



U.S. DEPARTMENT OF  
**ENERGY**

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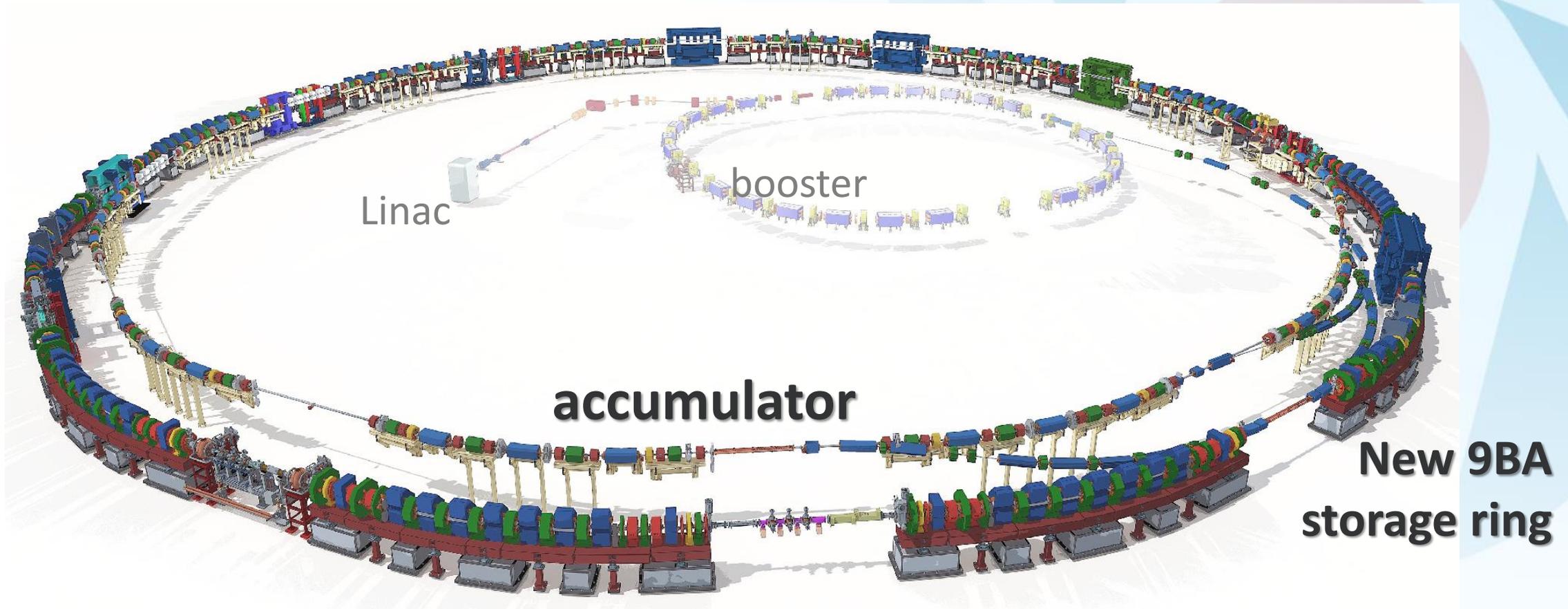


# When Landau doesn't damp

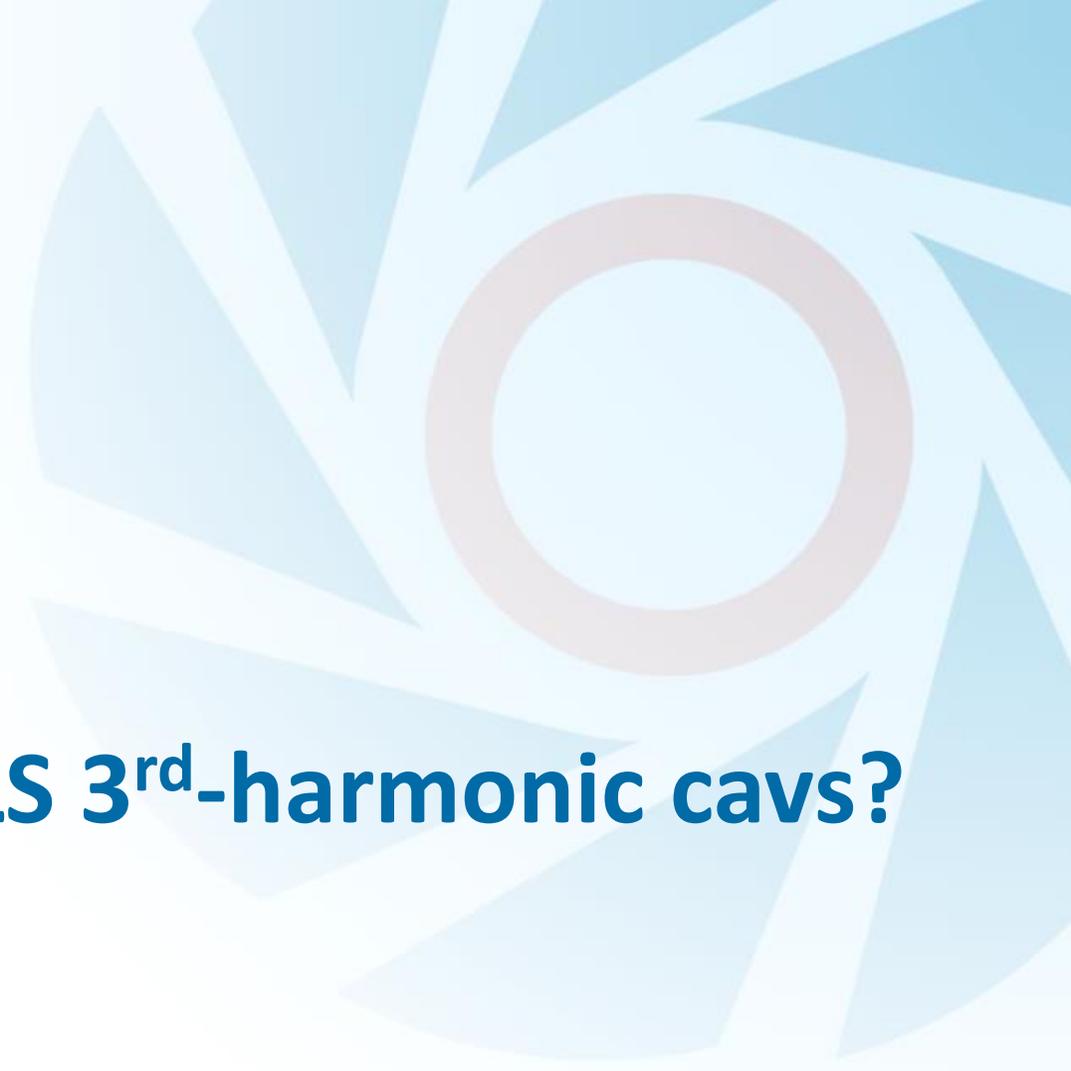
*Passive higher-harmonic RF cavities with general settings and longitudinal multi-bunch instabilities in electron storage rings*

**Marco Venturini, LBNL**

# The Advanced Light Source Upgrade is underway at LBNL



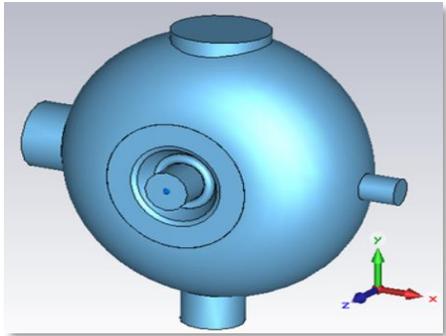
- Replace (**2nm** emittance) **Triple-Bend Achromat** lattice ring with a **100pm** 9BA ring
- Goal: recycle existing RF system. Two 500MHz main + three 3<sup>rd</sup>-harmonic passive (normal conductive) cavities



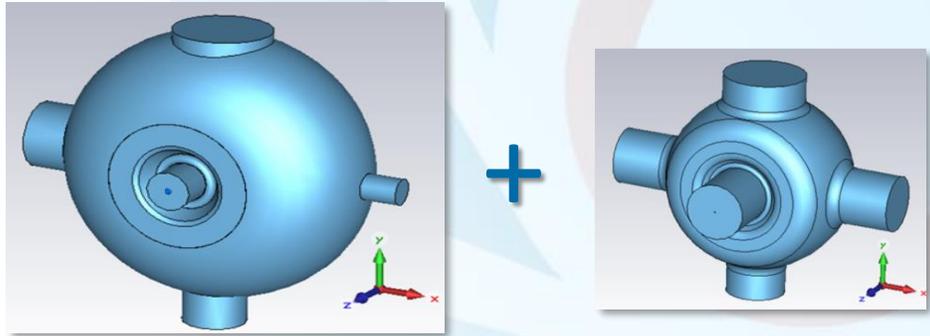
**The \$1M question:  
can ALS-U re-use the existing ALS 3<sup>rd</sup>-harmonic cavs?**

