Suppression of the Fast Beam-Ion Instability by Tune Spread in the Electron Beam due to Beam-Beam Effects

G. Stupakov

SLAC National Accelerator Laboratory, Menlo Park, CA 94025

ICFA mini-Workshop on "Mitigation of Coherent Beam Instabilities in particle accelerators" MCBI 2019, 23-27 September 2019
Zermatt, CH

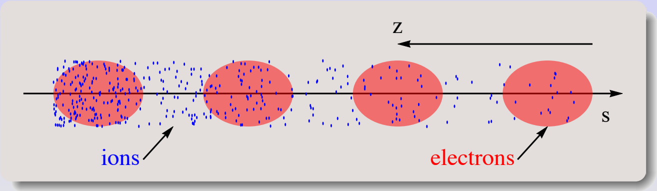
Introduction

- Conventional (electron) beam-ion instability: ion trapping over many turns $\rightarrow$ ions drive the instability; can be cured by clearing gaps.
- For high current and small emittance beams an instability can develop within a single bunch train ("fast beam-ion instability" or FII[1]).

[1] T. Raubenheimer, F. Zimmermann, Phys. Rev. E **52**, 5487 (1995); G. Stupakov, T. Raubenheimer, F. Zimmermann, Phys. Rev. E **52**, 5499 (1995).

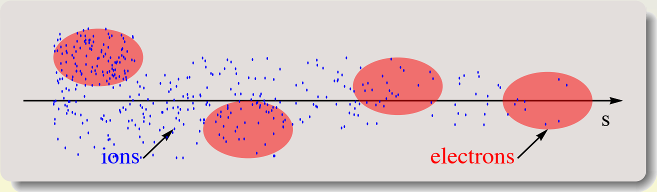
Mechanism of the instability

Equilibrium state:



Electrons oscillate transversely with the betatron frequency, ω_β; ions oscillated with ω_i in the potential well of the electron beam. An initial resonant perturbation is amplified downstream.

Instability:

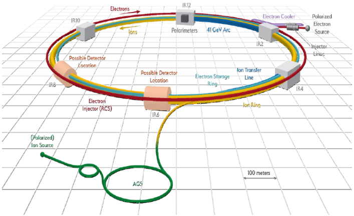


FII at eRHIC

eRHIC Layout

- **Hadrons up to 275 GeV**

  eRHIC is using the existing RHIC complex: Storage ring (Yellow Ring), injectors, ion sources, infrastructure
- Need only few modifications for eRHIC
- Todays RHIC beam parameters are close to what is required for eRHIC



- **Electrons up to 18 GeV**

- Electron storage ring with up to 18GeV ➔ E_{cm} = 20 GeV -141 GeV installed in RHIC tunnel. Beam current are limited by the choice of installed RF power 10 MW
- Electron beams with a variable spin pattern accelerated in the on-energy, spin transparent injector: Rapid Cycling Synchrotron with 1-2 Hz cycle frequency in the RHIC tunnel
- Polarized electron source and 400 MeV s-band injector linac in existing tunnel
- Design meets the high luminosity goal of **$L = 10^{34} cm^{-2} s^{-1}$**

7

Question (M. Blaskiewicz): how the tune spread from beam-beam collisions in electron-ion collider eRHIC affects FII?

Approximations of a simple model (1995)

- One dimensional, the electron beam offset is y
- Continuous electron beam ($\ell_b \ll c/\omega_i, c/\omega_\beta$ with ℓ_b the distance between the bunches)
- Small displacements, $y \ll \sigma_y$, $\rightarrow$ linear theory
- Ion are completely cleared by the gap
- Constant ω_i, ω_β (ω_i does not depend on s)

The governing equation (RZ)

y – the beam centroid offset, $z = ct - s$ – the coordinate in the bunch ($v = c$)

$$\frac{\partial^2 y(s,z)}{\partial s^2} + \frac{\omega_\beta^2}{c^2} y(s,z) = -\varkappa \int_0^z z' \frac{\partial y(s,z')}{\partial z'} \cos\left[\omega_i (z - z')/c\right] dz'$$

