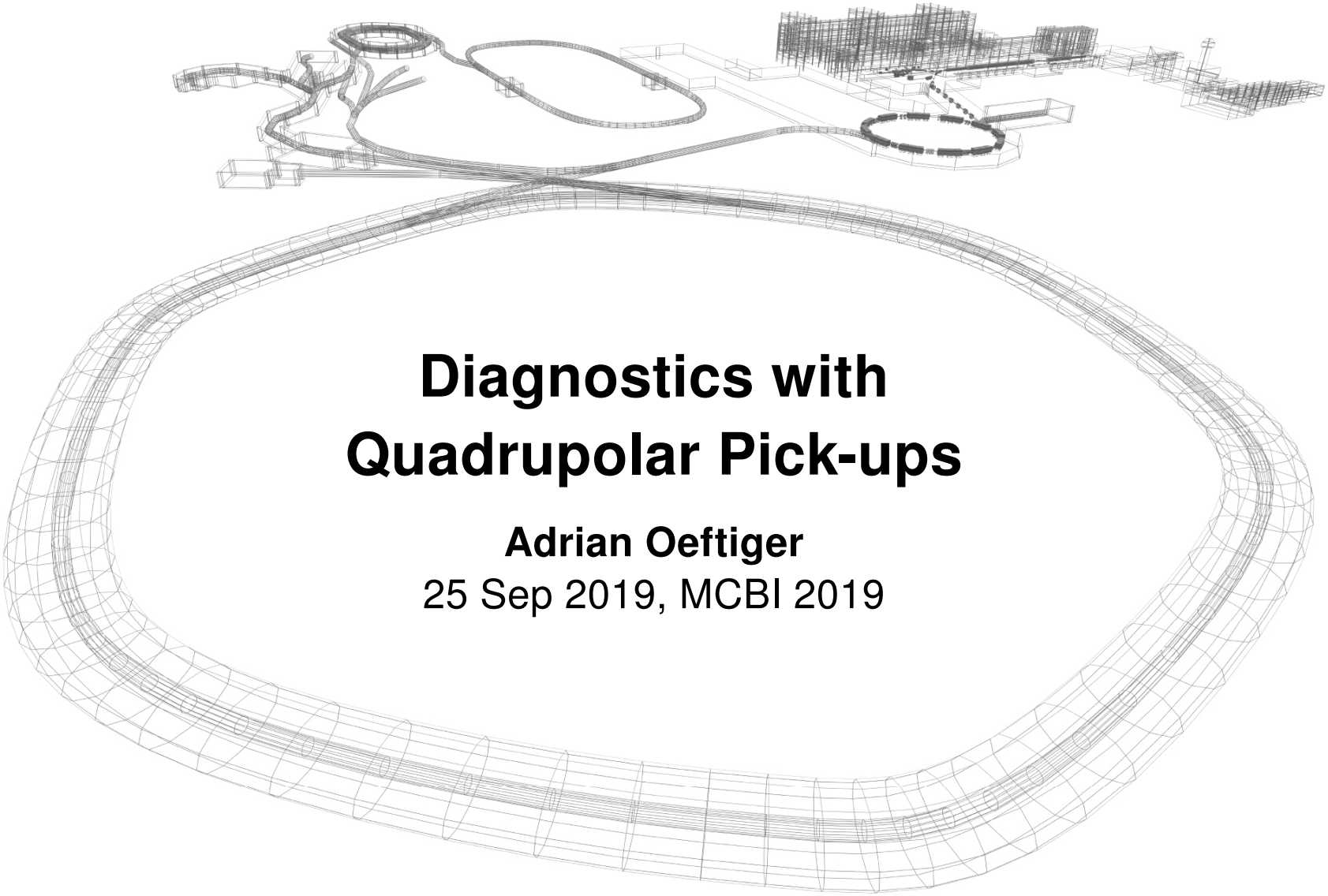

Diagnostics with Quadrupolar Pick-ups

Adrian Oeftiger

25 Sep 2019, MCBI 2019

Motivation

1st order

rigid dipolar centroid oscillation:

- Newton's third law,
  actio = reactio

⟶ no direct space charge (SC)
[except higher-order projections]

Motivation

1st order

rigid dipolar centroid oscillation:

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2nd order

quadrupolar envelope oscillation:

- defocused by transverse space charge force

⟶ frequency of envelope oscillation decreases with SC

⟹ measure direct space charge through frequency shift of beam size oscillations about matched $\sigma_{x,y}$

⇝ during this campaign, found many new insights on QPU spectrum

Context

What about quadrupolar pick-ups?

- non-invasive (and thus non-destructive) measurement
  - ⟶ contains info on transverse emittances $\epsilon_{x,y}$ and linear coupling
  - ⟶ even with quadrupolar excitation: beam transfer function measurement without significant emittance growth ($\Delta\epsilon/\epsilon < 5\%$)
  - ⟶ modes and their amplitudes in quadrupolar spectrum provide hint at beam dynamics

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- measures *coherent* mode tunes, not incoherent spreads
  - ⟶ cannot infer transverse distribution type!
  - ⟶ can express coherent tune shifts in units of RMS equivalent K-V SC tune shift
  - ⟹ different transverse distributions with same RMS characteristics *yield same coherent tune shifts*

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  - ⟶ can express coherent tune shifts in units of RMS equivalent K-V SC tune shift
  - ⟹ different transverse distributions with same RMS characteristics *yield same coherent tune shifts*
- recent push from coasting beam to bunched beam!
  - ⟶ synchrotron motion ⇝ much more dynamics
  - ⟶ chromaticity, synchrotron sidebands (cf. my HB2018 contribution ↗)
  - ⟶ started to study head-tail instabilities vs. second-order modes, e.g. $\left\langle x \cdot \frac{\Delta p}{p_0} \right\rangle$
  - ⟹ new perspectives!? In particular for space charge vs. head-tail instabilities ⟹ a lot remains to be done!

Quadrupolar Pick-up

Schematics of a quadrupolar pick-up (see Ref. [1]):

Induced voltage on electrodes:

$$
\begin{aligned}
U_{right} &\propto I_{\text{beam}}\left(1 + z_{1x} + z_2 + \ldots\right) \\
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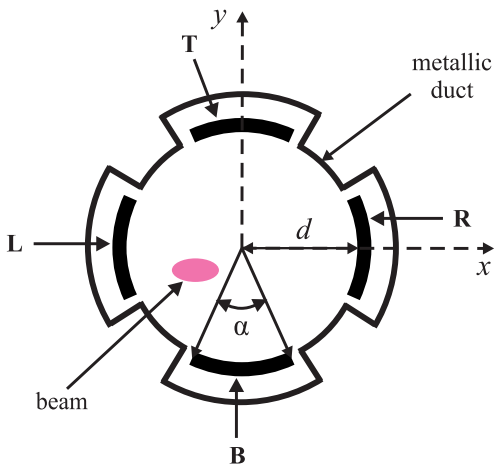

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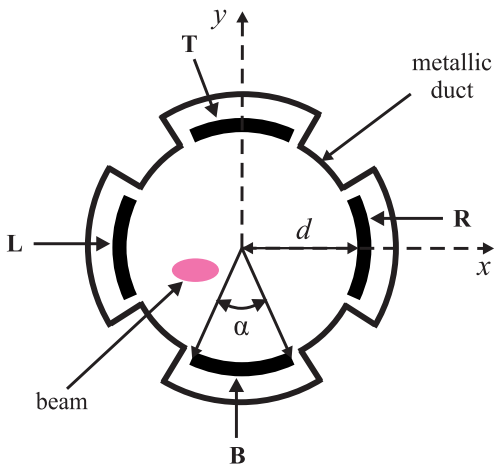

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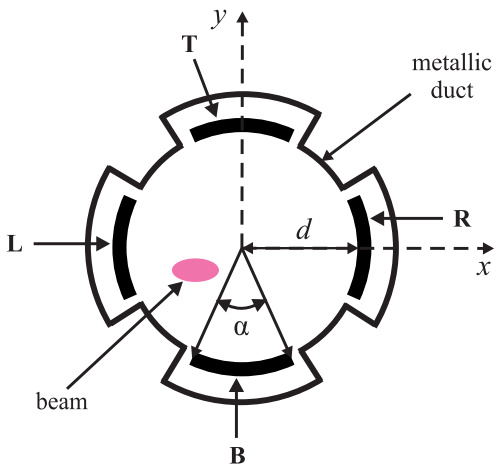

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z_2 &\propto \frac{\langle x^2 \rangle - \langle y^2 \rangle}{d^2} = \frac{\sigma_x^2 - \sigma_y^2 + \langle x \rangle^2 - \langle y \rangle^2}{d^2} \quad \text{(neglecting dispersion)}
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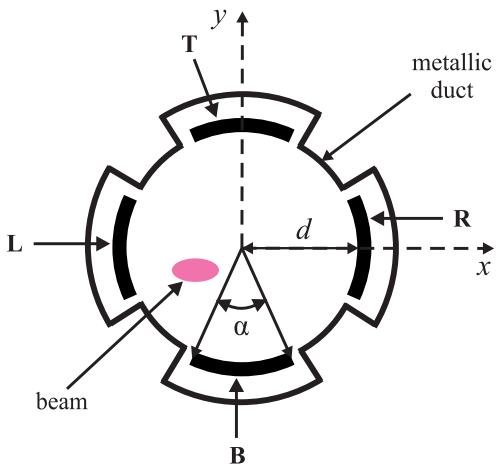

$\Longrightarrow$ combine voltages to measure dipolar beam moments (usual BPM):

$$\langle x \rangle \propto U_{right} - U_{left}$$
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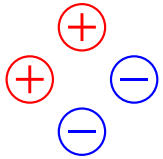

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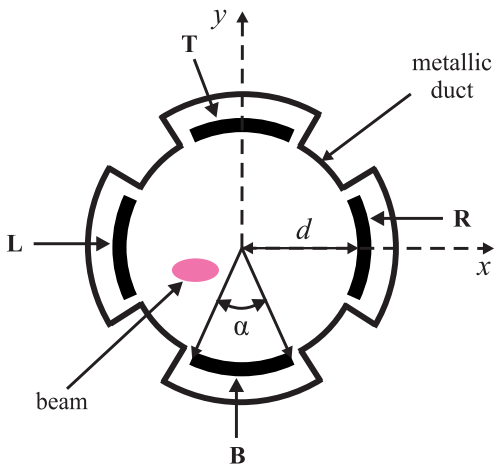

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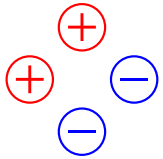
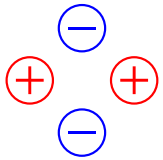


$\Longrightarrow$ or combine voltages to measure quadrupolar beam moments:

$$z_2 \propto \sigma_x^2 - \sigma_y^2 + \langle x \rangle^2 - \langle y \rangle^2$$
$$\propto U_{right} + U_{left} - U_{top} - U_{bottom}$$



Time Domain $\rightsquigarrow$ Frequency Domain

Time domain very challenging:

- accurately resolving the beam size moment $\sigma_x^2 - \sigma_y^2$ requires removal of (strong) dipolar component in $z_2 \propto \sigma_x^2 - \sigma_y^2 + \langle x \rangle^2 - \langle y \rangle^2$
- Ref. [2] A. Jansson's PhD thesis: clever idea, magnetic quadrupolar pick-up
  - $\longrightarrow$ beautiful measurements of injection mismatch and transverse emittances $\epsilon_{x,y}$ (no dipolar contribution to magnetic signal)
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- lately also differential measurements in LHC, cf. Ref. [3]

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Frequency domain more accessible:

- spectral measurements don't care about differential offsets, dipolar contribution oscillates at distinct frequency etc.
  - $\longrightarrow$ idea: resolve quadrupolar beam oscillation modes accurately!
    - $\Longrightarrow$ determine frequency shifts due to space charge
    - $\Longrightarrow$ determine energy/amplitudes in modes as measure of mismatch
- CERN LHC injectors equipped with stripline pick-ups, new 3-channel frontend: horizontal and vertical dipolar + quadrupolar signals!
- GSI SIS-18 also equipped with 2 QPUs!

Overview

Frequency domain measurements in PS:

- to my surprise, I found super many modes in the spectrum when I first looked at injection...
  - ⟶ in the meantime, quite a few experiments identified them!
- beam size oscillation itself difficult to observe at injection
  - ⟶ developed beam transfer function (BTF) measurement technique with transverse feedback as quadrupolar kicker

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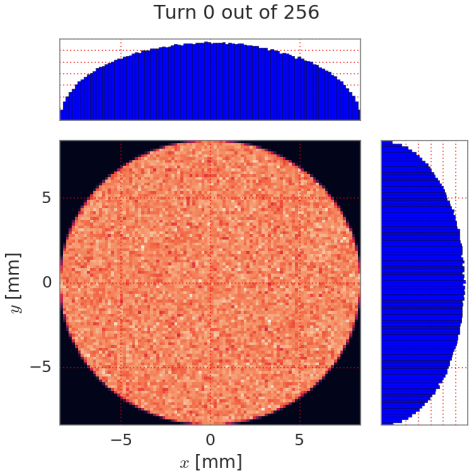
Structure of this talk:

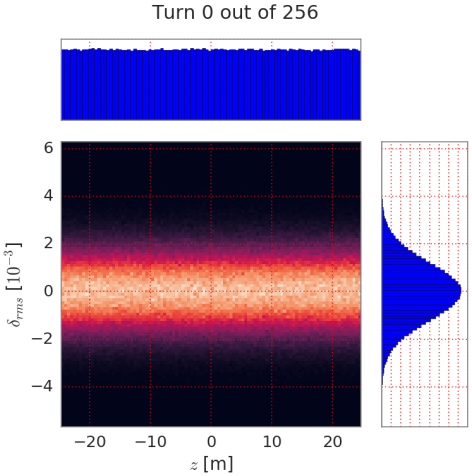
1. second-order mode overview in simulations
2. mode identification at CERN PS injection
   - dipolar modes
   - Chernin's odd (tilting) modes
   - coherent dispersion modes
3. the missing puzzle piece: even envelope modes $\sigma_{x,y}$?
4. quadrupolar BTF and space charge

Part I:
Second-order Modes in Simulations

Quad. Spectrum: Coasting KV Beam

bunched	transv. distr.	synchrotron motion	dispersion	chromaticity
no	KV (uniform)	no	no	no





