Identification and Reduction of Space-Charge and Beam-Beam Effects

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Method

- It is 30 years since publication by K. Yokoya et al. the KEK report “Tune Shift of Coherent Beam-Beam Oscillations” which introduced the method of eigenmodes of the Vlasov equation into the beam-beam interaction theory.
- It filled an important niche between analytical calculations and massive tracking simulations providing the insight of the former and accuracy of the latter.
- Recently the eigenmode approach was applied to coherent oscillations in beams with space charge.

In this report:
- Spectra of coherent beam-beam oscillations, instabilities and their damping
- Transverse oscillations in a bunched beam with space charge (SC), including TMCI in the in the case of strong SC
**Vlasov Perturbation Theory**

\[
\frac{\partial}{\partial \theta} F_1^{(k)} + Q^{(k)} (I) \frac{\partial}{\partial \psi} F_1^{(k)} = - \frac{r_p N_{3-k}}{\gamma} \delta_p(\theta) F_0 \varepsilon^{-1} \cdot \frac{\partial}{\partial \psi} \int G^{(k)} F_1^{(3-k)} d^3 l' d^3 \psi' + \varepsilon^{-1} \cdot \mathbf{l}^{(ext)} F_0
\]

\[
G = -\ln \left\{ \left[ x - x' + \left( \alpha + \frac{p_x + p'_x}{2} (z - z') \right)^2 \right] + \left[ y - y' + \frac{p_y + p'_y}{2} (z - z') \right]^2 \right\}
\]

The incoherent tunes \( Q = Q(l) \) may include contribution from lattice multipoles, e-lens etc.

There is no attempt to replace the integral operator with something else, but apart from coherent resonances \( \delta_p(\theta) \rightarrow 1/2\pi \)

Expansion in angle variables \( \psi \) (azimuthal mode expansion)

\[
F_1^{(k)} = \exp \left( -\varepsilon^{-1} \cdot \mathbf{l} / 2 \right) \sum_m \exp(i m \cdot \psi) f_m^{(k)} (I, \theta)
\]

e.g. for horz dipole oscillations \( m=(1, 0, 0) \).

Outside of resonances the azimuthal modes can be considered uncoupled. Assuming \( f \sim \exp(-i \xi \alpha \theta) \) we arrive at the eigenvalue problem

\[
\lambda f = \hat{A} f
\]

small perturbation to be treated last

finite bunch length effect

\( k = 1,2 \) is beam number,

\( z = \) long. displacement from the bunch center

\( \alpha = \) half crossing angle (horizontal here)

Gaussian equilibrium distribution assumed

The finite bunch length effect is the incoherent tune variation due to finite bunch length.
Spectral Coefficients

Beam oscillation as a whole (and a dipole kick at the beam) is described by function:

\[ \Psi_0(I_x, I_y) = \sqrt{I_x} e^{-(I_x + I_y)/2}. \]

Spectral coefficient of eigenfunction

\[ c(\lambda) = \int \Psi_0(I_x, I_y) \Psi_\lambda(I_x, I_y) dI_x dI_y \]

\( s(\lambda) = c^2(\lambda) \) gives energy density that eigenmodes in \((\lambda, \lambda+d\lambda)\) receive from a kick.

Integration over the spectrum of \(\pi\) (-) and \(\Sigma\) (+) modes with weight functions

\[ w_+ (\lambda) = \begin{cases} 
0, & \lambda < 0 \\
1 + \lambda, & 0 \leq \lambda < 1 \\
2, & 1 \leq \lambda 
\end{cases} \]

\[ w_- (\lambda) = \begin{cases} 
0, & \lambda < 0 \\
\lambda, & 0 \leq \lambda < 1 \\
1, & 1 \leq \lambda < \lambda_0 \\
2, & \lambda_0 \leq \lambda 
\end{cases} \]

Stability Diagram

\[ D(\nu, \zeta) = 1 - \zeta \int \frac{s(\lambda)}{\nu - \lambda} d\lambda = 0 \]

\( \lambda_0 = 1.214 \) for round beams, \( \zeta = \) complex tuneshift due to impedance

Head-On Beam-Beam Modes

Spectral density of horizontal oscillations in flat beams after a kick

Spectra of horizontal oscillations in LEP of two bunches colliding at two IPs: left – electron beam, right – positron beam (courtesy of G. Morpurgo).

