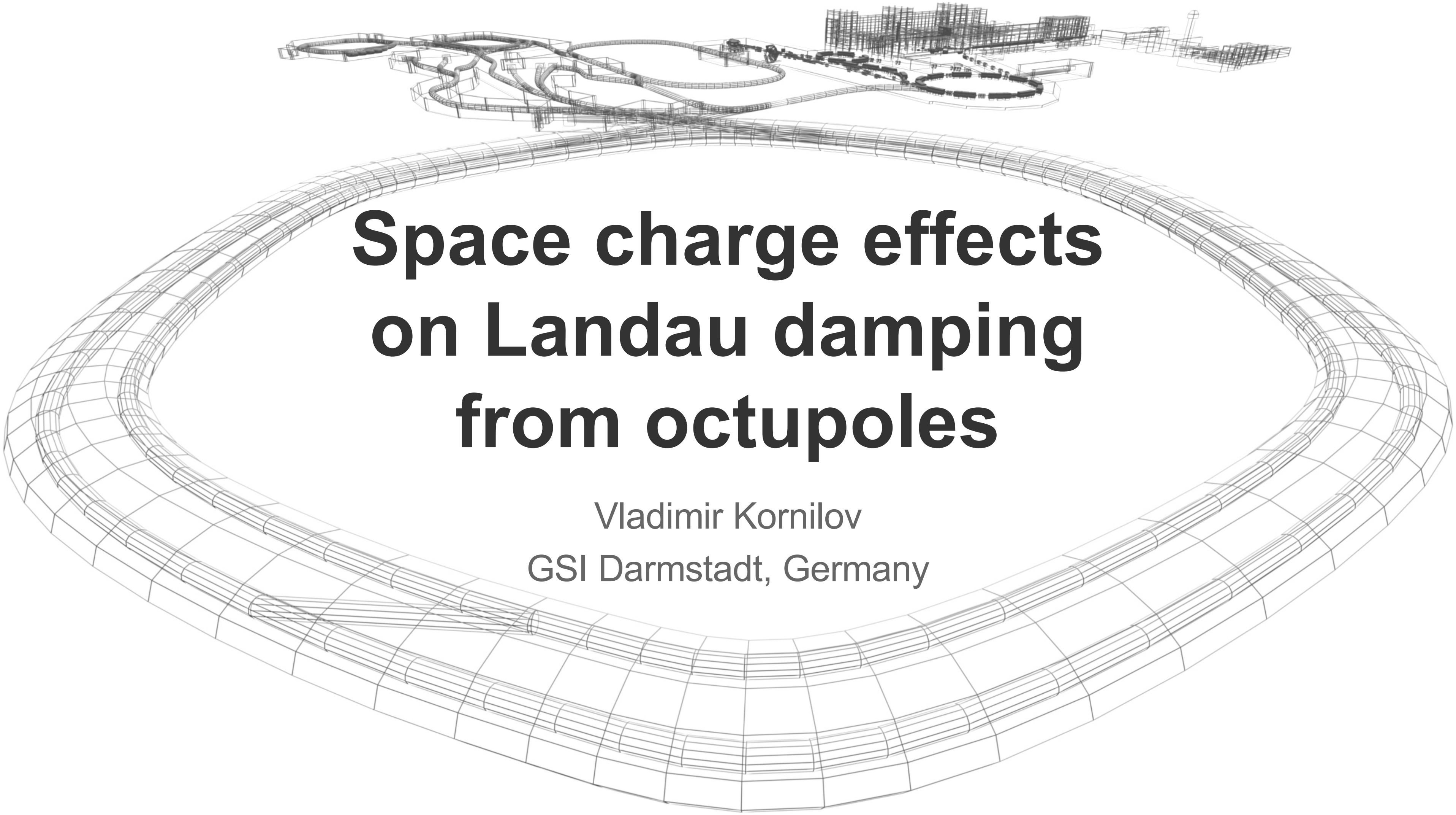


Space charge effects on Landau damping from octupoles

Vladimir Kornilov
GSI Darmstadt, Germany

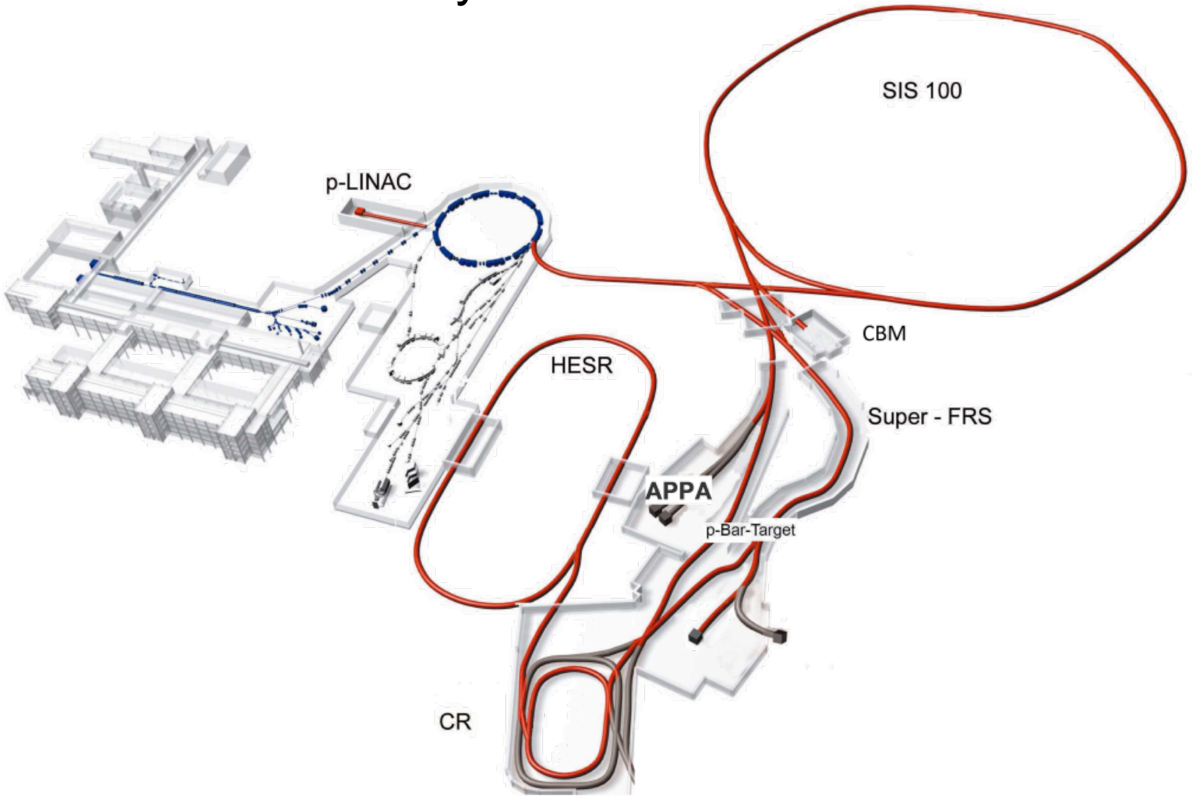
FAIR project at GSI

SIS100

- C: 5 x SIS18 = 1 083.6 m
- Ions H−U.
- U^{28+} 0.2→1.5 GeV/u
- p^{+} 4→29 GeV
- High-intensity, low-loss operation
- Coherent instabilities (head-tail, see the poster) is an important issue
- Space charge ΔQ_{sc} up to 0.3

- Under construction, commissioning 2025.

The SIS100 synchrotron is the central accelerator of the FAIR Project at GSI Helmholtzzentrum in Darmstadt, Germany



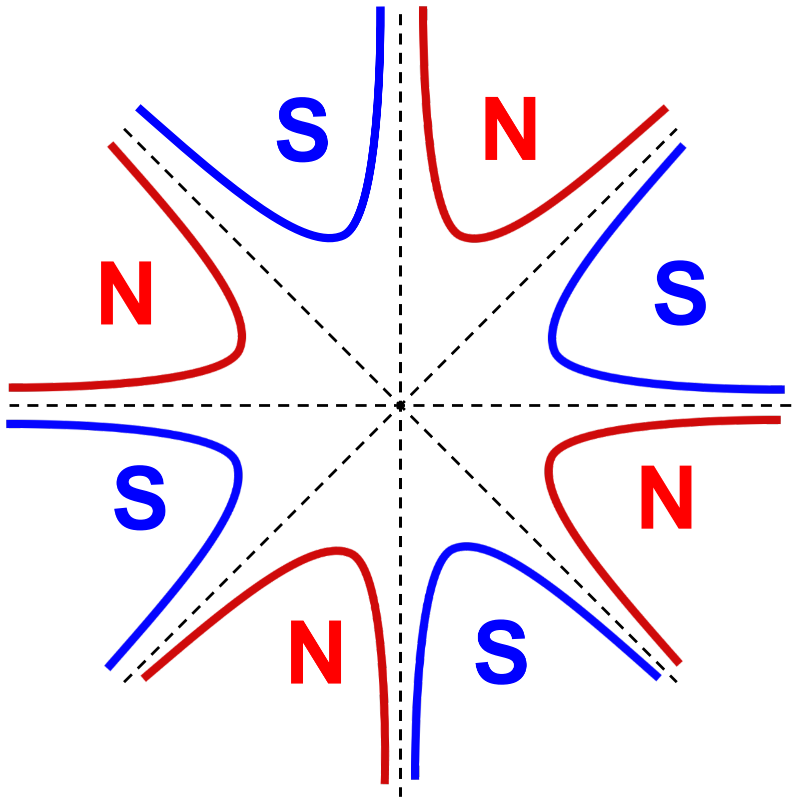
Octupole magnets

$$
\begin{aligned}
B_x &= O_3(3x^2y - y^3) \\
B_y &= O_3(x^3 - 3xy^2)
\end{aligned}
$$

$$
\Delta Q_x = \left(\int \frac{K_3\beta_x^2}{16\pi} \mathrm{d}s\right) J_x - \left(\int \frac{K_3\beta_x\beta_y}{8\pi} \mathrm{d}s\right) J_y
$$

$$
\Delta Q_y = \left(\int \frac{K_3\beta_y^2}{16\pi} \mathrm{d}s\right) J_y - \left(\int \frac{K_3\beta_x\beta_y}{8\pi} \mathrm{d}s\right) J_x
$$

