Noise and possible loss of Landau damping

Noise Excited Wakefields

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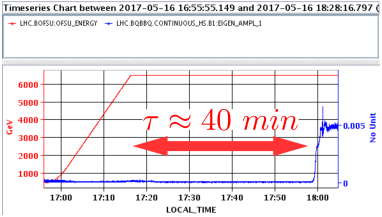
Handout

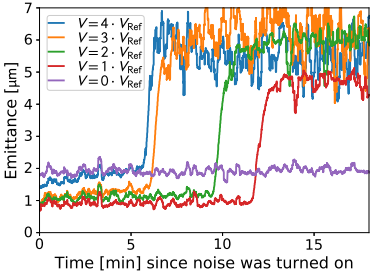
This is the handout, not the actual presentation.
Questions and feedback can be sent to sondre.vik.furuseth@cern.ch.

Abstract: The effect of transverse Landau damping in circular hadron colliders, depends strongly on the bunch distribution. The bunches are often assumed to be Gaussian in the transverse dimensions, as it fits well to measurements and it is the expected effect of intra-beam scattering. However, a small change of the distribution can cause a loss of stability. We study the effect that external noise excites the transverse motion of the beam, which produces wakefields, which act back on the beam and cause a diffusion of incoherent particles. The diffusion is narrow in frequency space, and thus also in action space. Macroparticle simulations have shown a similar change of the distribution, which is only detectable in action space, not projected in position space. The narrow diffusion efficiently drills a hole in the stability diagram, at the location of the unstable mode. The advised mitigation technique is to reduce the drilling rate by operating with a stability margin.

Instability Latency

- Instabilities of high latencies have been observed in the LHC.
- Reproduced in dedicated experiments. (S.V. Furuseth *et al.*, WEPTS044, IPAC 2019.)
- The instabilities cannot be explained by machine variations.
- This mechanism is linked to the discrepancy between the predicted and required octupole current in the LHC. (X. Buffat *et al.*, Evian Workshop 2019).

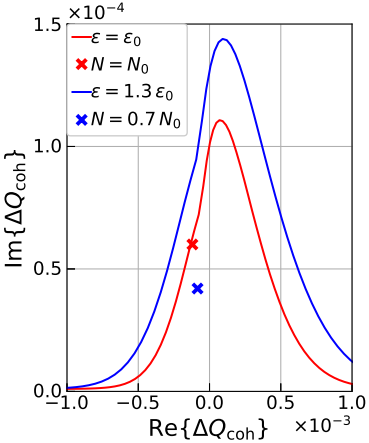




Beam Stability

- Landau damping prevents self-amplification of coherent motion.
  - Energy transferred to the beam, mostly at the resonant frequencies.
- Stability of a mode can be expressed through the stability diagram.

$$\frac{-1}{\Delta Q_{\mathrm{coh},\substack{x\\y}}} = \int_0^\infty \int_0^\infty \mathrm{d}J^2 \, \frac{J_{\substack{x\\y}} \frac{\mathrm{d}\Psi(J_x,J_y)}{\mathrm{d}J_{\substack{x\\y}}}}{Q_{LD,\substack{x\\y}} - Q_{\substack{x\\y}}(J_x, J_y)}$$