The noise to the left of the \( \Sigma \)-mode is likely the vertical spectrum seen due to coupling.
Why \( \pi \) Mode not Always Seen?

In round beams the discrete \( \pi \)-mode emerges from the continuum at intensity ratio \( r=0.6 \) (YA, 1996)

Spectra of two colliding p-bunches in RHIC (courtesy of W. Fischer)

Simulation by M. Vogt et al. (2002)

Simulation by the Hybrid Fast Multipole Method (Herr, Jones & Zorzano, 2001) confirms this result
Coherent Beam-Beam Modes Suppression

The very existence of the discrete modes outside the incoherent tunespread implies possible instability due to lack of Landau damping.

Cures for head-on colliding beams:

- Splitting bare lattice tunes (A. Hoffman)
- Redistribution of phase advances between IPs (A. Temnykh, J. Welch)
- Different parity of integer parts of the tunes in separate rings (W. Herr)
- Octupoles to increase the incoherent tunespread
- Electron lens for the same purpose
- Transverse feedback!
- Chromaticity
- Landau damping by synchrotron sidebands

\[ m=0 \quad m=1 \quad m=2 \]
\[ \text{incoherent tunespread} \quad Q_s \quad 2Q_s \]
Effects of Chromaticity

- Reduction in coherency of oscillations → weakening of the discrete modes
- Coupling to synchrotron sidebands → Landau damping
- Head-tail damping in the presence of (inductive) impedance

Coupling to synchrotron sidebands in short bunches is characterized by parameter

$$\kappa = (\frac{1}{\beta_x^*} - \frac{Q'_x}{\alpha_c R})\sigma_s$$

If the synchrotron tune is high ($\Delta Q_{BB} < Q_s$) – like it was in LEP – the coupling weakens the discrete modes by factor

$$\lambda_\parallel = e^{-\kappa^2} I_{m_s}(\kappa^2)$$

Coherent modes $m_s \neq 0$ are strongly suppressed

*) This parameter pertains to the phase modulation of the beam-beam kick, no matter how weak or strong it is. It does NOT present a beam-beam contribution to chromaticity!
However, if the beam-beam effect is strong ($\Delta Q_{BB} >> Q_s$), it changes the head-tail damping
Effects of Chromaticity on HO collisions

- Head-tail damping in the presence of an external wake

\[
\Delta v_x \sim \beta_x \int dJ_x d\psi_x dJ_x' d\psi_x' e^{-(J_x + J_x')} W_1(z - z') \times \begin{cases} 
  \cdots e^{-i \chi(z-z')} & Q_s \geq \xi \\
  e^{-i(z-z')/\beta^*} & \xi \gg Q_s 
\end{cases}
\]

stability condition, \(\text{Im}\Delta v_x < 0\), does not depend on chromaticity \(\chi\) for large \(\xi\)!

From the Tevatron RunIb summary (Fermilab-TM-1970, 1996):

“At collisions, the chromaticities are quite small and may even be slightly negative. (They are probably between about -5 and 5 units in both planes.) This seems to be a requirement for good particle and luminosity lifetimes. It has also been observed that when the beams are colliding they can tolerate chromaticities that would make a single beam unstable.”
LR Interactions & Phase Advance Difference

- The Yokoya factor for long-range interactions with separation in just one plane would be $Y = 2$ with both horizontal and vertical separation.
- With alternating separation the incoherent tuneshift cancels out, but the coherent kicks may not due to phase advance difference for beams 1 and 2 – potentially a trouble.

In the LHC:
- $\phi_x^{(1)} - \phi_x^{(2)} = 0.54\pi$, $\phi_y^{(1)} - \phi_y^{(2)} = -0.18\pi$,
Long-range interactions @ LHC

from X. Buffat et al. PRSTAB 17, 111002 (2014)

Long-range interactions at IP1 & IP5

\[ d_{\text{average}} = 1/\sqrt{\sum_i 1/d_i^2} \approx 9.3\sigma \]  (two nearest PIPs excluded)

Scaling with \( E, \beta^* \) and \( \varepsilon \) gives for \( \beta^*=0.8m \) and \( \varepsilon=3.5e^{-6} \)

\[ d_{\text{average}} \approx 11.2\sigma \]

I had it 12\( \sigma \) and to have the same effect increased the number of lumped NLR 28 \( \rightarrow \) 32