- Amplitude-dependent betatron tune shifts
- Tune spread provides Landau damping
- Used since 70s in many machines
- The cornerstone of the mitigation scheme for SIS100 (beam Ø ≈ 30mm)
- Can reduce the Dynamic Aperture



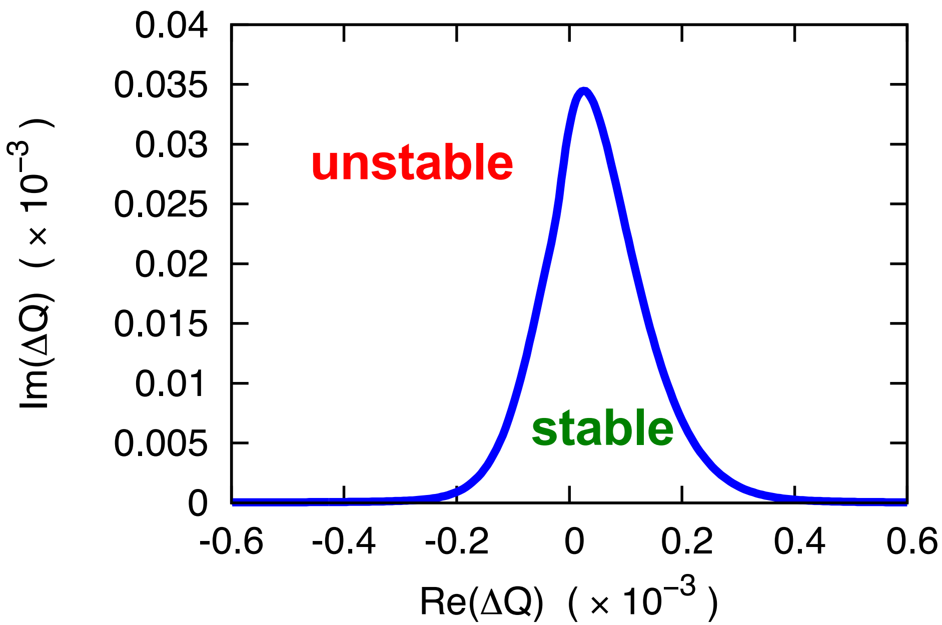
Schematic yoke profile of an octupole magnet

Analytic calculations

For Landau damping due to octupoles only, the dispersion relation has been commonly used

$$\Delta Q_{\mathrm{coh}} \int \frac{1}{\Delta Q_{\mathrm{oct}} - \Omega/\omega_0} J_x \frac{\partial \psi_\perp}{\partial J_x} dJ_x dJ_y = 1$$

Discussed here extensively



But, with space charge, it is more difficult

Landau damping with space charge

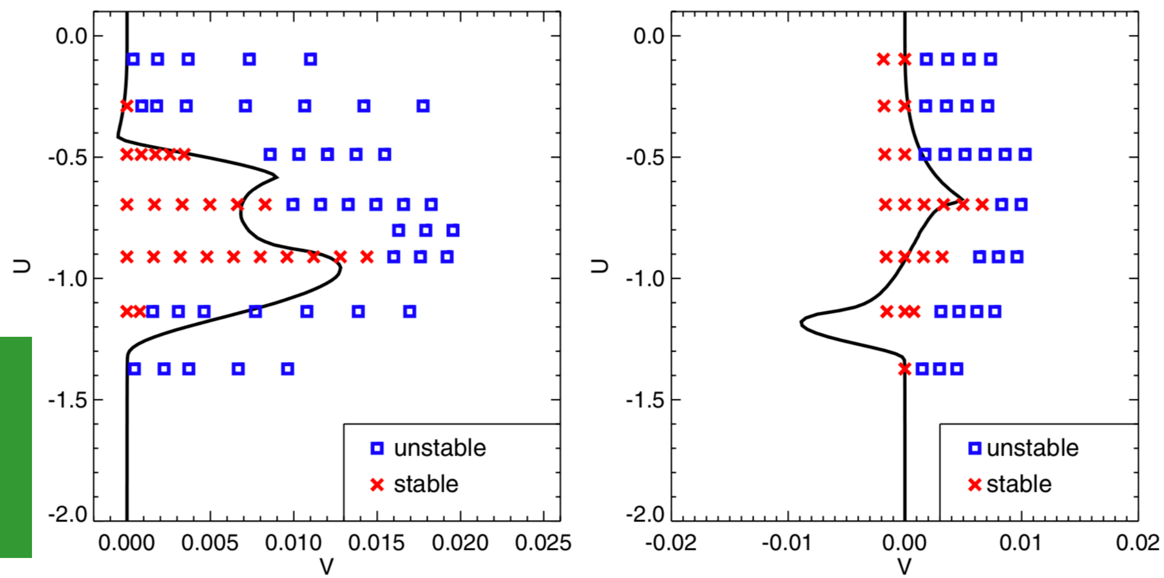
The dispersion relation (D.Möhl, H.Schönauer,1974)

$$\int \frac{\Delta Q_{\rm coh} - \Delta Q_{\rm sc}}{\Delta Q_{\rm ex} + \Delta Q_{\rm sc} - \Omega/\omega_0} J_x \frac{\partial f}{\partial J_x} {\rm d}J_x {\rm d}J_y = 1$$

which is very good
for monotonic (ΔQ_{ex}+ΔQ_{sc}),
but can predict
an antidamping
for a complicated
(ΔQ_{ex}+ΔQ_{sc})(J_x,J_y)

Note the negative shift
U~Im(ΔQ) due to (linear)
effect of space charge!

V.Kornilov, O.Boine-Frankenheim, I.Hofmann, PRSTAB 11, 014201 (2008)



Landau damping with space charge

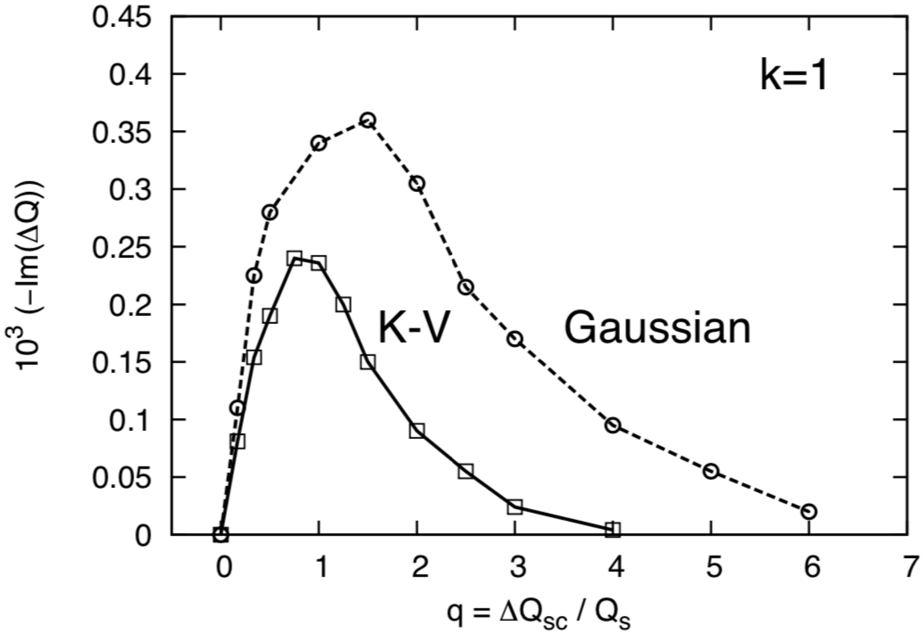
The dispersion relation (D.Möhl, H.Schönauer,1974)

$$\int \frac{\Delta Q_{\mathrm{coh}} - \Delta Q_{\mathrm{sc}}}{\Delta Q_{\mathrm{ex}} + \Delta Q_{\mathrm{sc}} - \Omega/\omega_0} J_x \frac{\partial f}{\partial J_x} \mathrm{d}J_x \mathrm{d}J_y = 1$$

Predicts (correctly!) for coasting beams, that there is NO Landau damping due to space charge only (even with the overlap coh. mode ↔ inc. spectrum). But, there is damping due to space charge only in bunches.

