BNS DAMPING

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“Mitigation of Coherent Beam Instabilities in particle accelerators"

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Some history

- In the middle of 70s of the last century the director of BINP **Andrei Mikhailovich Budker** esteblished a new small group of researchers under the leadership of Vladimir Balakin to study a new type of an accelerator for colliding beams. He called this machine a "SuperLinac", what we now call "Linear Colliders"
- The main task of the study was
  - To demonstrate a possibility to achieve a high gradient of acceleration of order of 1 MV/cm
  - And to solve all beam dynamics problems
- It happened that it was not the only task. In parallel we worked also on the proton ironless ring of 10 GeV, which Andrei Mikhailovich extremely want to build at the end of his life to study very exciting physics of heavy nucleolus, predicted by Spartak Belayev, who was at that time a president of the Novosibirsk State University.

Some history. Continue.

- However all studies were kept in secrete and it was not allowed to show or publish results outside the laboratory.
- What happened later. We solved practically all problems;
  - We developed a technology of fabricating high gradient accelerating structure. We managed to achieve almost 2 MV/cm in a cavity.
  - We understood main beam dynamics problems and found solutions for almost all of them
- Unfortunately Andrei Mikhailovich died in 1977 just before he became a sixty.
- A new director of BINP, a real successor of Budker, Alexander Nikolayevich Skrinsky greatly supported the activity on Linear Colliders.
- The project of the BINP Linear Collider (VLEPP) was first presented at the International Symposium devoted to 60 year anniversary of A. M. Budker and All-union particle accelerator Conference in Dubna. The results published at BINP in Russian were also translated to English at SLAC in 1978.

Single bunch instability and BNS damping

- One of the main beam dynamics problems was a transverse beam instability or beam break-up effect, which limited the acceleration of the very high intense beams, needed to achieve high luminosity.
- We developed a method to calculate electromagnetic interaction of the beam with a metal accelerating structure.
- With a very precise description of the forces acting on the bunch particles we started doing beam dynamics simulation and immediately found a very strong transverse instability of a single bunch.
- Careful analyses of the beam dynamics showed the resonant structure of the instability.
- This important feature was the most impotent point in invention of a new method to damp the transverse instability, which later got the name "BNS Damping" by the first letters of the inventors Balakin, Novokhatski and Smirnov.
- Later BNS Damping has been successfully implemented into the Stanford Linear Collider

Following discussion

- We will start with the description of the electromagnetic force
- Then we present an equation for particle motion
- Solution for the case with a strong focusing
- Physics of BNS damping
- Two particle model
- Exact solution in some special cases
- Efficiency of the method
- Comparison with Landau damping

Electromagnetic fields and wake field code

- At that time it was not so much clear about the action of the electromagnetic force on the particles of a single bunch. To understand the behavior of these forces we developed a computer code for the time domain electromagnetic calculations. It was the first wake field code in the world.
- With this code we calculate all components of the electromagnetic field excited by a single bunch in an accelerating structure and evaluate the averaged longitudinal and transverse force

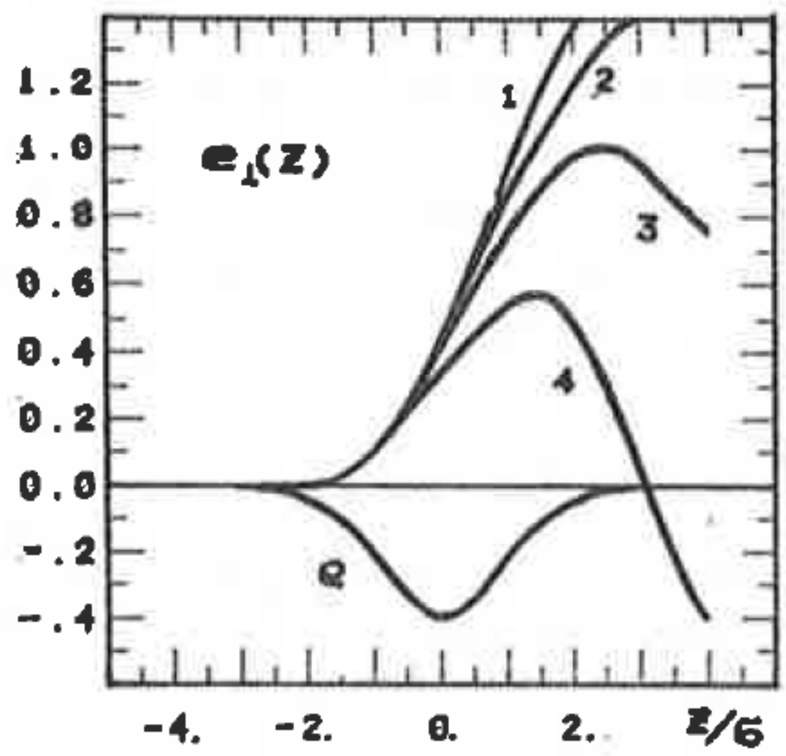
$$F_{\parallel}(s,r_0) = \frac{1}{L}\int_{-L/2}^{L/2} E_z(t, z = ct - s,\ r_0)\, cdt \qquad L \to \infty$$