Octupoles

\[
\begin{align*}
\Delta Q_x &= a \cdot J_x + b \cdot J_y \\
\Delta Q_y &= b \cdot J_x + a \cdot J_y \\
a &= 3.28 \cdot \frac{I_{\text{oct}}[A] \cdot \varepsilon[m]}{E_{\text{beam}}^2[\text{TeV}^2]} \\
b &= -2.32 \cdot \frac{I_{\text{oct}}[A] \cdot \varepsilon[m]}{E_{\text{beam}}^2[\text{TeV}^2]}
\end{align*}
\]

\[ a = 1.5 \text{ e-4} \text{ for } I_{\text{oct}} = 550 \text{ A} \]

\[ b = -0.71 \text{ a} \]
Alternating Crossing + Unequal Phase Advance

- No phase advance difference $\rightarrow$ coherent modes suppressed
- Total phase advance difference $= \pi$ $\rightarrow$ very large coherent tuneshifts
  $\rightarrow$ Landau damping switched off
- Octupoles do not help much (included in calculations)
- There can be coherent head-tail modes ($m_s \neq 0$) $-$ see next slide

$\xi_0 = \text{beam-beam parameter} / \text{head-on IP} (\approx 0.004)$
Piwinsky (half) angle = 0.55
... + Chromaticity = 10

- Coherent tuneshifts are significantly reduced → facilitated Landau damping
- The peak height is also reduced by a factor >2 → equivalent to reduced impedance
- Chromaticity ~10 should be enough unless there is:
  - large tuneshift by impedance
  - large beam-beam contribution to chromaticity

- \( Q_s / \xi_0 \)
- \( -Q_s / \xi_0 \)
- coherent HT modes
- incoherent tunespread

cropped!
Many Things Left Out

Unfortunately I had to skip important issues like:

- Coherent beam-beam resonances (observed at LEP)

Spontaneous excitation of $\pi$-mode observed in LEP (courtesy of K.Cornelis)

Explained (YA, 1999) by coupling (offset needed!) of dipole $\pi$-mode

$$\nu = \nu_0 + 1.33\xi$$

to quadrupole $\Sigma$-mode

$$2\nu \approx 2\nu_0 + \xi$$

$$\nu_0 \approx \frac{n}{3} - \frac{1+1.33}{3} \xi_x \approx .265 \quad \text{at} \quad \xi_x = 0.088$$

(4 IPs)

- Feedback operation in the case of strong-strong BB: energy partitioning between discrete and continuum modes.
Space Charge Modes in Bunched Beams

Look similar to the beam-beam $\Sigma$-mode with one important distinction: the SC kick is longitudinally local, Green’s function $\sim \delta(z - z') \rightarrow$

- Coupling of many longitudinal modes
- Action-angle variables work poorly for the longitudinal DoF, instead

$$ F_1 = e^{i\psi_x - J_x/2 - (\tau^2 + \nu^2)/2} f(J_x, \tau, \nu; \theta) / (2\pi)^2 \varepsilon_x \varepsilon_z + c.c. $$

where $\tau = z/\sigma_z$ and relative momentum deviation $\nu = (p - p_0)/\sigma_p$, \quad $J_x = a_x^2 / 2\sigma_x^2$

The Vlasov equation as the eigenvalue problem:

$$ i \frac{\partial}{\partial \theta} f = \hat{A} f $$

$$ \hat{A}_0 f = -i\nu_x (\nu \frac{\partial}{\partial \tau} - \tau \frac{\partial}{\partial \nu}) f + v^{(ext)}_x f + e^{-\tau^2/2} \left[ v^{(SC)}_x (J_x) f + \frac{1}{\sqrt{2\pi}} \hat{G} \int_{-\infty}^{\infty} e^{-\nu'^2/2} f(J_x, \tau, \nu'; d\nu') \right] $$

eigenvectors are sought as expansion in some basis functions:

$$ V_m(J_x, \tau, \nu) = \sum_{k,l} V_{m; k,l} (J_x) \Phi_k(\nu) \Phi_l(\tau) $$
Longitudinal Basis Functions

