Overview of Event Generation

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CP3/CISM

Plan

- Overview of Monte-Carlo
- Details of the computation
    - Evaluation of matrix-element
    - Phase-Space integration

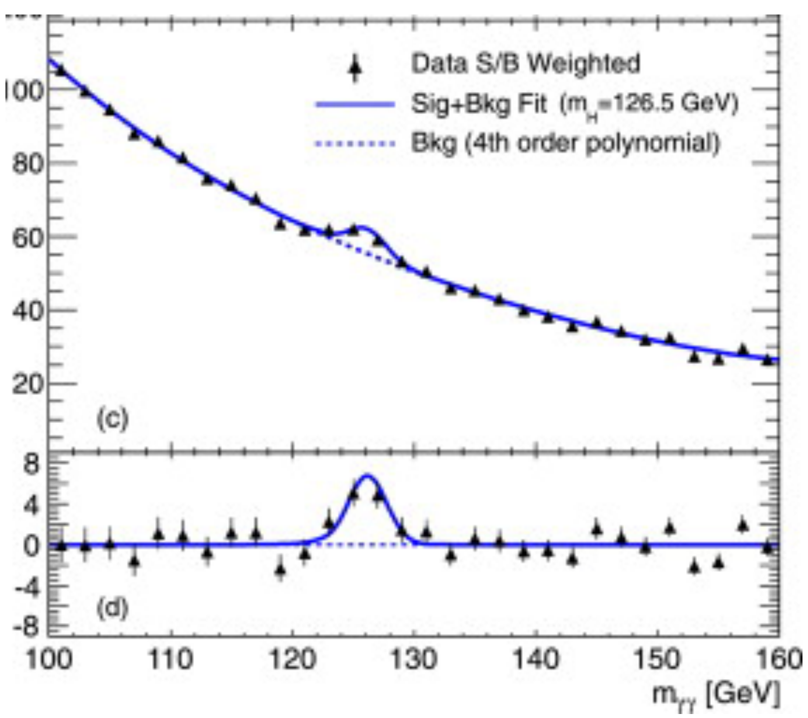
What are my goals

Title

- Overview of Monte-Carlo Field
  ➡ We split the computation by scales
- Justify why **analytic** computation are **SLOWER** than **numerical** computation
- Justify why **adding cuts** to the code are **POSSIBLE** but can lead to **PROBLEM**

Kind of measurement

Peak



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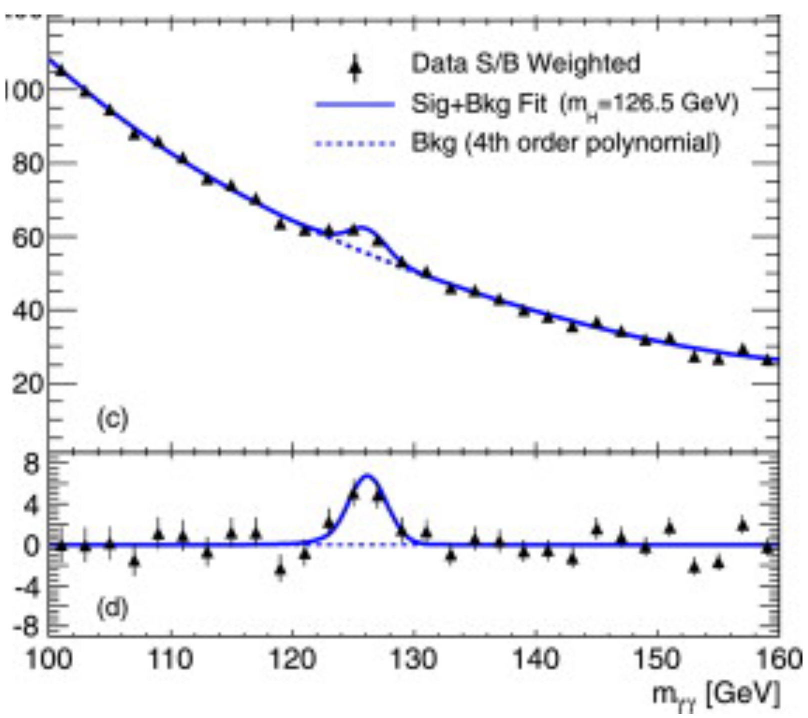

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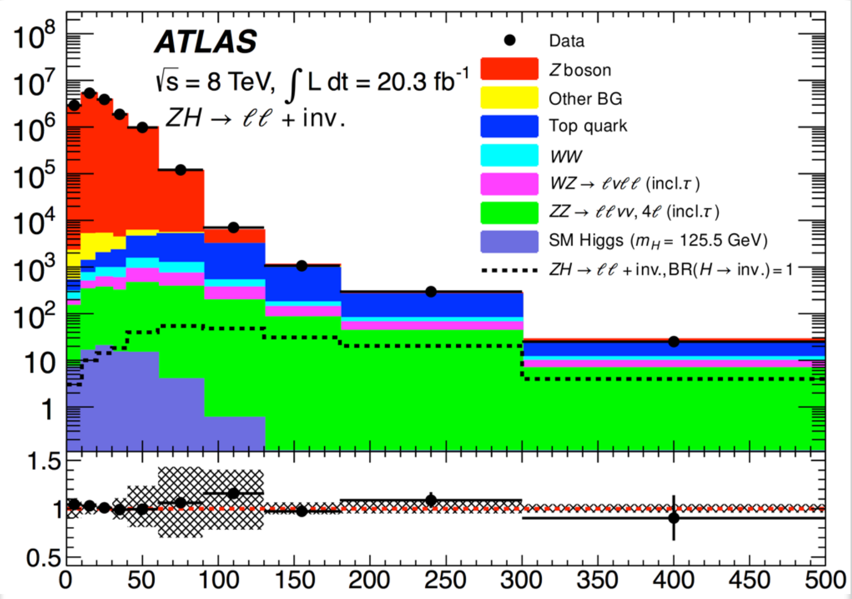

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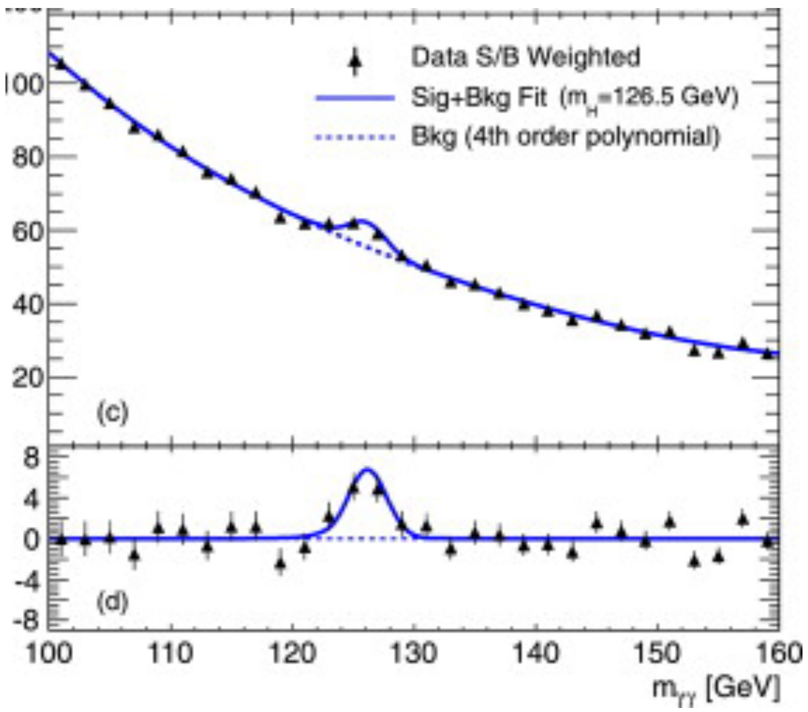

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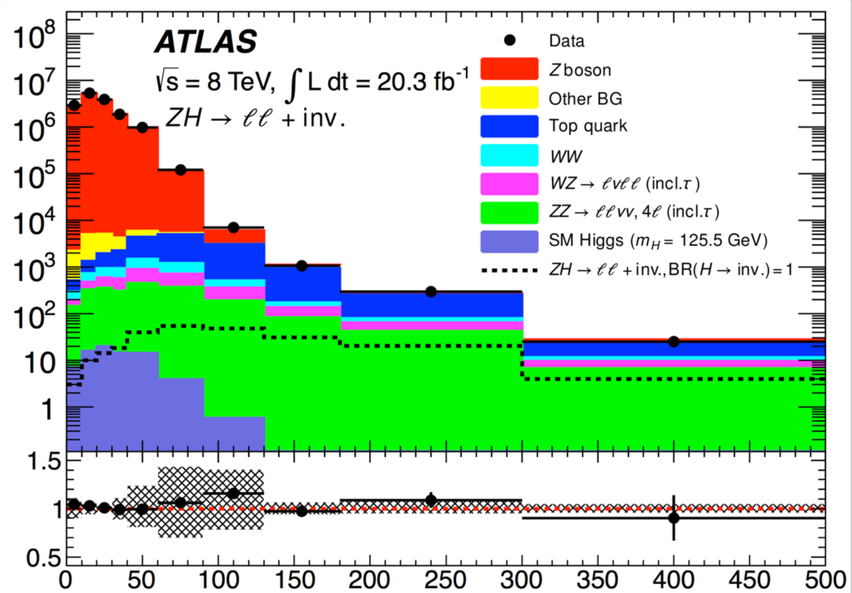

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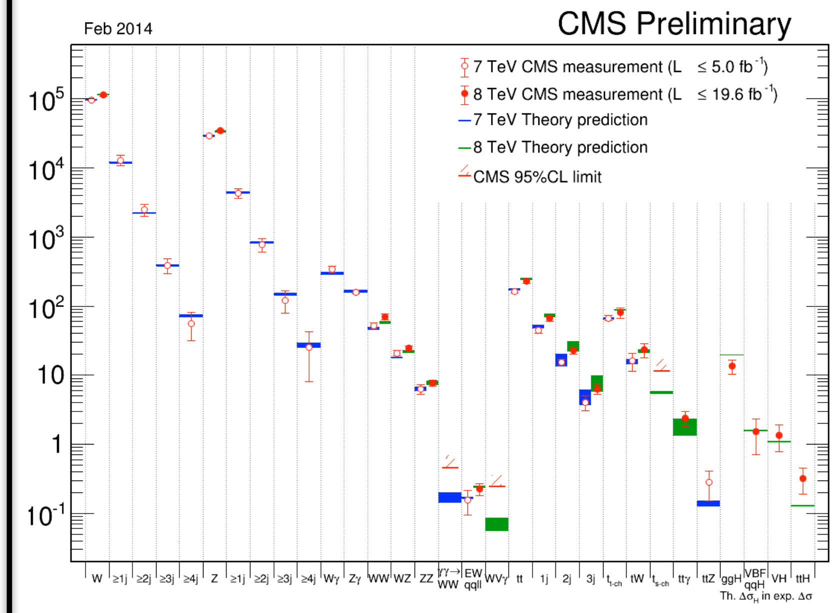

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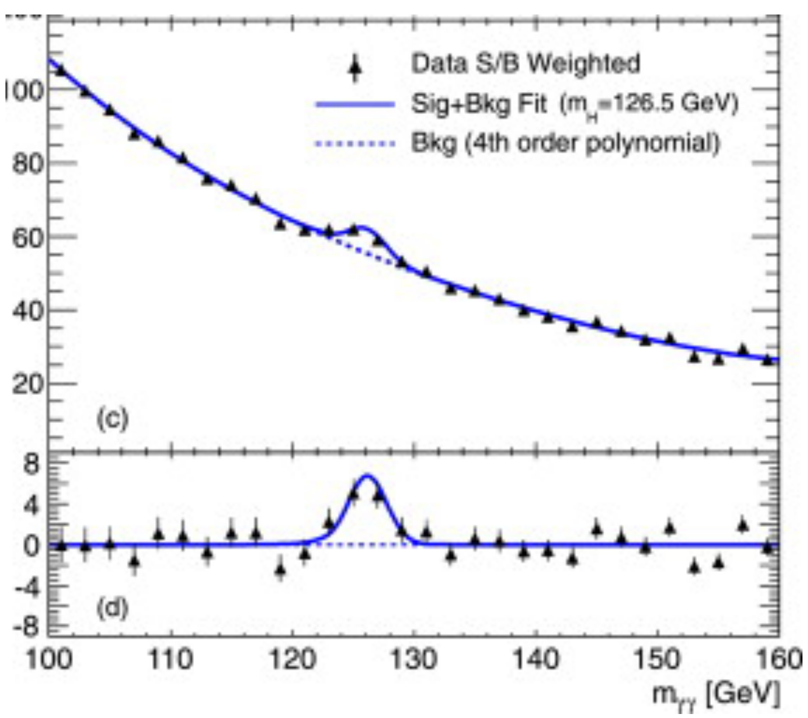
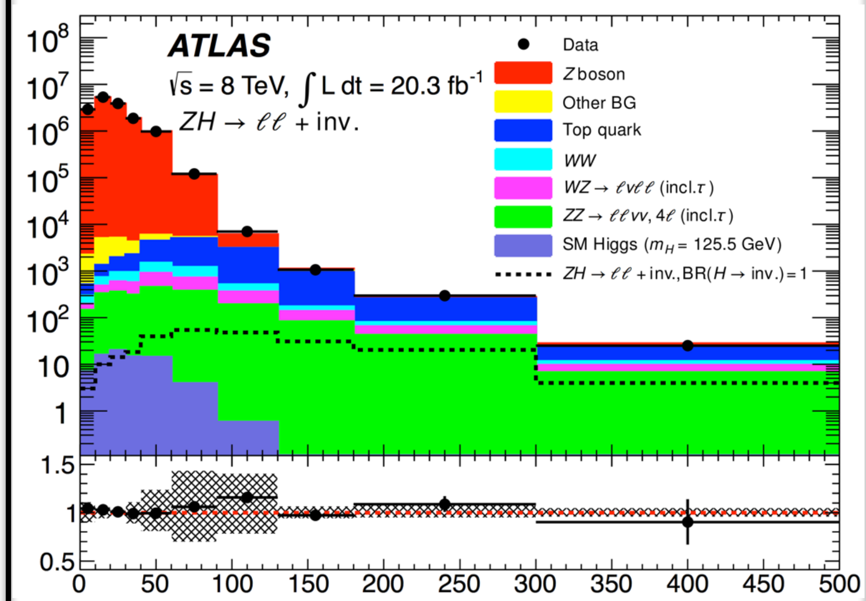


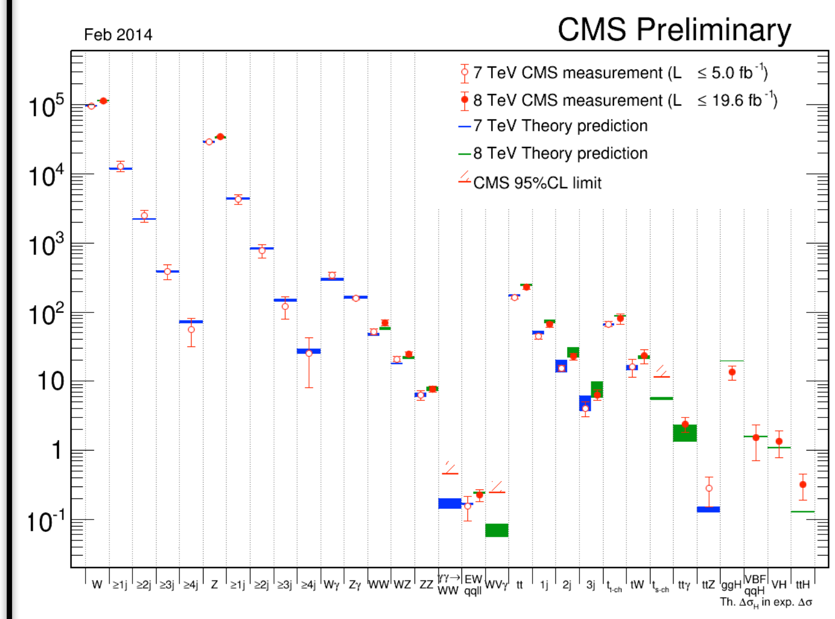
"EASY"

Shape



"HARD"

Rate



"VERY HARD"

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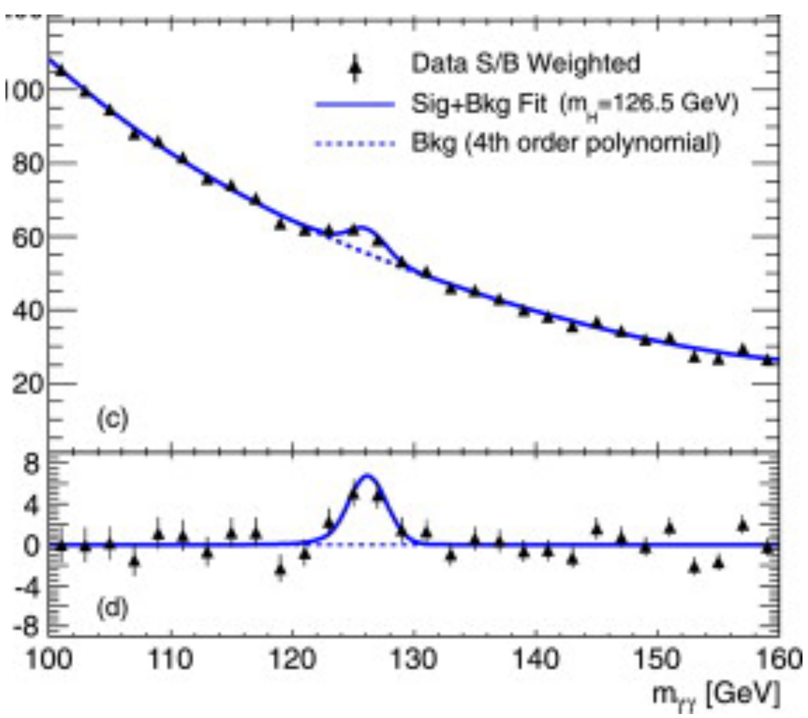
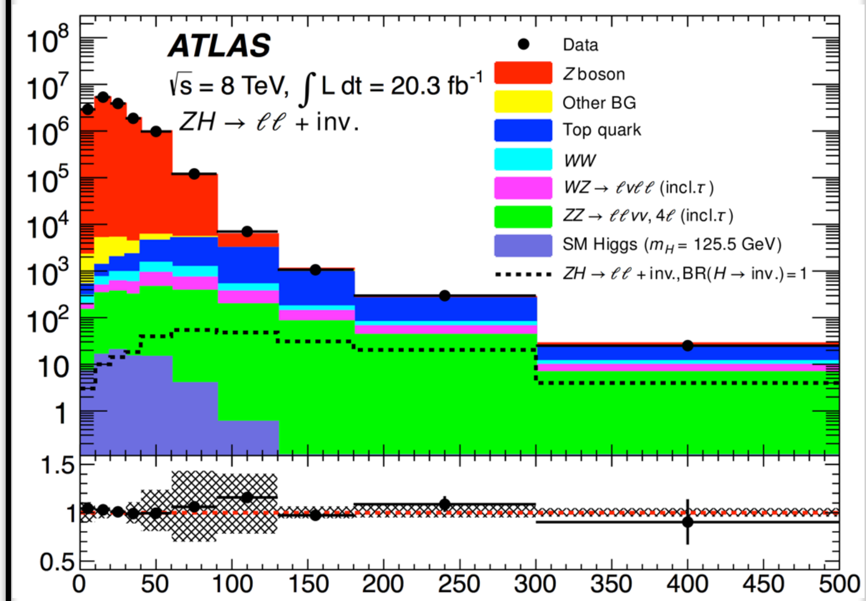


“EASY”

Background directly measured from **data**. Theory needed only for parameter extraction

Shape



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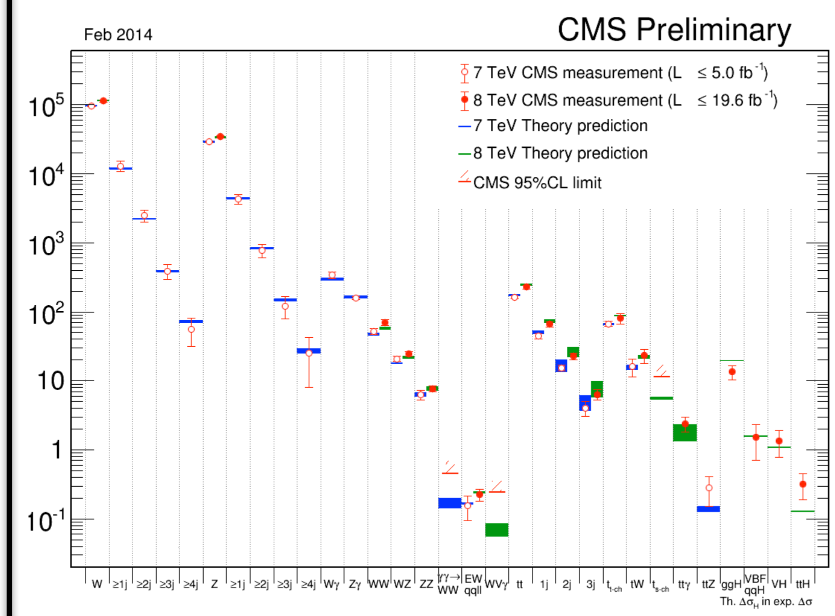

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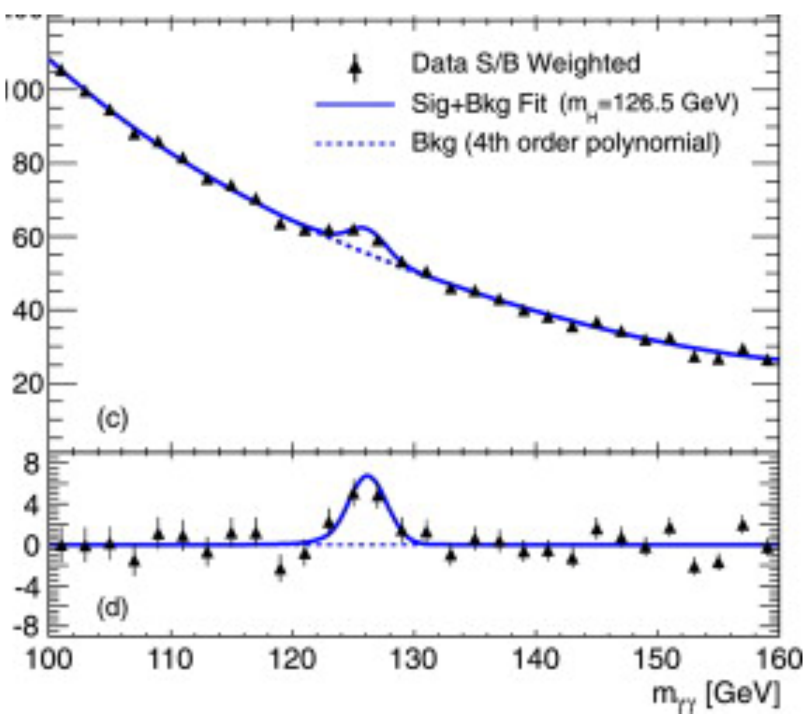

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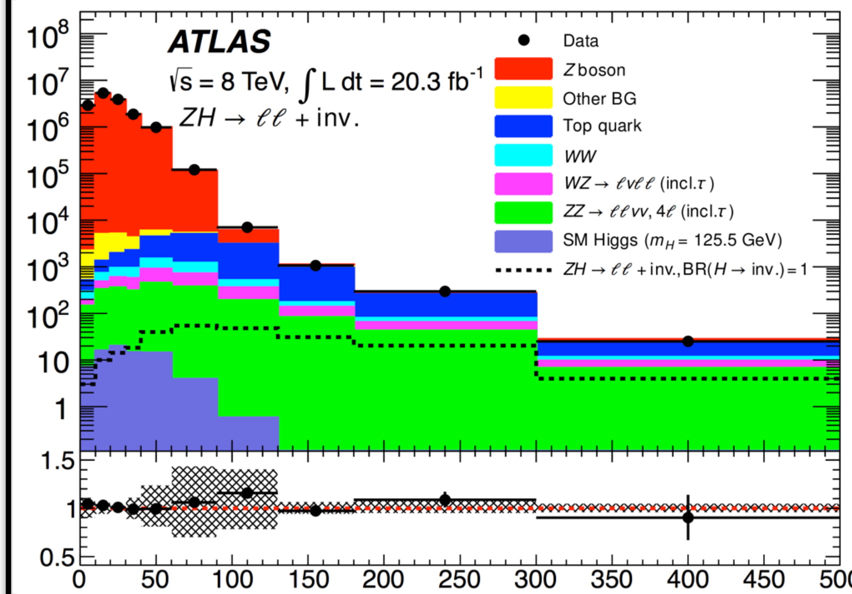
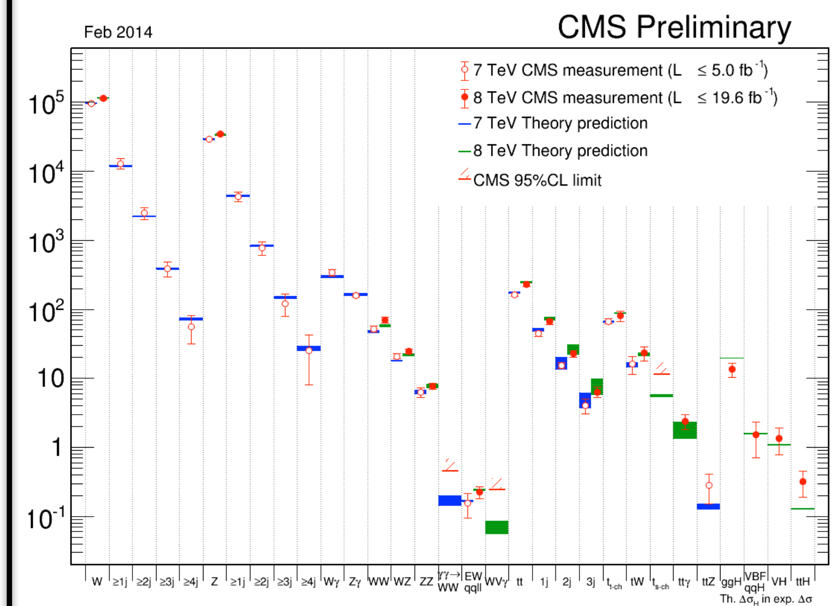


"HARD"

Background **SHAPE** needed. Flexible MC for both signal and background validated and tuned to data

Rate



"VERY HARD"

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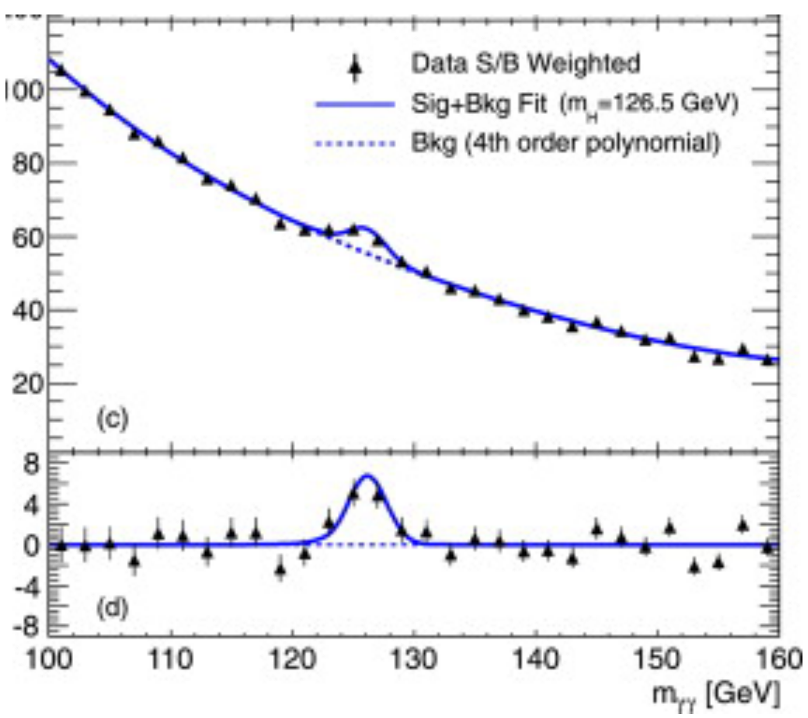

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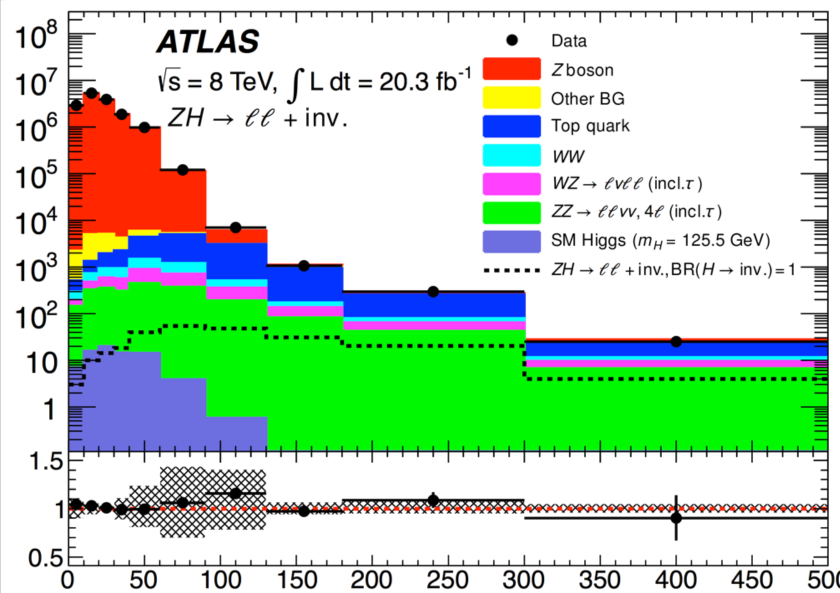

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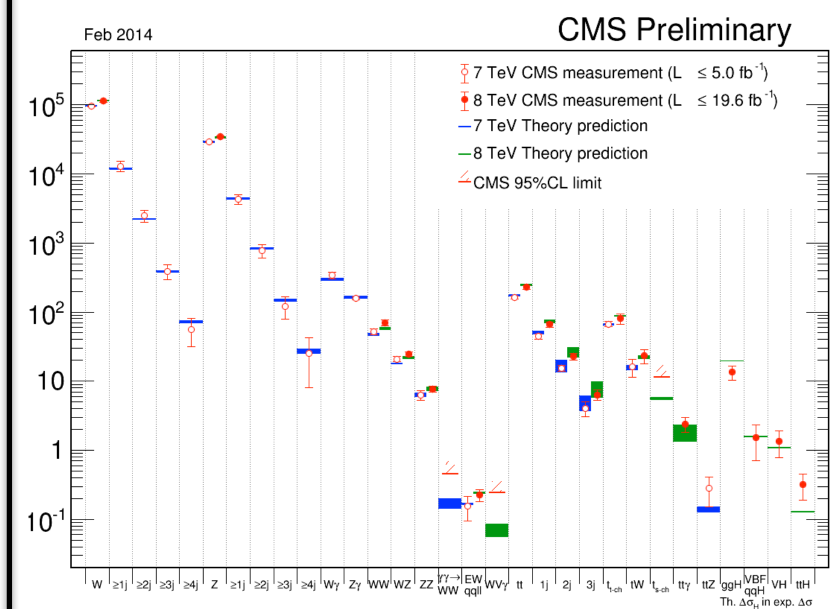
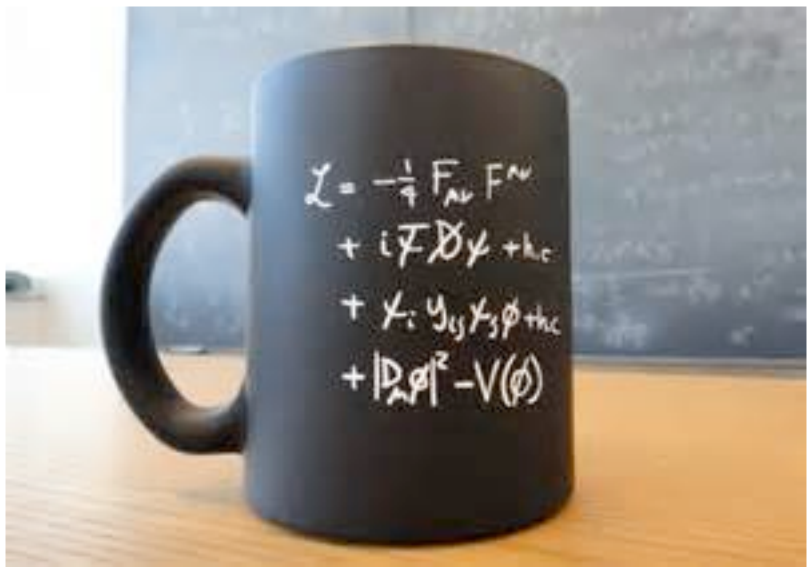


"VERY HARD"

Relies on prediction for both **shape** and **normalization**. Complicated interplay of best simulations and data

Theory side

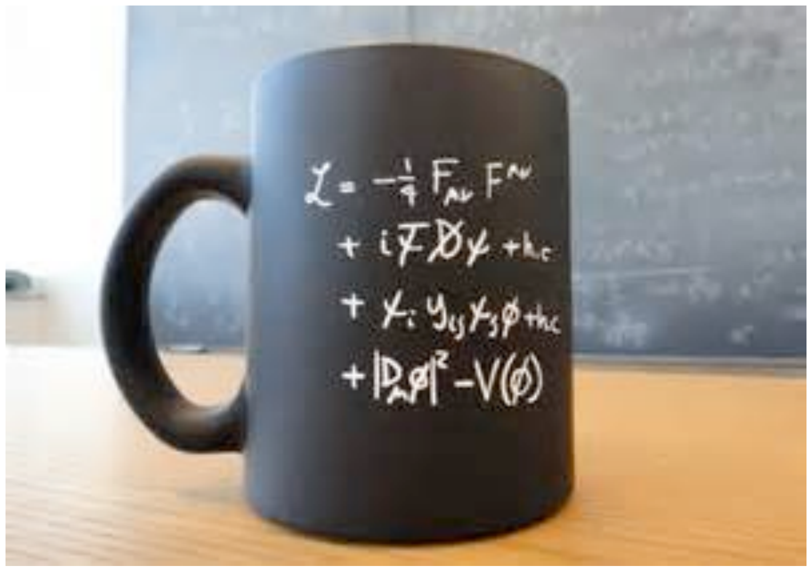
Lagrangian



- This is Where the new idea are expressed

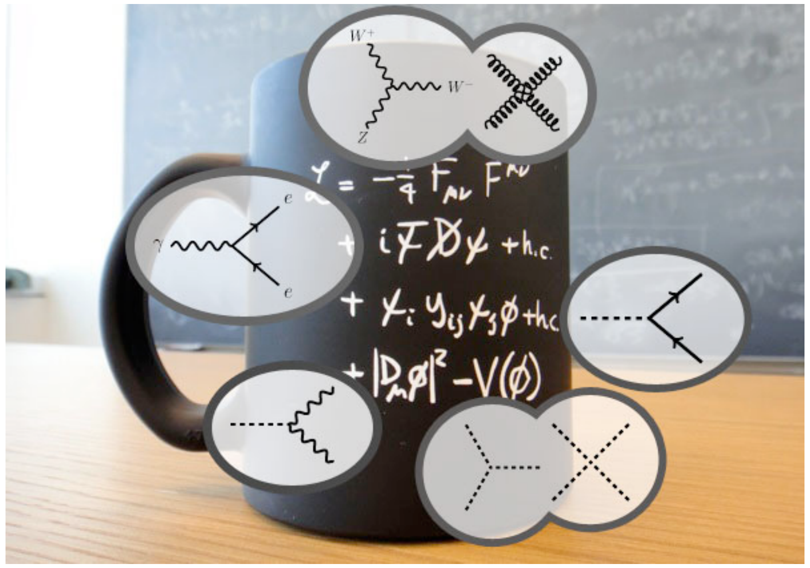
Theory side

Lagrangian



- This is Where the new idea are expressed

Feynman Rule

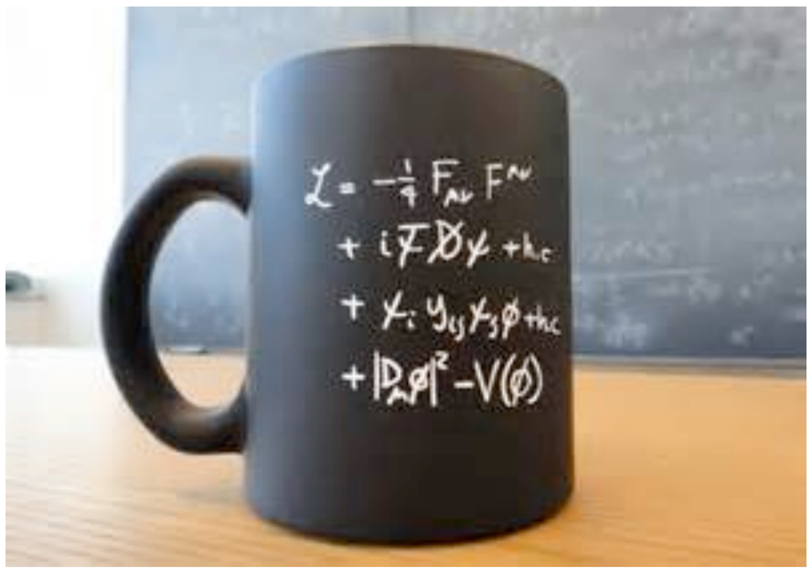


- Same information as the Lagrangian

FeynRules

Theory side

Lagrangian



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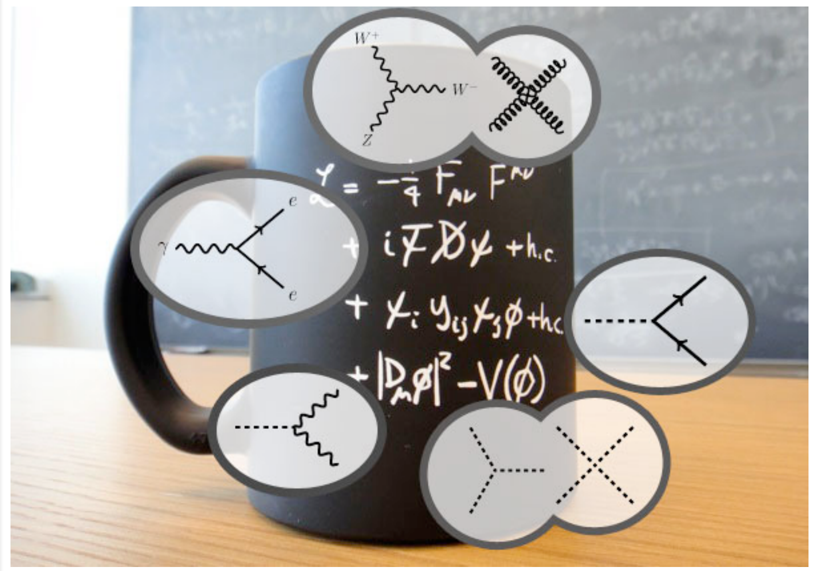
Feynman Rule