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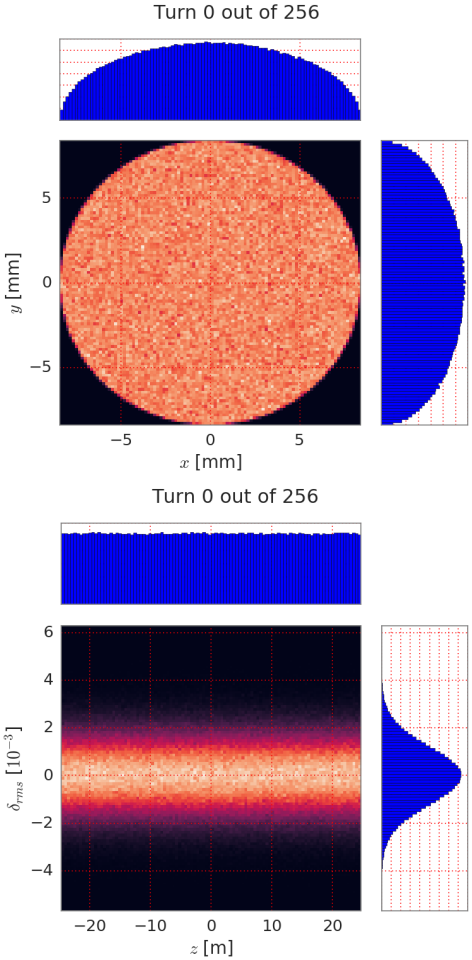
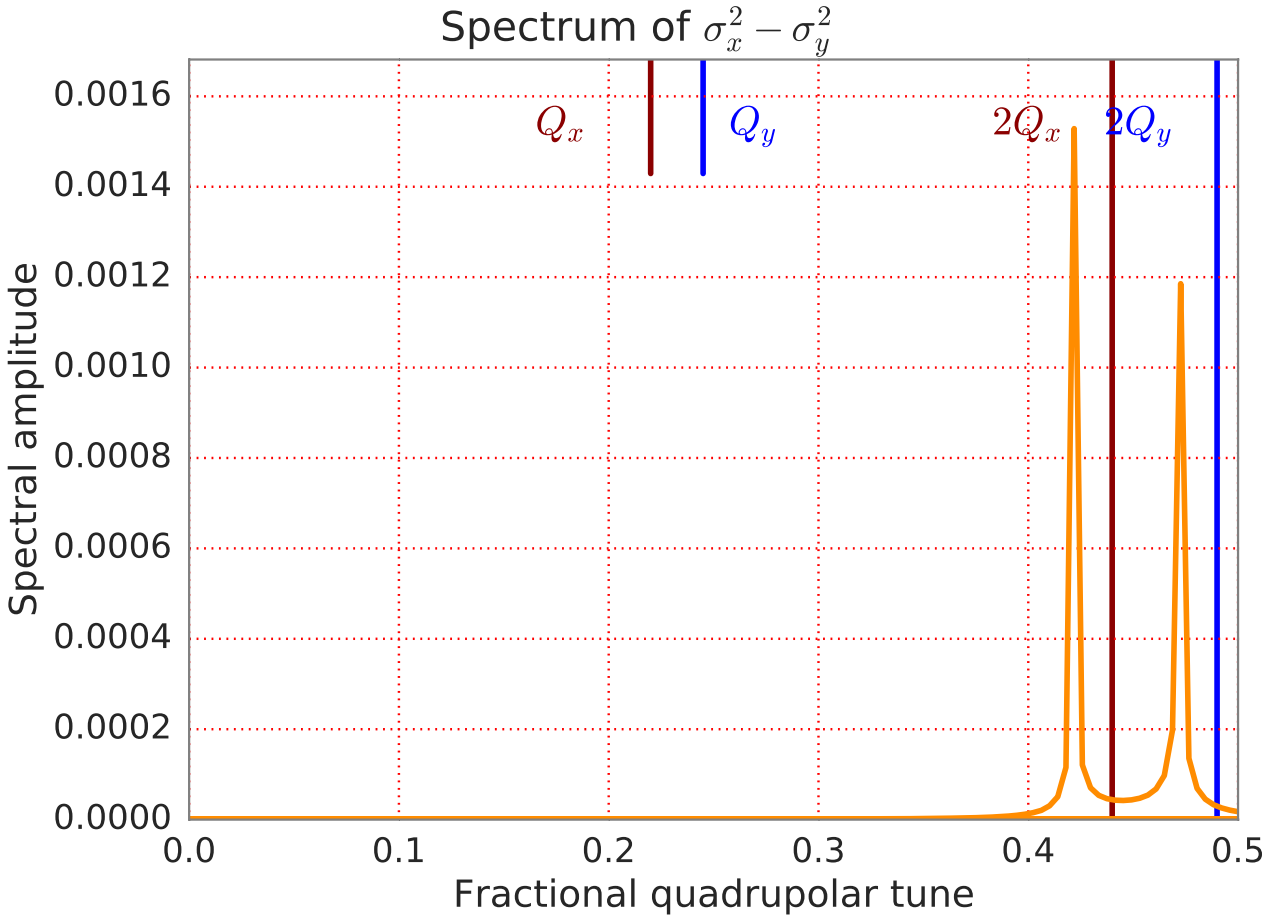
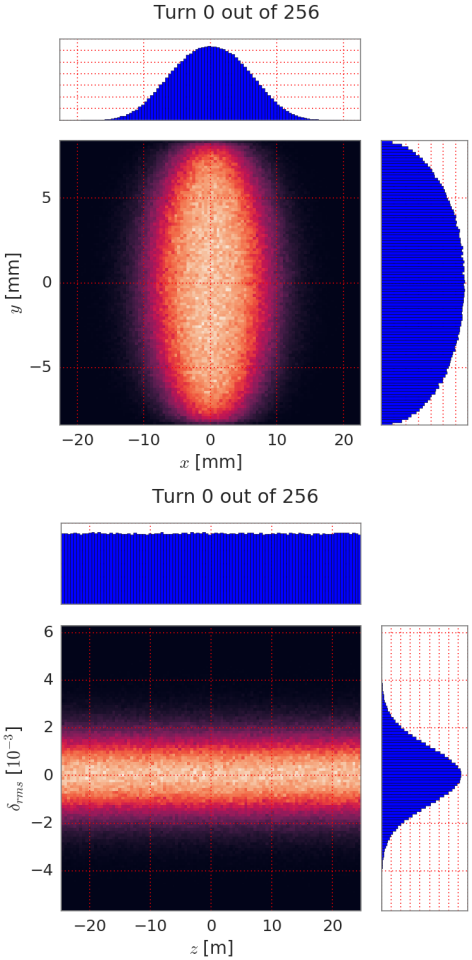
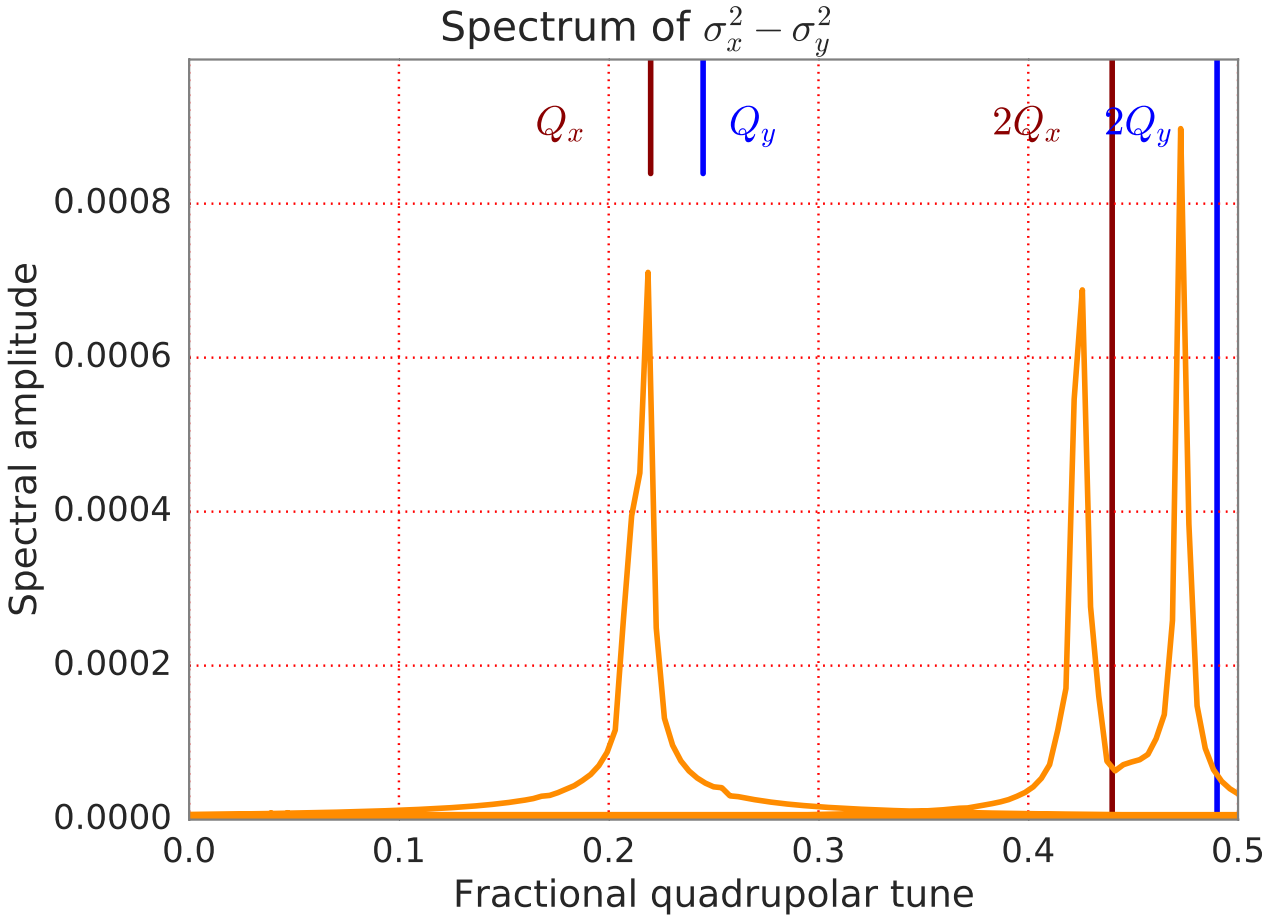




Quad. Spectrum: + Dispersion

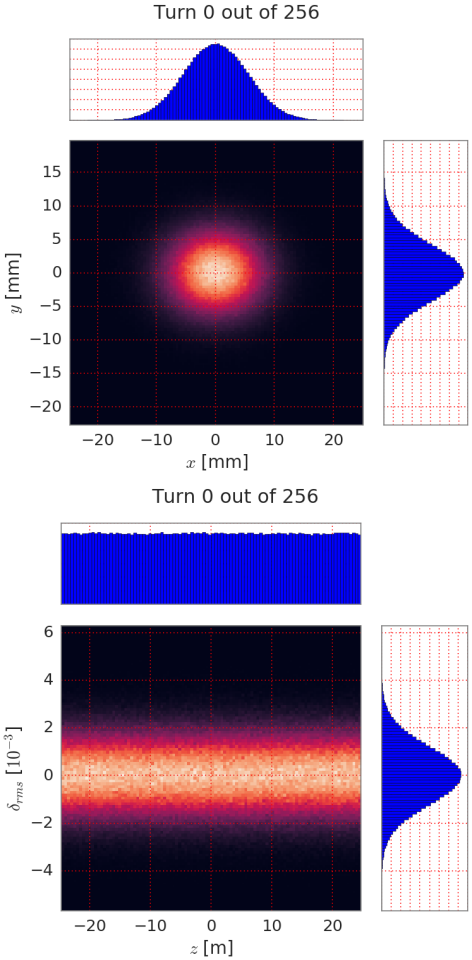
bunched	transv. distr.	synchrotron motion	dispersion	chromaticity
no	KV (uniform)	no	yes	no

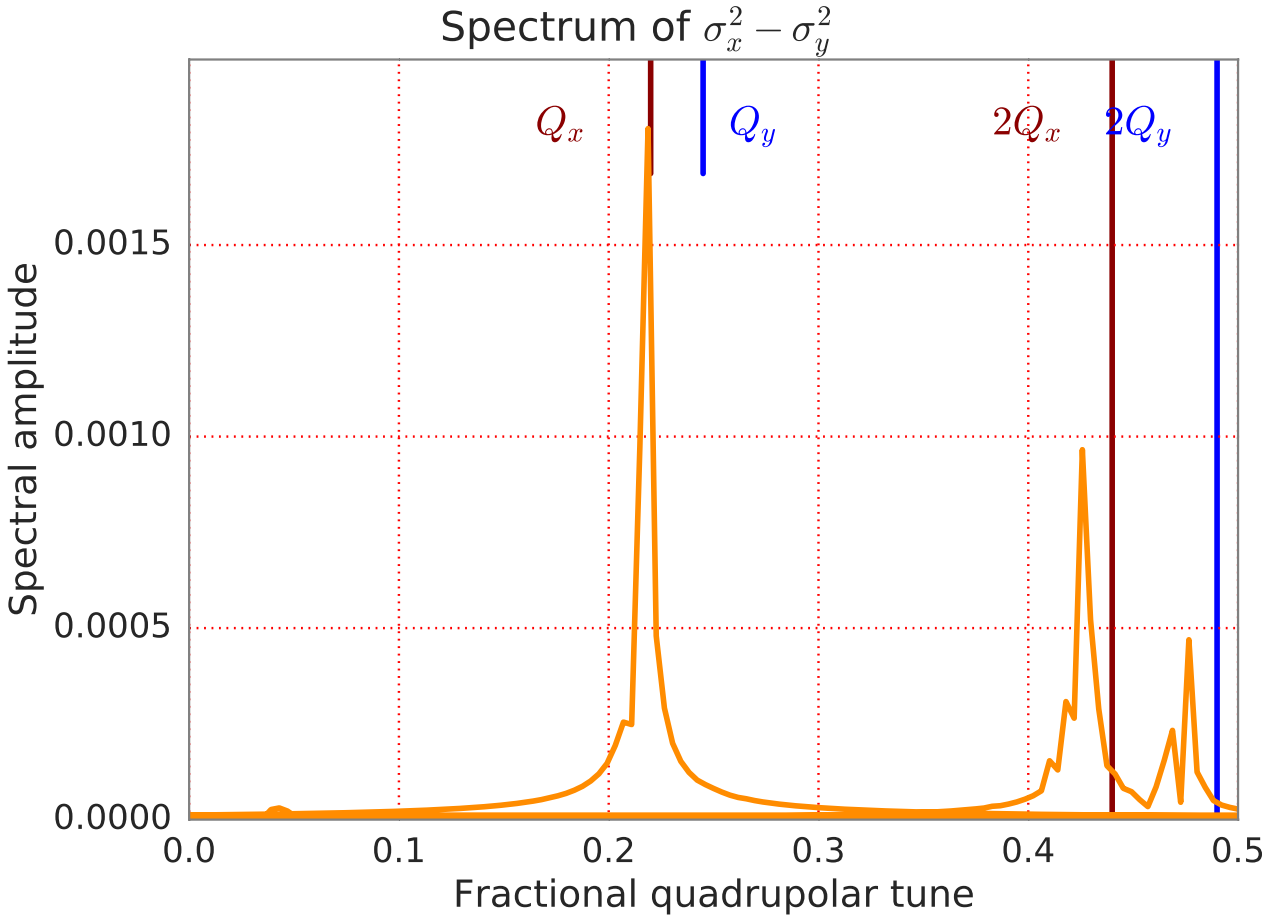




Quad. Spectrum: RMS-equiv. Gaussian

bunched	transv. distr.	synchrotron motion	dispersion	chromaticity
no	Gaussian	no	yes	no





Quad. Spectrum: Bunched Beam

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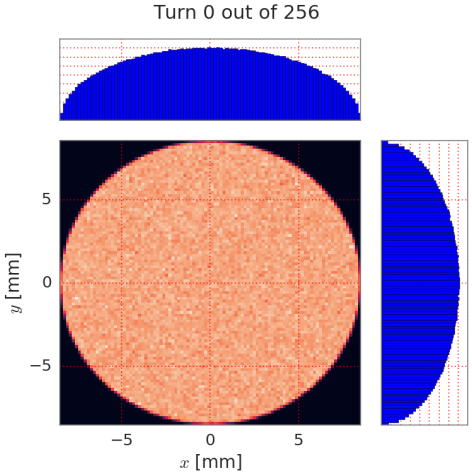