# In light sources HCs are for bunch lengthening (better lifetime)

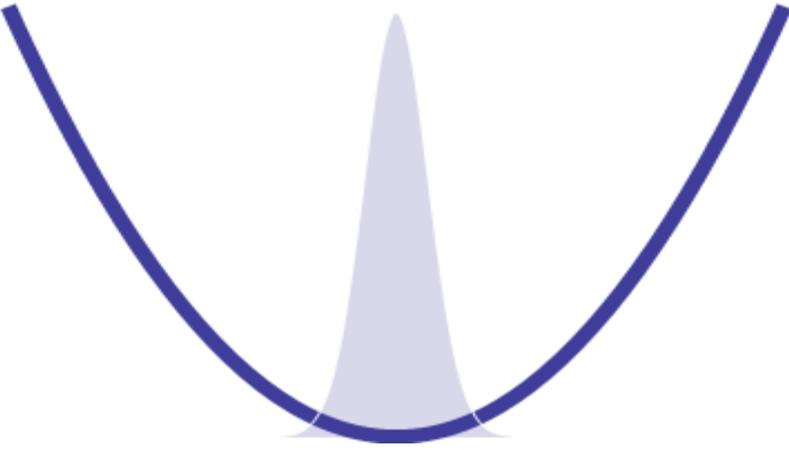
Main RF cavity only



Main + 3<sup>rd</sup> Harmonic Cavity



$U(z)$



Quadratic RF potential well



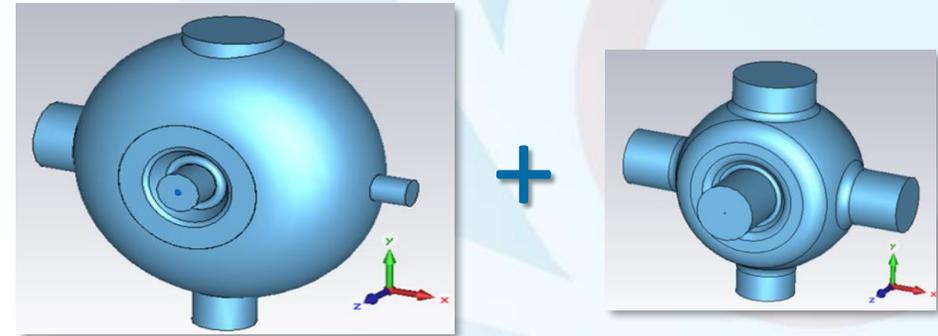
Non-quadratic RF potential well

Longer bunch

# With “optimum” HC settings the bunch core is flat

- Vanishing derivatives of RF potential:  
 $U', U'', U'''$
- $U \sim$  quartic ( $\sim z^4$ )

Main + 3<sup>rd</sup> Harmonic Cavity



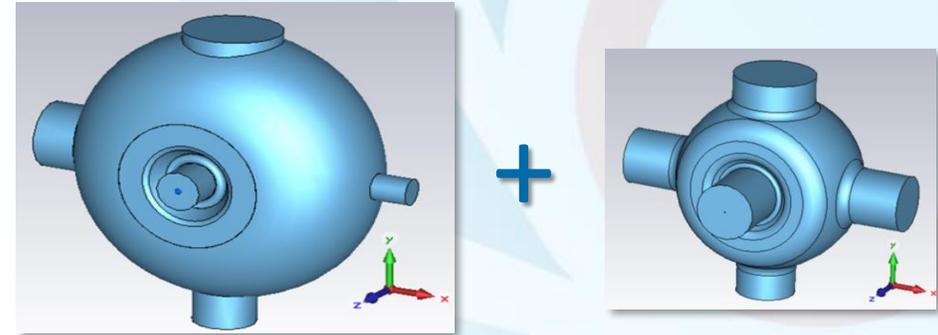
Quartic RF potential well

# The beam-HC interaction is well described by a narrow-band resonator-model impedance

shunt impedance

$$Z(\omega) = \frac{R_s}{1 + iQ \left( \frac{\omega_r}{\omega} - \frac{\omega}{\omega_r} \right)} \equiv R_s \cos \psi_\omega e^{-i\psi_\omega},$$

Main + 3<sup>rd</sup> Harmonic Cavity



$\psi$  (detuning angle) measures proximity of HC resonance to 3<sup>rd</sup> harmonic of RF



$$\tan \psi \simeq 2Q \frac{\omega_r - 3\omega_{rf}}{\omega_r}$$

HHC resonance frequency  $\omega_r$

RF generator frequency (500 MHz)  $\omega_{rf}$

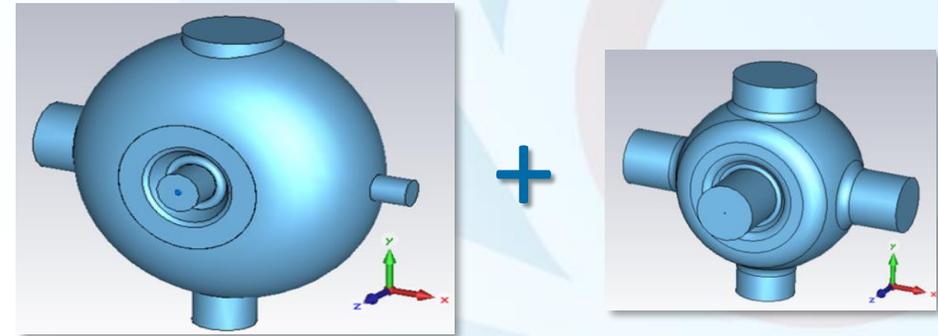
Quality factor  $Q \sim 20,000$  (NC)

# Choosing $R_s$ for optimum

Only if  $R_s$  has been chosen appropriately can the HCs be tuned to yield a flat beam (optimum setting)