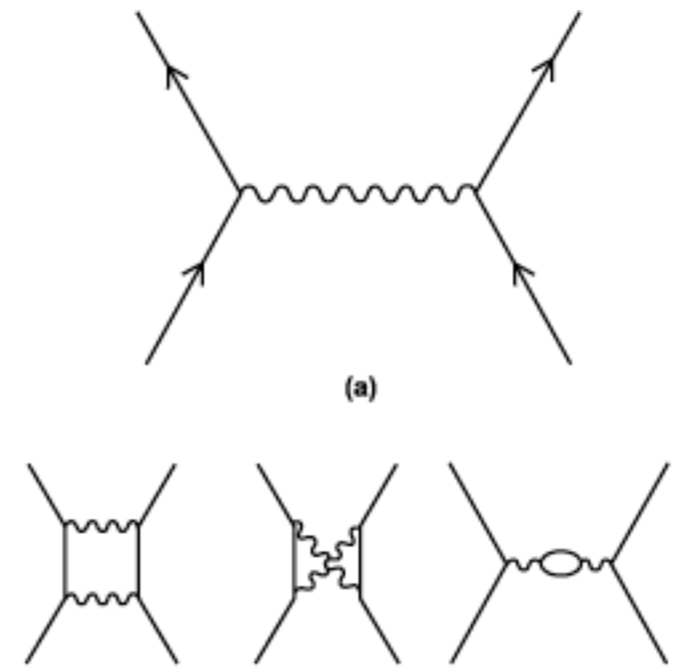


- Same information as the Lagrangian

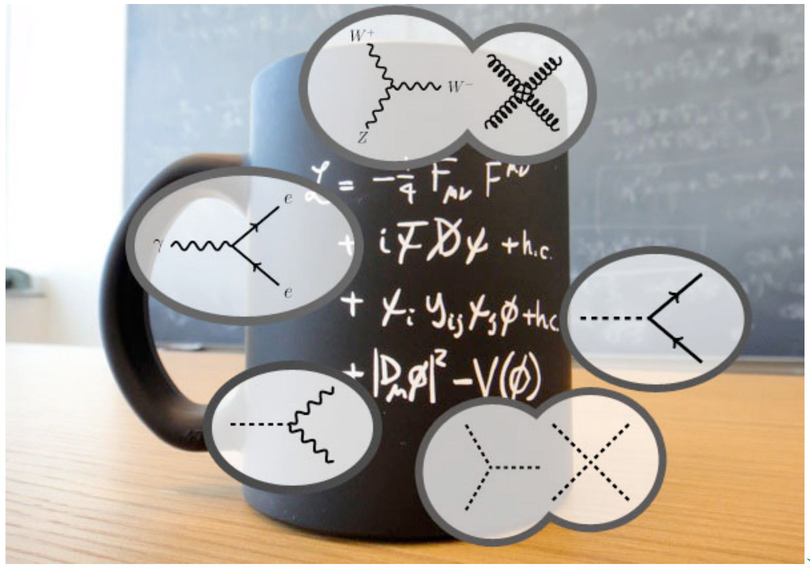
FeynRules

Cross-section

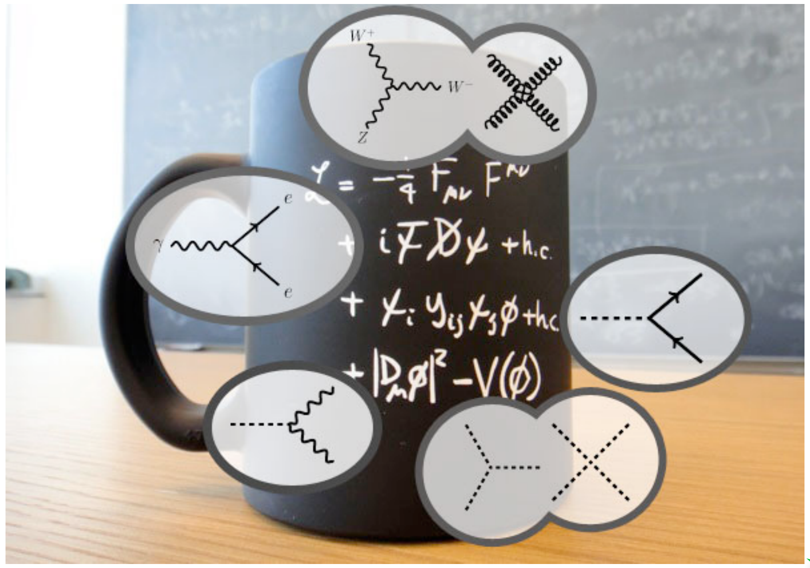
$$\frac{d\sigma}{d\ \cos\theta} = \left(\frac{d\sigma}{d\ \cos\theta}\right)_R \frac{(1+\cos\theta)/2}{\left[1+\frac{(1-\cos\theta)KE}{Mc^2}\right]}$$



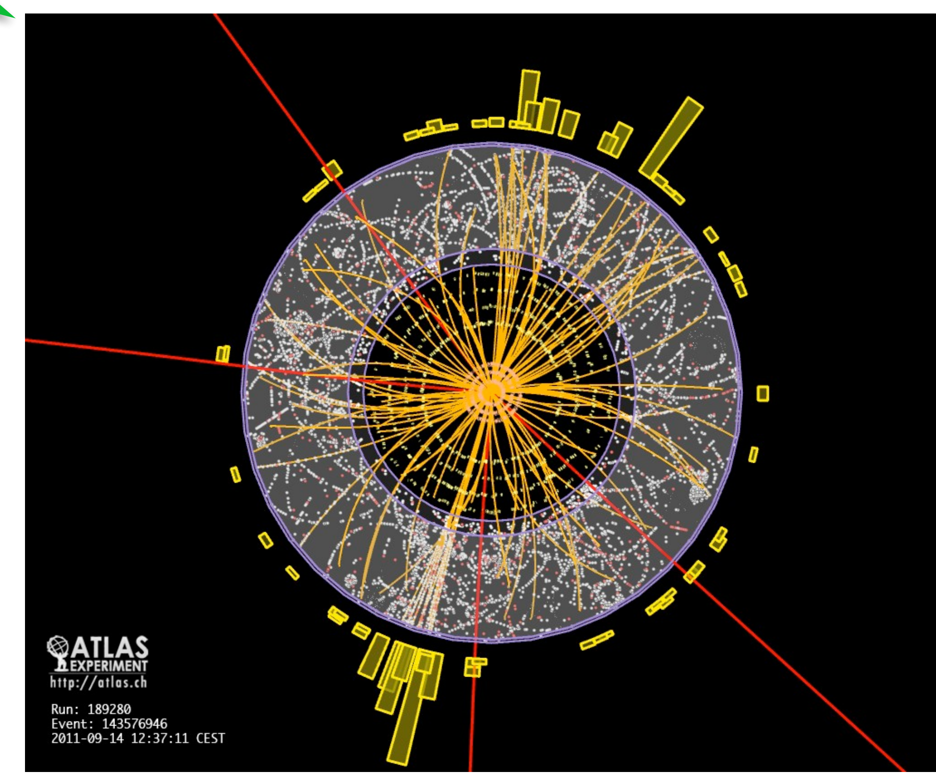
- What is the precision?



Monte-Carlo Physics

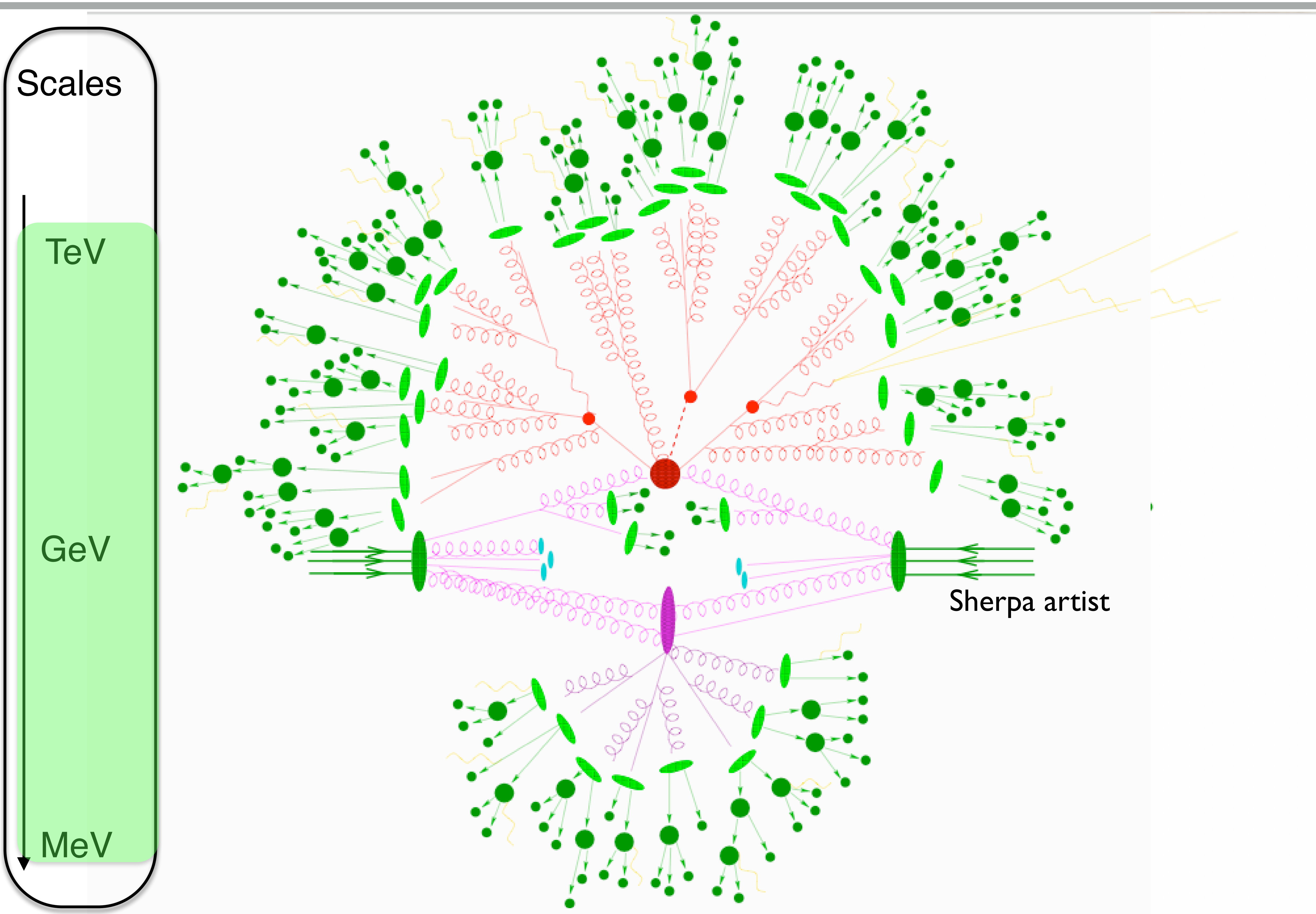


Monte-Carlo Physics



Simulation of collider events

What are the MC for?



What are the MC for?

Scales

TeV

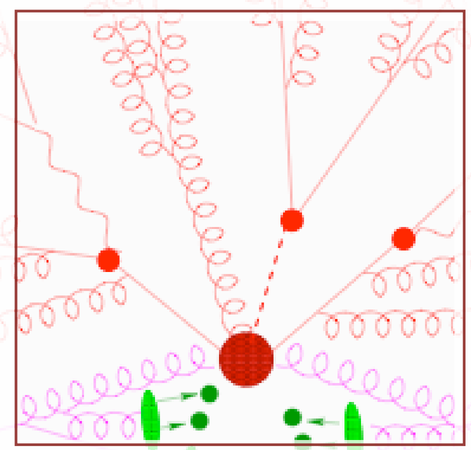
GeV

MeV

1. High-Q^2 Scattering

2. Parton Shower

☞ where BSM physics lies



☞ process dependent

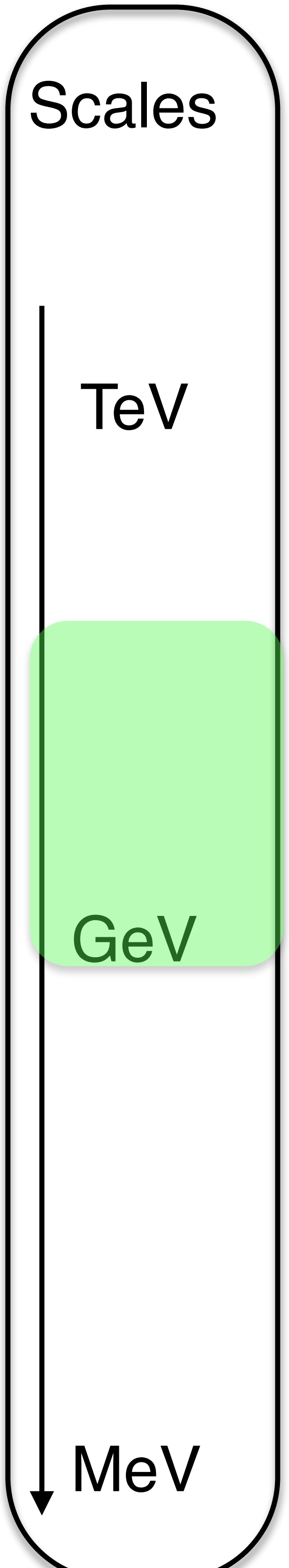
☞ first principles description

☞ it can be systematically improved

3. Hadronization

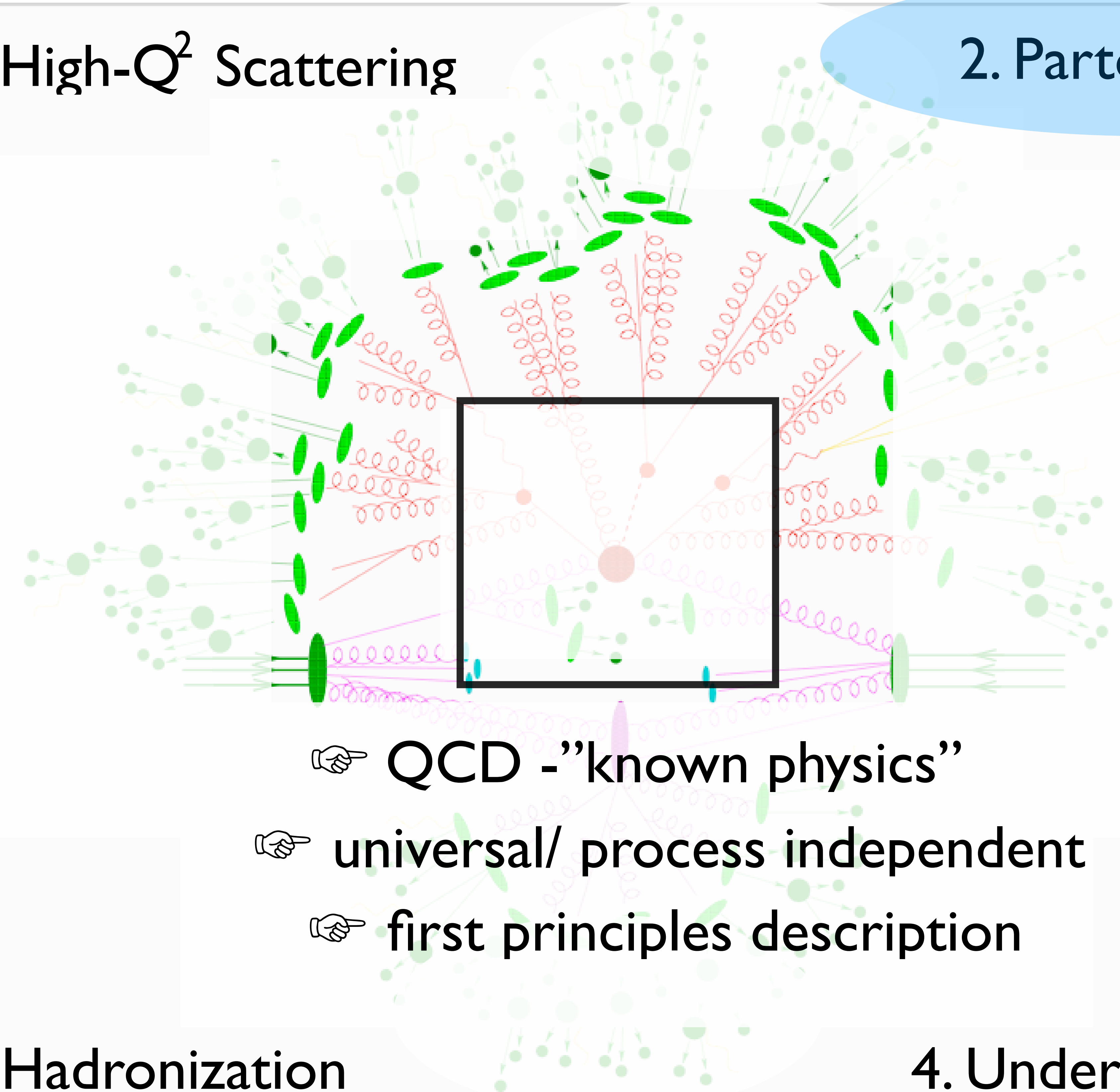
4. Underlying Event

What are the MC for?



1. High-Q^2 Scattering

2. Parton Shower



- QCD -"known physics"
- universal/ process independent
- first principles description

3. Hadronization

4. Underlying Event

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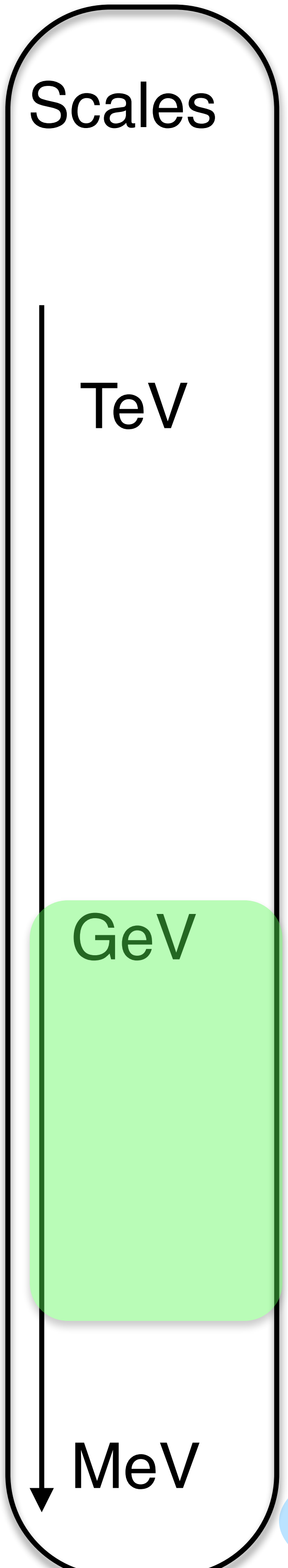
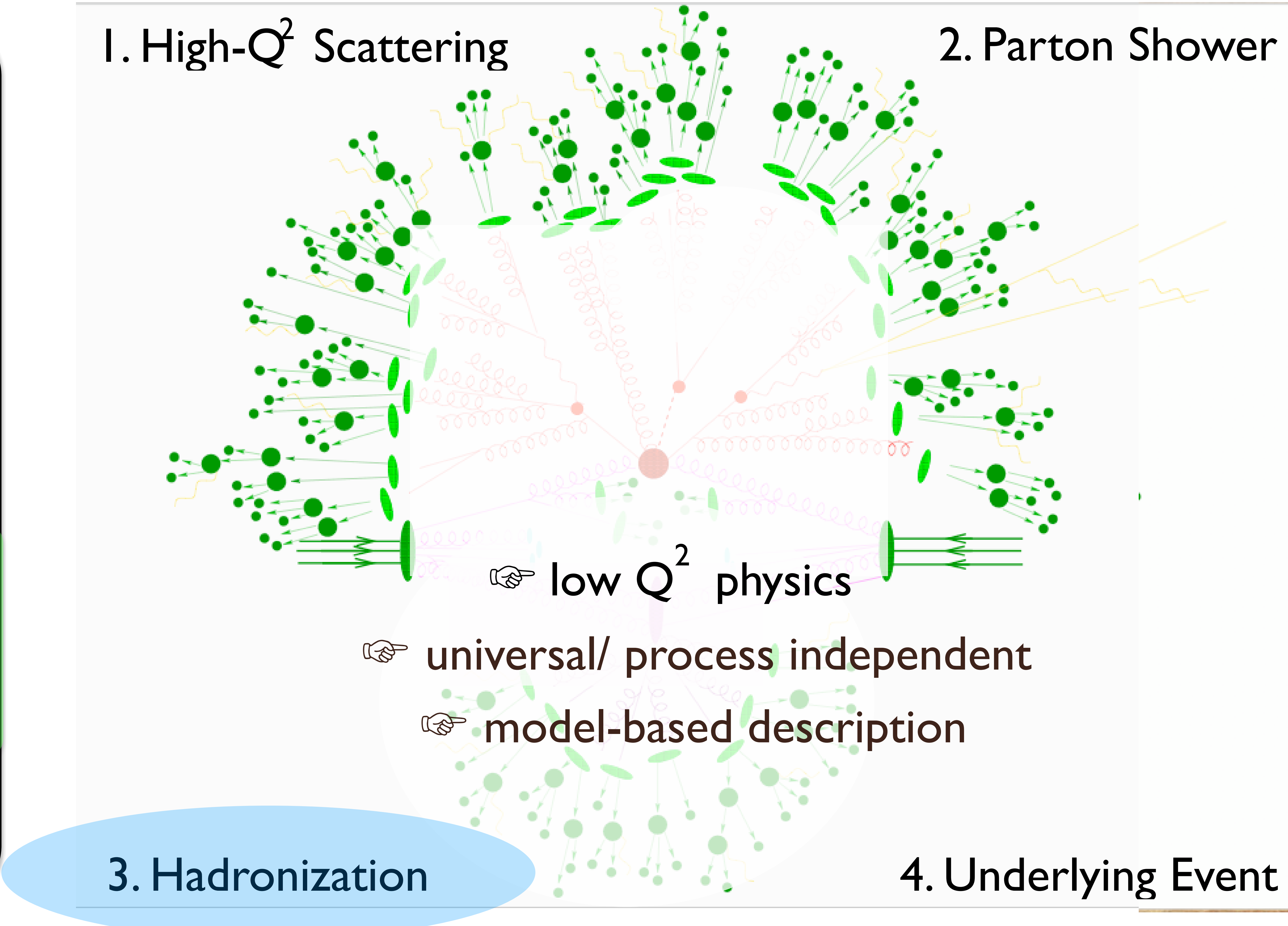
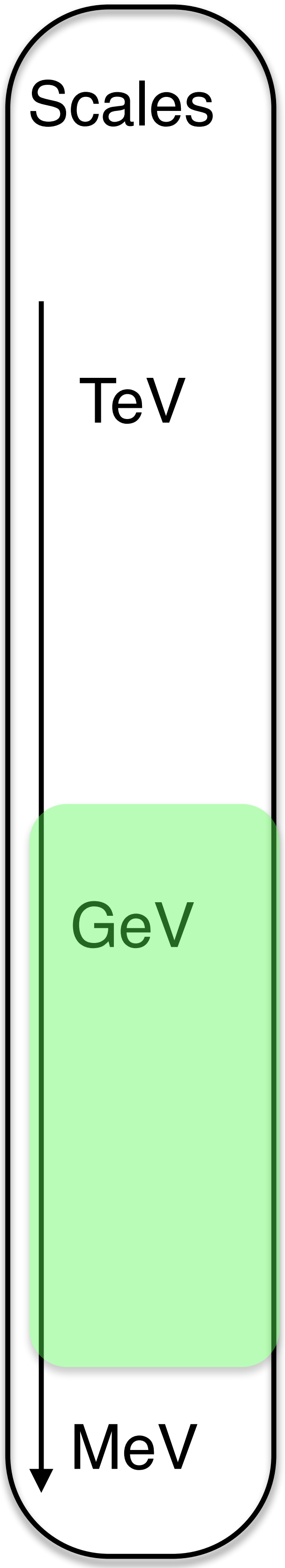
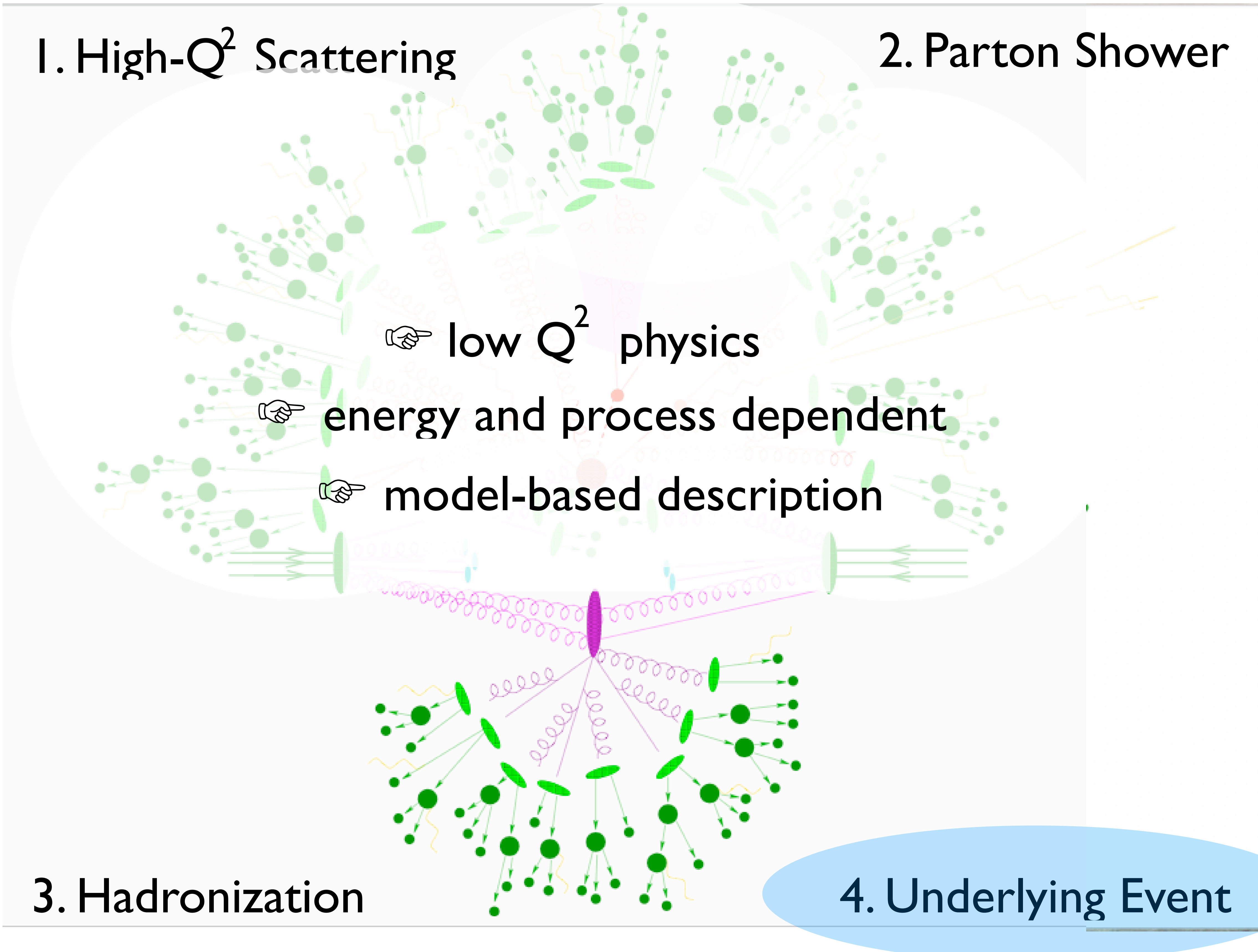




- ☞ low Q^2 physics
- ☞ universal/ process independent
- ☞ model-based description

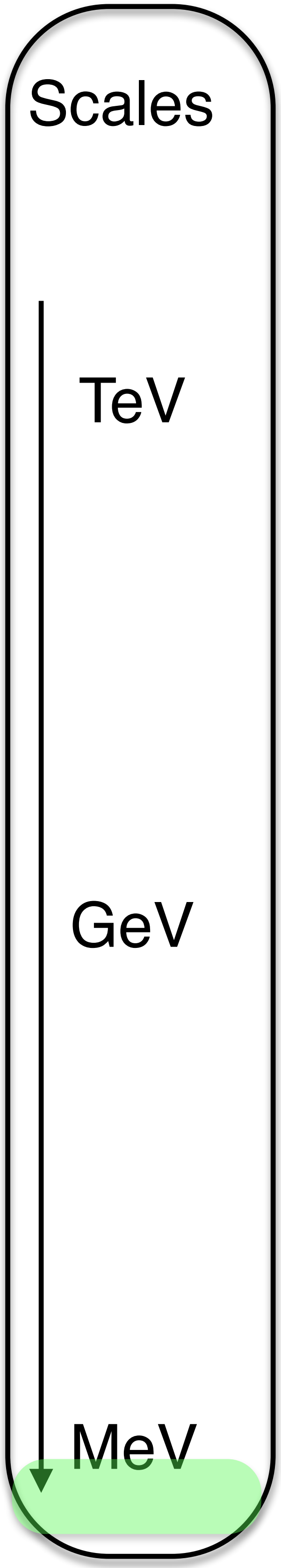
What are the MC for?

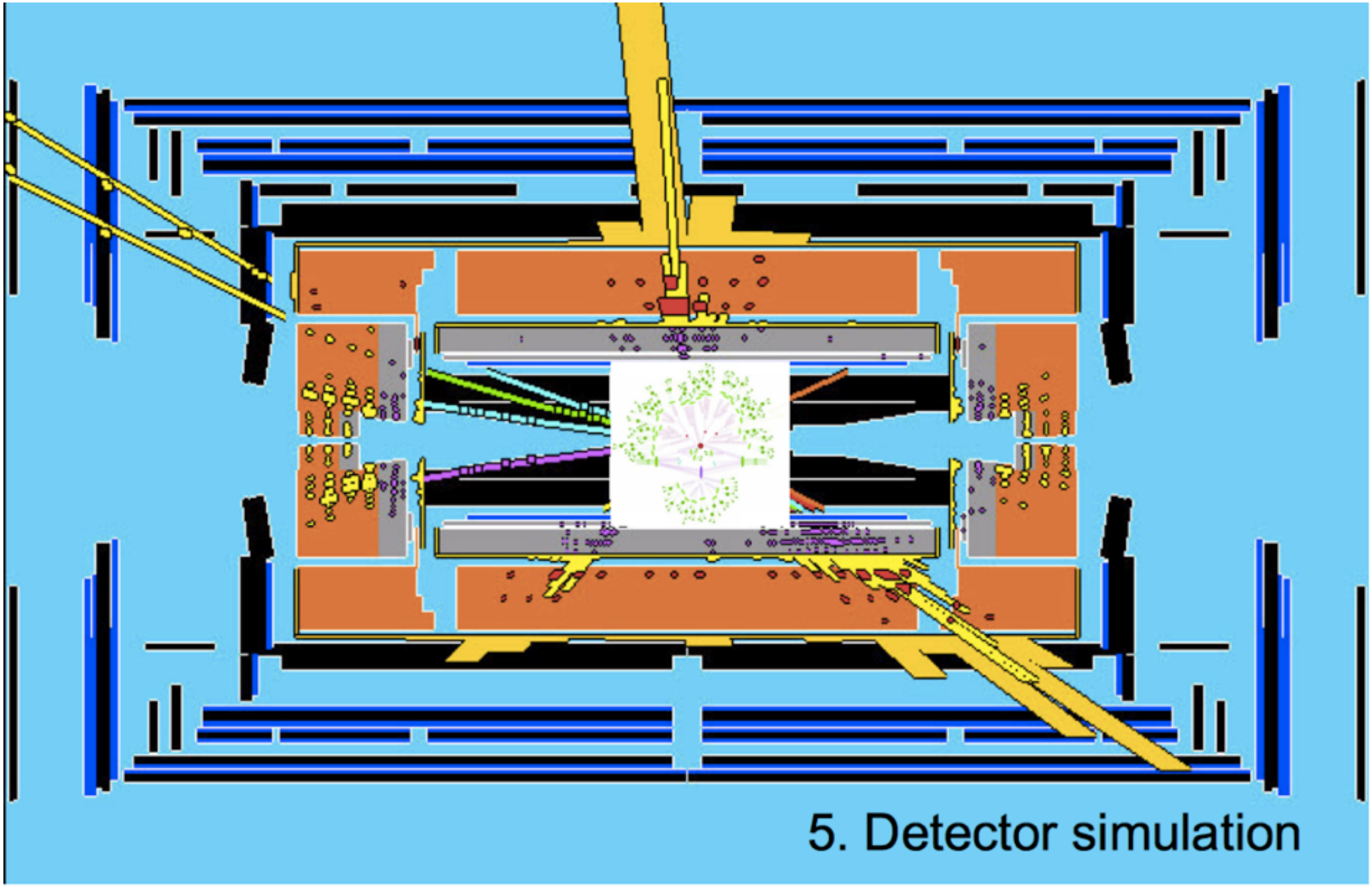




- ☞ low Q^2 physics
- ☞ energy and process dependent
- ☞ model-based description

What are the MC for?