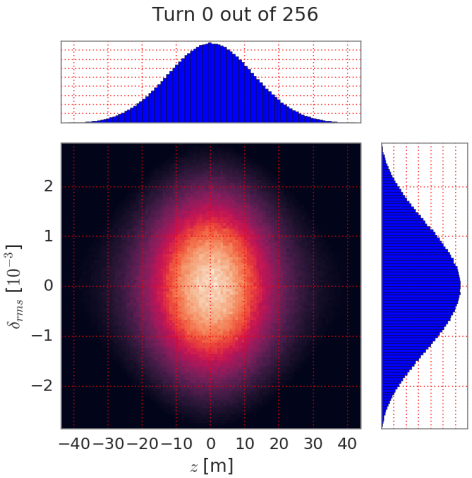

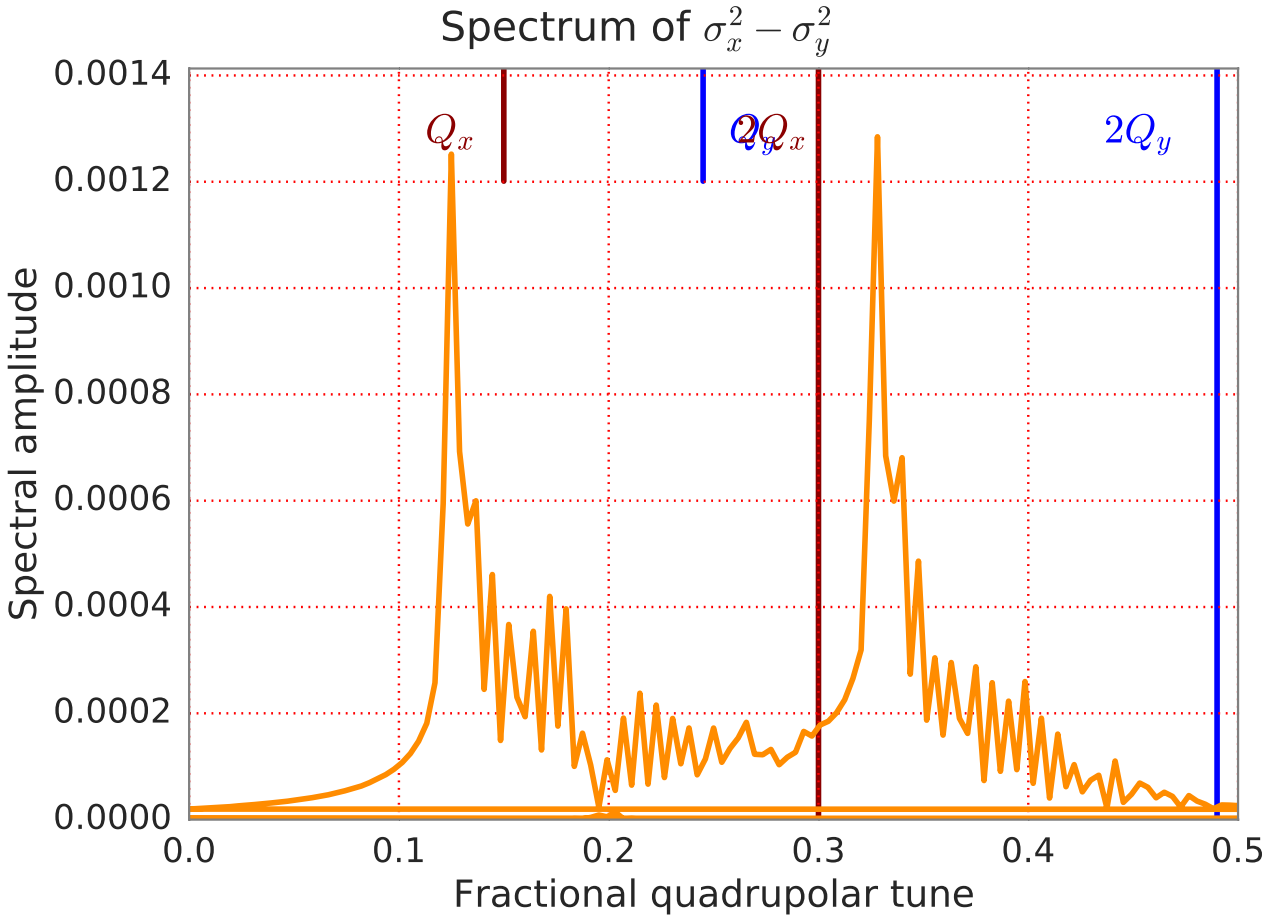

Quad. Spectrum: + Finite Q_s

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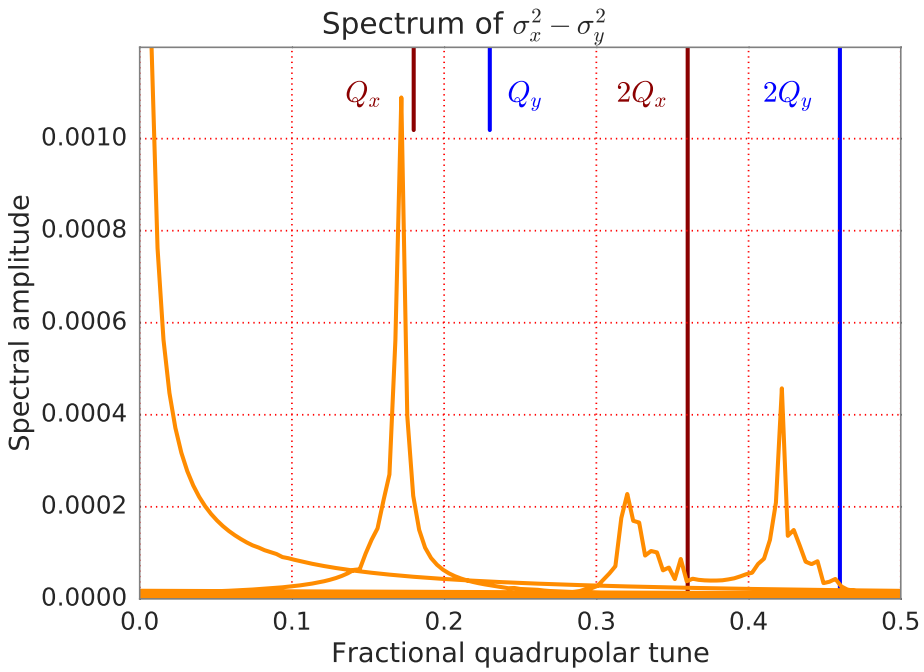

- experimental parameters (here $N = 4 \times 10^{11}$ ppb)
- evident quadrupolar betatron bands below $2Q_{x,y}$
- coherent dispersive mode slightly below Q_x (shifted by space charge!)
  - ⟶ **single peak-like** – unlike experimental observation

Quad. Spectrum: + Finite Chromaticity

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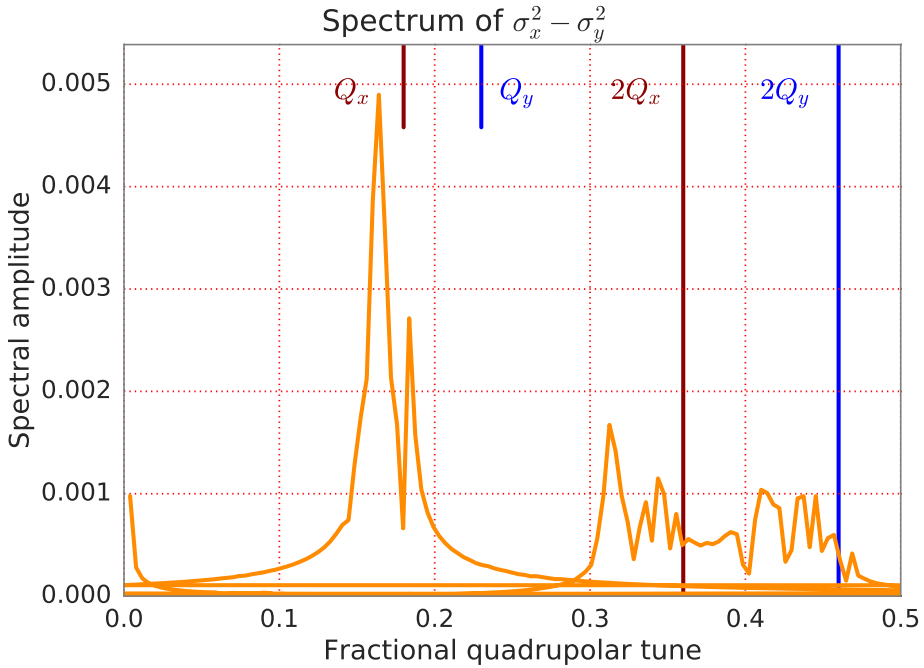
yes	KV (uniform)	yes	yes	yes



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- broadens dispersive peak (here FFT undersamples sidebands)
- produces additional peaks, shifted dominant peak

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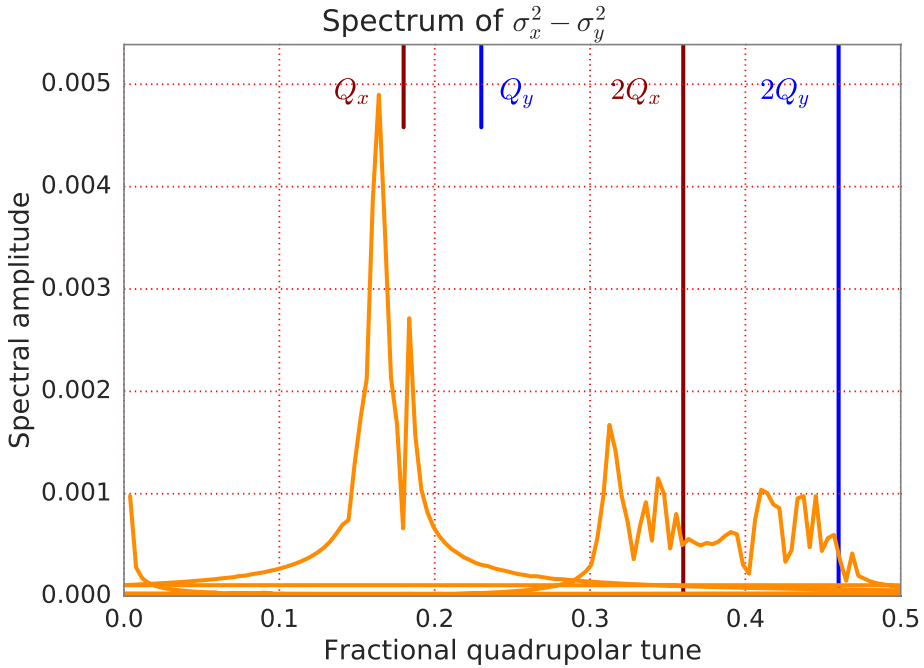

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NB: simulations ran with 10×10^6 macro-particles on 150 longitudinal slices across the RF bucket (≈ 80 m) where space charge is solved on 128×128 grids (no significant transverse difference between 2.5D / 3D PIC)

Part II:
Measurements –
Mode Identification
at CERN PS Injection

... and there is much to see!

The quadrupolar spectrum features many collective second-order modes:

$$\begin{aligned} S_{QPU} &\propto U_{right} + U_{left} - U_{top} - U_{bottom} \\ &\propto \langle x^2 \rangle - \langle y^2 \rangle \end{aligned}$$

with $x = \overline{x} + x_\beta + D_x \frac{\Delta p}{p_0}$ (likewise for y) this becomes

$$S_{QPU} \propto \underbrace{\left(\langle x_\beta^2 \rangle - \langle y_\beta^2 \rangle\right)}_{\sigma_x^2 - \sigma_y^2 + \langle x_\beta \rangle^2 - \langle y_\beta \rangle^2} + 2 \cdot \overline{x} \langle x_\beta \rangle + 2 \cdot \overline{y} \langle y_\beta \rangle$$

$$+ \left\langle x_\beta \frac{\Delta p}{p_0} \right\rangle - \left\langle y_\beta \frac{\Delta p}{p_0} \right\rangle + \text{longitudinal } \frac{\Delta p}{p_0} \text{ terms}$$

If linear coupling present, x, y rotate and additional $\langle x\, y \rangle$ cross-terms appear.

Relevant for CERN PS

At CERN PS injection:

- $\overline{x}\langle x_\beta\rangle, \langle x_\beta\rangle^2$: dipolar horizontal motion $\Longrightarrow$ $Q_x, 2Q_x$ lines
- $\overline{y}\langle y_\beta\rangle, \langle y_\beta\rangle^2$: dipolar vertical motion $\Longrightarrow$ $Q_y, 2Q_y$ lines

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- $\overline{y}\langle y_\beta\rangle, \langle y_\beta\rangle^2$: dipolar vertical motion $\Longrightarrow$ $Q_y, 2Q_y$ lines
- $\left\langle y_\beta \frac{\Delta p}{p_0} \right\rangle$: coherent vertical dispersion mode $\Longrightarrow$ $Q_y \pm Q_s - \Delta Q_{SC}$ line

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- σ_x^2: horizontal envelope mode[1] $\Longrightarrow 2Q_x - \Delta Q_{SC}$ line
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[1] in coupled case, betatron envelope modes rotate away from horizontal / vertical reference

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- σ_y^2: vertical envelope mode[1] $\Longrightarrow 2Q_y - \Delta Q_{SC}$ line
- $\langle x\,y\rangle$: Chernin's odd (tilting) modes $\Longrightarrow Q_x - Q_y$ and $Q_x + Q_y$ lines

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- σ_y^2: vertical envelope mode[1] $\Longrightarrow$ $2Q_y - \Delta Q_{SC}$ line
- $\langle x\, y\rangle$: Chernin's odd (tilting) modes $\Longrightarrow$ $Q_x - Q_y$ and $Q_x + Q_y$ lines

$\Longrightarrow$ in general, second-order modes affected by space charge!