- *optimum  $R_s$  depends on beam current, main cav.'s  $V$*

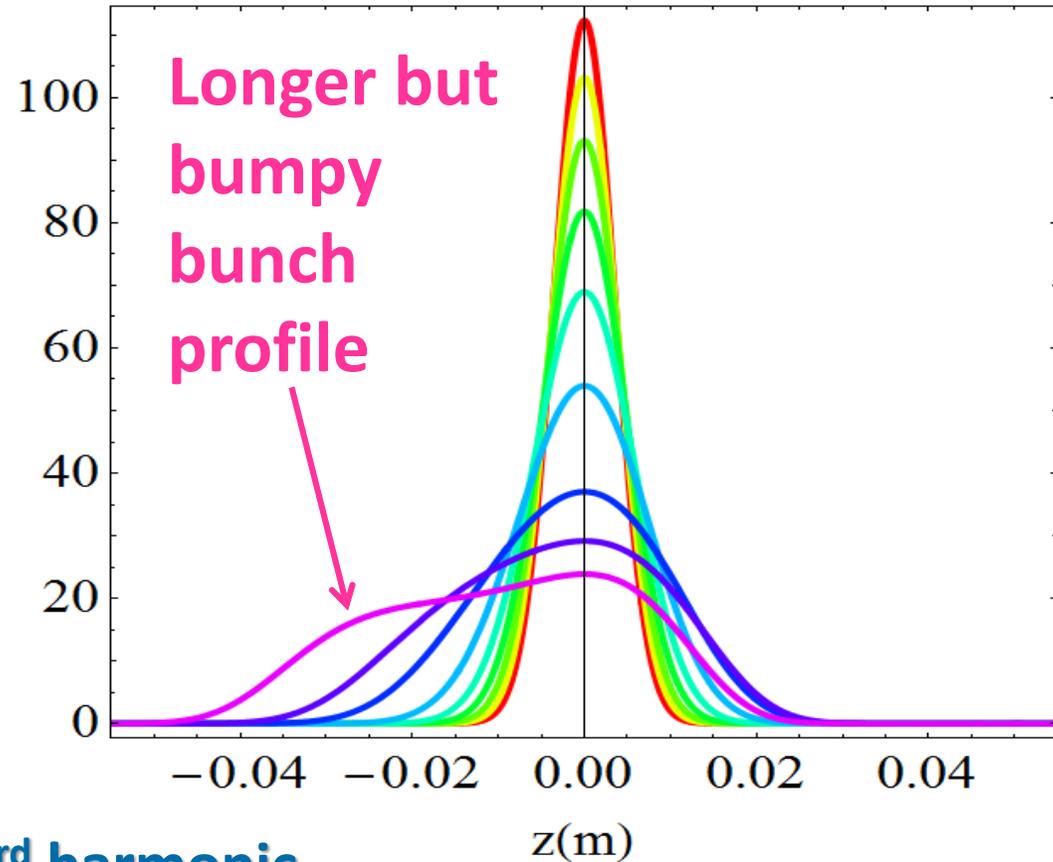
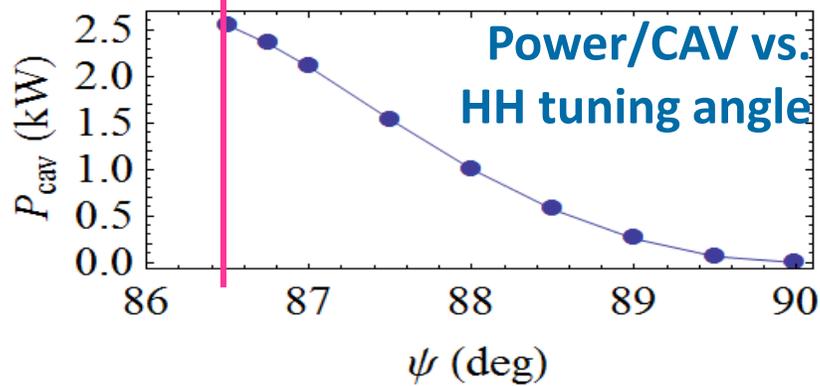
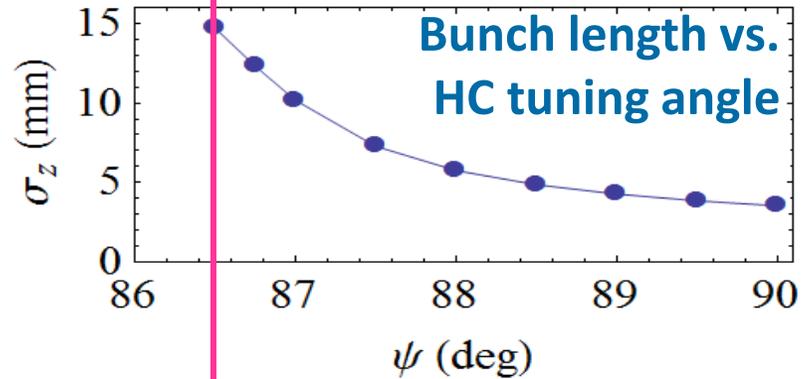
Main + 3<sup>rd</sup> Harmonic Cavity



Quartic RF potential well

# With non-optimum setting, HCs still lengthen the bunch, but the bunch profile will be bumpy. OK for lifetime.

Design  
detuning



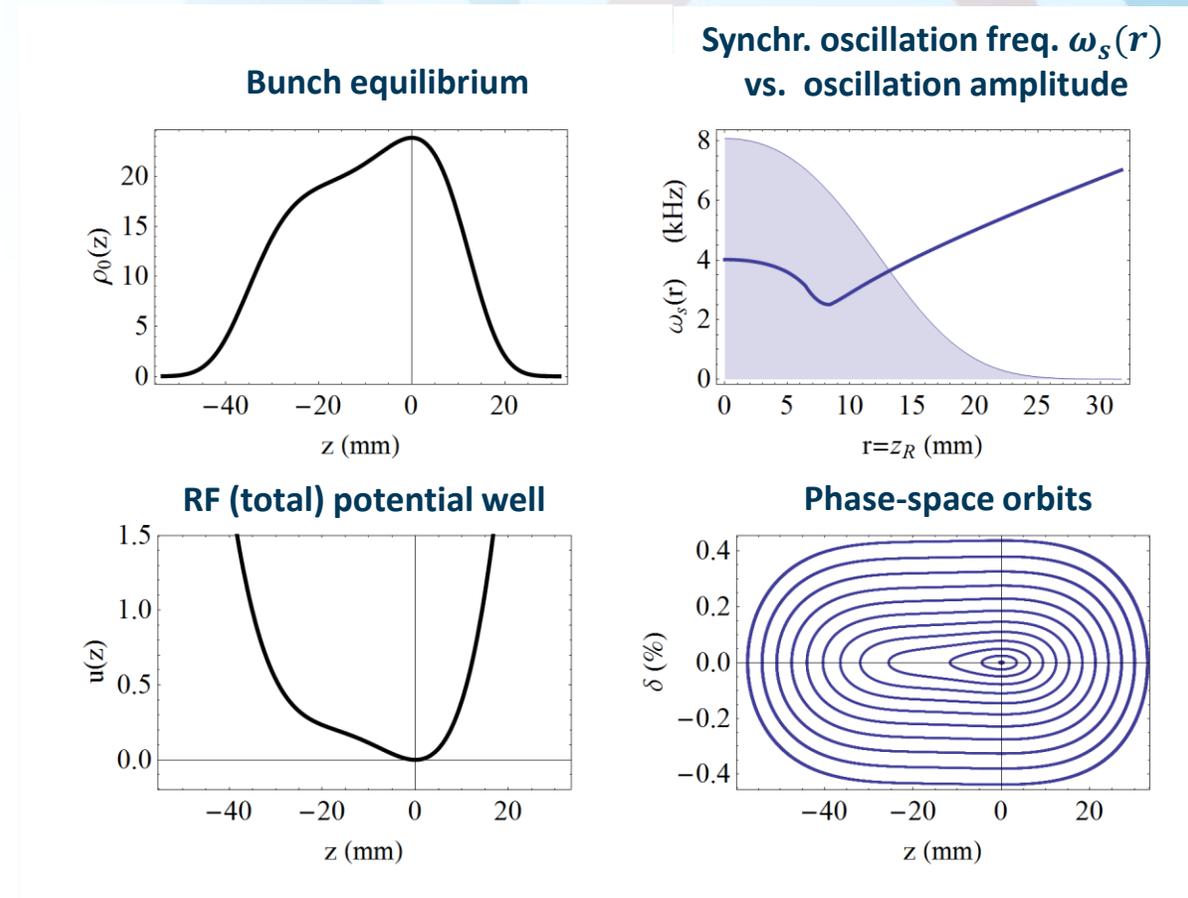
← HC resonance closer to 3<sup>rd</sup> harmonic

# Two options on the table for reusing the ALS HCs

- Optimum ALS-U 3<sup>rd</sup>-HC shunt impedance:  $R_s = 1.4 M\Omega$
- One ALS 3<sup>rd</sup>-HC:  $R_s = 1.7 M\Omega$
- **Possible solutions:**
  - 1. Use a single ALS HC**
    - shunt impedance fairly close to optimum 😊
    - power dissipated on single HC is too high 😞😞
  - 2. Use two ALS HCs**
    - power problem solved 😊;
    - shunt impedance far from optimum. Bumpy profile OK but is it otherwise an issue? Instabilities? Robinson?