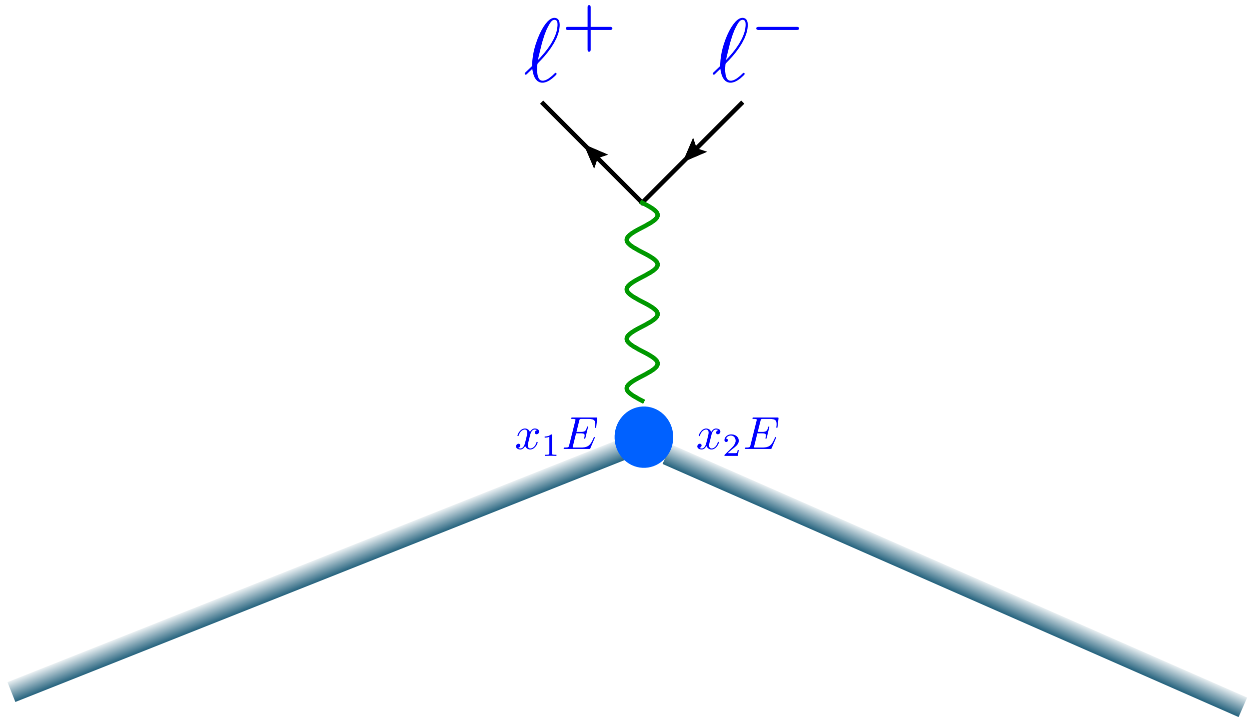




To Remember

- Multi-scale problem
  - ➡ New physics visible only at High scale
  - ➡ Problem split in different scale

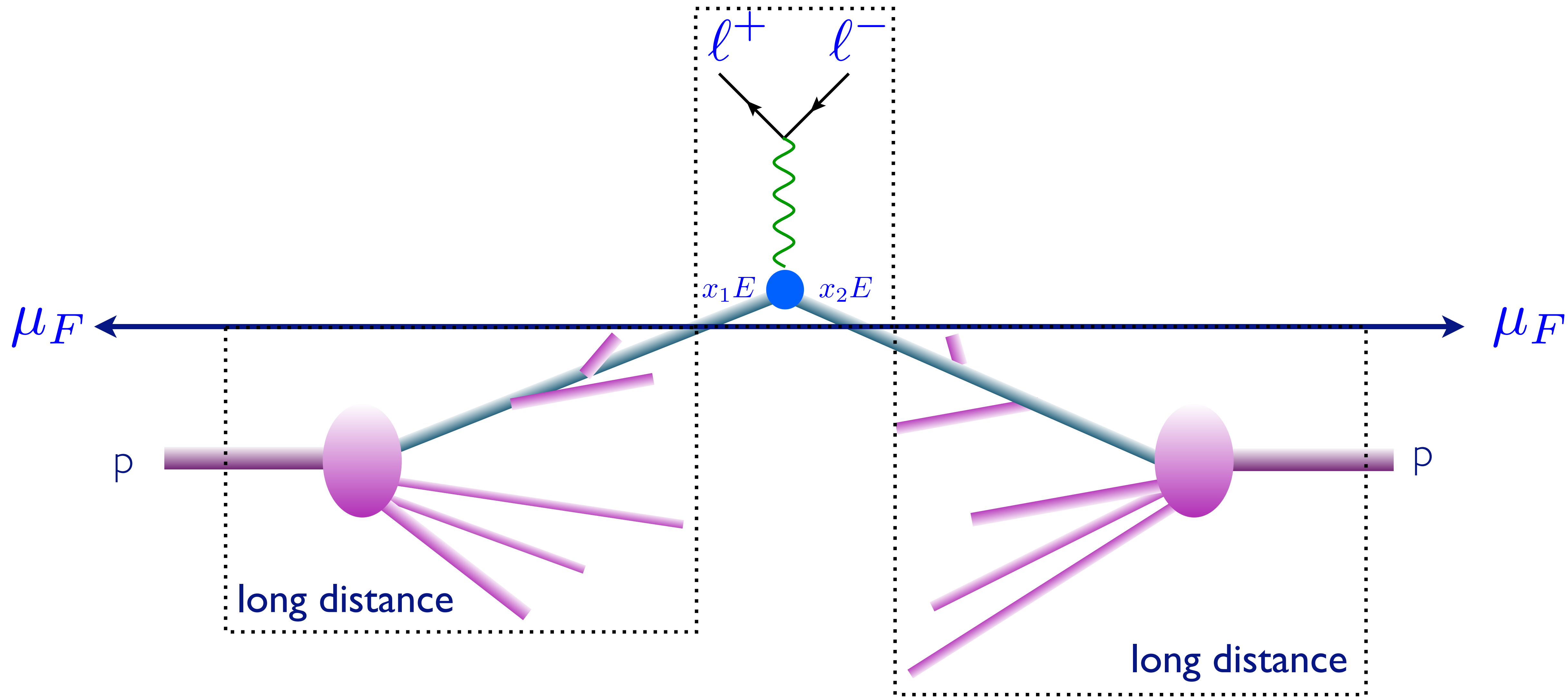
Master formula for the LHC



$$\hat{\sigma}_{ab \to X}(\hat{s}, \mu_F, \mu_R)$$

Parton-level cross section

Master formula for the LHC

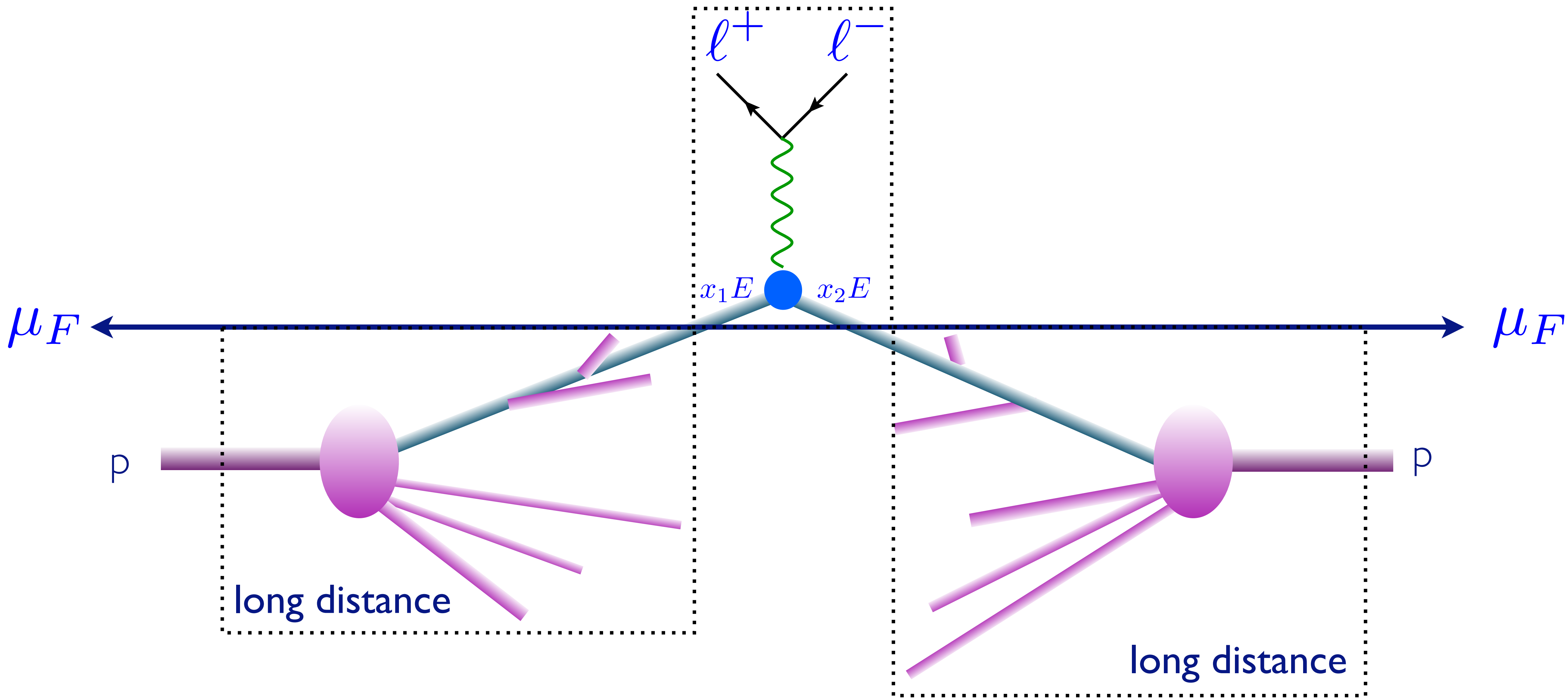


$$f_a(x_1, \mu_F) f_b(x_2, \mu_F) \, \hat{\sigma}_{ab \to X}(\hat{s}, \mu_F, \mu_R)$$

Parton density functions

Parton-level cross section

Master formula for the LHC



$$\sum_{a,b} \int dx_1 dx_2 d\Phi_{\rm FS}\, f_a(x_1,\mu_F) f_b(x_2,\mu_F)\, \hat{\sigma}_{ab\to X}(\hat{s},\mu_F,\mu_R)$$

Phase-space integral — Parton density functions — Parton-level cross section

Perturbative expansion

$d\hat{\sigma}_{ab\to X}(\hat{s}, \mu_F, \mu_R)$ **Parton-level cross section**

- The parton-level cross section can be computed as a series in perturbation theory, using the coupling constant as an expansion parameter, schematically:

$$\hat{\sigma} = \sigma^{\text{Born}}\left(1 + \frac{\alpha_s}{2\pi}\sigma^{(1)} + \left(\frac{\alpha_s}{2\pi}\right)^2\sigma^{(2)} + \left(\frac{\alpha_s}{2\pi}\right)^3\sigma^{(3)} + \dots\right)$$

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LO predictions

Perturbative expansion

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LO predictions

NLO corrections

Perturbative expansion

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LO predictions | NLO corrections | NNLO corrections | N3LO or NNNLO corrections

Perturbative expansion

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LO predictions | NLO corrections | NNLO corrections | N3LO or NNNLO corrections

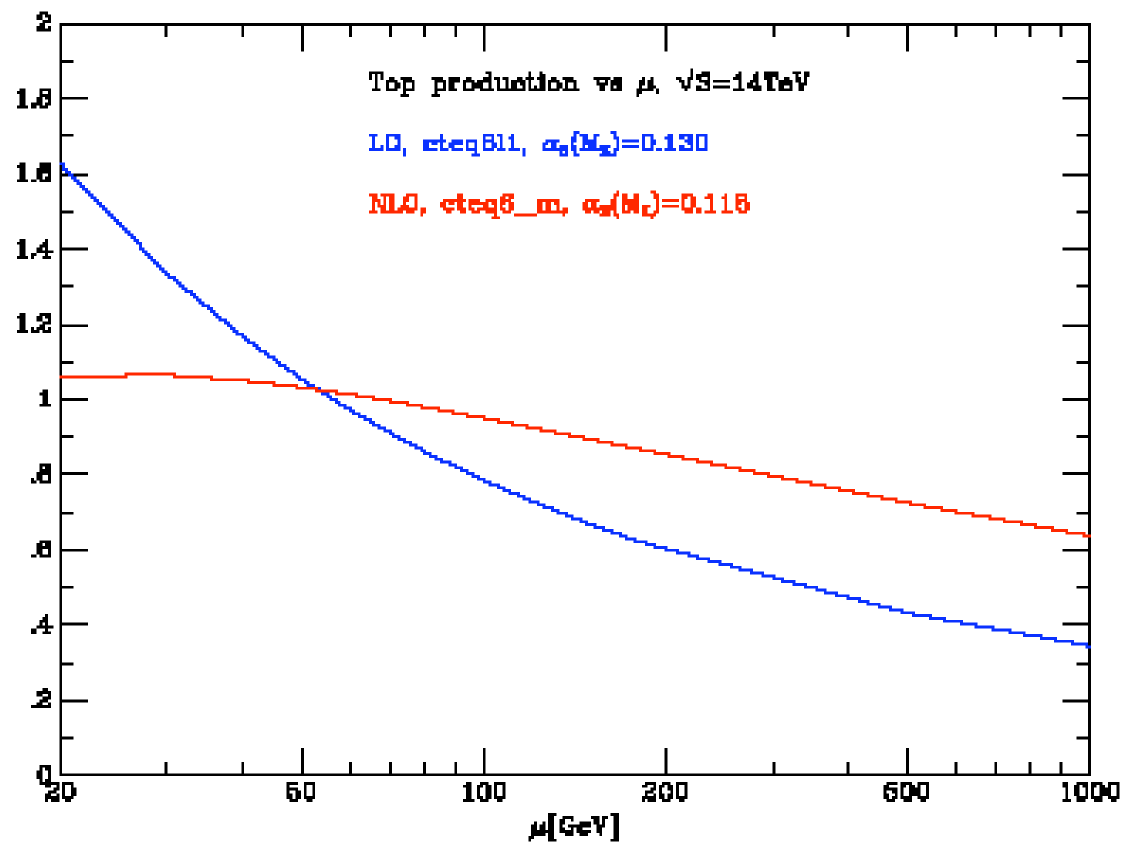
- Including higher corrections improves predictions and reduces theoretical uncertainties

Improved predictions

$$d\sigma = \sum_{a,b} \int dx_1 dx_2 \;\; f_a(x_1, \mu_F) f_b(x_2, \mu_F) \, d\hat{\sigma}_{ab\to X}(\hat{s}, \mu_F, \mu_R)$$

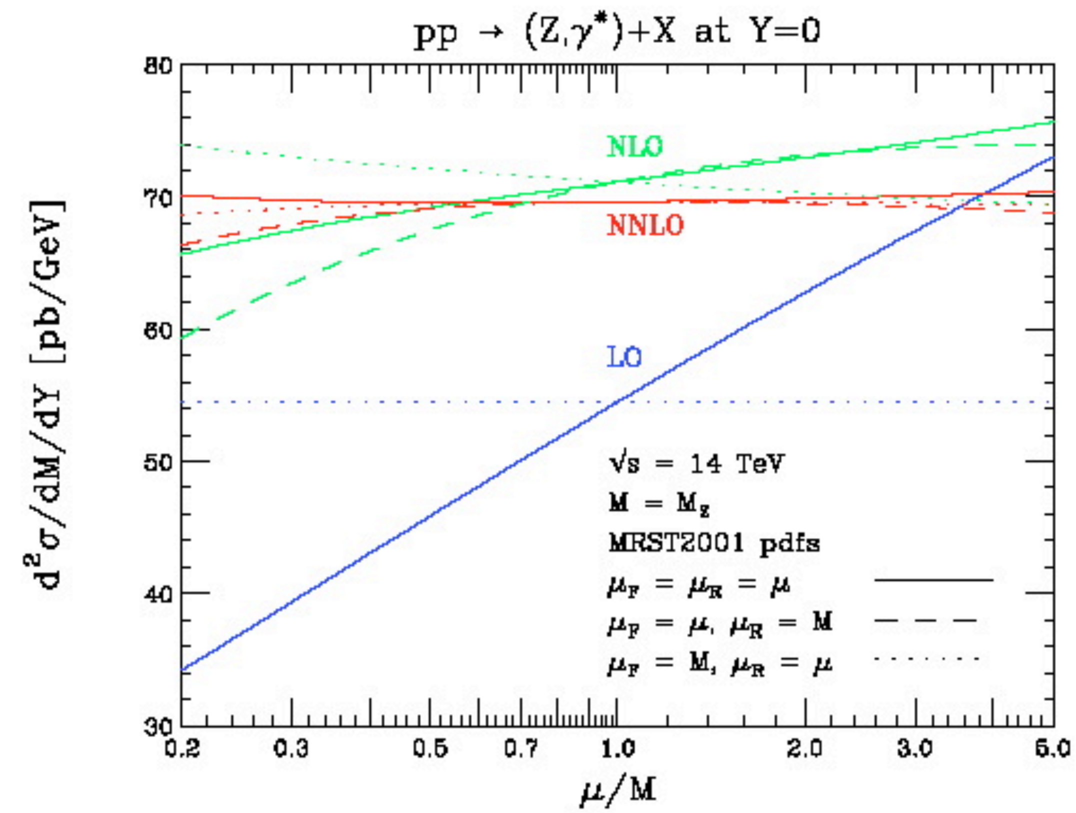
$$\hat{\sigma} = \sigma^{\text{Born}} \left(1 + \frac{\alpha_s}{2\pi} \sigma^{(1)} + \left(\frac{\alpha_s}{2\pi}\right)^2 \sigma^{(2)} + \left(\frac{\alpha_s}{2\pi}\right)^3 \sigma^{(3)} + \dots \right)$$

- Leading Order predictions can depend strongly on the renormalization and factorization scales
- Including higher order corrections reduces the dependence on these scales



Going NNLO...?

- **NNLO is the current state-of-the-art. There are only a few results available: Higgs, Drell-Yan, ttbar**
- **Why do we need it?**
  - control of the uncertainties in a calculation
  - It is "mandatory" if NLO corrections are very large to check the behavior of the perturbative series
  - It is needed for Standard Candles and very precise tests of perturbation theory, exploiting all the available information, e.g. for determining NNLO PDF sets



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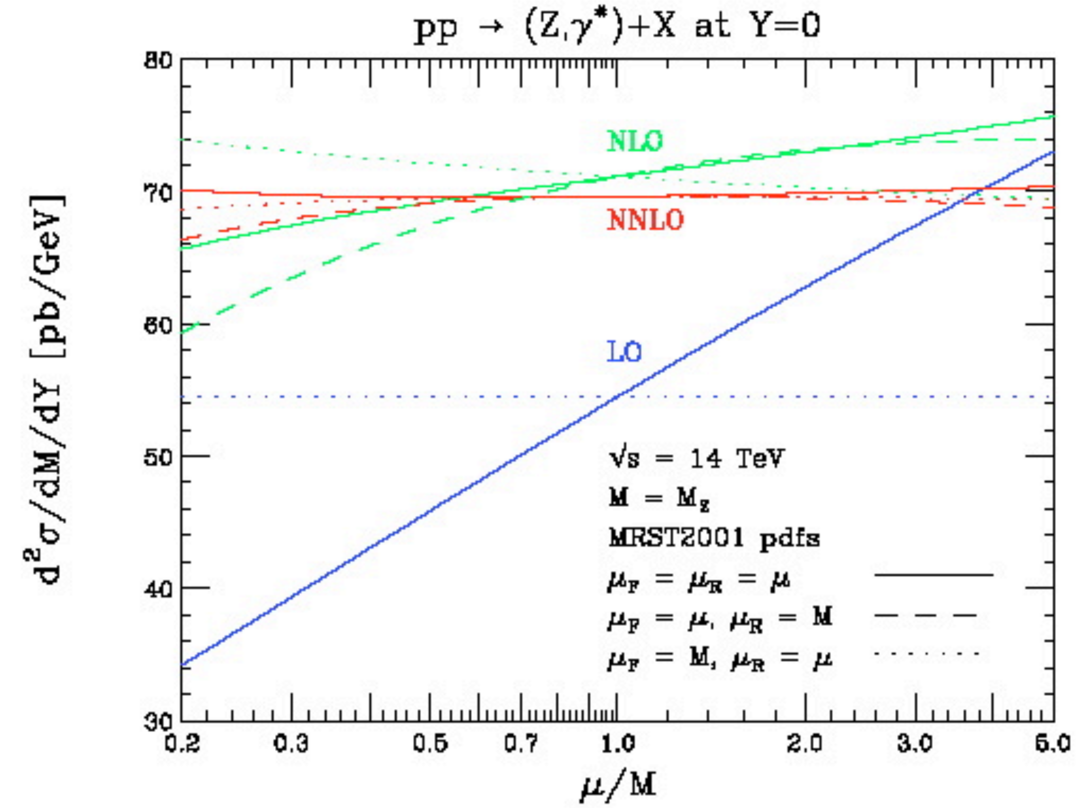
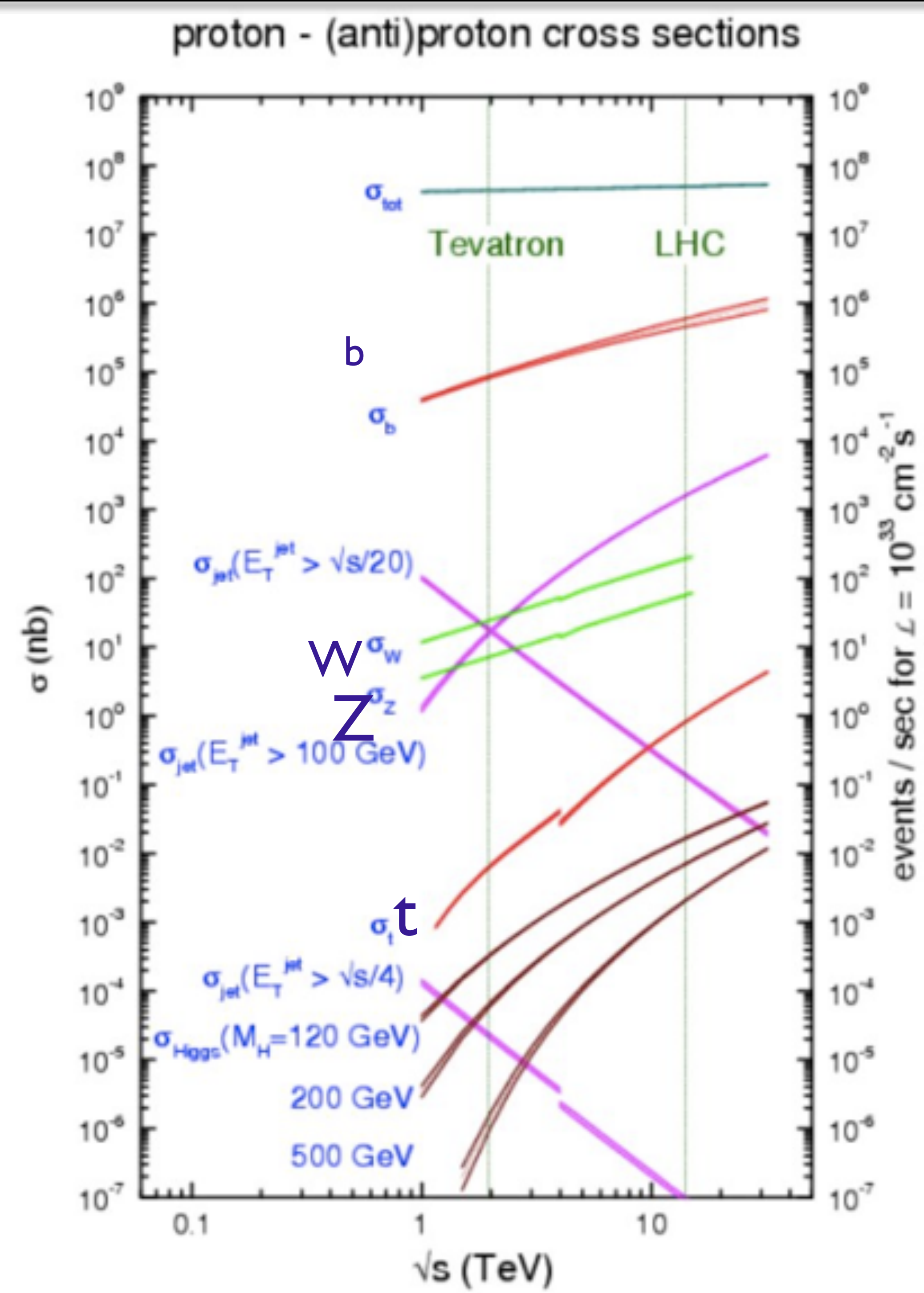


Let's focus on LO

Hadron Colliders

$$\sum_{a,b} \int dx_1 dx_2 d\Phi_{\rm FS}\, f_a(x_1,\mu_F) f_b(x_2,\mu_F)\, \hat{\sigma}_{ab\to X}(\hat{s},\mu_F,\mu_R)$$



To Remember

$$\sum_{a,b} \int dx_1 dx_2 d\Phi_{\text{FS}}\, f_a(x_1, \mu_F) f_b(x_2, \mu_F)\, \hat{\sigma}_{ab\to X}(\hat{s}, \mu_F, \mu_R)$$

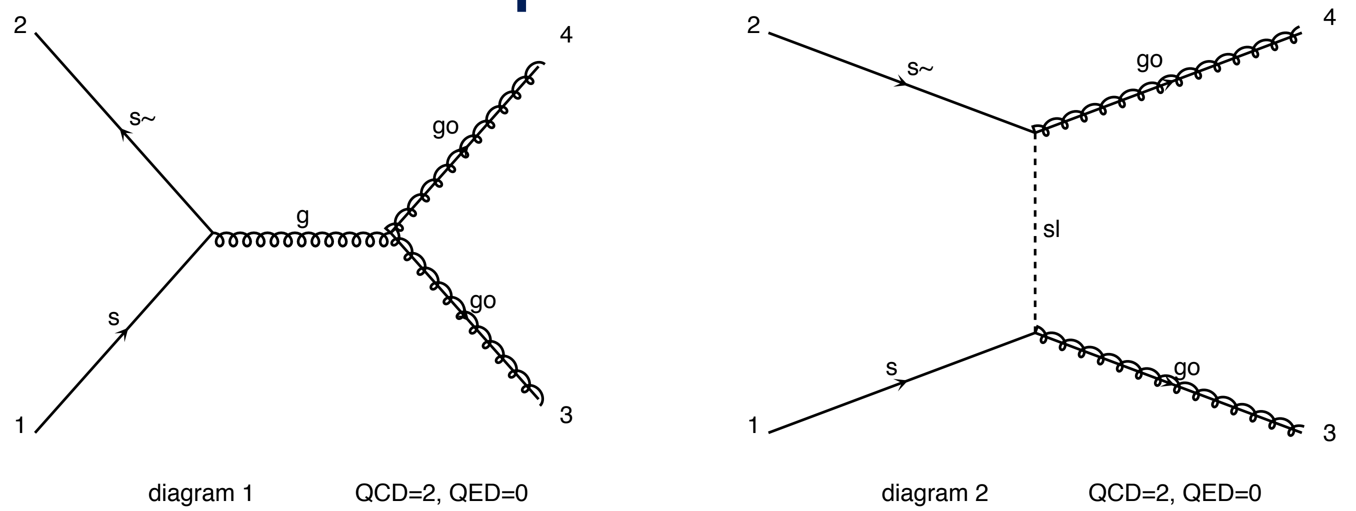
Phase-space integral — Parton density functions — Parton-level cross section

- PDF: content of the proton
  - ➡ Define the physics/processes that will dominate on your accelerator
- NLO/NNLO: Reduce scale uncertainty linked to your division of your multi-scale problem

Matrix-Element

Calculate a given process (e.g. gluino pair)

- Determine the production mechanism



- Evaluate the matrix-element

$$|\mathcal{M}|^2$$

- Phase-Space Integration

$$\sigma = \frac{1}{2s} \int |\mathcal{M}|^2 d\Phi(n)$$

Matrix-Element

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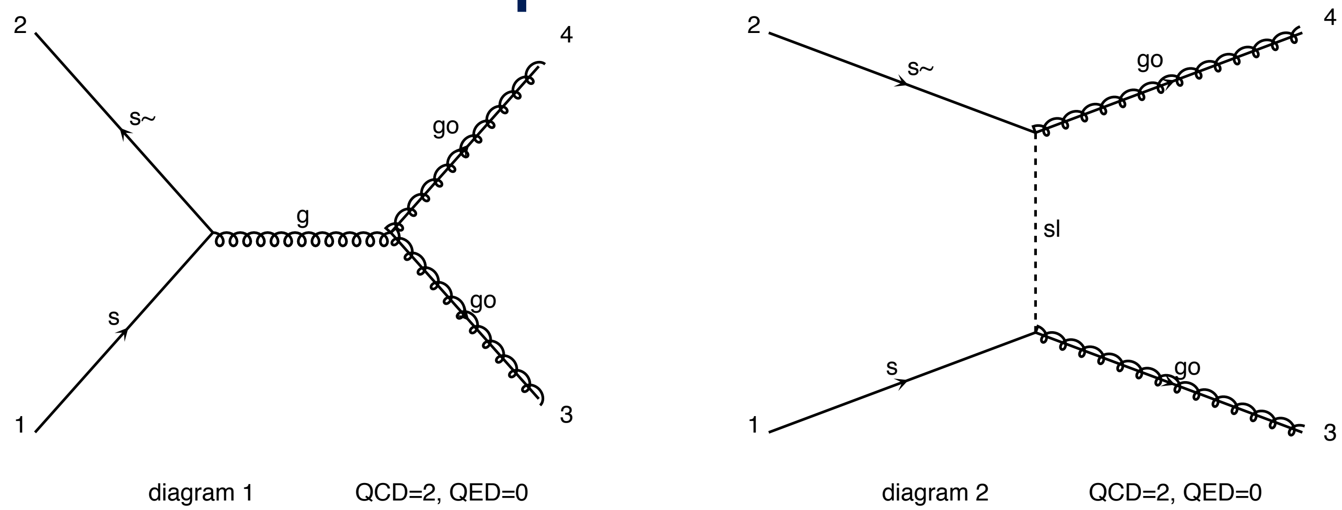


- Evaluate the matrix-element

$$|\mathcal{M}|^2$$

- Phase-Space Integration

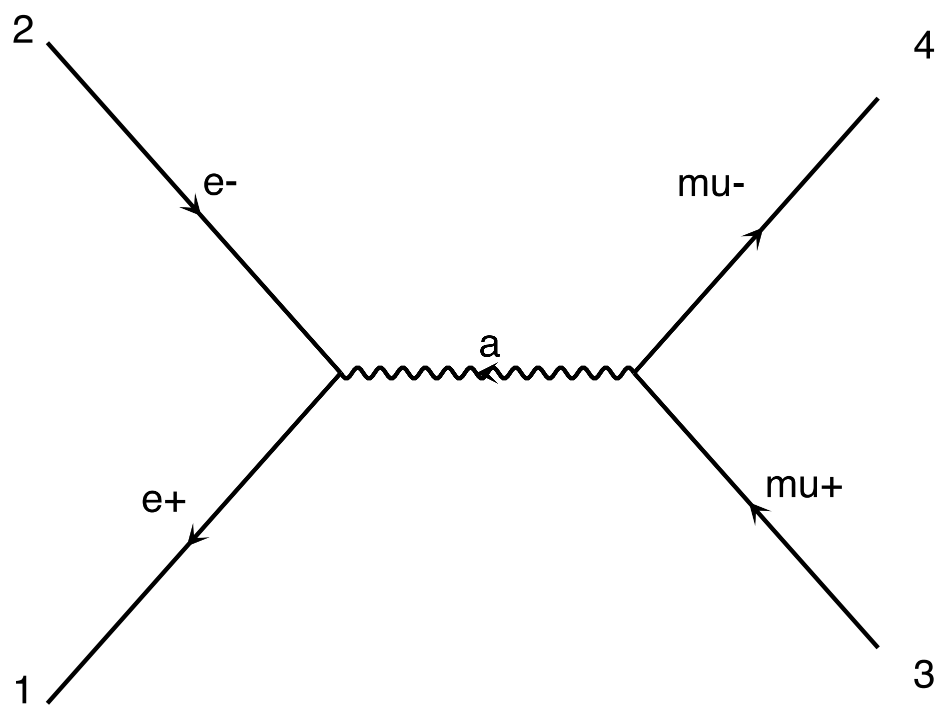
$$\sigma = \frac{1}{2s}\int |\mathcal{M}|^2 d\Phi(n)$$

Easy

Hard

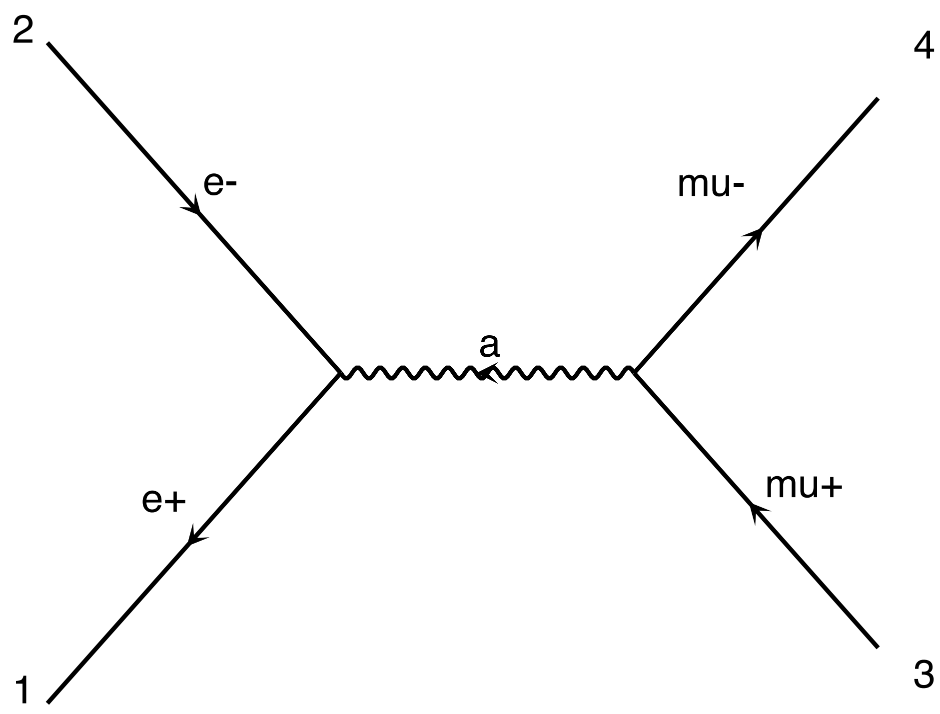
Very Hard
(in general)

Matrix Element



$$\mathcal{M} = e^2 (\bar{u}\gamma^\mu v) \frac{g_{\mu\nu}}{q^2} (\bar{v}\gamma^\nu u)$$

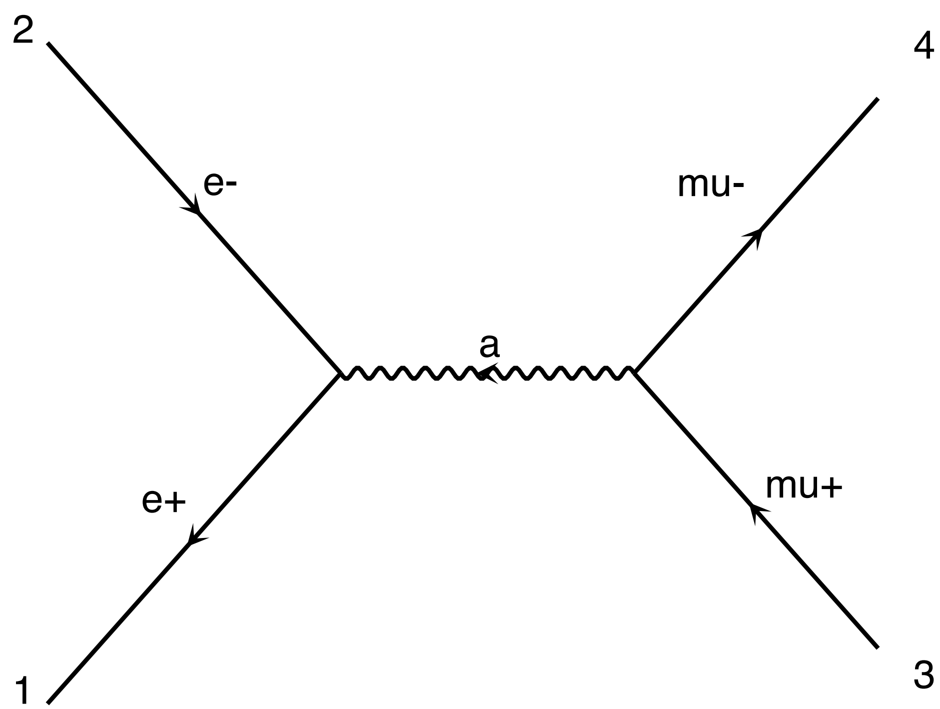
Matrix Element



$$\mathcal{M} = e^2 (\bar{u}\gamma^\mu v) \frac{g_{\mu\nu}}{q^2} (\bar{v}\gamma^\nu u)$$

$$\frac{1}{4}\sum_{pol} |\mathcal{M}|^2 = \frac{1}{4}\sum_{pol} \mathcal{M}^*\mathcal{M}$$

Matrix Element

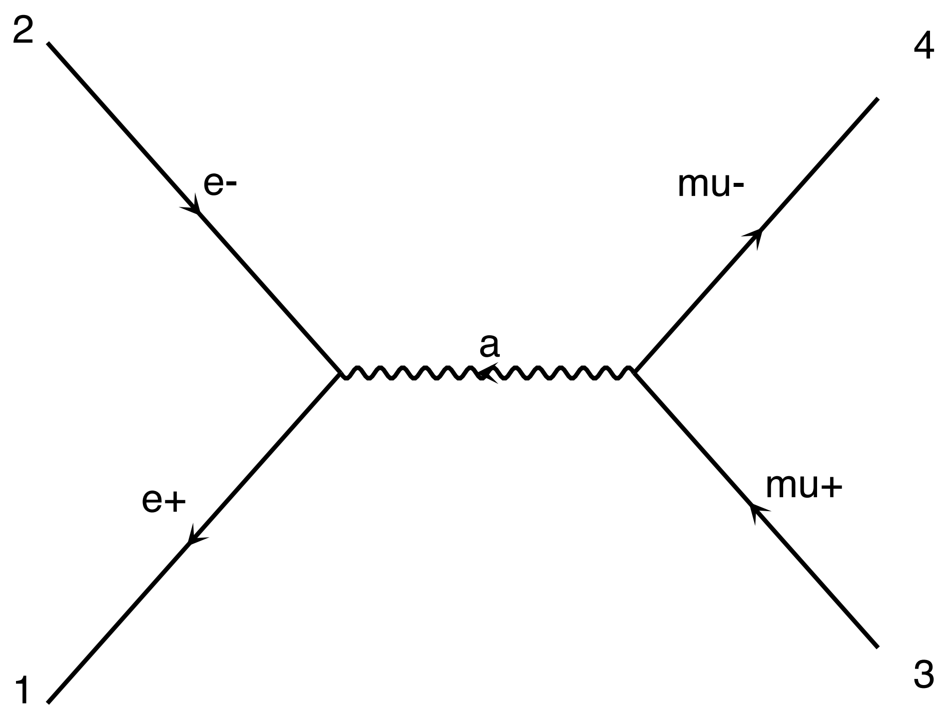


$$\mathcal{M} = e^2 (\bar{u}\gamma^\mu v)\frac{g_{\mu\nu}}{q^2}(\bar{v}\gamma^\nu u)$$

$$\frac{1}{4}\sum_{pol}|\mathcal{M}|^2 = \frac{1}{4}\sum_{pol}\mathcal{M}^*\mathcal{M}$$

$$\sum_{pol}\bar{u}u = \not{p} + m$$

Matrix Element



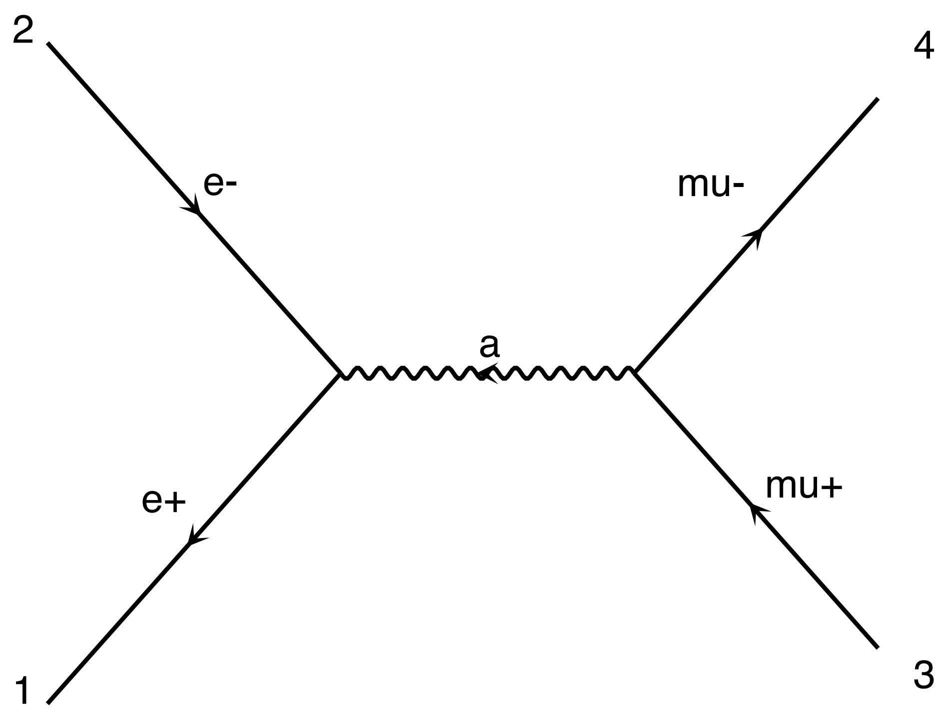
$$\mathcal{M} = e^2(\bar{u}\gamma^\mu v)\frac{g_{\mu\nu}}{q^2}(\bar{v}\gamma^\nu u)$$

$$\frac{1}{4}\sum_{pol}|\mathcal{M}|^2 = \frac{1}{4}\sum_{pol}\mathcal{M}^*\mathcal{M}$$

$$\sum_{pol}\bar{u}u = \not{p} + m$$

$$\rightarrow \frac{e^4}{4q^4}Tr[\not{p}_1\gamma^\mu\not{p}_2\gamma^\nu]Tr[\not{p}_3\gamma_\mu\not{p}_4\gamma_\nu]$$

