Diagnostics with Quadrupolar Pick-ups

Adrian Oeftiger
25 Sep 2019, MCBI 2019
Motivation

1\textsuperscript{st} order

rigid dipolar centroid oscillation:

- Newton’s third law, \( \text{actio} = \text{reactio} \)
- no direct space charge (SC)
  [except higher-order projections]
Motivation

1\textsuperscript{st} order

rigid dipolar centroid oscillation:
- Newton’s third law, actio = reactio
- no direct space charge (SC) [except higher-order projections]

⇒ measure direct space charge through frequency shift of beam size oscillations about matched $\sigma_{x,y}$

⇒ during this campaign, found many new insights on QPU spectrum

2\textsuperscript{nd} order

quadrupolar envelope oscillation:
- defocused by transverse space charge force
- frequency of envelope oscillation decreases with SC
What about quadrupolar pick-ups?

- non-invasive (and thus non-destructive) measurement
  - contains info on transverse emittances $\epsilon_{x,y}$ and linear coupling
  - even with quadrupolar excitation: beam transfer function measurement without significant emittance growth ($\Delta \epsilon / \epsilon < 5\%$)
  - modes and their amplitudes in quadrupolar spectrum provide hint at beam dynamics
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- measures *coherent* mode tunes, not incoherent spreads
  - cannot infer transverse distribution type!
  - can express coherent tune shifts in units of RMS equivalent K-V SC tune shift
  - different transverse distributions with same RMS characteristics *yield same coherent tune shifts*
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- recent push from coasting beam to bunched beam!
  - synchrotron motion $\rightsquigarrow$ much more dynamics
  - chromaticity, synchrotron sidebands (cf. my HB2018 contribution ↗)
  - started to study head-tail instabilities vs. second-order modes, e.g. $\langle x \cdot \frac{\Delta p}{p_0} \rangle$
  - new perspectives!? In particular for space charge vs. head-tail instabilities
    $\implies$ a lot remains to be done!
Quadrupolar Pick-up

Schematics of a quadrupolar pick-up (see Ref. [1]):

Induced voltage on electrodes:

\[ U_{\text{right}} \propto I_{\text{beam}} (1 + z_1 x + z_2 + ...) \]
\[ U_{\text{left}} \propto I_{\text{beam}} (1 - z_1 x + z_2 + ...) \]
\[ U_{\text{top}} \propto I_{\text{beam}} (1 + z_1 y - z_2 + ...) \]
\[ U_{\text{bottom}} \propto I_{\text{beam}} (1 - z_1 y - z_2 + ...) \]

where

\[ z_{1x} \propto \frac{\langle x \rangle}{d}, \]

\[ z_{1y} \propto \frac{\langle y \rangle}{d}. \]
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where

\[ z_{1x} \propto \frac{\langle x \rangle}{d}, \]
\[ z_{1y} \propto \frac{\langle y \rangle}{d}, \quad \text{and} \]
\[ z_2 \propto \frac{\langle x^2 \rangle - \langle y^2 \rangle}{d^2} = \frac{\sigma_x^2 - \sigma_y^2 + \langle x \rangle^2 - \langle y \rangle^2}{d^2} \] (neglecting dispersion)
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\[ \Rightarrow \] combine voltages to measure dipolar beam moments (usual BPM):

\[ \langle x \rangle \propto U_{\text{right}} - U_{\text{left}} \]
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\[ \langle x \rangle \propto U_{right} - U_{left} \]
\[ \langle y \rangle \propto U_{top} - U_{bottom} \]

\[ \Rightarrow \text{or combine voltages to measure quadrupolar beam moments:} \]
\[ z_2 \propto \sigma_{x}^2 - \sigma_{y}^2 + \langle x \rangle^2 - \langle y \rangle^2 \]
\[ \propto U_{right} + U_{left} - U_{top} - U_{bottom} \]
Time Domain $\leftrightarrow$ Frequency Domain

**Time domain** very challenging:

- accurately resolving the beam size moment $\sigma_x^2 - \sigma_y^2$ requires removal of (strong) dipolar component in $z_2 \propto \sigma_x^2 - \sigma_y^2 + \langle x \rangle^2 - \langle y \rangle^2$

  $\rightarrow$ beautiful measurements of injection mismatch and transverse emittances $\epsilon_{x,y}$
  (no dipolar contribution to magnetic signal)
  $\rightarrow$ unfortunately equipment difficult to operate, later removed from PS

