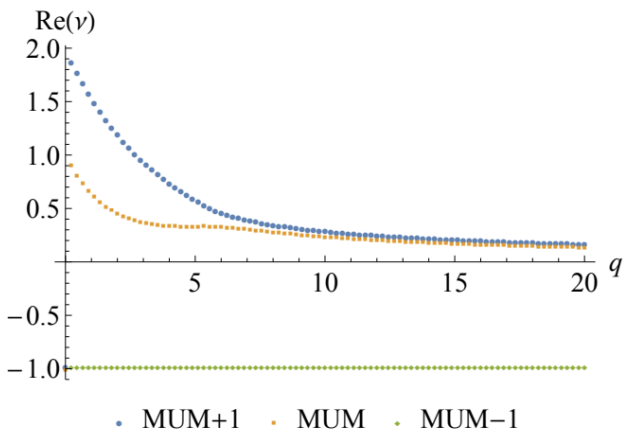
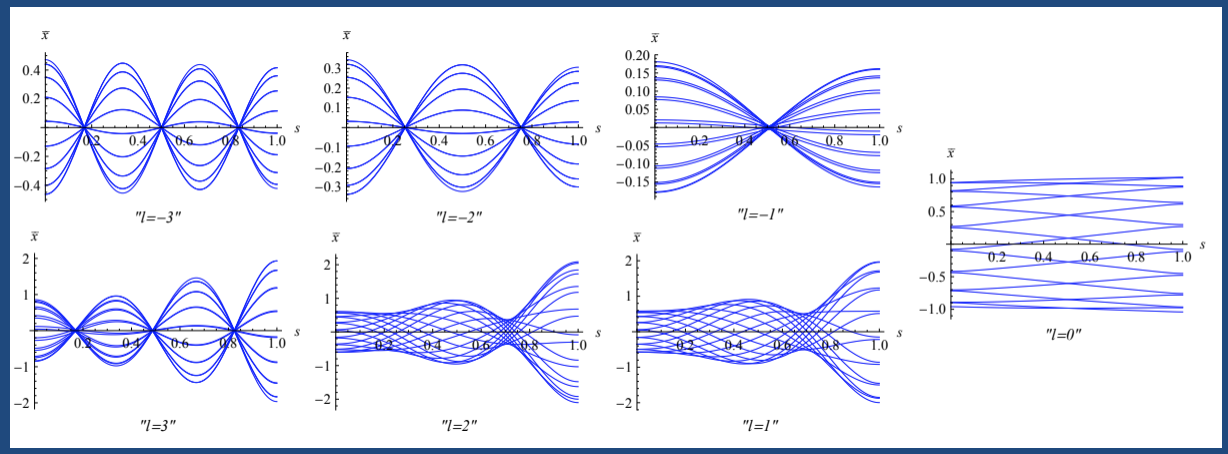
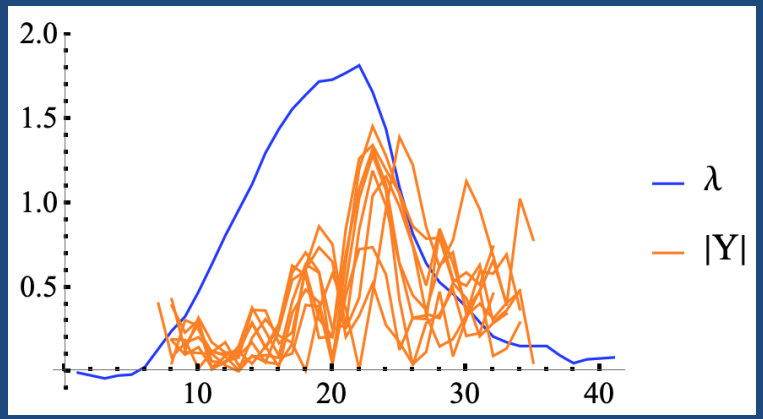
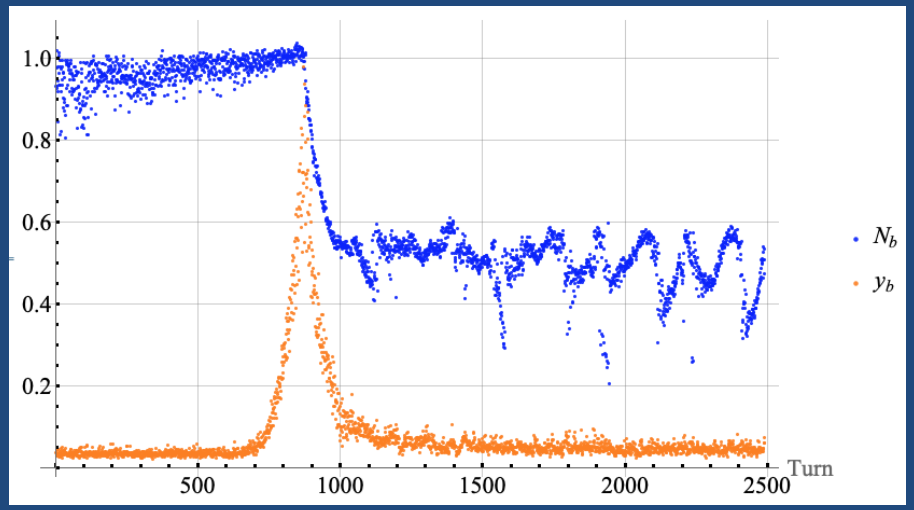
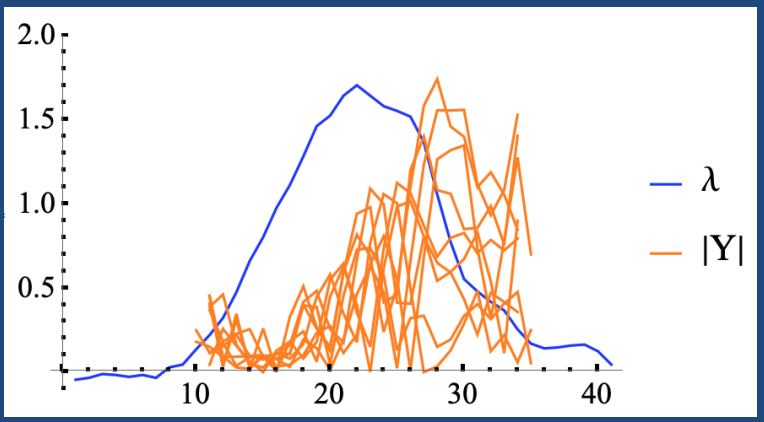