Matrix Element



$$\mathcal{M} = e^2(\bar{u}\gamma^\mu v)\frac{g_{\mu\nu}}{q^2}(\bar{v}\gamma^\nu u)$$

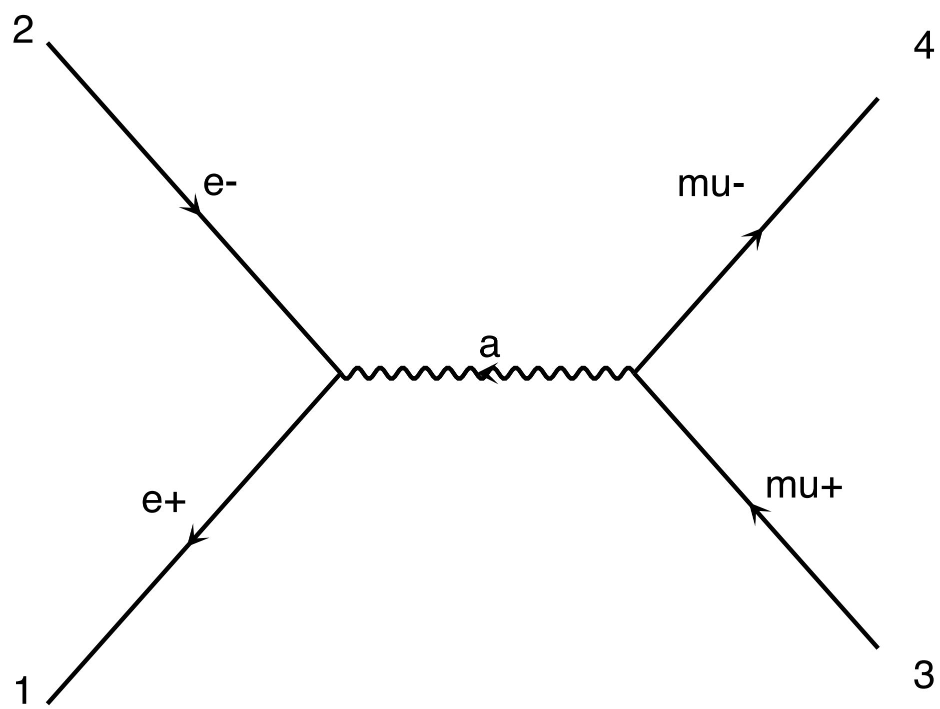
$$\frac{1}{4}\sum_{pol}|\mathcal{M}|^2 = \frac{1}{4}\sum_{pol}\mathcal{M}^*\mathcal{M}$$

$$\sum_{pol}\bar{u}u = \not{p} + m$$

$$\rightarrow \frac{e^4}{4q^4}Tr[\not{p}_1\gamma^\mu\not{p}_2\gamma^\nu]Tr[\not{p}_3\gamma_\mu\not{p}_4\gamma_\nu]$$

$$\rightarrow \frac{8e^4}{q^4}[(p_1.p_3)(p_2.p_4) + (p_1.p_4)(p_2.p_3)]$$

Matrix Element



$$\mathcal{M} = e^2(\bar{u}\gamma^\mu v)\frac{g_{\mu\nu}}{q^2}(\bar{v}\gamma^\nu u)$$

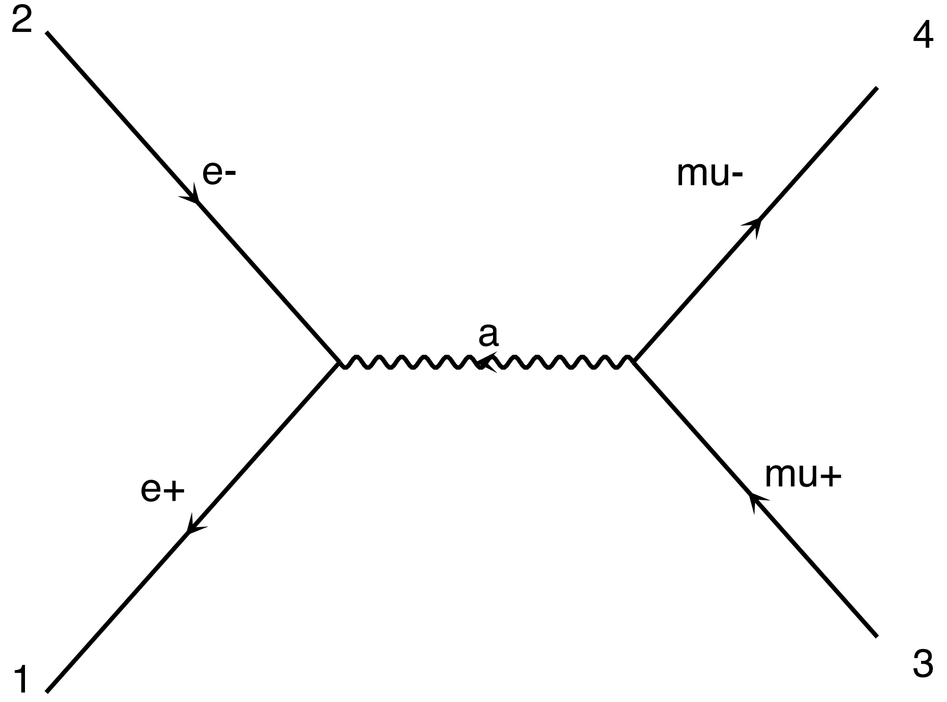
$$\frac{1}{4}\sum_{pol}|\mathcal{M}|^2 = \frac{1}{4}\sum_{pol}\mathcal{M}^*\mathcal{M}$$

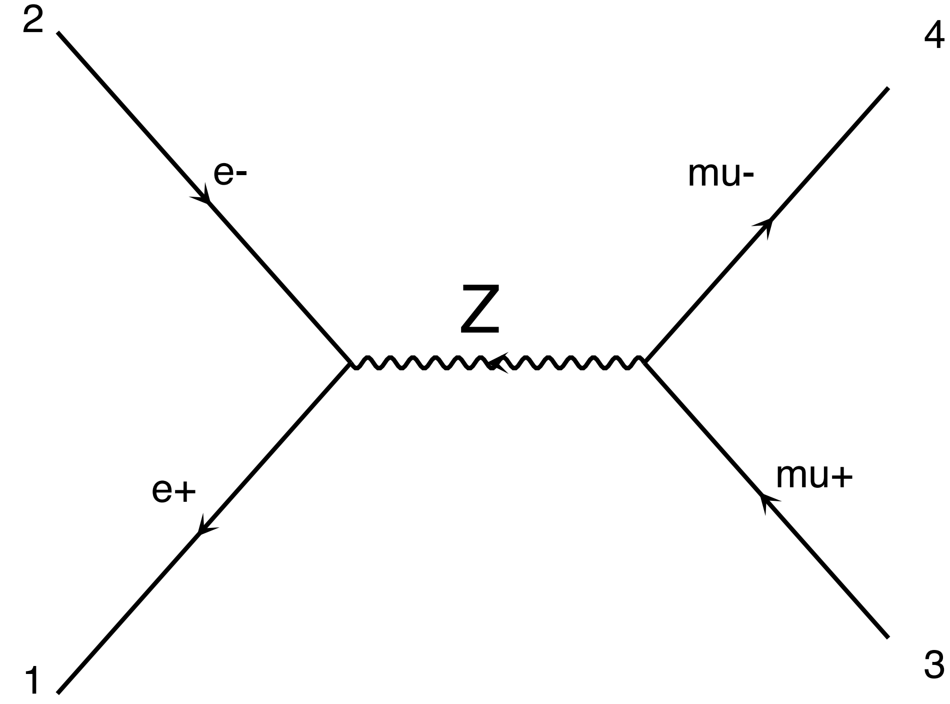
$$\sum_{pol}\bar{u}u = \not{p} + m$$

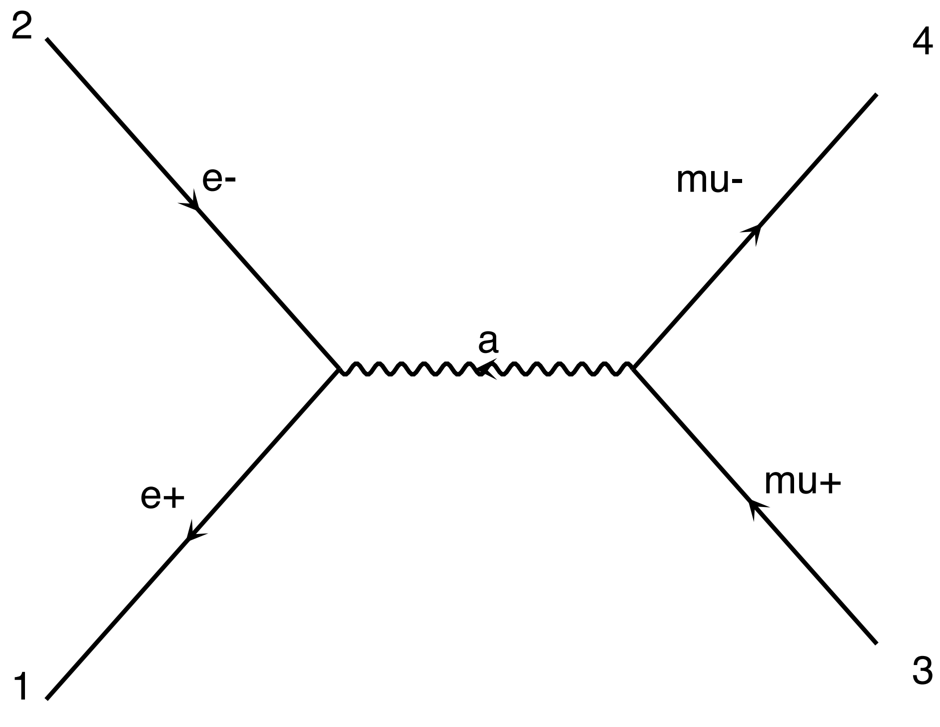
$$\rightarrow \frac{e^4}{4q^4}Tr[\not{p}_1\gamma^\mu\not{p}_2\gamma^\nu]Tr[\not{p}_3\gamma_\mu\not{p}_4\gamma_\nu]$$

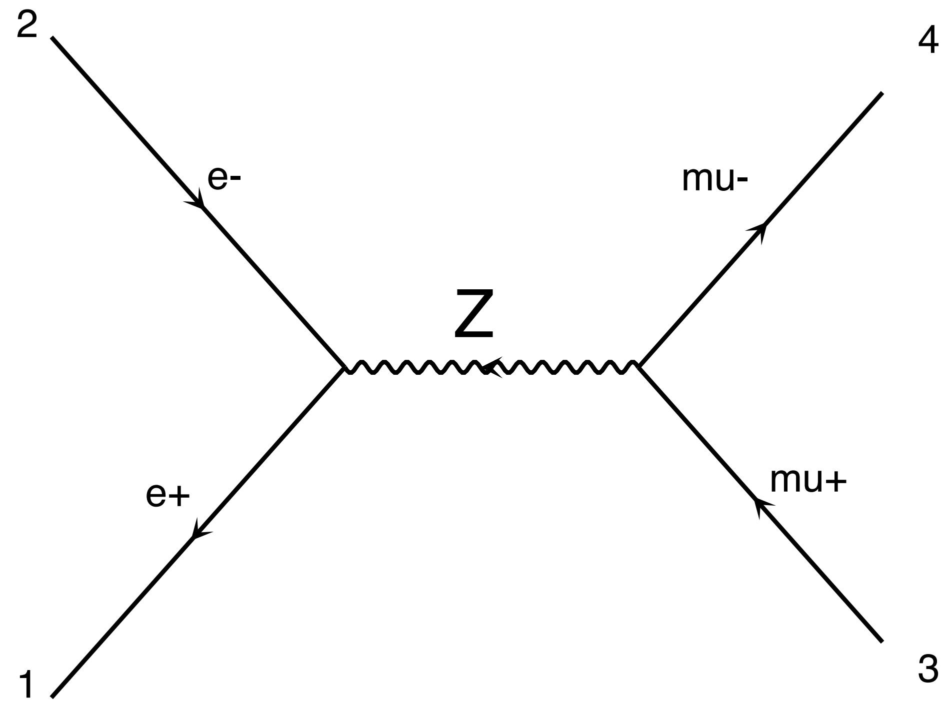
$$\rightarrow \frac{8e^4}{q^4}[(p_1.p_3)(p_2.p_4)+(p_1.p_4)(p_2.p_3)]$$

Very Efficient
(few computation to perform to get that number)

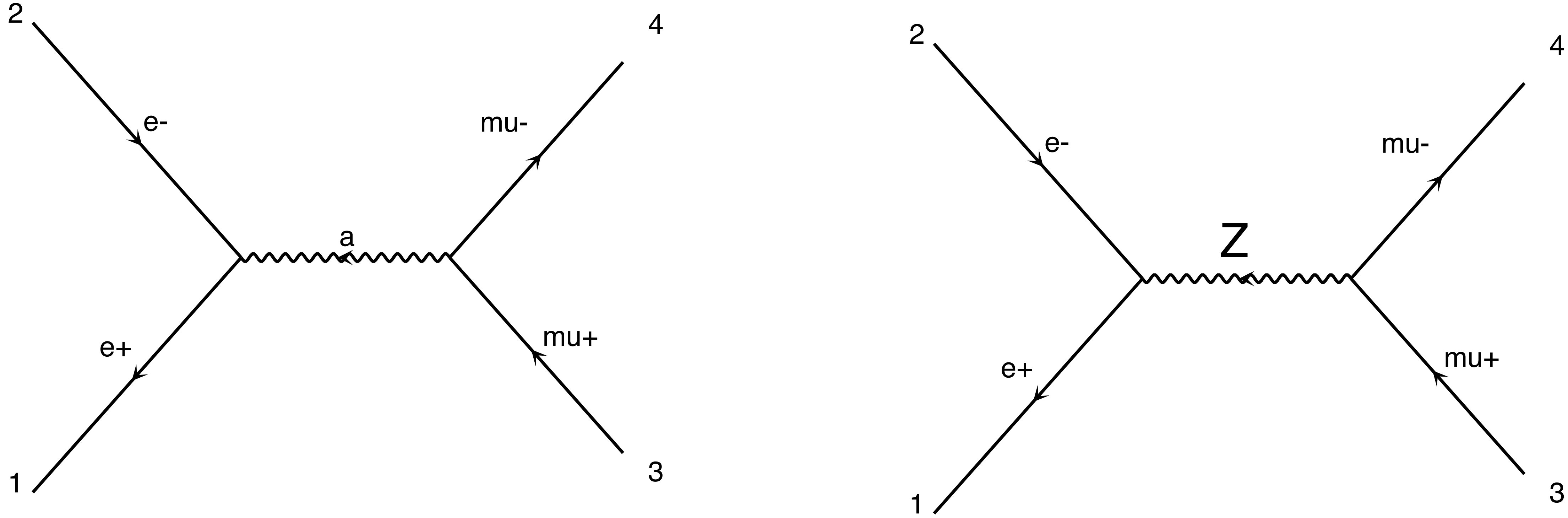

2
e-
mu-
4
a
e+
mu+
1
3


2
e-
mu-
4
Z
e+
mu+
1
3



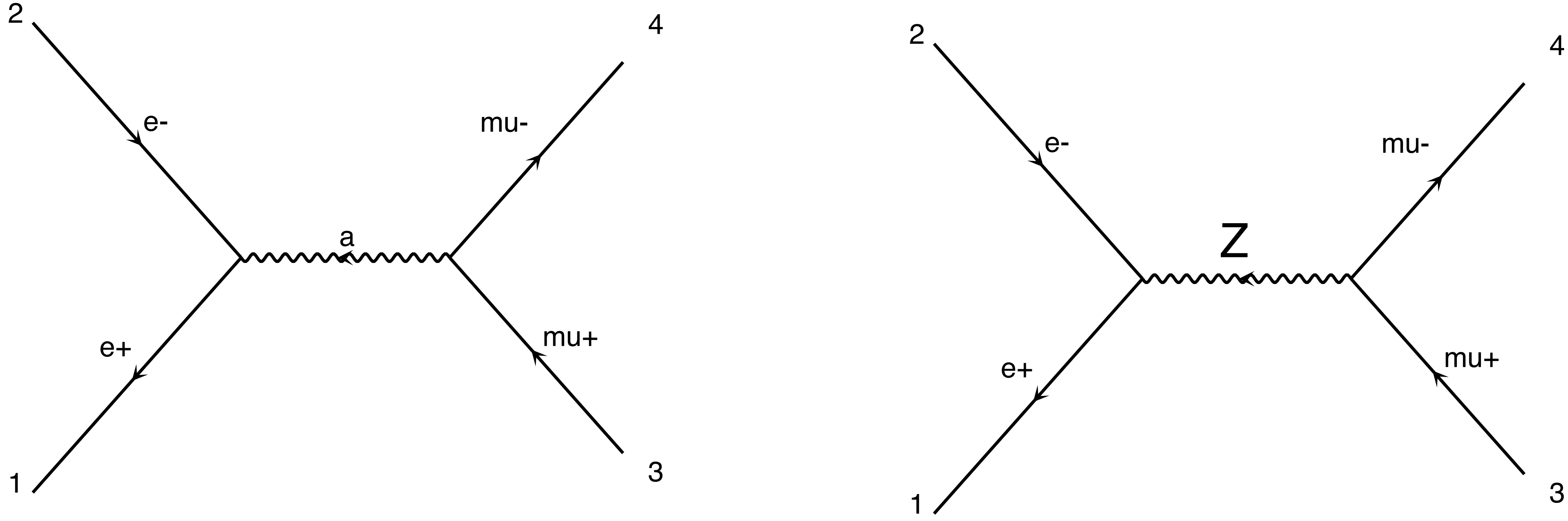


Need to compute $|M_a|^2$ $|M_z|^2$ $2Re(M_a^* M_z)$



Need to compute $|M_a|^2$ $|M_z|^2$ $2Re(M_a^* M_z)$

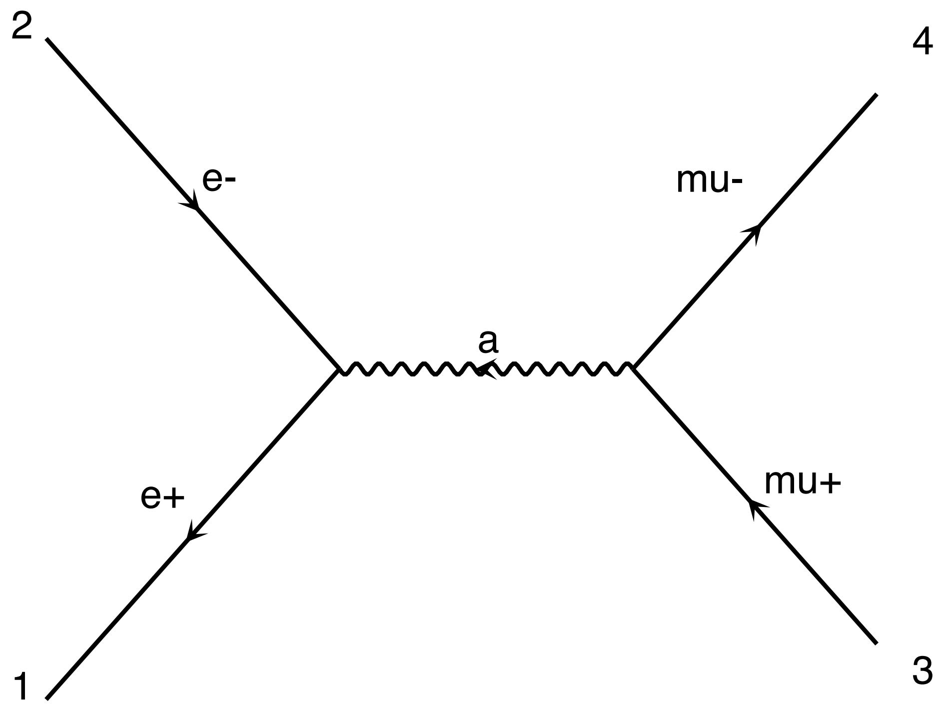
So for M Feynman diagram we need to compute M^2 different term

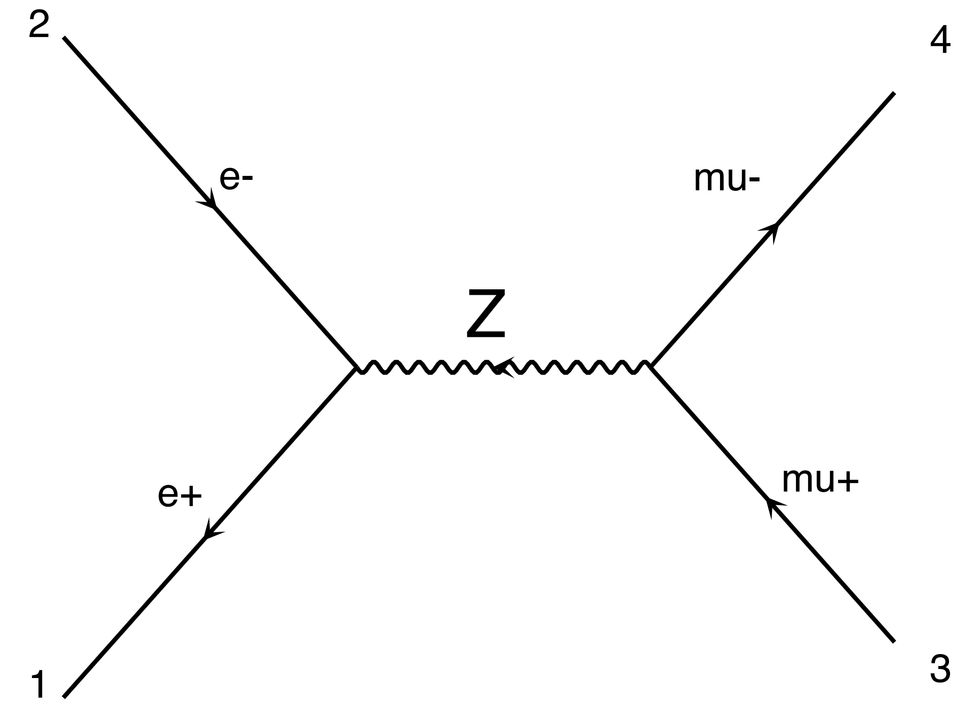


Need to compute $|M_a|^2$ $|M_z|^2$ $2Re(M_a^* M_z)$

So for M Feynman diagram we need to compute M^2 different term

The number of diagram scales factorially with the number of particle





Need to compute $|M_a|^2$ $|M_z|^2$ $2Re(M_a^* M_z)$

So for M Feynman diagram we need to compute M^2 different term

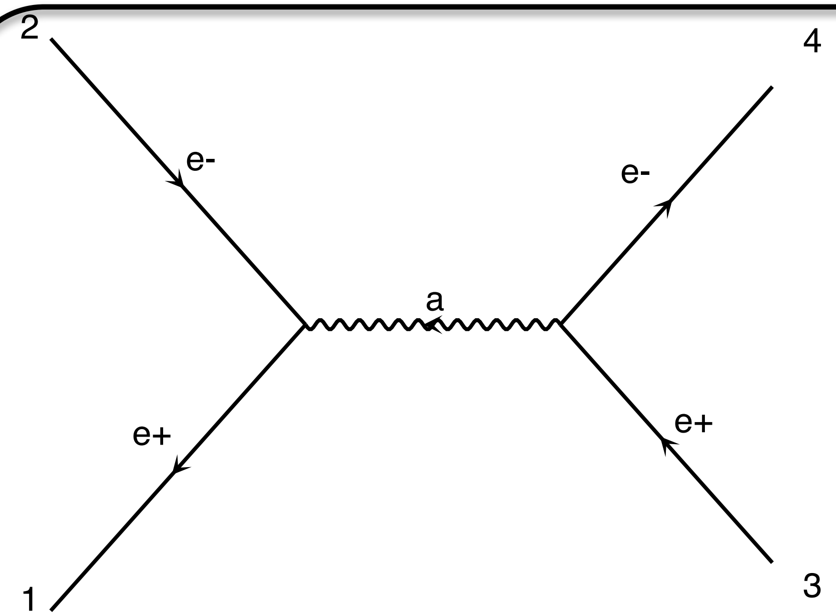
The number of diagram scales factorially with the number of particle

In practise possible up to 2>4

Helicity

Idea

- Evaluate $\mathcal{M}$ for fixed helicity of external particles
  - ➡ Multiply $\mathcal{M}$ with $\mathcal{M}^*$ -> $|\mathcal{M}|$^2
  - ➡ Loop on Helicity and average the results



$$\mathcal{M} = (\bar{u}\, e\, \gamma^{\mu} v)\frac{g_{\mu\nu}}{q^2}(\bar{v}\, e\, \gamma^{\nu} u)$$

Helicity

Idea

- Evaluate $\mathcal{M}$ for fixed helicity of external particles
  - ➡ Multiply $\mathcal{M}$ with $\mathcal{M}^*$ -> $|\mathcal{M}|$^2
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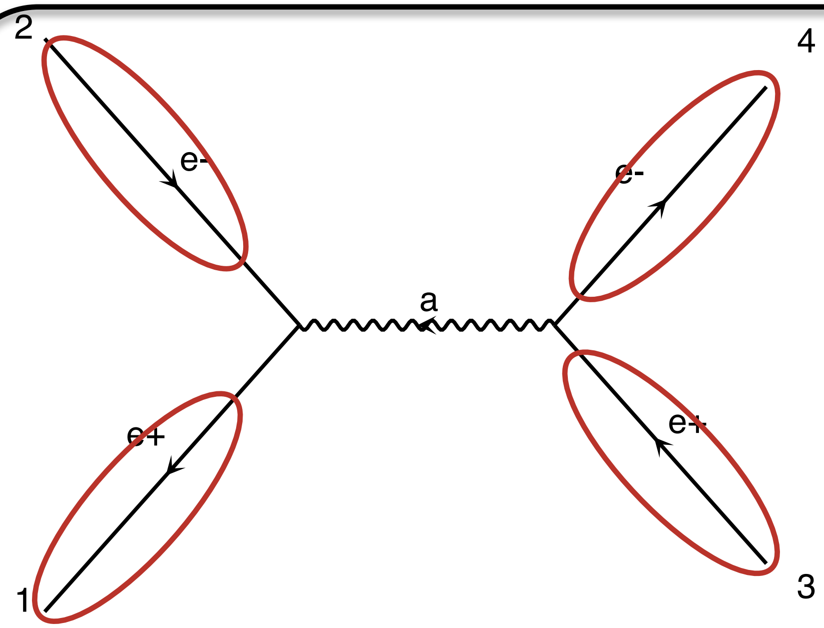


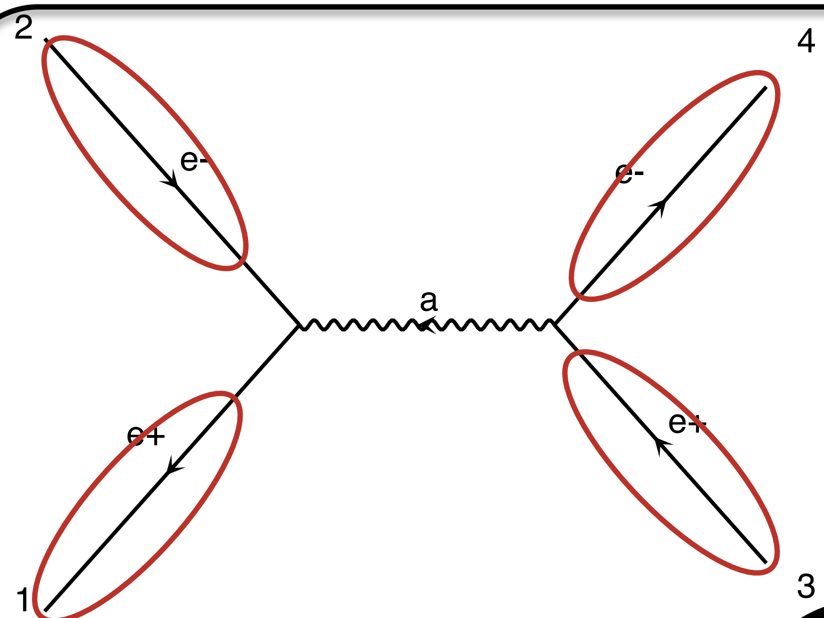
$$\mathcal{M} = (\bar{u}\, e\, \gamma^{\mu}\, v)\, \frac{g_{\mu\nu}}{q^2}\, (\bar{v}\, e\, \gamma^{\nu}\, u)$$

Numbers for given helicity and momenta

Helicity

Idea

- Evaluate $\mathcal{M}$ for fixed helicity of external particles
  - ➡ Multiply $\mathcal{M}$ with $\mathcal{M}^*$ -> $|\mathcal{M}|$^2
  - ➡ Loop on Helicity and average the results



$$\mathcal{M} = (\bar{u}\, e\, \gamma^{\mu} v) \frac{g_{\mu\nu}}{q^2} (\bar{v}\, e\, \gamma^{\nu} u)$$

Numbers for given helicity and momenta

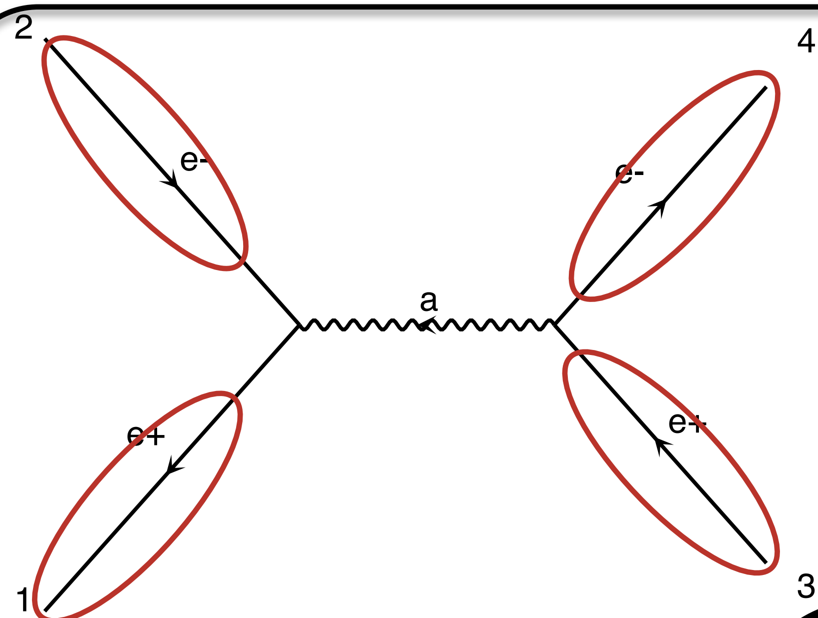
Lines present in the code.

$$\left\{ \begin{array}{l} \bar{v}_1 = fct(\vec{p}_1, m_1) \\ u_2 = fct(\vec{p}_2, m_2) \\ v_3 = fct(\vec{p}_3, m_3) \\ \bar{u}_4 = fct(\vec{p}_4, m_4) \end{array} \right.$$

Helicity

Idea

- Evaluate $\mathcal{M}$ for fixed helicity of external particles
  - ➡ Multiply $\mathcal{M}$ with $\mathcal{M}^*$ -> $|\mathcal{M}|$^2
  - ➡ Loop on Helicity and average the results



$$\mathcal{M} = (\bar{u}\, e\, \gamma^\mu v) \frac{g_{\mu\nu}}{q^2} (\bar{v}\, e\, \gamma^\nu u)$$

Numbers for given helicity and momenta

Lines present in the code.

$$\begin{cases} \bar{v}_1 = fct(\vec{p}_1, m_1) \\ u_2 = fct(\vec{p}_2, m_2) \\ v_3 = fct(\vec{p}_3, m_3) \\ \bar{u}_4 = fct(\vec{p}_4, m_4) \end{cases}$$

$$u(p) = \begin{pmatrix} \omega_{-\lambda}(p)\chi_\lambda(\vec{p}) \\ \omega_\lambda(p)\chi_\lambda(\vec{p}) \end{pmatrix}$$

$$\omega_\pm(p) \equiv \sqrt{E \pm |\vec{p}|}.$$

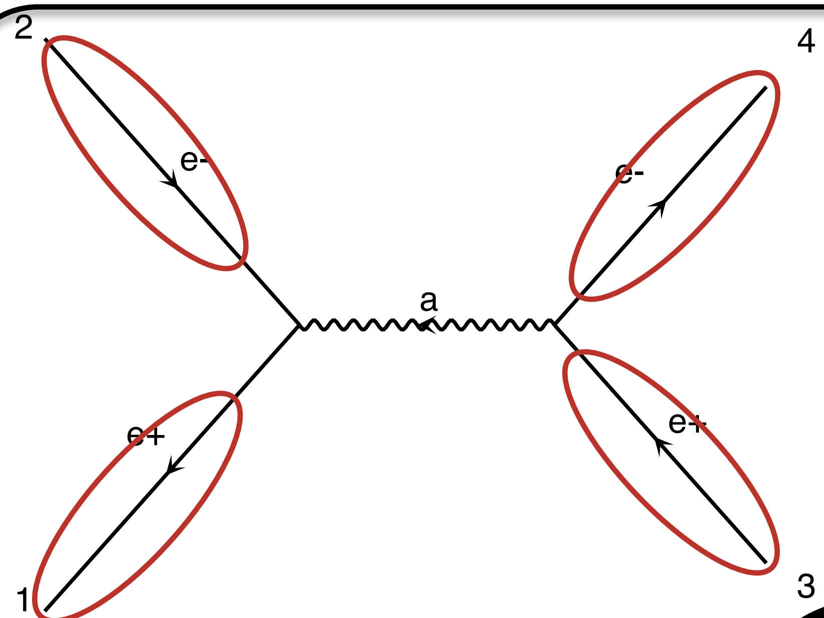
$$\chi_+(\vec{p}) = \frac{1}{\sqrt{2|\vec{p}|(|\vec{p}|+p_z)}} \begin{pmatrix} |\vec{p}|+p_z \\ p_x + i p_y \end{pmatrix},$$

$$\chi_-(\vec{p}) = \frac{1}{\sqrt{2|\vec{p}|(|\vec{p}|+p_z)}} \begin{pmatrix} -p_x + i p_y \\ |\vec{p}|+p_z \end{pmatrix}.$$

Helicity

Idea

- Evaluate $\mathcal{M}$ for fixed helicity of external particles
  - ➡ Multiply $\mathcal{M}$ with $\mathcal{M}^*$ -> $|\mathcal{M}|$^2
  - ➡ Loop on Helicity and average the results



$$\mathcal{M} = (\bar{u}\, e\, \gamma^\mu v)\, \frac{g_{\mu\nu}}{q^2}\, (\bar{v}\, e\, \gamma^\nu u)$$

Numbers for given helicity and momenta

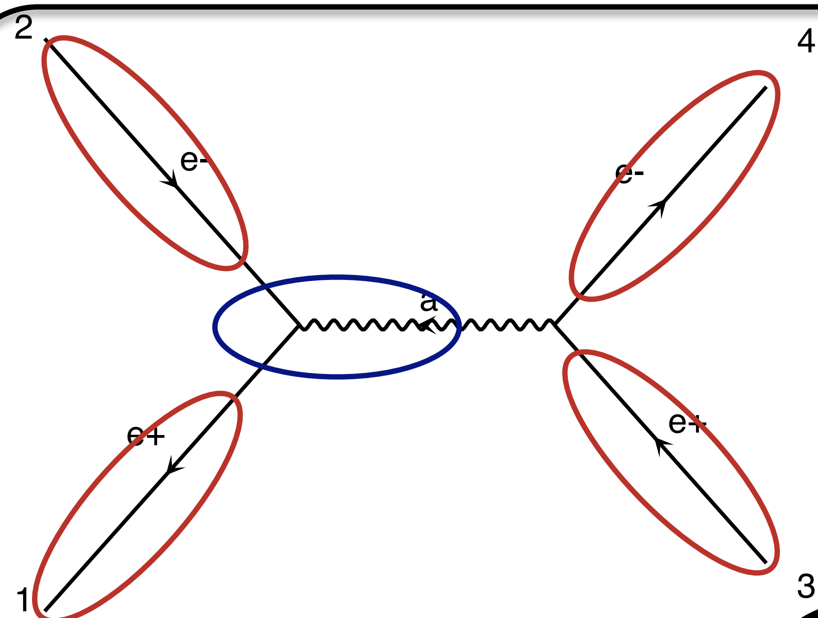
Lines present in the code.

$$\begin{cases} \bar{v}_1 = fct(\vec{p}_1, m_1) \\ u_2 = fct(\vec{p}_2, m_2) \\ v_3 = fct(\vec{p}_3, m_3) \\ \bar{u}_4 = fct(\vec{p}_4, m_4) \end{cases}$$

Helicity

Idea

- Evaluate $\mathcal{M}$ for fixed helicity of external particles
  - ➡ Multiply $\mathcal{M}$ with $\mathcal{M}^*$ -> $|\mathcal{M}|$^2
  - ➡ Loop on Helicity and average the results



$$\mathcal{M} = (\bar{u}\, e\, \gamma^{\mu} v)\, \frac{g_{\mu\nu}}{q^2}\, (\bar{v}\, e\, \gamma^{\nu} u)$$

Numbers for given helicity and momenta

Calculate propagator wavefunctions

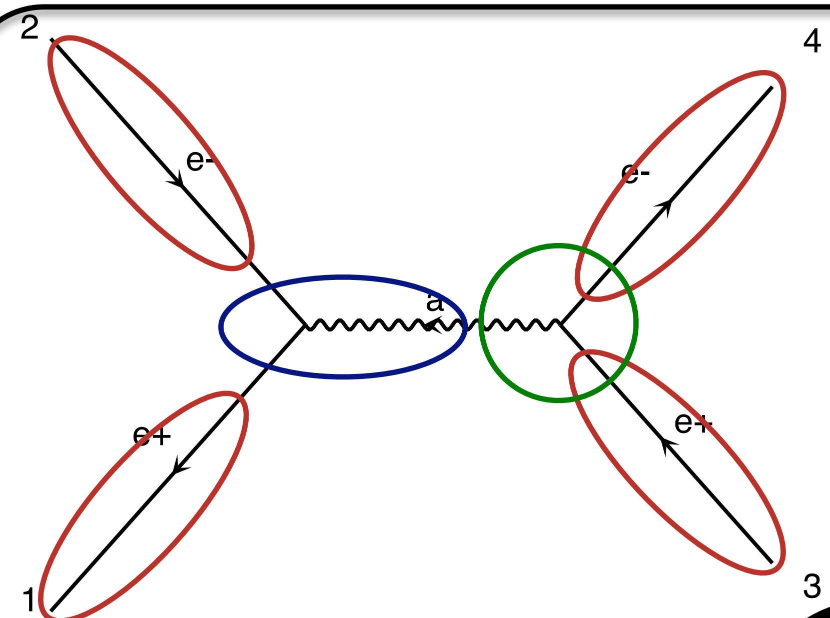
Lines present in the code.

$$\left\{ \begin{array}{l} \bar{v}_1 = fct(\vec{p}_1, m_1) \\ u_2 = fct(\vec{p}_2, m_2) \\ v_3 = fct(\vec{p}_3, m_3) \\ \bar{u}_4 = fct(\vec{p}_4, m_4) \\ W_a = fct(\bar{v}_1, u_2, m_a, \Gamma_a) \end{array} \right.$$

Helicity

Idea

- Evaluate $\mathcal{M}$ for fixed helicity of external particles
  - ➡ Multiply $\mathcal{M}$ with $\mathcal{M}^*$ -> $|\mathcal{M}|$^2
  - ➡ Loop on Helicity and average the results



$$\mathcal{M} = (\bar{u}\, e\, \gamma^{\mu} v)\, \frac{g_{\mu\nu}}{q^2} (\bar{v}\, e\, \gamma^{\nu} u)$$

Numbers for given helicity and momenta

Calculate propagator wavefunctions

Finally evaluate amplitude (c-number)

Lines present in the code.