- lately also differential measurements in LHC, cf. Ref. [3]
Time Domain \xrightarrow{\sim} Frequency Domain

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- Ref. [2] A. Jansson’s PhD thesis: clever idea, magnetic quadrupolar pick-up → beautiful measurements of injection mismatch and transverse emittances \( \epsilon_x, \epsilon_y \) (no dipolar contribution to magnetic signal)
  → unfortunately equipment difficult to operate, later removed from PS

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**Frequency domain** more accessible:

- spectral measurements don’t care about differential offsets, dipolar contribution oscillates at distinct frequency etc.
  → idea: resolve quadrupolar beam oscillation modes accurately!
    → determine frequency shifts due to space charge
    → determine energy/amplitudes in modes as measure of mismatch

- CERN LHC injectors equipped with stripline pick-ups, new 3-channel frontend: horizontal and vertical dipolar + quadrupolar signals!

- GSI SIS-18 also equipped with 2 QPUs!
Overview

Frequency domain measurements in PS:

- to my surprise, I found super many modes in the spectrum when I first looked at injection...
  - in the meantime, quite a few experiments identified them!

- beam size oscillation itself difficult to observe at injection
  - developed beam transfer function (BTF) measurement technique with transverse feedback as quadrupolar kicker
Overview

Frequency domain measurements in PS:

- to my surprise, I found super many modes in the spectrum when I first looked at injection...
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Structure of this talk:

1. second-order mode overview in simulations
2. mode identification at CERN PS injection
   - dipolar modes
   - Chernin’s odd (tilting) modes
   - coherent dispersion modes
3. the missing puzzle piece: even envelope modes $\sigma_{x,y}$?
4. quadrupolar BTF and space charge
Part I: Second-order Modes in Simulations
## Quad. Spectrum: Coasting KV Beam

<table>
<thead>
<tr>
<th>bunched</th>
<th>transv. distr.</th>
<th>synchrotron motion</th>
<th>dispersion</th>
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<tbody>
<tr>
<td>no</td>
<td>KV (uniform)</td>
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### Turn 0 out of 256

*Diagram showing the evolution of the beam distribution over a turn.*
Quad. Spectrum: Coasting KV Beam

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Spectrum of $\sigma^2_x - \sigma^2_y$

Fractional quadrupolar tune

Spectral amplitude

$Q_x$, $Q_y$, $2Q_x$, $2Q_y$
Quad. Spectrum: + Dispersion

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Spectrum of $\sigma_x^2 - \sigma_y^2$

Fractional quadrupolar tune

Spectral amplitude

Turn 0 out of 256

$Q_x$ $Q_y$ $2Q_x$ $2Q_y$
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<td>no</td>
<td>Gaussian</td>
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Quad. Spectrum: RMS-equiv. Gaussian

Spectrum of \( \sigma_x^2 - \sigma_y^2 \)

Fractional quadrupolar tune

<table>
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<th>Spectral amplitude</th>
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<tbody>
<tr>
<td>0.0015</td>
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<tr>
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<tr>
<td>0.0005</td>
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<td>0.0000</td>
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Quad. Spectrum: Bunched Beam

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Spectrum of $\sigma^2_x - \sigma^2_y$
Quad. Spectrum: + Finite $Q_s$

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- experimental parameters (here $N = 4 \times 10^{11}$ ppb)
- evident quadrupolar betatron bands below $2Q_{x,y}$
- coherent dispersive mode slightly below $Q_x$ (shifted by space charge!)

→ **single peak-like** – unlike experimental observation
Quad. Spectrum: + Finite Chromaticity

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including natural chromaticity ($Q'_x = -0.83Q_x$ and $Q'_y = -1.12Q_y$):
- broadens dispersive peak (here FFT undersamples sidebands)
- produces additional peaks, shifted dominant peak
Quad. Spectrum: + Finite Chromaticity

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  - broadens dispersive peak (here FFT undersamples sidebands)
  - produces additional peaks, shifted dominant peak

NB: simulations ran with $10 \times 10^6$ macro-particles on 150 longitudinal slices across the RF bucket ($\approx 80$ m) where space charge is solved on $128 \times 128$ grids (no significant transverse difference between 2.5D / 3D PIC)
Part II: Measurements – Mode Identification at CERN PS Injection
... and there is much to see!