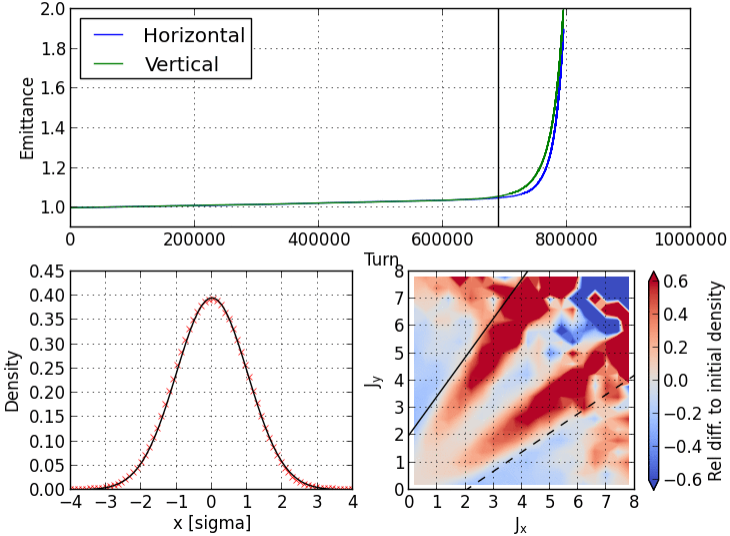


Noise Excited Wakefields – Numerical Model

- The diffusion due to wakefields was studied (mostly) numerically in X. Buffat, PhD thesis, 2015.
- Macroparticle tracking simulations (COMBI) have been run with external noise, wakefields and linear detuning.
  - LHC-like setup at top energy.

$$Q_x = Q_{x0} + aJ_x + bJ_y$$
$$Q_y = Q_{y0} + bJ_x + aJ_y$$

- See on the next slide particle transport across the black solid line horizontally and dashed line vertically. These frequencies correspond to the most unstable wakefield modes. The projection on the x-axis does not show a visible change of the distribution.


Horizontal
Vertical
Emittance
Turn
Density
x [sigma]
J_y
J_x
Rel diff. to initial density

Noise Excited Wakefields – Analytical Model

- Goal: Derive an analytical model to guide the search for optimal machine and beam parameters, mitigating this mechanism and maximising the latency, relevant for HL-LHC and future projects.

Noise Excited Wakefields

1. **Transverse wakefields drive eigenmodes of eigenfrequencies ω_j.**
2. **Landau damping changes the eigenfrequencies to Ω_j.**
3. **External noise, $\xi(t)$, drives the stable eigenmodes to nonzero amplitudes χ_j.**
4. **The wakefields act on all single particles.**
5. **From Liouville equation, derive a diffusion equation.**

Step 1: Impedance Modes

Find impedance eigenmodes $\Delta\Psi_j$ with eigenvalues $\omega_j = \omega_0 + \Delta\omega_j$, amplitudes χ_j, and dipole moments η_j (dependent on impedance, chromaticity and transverse feedback). The amplitudes behave like

$$\ddot{\chi}_j + \omega_j^2 \chi_j = 0.$$

Can easily find the wake force, F_j, from that eigenmode

$$\ddot{\chi}_j + \omega_0^2 \chi_j = \omega_j F_j = (\omega_0^2 - \omega_j^2)\chi_j.$$

(The frequency in $\omega_j F_j$ is due to choice of parameters, see backup).

Step 2: Landau Damped Modes (1/2)

If inside stability diagram, the discrete modes will not exist, the continuous modes will be damped by filamentation

$$\Delta\Psi = \int \chi(\Omega)\Delta\Psi(\Omega)\mathrm{e}^{-\mathrm{i}\Omega t}\mathrm{d}\Omega.$$

"*An arbitrary initial distribution behaves (after a short transient time) like a superposition of [slightly damped plane waves, which do obey the dispersion equation], as far as the density is concerned.*"

N. G. Van Kampen, On the theory of stationary waves in plasmas, 1955.

Assumption: The damped plane waves correspond to the most unstable modes without Landau damping.

Step 2: Landau Damped Modes (2/2)