# Stripped-down semi-analytical model to study multi-bunch instabilities

- Uniform beam fill, nominal ALS/ALS-U average current (500mA)
  - No transients, etc.
- Main cavities as static objects (no beam-loading effects)
  - Expect main cavity fundamental mode to counter Robinson instability from HC
- **Perturbation theory based on exact (numerical) single-particle motion in RF bucket**
  - Is it important to capture exact single-particle dynamics?
- Compare
  1. Two ALS-like cavities
  2. Optimum cavities



# Aside: Longitudinal Coupled-Bunch Instability (CBI) modes (uniform beam fill with $M$ bunches)

There are  $M$  CBI modes, characterized by integer  $\ell = 0, 1, 2, \dots, M - 1$

## CBI mode $\ell = 0$

(All bunches move  
in phase,  
Robinson instability)



Mode  $\ell$  corresponds to  $\frac{2\pi\ell}{M}$  bunch-to-bunch phase difference

**Will be interested in  $\ell = 0$  and  $\ell = 1$**

# The truth is in the dispersion-relation (dipole approx.)

effective impedance:

$$Z_{\text{eff}}(\Omega) = \sum_{p=\pm 3} \frac{\omega_{p,\ell}}{\omega_1} Z(\omega_{p,\ell} + \Omega)$$

$Z$  is peaked at 3<sup>rd</sup> harmonic,  
only relevant terms are  $p = \pm 3$

$Z$  of HC fundamental mode

$$\omega_{p,\ell} = p h \omega_0 + \ell \omega_0 = p \omega_1 + \ell \omega_0$$

This is  $\omega_{\text{rf}}$

CBI mode  
no.

$$1 + 4\pi i \frac{e I_{\text{avg}} c Z_{\text{eff}}(\Omega)}{9 E_0 T_0 \omega_1} \int_0^\infty dJ \frac{\partial \Psi_0(J)}{\partial J} \frac{|H_{1,3}(J)|^2 \omega_s(J)}{\Omega^2 - \omega_s(J)^2} = 0.$$

$$H_{m,p}(J) = \frac{1}{2\pi} \int_0^{2\pi} e^{im\varphi + i\omega_p \zeta(J,\varphi)/c} d\varphi,$$

Single-particle  
synchr. oscillation  
frequency

Canonical transformation  
(unperturbed single-particle  
motion)

- Equation valid for  $\text{Im } \Omega > 0$
- $\ell$ -dependence is through  $Z_{\text{eff}}$  and  $H_{1,3}$

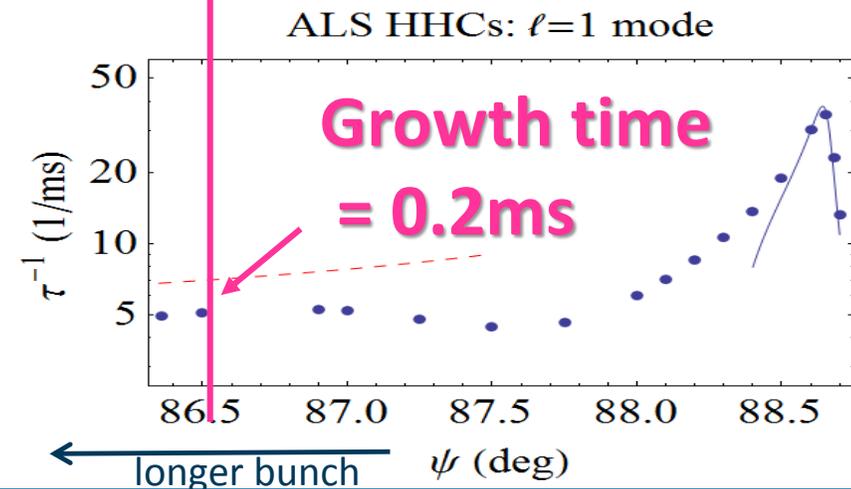
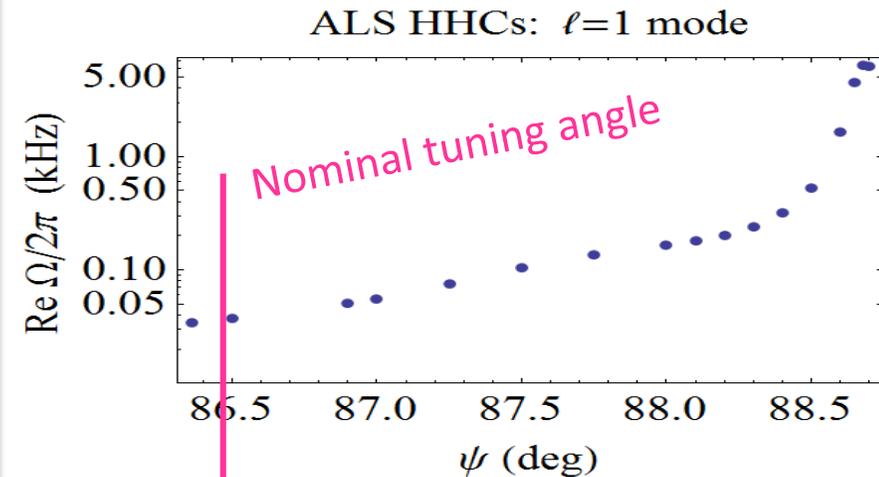
# The main result: The CBI-mode $\ell = 1$ is a killer: the two ALS-HC solution won't work

$\ell = 0$  mode (Robinson)

$\ell = 1$  mode

Mode  
oscillation  
frequency

Mode  
growth rate

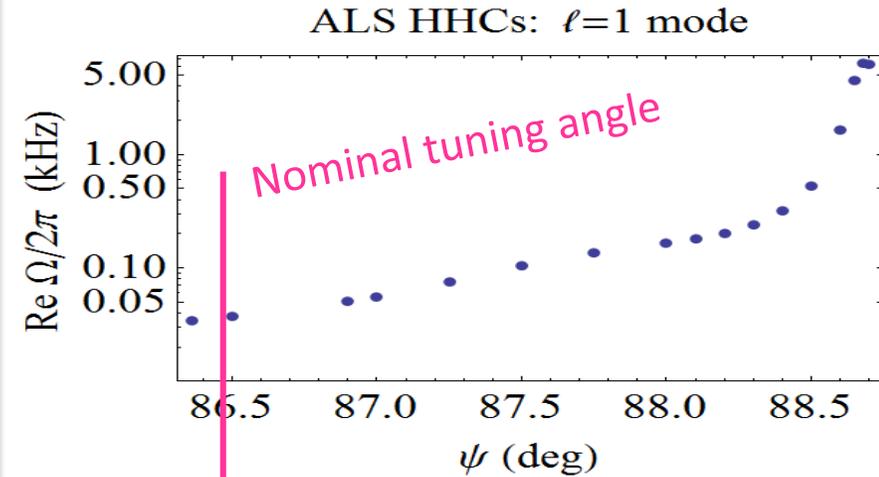
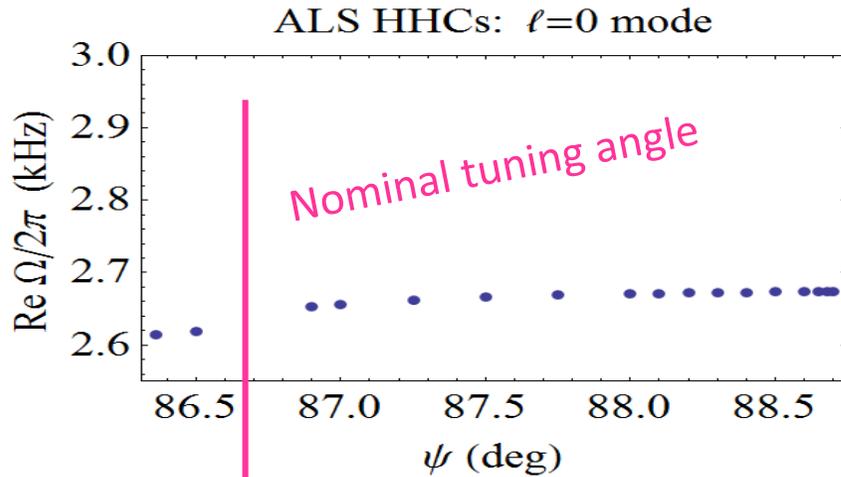