Cannot be directly extended to bunches

V.Kornilov, O.Boine-F, arXiv:1709.01425 (2017)
V.Kornilov, O.Boine-F, PRSTAB 13, 114201 (2010)



Landau damping decrement

Particle-in Cell Tracking

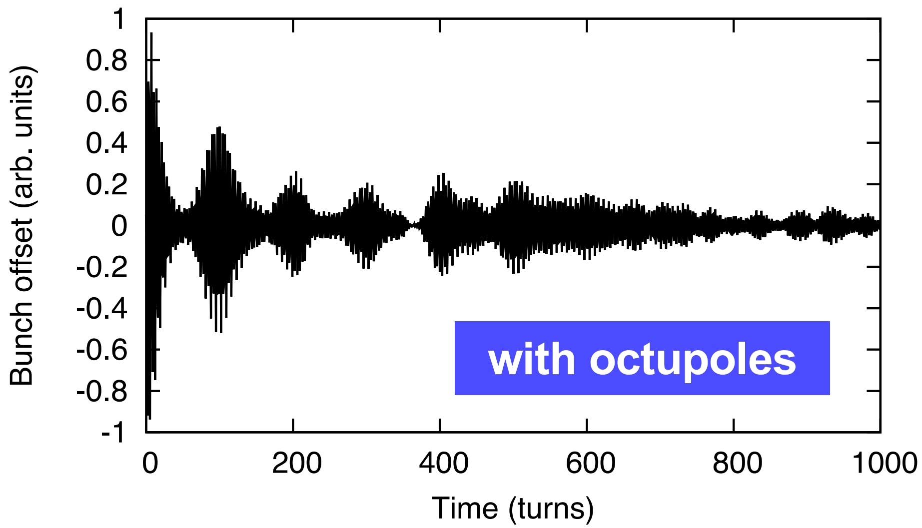
For the mitigation in SIS100, we need the quantitative predictions for Landau damping due to combinations of octupoles and space charge

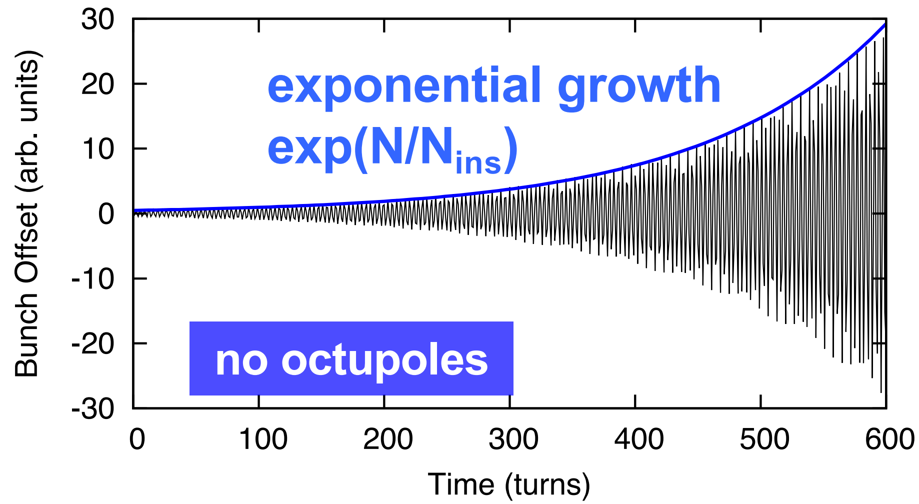
The PIC code PATRIC (development GSI Darmstadt)

- 2.5D sliced bunches
- Self-consistent space-charge
- Impedances, Wakes, Image Charges
- Tune shifts, spectra, instabilities, decoherence (bunched beams, coasting beams, with space charge, without space charge) verified with analytical theories
  V. Kornilov and O. Boine-Frankenheim, Proc. of ICAP2009, San Francisco (2009)
  O.Boine-Frankenheim, V.Kornilov, Proc. of ICAP2006 (2006)
  V.Kornilov, HB2016, July 3-8, Malmö, Sweden (2016)
- Verified vs. HEADTAIL (2005)
- Landau damping simulations, head-tail modes with space-charge
  V.Kornilov, O.Boine-Frankenheim, PRSTAB 13, 114201 (2010)
  V.Kornilov, O.Boine-Frankenheim, PRSTAB 15, 114201 (2012)
  V.Kornilov, O.Boine-Frankenheim, arXiv:1709.01425 (2017)
- This work: the constant focusing, linear rf bucket, 3D Gaussian 3.5σ

Particle tracking simulations

- Start with a tiny perturbation
- Apply a wake field (resistive-wall $W(z) = w_0/\sqrt{z}$)
- Apply octupoles