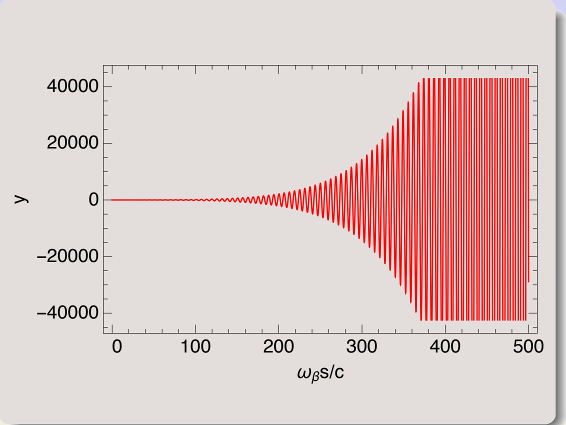
e^- betatron oscillations — coupling — linear ion profile — ion oscillations

$$\omega_i = \left[\frac{4 n_e r_p c^3}{3 A \sigma_y (\sigma_x + \sigma_y)}\right]^{1/2}$$

$$\varkappa = \frac{4 \dot{\lambda}_{ion} r_e}{3 \gamma c \sigma_y (\sigma_x + \sigma_y)}, \qquad \dot{\lambda}_{ion}[\mathrm{m}^{-1}\mathrm{s}^{-1}] \approx 9 \times 10^8 \sigma_i n_e p_{gas}$$

s is the path length in the ring, n_e – the number of electron per meter; A – atomic mass number; $\dot{\lambda}_{ion}$ – number of ions per meter per unit time generated by the beam; σ_i – ionization cross section (in Mbarn); p_{gas} – residual gas pressure (in torr); $\varkappa$ [m^{-3}] – the interaction parameter.

A numerical example of instability



Oscillations of the last bunch, $y(z = \ell_{tr}, s)$, for $\ell_{tr} c^2 \varkappa / \omega_\beta^2 = 0.05$, $\ell_{tr} \omega_i / c = 20$, and initial condition $y(z, s = 0) = \cos(\omega_i z / c)$ (ℓ_{tr} is the length of the train).

Waves in the beam

The interaction between the electrons and ions is established in the form of a *wave*:

$$y(s,t) = Ye^{-iks+i\omega_i t}$$

A moving electron, $s = -z + ct$, "sees" a Doppler shifted frequency $\omega_i - kc$ which should be equal to $\pm\omega_\beta$, hence $k = \omega_i \pm \omega_\beta$:

$$y(s,t) = Ye^{\pm i\frac{\omega_\beta}{c}s + i\omega_i\left(t - \frac{s}{c}\right)}$$

It turns out that the "fast" wave $(+)$ is stable, the slow wave $(-)$ is unstable.

Assume a weak instability

$$\varkappa\ell_{tr} \ll \frac{\omega_\beta^2}{c^2}, \frac{\omega_i^2}{c^2}$$

and look for a solution

$$y(s,z) = \mathrm{Re}\, Y(s,z)e^{-i\frac{\omega_\beta}{c}s + i\frac{\omega_i}{c}z}$$

with slowly varying amplitude Y (SVEA). Average over fast oscillations and keep resonant terms only.

The instability

The equation for A becomes

$$\frac{\partial^2 Y(s,z)}{\partial s \partial z} = \frac{\varkappa \omega_i}{4\omega_\beta} z Y(s,z)$$