# Main result: The CBI-mode $\ell = 1$ is a killer: the two ALS-HC solution won't work

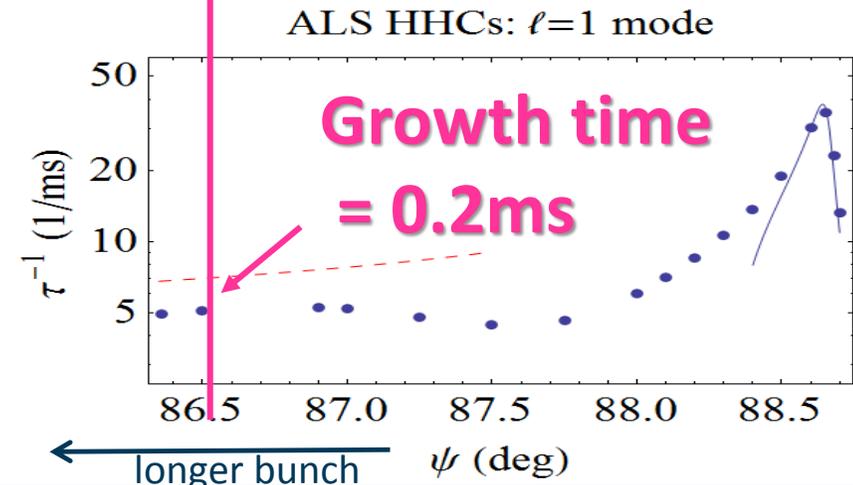
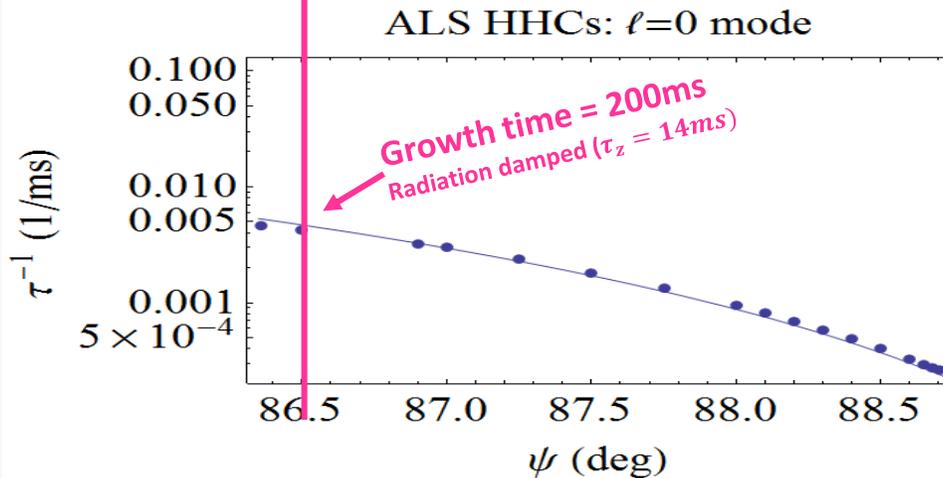
$\ell = 0$  mode (Robinson)

$\ell = 1$  mode

Mode  
oscillation  
frequency



Mode  
growth rate



# Lesson learned no. 1: there is no Landau damping of the Robinson instability associated with HCs

- For optimum HCs, it can be analytically demonstrated that the oscillation frequency of the Robinson mode is always a bit larger than the single-particle synchr. oscillation frequency
  - For ALS-U:  $\Omega_r \equiv \text{Re } \Omega / 2\pi \simeq 2.5 \text{ kHz}$  vs.  $\langle \omega_s \rangle / 2\pi \simeq 500 \text{ Hz}$

$$1 + 4\pi i \frac{eI_{\text{avg}} c Z_{\text{eff}}(\Omega)}{9E_0 T_0 \omega_1} \int_0^\infty dJ \frac{\partial \Psi_0(J)}{\partial J} \frac{|H_{1,3}(J)|^2 \omega_s(J)}{\Omega^2 - \omega_s(J)^2} = 0.$$

*$\Omega$  can be pulled out from under the integral; Eq. simplifies*

$$\text{Growth rate: } \Omega_i \simeq \frac{3\hat{I}}{2\Omega_r} \text{Re} [ Z(3\omega_{\text{rf}} + \Omega_r) - Z(3\omega_{\text{rf}} - \Omega_r) ]$$

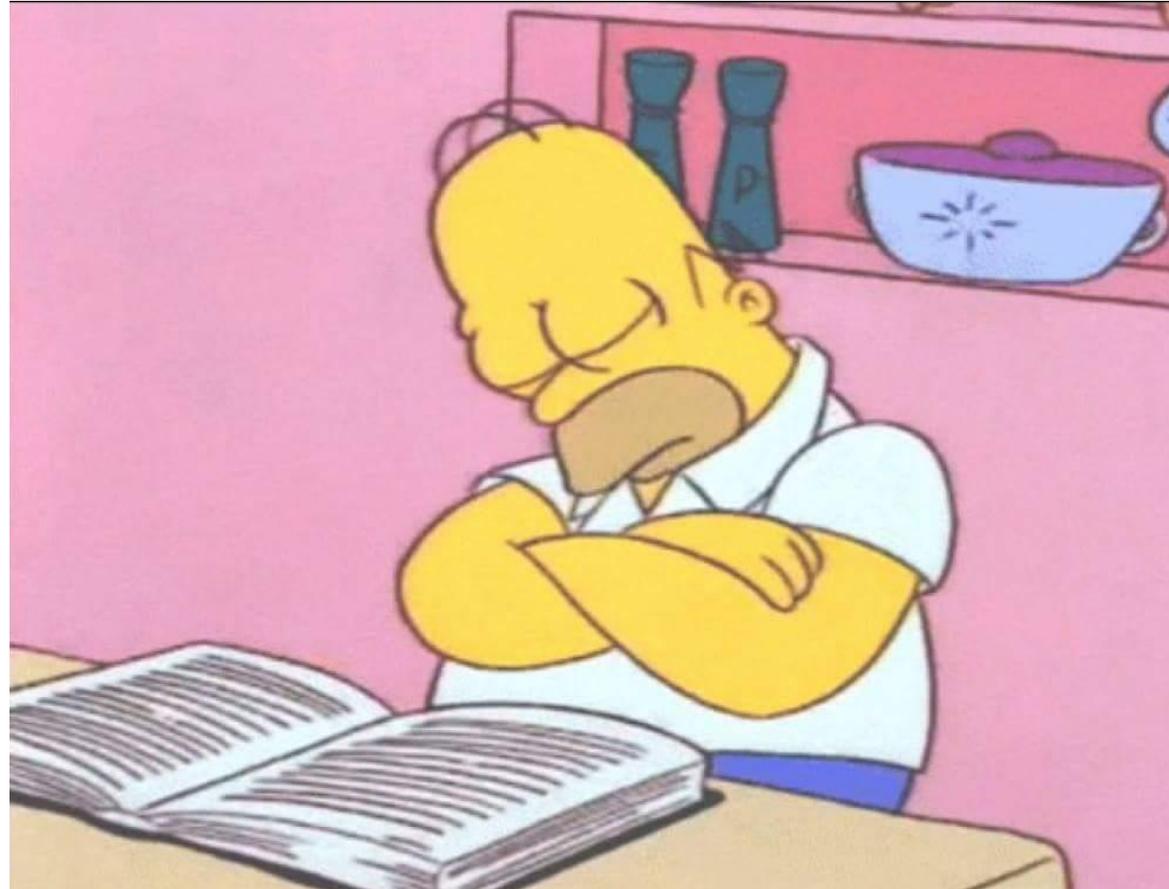
*Essentially independent of  $\Omega_r$  since  $\Omega_r \ll \omega_{\text{rf}}$*

# Much ado about nothing?

- Formally, same result as formula for case where particle-motion in RF bucket is linear (all particles have same synchrotron oscillation frequency)
- A popular "hand waving" derivation assumes "for simplicity" that one can simply ignore the effect of the HC on single-particle dynamics
  - Result is accurate within 10% (ALS-U)
- Problem is, occasionally a "hand waver" may feel that important physics is left out if HC nonlinearities are simply ignored (next slide)

# Lesson learned no. 2: “Quandoquidem bonus dormitat Homerus” (Sometimes even good ol’ Homer falls asleep)

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# Lesson learned no. 2: “Quandoquidem bonus dormitat Homerus” (Sometimes even good ol’ Homer falls asleep)

$$\frac{1}{\tau} = \text{Im } \Omega \approx \frac{\eta e I_0 \omega_{\text{rf}}}{2 E_0 T_0 \bar{\omega}_s} \left\{ \left[ \text{Re } Z_0^{\parallel}(\omega_{\text{rf}} + \bar{\omega}_s) - \text{Re } Z_0^{\parallel}(\omega_{\text{rf}} - \bar{\omega}_s) \right] \leftarrow \begin{array}{l} \text{Main Cavs} \\ \text{Harmonic Cavs} \end{array} \right. \\ \left. + m \left[ \text{Re } Z_0^{\parallel}(m\omega_{\text{rf}} + \bar{\omega}_s) - \text{Re } Z_0^{\parallel}(m\omega_{\text{rf}} - \bar{\omega}_s) \right] \right\}, \quad (8.110)$$

very large. As a result, Eq. (8.110) can only be viewed as an estimate.