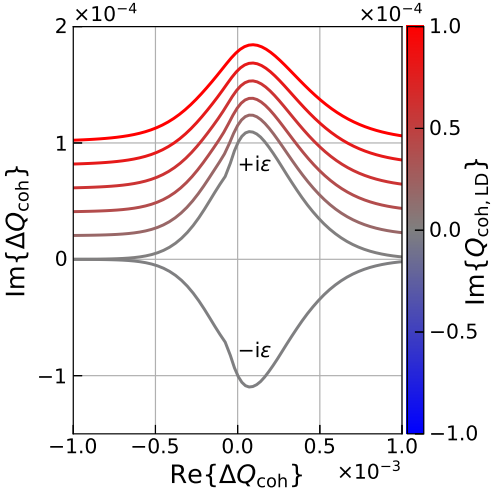
Find Ω_j including the Landau damping, from the initial ω_j, by integrating the dispersion integral numerically with `PySSD`

$$\frac{-1}{\Delta\omega_{j,\substack{x\\y}}} = \int_0^\infty \int_0^\infty \mathrm{d}J^2 \, \frac{J_{\substack{x\\y}} \frac{\mathrm{d}\Psi(J_x, J_y)}{\mathrm{d}J_{\substack{x\\y}}}}{\Omega_{j,\substack{x\\y}} - \omega_{\substack{x\\y}}(J_x, J_y)}.$$

This is actually even less straightforward than it looks like...

Step 2: Landau Damped Modes - A challenge

- Can easily get the growth rate $\mathrm{Im}\{\Omega_j\} > 0$.
- There is a hole in the mapping by the dispersion integral.
- Currently we extrapolate from the stability limit. This requires further investigation.



Step 3: Excitation of Modes

The external noise is expected to be low-frequency and thus dipolar in nature (affects all particles along the bunch equally). Thus, mode $\Delta\Psi_j$ will on average be affected proportional to η_j

$$\ddot{\chi}_j + \Omega_j^2 \chi_j = \Omega_j \xi_j(t) = \Omega_j \eta_j \xi(t).$$

Take the Fourier transform to get the frequency spectrum of χ_j

$$\mathcal{F}(\chi_j) = \frac{\Omega_j \eta_j \mathcal{F}(\xi)}{\Omega_j^2 - \omega^2} = \frac{\Omega_j \eta_j \mathcal{F}(\xi)}{\mathrm{Re}\left\{\Omega_j^2\right\} - \omega^2 + \mathrm{i}\,\mathrm{Im}\left\{\Omega_j^2\right\}}.$$

This is superposition of singular modes in a continuous spectrum.

Step 4: Excitation of Single Particles

The wakefields act on all incoherent particles, described by transverse amplitudes y and betatron frequencies $\omega = \omega(J_x, J_y)$,

$$\begin{aligned}\ddot{y} + \omega^2 y &= \omega\xi(t) + \omega F_{\text{wake}} \\ &= \omega\xi(t) + \omega \sum_j F_j \\ &= \omega\xi(t) + \sum_j \tfrac{\omega}{\omega_j}(\omega_0^2 - \omega_j^2)\chi_j.\end{aligned}$$

In the following we will consider the second term.

The first term was studied in S.V. Furuseth *et al.*, MOPGW07, IPAC 2019.

Step 5: Wakefield driven Diffusion (1/2)

Consider the dynamics of y by a Hamiltonian with a stochastic perturbation given by the weak wakefields, as

$$\begin{aligned}\mathcal{H} &= \mathcal{H}_0(J) + \epsilon\mathcal{H}_1(\theta, J),\\ &= \left(\omega_0 + \tfrac{1}{2}aJ\right)J - y \cdot F_{\text{wake}}.\end{aligned}$$

Can in this case calculate the diffusion coefficient as

(A. Bazzani *et al.*, Diffusion in Hamiltonian systems driven by harmonic noise, 1998)

$$\begin{aligned}D &= \frac{\epsilon^2}{2}\left\langle \frac{\partial\mathcal{H}_1}{\partial\theta}(t)\frac{\partial\mathcal{H}_1}{\partial\theta}(t')\right\rangle\\ &= \frac{1}{2}J\left|\mathcal{F}(F_{\text{wake}})\right|^2.\end{aligned}$$

Step 5: Wakefield driven Diffusion (2/2)

This leads to the diffusion equation, which for one dominant mode can be written as