For the initial condition $Y = y_0$ at $s = 0$, the solutions is

$$Y(s,z) = Y_0 I_0 \left(\frac{z}{\ell_{tr}} \sqrt{\frac{s}{c\tau}} \right)$$

where

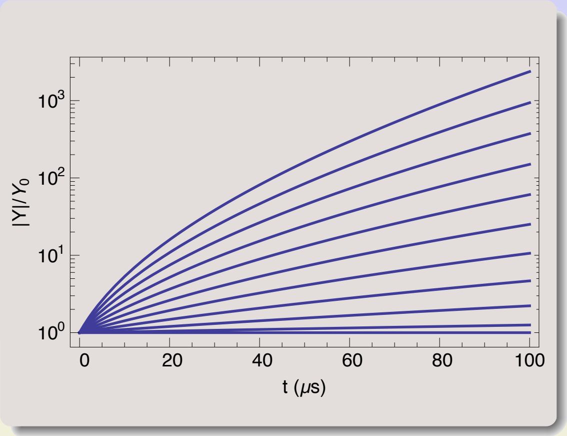
$$\tau = \frac{2\omega_\beta}{c \varkappa \omega_i \ell_{tr}^2}$$

Asymptotically, for large s

$$Y \propto Y_0 \sqrt{\frac{c\tau}{s}} \exp\left(\frac{z}{\ell_{tr}} \sqrt{\frac{s}{c\tau}} \right)$$

For eRHIC parameters $\tau \approx 1\,\mu\text{s}$.

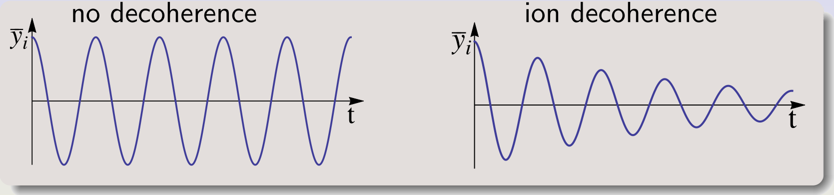
FII at eRHIC, no decoherence



Growth of an initial unit offset in the eRHIC electron ring at 11 different points in the train (the line corresponding to the first point is superimposed on the abscissa) without decoherence.

Ion decoherence

Ions have a distribution function over frequency, $f_i(\omega_i)$ with a characteristic spread $\Delta\omega_i$ (we assume $\Delta\omega_i \ll \omega_{i0}$). This causes filamentation of ion oscillations and effective (Landau) damping.

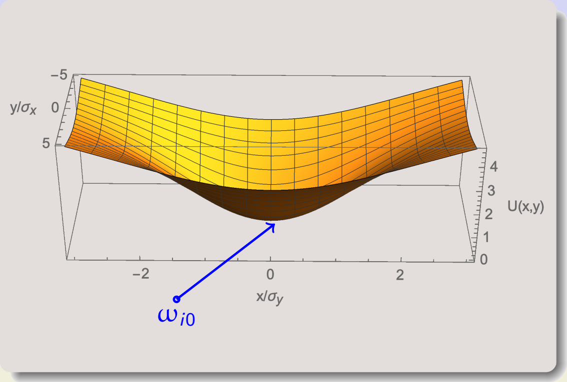


The decoherence function $\bar{y}_i(ct) = D(ct)$ is the response of the ion centroid to an initial kick (offset),

$$D(z) = \int d\omega_i f_i(\omega_i) e^{i(\omega_i - \omega_{i0})z/c}$$

Sources of ion decoherence

There are two sources of the ion frequency spread:



1. Nonlinearity of the potential well
2. Variation of the y potential (for vertical oscillations) along x

$$n_e(x) \propto e^{-x^2/2\sigma_x^2} \rightarrow \omega_i \propto e^{-x^2/4\sigma_x^2}$$

Ion decoherence in FII equations

Ion centroid oscillations: $\cos(\omega_i z/c) \to D_i(z)$

$$\frac{\partial^2 y(s,z)}{\partial s^2} + \frac{\omega_\beta^2}{c^2} y(s,z) = -\varkappa \int_0^z z' \frac{\partial y(s,z')}{\partial z'} D_i(z-z') dz'$$

The equation for the slowly varying amplitude

$$\frac{\partial Y(x,z)}{\partial s} = \frac{\varkappa \omega_{i0}}{4\omega_\beta} \int_0^z z' Y(x,z') D_i(z-z') dz'$$