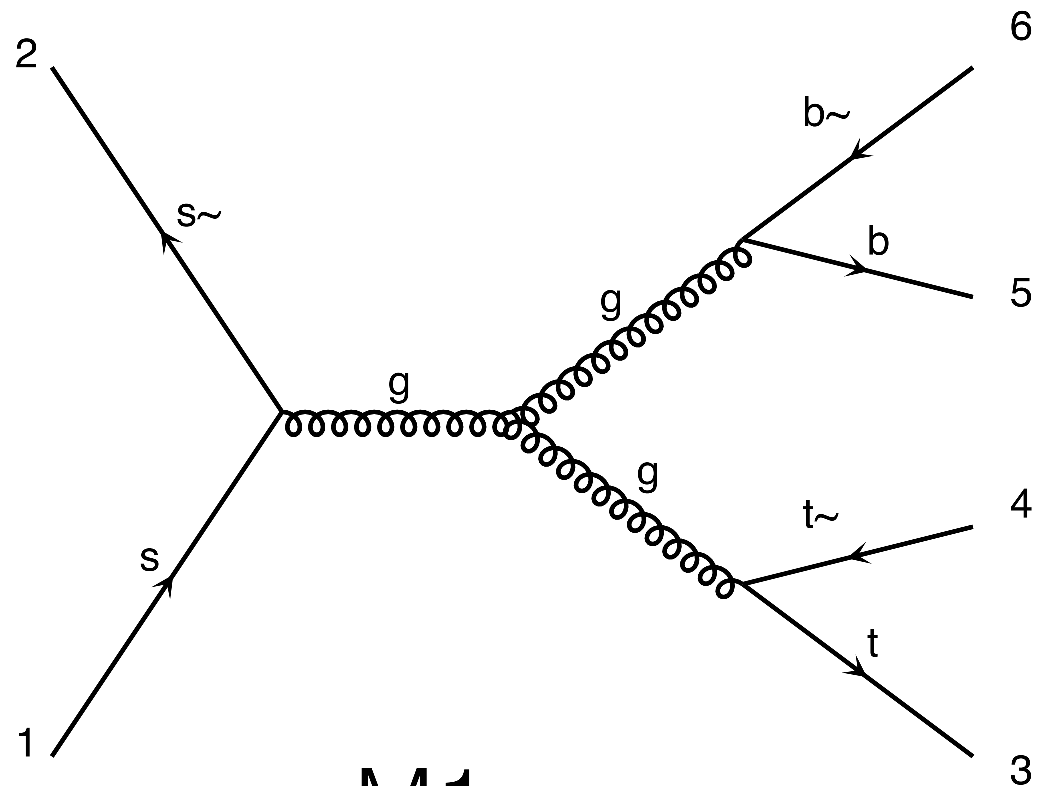
$$\begin{cases} \bar{v}_1 = fct(\vec{p}_1, m_1) \\ u_2 = fct(\vec{p}_2, m_2) \\ v_3 = fct(\vec{p}_3, m_3) \\ \bar{u}_4 = fct(\vec{p}_4, m_4) \\ W_a = fct(\bar{v}_1, u_2, m_a, \Gamma_a) \\ \mathcal{M} = fct(v_3, \bar{u}_4, W_a) \end{cases}$$