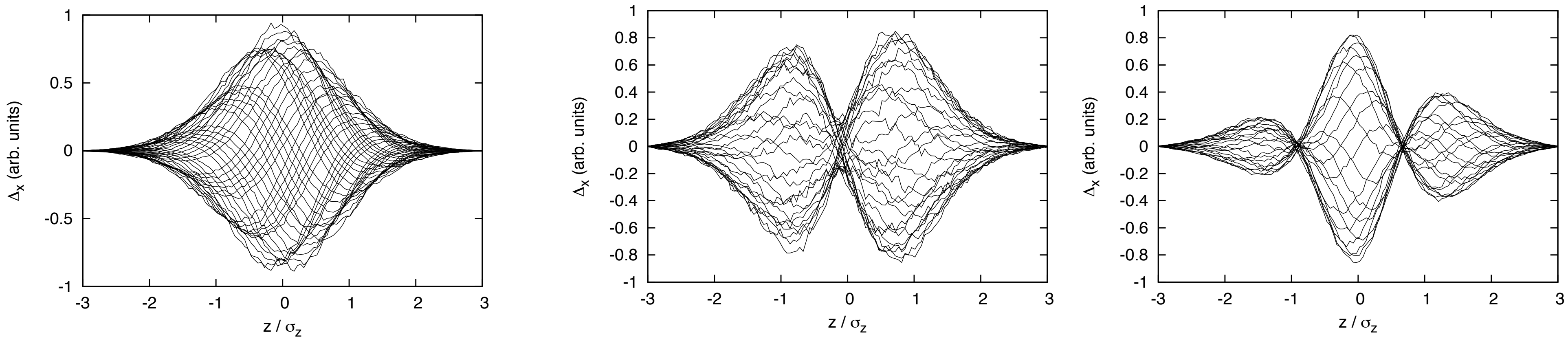


Stabile due to octupoles

Unstable

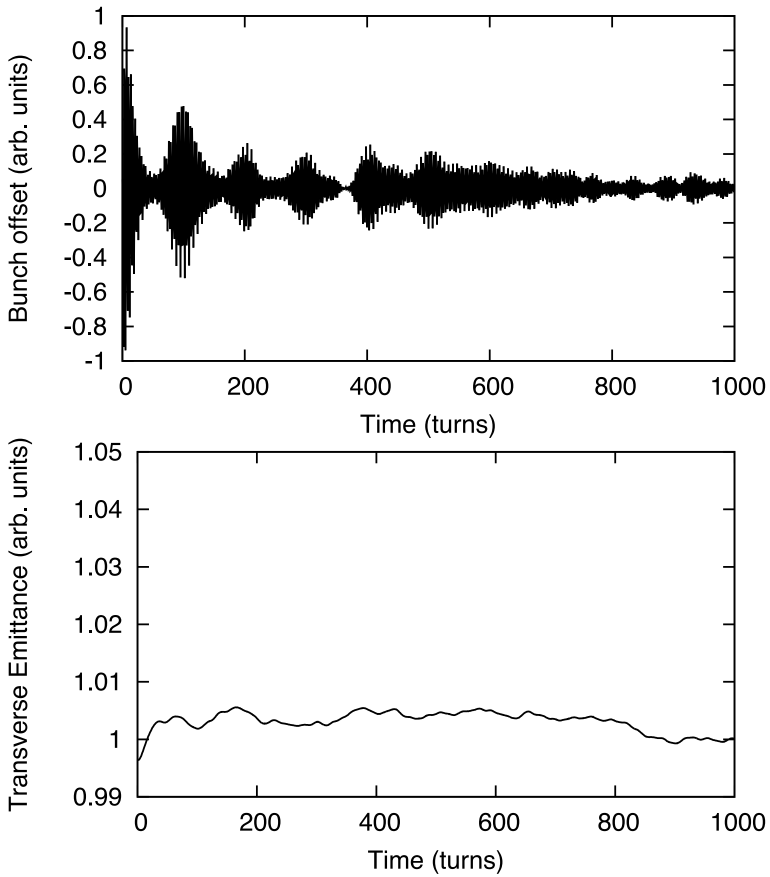
Particle tracking simulations

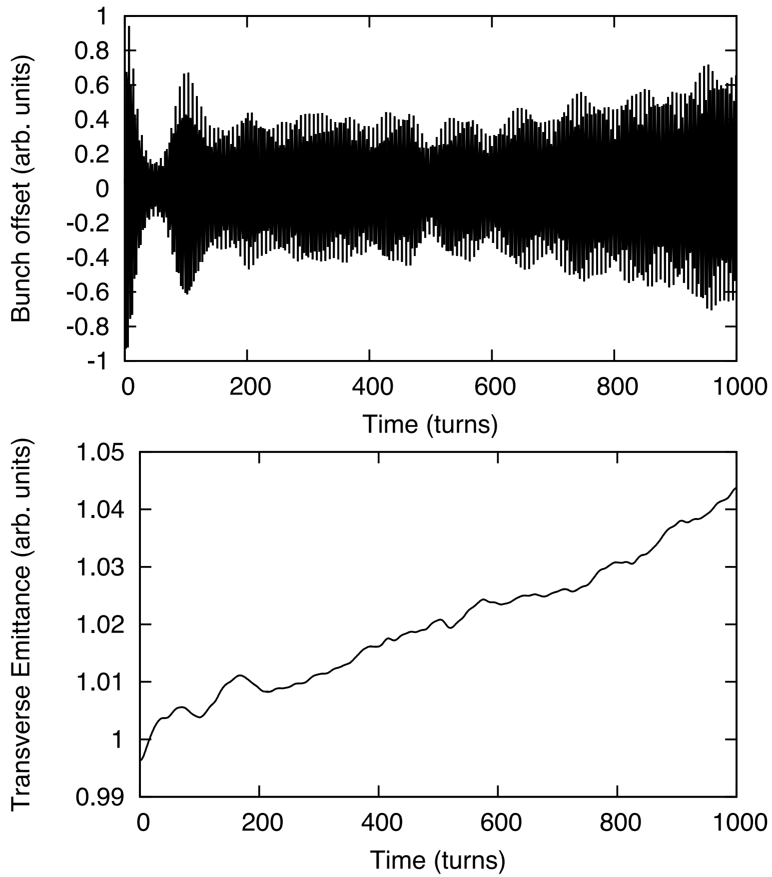
Simulation scans for k=0, k=1 and k=2 modes



- Varying the chromaticity, the same RW wake
- Intra-Bunch oscillation
- Even the k=0 mode is not a rigid dipole mode

Particle tracking simulations



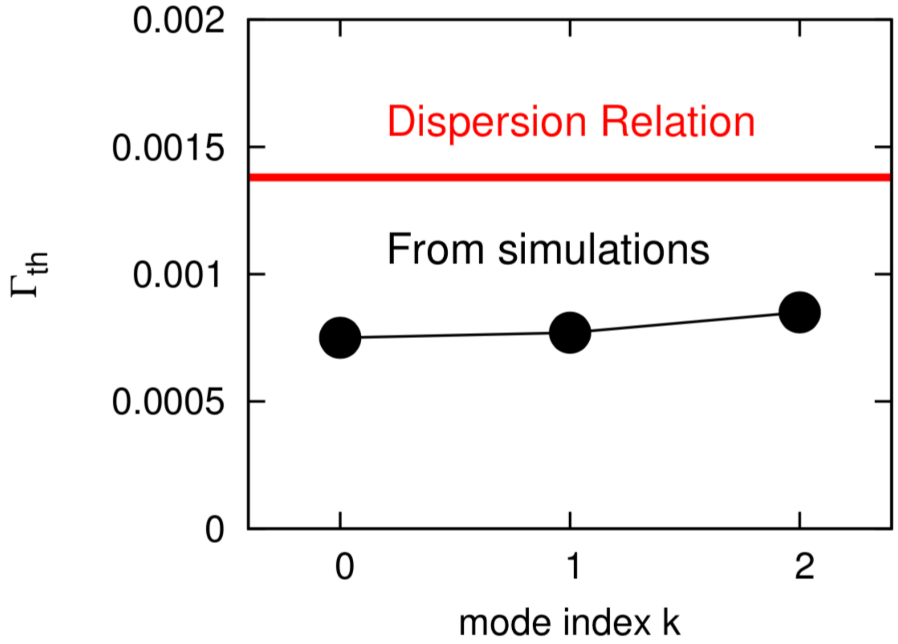


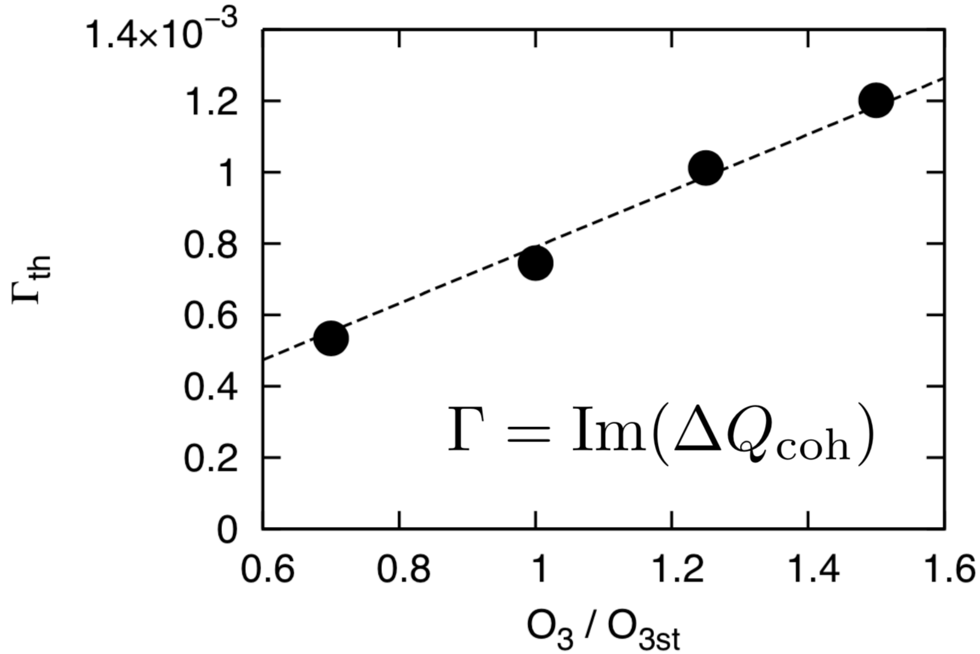
Stabile due to octupoles

Above the threshold

Damping due to octupoles

Simulation scans

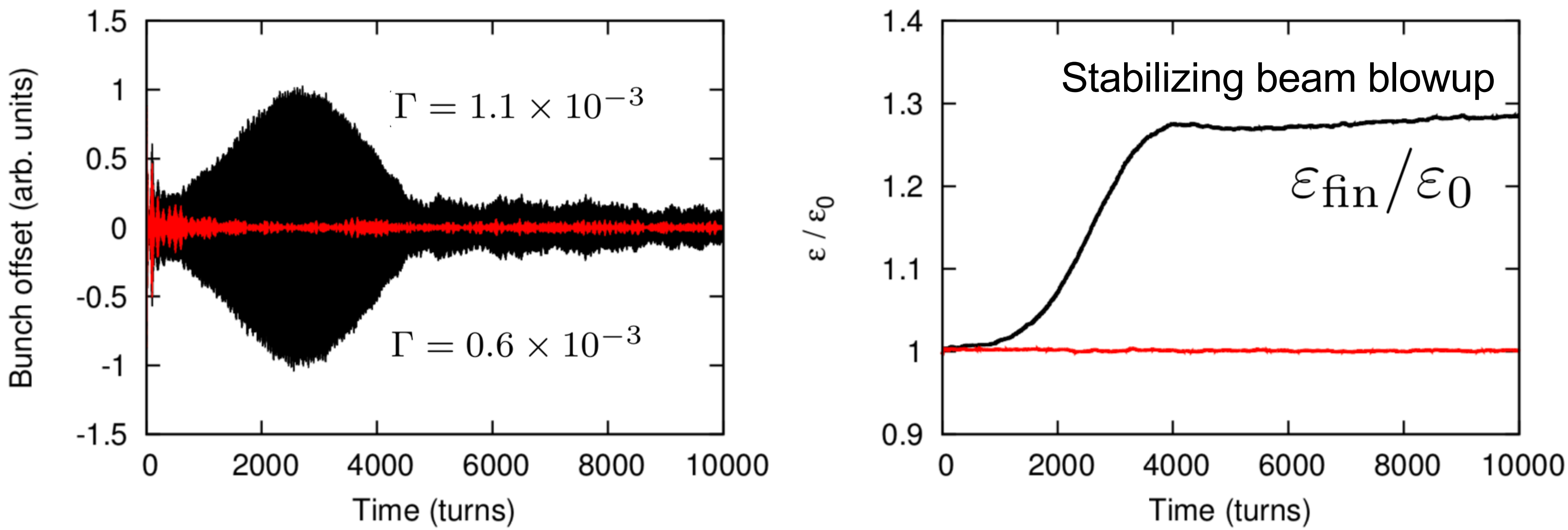




- Γ_{th} is the stability threshold for a fixed octupole scheme
- Octupoles provide similar Landau damping to k=0, k=1, k=2.
- Clear linear dependency of the damping on the octupole power