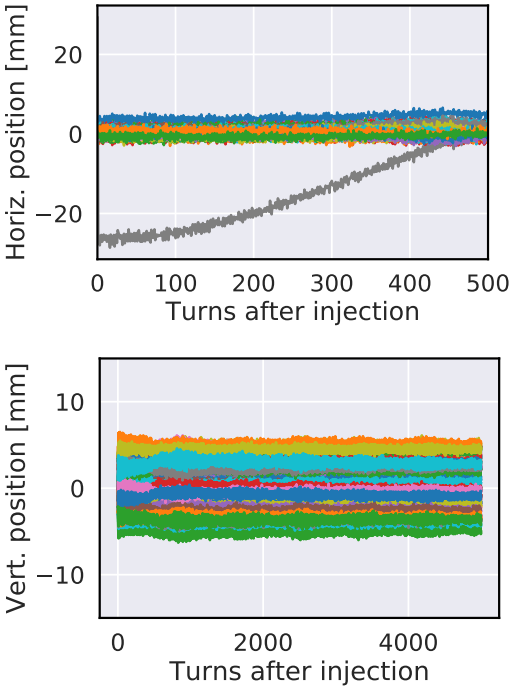
[1] in coupled case, betatron envelope modes rotate away from horizontal / vertical reference

Dipolar Mismatch

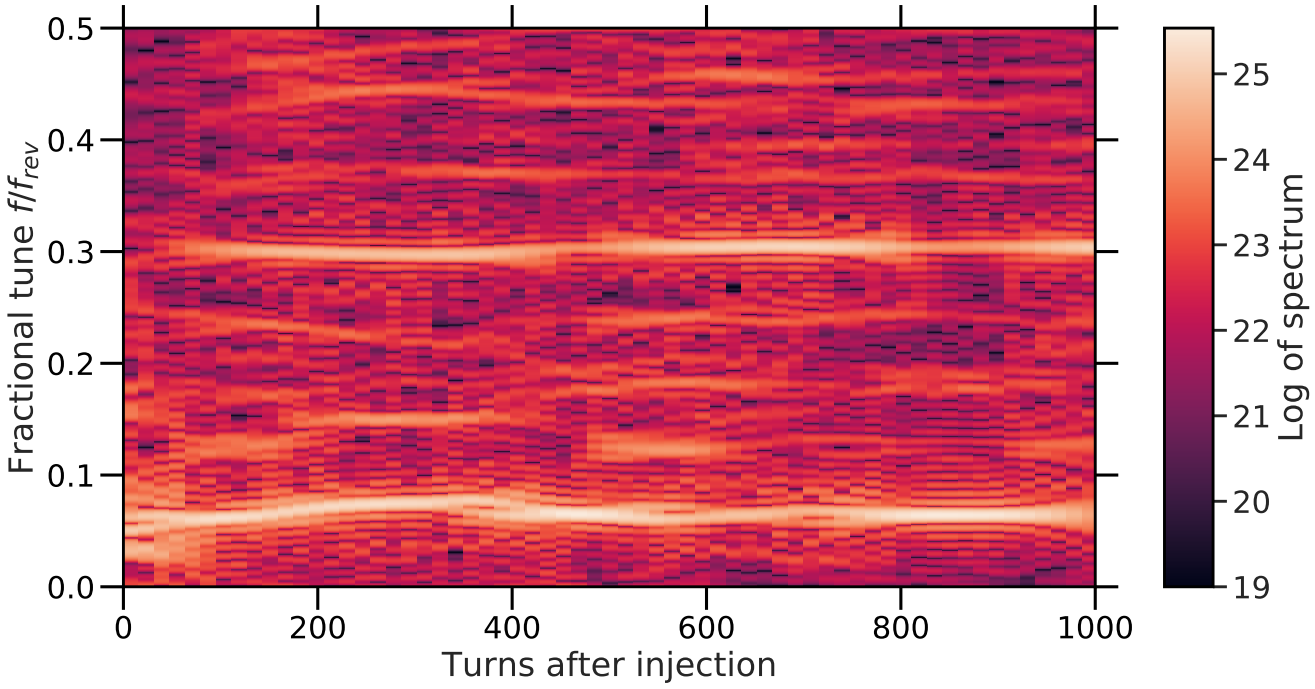
Dipolar mismatch contributes to amplitude of

⟶ dipolar modes $\Longrightarrow$ $Q_{x,y}$, $2Q_{x,y}$

set-up with $Q_y \approx 6.05$ and $Q_x \approx 6.3$, dipolar injection mismatch corrected:



(a) BPM measurements



(b) QPU spectogram

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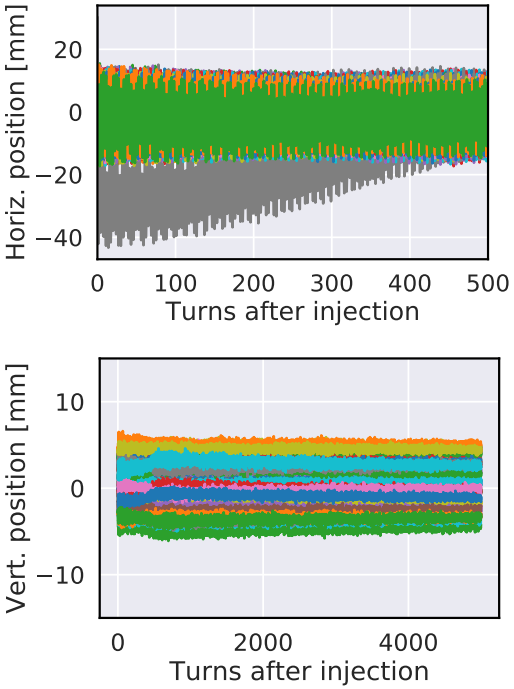
horizontal dipolar injection mis-steering:



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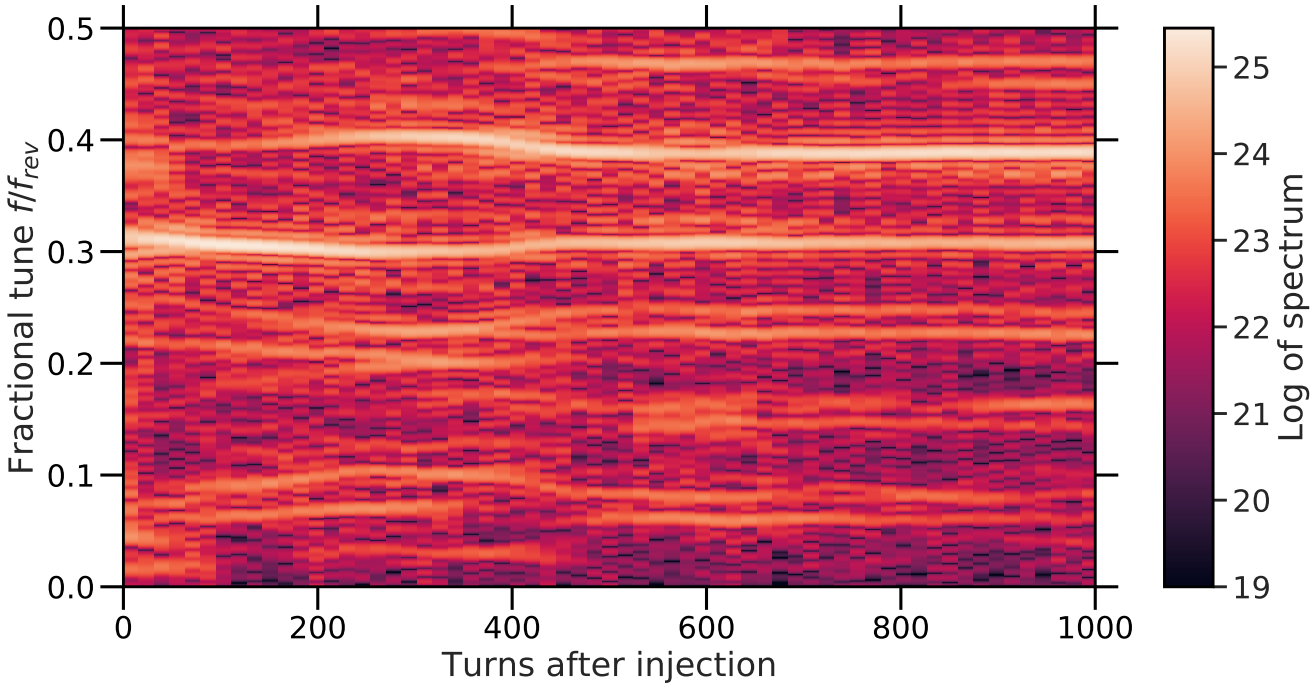


(b) QPU spectogram

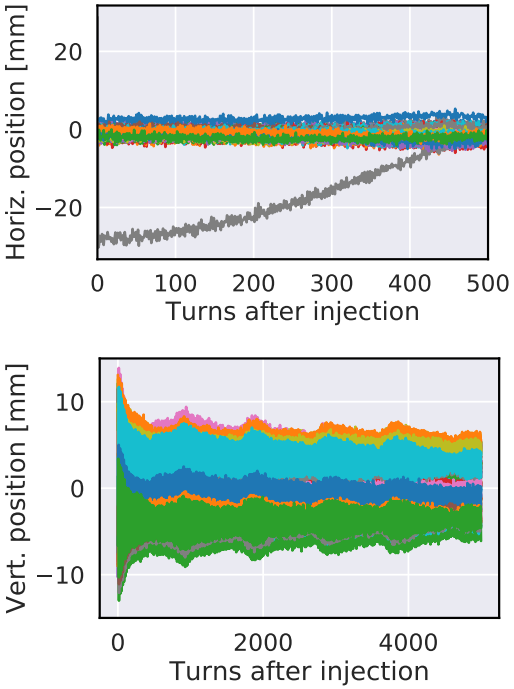
$\Longrightarrow$ dipolar mode $\langle x \rangle^2$ mirrored at 0.5: $2Q_x$ line at $1 - 2 \times 0.3 = 0.4$

Dipolar Mismatch

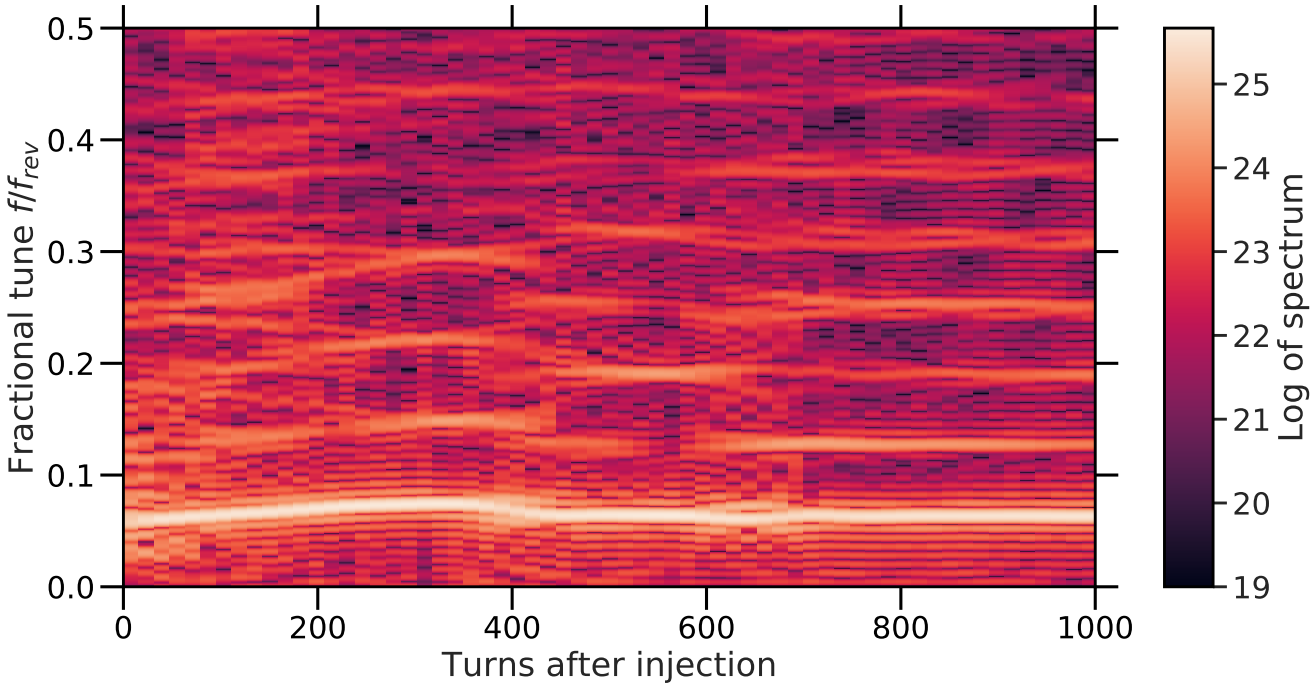
Dipolar mismatch contributes to amplitude of

⟶ dipolar modes $\Longrightarrow$ $Q_{x,y}$, $2Q_{x,y}$

vertical dipolar injection mis-steering:



(a) BPM measurements



(b) QPU spectogram

Odd Modes due to Linear Coupling

The beam features 2 odd (tilting) eigenmodes: 2nd order resonances due to linear coupling between the transverse planes (Chernin, Ref. [4, 5]).

1. low-frequency eigenmode: difference resonance $Q_x - Q_y$
2. high-frequency eigenmode: sum resonance $Q_x + Q_y$

$\longrightarrow$ driving terms for these originate from

I. skew quadrupole component in optics
II. space charge coupling in case of unequal beam sizes (e.g. $\epsilon_x \neq \epsilon_y$!)

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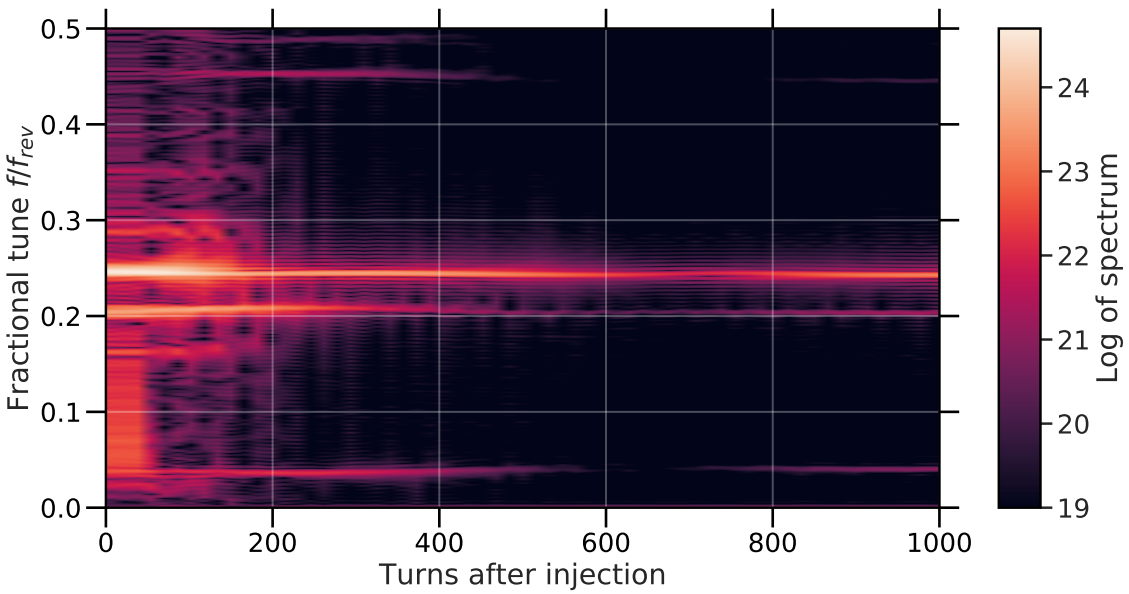