Comparison

	M diag	N particle
Analytical	M^2	$(N!)^2$
Helicity	M	$(N!)\,2^N$

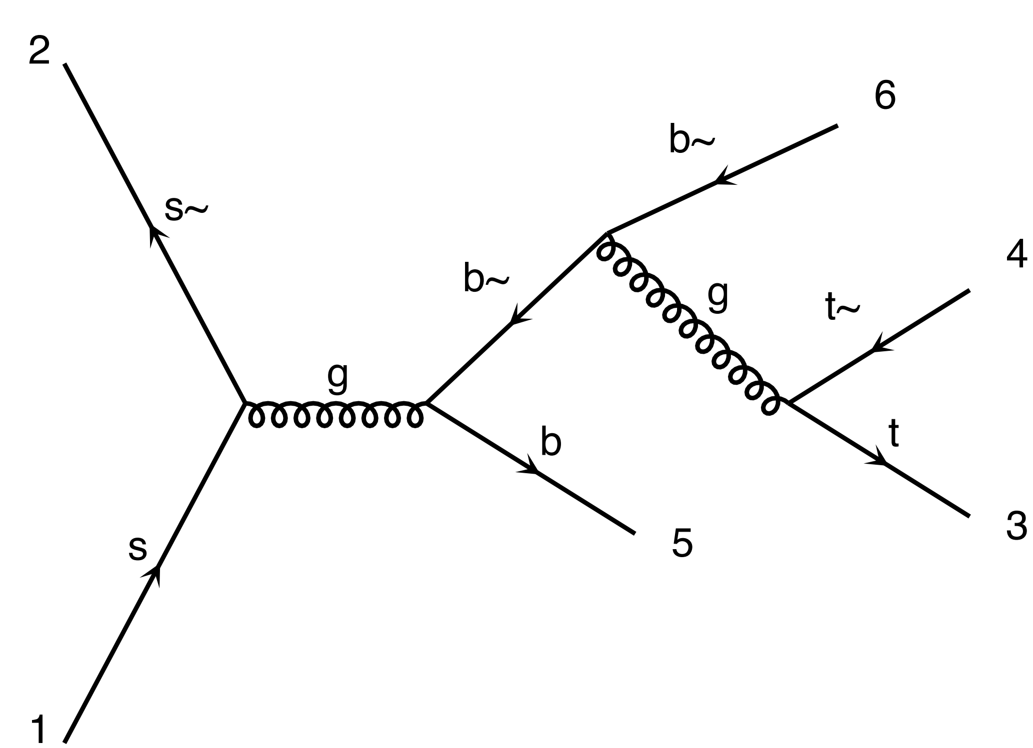
Real case

Known



M1

Number of routines: 0



M2

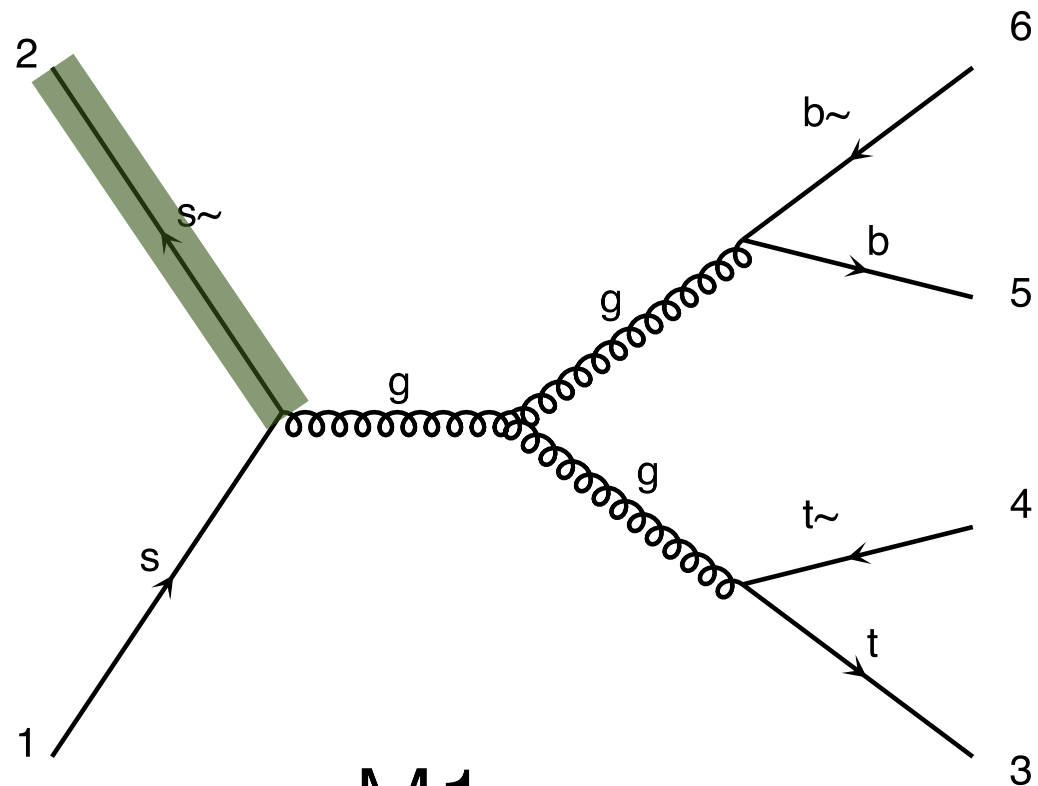
Number of routines: 0

Number of routines for both: 0

$$|M|^2 = |M_1 + M_2|^2$$

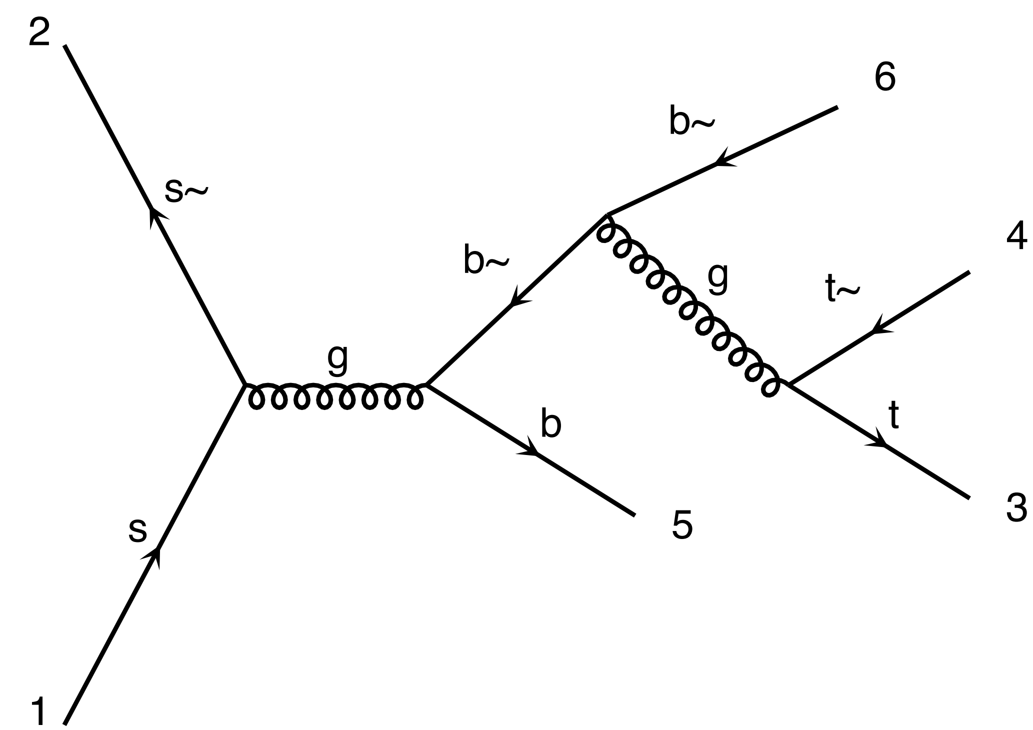
Real case

Known



M1

Number of routines: 1



M2

Number of routines: 0

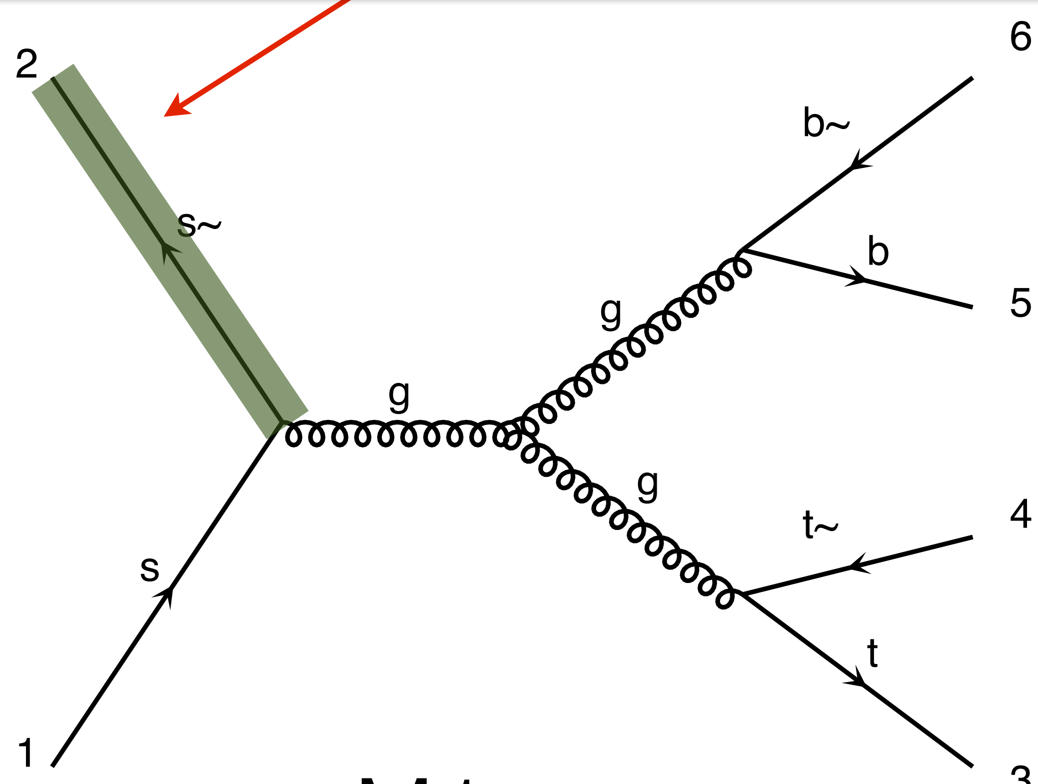
Number of routines for both: 1

$$|M|^2 = |M_1 + M_2|^2$$

Real case

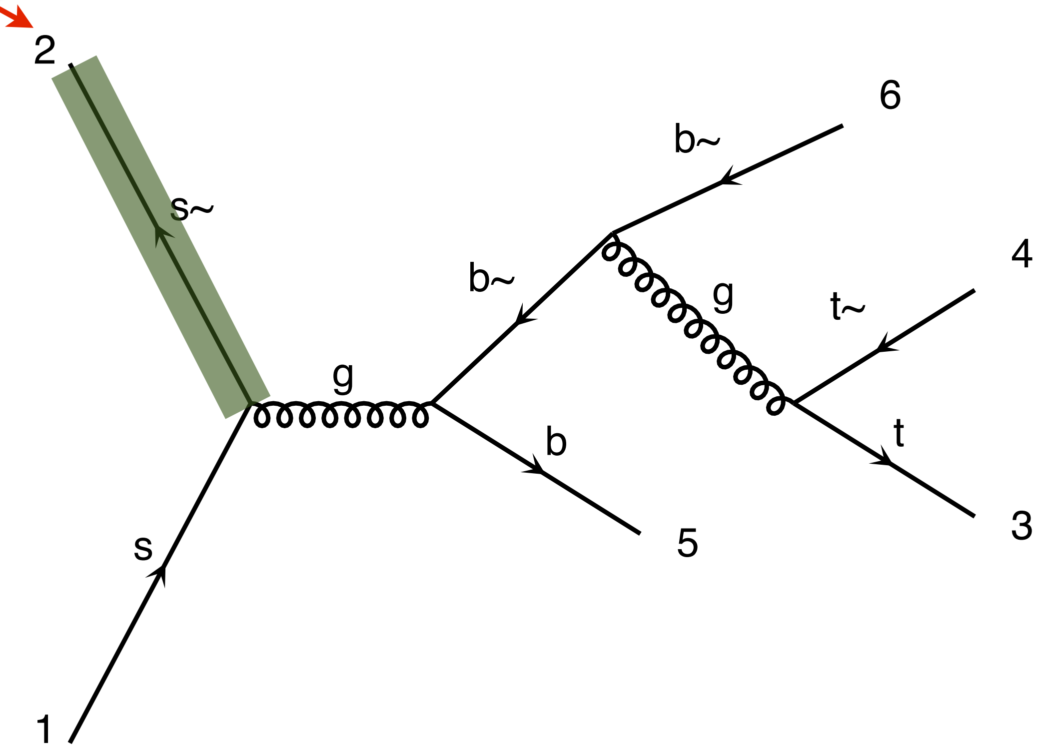
Identical

Known



M1

Number of routines: 1



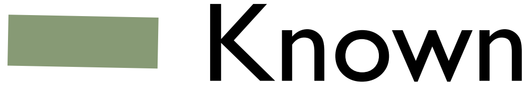
M2

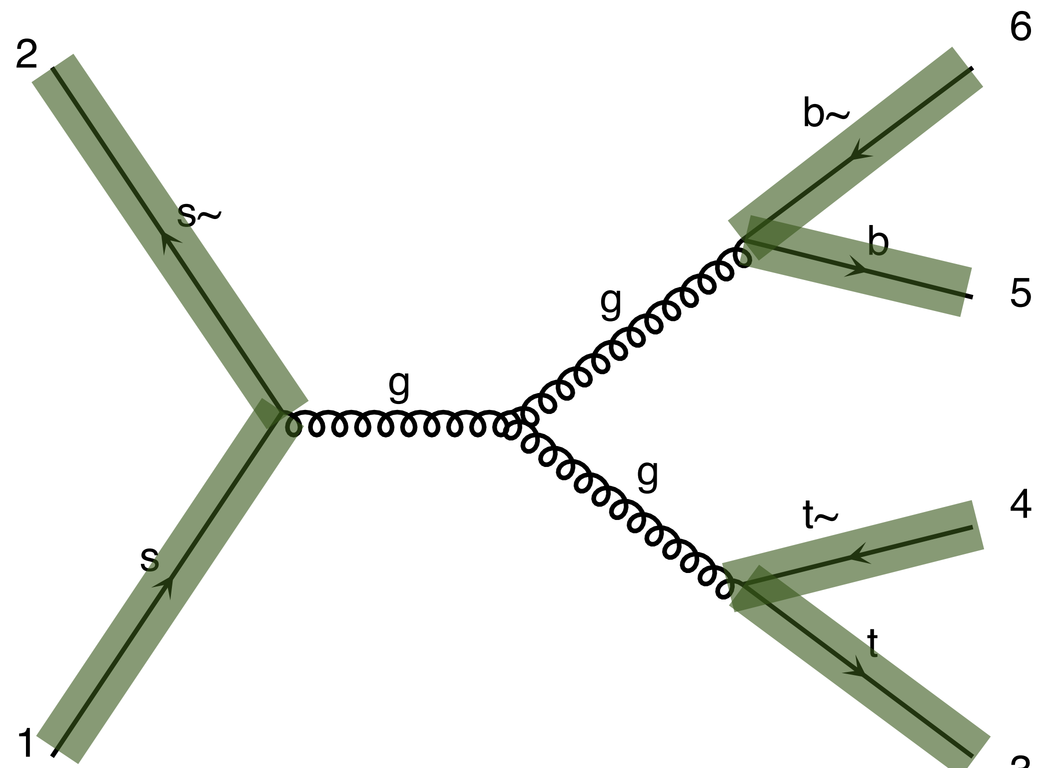
Number of routines: 1

Number of routines for both: 1

$$|M|^2 = |M_1 + M_2|^2$$

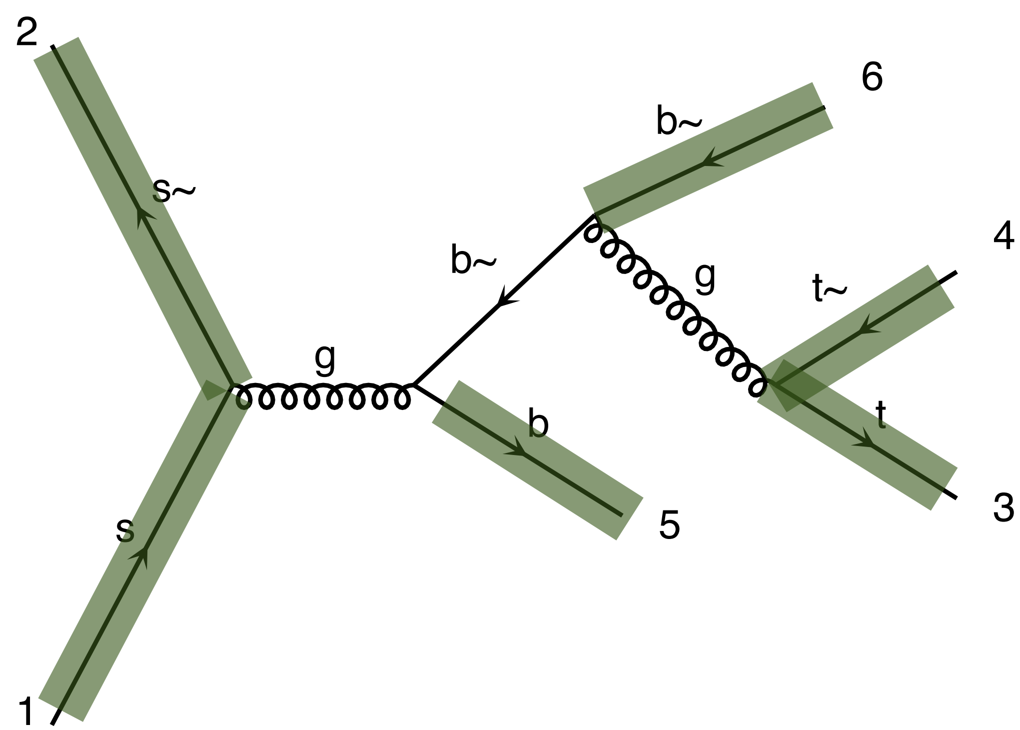
Real case





M1

Number of routines: 6



M2

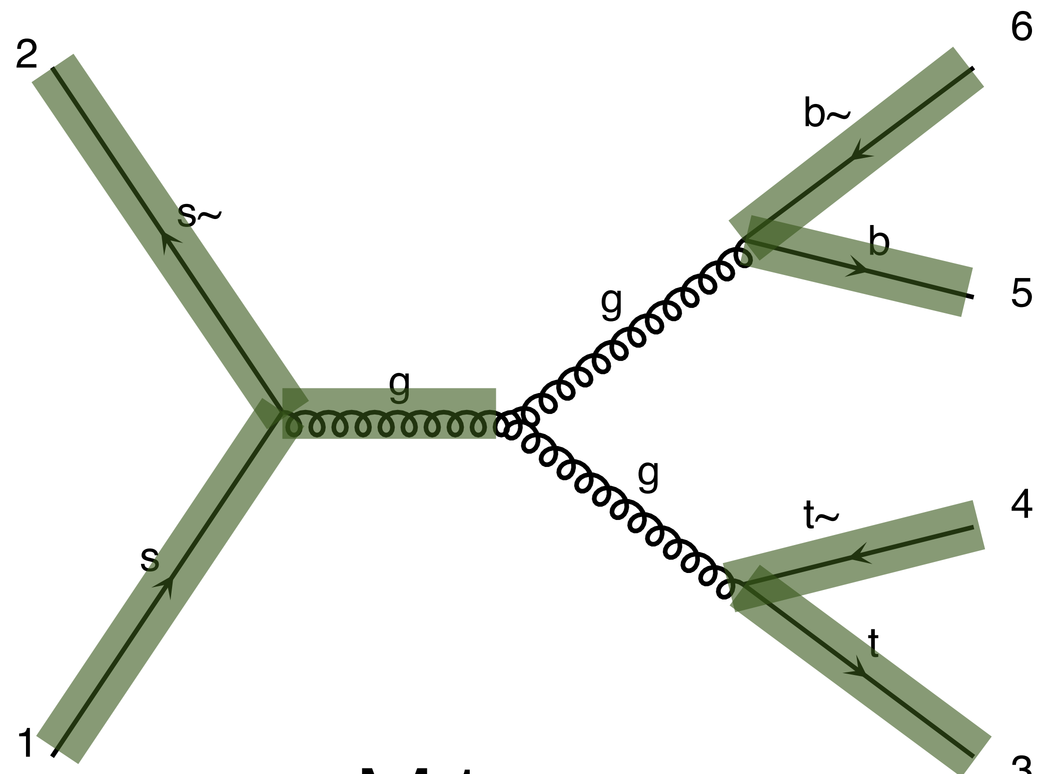
Number of routines: 6

Number of routines for both: 6

$$|M|^2 = |M_1 + M_2|^2$$

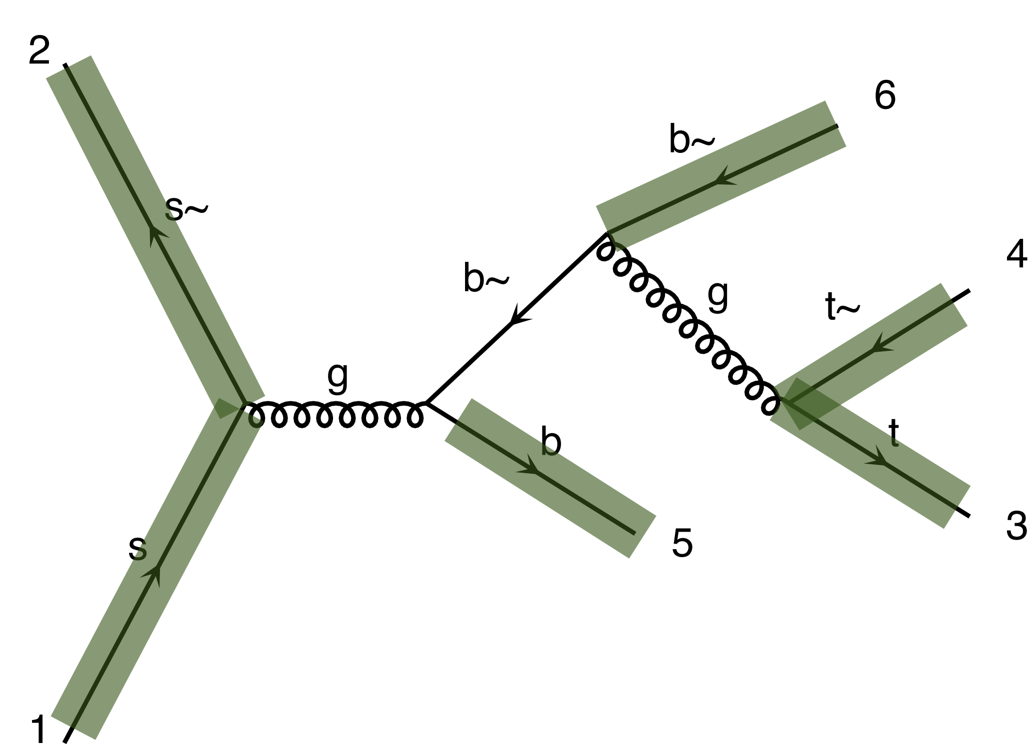
Real case

Known



M1

Number of routines: 7



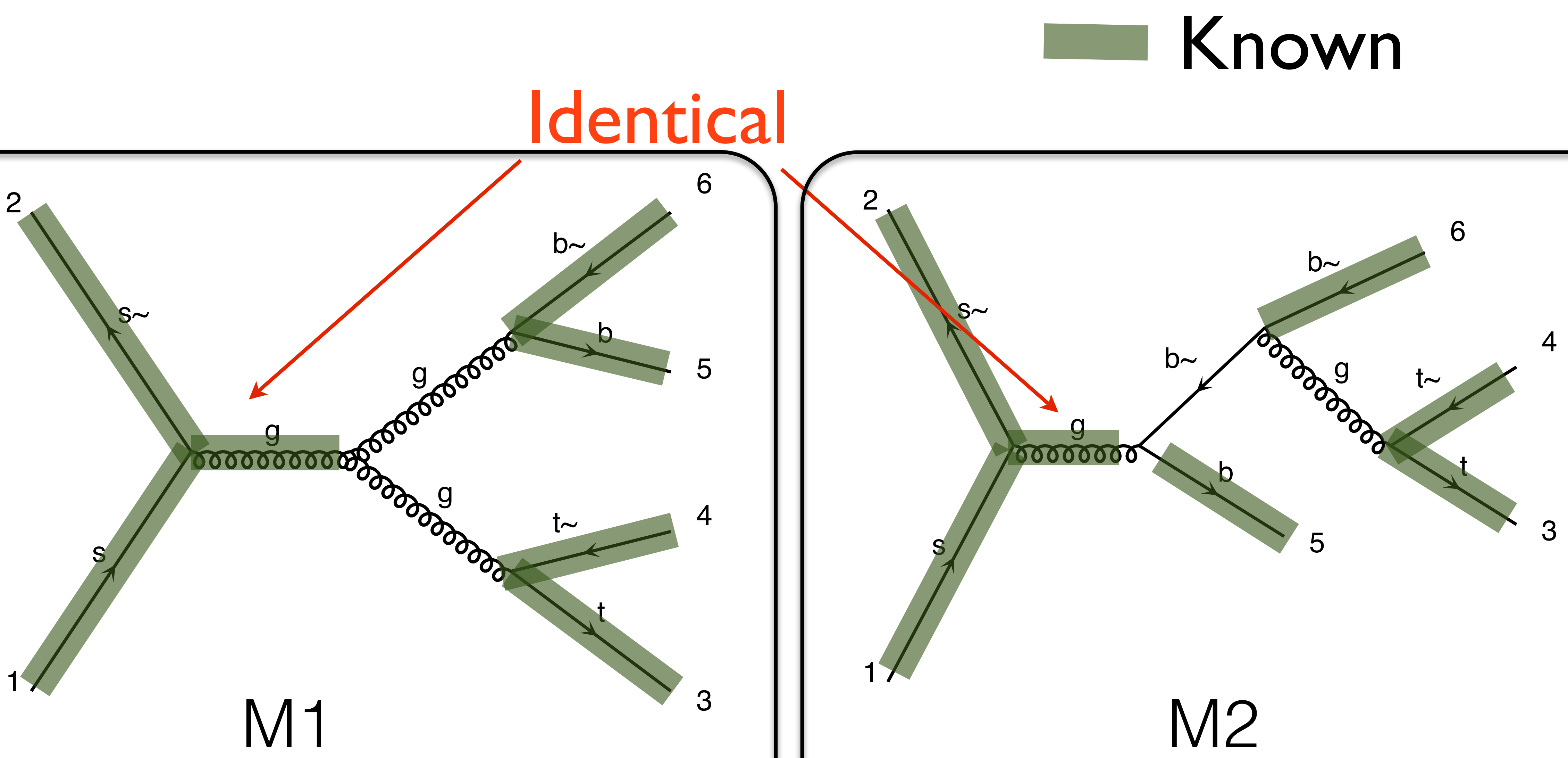
M2

Number of routines: 6

Number of routines for both: 7

$$|M|^2 = |M_1 + M_2|^2$$

Real case



Number of routines: 7

Number of routines: 7

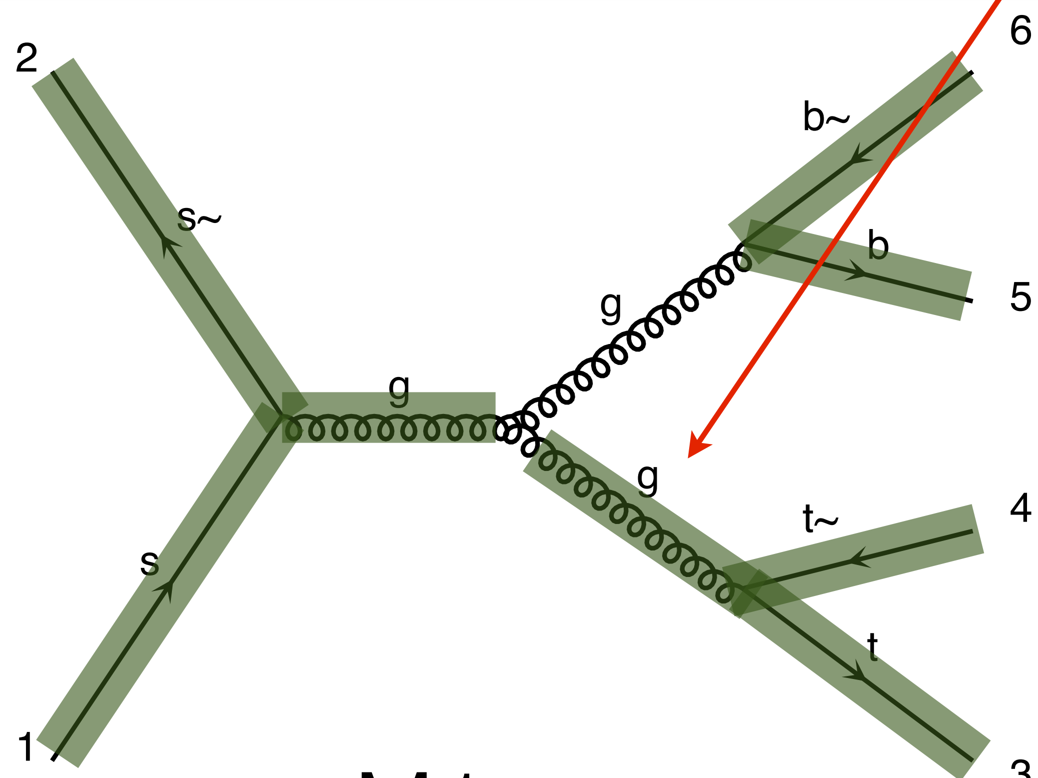
Number of routines for both: 7

$$|M|^2 = |M_1 + M_2|^2$$

Real case

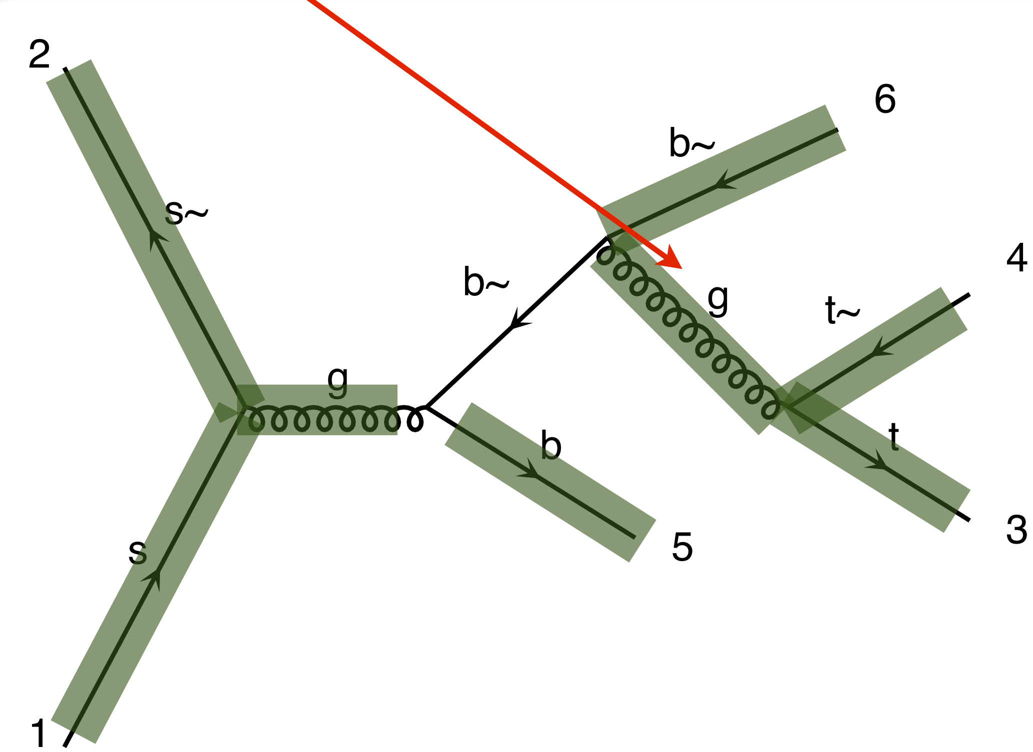
Identical

Known



M1

Number of routines: 8



M2

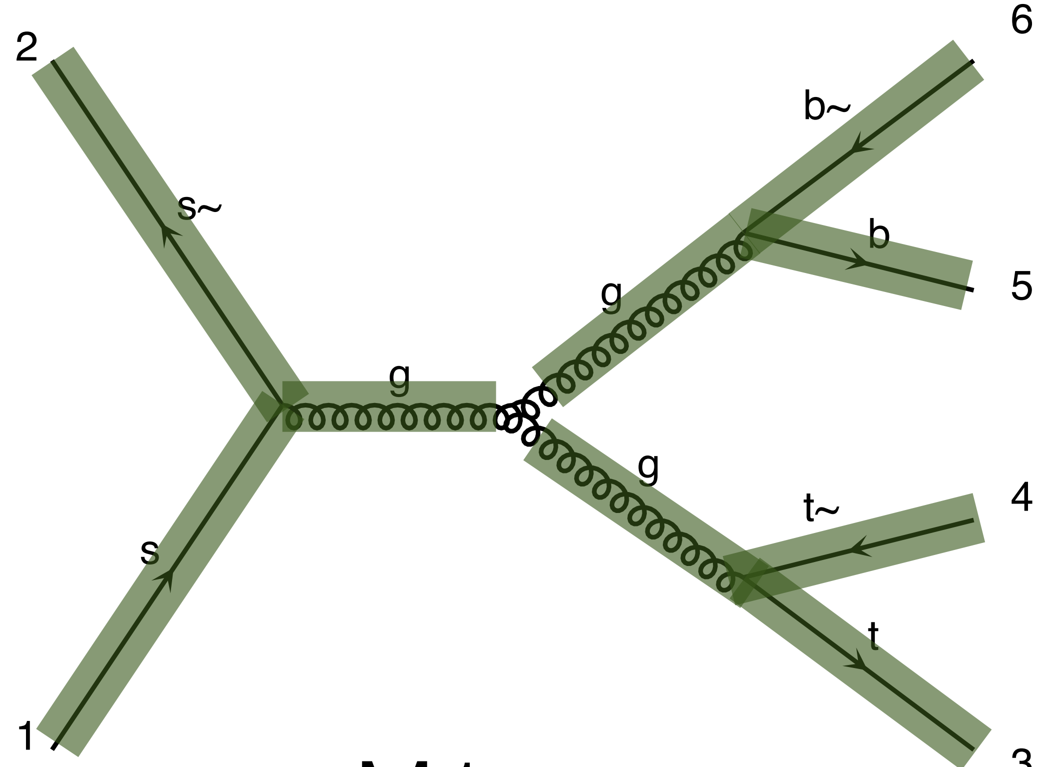
Number of routines: 8

Number of routines for both: 8

$$|M|^2 = |M_1 + M_2|^2$$

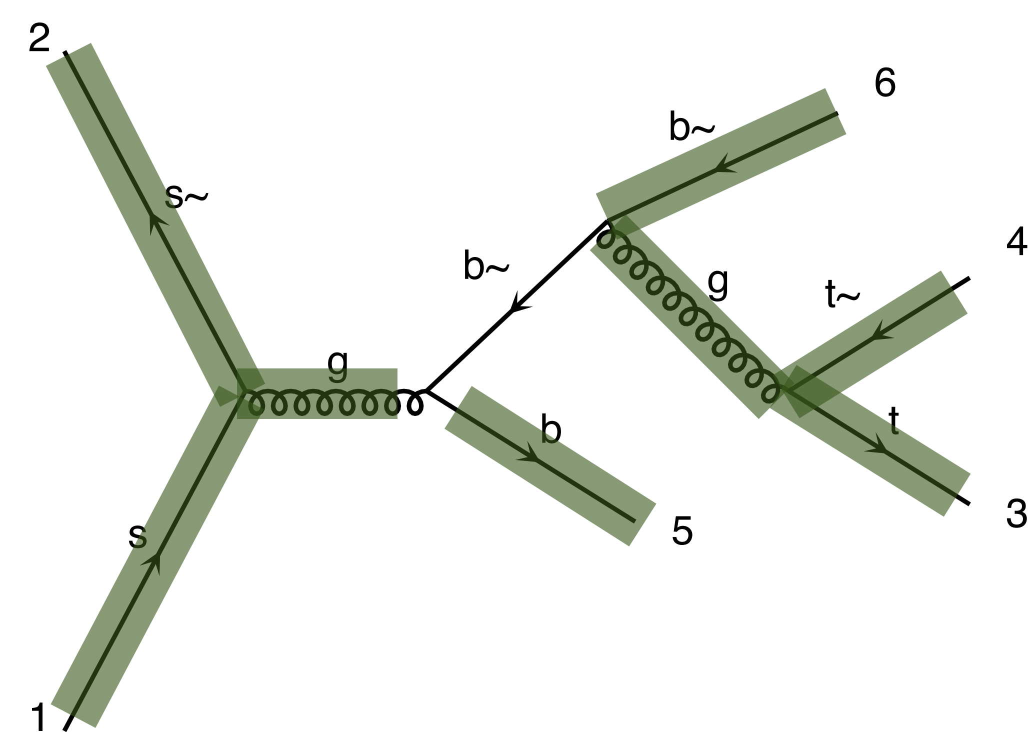
Real case

Known



M1

Number of routines: 9



M2

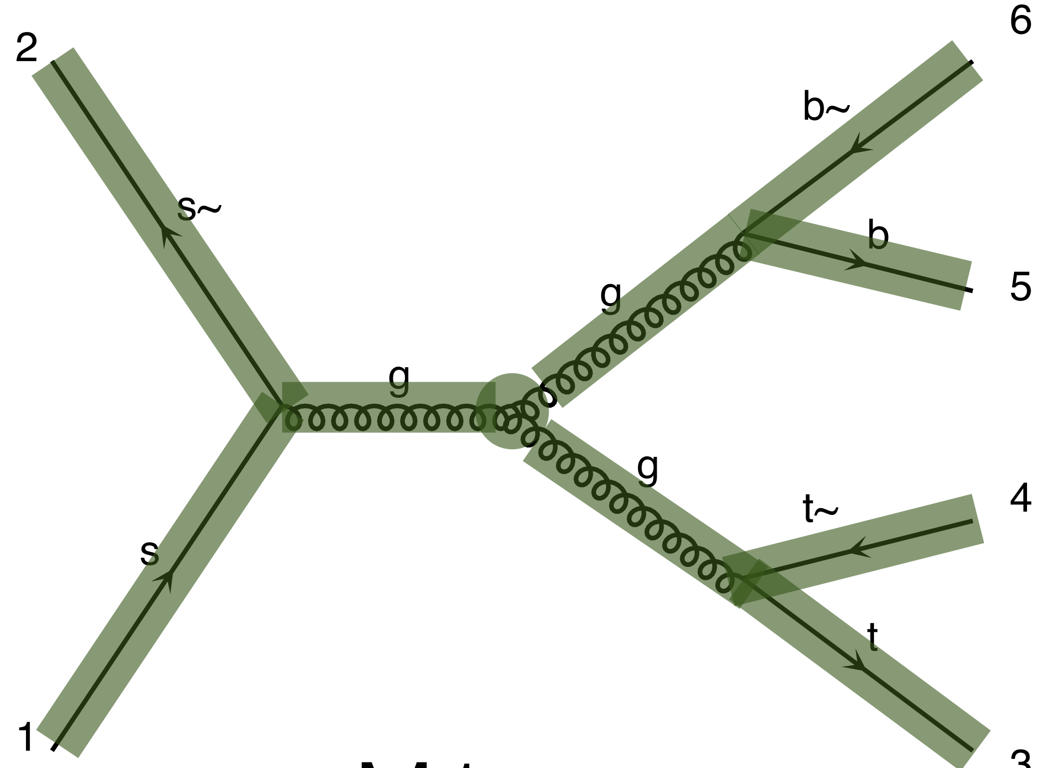
Number of routines: 8

Number of routines for both: 9

$$|M|^2 = |M_1 + M_2|^2$$

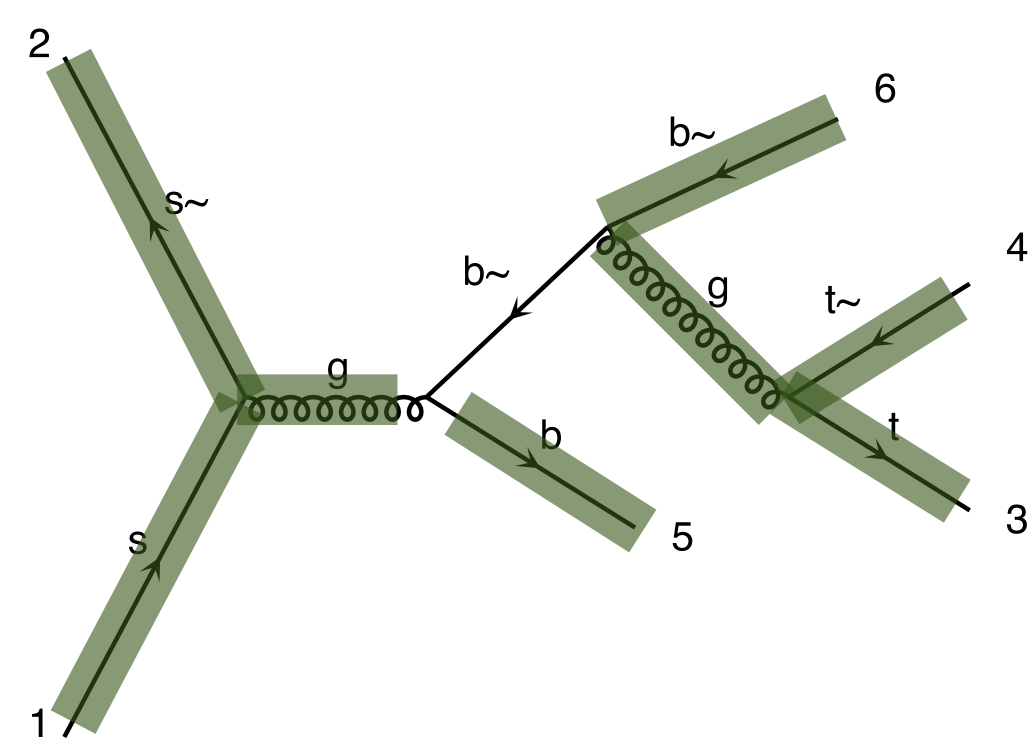
Real case

Known



M1

Number of routines: 10



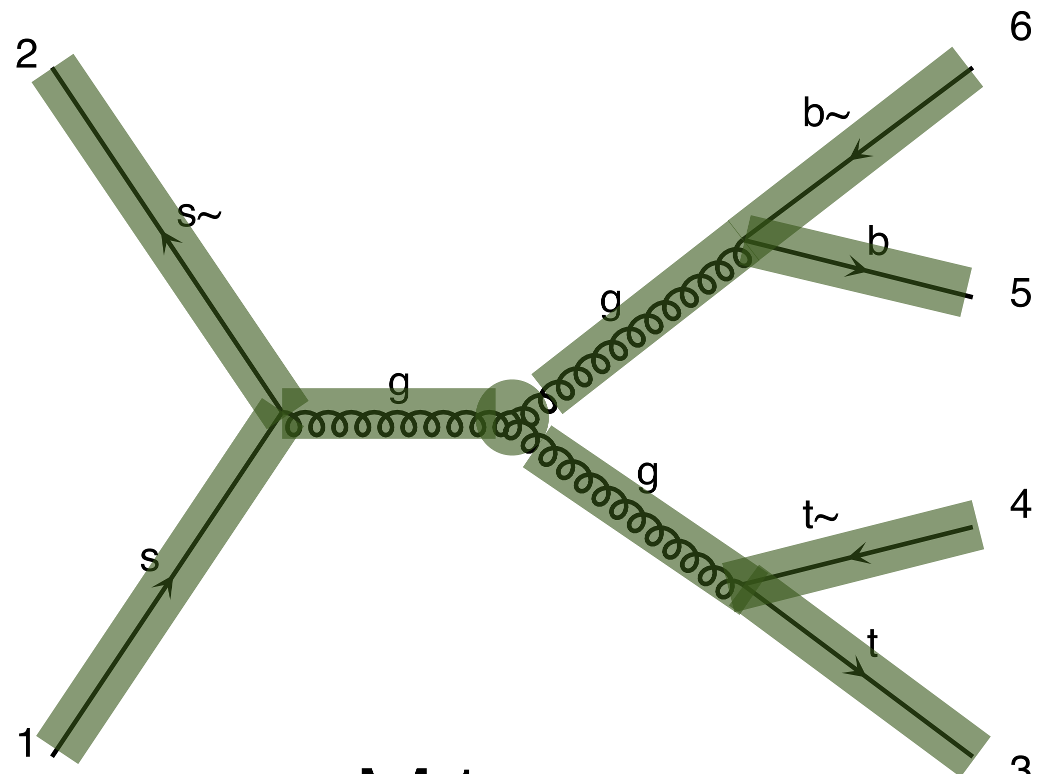
M2

Number of routines: 8

Number of routines for both: 10

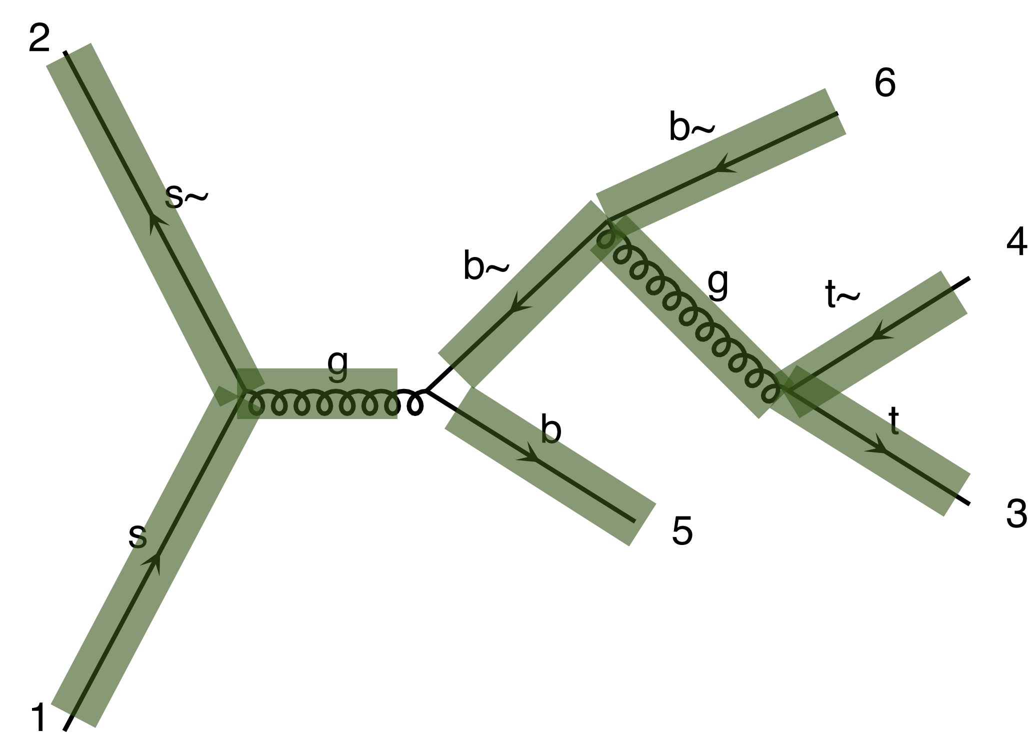
$$|M|^2 = |M_1 + M_2|^2$$

Real case



M1

Number of routines: 10



M2

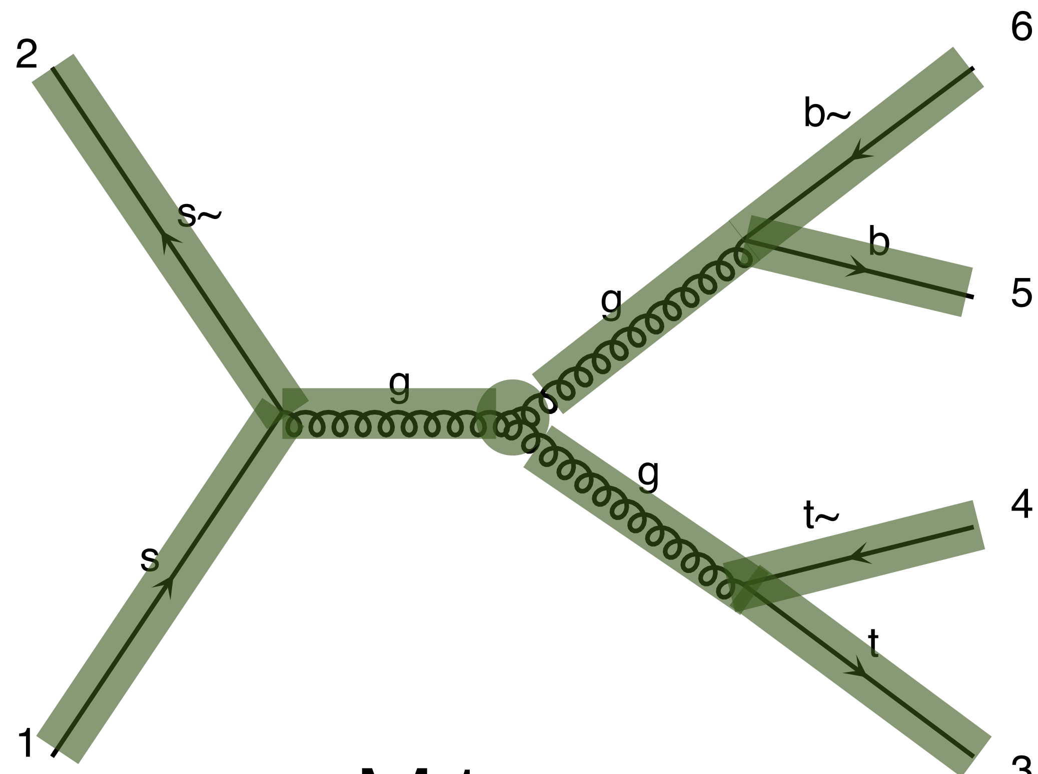
Number of routines: 9

Number of routines for both: 11

$$|M|^2 = |M_1 + M_2|^2$$

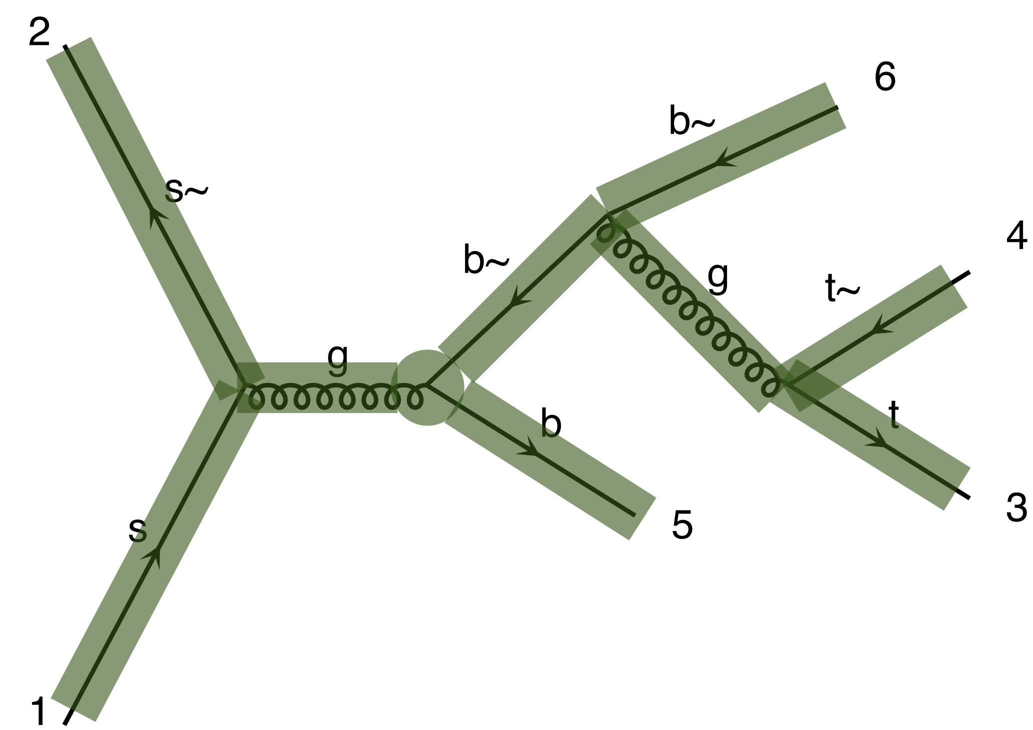
Real case

Known



M1

Number of routines: 10



M2

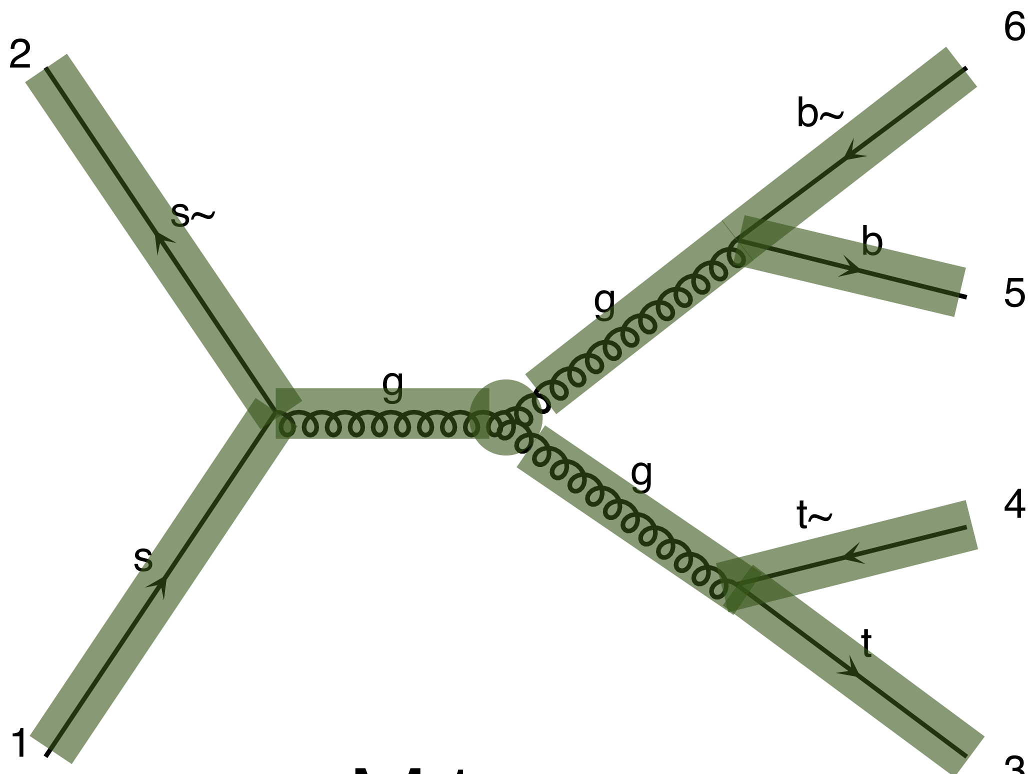
Number of routines: 10

Number of routines for both: 12

$$|M|^2 = |M_1 + M_2|^2$$

Real case

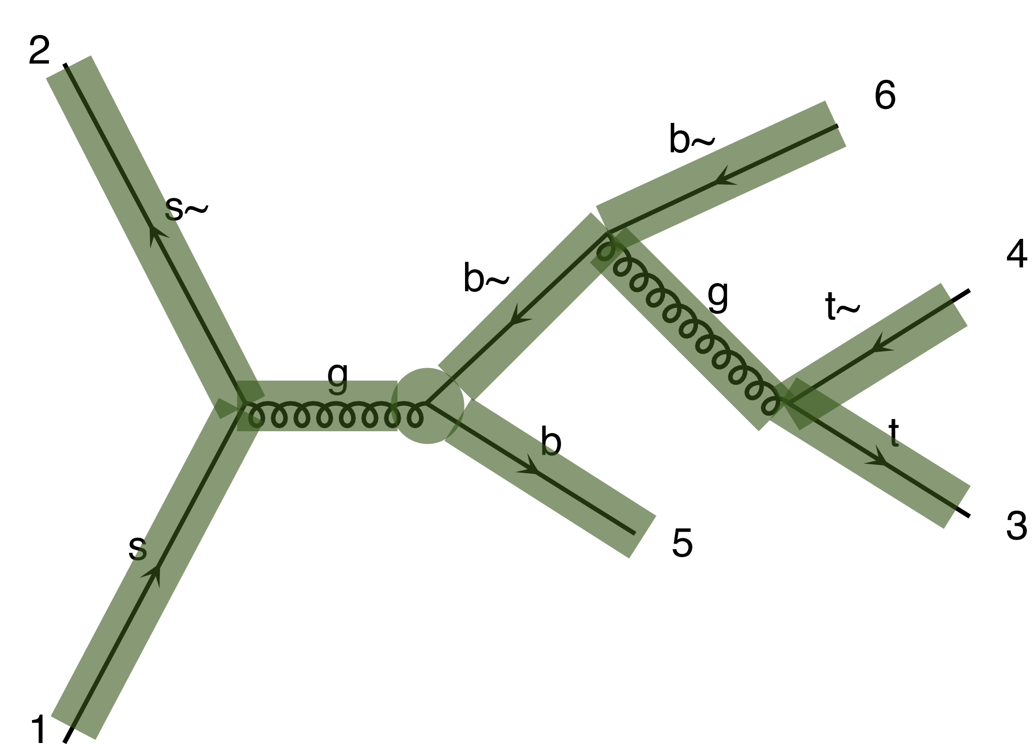
Known



M1

Number of routines: 10

2(N+1)



M2

Number of routines: 10

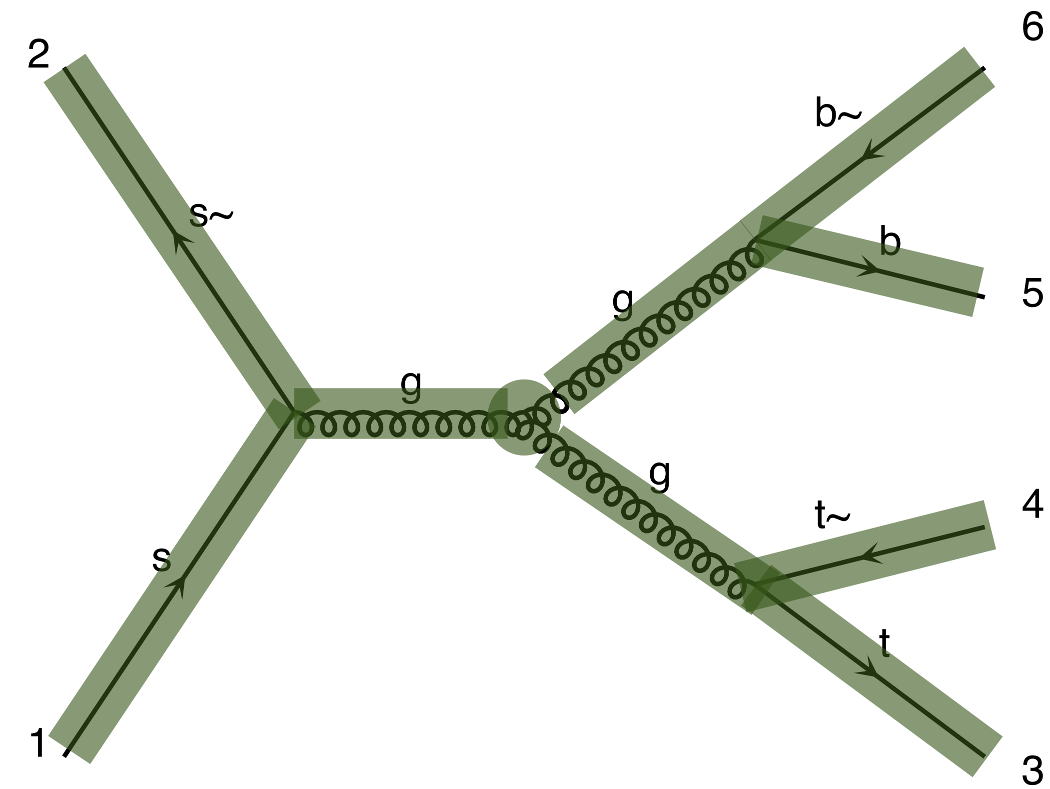
2(N+1)

Number of routines for both: 12

$$|M|^2 = |M_1 + M_2|^2$$

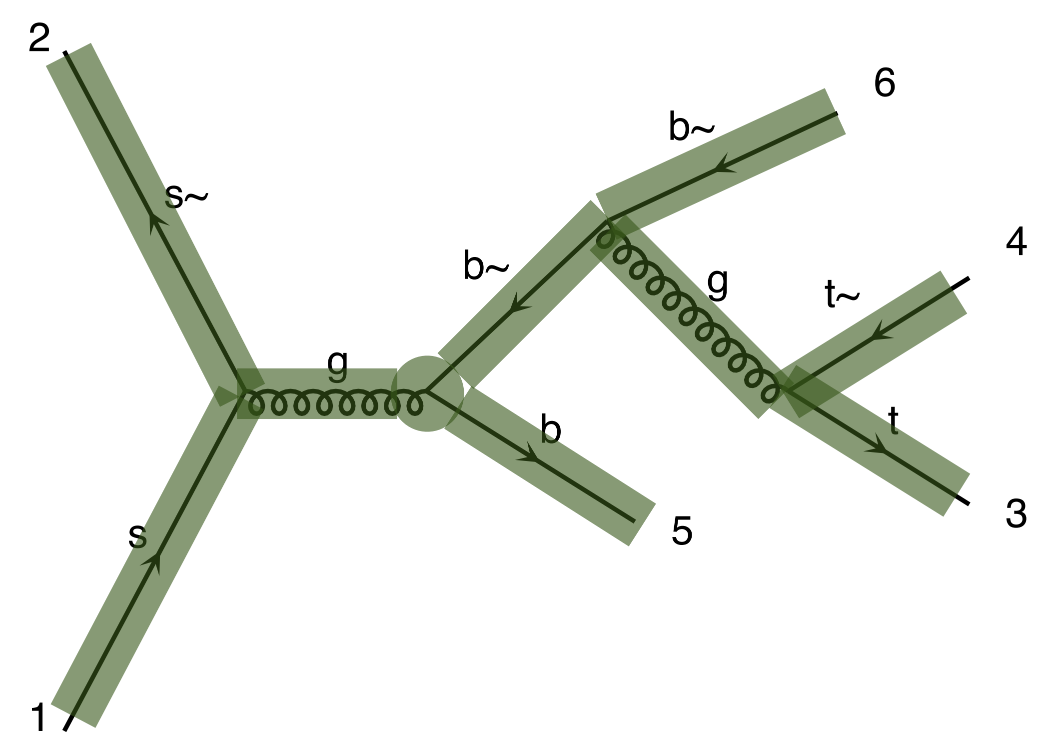
Real case

Known



Number of routines: 10

2(N+1)



Number of routines: 10

2(N+1)

Number of routines for both: 12

N!*2(N+1) ⟶ N!

Comparison

	M diag	N particle
Analytical	M^2	$(N!)^2$
Helicity	M	$(N!)\,2^N$
Recycling	M	$(N-1)!\,2^{(N-1)}$

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Recursion Relation	$log(M)$	$2^N\,2^{(N-1)}$	Not used in Madgraph

Comparison

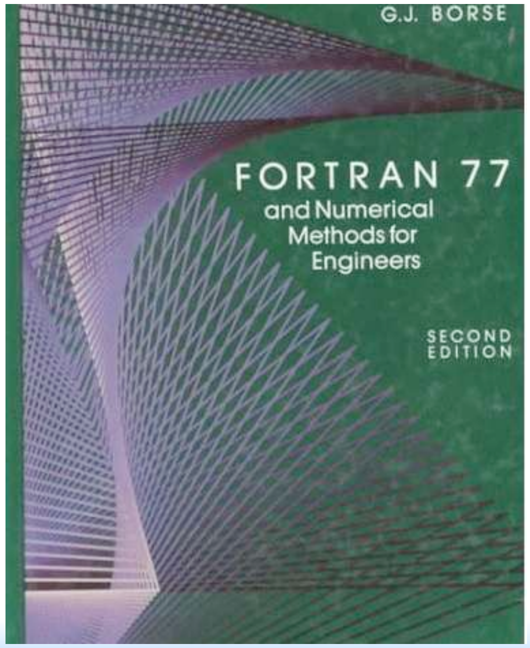
	M diag	N particle	2 > 6
Analytical	M^2	$(N!)^2$	1.6e9
Helicity	M	$(N!)\,2^N$	1.0e7
Recycling	M	$(N-1)!\,2^{(N-1)}$	6.5e5
Recursion Relation	$log(M)$	$2^N\,2^{(N-1)}$	3.2e4

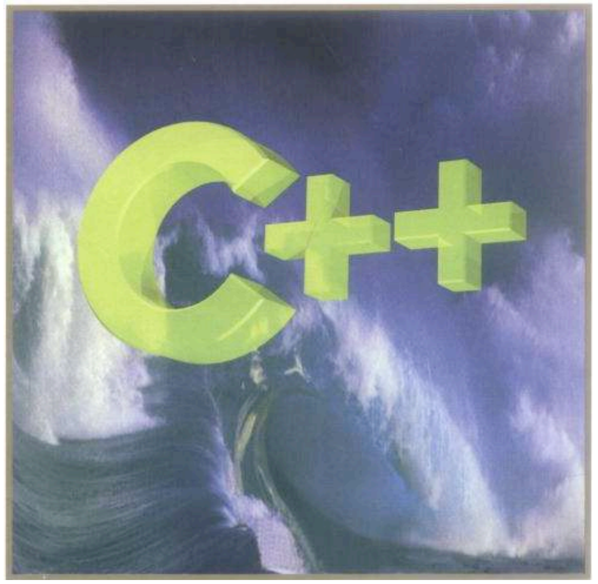
ALOHA

ALOHA ~~Google~~ translate

From: UFO To: Helicity Translate

Type text or a website address or translate a document.





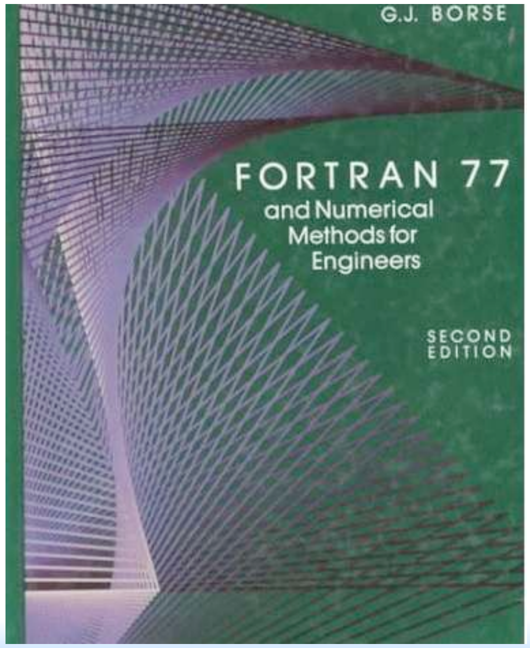
ALOHA

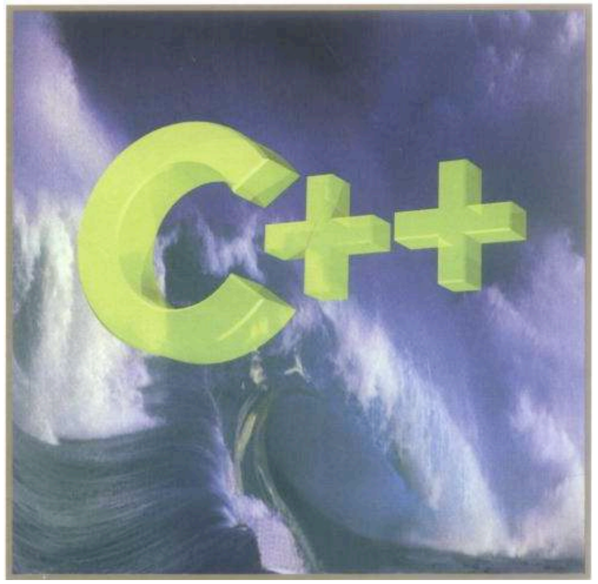
ALOHA ~~Google~~ translate

From: UFO ▾ To: Helicity Translate

Basically, any new operator can be handle by MG5/Pythia8 out of the box!

Type text or a website address or translate a document.