Damping due to octupoles

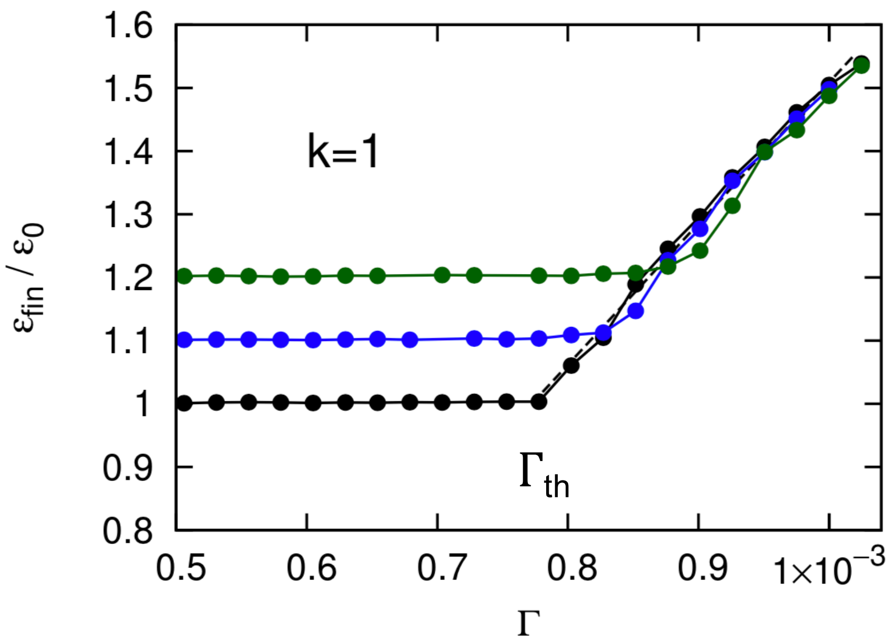
<u>Octupoles for modes slightly above the stability threshold</u>



Advantage of the octupoles:
The tune spread is proportional to the transverse emittance.
Oscillations cause a beam blowup and stabilize the beam.

Damping due to octupoles

Octupoles for modes slightly above the stability threshold



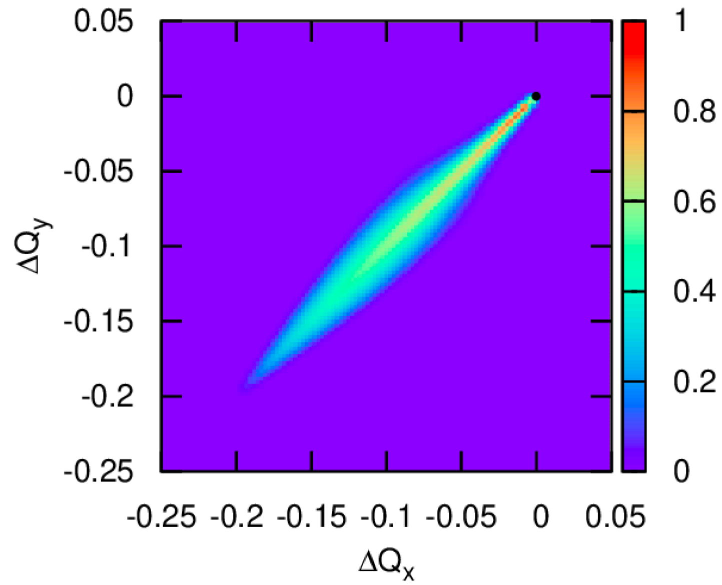
Stabilizing beam blowup

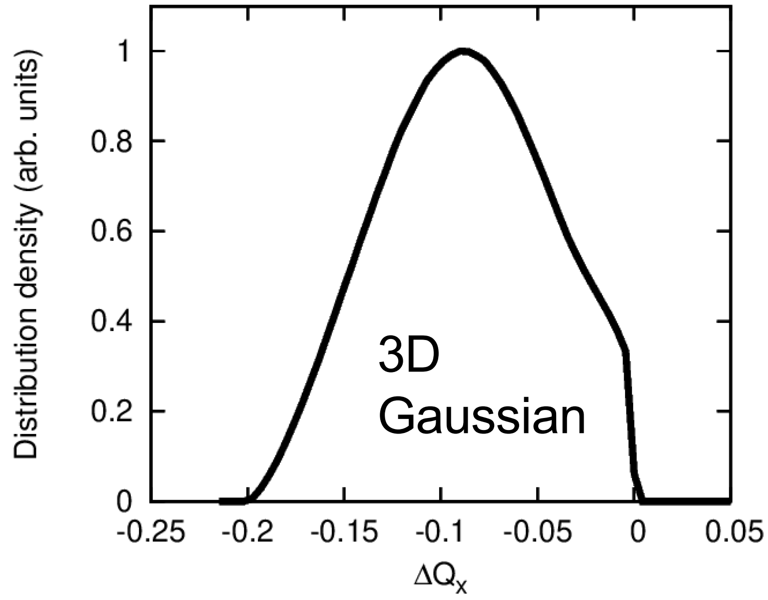
$$\Gamma = \mathrm{Im}(\Delta Q_{\mathrm{coh}})$$

Advantage of the octupoles:
The tune spread is proportional to the transverse emittance.
Oscillations cause a beam blowup and stabilize the beam.

Tune shifts due to space charge

Tune footprints: space charge





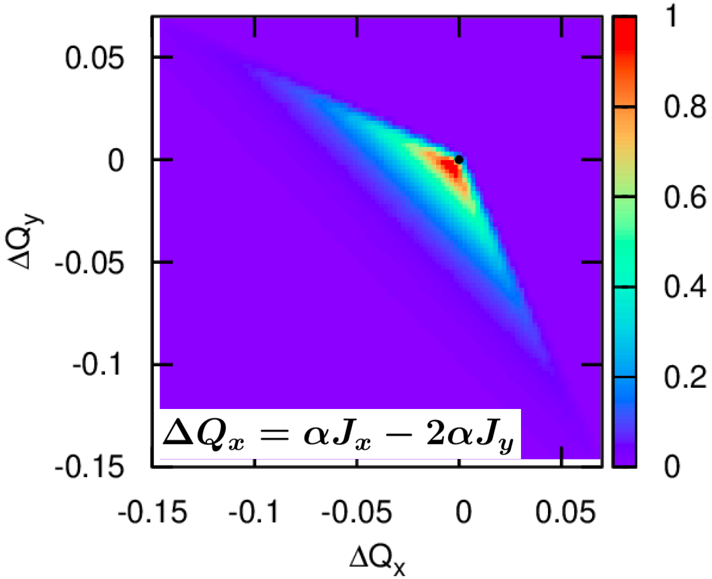
Space charge $$q = \frac{\Delta Q_{\mathrm{sc}}}{Q_s}$$

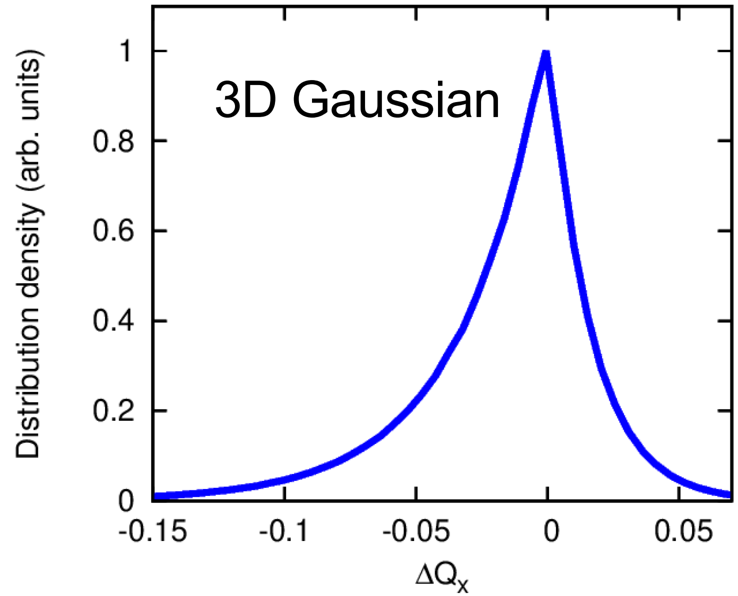
Octupoles $$q_4 = \frac{\Delta Q_\sigma}{Q_s}$$

Average $\langle \Delta Q_x \rangle = -0.0878$
RMS $\delta(\Delta Q_x) = 0.0429$
q=10
for Q_s=0.01