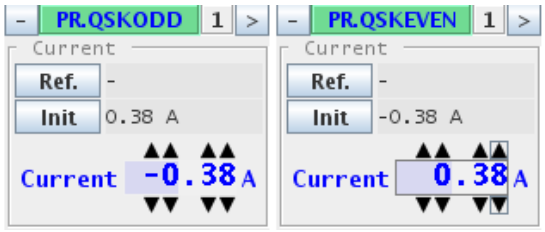

Figure: skew quadrupoles in max. coupling

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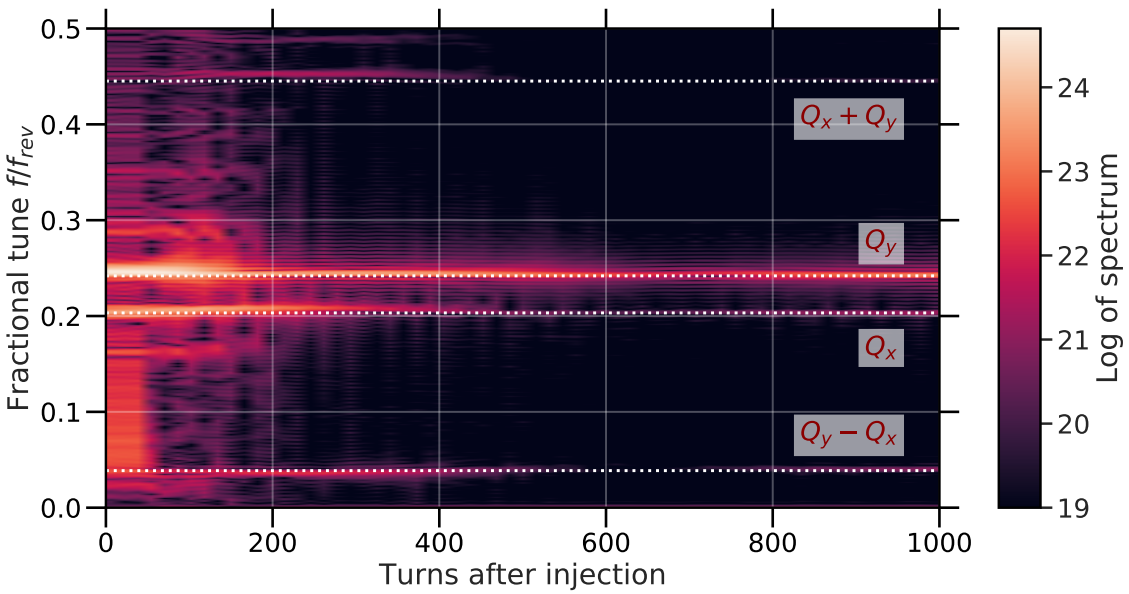


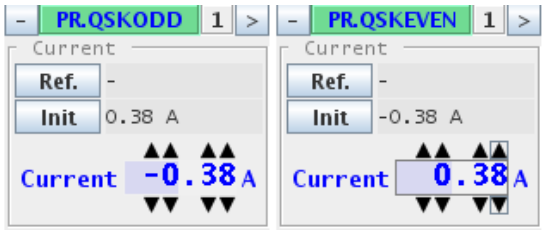


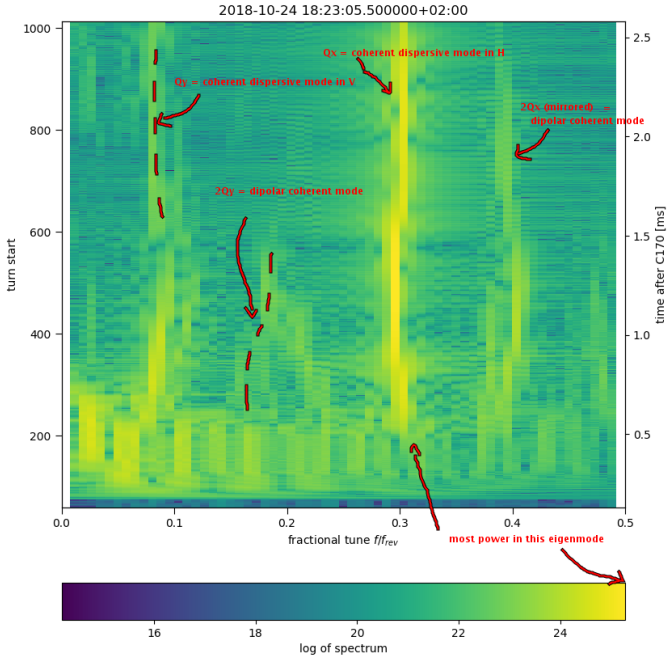
Figure: skew quadrupoles in max. coupling

Dispersion Mode: Head-tail Motion!

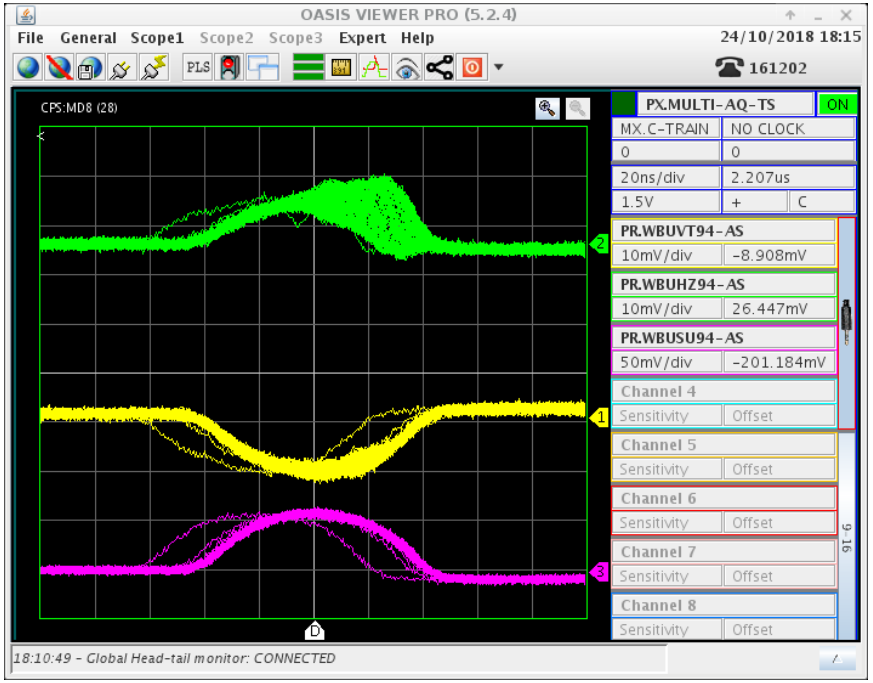
- coherent dispersion mode: measure correlation $\left\langle x \frac{\Delta p}{p_0} \right\rangle$, $\left\langle y \frac{\Delta p}{p_0} \right\rangle$
- in PS at natural chroma, have (higher-order) horizontal instability

$\longrightarrow$ can be cured with transverse feedback

$\Longrightarrow$ idea: identify dispersion mode, $\rightsquigarrow$ space charge shift?



(a) quadrupolar spectrum



(b) wideband pick-up

Figure: without transverse feed-back

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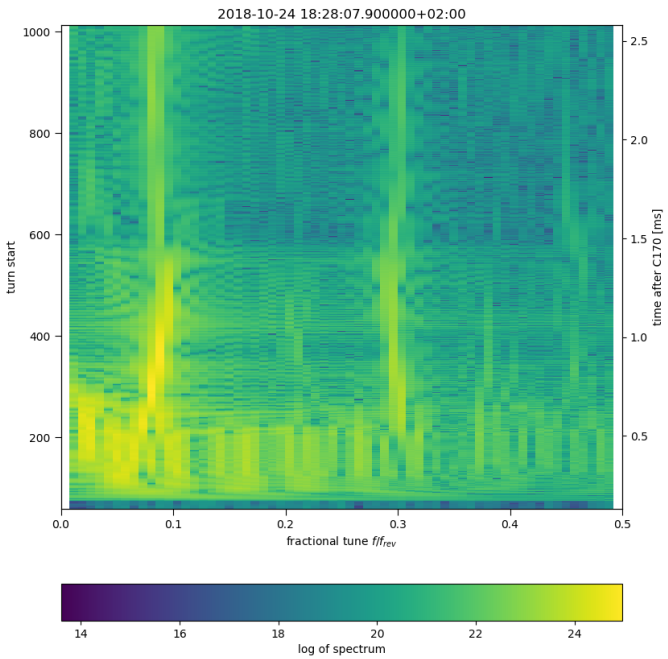

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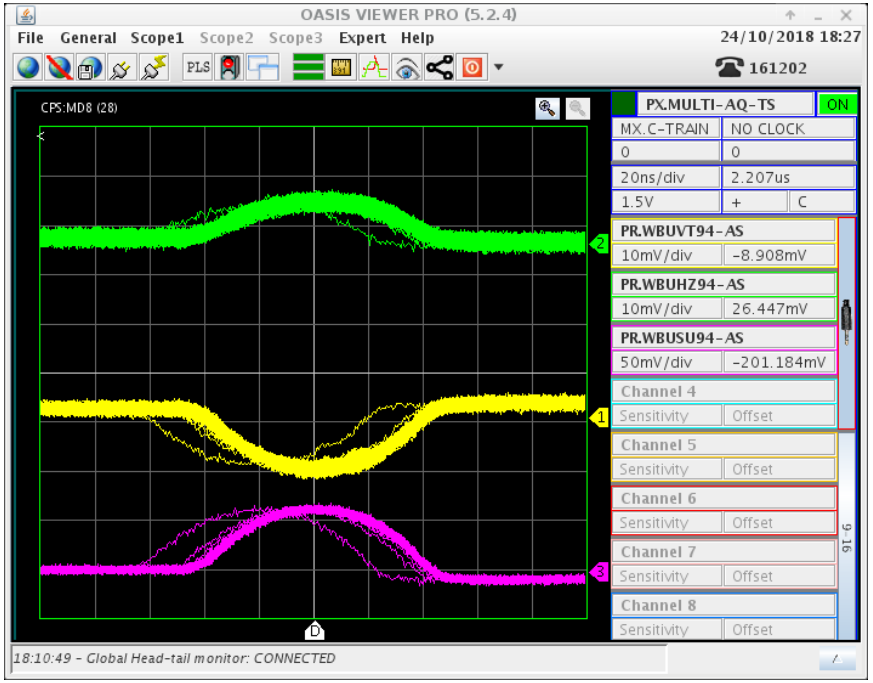


(b) wideband pick-up

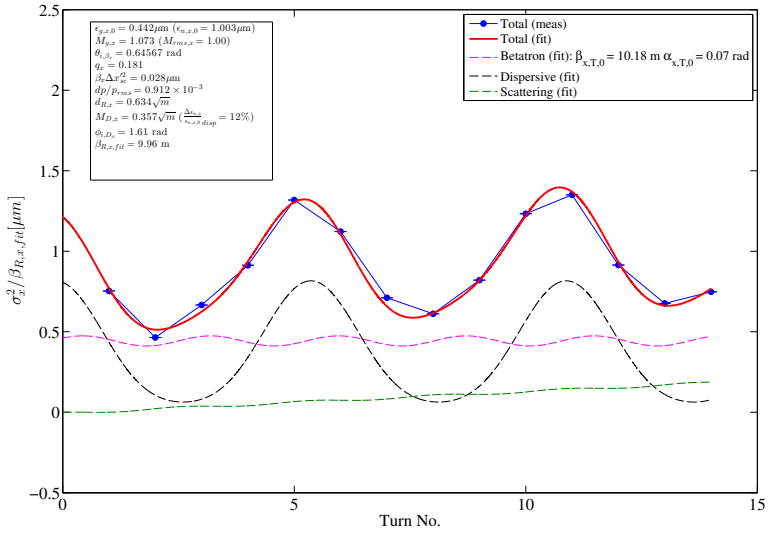
Figure: with transverse feed-back switched on

Part III:
Missing Puzzle Piece...

the Even Envelope Modes $\sigma_{x,y}$

Betatron Mismatch from Transfer Line

At PS injection, SEM[2] grid measurements provide turn-by-turn data for transverse profiles (cf. CERN talk ↗), also quadrupolar pick-up data recorded.



(a) Operational optics: optimised injection

⟶ both operational and dispersion-optimised transfer line optics lead to minimal betatron mismatch in the PS (cf. pink curve)

⟹ typically the corresponding even envelope oscillation is minimal

[2] **S**econdary **E**lectron E**M**ission

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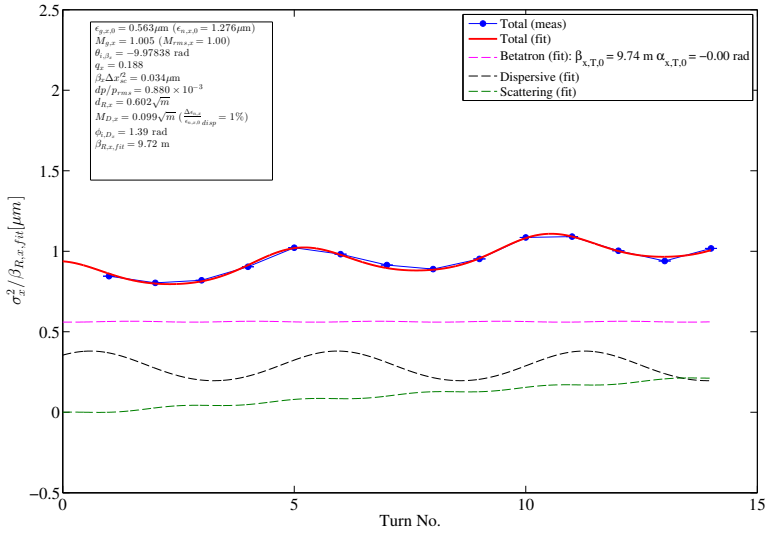

(b) Dispersion-optimised optics: optimised injection

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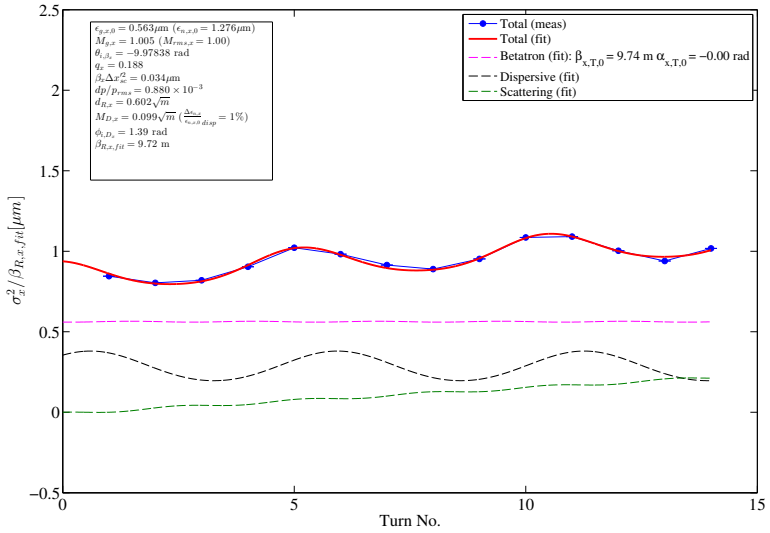
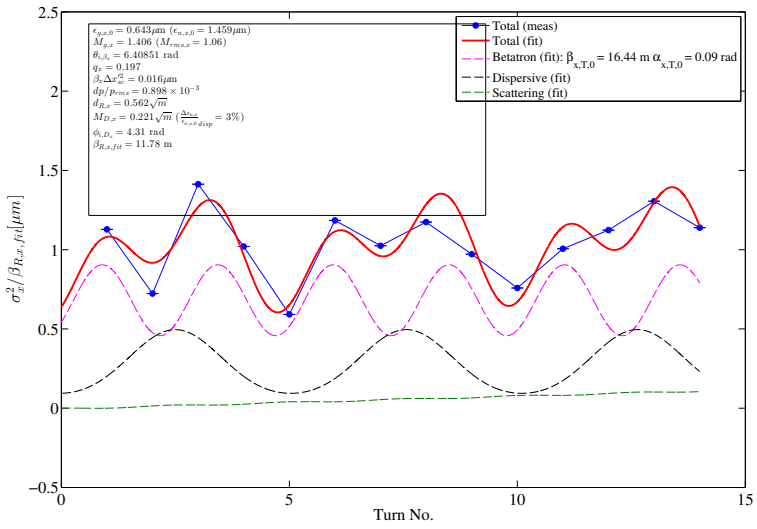