$$F_{\perp}(s,r_0) = \frac{1}{L}\int_{-L/2}^{L/2} \{E_r(t, z = ct - s,\ r_0) - H_{\varphi}(t, z = cts,\ r_0)\}\, cdt$$

Transverse force Green's function

- The magnitude of the transverse force is determined by the parameters of a bunch and by the structure geometry.
- With our code we calculated electromagnetic fields of the very short bunches
- This gave us a possibility to derive an approximation for the Green's function.

- This code is still the best code for calculating electromagnetic fields of extremely short bunch, as it has a special algorithm.



Transverse force presentation

- Electromagnetic fields generated by a bunch in an accelerating structure have a defocusing action on the bunch particles if they travel off axes. The more leading particles are far away of axes the more effect on the following particles. Naturally the force grows along the bunch.
- Usually a particle trajectory is not very from the axes, so the dipole component plays the main role. With a Green's function we can present the dipole transverse force in the following way

$$F_{\perp}(s) = \frac{eQ}{4\pi\epsilon_0\, a_w^3} \int_{-\infty}^{s} \varrho(\chi)\, X(\xi) g_{\perp}(\xi - s) d\xi$$

Here Green's function $g_{\perp}(s)$ is a dimensionless function , a_w is an effective parameter

Bunch charge distribution is normalized like this

$$\int_{-\infty}^{-\infty} \varrho(\xi)\, \mathrm{d}\xi = 1$$

Equation for the transverse motion

$$\frac{\partial}{\partial \tau}\left(\frac{\gamma}{\gamma_0}\frac{\partial X(\tau,s)}{\partial \tau}\right)+\frac{\gamma}{\gamma_0}\mu^2 X(\tau,s)=\int_{-\infty}^{s}\varrho(\xi)\,X(\tau,\xi)g_{\perp}(s-\xi)\mathrm{d}\xi$$

We introduce a characteristic length

$$L^* = a_w\sqrt{\frac{\gamma_0 mc^2}{\frac{eQ}{4\pi\varepsilon_0 a_w}}}$$

Time and betatron frequency is measured in the characteristic lengths

$$\tau = \frac{ct}{L^*} \qquad \mu = \nu * L^*$$

Instability growth

The characteristic length L^* may be consider to be an instability growth rate parameter

In the case of a strong focusing when $\mu = \nu * L^* \geq 1$

we can evaluate the emittance growth

$$\varepsilon \sim exp\left(\sqrt{\frac{\tau}{\mu}}\right) = exp\left(\frac{1}{L^*}\sqrt{\frac{ct}{\nu}}\right)$$

Physics of the BNS damping

- The particles of the head of a bunch do not experienced action of the wake field and freely oscillate in the focusing lattice at the betatron frequencies.
- However this oscillations produce a periodical force for the particles of the tail of the bunch, which experience the action of the wake field.
- As the frequency of the force and the frequency of free oscillations are the same then amplitude of oscillations of the tail's particles will grow in time because of the resonance.
- An immediate solution for this situation is to destroy the resonance, than means to give different betatron frequencies to the particle of the bunch head and particles of the bunch tail.
- It can be done in many different way, but a simple solution is to utilize the fact that the betatron oscillation frequency depends by virtue of the chromaticity on the energy of the beam particles.

Physics of the BNS damping

- Since the transverse wake field introduces defocusing this additional chromatic focusing can be used for compensation.
- By accelerating the bunch behind the crest of the accelerating field, the tail particles gain less energy than the head. Therefore, the tail particles are focused more by the quadrupoles than the head.
- The longitudinal wake field actually helps to increase the energy spread. The tail particles loss more energy due to the action of this field.
- With increasing of the particle energy during the acceleration, the energy difference can be reduced. The beam break up effect becomes small $\sim\frac{\gamma_0}{\gamma}$ and the bunch is now moved ahead of the crest to reduce the energy spread in the beam.