Now let us estimate how large a Landau damping we obtain from the passive Landau cavity coming from the spread of the synchrotron frequency. The stability criterion is roughly

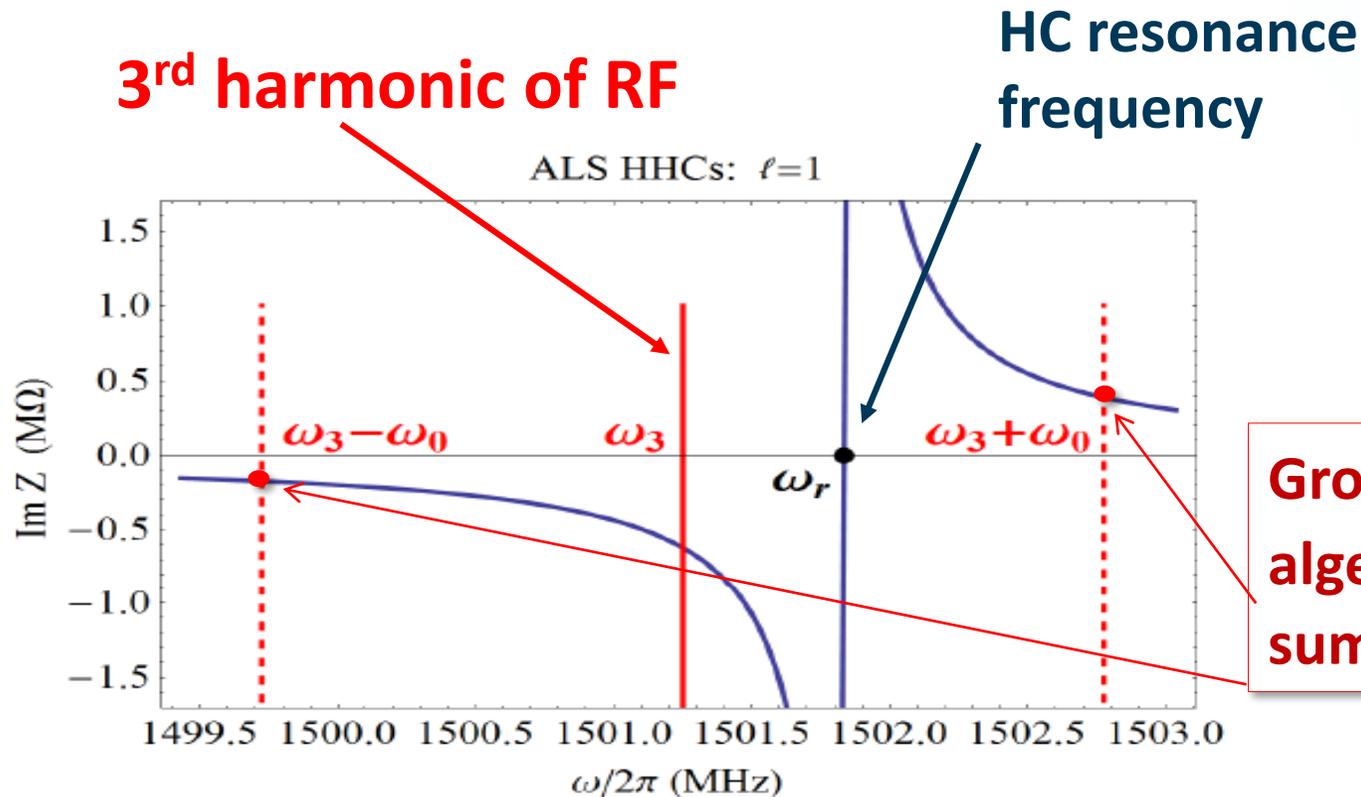
$$\frac{1}{\tau} \lesssim 4\sigma_{\omega_s}, \quad (8.116)$$

where the synchrotron angular frequency spread is given by Eq. (8.109), and we have taken approximately  $4\sigma_{\omega_s} = 29.4$  kHz as the total spread. In other words, the higher-harmonic cavity is able to damp an instability that has a growth time longer than 0.034 ms, an improvement that is about 160 folds better than when the higher-harmonic cavity is absent. Thus, theoretically, this Landau damping is large enough to alleviate the Robinson’s antidamping of the higher-harmonic cavity as well. We can now rewrite the growth rate of Eq. (8.115) in terms of the

# Lesson learned no. 3: the CBI mode $\ell = 1$ instability is associated with $ImZ$ rather than $ReZ$

$$\Omega_i \simeq \sqrt{3\hat{I} \text{Im}[Z(3\omega_{rf} + \omega_0) + Z(3\omega_{rf} - \omega_0)]}$$

- Relevant frequencies are the satellite beam harmonics at  $\omega_3 \pm \omega_0$
- Formula valid if growth rate is large



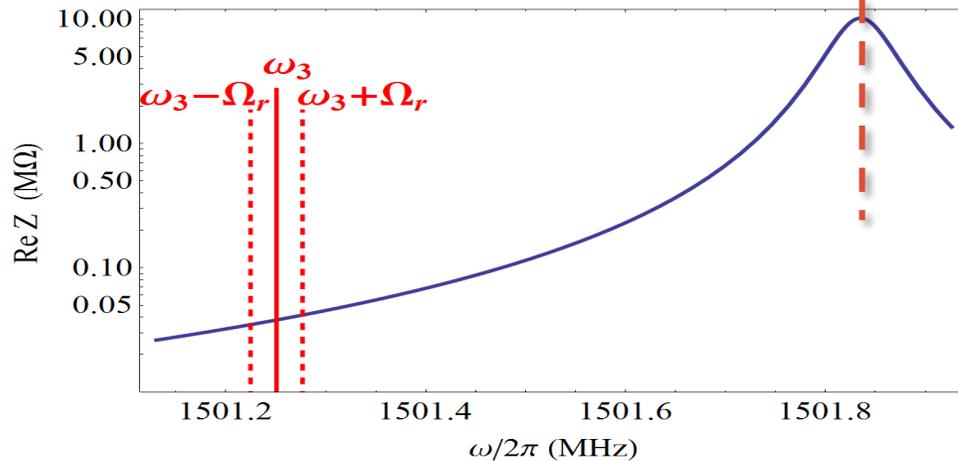
**Growth rate proportional to  $\sqrt{\text{algebraic sum of Im Z at two harmonics: sum} > 0 \rightarrow \text{instability}}$**

# Optimal HCs are $\ell = 1$ stable ( $R_s$ is smaller; HCs have to be detuned less)

Optimal  
3<sup>rd</sup> HCs

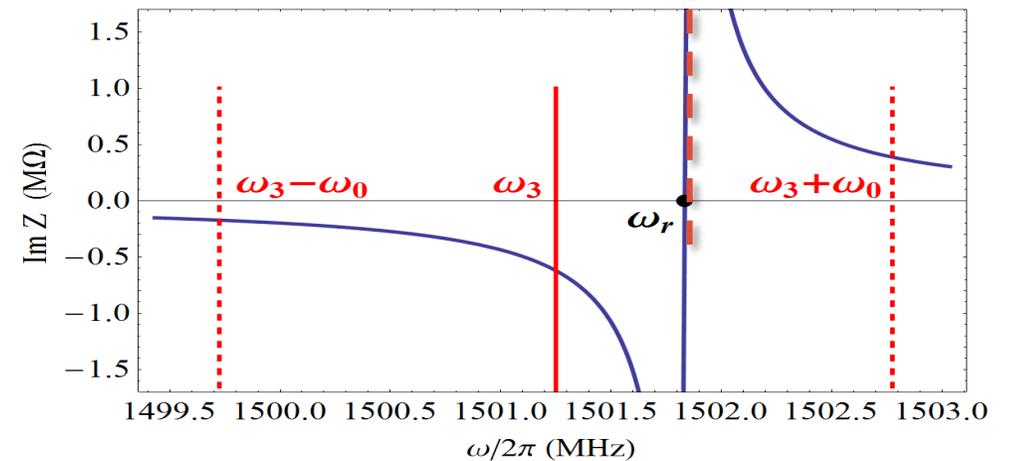
$\ell = 0$  (Robinson)