The quadrupolar spectrum features many collective second-order modes:

\[ S_{QPU} \propto U_{right} + U_{left} - U_{top} - U_{bottom} \]
\[ \propto \langle x^2 \rangle - \langle y^2 \rangle \]

with \( x = \bar{x} + x_\beta + D_x \frac{\Delta p}{p_0} \) (likewise for \( y \)) this becomes

\[ S_{QPU} \propto \left( \langle x_\beta^2 \rangle - \langle y_\beta^2 \rangle \right) + 2 \cdot \bar{x} \langle x_\beta \rangle + 2 \cdot \bar{y} \langle y_\beta \rangle \]
\[ + \langle x_\beta \frac{\Delta p}{p_0} \rangle - \langle y_\beta \frac{\Delta p}{p_0} \rangle + \text{longitudinal} \frac{\Delta p}{p_0} \text{ terms} \]

If linear coupling present, \( x, y \) rotate and additional \( \langle x \, y \rangle \) cross-terms appear.
Relevant for CERN PS

At CERN PS injection:

- $\bar{x}\langle x_\beta \rangle, \langle x_\beta \rangle^2$: dipolar horizontal motion
  $\Rightarrow Q_x, 2Q_x$ lines

- $\bar{y}\langle y_\beta \rangle, \langle y_\beta \rangle^2$: dipolar vertical motion
  $\Rightarrow Q_y, 2Q_y$ lines

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At CERN PS injection:

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- $\langle x_\beta \frac{\Delta p}{p_0} \rangle$: coherent horizontal dispersion mode
  $\implies Q_x \pm Q_s - \Delta Q_{SC}$ line

- $\bar{y}\langle y_\beta \rangle, \langle y_\beta \rangle^2$: dipolar vertical motion
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$\langle x y \rangle$: Chernin’s odd (tilting) modes
$\implies Q_x - Q_y$ and $Q_x + Q_y$ lines

in general, second-order modes affected by space charge!

1 in coupled case, betatron envelope modes rotate away from horizontal / vertical reference
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- $\sigma_x^2$: horizontal envelope mode
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- $\sigma_y^2$: vertical envelope mode
  $\Rightarrow 2Q_y - \Delta Q_{SC}$ line

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$\implies$ in general, second-order modes affected by space charge!

---

$^1$ in coupled case, betatron envelope modes rotate away from horizontal / vertical reference
Dipolar Mismatch

Dipolar mismatch contributes to amplitude of dipolar modes \( \rightarrow Q_{x,y}, 2Q_{x,y} \)

set-up with \( Q_y \approx 6.05 \) and \( Q_x \approx 6.3 \), dipolar injection mismatch corrected:

(a) BPM measurements

(b) QPU spectogram
Dipolar Mismatch

Dipolar mismatch contributes to amplitude of dipolar modes $\implies Q_{x,y}, 2Q_{x,y}$

horizontal dipolar injection mis-steering:

$$\implies \text{dipolar mode } \langle x \rangle^2 \text{ mirrored at } 0.5: 2Q_x \text{ line at } 1 - 2 \times 0.3 = 0.4$$
Dipolar Mismatch

Dipolar mismatch contributes to amplitude of dipolar modes \( \Rightarrow Q_{x,y}, 2Q_{x,y} \)

vertical dipolar injection mis-steering:

(a) BPM measurements

(b) QPU spectogram
Odd Modes due to Linear Coupling

The beam features 2 odd (tilting) eigenmodes: 2nd order resonances due to linear coupling between the transverse planes (Chernin, Ref. [4, 5]).

1. low-frequency eigenmode: difference resonance $Q_x - Q_y$
2. high-frequency eigenmode: sum resonance $Q_x + Q_y$

→ driving terms for these originate from
   I. skew quadrupole component in optics
   II. space charge coupling in case of unequal beam sizes (e.g. $\epsilon_x \neq \epsilon_y$)
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Figure: skew quadrupoles in max. coupling
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Figure: skew quadrupoles in max. coupling
Dispersion Mode: Head-tail Motion!

- coherent dispersion mode: measure correlation $\langle x \frac{\Delta p}{p_0} \rangle$, $\langle y \frac{\Delta p}{p_0} \rangle$
- in PS at natural chroma, have (higher-order) horizontal instability
  $\longrightarrow$ can be cured with transverse feedback
  $\implies$ idea: identify dispersion mode, $\sim$ space charge shift?

**(a)** quadrupolar spectrum

**(b)** wideband pick-up

*Figure: without transverse feedback*
Dispersion Mode: Head-tail Motion!