Tune shifts due to octupoles

Tune footprints: octupoles





Space charge $$q = \frac{\Delta Q_{sc}}{Q_s}$$

Octupoles $$q_4 = \frac{\Delta Q_\sigma}{Q_s}$$

Average $\langle \Delta Q_x \rangle = -0.0157$

RMS $\delta(\Delta Q_x) = 0.034$

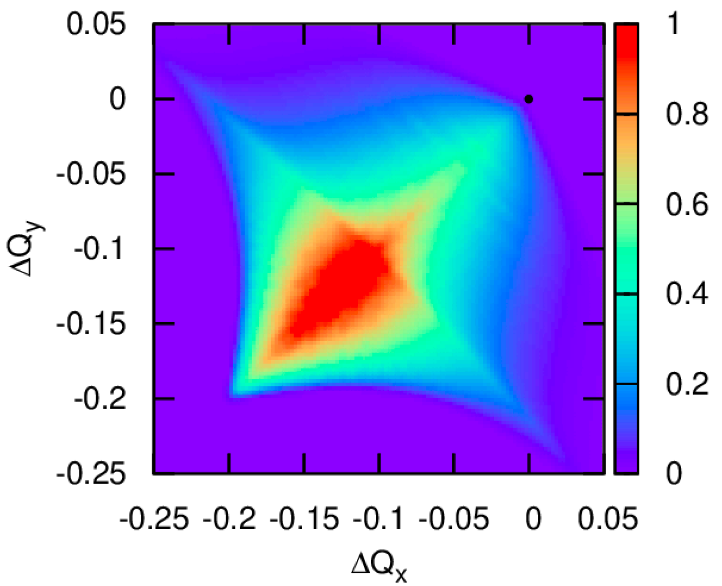
$\Delta Q_\sigma = 0.008$

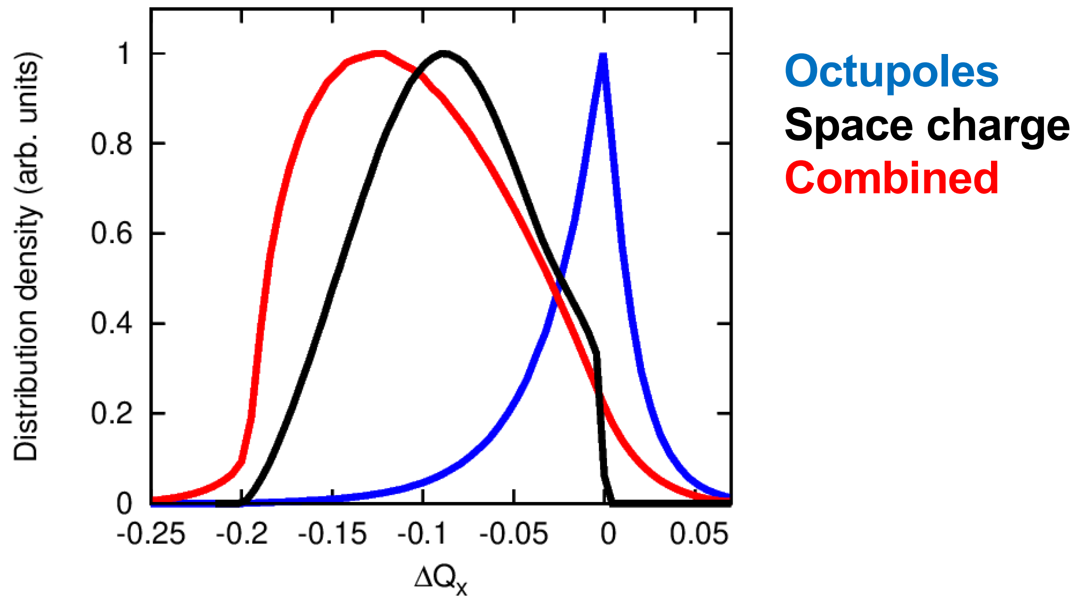
q_4=0.8

ΔQ_σ for a particle with $a_x = \sigma_x, a_y = 0$

Tune shifts due to octupoles

Tune footprints: octupoles + space charge





Space charge $q = \dfrac{\Delta Q_{sc}}{Q_s}$

Octupoles $q_4 = \dfrac{\Delta Q_\sigma}{Q_s}$

Average $\langle \Delta Q_x \rangle = -0.104$
RMS $\delta(\Delta Q_x) = 0.0534$
q_4=0.8
q=10

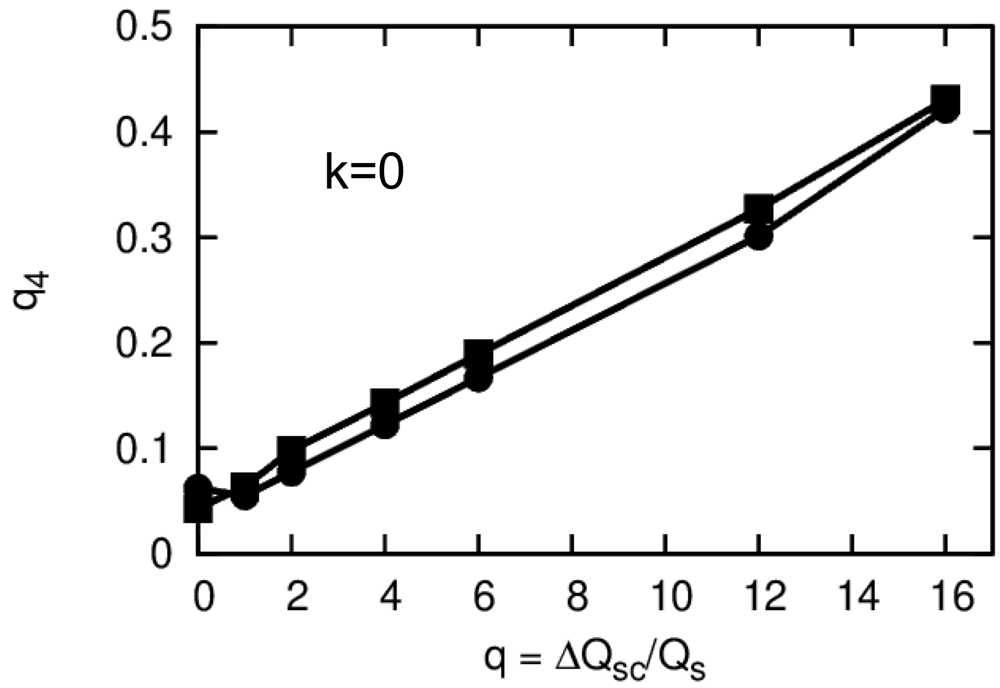
Landau damping for the k=0 mode

Results of the simulation scans

$$q_4 = \frac{\Delta Q_\sigma}{Q_s}$$

Minimal octupole power for the stability as a function of the space-charge strength

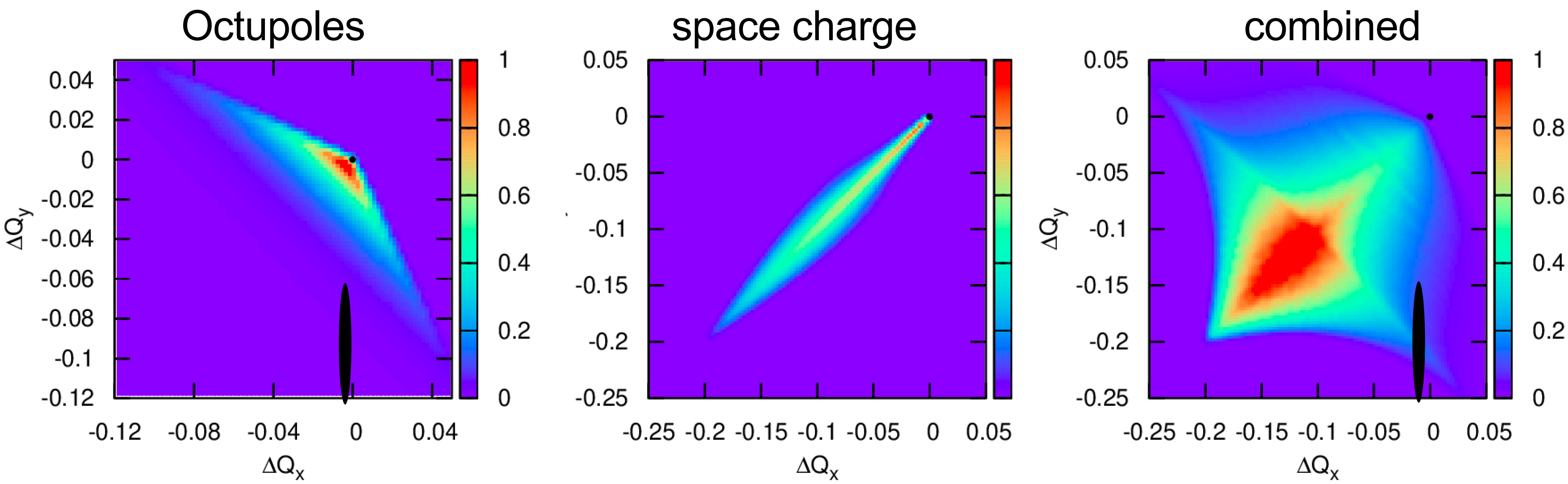
Octupole polarity:
Circles: positive q_4
Squares: negative q_4



- For stability, higher octupole power at stronger space charge is needed
- Loss of Landau damping due to space charge
- As predicted, no Landau damping due to space charge for k=0