Two particle model

- Two particles (head and tail) have different betatron frequencies and $\gamma = const,\ g_{\perp}(s) = 2$

$$\frac{\partial^2}{\partial \tau^2} X_H(\tau) + \mu_H^2\, X_H(\tau) = 0 \qquad X_H(\tau) = cos(\mu_H \tau)$$

$$\frac{\partial^2}{\partial \tau^2} X_T(\tau) + \mu_T^2\, X_T(\tau) = X_H(\tau) \qquad X_T(\tau) = 1 + \frac{cos(\mu_H \tau) - cos(\mu_T \tau)}{\mu_H^2 - \mu_T^2}$$

- To keep the amplitude of $X_T(\tau)$ around 1 we need $\Delta\mu = \mu_T - \mu_h \geq \frac{1}{2\mu}$

$$\frac{\Delta\nu}{\nu} = \frac{\Delta\mu}{\mu} \geq \frac{1}{2\,\mu^2} = \frac{1}{2\,\nu^2\,(L^*)^2} = \frac{eQ}{2\,\nu^2 4\pi\varepsilon_0 a_W^3\, \gamma_0 m c^2}$$

BNS damping condition

Analytical solutions

- The equation for particle motion (slide 9) is rather complicated for analytical solution but can be solved by using numerical methods.
- However, some properties of the BNS damping can be found on the basis of the solutions of a more simple equation with the following assumption

$$\frac{\partial \gamma}{\partial \tau} = 0 \quad \rho(\xi) = const\ [0,1] \ \ \mathrm{g}(\xi) = const \ \ \mu(s) = \mu_0\left(1 + \frac{\Delta\mu}{\mu_0} s\right) \quad \gamma(s) = \gamma_0\left(1 - \frac{\Delta\mu}{\mu_0} s\right)$$

- Now the equation takes the following form

$$\frac{\partial^2 X(\tau,s)}{\partial \tau^2} + \mu^2 X(\tau,s) = \frac{\gamma_0}{\gamma(s)} \int_0^s X(\tau,\xi)\mathrm{d}\xi$$

We found the way to solve this equation analytically using the Laplace transformation because this problem is a problem with initial conditions

Analytical solutions

- The Laplace transform

$$\mathrm{V}(s,p) = \int_0^\infty X(s,\tau)\, e^{-p\tau} d\tau$$

We get an analytical solution in the Laplace presentation

$$V(p) = \frac{\gamma_0}{\gamma} \frac{pX_0}{\mu^2 + p^2} \left(\frac{\mu^2 + p^2}{\mu_0^2 + p^2}\right)^{\eta} \left(1 - \mu_0 \int_{\mu_0}^{\mu} \left(\frac{\mu^2 + p^2}{\mu_0^2 + p^2}\right)^{-\eta} \frac{d\mu}{\mu^2}\right)$$

$$\eta = \frac{2\mu_0^2}{\frac{\Delta\mu}{\mu_0}}$$

Ratio of the BNS damping to a total betatron spread

Solutions for integral value of η

- The inverse Laplace transform is easy to derive for integer values of η

- $\eta = 0$ There are no transverse forces. Just to check the model.
  Free oscillations with natural frequencies

$$X(s,\tau) = X_0 \frac{\gamma_0}{\gamma(s)} Cos(\mu(s)\tau)$$

- $\eta = 2$ Instability.
  Build-up of oscillations at a frequency of the 'head'

Resonant growth

$$X(s,\tau) = X_0 \frac{\gamma_0}{\gamma(s)} \left\{ Cos(\mu_0\tau) + \frac{\mu^2(s) - \mu_0^2}{2\mu_0^2} (\mu_0\tau)\, Sin(\mu_0\tau) \right\}$$

Solutions for integral value of η

- $\eta = -1$ Instability.
  Build-up of oscillations at natural frequencies

$$X(s,\tau) = X_0 \frac{\gamma_0}{\gamma(s)} \left\{ Cos(\mu_0 \tau) + \frac{\mu^2(s) - \mu_0^2}{2\mu_0^2} (\mu(s)\tau)\, Sin(\mu(s)\tau) \right\}$$