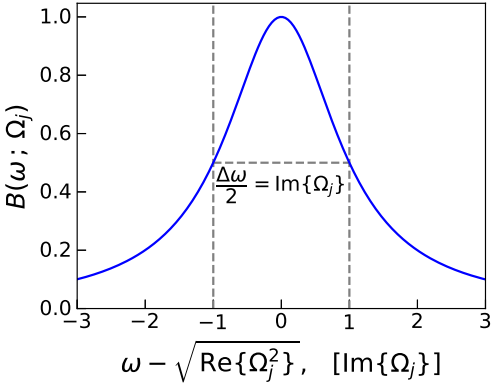
$$\frac{\partial \Psi}{\partial t} = \frac{\partial}{\partial J}\left[D \cdot \frac{\partial \Psi}{\partial J}\right]$$

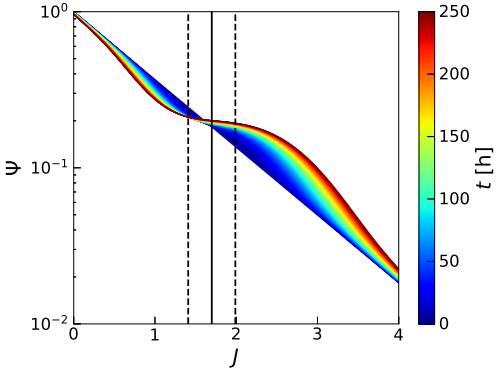
$$D(\omega) = J \cdot \frac{\omega_{\mathrm{rev}}}{2} \cdot \frac{\eta_j^2 \sigma_\xi^2 |\Delta\omega_j|^2}{\mathrm{Im}\{\Omega_j\}^2} \cdot B(\omega;\Omega_j) \cdot C(\omega_j,\Omega_j)$$

$$B(\omega;\Omega_j) = \frac{\mathrm{Im}\left\{\Omega_j^2\right\}^2}{\left(\mathrm{Re}\left\{\Omega_j^2\right\} - \omega^2\right)^2 + \mathrm{Im}\left\{\Omega_j^2\right\}^2}\ ,\ \ C \sim 1.$$

Diffusion in 1 Transverse plane

The diffusion is centred around the mode frequency. It leads to a local flattening of the distribution (exaggerated here).





Loss of Landau damping driven by diffusion

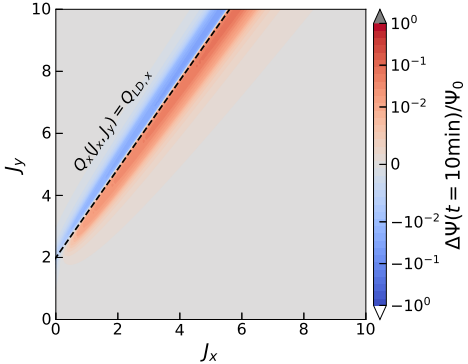
Drilling for Instabilities

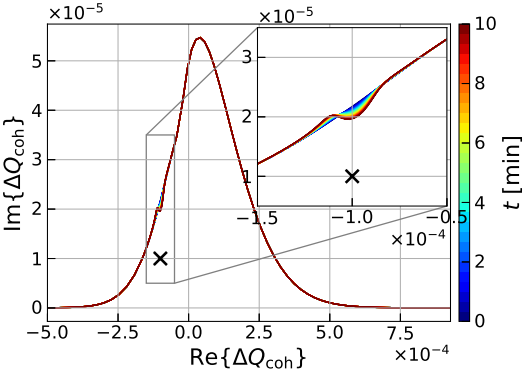
- Only consider the most unstable mode, in a configuration with noise and linear detuning.

$$\boxed{\begin{aligned} Q_x &= Q_{x0} + aJ_x + bJ_y \\ Q_y &= Q_{y0} + bJ_x + aJ_y \end{aligned}}$$

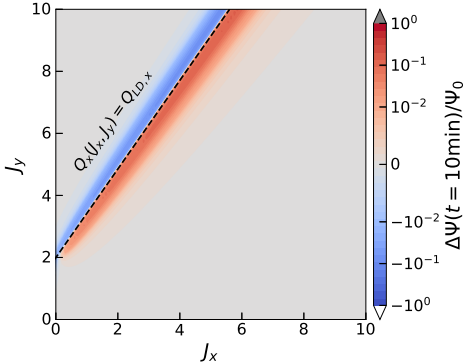
- Get distribution evolution with a FVM solver - `PyRADISE` (Python Radial or Action Dependent dIffusion and Stability Evolution).
- Calculate the stability diagram with `PySSD`.