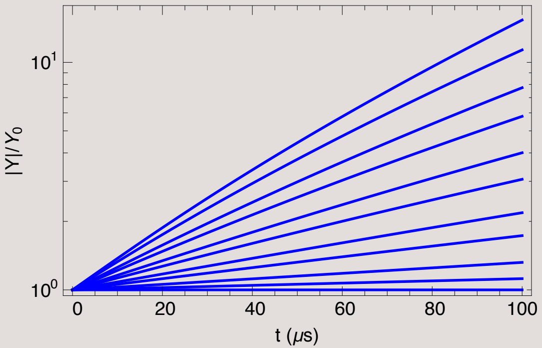
The decoherence function can be approximated[2]

$$D_i(z) \approx (1 + i\alpha\omega_{i0} z/c)^{-1/2}$$

with $\alpha \approx 0.25$. There is also a lattice effect: ω_i varies along the circumference of the ring (ignored in this analysis).

[2] G. Stupakov, T. Raubenheimer, F. Zimmermann, Phys. Rev. E **52**, 5499 (1995).

FII at eRHIC – with ion decoherence



Growth of an initial unit offset in the eRHIC at ten different points in the train with ion decoherence taken into account.

Beam-beam tune spread and e^- beam decoherence

Electron beam oscillations decohere because of the tune spread due to beam-beam collisions. The decoherence function $D_e(s)$ for the case when the betatron tune spread is due to the beam-beam collisions at the interaction point in a collider was derived in[3]. Assuming round beams at the interaction point:

$$D_e(s) = 4\int_0^\infty \int_0^\infty dx\, dy\, \exp\left[-2(x+y) + i(\omega_{\mathrm{rev}} s/c)\Delta\nu_y(x,y)\right]$$

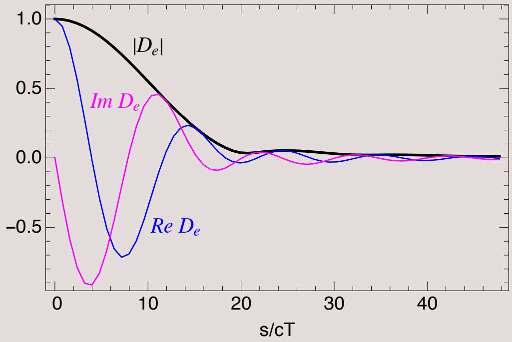
where $\omega_{\mathrm{rev}} = 2\pi/T$ is the revolution frequency. The tune shift $\Delta\nu_y$ is given by

$$\Delta\nu_y(x,y) = -\xi \int_0^1 du\, e^{-u(x+y)} I_0(yu)\left[I_0(xu) - I_1(xu)\right]$$

where x and y are the dimensionless amplitudes of the betatron oscillations, ξ is the tune shift parameter, $\xi = N_p r_e/4\pi\epsilon$ with N_p the number of particles in the bunch, r_e the classical electron radius and ϵ the normalized beam emittance. For eRHIC, $\xi = 0.1$.

[3] G. Stupakov, V. Parkhomchuk, and V. Shiltsev, Preprint SSCL-Preprint-495 (1993).

e^- beam decoherence



Plot of real (blue) and imaginary (magenta) parts of the function $D_e(s)$. The black line is the absolute value $|D_e(s)|$. For eRHIC, $T = 12.5\,\mu$s.

FII with electron and ion decoherence

The following equation for Y can be derived

$$\frac{\partial}{\partial s} Y(s,z) = Y_0(z) D_e'(s) + \frac{\kappa \omega_{i0}}{4\omega_{\beta 0}} \int_0^z z' Y(s,z') D_i(z-z') dz'$$
$$+ \frac{\kappa \omega_{i0}}{4\omega_{\beta 0}} \int_0^s ds' D_e'(s-s') \int_0^z z' Y(s',z') D_i(z-z') dz',$$

For a model electron decoherence function (and no ion decoherence)

$$D_e(s) = e^{-s/s_0}$$

this equation can be solved analytically

$$Y(s,z) = Y_0 e^{-s/s_0} I_0\left(\frac{z}{\ell_{tr}}\sqrt{\frac{s}{c\tau}}\right)$$

Asymptotically, in the limit $s \to \infty$, the exponential factor e^{-s/s_0} overcomes the growing Bessel function, and hence, suppresses the instability.