To Remember

- Numerical computation faster than analytical computation
- We are able to compute matrix-element
    - ➡ for large number of final state
    - ➡ for any BSM theory

Plan

- Overview of Monte-Carlo
- Details of the computation
    - Evaluation of matrix-element
    - Phase-Space integration

Monte Carlo Integration

Calculations of cross section or decay widths involve integrations over high-dimension phase space of very peaked functions:

$$\sigma = \frac{1}{2s}\int |\mathcal{M}|^2 d\Phi(n)$$

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$$\sigma = \frac{1}{2s} \int |\mathcal{M}|^2 d\Phi(n)$$

$Dim[\Phi(n)] \sim 3n$

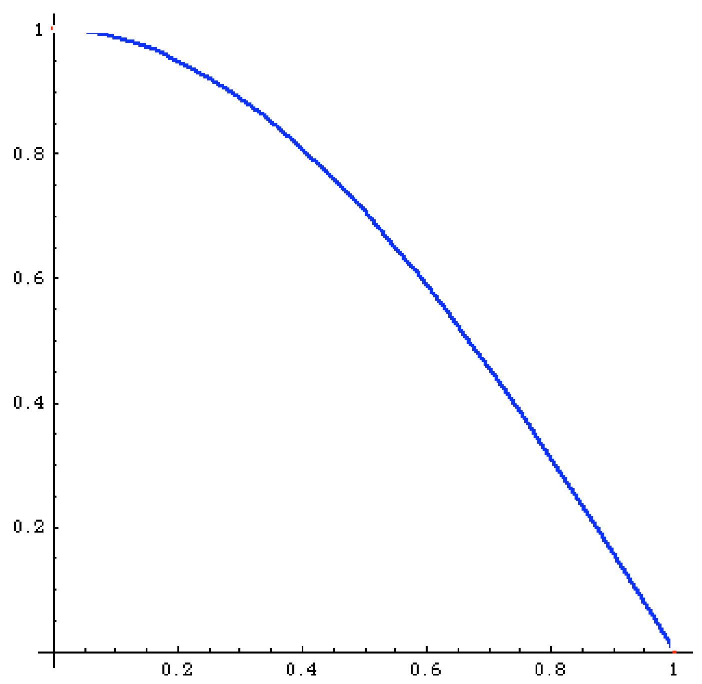
General and flexible method is needed

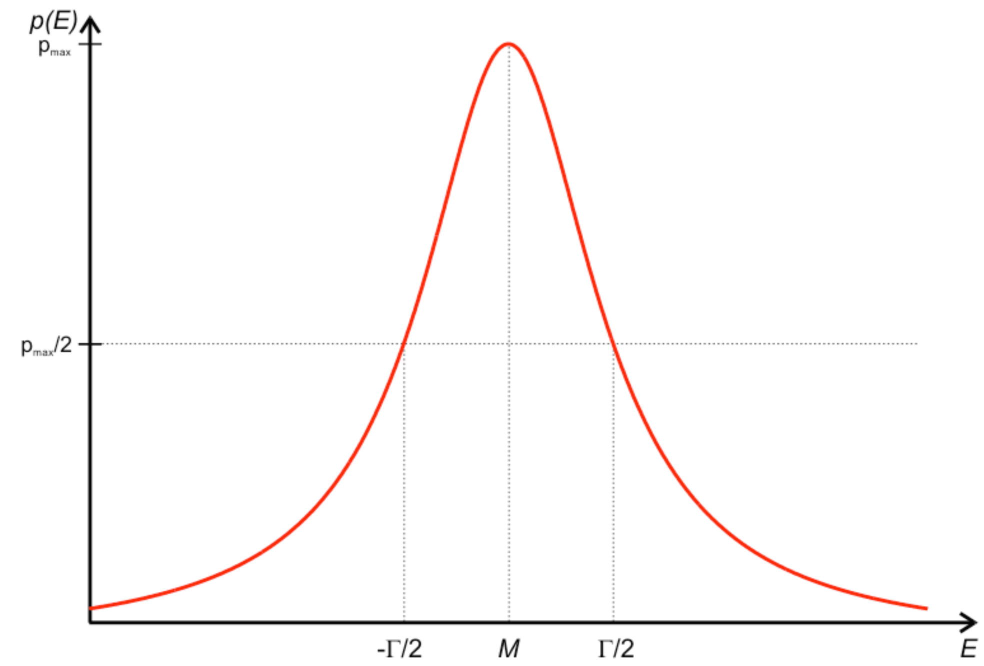
Integration

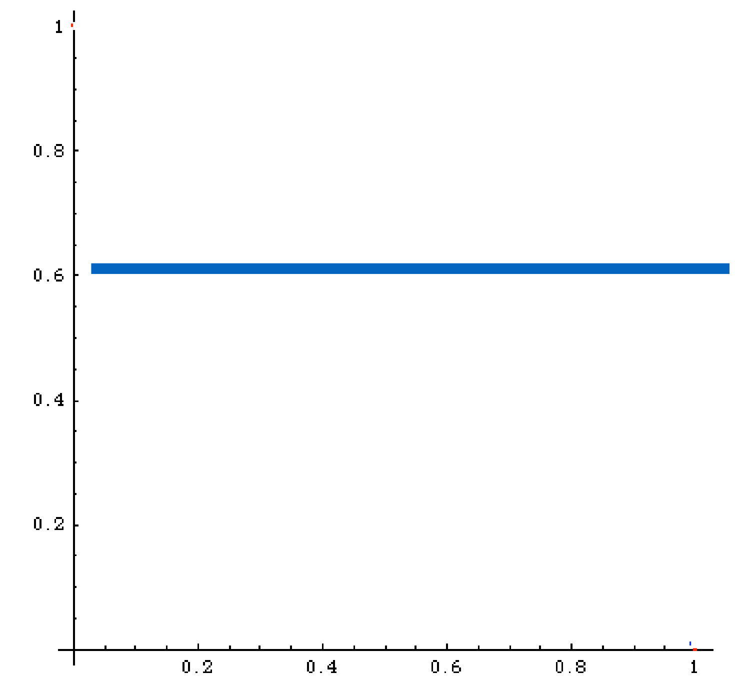
$$I = \int_0^1 dx \cos \frac{\pi}{2} x$$

$$\int \frac{dq^2}{(q^2 - M^2 + iM\Gamma)^2}$$

$$\int dx\, C$$







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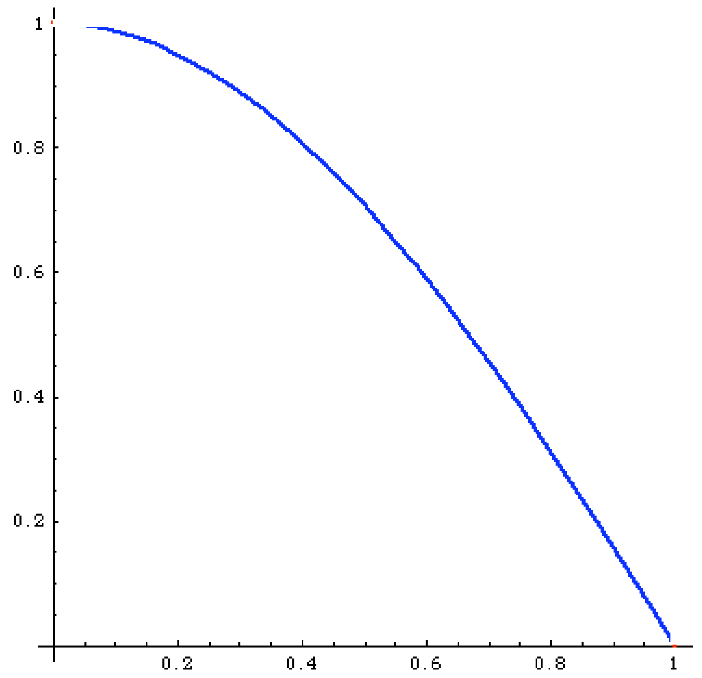
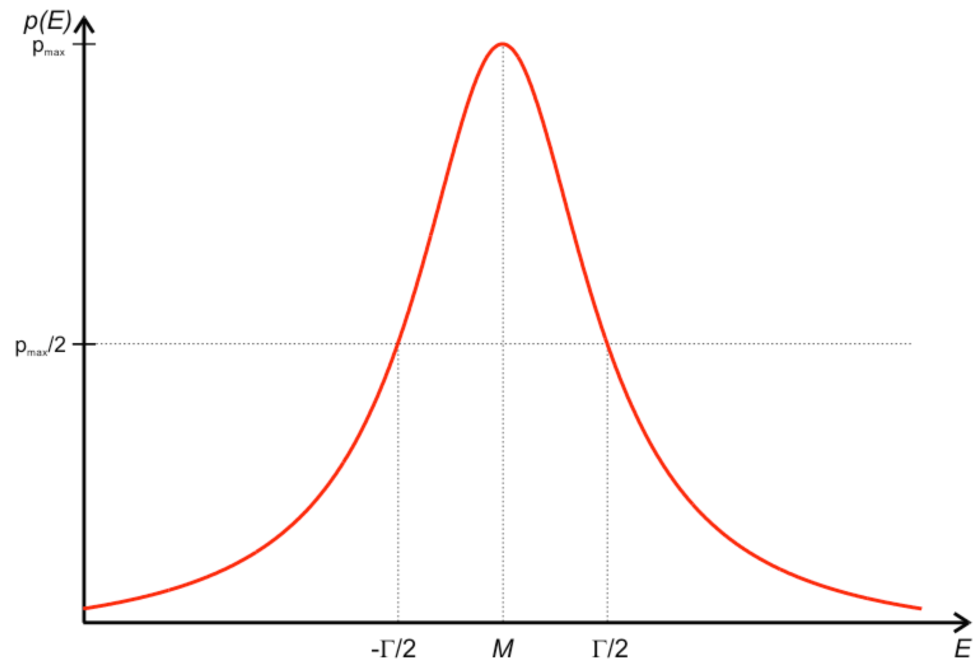
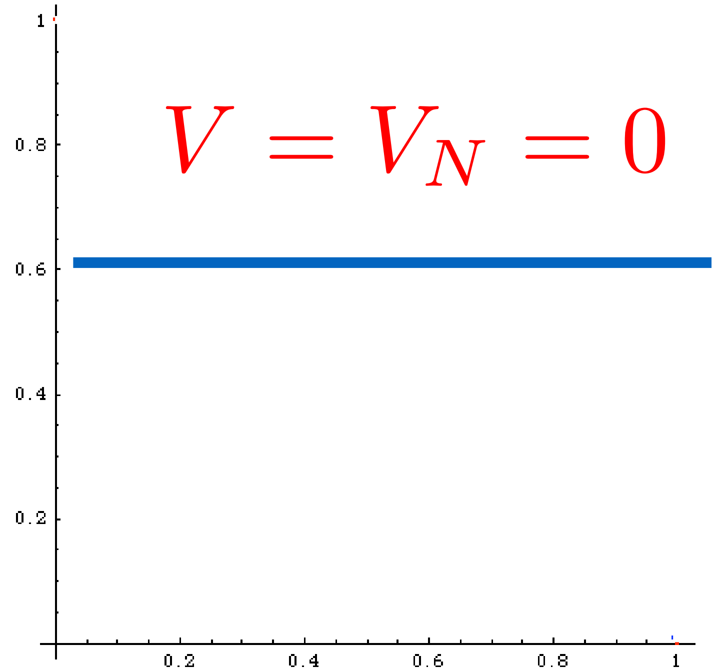






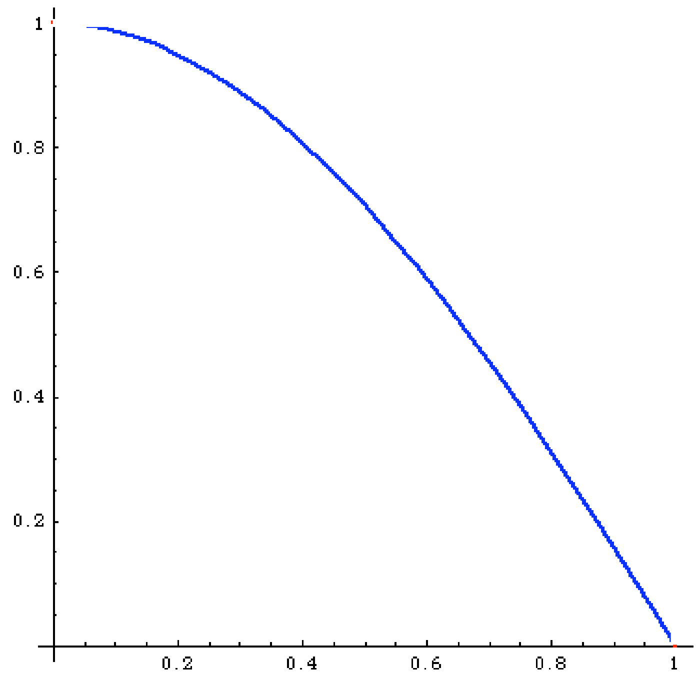
$V = V_N = 0$

Method of evaluation

- MonteCarlo $1/\sqrt{N}$
- Trapezium $1/N^2$
- Simpson $1/N^4$

Integration

$$I = \int_0^1 dx \cos\frac{\pi}{2}x$$



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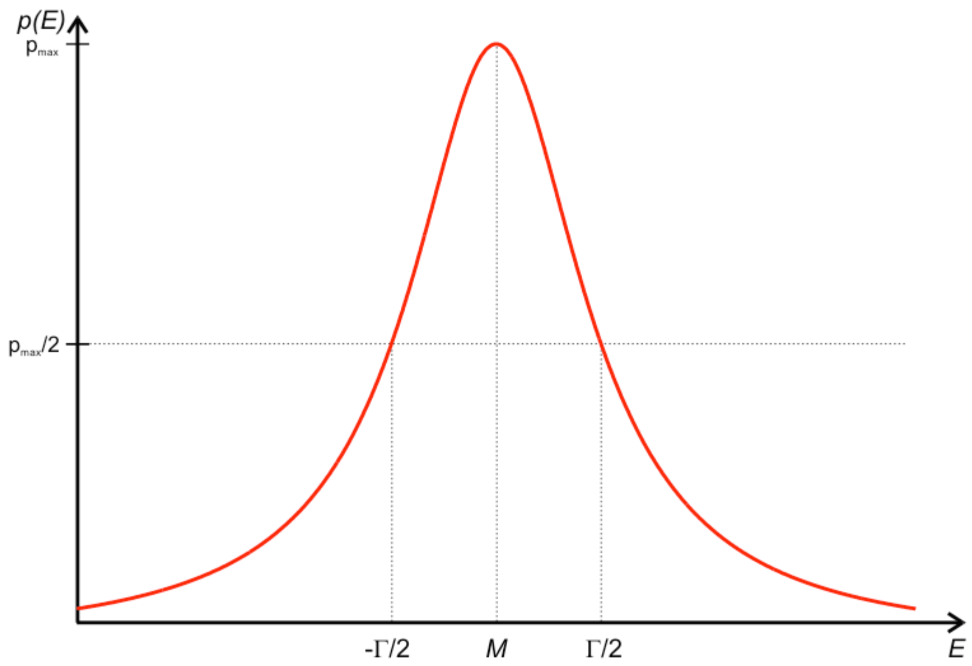
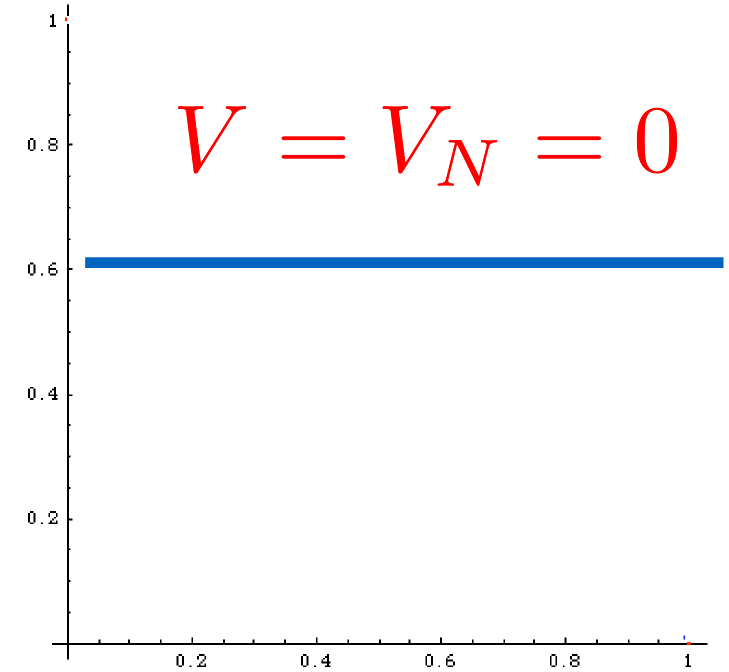


$$\int dx\, C$$



$V = V_N = 0$

	simpson	MC
3	0,638	0,3
5	0,6367	0,8
20	0,63662	0,6
100	0,636619	0,65
1000	0,636619	0,636

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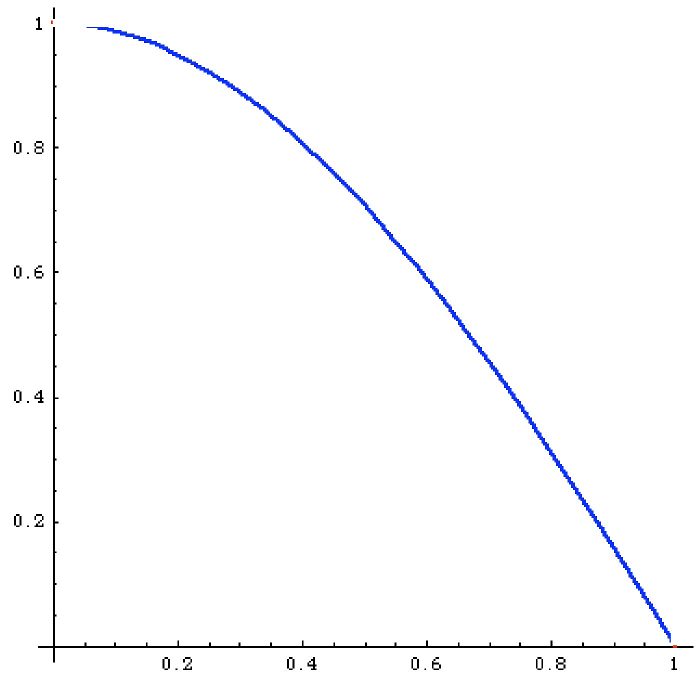

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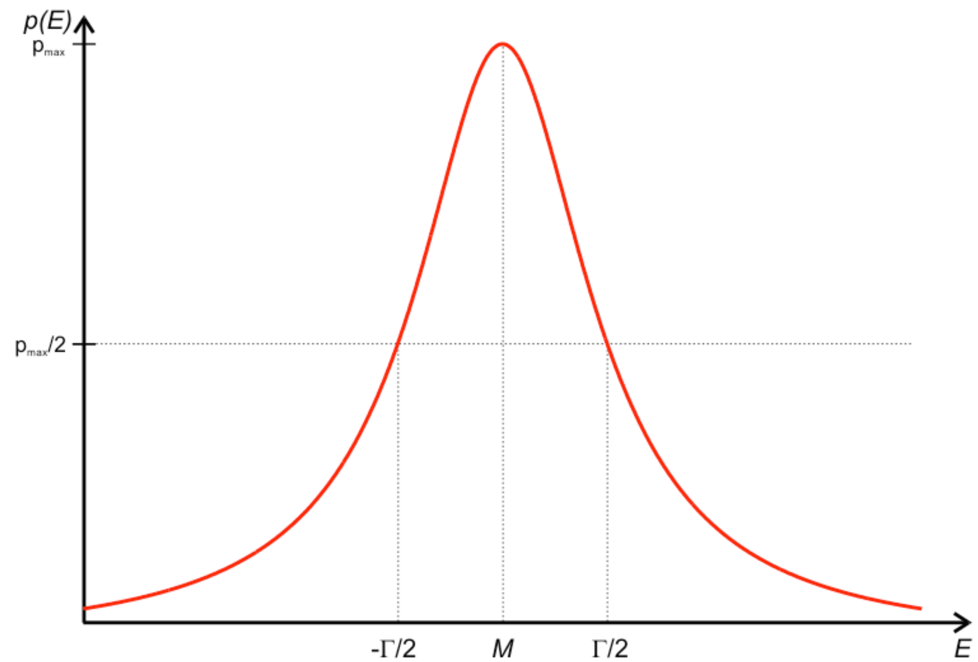
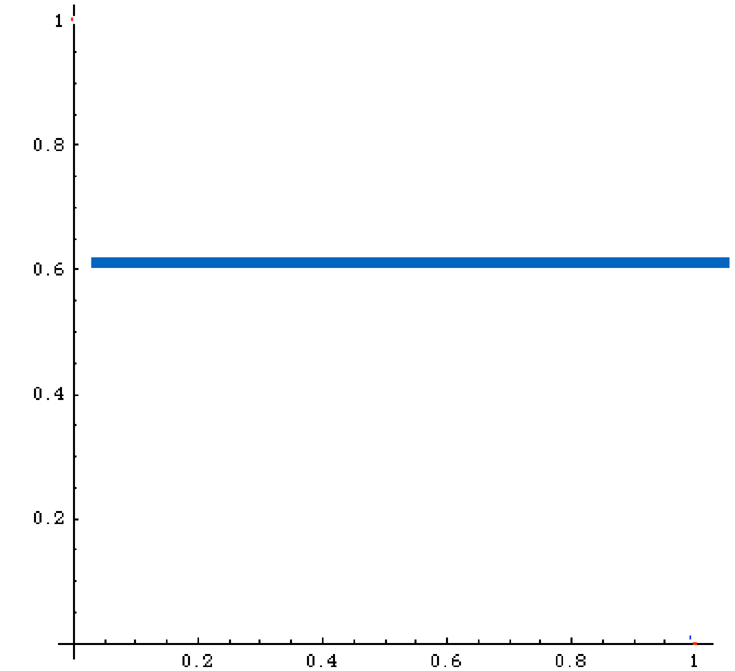


$$\int dx\, C$$



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More Dimension →

$1/\sqrt{N}$

$1/N^{2/d}$

$1/N^{4/d}$

Integration

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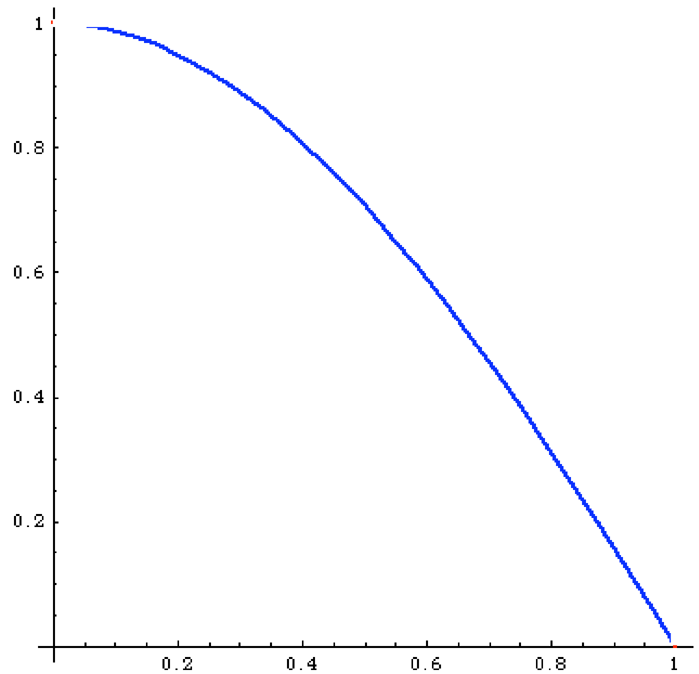

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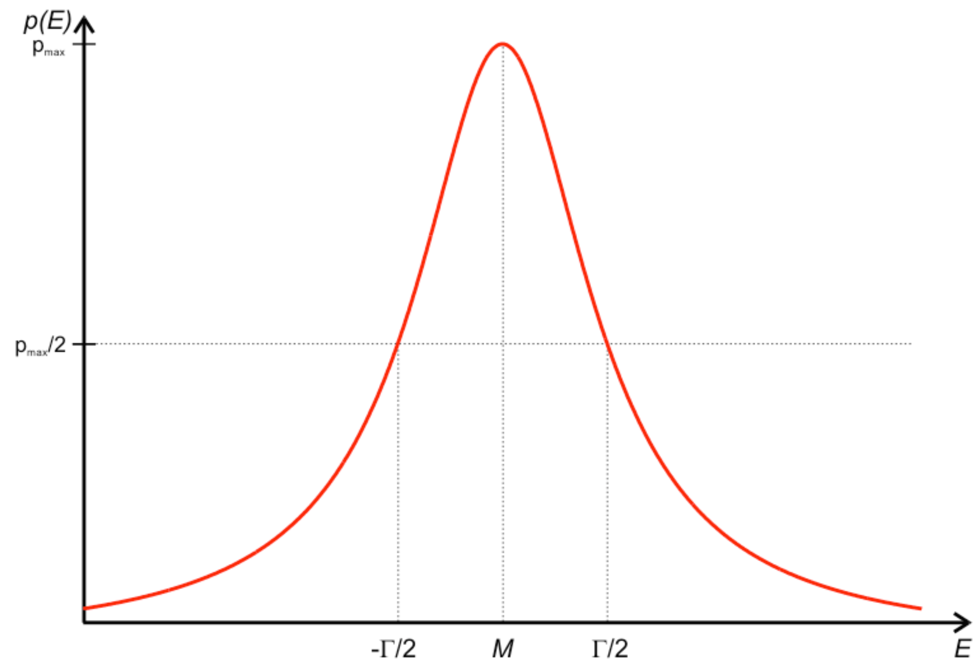

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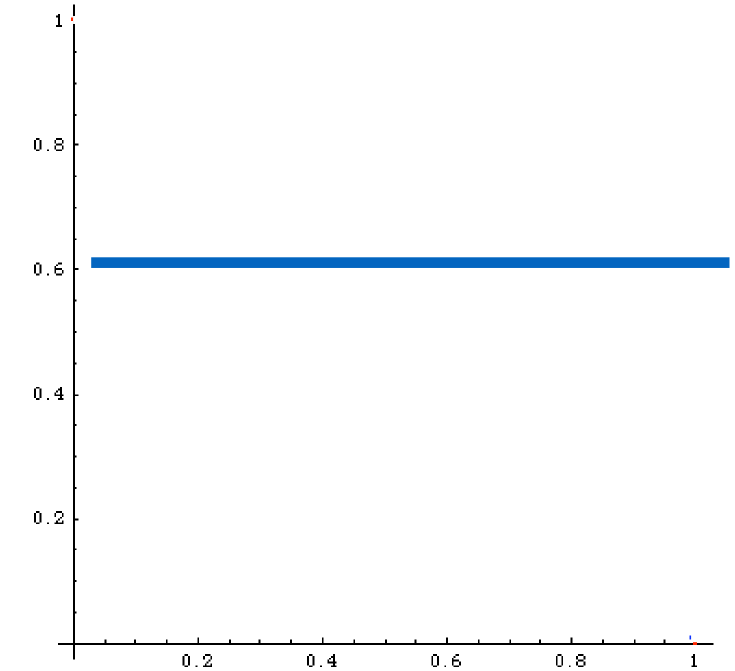

$$I = \int_{x_1}^{x_2} f(x)dx \Rightarrow I_N = (x_2 - x_1)\frac{1}{N}\sum_{i=1}^{N} f(x)$$

$$V = (x_2 - x_1)\int_{x_1}^{x_2} [f(x)]^2 dx - I^2 \Rightarrow V_N = (x_2 - x_1)^2 \frac{1}{N}\sum_{i=1}^{N}[f(x)]^2 - I_N^2$$

Integration

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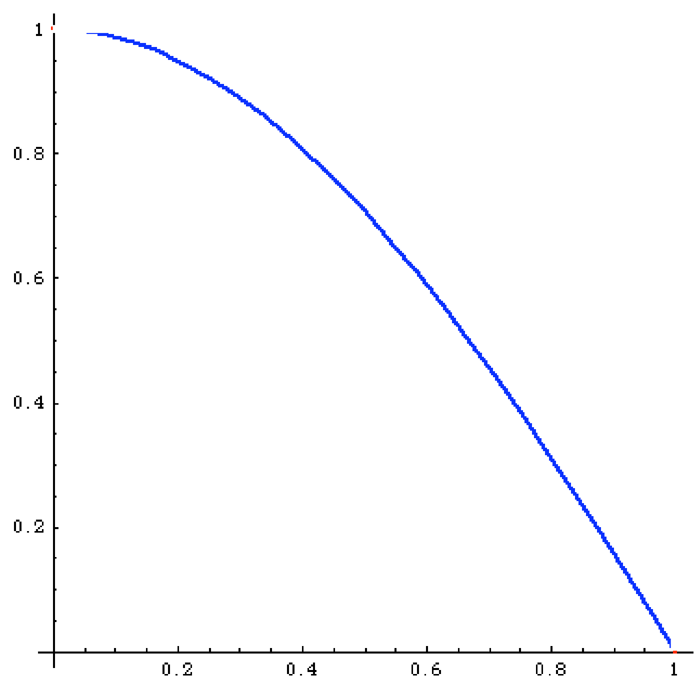

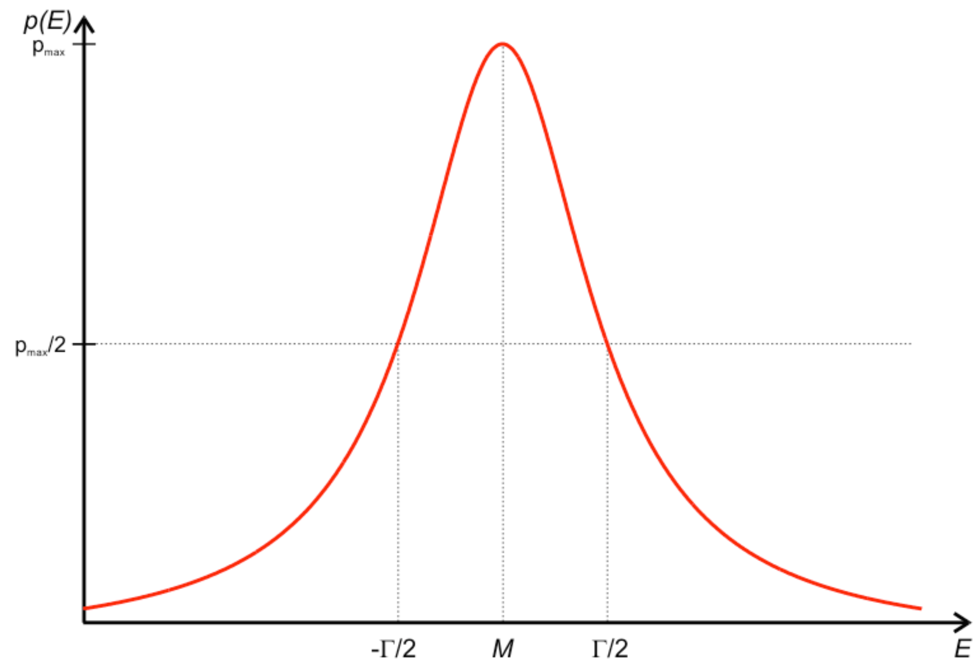

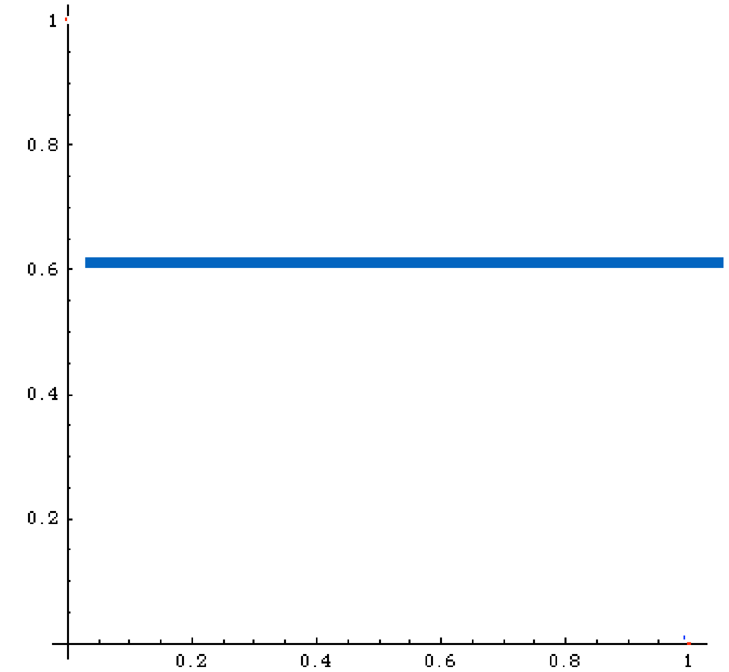

$$I = \int_{x_1}^{x_2} f(x)dx \quad \Rightarrow \quad I_N = (x_2 - x_1)\frac{1}{N}\sum_{i=1}^{N} f(x)$$

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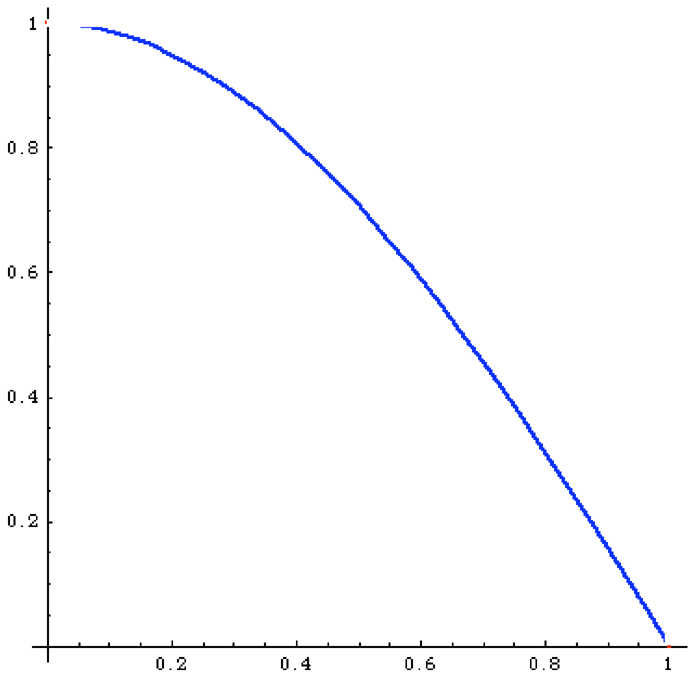
$$I = I_N \pm \sqrt{V_N/N}$$

Integration

$$I = \int_0^1 dx \cos\frac{\pi}{2}x$$



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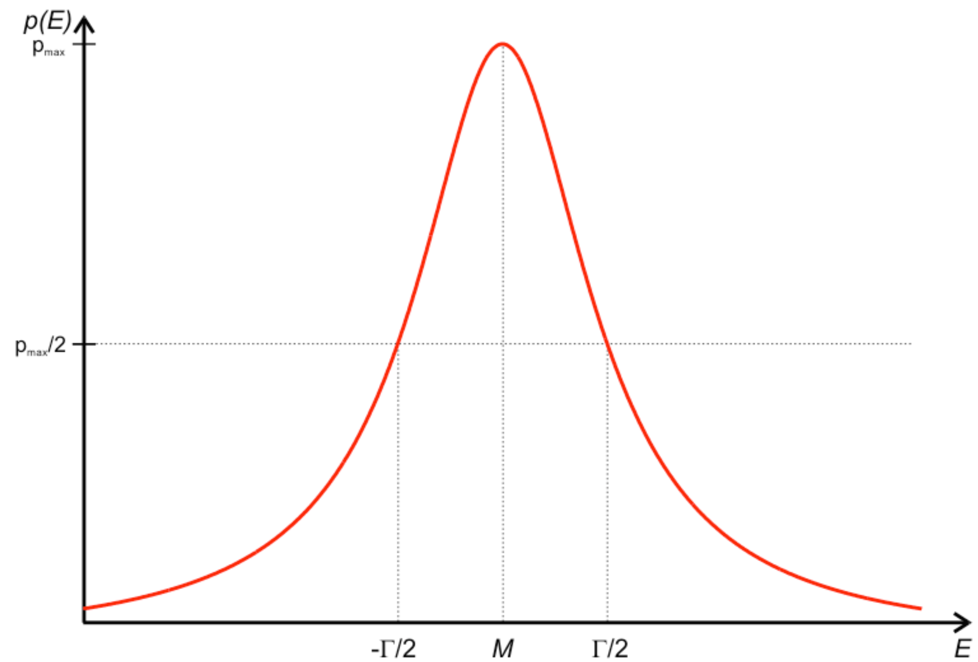

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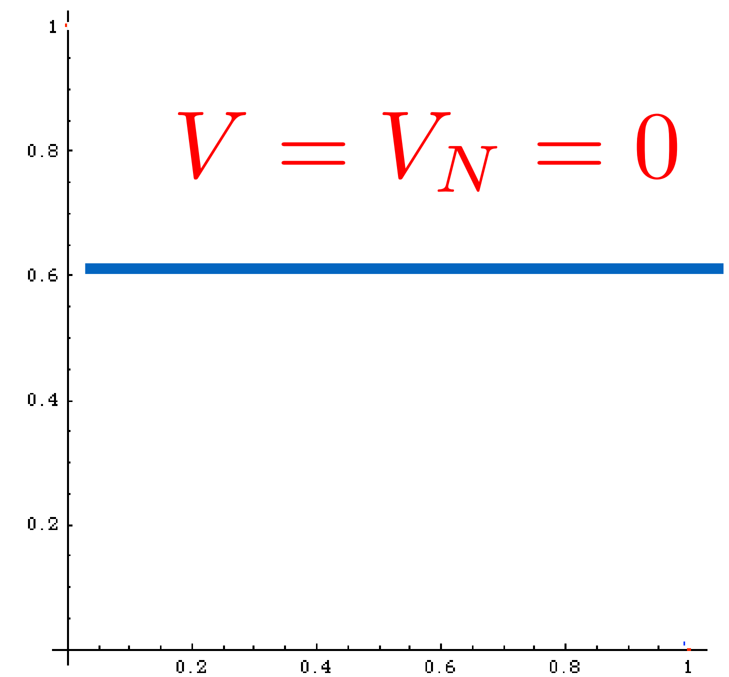

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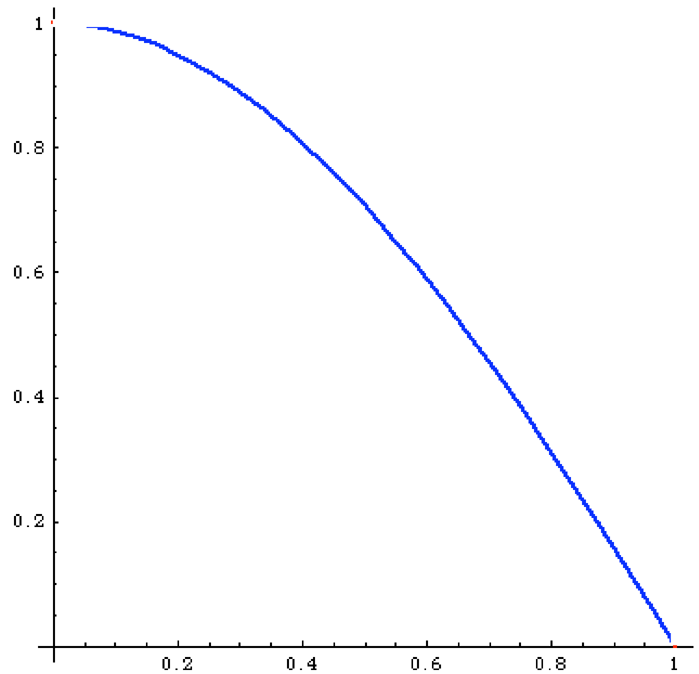

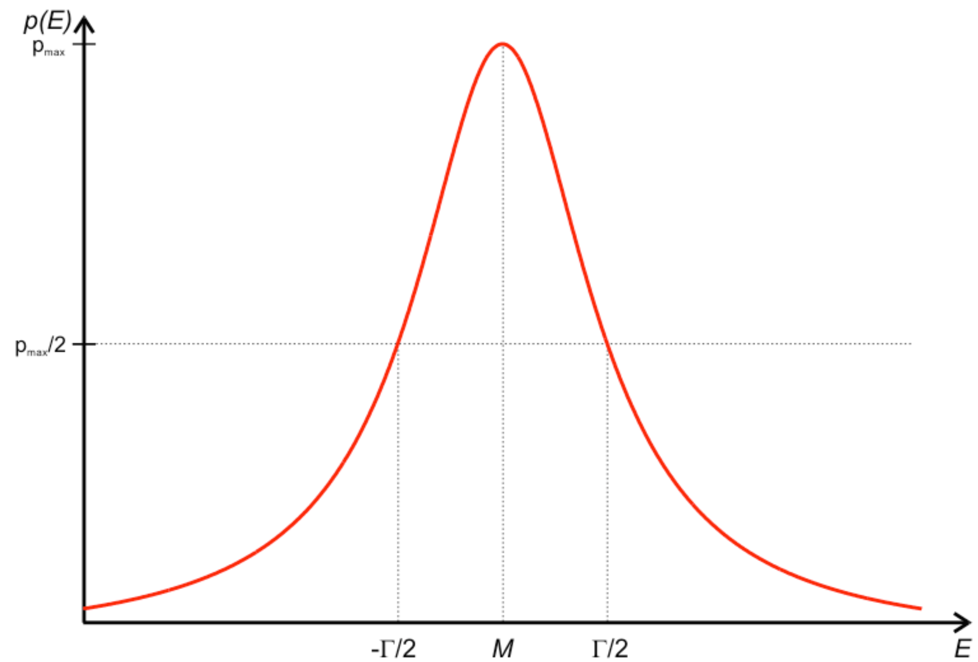

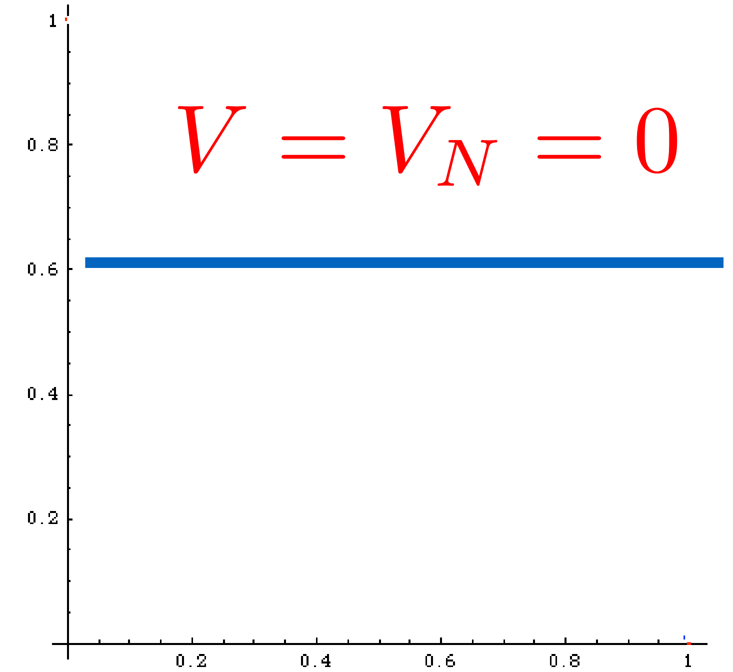

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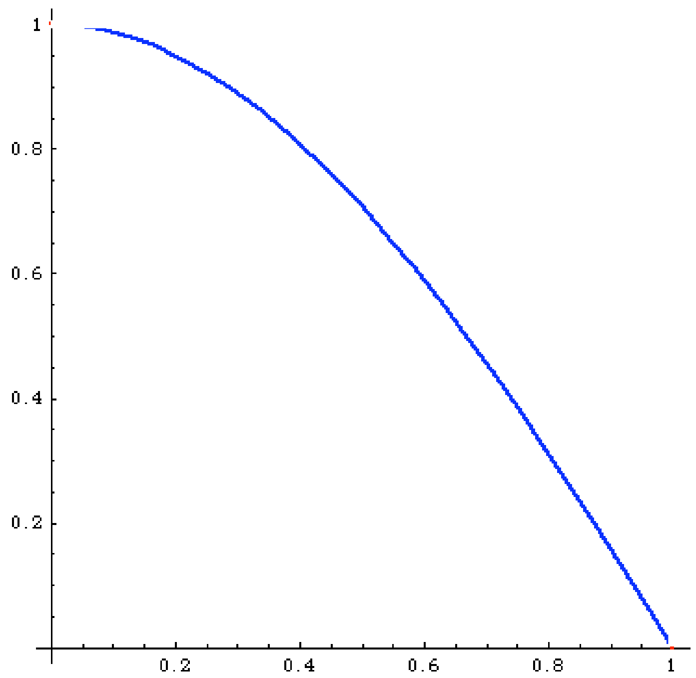
$$I = I_N \pm \sqrt{V_N/N}$$

Can be minimized!