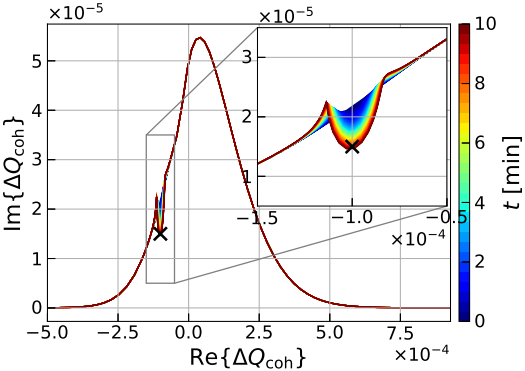
$\Delta Q_{\mathrm{coh}} = -1.0 \times 10^{-4} + \mathrm{i} \cdot 1.0 \times 10^{-5}$





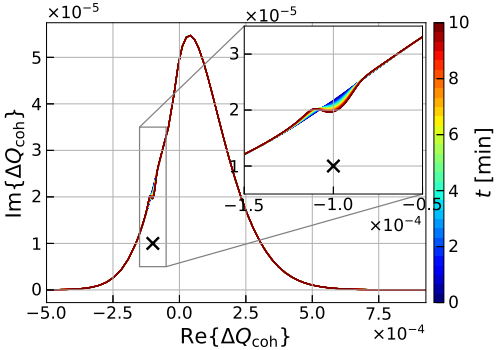
$\Delta Q_{\mathrm{coh}} = -1.0 \times 10^{-4} + \mathrm{i} \cdot 1.5 \times 10^{-5}$

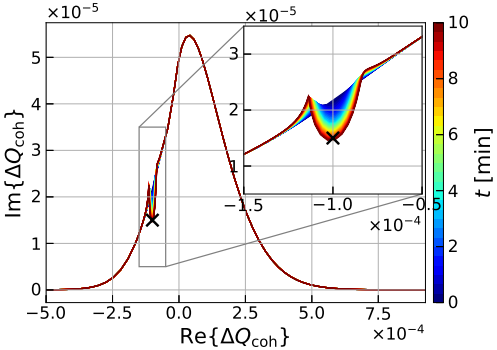




Comparison

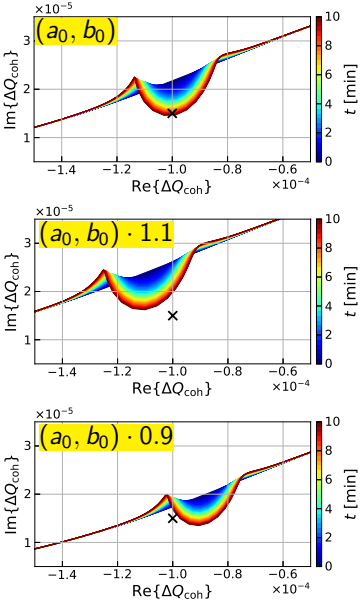
The rate of drilling in the stability diagram is higher as you are closer to the stability threshold.





Mitigation

- Reduce drilling rate, $D \propto \frac{\eta_j^2 \sigma_\xi^2 |\Delta\omega_j|^2}{\mathrm{Im}\{\Omega_j\}^2}$.
  - Reduce the noise.
  - Reduce the impedance.
  - Increase margin to threshold (LHC).
- Dig everywhere.
  - Vary the detuning strength.
- Increase incoherent noise to counteract the drilling.
  - Intra-beam scattering and Quantized Synchrotron radiation.