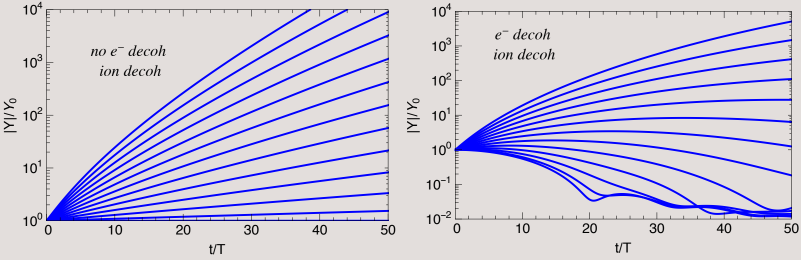
Parameters of the eRHIC collider relevant for FII

Electron beam energy	10 GeV
Vertical beam emittance, ϵ_y	4.9 nm
Horizontal beam emittance, ϵ_x	20 nm
Residual gas pressure, p	0.75 nTorr
Averaged beta function, β_x, β_y	18 m
Vertical betatron tune, ν_y	31.06
Number of electron bunches, N_b	567
Length of the bunch train, ℓ_b	3451 m
Atomic mass number for ions, A	28
Number of electrons per unit length, n_e	5.6×10^{10} m^{-1}
Beam-beam tune shift, ξ	0.1

Using the beam emittance and the value for the averaged beta functions we find the characteristic beam sizes in the vertical and horizontal directions, $\sigma_y = 0.3$ mm and $\sigma_x = 0.6$ mm. The ion frequency is $\omega_{i0} = 4.5 \times 10^7$ s^{-1}. The vertical betatron frequency is $\omega_\beta = 1.5 \times 10^7\ \mathrm{s}^{-1}$. The parameter $\dot{\lambda}$ is $\dot{\lambda} = 7.5 \times 10^{10}\ \mathrm{m}^{-1}\mathrm{s}^{-1}$.

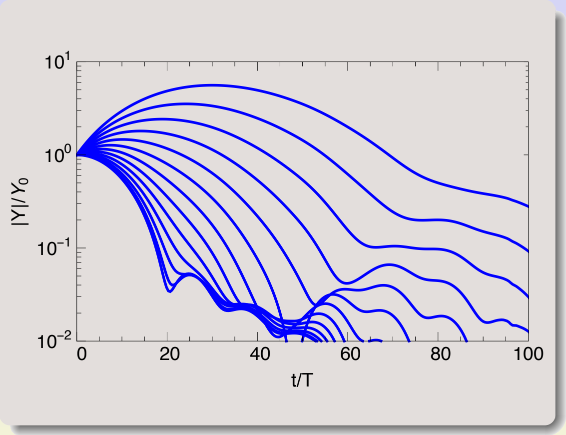
FII in eRHIC

We numerically solved the FII equations for parameters of eRHIC taking into account both the ion and electron decoherence. One can see that the decoherence slightly suppresses the instability, but does not kill it.



FII in eRHIC, smaller gas pressure

Make gas pressure 2.5 times smaller (0.3 nTorr)



Summary/Discussion/Acknowledgements

- Effect of the electron beam decoherence due to beam-beam induced tune spread is now included into FII theory. As expected, it mitigates the instability and can suppress FII for a smaller residual vacuum pressure.
- These results are in qualitative agreement with recent study of M. Blaskiewics[4].
- The continuous beam model for FII requires ($\ell_b \ll c/\omega_i, c/\omega_\beta$ with ℓ_b the distance between the bunches) which are not well satisfied for eRHIC parameters.
- While the electron beam decoherence can suppress FII, it converts the amplified noise in the electron beam into its emittance growth. This effect needs to be evaluated (the transverse damping time is 5.4×10^3 turns).

I would like to thank M. Blaskiewicz for formulating the problem and providing relevant numerical parameters of eRHIC.

[4] M. Blaskiewics, paper TUPLM11 at NAPAC19.