The best solution found – Legendre polynomials of error function:

\[ \Phi_l(z) = P_l(u), \quad u = \text{erf}(\frac{z}{\sqrt{2}}) = \sqrt{\frac{2}{\pi}} \int_0^z e^{-x^2/2} dx \]

\[ \int_{-\infty}^{\infty} \Phi_k(z) \Phi_l(z) e^{-z^2/2} \, dz = \sqrt{\frac{\pi}{2}} \int_{-1}^{1} P_k(u) P_l(u) \, du = \frac{\sqrt{2\pi}}{2k+1} \delta_{kl} \]

Normalized functions:

\[ \Phi_k(z) = \sqrt{\frac{2k+1}{\sqrt{2\pi}}} \tilde{\Phi}_k(z) \]

Normalized eigenvalues: \( \mu_0 = 0, \mu_1 = 1.342, \mu_2 = 4.325, \mu_3 = 8.898, \mu_4 = 15.053, \ldots \)
Radial Modes

Plots on the left present response to a kick depending on z as basis functions $l_0=1$ (top) and $l_0=2$ (bottom). The response was convoluted with the same basis functions.

The eigenvalues $\lambda_m$ are coherent tuneshifts in units of maximum abs. value of SC tuneshift.

With $Q_{SC}/Q_s = 5$ the peak positions in units of $Q_s$ are $\nu_1 = 0.49, \nu_{-1} = -1.8, \nu_2 = 1.21, \nu_0 = -0.88, \nu_{-2} \approx -2.7$

An eigenfunction in long. phase space from $l=0$ peak. 21 basis functions in both z and p direction, $21 \times 21 = 441$ total long.
"Intrinsic" Landau Damping

To get some idea of the spectra let us look at the contribution from the same $l = l_0$.
The spectral density of such contributions for $l_0 = 2$ and $l_0 = 4$ are shown below for $Q_s = 0.2$.

Every eigenmode (except $\lambda_0 = 0$) includes all harmonics in long. phase space which are difficult to separate.

Simple projection on basis functions can be used for a rough estimate.

The peak positions coincide almost exactly with Alex’ result.
“Vanishing” TMCI

**History:**
- M. Blaskiewicz in PRSTAB 1, 044201 (1998) predicted complete suppression of TMCI in a wide range of $Q_{SC}/Q_s>1$.
- V. Balbekov in his papers basically confirmed this prediction for simple forms of wake function and RF bucket (transverse nonlinearity ignored)

![Graph showing behavior of TMCI threshold](image)

- A. Burov in PRSTAB 12, 044202 (2009) from qualitative considerations suggested the shown behavior of the TMCI threshold at large $Q_{SC}/Q_s$,
- later – based on some numerical simulations – he had withdrawn his prediction.

**The reason for controversy:**
- large number of eigenmodes should be taken into account with $Q_{SC}/Q_s>>1$ (slow convergence)
- transverse nonlinearity not properly taken into account
There is indeed suppression of TMCI by space charge, but not as drastic as the “rigid slice” approximation predicts.

Model:

\[ W1(z) = -cR/Q * \exp[-dec*z] * \sin[om*z]/\sqrt{1 - 1/(2Q)^2}, \]

\[ dec = om*\sigma_z/(2Qv); \]

\[ om = om*\sigma_z*Sqrt[1 - 1/(2Q)^2]/v; \]

with \( Q=1 \) and \( \lambda = \sigma_z \rightarrow \text{omega} * \sigma_z/v = 2\pi, \) \( dec = \pi, \) \( om = \pi*Sqrt[3] \)
TMCI with Resonator Wake

The TMCI threshold defined as \( \text{Im} \, \nu/Q_s > 0.1 \) vs the space charge tuneshift.

What this means in real life parameters? Assume:

- \( Q_s = 3.24 \times 10^{-3} \), \( Q_x = 26.13 \), \( E = 26 \text{ GeV} \),
- \( Q_{\text{SC}} = 0.16 = 50Q_s \rightarrow \)
- \( N_p = 2.4 \times 10^{11} \), \( Q_w/Q_s = 2 \) reached at \( R_s = 8 \text{ M\Omega/m}^2 \),
- \( Q_{\text{w}}/Q_s = 2 \) reached at \( R_s = 8 \text{ M\Omega/m}^2 \),

From Adrian’s simulations with \( R_s = 10 \text{ M\Omega/m}^2 \)

Most unstable mode (\( \text{Im} \, \nu/Q_s = 0.21 \)) dipole moment profile for \( Q_{\text{SC}}/Q_s = 50 \), \( Q_w/Q_s = 2 \).
Thank you for attention!