Conclusion

- Instabilities of **high latencies** can develop in high-energy hadron machines with noise and impedance, by changing the distribution.
  - An external source of **noise excites the beam**, which in return affects incoherent particles through wakefields.
  - These **wakefields cause diffusion** in a narrow frequency range centred at the eigenfrequency of the beam mode.
- The diffusion efficiently **drills a hole in the stability diagram** at the eigenfrequency of the corresponding mode.
- The recommended **mitigation technique** is to reduce the drilling rate, $D \propto \eta_j^2 \sigma_\xi^2 |\Delta\omega_j|^2 / \mathrm{Im}\{\Omega_j\}^2$.
- The interplay with IBS and external noise has been studied with `PyRADISE`, and it does not mitigate the drilling process (backup).

Outlook

- Improve the description of the damped modes (Van Kampen).
- Consider other types of noise, e.g. Crab-cavity amplitude noise.
- Include single particle chromatic tuneshift.
- Study the stability evolution self-consistently as the Landau damping varies during the drilling process.
- Compare quantitatively to Instability Latency experiments in the LHC.

Thank you for your attention!

Backup Overview

Backup: Explanation of $\omega\xi$ in E.o.m.

In the normalized coordinate system used, the position x and momentum p have equal units,

$$\mathcal{H} = \omega J = \omega \cdot \frac{x^2 + p^2}{2} - x\xi(t).$$

Using Hamiltons' equations

$$\dot{x} = \frac{\partial \mathcal{H}}{\partial p} = \omega p \quad , \quad \dot{p} = -\frac{\partial \mathcal{H}}{\partial x} = -\omega x + \xi(t) \ ,$$

we immediately get to the equation of motion

$$\ddot{x} + \omega^2 x = \omega \xi(t).$$

Backup: Correction factor

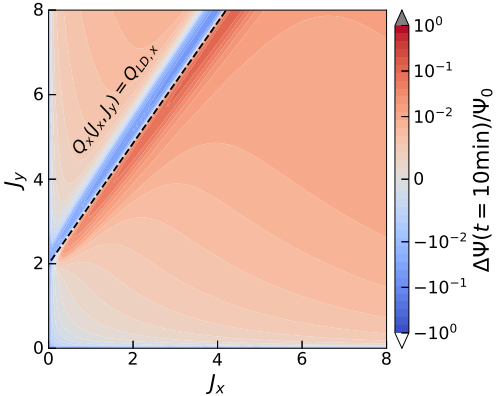
The additional correction factor is close to 1 for small wakefield induced tune shifts, and is given by

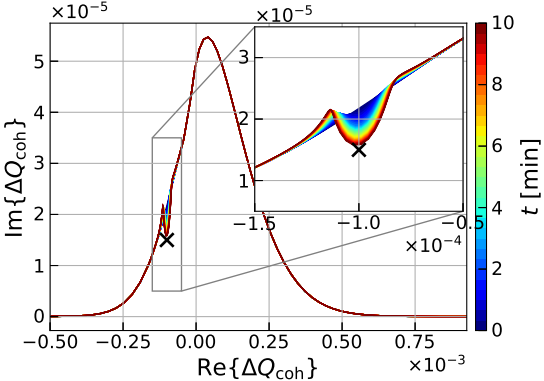
$$C(\omega_j, \Omega_j) = \frac{\mathrm{Re}\{\omega_j\}\omega_0 + |\Delta\omega_j|^2/4}{\mathrm{Re}\{\Omega_j\}^2} \cdot \frac{|\Omega_j|^2}{|\omega_j|^2} = 1 + \mathcal{O}\left(\frac{\mathrm{Re}\{\Delta\omega_j + \Delta\Omega_j\}}{\omega_0}\right),$$

where $\omega_j = \omega_0 + \Delta\omega_j$ and $\Omega_j = \omega_0 + \Delta\Omega_j$. The tune shifts have been assumed small relative to ω_0 for the second expression.