ALS HHCs:  $\ell=0$



$\ell = 1$

ALS HHCs:  $\ell=1$



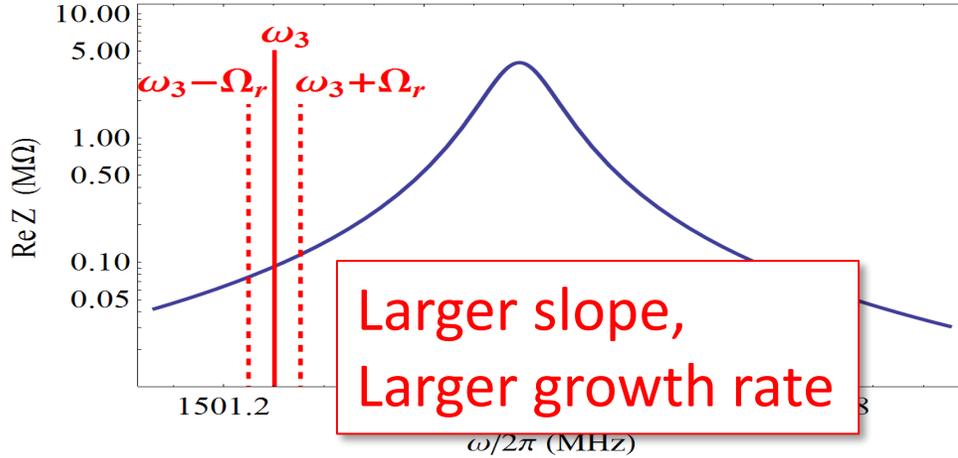
Two ALS  
3<sup>rd</sup> HCs

# Optimal HCs are $\ell = 1$ stable (mostly) because they have to be detuned less, having smaller $R_s$

$\ell = 0$  (Robinson)

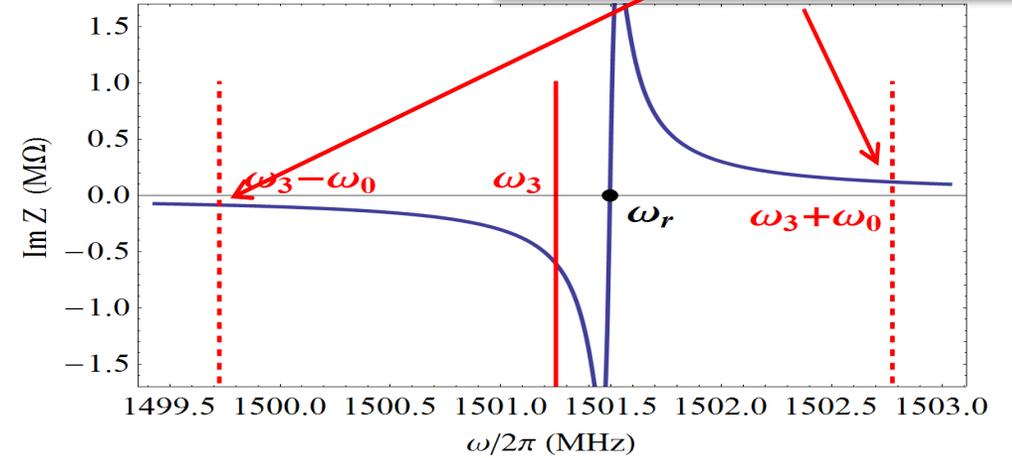
Optimal HHCs:  $\ell=0$

Optimal  
3<sup>rd</sup> HCs



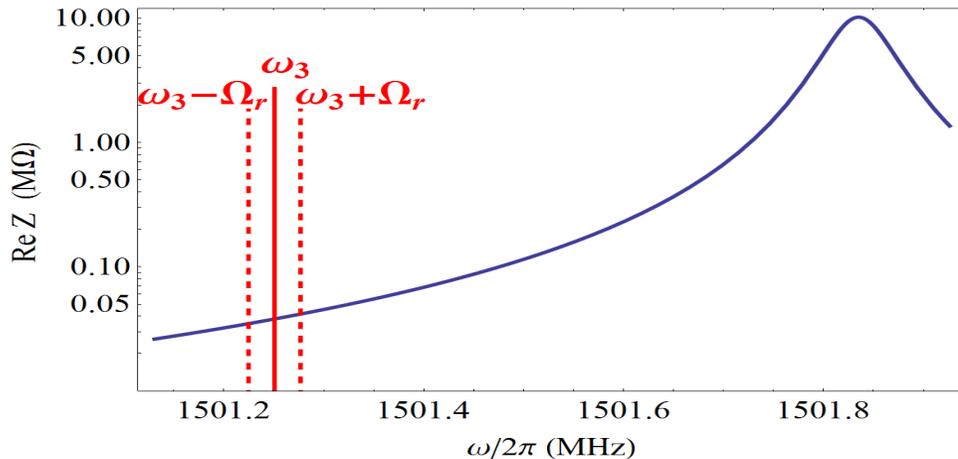
smaller growth rate

Optimal

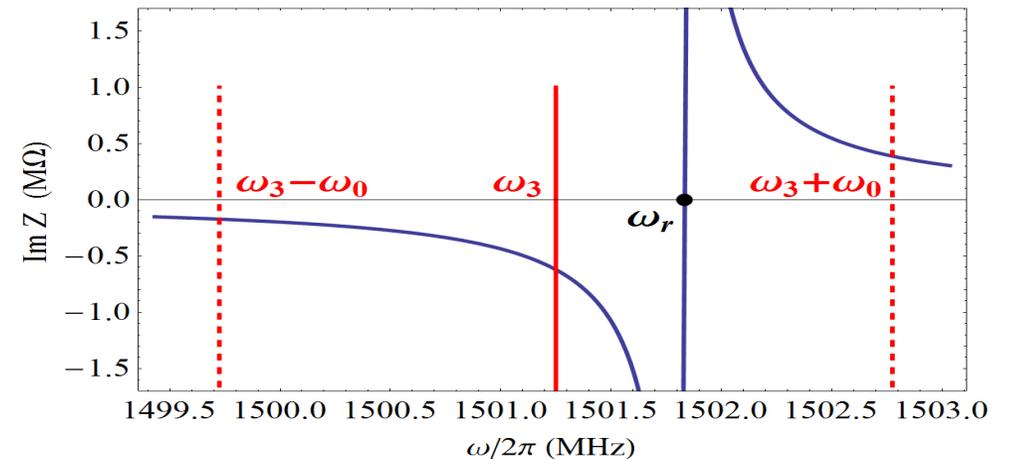


ALS HHCs:  $\ell=0$

Two ALS  
3<sup>rd</sup> HCs

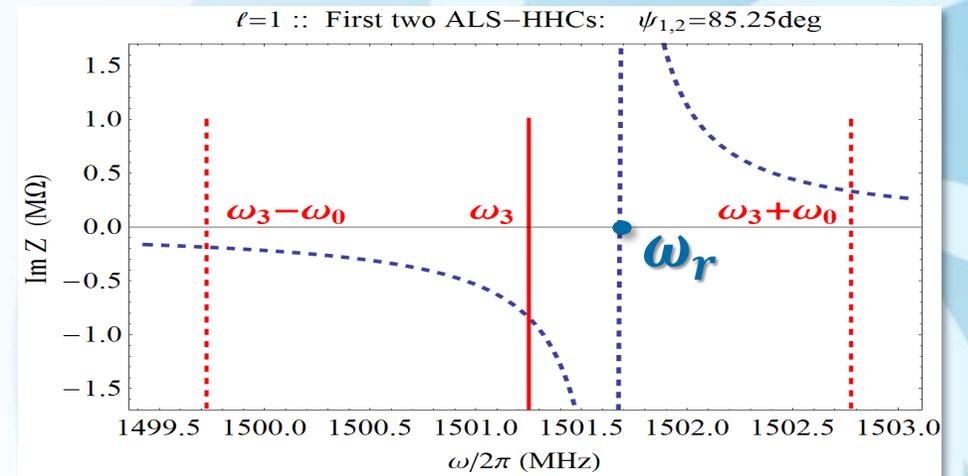


ALS HHCs:  $\ell=1$



# Lesson learned no. 4: Using the third ALS HC in bunch shortening mode would stabilize the system

Two HCs with  $\omega_r > 3\omega_{rf}$  (bunch lengthening)