FIG. 3. With sufficient gain and wake, SC becomes a mode coupling factor, as in here, with $+1^{st}$ and $+2^{nd}$ modes coupling. The wake is same, $w = 2$; the gain $g = 5$. MUM is $+1^{st}$. Mode 0 (green dots) is insensitive to SC, being just shifted down by $w/2 = 1$.



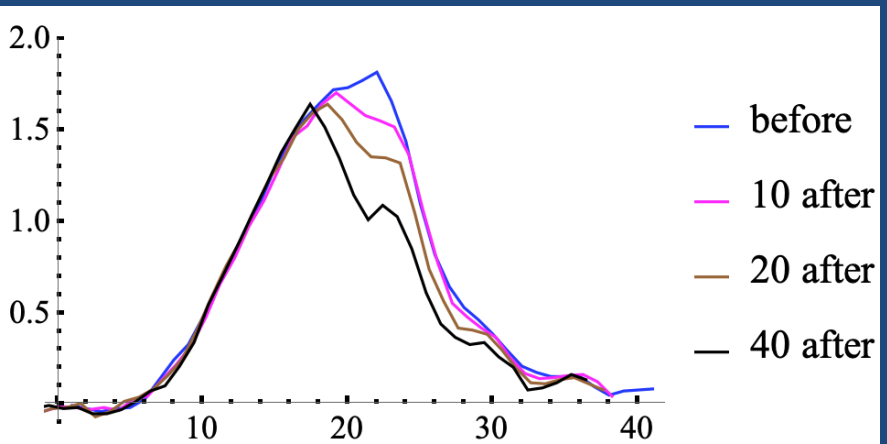



Convective instability at the FNAL Booster,

July 2nd, 2019



Mitigations are left totally out of my talk:

Chromaticity (limited by incoherent SC)

Octupoles (hardly useful at SSC)