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- in PS at natural chroma, have (higher-order) horizontal instability
  → can be cured with transverse feedback
  ⟷ idea: identify dispersion mode, ~→ space charge shift?

(a) quadrupolar spectrum
(b) wideband pick-up

**Figure:** with transverse feed-back switched on
Part III:
Missing Puzzle Piece...
the Even Envelope Modes $\sigma_{x,y}$
Betatron Mismatch from Transfer Line

At PS injection, SEM² grid measurements provide turn-by-turn data for transverse profiles (cf. CERN talk ↩), also quadrupolar pick-up data recorded.

(a) Operational optics: optimised injection

→ both operational and dispersion-optimised transfer line optics lead to minimal betatron mismatch in the PS (cf. pink curve)

⇒ typically the corresponding even envelope oscillation is minimal

²Secondary Electron EMission
Betatron Mismatch from Transfer Line

At PS injection, SEM² grid measurements provide turn-by-turn data for transverse profiles (cf. CERN talk ↗), also quadrupolar pick-up data recorded.

(b) Dispersion-optimised optics: optimised injection

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Secondary Electron Emission
Betatron Mismatch from Transfer Line

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(b) Dispersion-optimised optics: optimised injection

(c) Dispersion-optimised optics: betatron mismatch

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⇒ typically the corresponding even envelope oscillation is minimal

⇒ intentional betatron mismatch ↘ significant even envelope oscillation

$^2$Secondary Electron Emission
QPU Spectrum for Betatron Mismatch

Looking at the quadrupolar pick-up for these 30 turns, we find a quickly damped even envelope oscillation (at $\approx 2Q_{x,y}$):

- With no betatron mismatch, the time signals show a smooth variation, and the frequency spectra are flat.
- With betatron mismatch, the time signals exhibit oscillations, and the frequency spectra show peaks, indicating the presence of the mismatch.

$\rightarrow$ clearly visible betatron mismatch amplitude in the quadrupolar signal

$\rightarrow$ frequency resolution at 30 turns is challenging

$\Rightarrow$ at usual minimised betatron mismatch: small amplitudes in $f/f_{rev} \approx 0.45$ region (tunes are usual operational $Q_{x,y} \approx 0.23$)

$\Rightarrow$ with intentional betatron mismatch: clearly observe energy in $f/f_{rev} \approx 0.45$ region

$\Rightarrow$ usual injection observations won’t reveal even envelope modes, too well matched
Looking at the quadrupolar pick-up for these 30 turns, we find a quickly damped even envelope oscillation (at $\approx 2Q_{x,y}$):

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- Frequency resolution at 30 turns is challenging
- At usual minimised betatron mismatch: small amplitudes in $f/f_{rev} \approx 0.45$ region (tunes are usual operational $Q_{x,y} \approx 0.23$)
- With intentional betatron mismatch: clearly observe energy in $f/f_{rev} \approx 0.45$ region
- Usual injection observations won’t reveal even envelope modes, too well matched
Part IV: Quadrupolar Beam Transfer Function
Transverse Feedback as Quad. Kicker

So we cannot measure the even envelope modes at injection...

→ excite the beam in quadrupolar mode during the cycle!

⇒ quadrupolar beam transfer function

Kicker in section 97 is part of the PS transverse feedback system:

courtesy Guido Sterbini
Quadrupolar Excitation: Chirp

- distinct peaks around machine tunes $f < 0.25f_{rev}$
- frequency bands around twice the machine tunes
- (disregard the constant frequencies, due to instrumentation)
Measured Quadrupolar BTF

For more details on this approach, cf. Ref. [6] and slides at HB2018 contribution.

$\Rightarrow$ I got intrigued by $\approx Q_x$ spectral content!
Dispersion Mode Revisited

- frequency analysis reveals regular sideband structure around dispersion mode (blue peaks), this measurement was for natural chroma

→ after LS2, new PS set-up: vanishing chroma + dipolar damping available!
Dispersion Mode Revisited

Simulations without chroma predict distinct narrow peak for dispersion mode
\( \sim \) measure SC shift much more accurately?

What happens if I excite higher-order head-tail modes?
\( \Longrightarrow \) energy in \( \left\langle x \frac{\Delta p}{p_0} \right\rangle \)? SC vs. instability?