(b) Dispersion-optimised optics: optimised injection



(c) Dispersion-optimised optics: betatron mismatch

⟶ both operational and dispersion-optimised transfer line optics lead to minimal betatron mismatch in the PS (cf. pink curve)

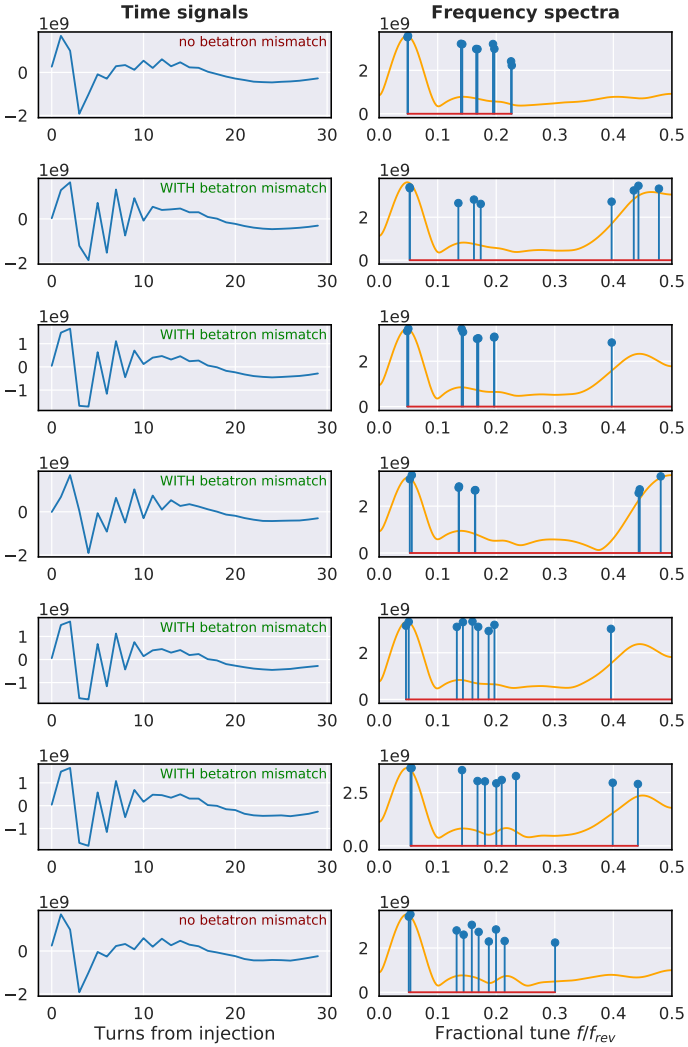
⟹ typically the corresponding even envelope oscillation is minimal

⟹ intentional betatron mismatch ⇝ significant even envelope oscillation

[2] **S**econdary **E**lectron E**M**ission

QPU Spectrum for Betatron Mismatch

Looking at the quadrupolar pick-up for these 30 turns, we find a quickly damped even envelope oscillation (at $\approx 2Q_{x,y}$):



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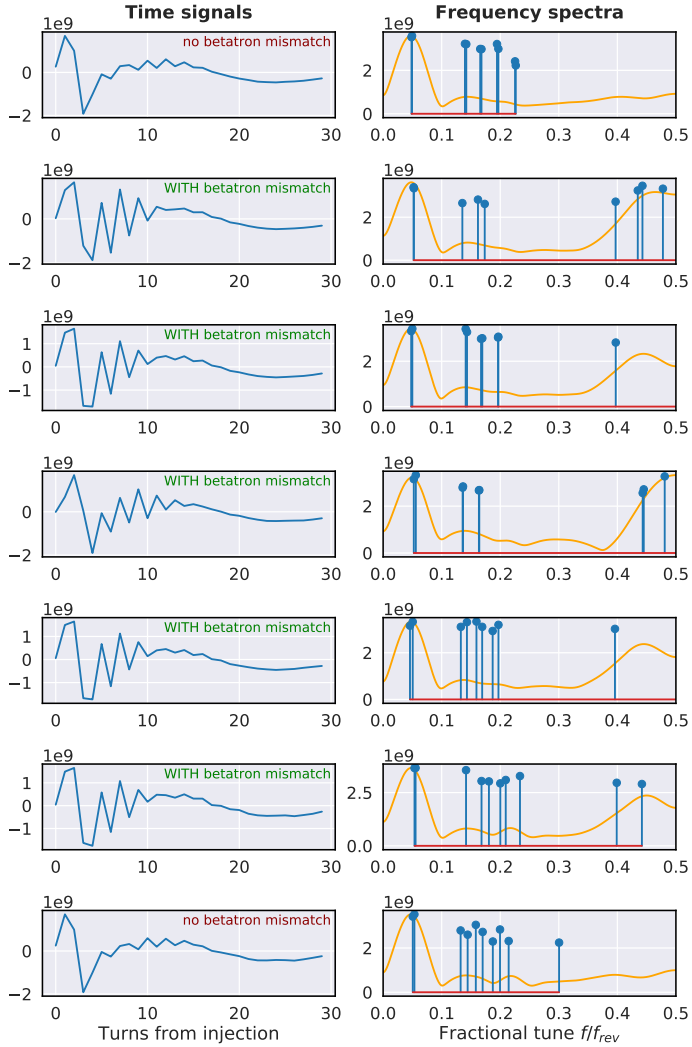


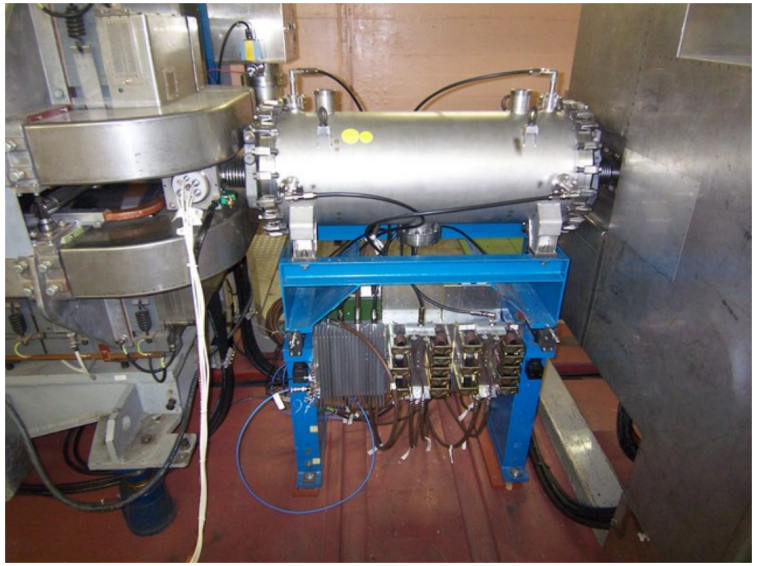
- clearly visible betatron mismatch amplitude in the quadrupolar signal
- frequency resolution at 30 turns is challenging
- ⟹ at usual minimised betatron mismatch: small amplitudes in $f/f_{rev} \approx 0.45$ region (tunes are usual operational $Q_{x,y} \approx 0.23$)
- ⟹ with intentional betatron mismatch: clearly observe energy in $f/f_{rev} \approx 0.45$ region
- ⟹ usual injection observations won't reveal even envelope modes, too well matched

Part IV:
Quadrupolar Beam Transfer Function

Transverse Feedback as Quad. Kicker

So we cannot measure the even envelope modes at injection...
$\longrightarrow$ excite the beam in quadrupolar mode during the cycle!
$\Longrightarrow$ quadrupolar beam transfer function

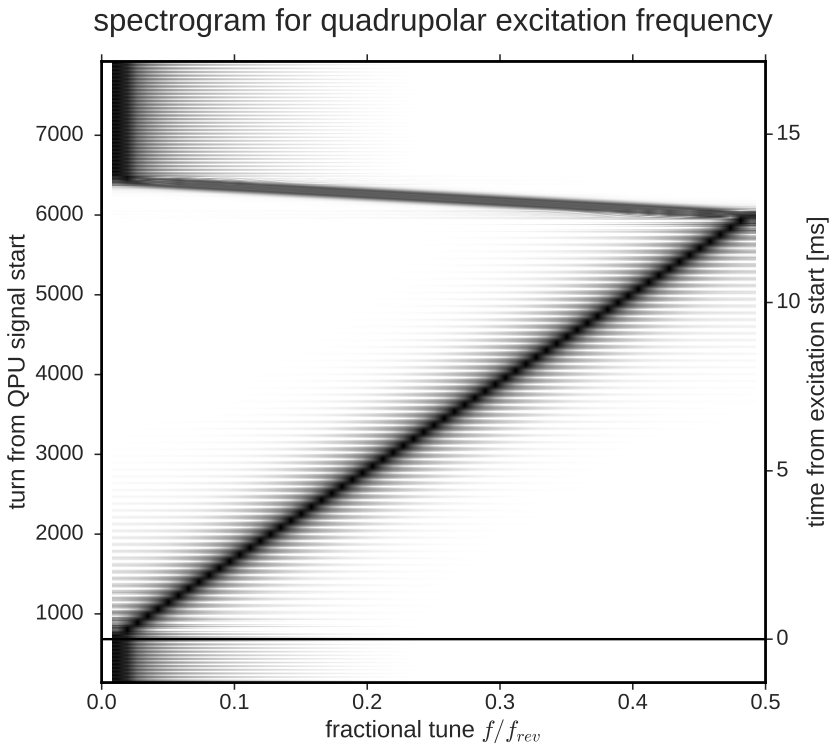
Kicker in section 97 is part of the PS transverse feedback system:



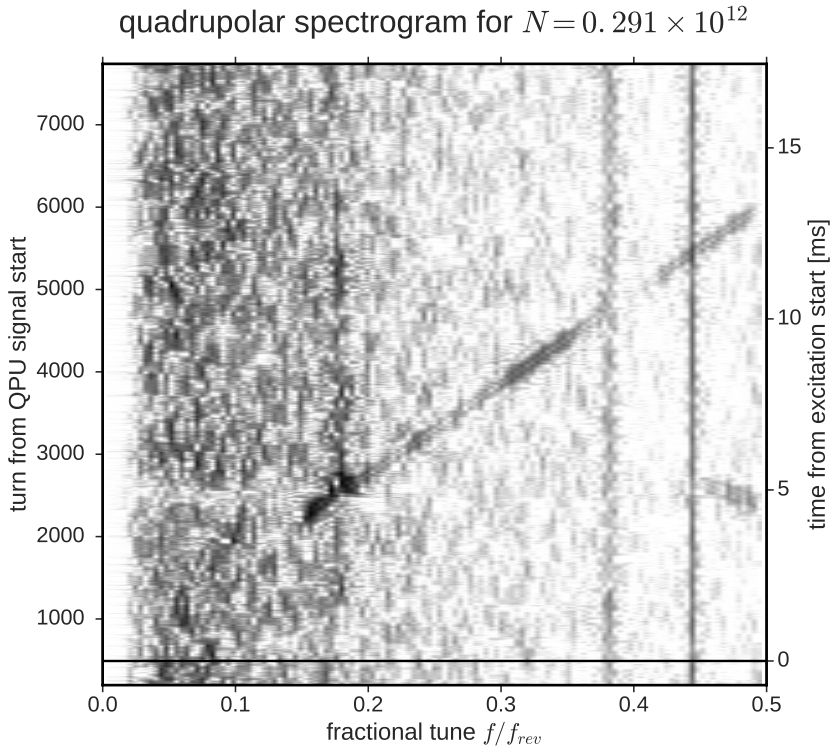
courtesy Guido Sterbini

Quadrupolar Excitation: Chirp

excitation signal

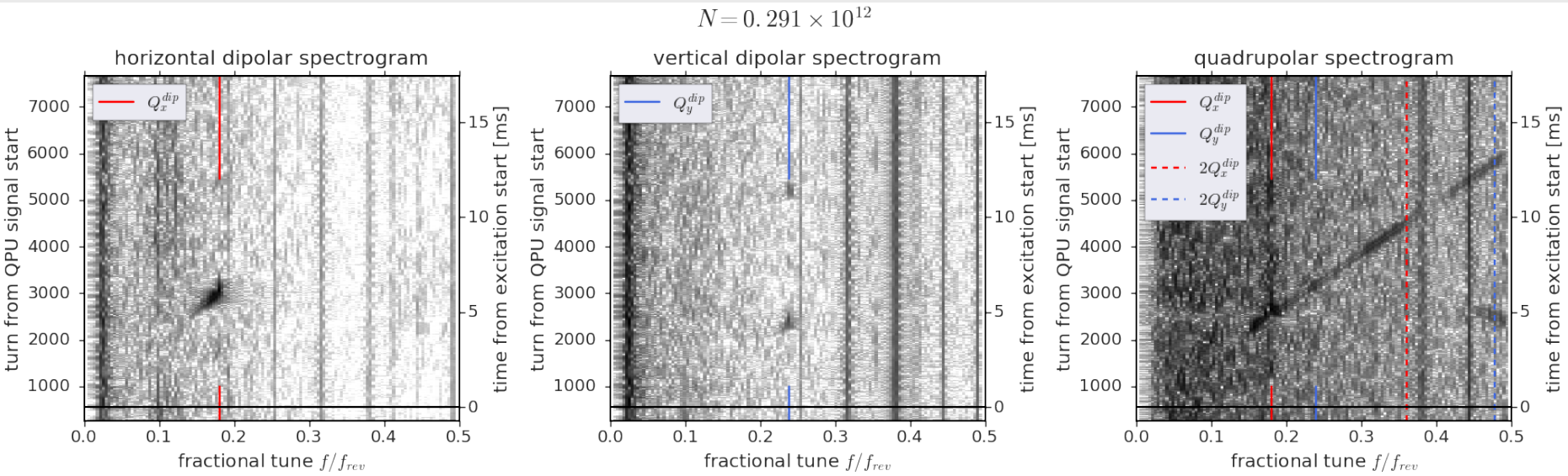


beam response (via QPU)



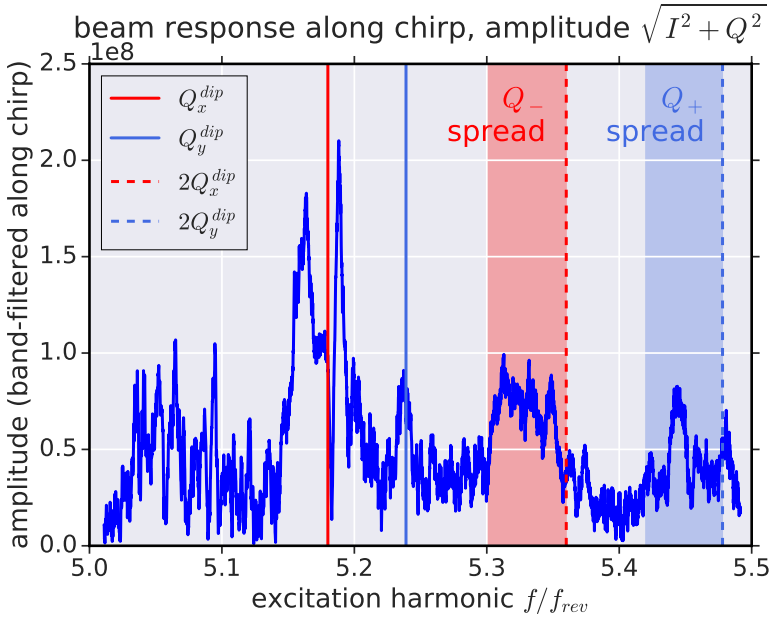
- distinct peaks around machine tunes $f < 0.25 f_{\text{rev}}$
- frequency bands around twice the machine tunes
- (disregard the constant frequencies, due to instrumentation)