Backup: NEW & IBS

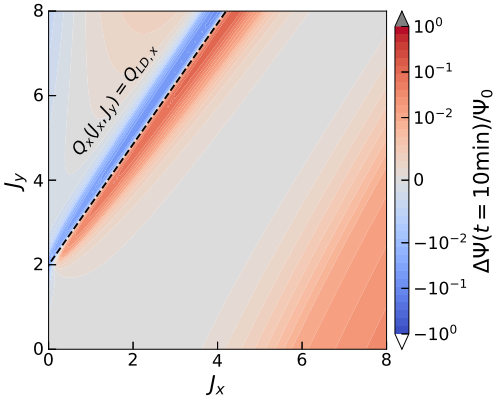
Change of distribution including the diffusion from intra-beam scattering, corresponding to an emittance growth of 2 %/h.

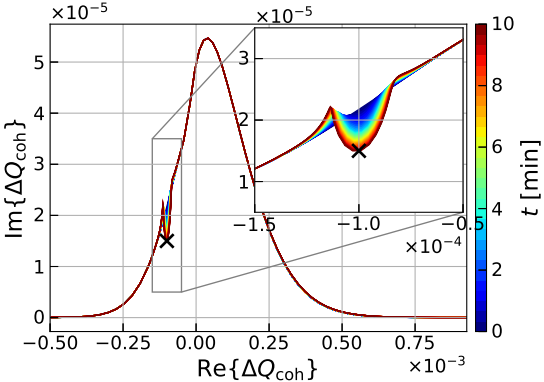




Backup: NEW & External noise (1/2)

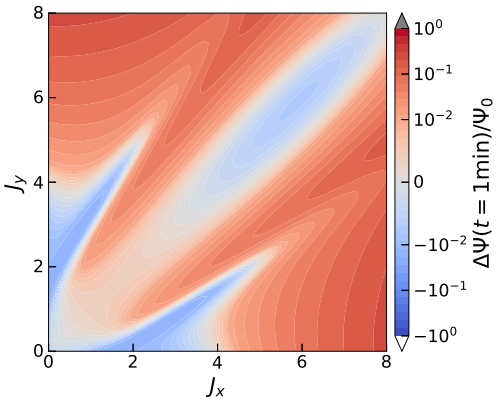
Change of distribution including the direct diffusion from the external noise on slide 18.

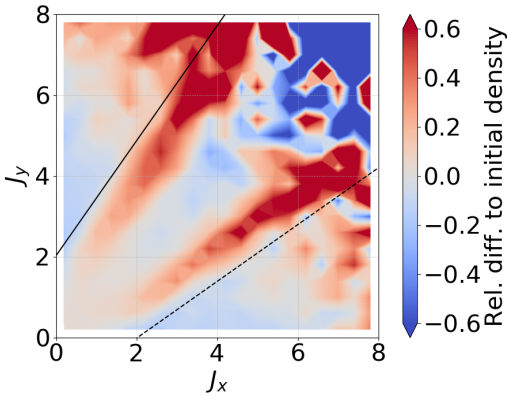




Backup: NEW & External noise (2/2)

Change of distribution including the direct diffusion from the external noise on slide 18. Parameters not equal.





Backup: PyRADISE Parameters

Typical parameters used in `PyRADISE` (except for in approximate comparison to simulation).

- $a = 5.0 \times 10^{-5}$
- $b = -3.5 \times 10^{-5} = -0.70 \cdot a$
- $\sigma_\xi = 10^{-3}$
- $\eta_j = 5 \times 10^{-3}$
- $\Delta Q_{\mathrm{coh}} = \dfrac{\omega_j - \omega_0}{2\pi\omega_{\mathrm{rev}}}$ is varied.

$$\boxed{\begin{aligned} Q_x &= Q_{x0} + aJ_x + bJ_y \\ Q_y &= Q_{y0} + bJ_x + aJ_y \end{aligned}}$$