Tune footprint for octupole + space charge

Tune footprints: octupoles + space charge



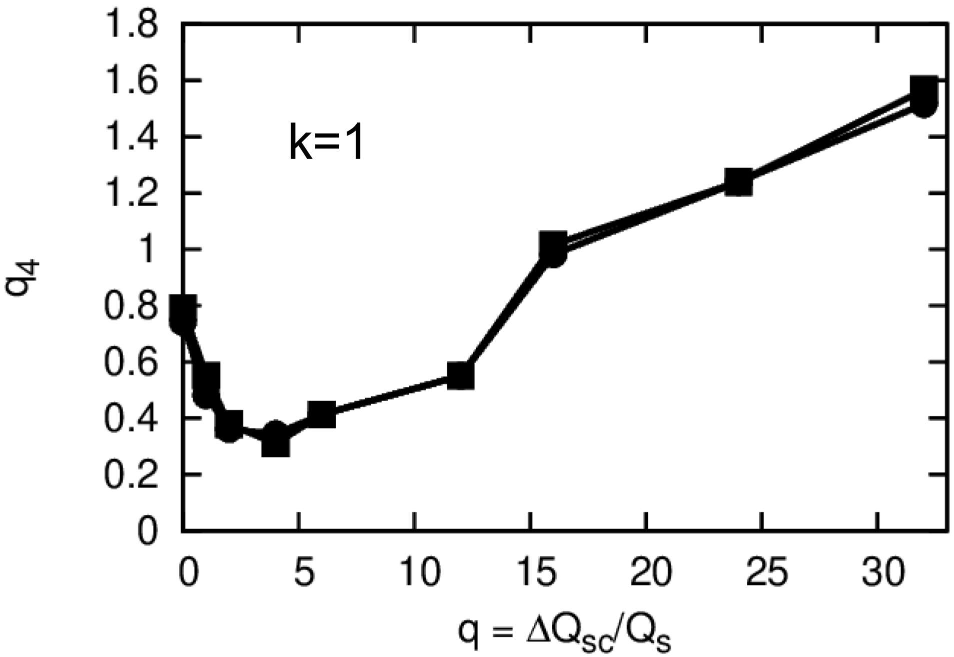
q_4=0.8
q=10

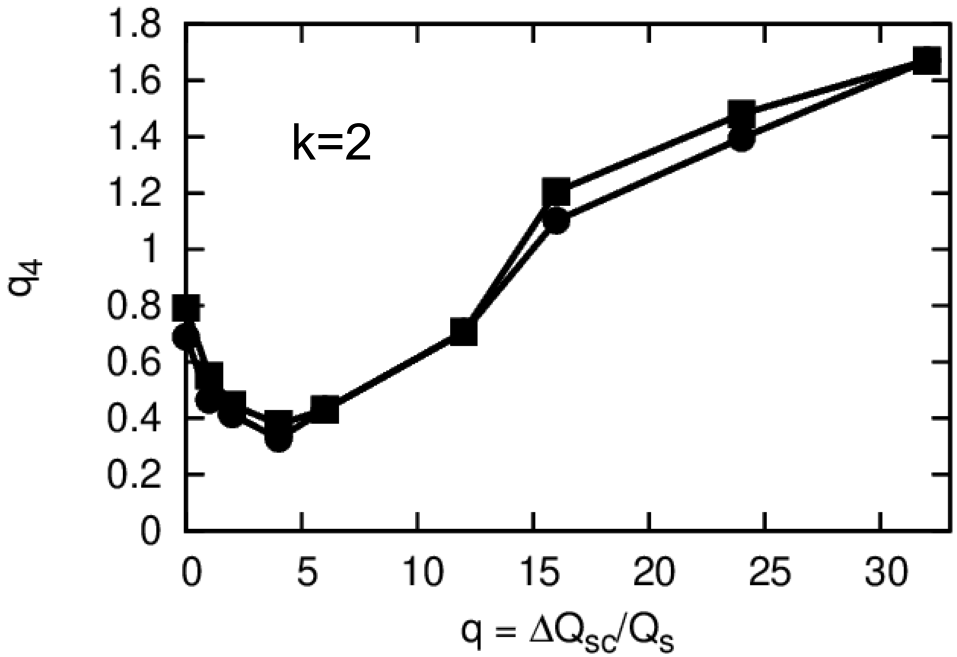
Black: schematic for the frequency of the coherent mode

Similar to the case of a coasting beam: loss of Landau damping due to space charge

Landau damping for k=1, k=2 modes

Results of the simulation scans





$$q_4 = \frac{\Delta Q_\sigma}{Q_s}$$

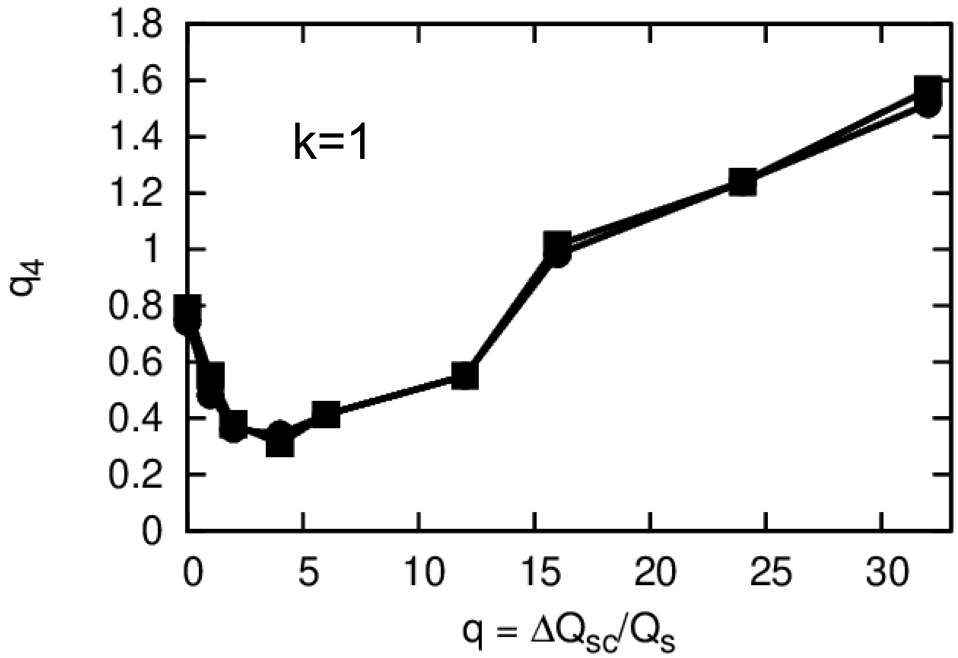
Minimal octupole power for the stability as a function of the space-charge strength

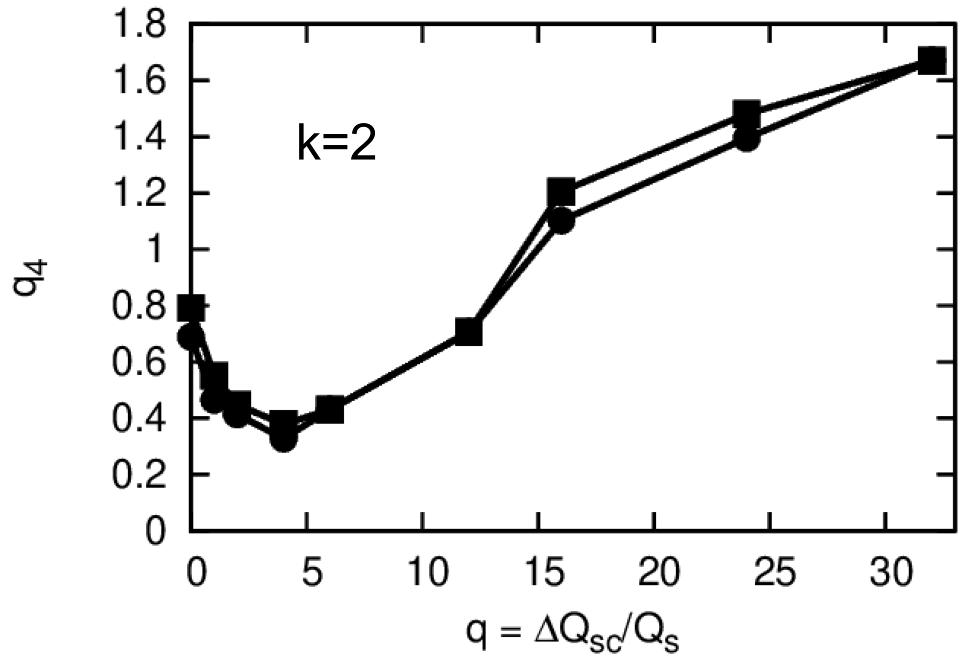
Octupole polarity: Circles: positive q_4