- $\eta = 1$ BNS damping. All particles oscillate at the frequency of the “head”

$$X(s,\tau) = X_0 \frac{\gamma_0}{\gamma(s)} Cos(\mu_0 \tau)$$

- This nice behavior possible for other Green’s function

One more exciting solution for η=0.5

- $\eta = 0.5$ The amplitude of oscillation is going down

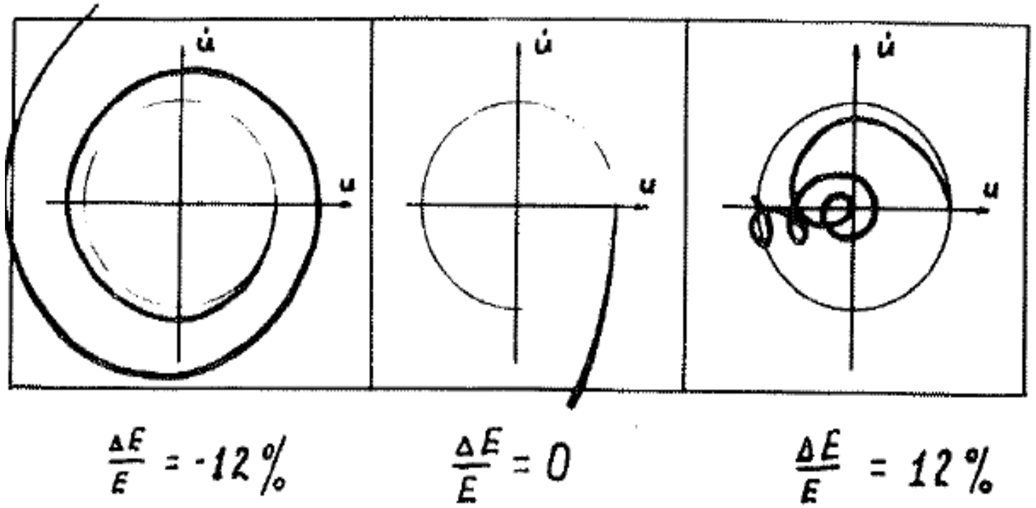
$$X(s,\tau) = X_0 \frac{\gamma_0}{\gamma(s)} J_0\left(\frac{\mu(s) - \mu_0}{2}\tau\right) Cos(\mu(s)\tau)$$

J_0 Bessel function.
The amplitude of oscillations is damped in time as $\frac{1}{\sqrt{\tau}}$

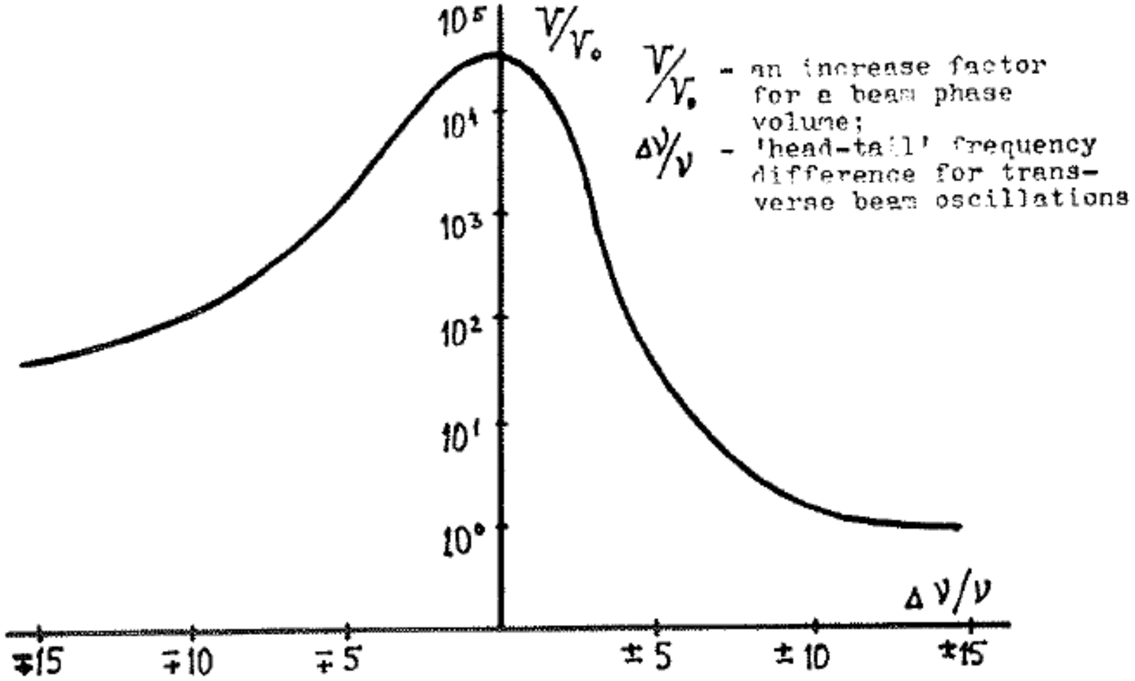
Unfortunately, analysis shows that this interesting result holds only for the constant Green's function

Results of computer simulations

Computer simulations with realistic Green's function and bunch distribution showed the same particle dynamics



X' X phase plot for different energy spreads.