Landau e-lenses (Alexahin's talk)

Feedbacks (Lebedev's talk)

Landau cavities (may be a bit helpful at SSC)

RFQ (should be studied for SSC)

*Many thanks!*

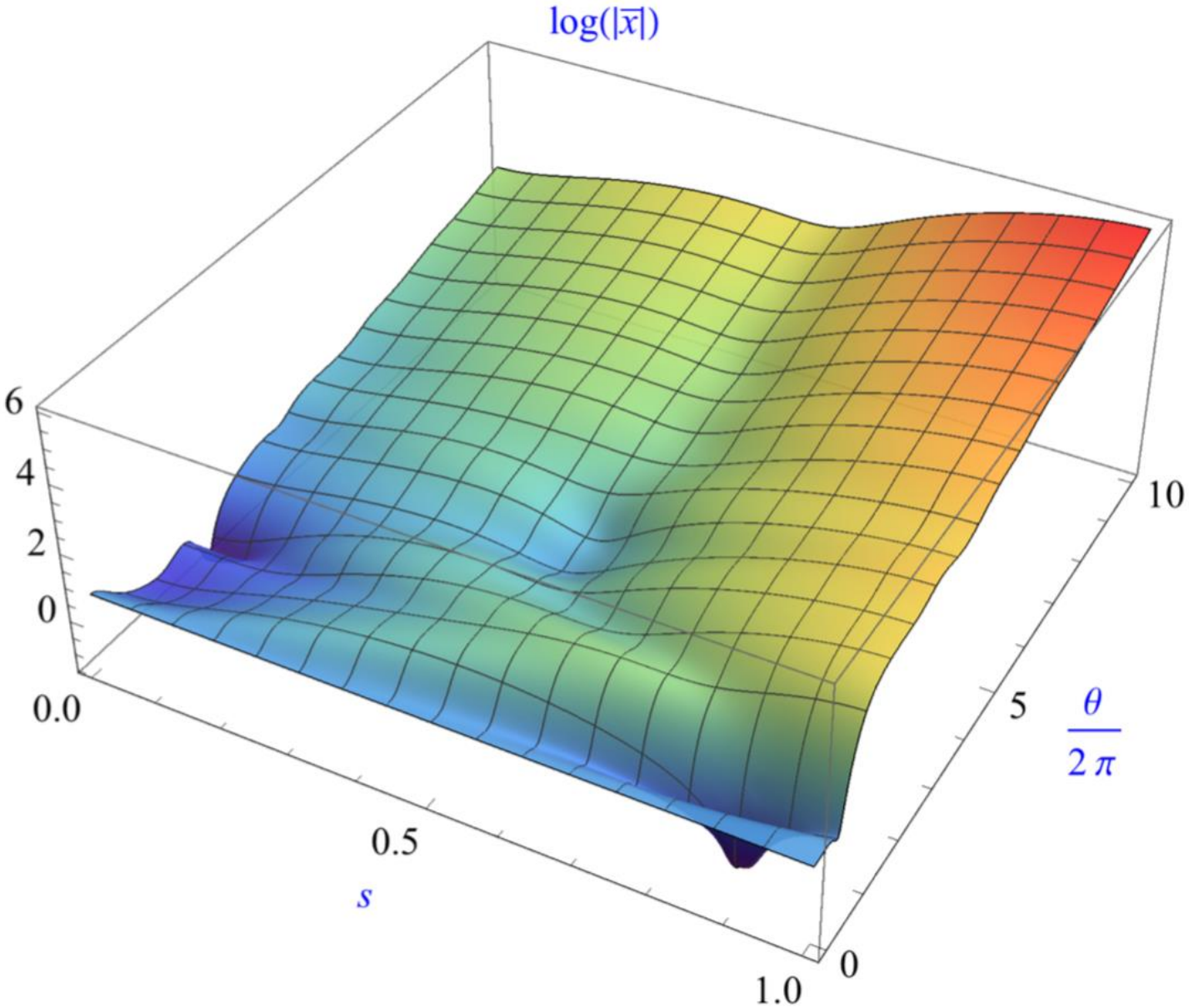



FIG. 17. Time evolution of the ACI for the same parameters as Fig. 16. An exponential growth is clearly seen.