Measured Quadrupolar BTF



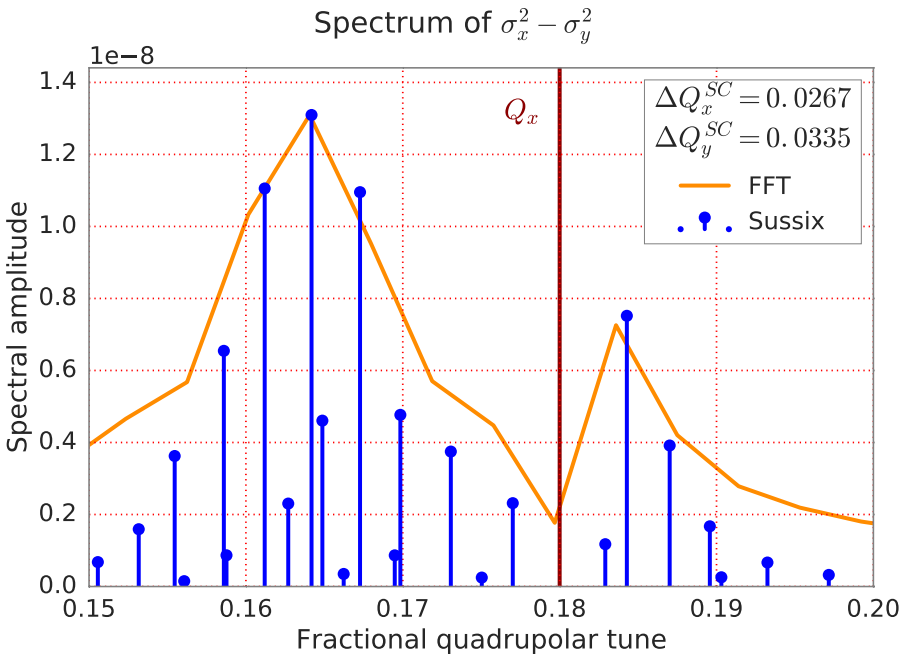
demodulation of QPU signal with excitation
$\Longrightarrow$ amplitude response of beam $\longrightarrow$

For more details on this approach, cf. Ref. [6] and slides at HB2018 contribution ↗

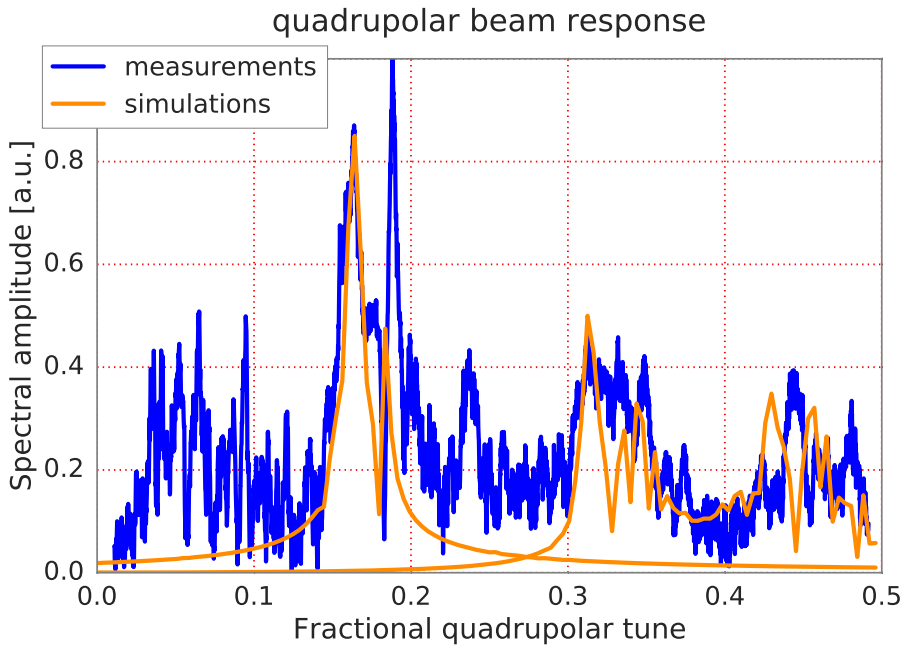


$\Longrightarrow$ I got intrigued by $\approx Q_x$ spectral content!

Dispersion Mode Revisited



(a) Q_x tune region (simulation)

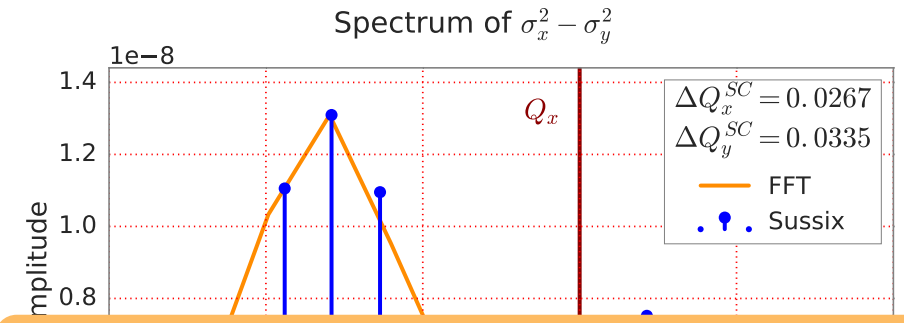


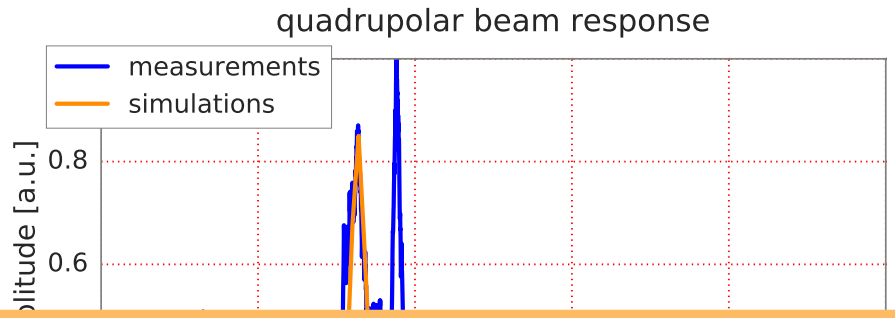
(b) experiment vs. simulation

- frequency analysis reveals regular sideband structure around dispersion mode (blue peaks), this measurement was for natural chroma

⟶ after LS2, new PS set-up: vanishing chroma + dipolar damping available!

Dispersion Mode Revisited





for vanishing chromaticity...

simulations without chroma predict distinct narrow peak for dispersion mode
$\rightsquigarrow$ measure SC shift much more accurately?

What happens if I excite higher-order head-tail modes?
$\Longrightarrow$ energy in $\left\langle x \frac{\Delta p}{p_0} \right\rangle$? SC vs. instability?

- frequency analysis reveals regular sideband structure around dispersion mode (blue peaks), this measurement was for natural chroma

$\longrightarrow$ after LS2, new PS set-up: vanishing chroma + dipolar damping available!

I hope, I could trigger some creative ideas...

Conclusion

Quadrupolar pick-ups

- provide non-invasive beam measurements
- can be “cheaply” recorded and long-term stored (standard operation?)

and they can provide rich information on the beam:

1. coherent dispersion mode: transverse-longitudinal correlation, e.g. due to
   - ⟶ dispersion mismatch
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3. even envelope mode: transverse beam size oscillation
   - ⟶ transverse emittance
   - ⟶ space charge tune shift

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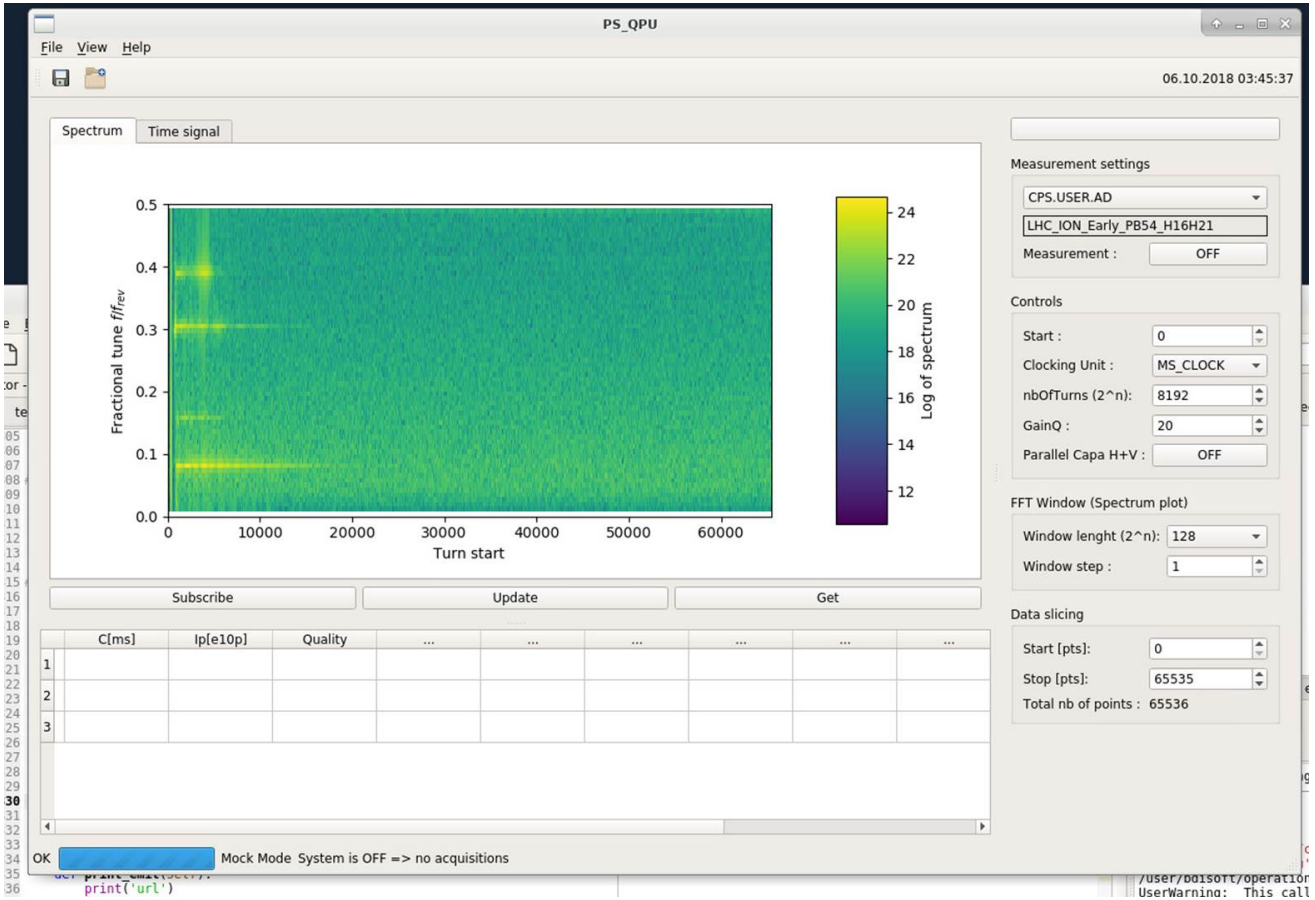
   - ⟶ transverse emittance
   - ⟶ space charge tune shift

Important: all these modes are 2nd order $\Longrightarrow$ all frequencies change with space charge!

Outlook: Control Room App Development

A first prototype of a control room app (CERN CC):

`https://gitlab.cern.ch/mcoly/ps_qpu`



Thank you for your attention!

Acknowledgements:
Simon Albright, Marcel Coly, Heiko Damerau, Vincenzo Forte,
Marek Gasior, Tom Levens, Elias Métral, Haroon Rafique,
Guido Sterbini, Malte Titze, Panagiotis Zisopoulos

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Some Historical Perspective

QPU in **time domain** for emittance measurements:

- 1983, R. H. Miller et al. at SLAC [7]
- 2002, A. Jansson at CERN in PS [2]

QPU in **frequency domain** for emittance measurements:

- 2007, C.Y. Tang at Fermilab [8]

QPU in **frequency domain** for space charge measurements:

- 1996, M. Chanel at CERN in LEAR [9]
- 1999, T. Uesugi et al. at NIRS in HIMAC [10]
- 2000, R. Bär at GSI in SIS-18 [11]
- 2014, R. Sing et al. at GSI in SIS-18 [12]

$\Longrightarrow$ all far away from coupling and coasting beams

$\Longrightarrow$ What about bunched beams? Close to coupling?

Incoherent KV Tune Shift

The Kapchinskij-Vladimirskij (KV) beam distribution has all particles at same incoherent space charge tune shift:

$$\Delta Q_{x,y}^{\mathrm{KV}} \doteq -\frac{K^{\mathrm{SC}} R^2}{4\sigma_{x,y}(\sigma_x + \sigma_y) Q_{x,y}} \tag{1a}$$

$$\doteq \frac{1 + \sigma_{x,y}/\sigma_{y,x}}{2Q_{x,y}} \Lambda \tag{1b}$$

space charge perveance $K^{\mathrm{SC}} \doteq \dfrac{q\lambda}{2\pi\epsilon_0 \beta\gamma^2 p_0 c}$

Incoherent KV Tune Shift

The Kapchinskij-Vladimirskij (KV) beam distribution has all particles at same incoherent space charge tune shift:

$$\Delta Q_{x,y}^{\mathrm{KV}} \doteq -\frac{K^{\mathrm{SC}} R^2}{4\sigma_{x,y}(\sigma_x+\sigma_y)Q_{x,y}} \tag{1a}$$

$$\doteq \frac{1+\sigma_{x,y}/\sigma_{y,x}}{2Q_{x,y}}\Lambda \tag{1b}$$

Connect Λ quantity to general 2D envelope mode expressions in terms of **observables**:

$$\Lambda = \frac{Q_+^2+Q_-^2-4(Q_x^2+Q_y^2)}{4+3(\sigma_x/\sigma_y+\sigma_y/\sigma_x)} \tag{2}$$

(Gaussian tune spread = 2x the RMS-equivalent KV tune shift!)

$$\text{space charge perveance } K^{\mathrm{SC}} \doteq \frac{q\lambda}{2\pi\epsilon_0\beta\gamma^2 p_0 c}$$

Envelope Equations

Envelope equations of motion (e.o.m.)