Relative emittance at the exit of the 100 GeV accelerator section versus the initial energy spread. With the initial energy spread of 12% the beam can reach at the section exit with a minimally achievable spread of 3%

BNS damping condition

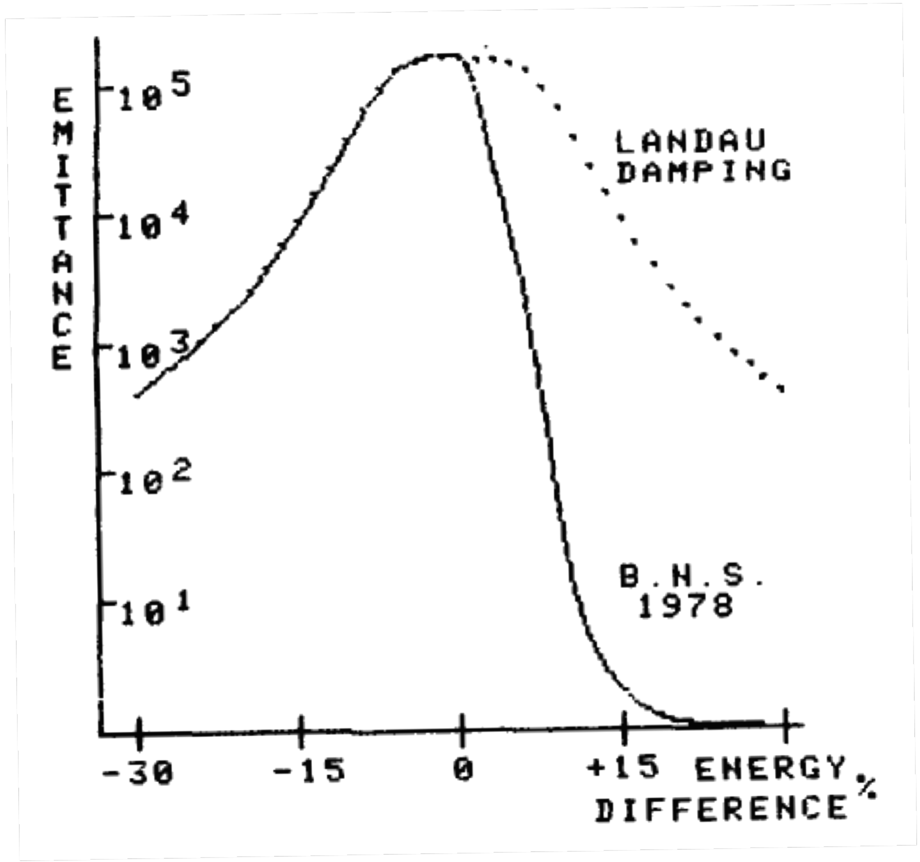
- To damp the transverse instability we introduce a linear variation in particle energy along the bunch by a phase tuning of the accelerating field. The energy variation along the bunch leads to variation of frequencies of the betatron oscillations and hence damping the transverse instability. The sign of the energy spread is very important and: The energy of the tail particles must be smaller than energy of the head particles.

$$\frac{\Delta E}{E} \geq \frac{1}{2\,\mu^2} = \frac{1}{2(\upsilon a_w)^2}\,\frac{\frac{eQ}{4\pi\varepsilon_0 a_w}}{\gamma_0 mc^2}$$

- The efficiency of the BNS damping is the higher for a larger number of periods of transverse oscillations.

Comparison BNS and Landau damping

It is possible to decrease the instability growth using also Landau damping, However a comparison with BNS damping showed that Landau damping is not so effective as BNS damping.



Landau damping works something like BNS damping but with an opposite sign of the energy spread and cannot damp instability completely

Summary

- BNS damping is a very efficient method for damping the transverse instability in a linear accelerator
- Naturally it works in the multi-bunch regime as well.
- SLC, the first linear collider using this method increased luminosity several times.
- BNS damping was effectively used in the injector of intense beams for the SLAC PEP-II B-factory.
- In the linacs BNS damping works better than Landau damping.

Acknowledgement

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