- Frequency analysis reveals regular sideband structure around dispersion mode (blue peaks), this measurement was for natural chroma

\( \rightarrow \) after LS2, new PS set-up: vanishing chroma + dipolar damping available!
I hope, I could trigger some creative ideas... 🤔
Conclusion

Quadrupolar pick-ups

- provide non-invasive beam measurements
- can be “cheaply” recorded and long-term stored (standard operation?)

and they can provide rich information on the beam:

1. coherent dispersion mode: transverse-longitudinal correlation, e.g. due to
   - dispersion mismatch
   - head-tail instabilities

2. odd (tilting) envelope mode: linear coupling
   - this is the real amplitude of linear coupling in beam

3. even envelope mode: transverse beam size oscillation
   - transverse emittance
   - space charge tune shift

Important: all these modes are 2nd order \Rightarrow all frequencies change with space charge!
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   - transverse emittance
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Important: all these modes are 2nd order \(\Rightarrow\) all frequencies change with space charge!
Outlook: Control Room App Development

A first prototype of a control room app (CERN CC):

https://gitlab.cern.ch/mcoly/ps_qpu
Thank you for your attention!

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Some Historical Perspective

QPU in **time domain** for emittance measurements:
- 1983, R. H. Miller et al. at SLAC [7]

QPU in **frequency domain** for emittance measurements:

QPU in **frequency domain** for space charge measurements:
- 1996, M. Chanel at CERN in LEAR [9]
- 1999, T. Uesugi et al. at NIRS in HIMAC [10]
- 2014, R. Sing et al. at GSI in SIS-18 [12]

⇒ all far away from coupling and coasting beams

⇒ What about bunched beams? Close to coupling?
Incoherent KV Tune Shift

The Kapchinskij-Vladimirskij (KV) beam distribution has all particles at same incoherent space charge tune shift:

\[
\Delta Q_{x,y}^{KV} = -\frac{K^{SC} R^2}{4\sigma_{x,y}(\sigma_x + \sigma_y)Q_{x,y}} \quad (1a)
\]

\[
\Delta Q_{x,y}^{KV} = 1 + \frac{\sigma_{x,y}}{\sigma_{y,x}} \Lambda \quad (1b)
\]

space charge perveance \( K^{SC} \) is given by

\[
K^{SC} = \frac{q\lambda}{2\pi\epsilon_0 \beta \gamma^2 p_0 c}
\]
Incoherent KV Tune Shift

The Kapchinskij-Vladimirskij (KV) beam distribution has all particles at same incoherent space charge tune shift:

\[ \Delta Q_{x,y}^{KV} = -\frac{K_{SC} R^2}{4\sigma_{x,y}(\sigma_x + \sigma_y)Q_{x,y}} \]

\[ = 1 + \frac{\sigma_{x,y}/\sigma_{y,x}}{2Q_{x,y}} \Lambda \]  \hspace{1cm} (1a)

\[ \Lambda = \frac{Q_+^2 + Q_-^2 - 4(Q_x^2 + Q_y^2)}{4 + 3(\sigma_x/\sigma_y + \sigma_y/\sigma_x)} \]  \hspace{1cm} (2)

(Gaussian tune spread = 2x the RMS-equivalent KV tune shift!)

space charge perveance \( K^{SC} \) = \( \frac{q\lambda}{2\pi\varepsilon_0 \beta \gamma^2 \rho_0 c} \)
Envelope Equations

Envelope equations of motion (e.o.m.)

\[
\begin{align*}
\sigma''_x + K_x(s) \sigma_x - \frac{\epsilon^2_{x,\text{geo}}}{\sigma^3_x} - \frac{K^{\text{SC}}}{2(\sigma_x + \sigma_y)} &= 0 , \\
\sigma''_y + K_y(s) \sigma_y - \frac{\epsilon^2_{y,\text{geo}}}{\sigma^3_y} - \frac{K^{\text{SC}}}{2(\sigma_x + \sigma_y)} &= 0
\end{align*}
\] (3a)

for transverse r.m.s. beam widths \(\sigma_{x,y}\) have equilibrium

\[
\begin{align*}
\frac{Q^2_x}{R^2} \sigma_{x,m} - \frac{\epsilon^2_{x,\text{geo}}}{\sigma^3_{x,m}} - \frac{K^{\text{SC}}}{2(\sigma_{x,m} + \sigma_{y,m})} &= 0 , \\
\frac{Q^2_y}{R^2} \sigma_{y,m} - \frac{\epsilon^2_{y,\text{geo}}}{\sigma^3_{y,m}} - \frac{K^{\text{SC}}}{2(\sigma_{x,m} + \sigma_{y,m})} &= 0
\end{align*}
\] (4a)
Smooth Approximation, Lin. Perturbation