Squares: negative q_4

Landau damping for k=1, k=2 modes

Results of the simulation scans

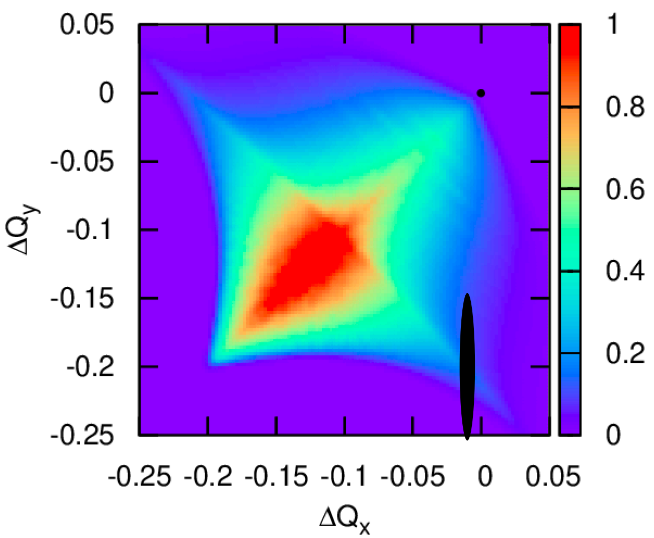


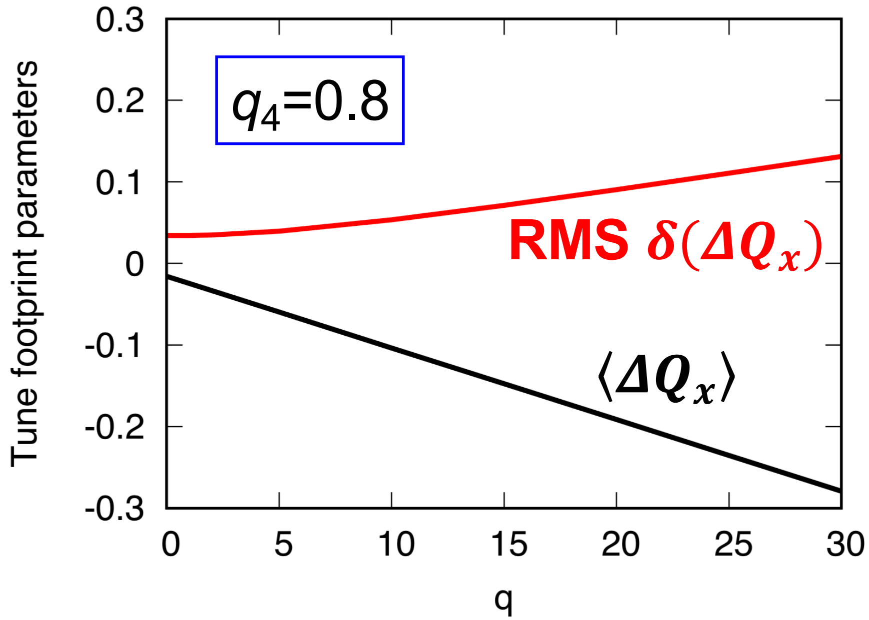


- For stability, higher octupole power at stronger space charge is needed
- Loss of Landau damping due to space charge
- Additional Landau damping due to space charge at medium q

Landau damping for k=1, k=2 modes

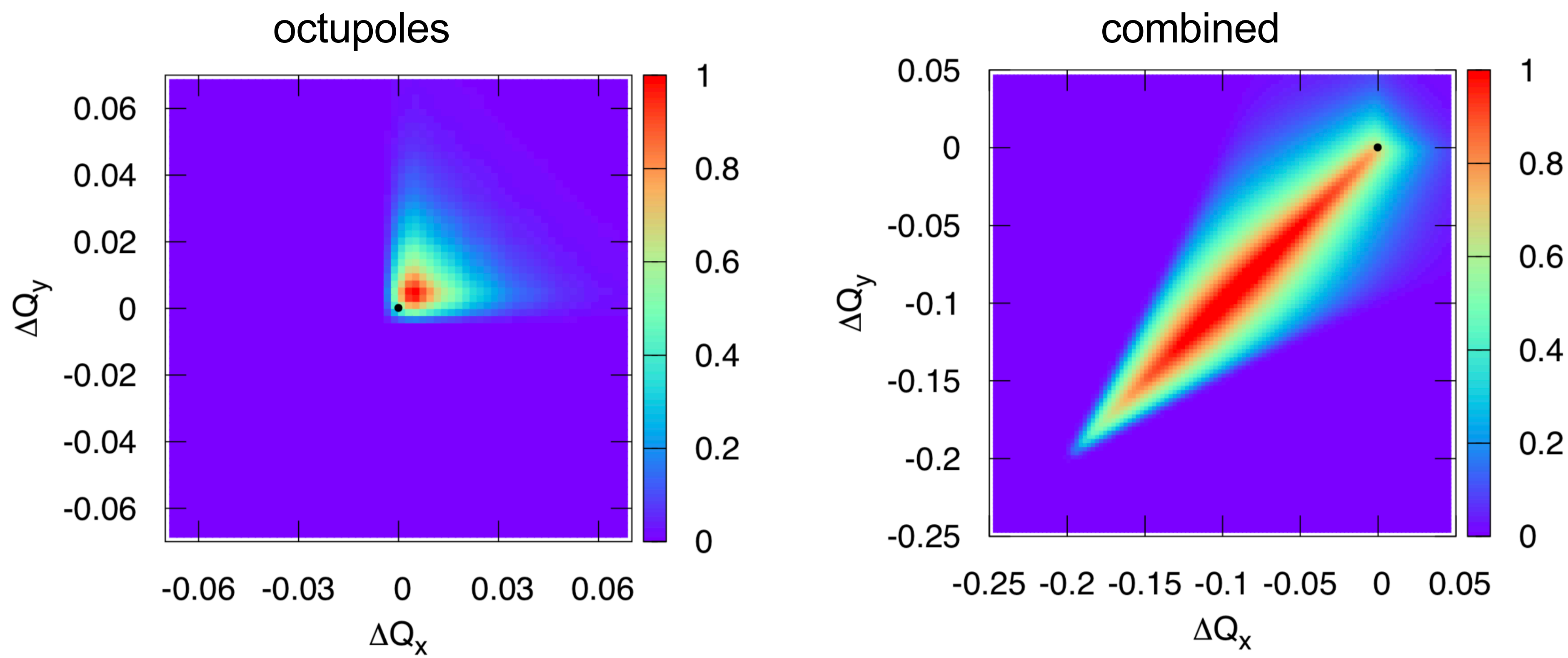
Tune footprints: octupoles + space charge





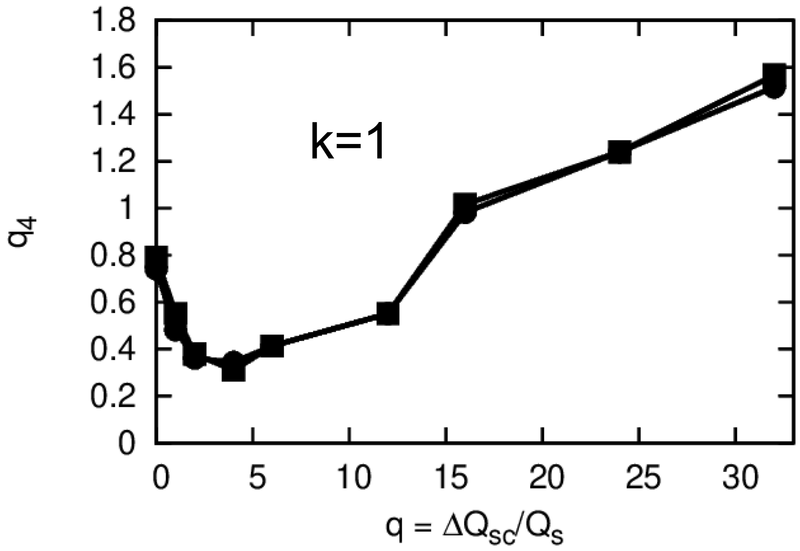
Loss of Landau damping due to space charge

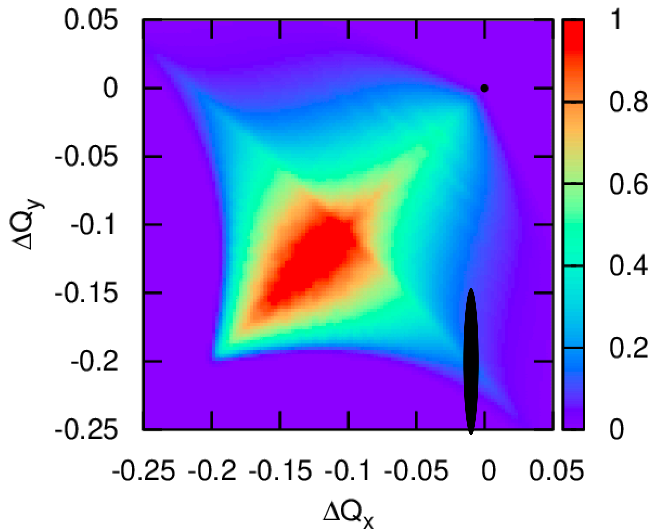
Octupole optimization



If groups of octupoles are available (SIS100: 2), optimization of the power scheme is possible

Conclusions





- Octupoles scheme is a cornerstone of the mitigation at SIS100
- Octupoles: the stabilizing beam blowup
- Important role of space charge for the mitigation using octupoles
- Loss of Landau damping due to space charge (still enough for SIS100)
- Additional Landau damping due to space charge at medium q for k>0
- The flexible octupole scheme provides optimized footprints for stability