$$\sigma_x'' + K_x(s)\sigma_x - \frac{\epsilon_{x,\mathrm{geo}}^2}{\sigma_x^3} - \frac{K^{\mathrm{SC}}}{2(\sigma_x + \sigma_y)} = 0 \quad , \tag{3a}$$

$$\sigma_y'' + K_y(s)\sigma_y - \frac{\epsilon_{y,\mathrm{geo}}^2}{\sigma_y^3} - \frac{K^{\mathrm{SC}}}{2(\sigma_x + \sigma_y)} = 0 \tag{3b}$$

for transverse r.m.s. beam widths $\sigma_{x,y}$ have equilibrium

$$\frac{Q_x^2}{R^2}\sigma_{x,\mathrm{m}} - \frac{\epsilon_{x,\mathrm{geo}}^2}{\sigma_{x,\mathrm{m}}^3} - \frac{K^{\mathrm{SC}}}{2(\sigma_{x,\mathrm{m}} + \sigma_{y,\mathrm{m}})} = 0 \quad , \tag{4a}$$

$$\frac{Q_y^2}{R^2}\sigma_{y,\mathrm{m}} - \frac{\epsilon_{y,\mathrm{geo}}^2}{\sigma_{y,\mathrm{m}}^3} - \frac{K^{\mathrm{SC}}}{2(\sigma_{x,\mathrm{m}} + \sigma_{y,\mathrm{m}})} = 0 \tag{4b}$$

Smooth Approximation, Lin. Perturbation

Constant focusing channel

$$K_{x,y} = \frac{1}{\beta_{x,y}^2} = \frac{Q_{x,y}^2}{R^2} = \text{const.} \tag{5}$$

gives linearised e.o.m. for perturbation around equilibrium $r = \sigma_\mathrm{m} + \delta r$

$$\frac{d^2}{ds^2}\begin{pmatrix}\delta r_x \\ \delta r_y\end{pmatrix} = -\underbrace{\begin{pmatrix}\kappa_x & \kappa_\mathrm{SC} \\ \kappa_\mathrm{SC} & \kappa_y\end{pmatrix}}_{\doteq(\kappa)} \cdot \begin{pmatrix}\delta r_x \\ \delta r_y\end{pmatrix} \tag{6}$$

$$\text{with} \quad \begin{cases} \kappa_{x,y} = 4\frac{Q_{x,y}^2}{R^2} - \frac{2\sigma_{x,y}+3\sigma_{y,x}}{\sigma_{x,y}}\kappa_\mathrm{SC} \\ \kappa_\mathrm{SC} \doteq \frac{K^\mathrm{SC}}{2(\sigma_x+\sigma_y)^2} \end{cases} \tag{7}$$

Venturini-Reiser / Lee-Okamoto Theory

Adding dispersion to envelope equations, studied by Venturini-Reiser [13] and, independently, by Lee-Okamoto [14]:

$$\sigma_x'' + \left(K_x(s) - \frac{K^{\mathrm{SC}}}{2r_x(r_x+\sigma_y)} \right) \sigma_x - \frac{\epsilon_{x,\mathrm{geo}}^2}{\sigma_x^3} = 0 \quad , \tag{8a}$$

$$\sigma_y'' + \left(K_y(s) - \frac{K^{\mathrm{SC}}}{2\sigma_y(r_x+\sigma_y)} \right) \sigma_y - \frac{\epsilon_{y,\mathrm{geo}}^2}{\sigma_y^3} = 0 \tag{8b}$$

$$D_x'' + \left(K_x(s) - \frac{K^{\mathrm{SC}}}{2r_x(r_x+\sigma_y)} \right) D_x = \frac{1}{\rho(s)} \tag{8c}$$

Generalised $r_x^2 \doteq \sigma_x^2 + D_x^2 \frac{\Delta p}{p_0}$ and $\epsilon_{x,\mathrm{geo}}$ only betatron emittance (no dispersive contribution). Linearisation around matched values gives a 3D matrix to be solved for eigenvalues $\rightsquigarrow$ mode tunes of the betatron envelopes $Q_\pm$ as well as the dispersion mode Q_d, cf. e.g. Ref. [15].

Far Away vs. On the Coupling Resonance

Two eigenmodes for coherent quadrupolar betatron oscillation:

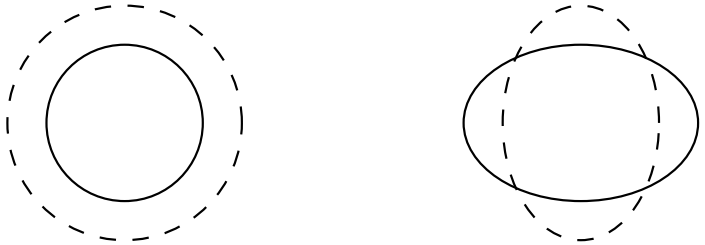
far away from coupling

(a) horizontal mode (b) vertical mode

Quadrupolar mode tunes:

$$Q_{\pm} = 2Q_{x,y} - \left|\Delta Q^{\mathrm{KV}}_{x,y}\right| \left(3 - \frac{\sigma_{x,y}}{\sigma_x + \sigma_y}\right)/2 \tag{9}$$

full coupling



(a) breathing mode (b) antisym. mode

Quadrupolar mode tunes:

$$Q_{+} = 2Q_0 - \left|\Delta Q^{\mathrm{KV}}_{x,y}\right| \tag{10a}$$

$$Q_{-} = 2Q_0 - \frac{3}{2}\left|\Delta Q^{\mathrm{KV}}_{x,y}\right| \tag{10b}$$

(assuming round beams, $Q_{x,y} \equiv Q_0$)

Space Charge from Odd Envelope Modes?

According to Aslaninejad and Hofmann [16], not possible to infer space charge from odd modes.

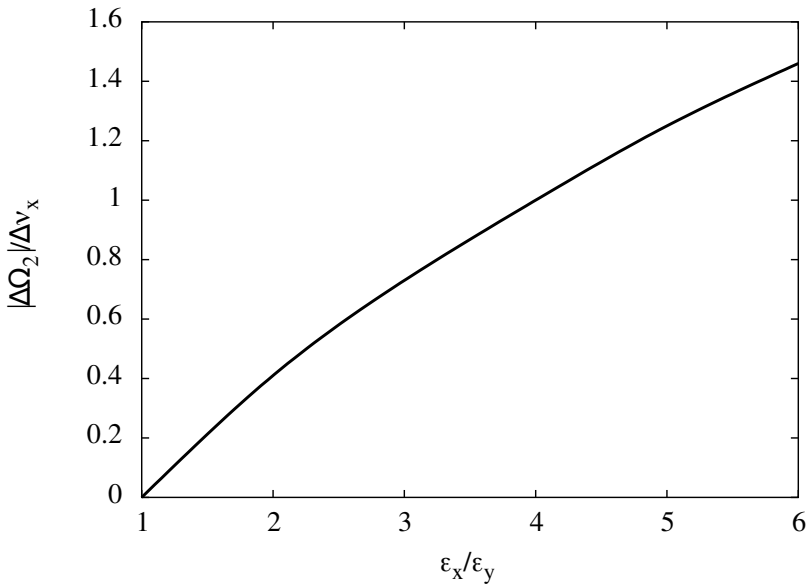
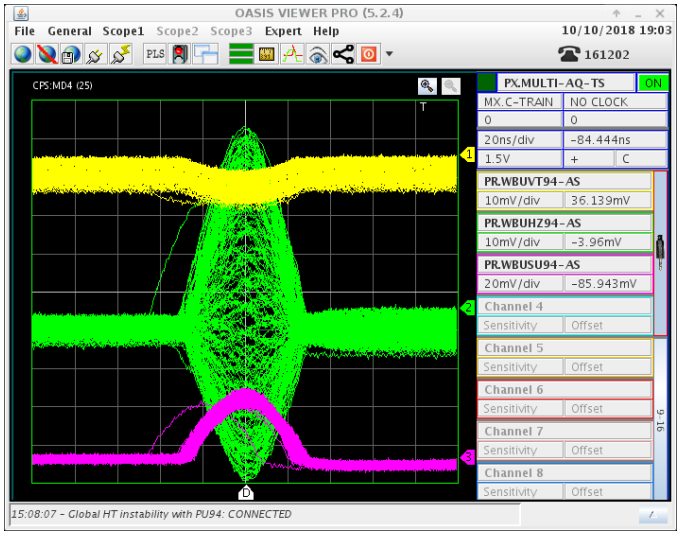


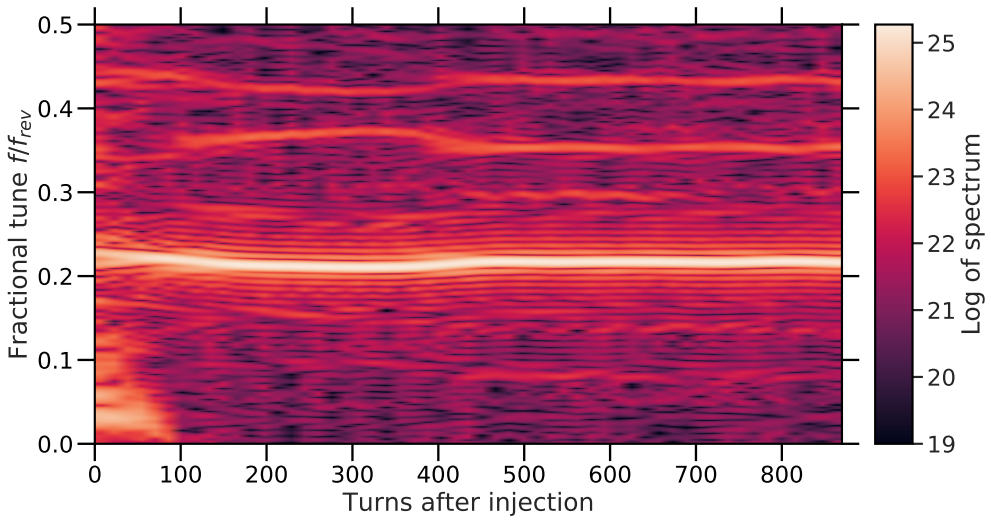
Figure: coherent shift in SC incoherent tune shift units vs. emittance ratio

"For weak space charge (as in rings) this coherent shift [of the linear coupling modes] is found to be approximately independent of the space-charge tune shift as well as the absolute tune values."

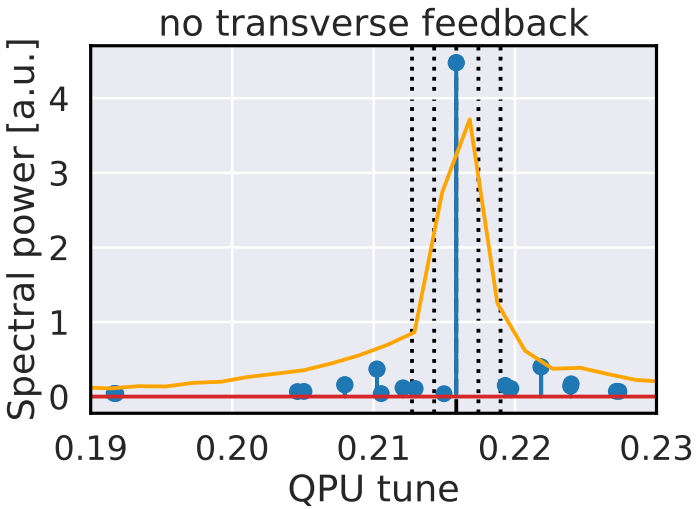
Vanishing Chroma, Mode 0



(a) Mode 0 in horizontal plane



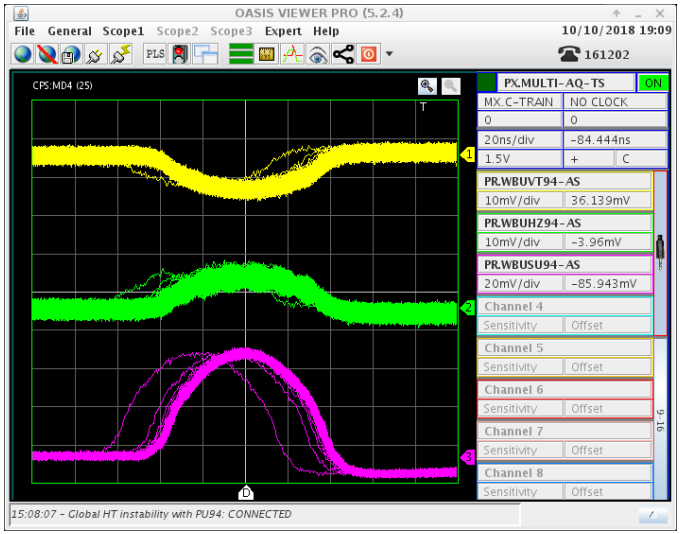
(b) QPU spectogram



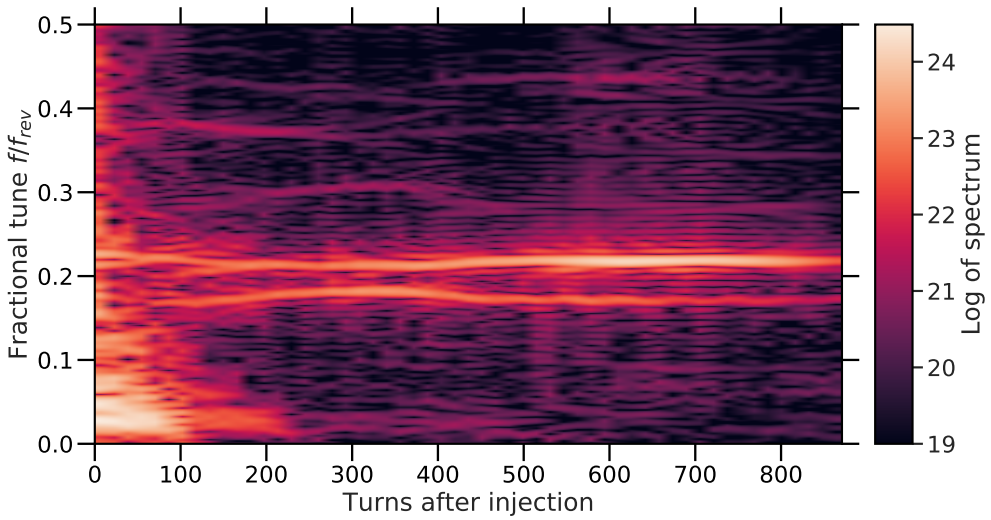
(c) QPU spectrum after injection bump (480 turns)

transverse feedback off at vanishing chromaticity $\Longrightarrow$ horizontal rigid head-tail mode

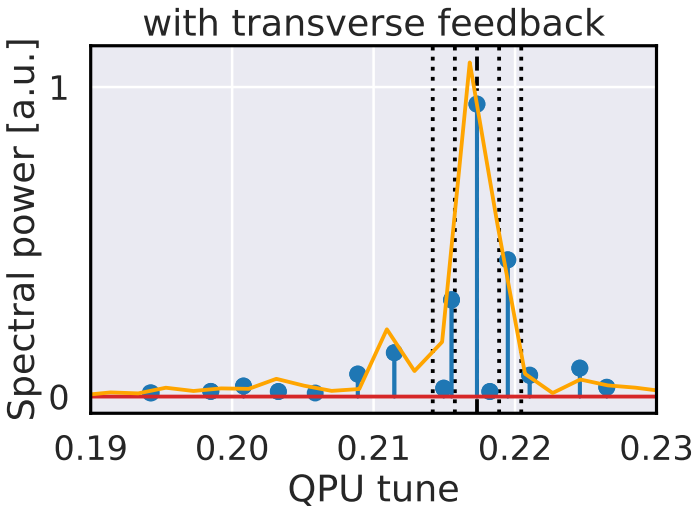
Vanishing Chroma, Stabilised



(a) Mode 0 in horizontal plane



(b) QPU spectogram



(c) QPU spectrum after injection bump (480 turns)

transverse feedback on at vanishing chromaticity $\Longrightarrow$ stabilised, shifted peak by about 0.006 $\rightsquigarrow$ coherent dispersion mode?