Constant focusing channel

\[ K_{x,y} = \frac{1}{\beta_{x,y}^2} = \frac{Q_{x,y}^2}{R^2} = \text{const.} \]  

(5)

gives linearised e.o.m. for perturbation around equilibrium \( r = \sigma_m + \delta r \)

\[ \frac{d^2}{ds^2} \begin{pmatrix} \delta r_x \\ \delta r_y \end{pmatrix} = - \begin{pmatrix} \kappa_x & \kappa_{SC} \\ \kappa_{SC} & \kappa_y \end{pmatrix} \cdot \begin{pmatrix} \delta r_x \\ \delta r_y \end{pmatrix} \]  

(6)

with

\[ \begin{cases} 
\kappa_{x,y} = 4 \frac{Q_{x,y}^2}{R^2} - \frac{2\sigma_{x,y} + 3\sigma_{y,x}}{\sigma_{x,y}} \kappa_{SC} \\
\kappa_{SC} = \frac{K_{SC}}{2(\sigma_x + \sigma_y)^2}
\end{cases} \]  

(7)
Adding dispersion to envelope equations, studied by Venturini-Reiser [13] and, independently, by Lee-Okamoto [14]:

\[
\sigma''_x + \left(K_x(s) - \frac{K^{SC}}{2r_x(r_x + \sigma_y)}\right) \sigma_x - \frac{\epsilon_{x,geo}^2}{\sigma_x^3} = 0,
\]

\[
(8a)
\]

\[
\sigma''_y + \left(K_y(s) - \frac{K^{SC}}{2\sigma_y(r_x + \sigma_y)}\right) \sigma_y - \frac{\epsilon_{y,geo}^2}{\sigma_y^3} = 0
\]

\[
(8b)
\]

\[
D''_x + \left(K_x(s) - \frac{K^{SC}}{2r_x(r_x + \sigma_y)}\right) D_x = \frac{1}{\rho(s)}
\]

\[
(8c)
\]

Generalised \( r_x^2 = \sigma_x^2 + D_x^2 \frac{\Delta p}{p_0} \) and \( \epsilon_{x,geo} \) only betatron emittance (no dispersive contribution). Linearisation around matched values gives a 3D matrix to be solved for eigenvalues \( \mapsto \) mode tunes of the betatron envelopes \( Q_\pm \) as well as the dispersion mode \( Q_d \), cf. e.g. Ref. [15].
Far Away vs. On the Coupling Resonance

Two eigenmodes for coherent quadrupolar betatron oscillation:

**far away from coupling**

- (a) horizontal mode
- (b) vertical mode

Quadrupolar mode tunes:

\[
Q_{\pm} = 2Q_{x,y} - \left| \Delta Q_{x,y}^{K\nu} \right| \left( 3 - \frac{\sigma_{x,y}}{\sigma_x + \sigma_y} \right) / 2 \quad (9)
\]

**full coupling**

- (a) breathing mode
- (b) antisym. mode

Quadrupolar mode tunes:

\[
Q_+ = 2Q_0 - \left| \Delta Q_{x,y}^{K\nu} \right| \\
Q_- = 2Q_0 - \frac{3}{2} \left| \Delta Q_{x,y}^{K\nu} \right| \quad (10a)
\]

(assuming round beams, \( Q_{x,y} \equiv Q_0 \))
According to Aslaninejad and Hofmann [16], not possible to infer space charge from odd modes.

"For weak space charge (as in rings) this coherent shift [of the linear coupling modes] is found to be approximately independent of the space-charge tune shift as well as the absolute tune values."

Figure: coherent shift in SC incoherent tune shift units vs. emittance ratio
Vanishing Chroma, Mode 0

(a) Mode 0 in horizontal plane

(b) QPU spectogram

(c) QPU spectrum after injection bump (480 turns)

transverse feedback off at vanishing chromaticity $\rightleftharpoons$ horizontal rigid head-tail mode
Vanishing Chroma, Stabilised

(a) Mode 0 in horizontal plane

(b) QPU spectogram

transverse feedback on at vanishing chromaticity $\Longrightarrow$ stabilised, shifted peak by about 0.006 $\sim\sim$ coherent dispersion mode?

(c) QPU spectrum after injection bump (480 turns)