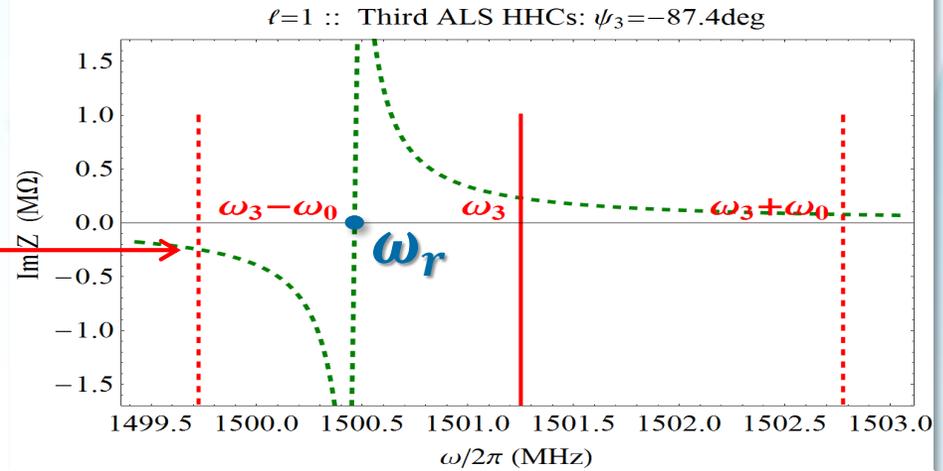
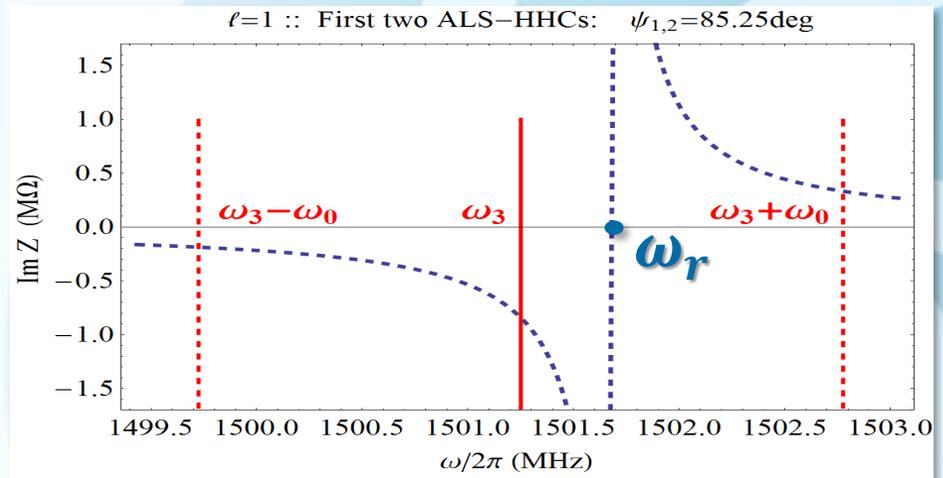


# Lesson learned no. 4: Using the third ALS HC in bunch shortening mode would stabilize the system

Two HCs with  $\omega_r > 3\omega_{rf}$  (bunch lengthening)

This large negative Im Z contribution is stabilizing

One HC with  $\omega_r < 3\omega_{rf}$  (bunch shortening)



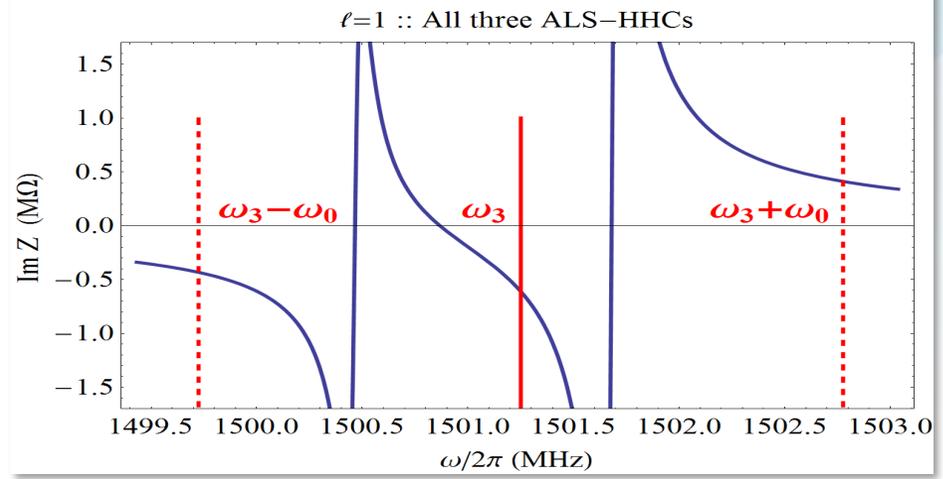
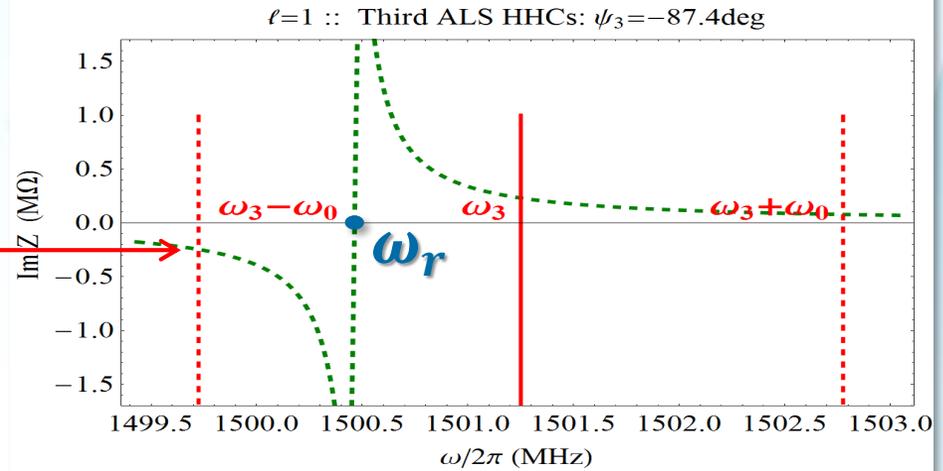
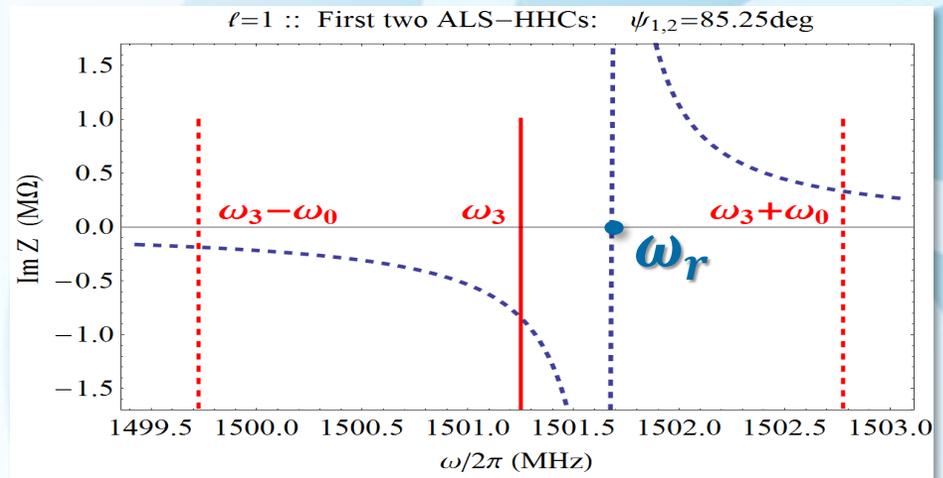
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Two HCs with  $\omega_r > 3\omega_{rf}$  (bunch lengthening)

This large negative Im Z contribution is stabilizing

One HC with  $\omega_r < 3\omega_{rf}$  (bunch shortening)

One + two HCs: overall bunch shortening & stability



# Lesson learned no. 5: In accelerator design, as in life, money is not everything ...

Project has opted for brand new 3<sup>rd</sup> HCs

# Conclusions

- **Studied perturbation-theory to multi-bunch instabilities driven by the fundamental mode of HHC**
  - Based on exact, numerical single-particle motion in RF bucket
  - Theory predict strong CBI  $\ell = 1$  mode instability
- **Instability confirmed by independent macroparticle simulations**
  - Courtesy of G. Bassi (BNL)
  - Growth-rate estimate  $\sim 20\%$  larger than theory's
- **For the details see <https://journals.aps.org/prab/abstract/10.1103/PhysRevAccelBeams.21.114404>**
- **Landau does not damp Robinson !**

# Bonus finding: simple formula for beam power dissipated to HC (optimum settings)

No explicit dependence (!) on shunt impedance, detuning angle, etc.

$$P_{\text{cav}} = \frac{F}{8} P_{\text{rad}} \simeq 10\% \times P_{\text{rad}}$$

3<sup>rd</sup> HCs

Form factor:  $F \simeq 1 - (3\omega_{\text{rf}}\sigma_t)^2/2 \simeq 0.9$