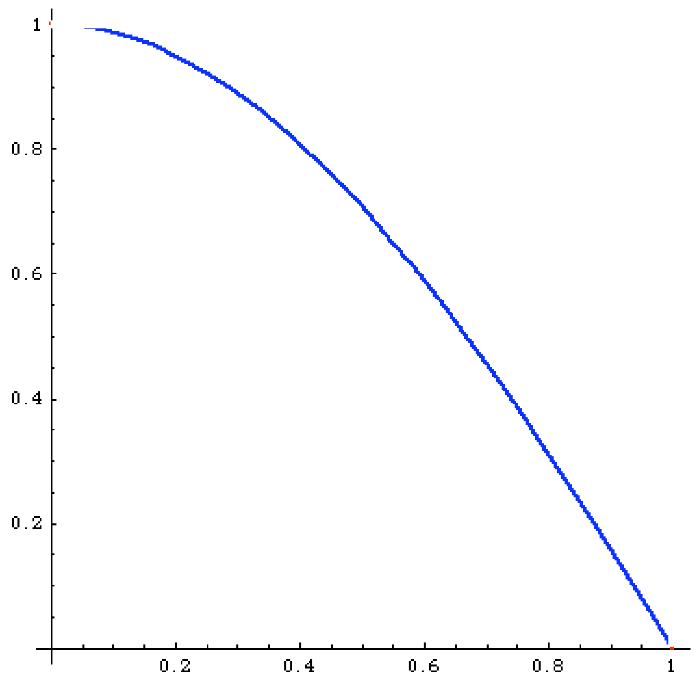
Importance Sampling



$$I = \int_0^1 dx \cos \frac{\pi}{2} x$$

$$I_N = 0.637 \pm 0.307/\sqrt{N}$$

Importance Sampling



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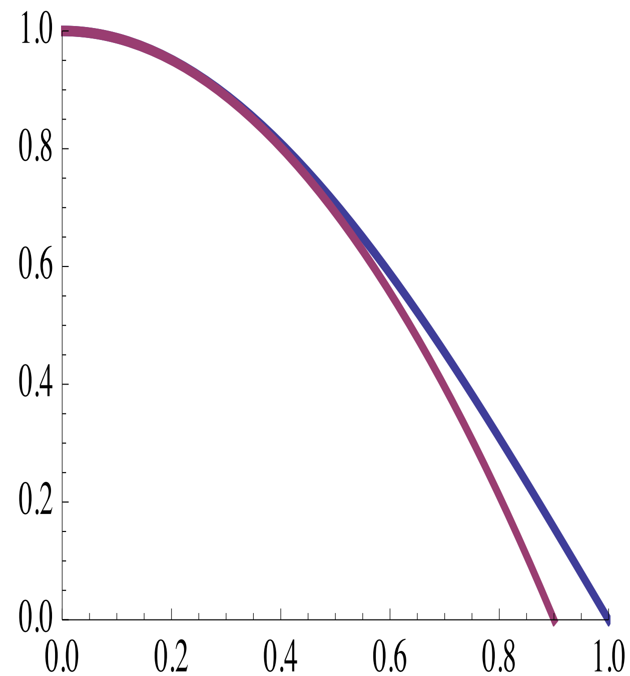
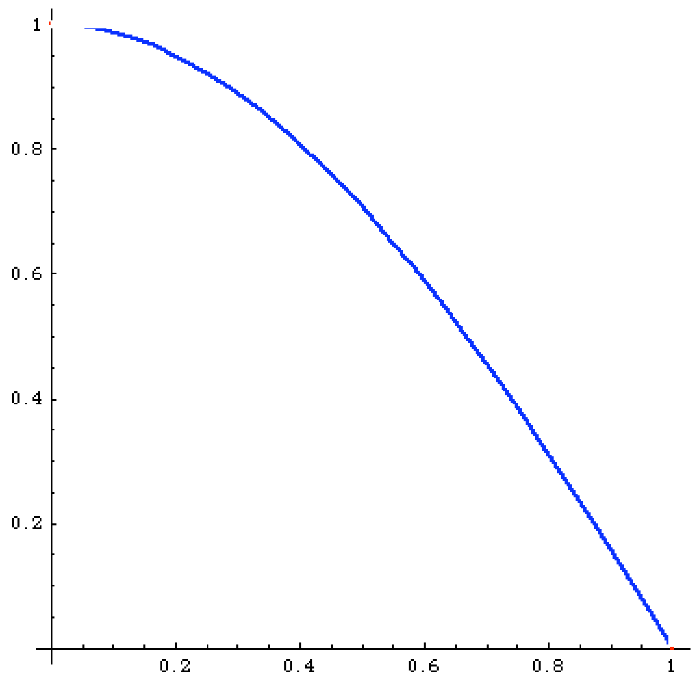


$$I = \int_0^1 dx (1 - cx^2) \frac{\cos\left(\frac{\pi}{2} x\right)}{(1 - cx^2)}$$

Importance Sampling



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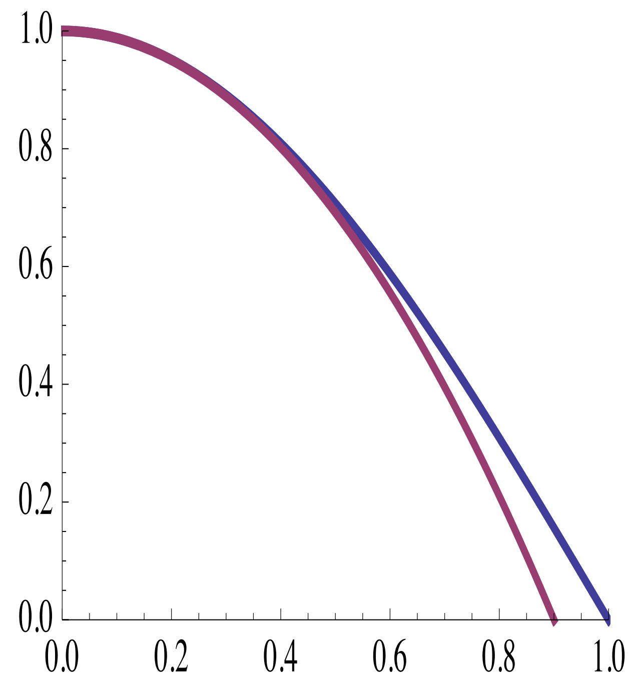

$$I = \int_0^1 dx(1-cx^2)\frac{\cos\left(\frac{\pi}{2}x\right)}{(1-cx^2)} = \int_{\xi_1}^{\xi_2} d\xi \frac{\cos\frac{\pi}{2}x[\xi]}{1-x[\xi]^2c}$$

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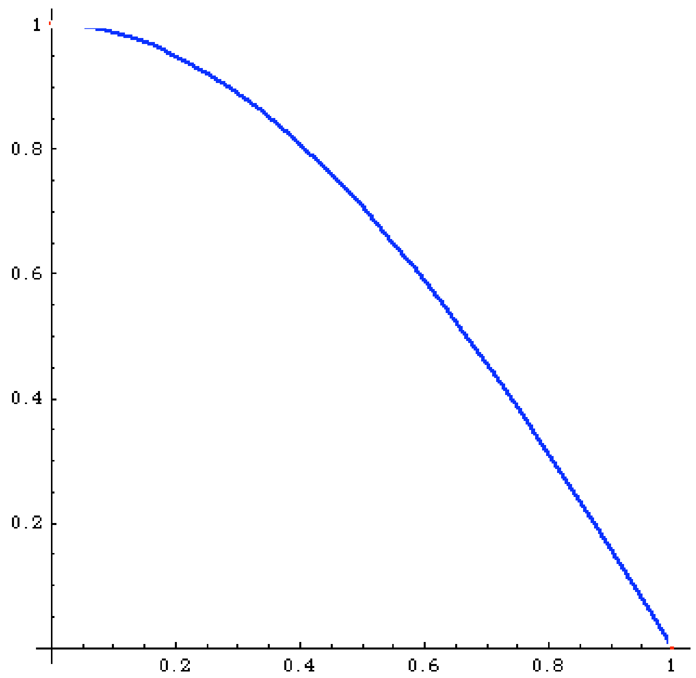

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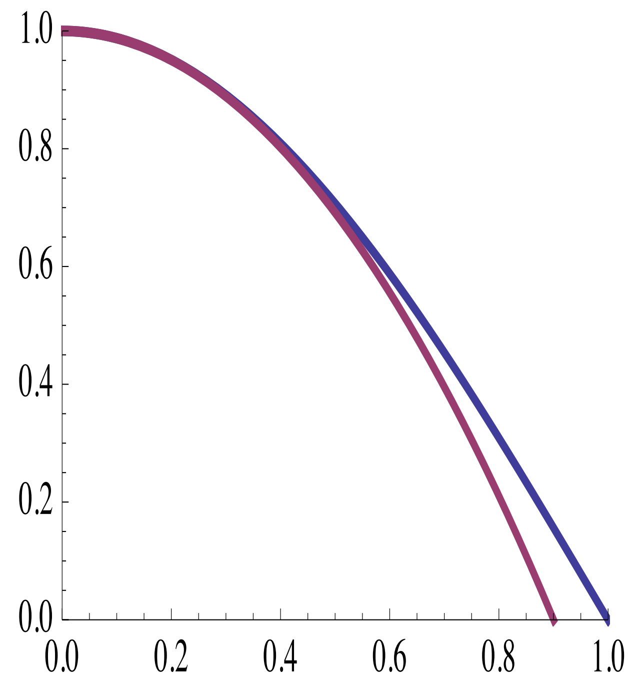

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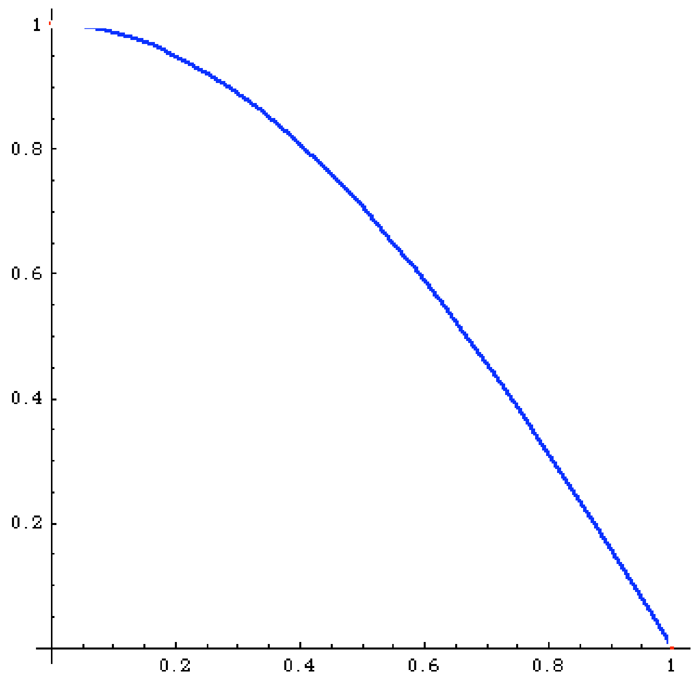
$\simeq 1$

Importance Sampling



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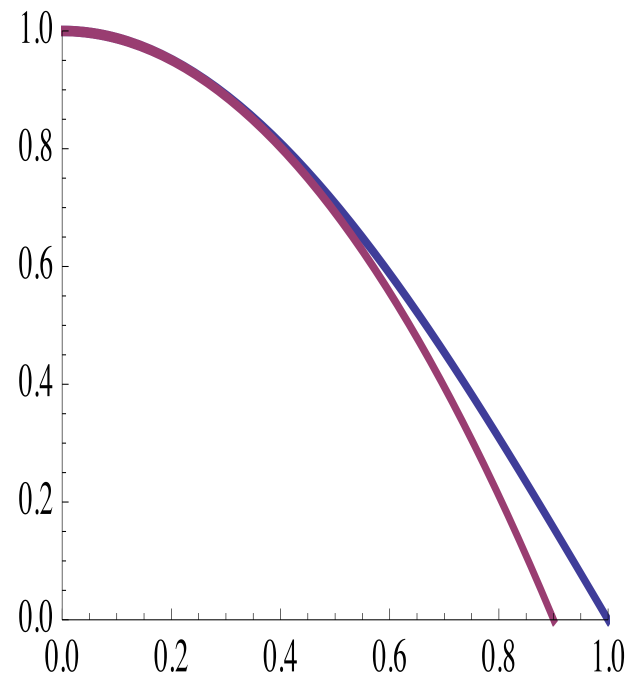

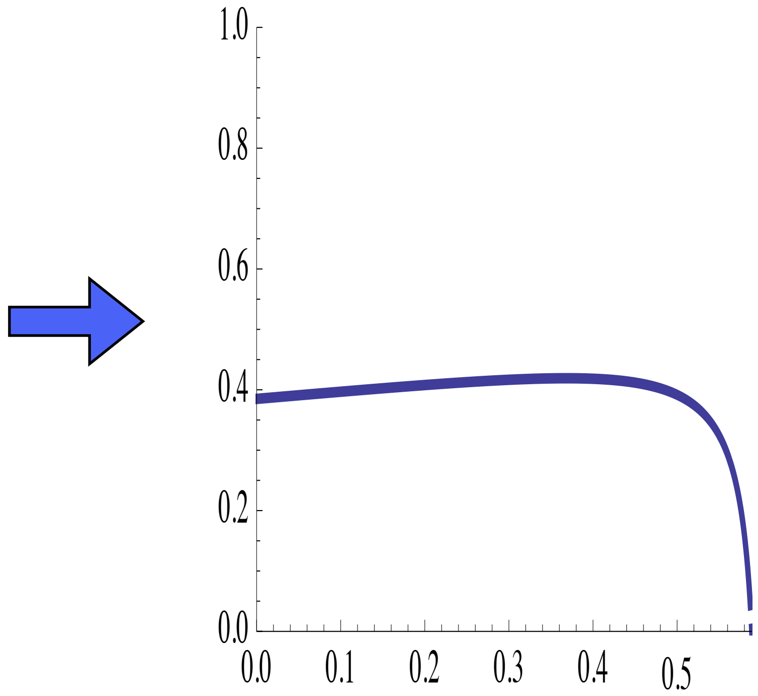

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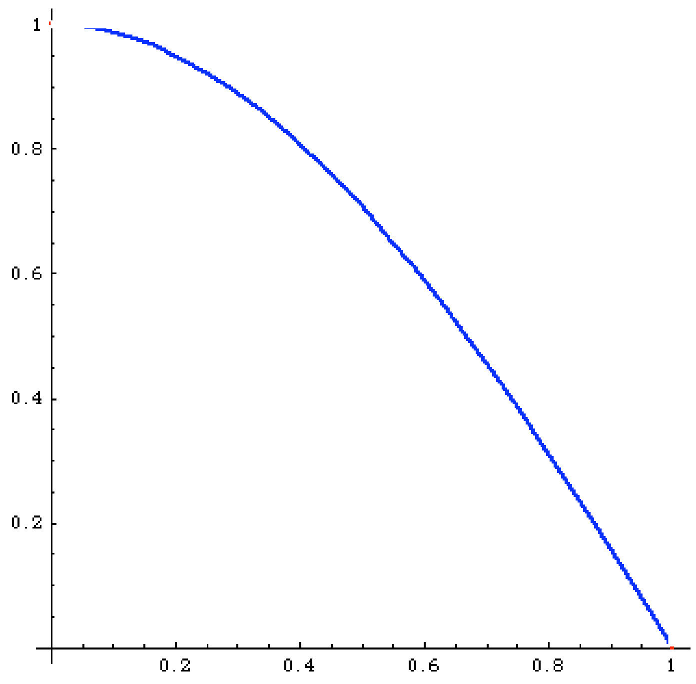

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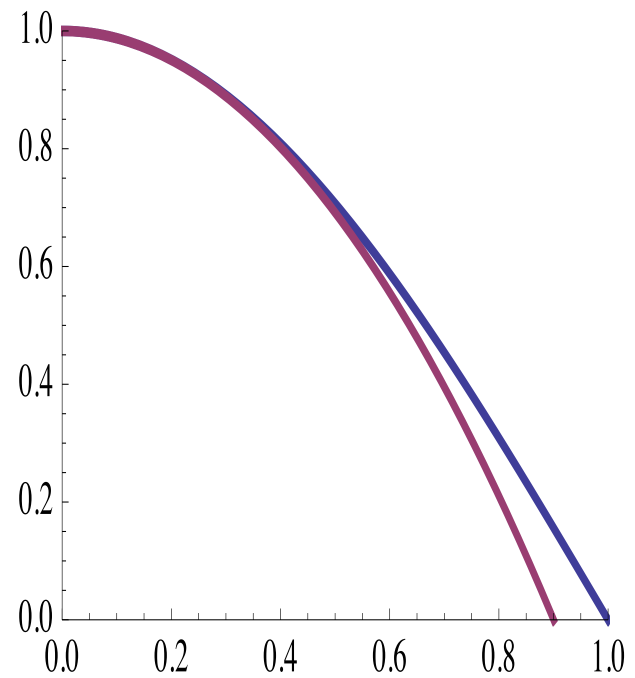

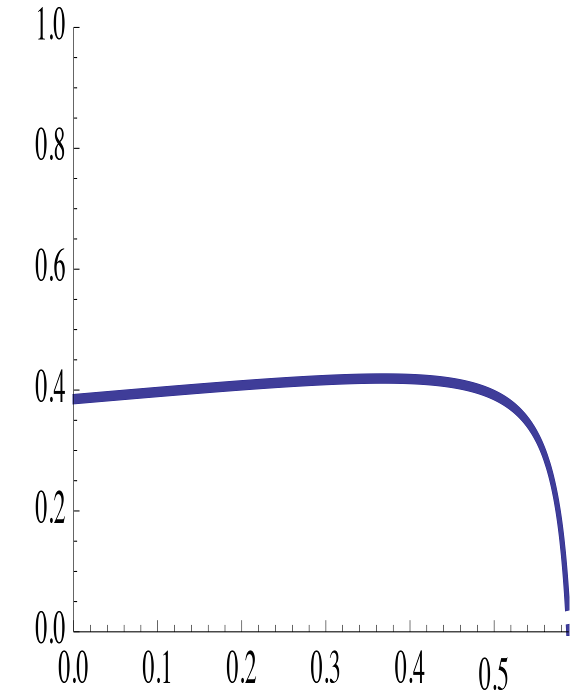

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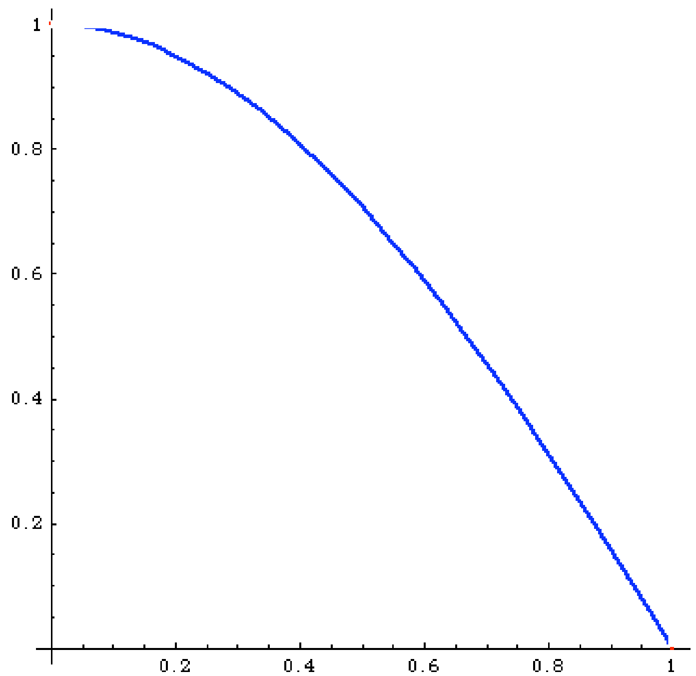
$\simeq 1$

$$I_N = 0.637 \pm 0.031/\sqrt{N}$$

Importance Sampling



$$I = \int_0^1 dx \cos \frac{\pi}{2} x$$

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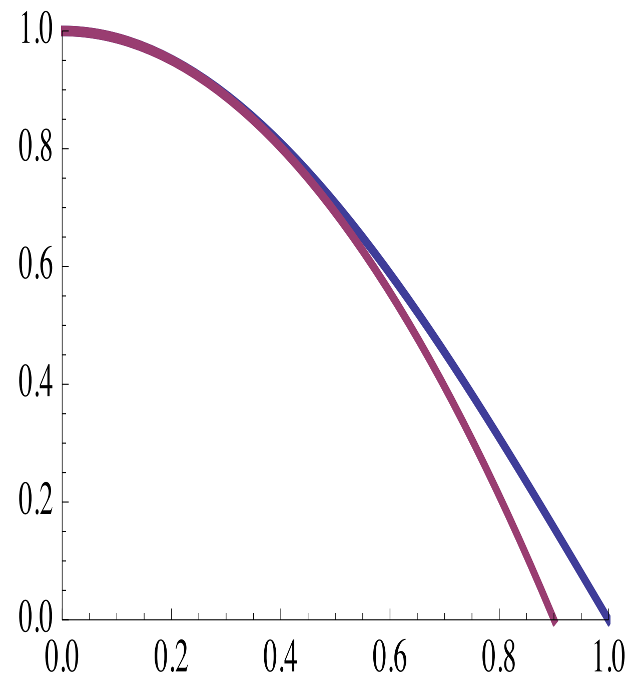

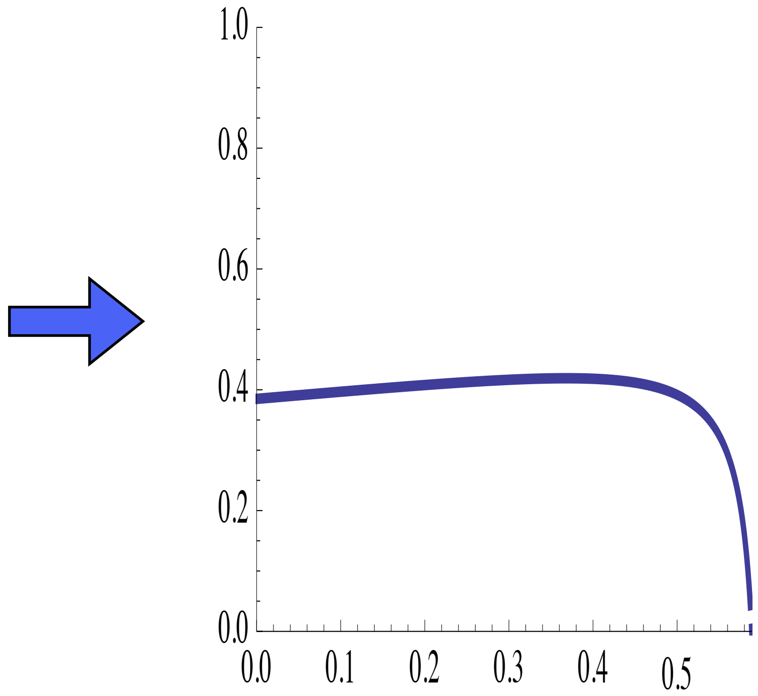

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$$\simeq 1$$

$$I_N = 0.637 \pm 0.031/\sqrt{N}$$

The Phase-Space parametrization is important to have an efficient computation!

Importance Sampling

Key Point

- Generate the random point in a distribution which is close to the function to integrate.
- This is a change of variable, such that the function is flatter in this new variable.
- Needs to know an approximate function.

Adaptative Monte-Carlo

- Create an approximation of the function on the flight!

Adaptative Monte-Carlo

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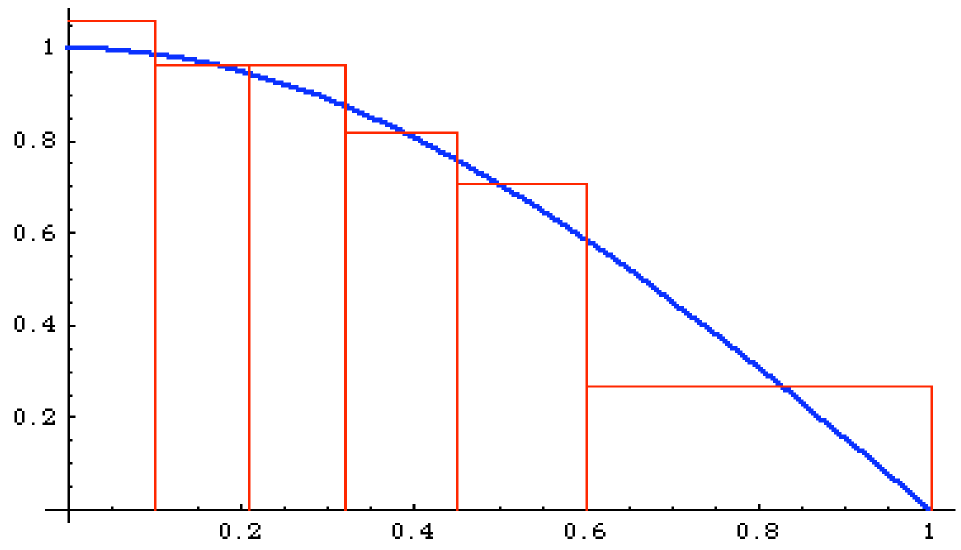


Algorithm

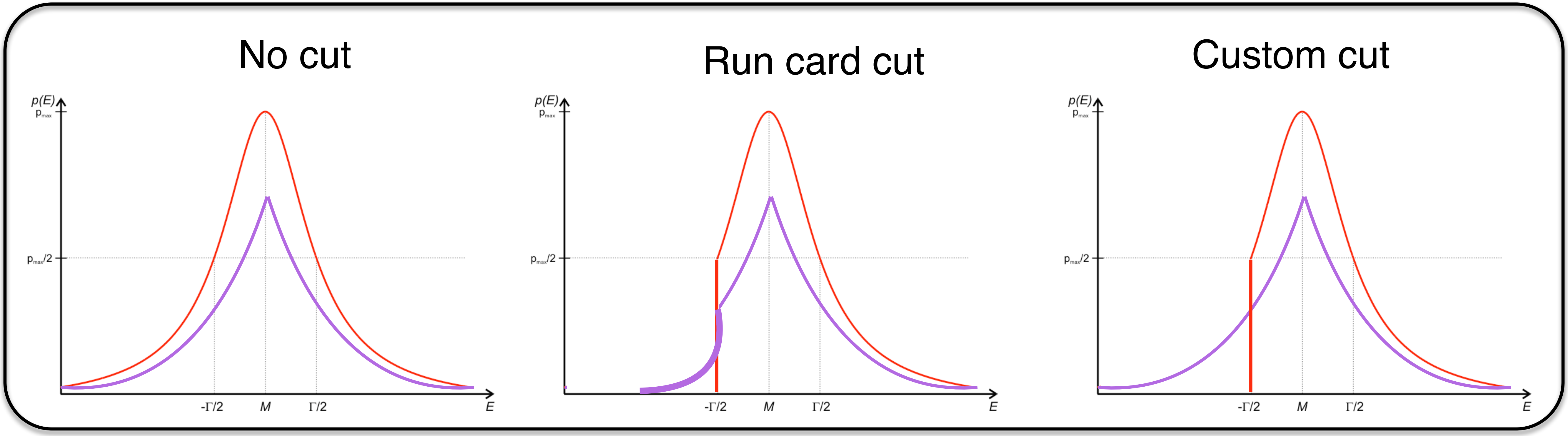
1. Creates bin such that each of them have the same contribution.

   ➡Many bins where the function is large

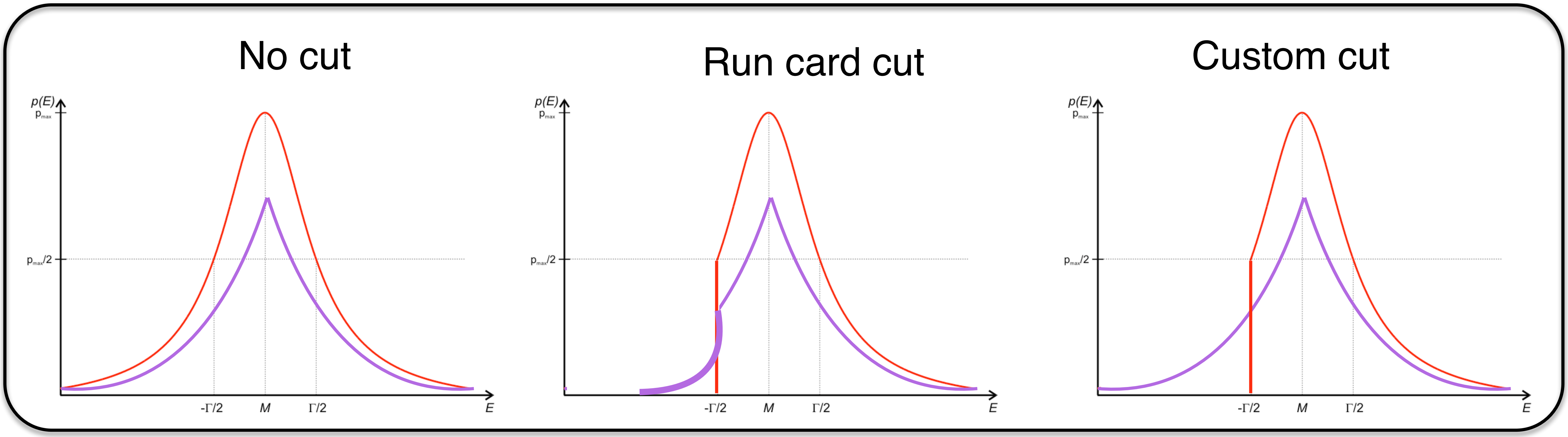
2. Use the approximate for the importance sampling method.

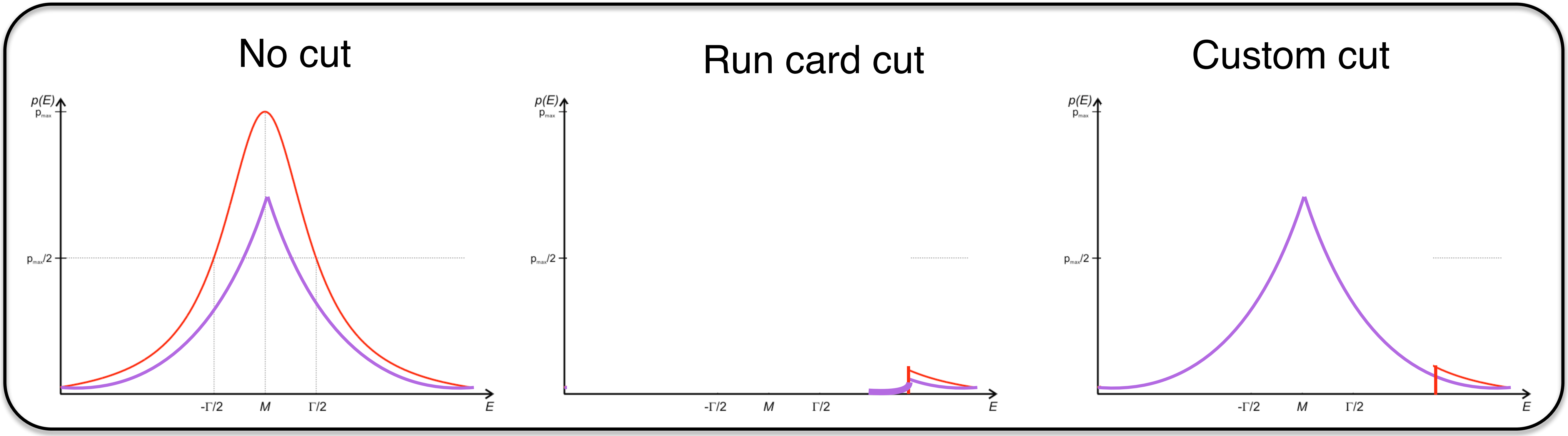
Cut Impact

- Events are generated according to our best knowledge of the function
  - ➡Basic cut include in this “best knowledge”
  - ➡Custom cut are ignored



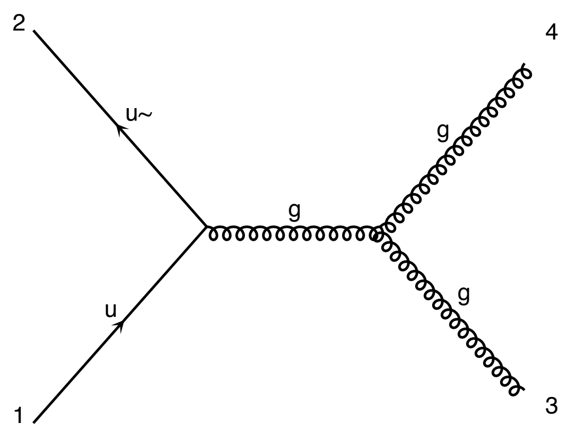
Cut Impact

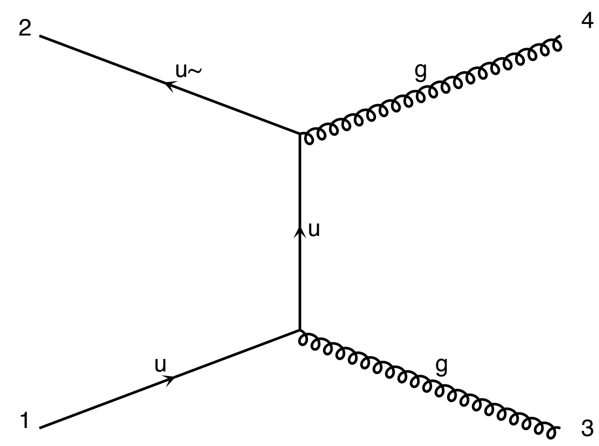


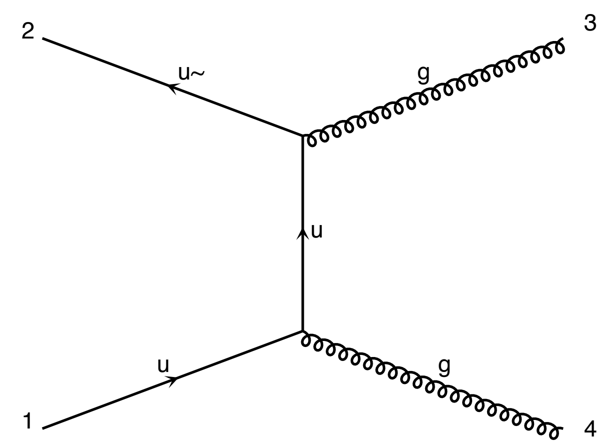


Might miss the contribution and think it is just zero.

Example: QCD 2 → 2







$$\propto \frac{1}{\hat{s}} = \frac{1}{(p_1 + p_2)^2} \qquad \propto \frac{1}{\hat{t}} = \frac{1}{(p_1 - p_3)^2} \qquad \propto \frac{1}{\hat{u}} = \frac{1}{(p_1 - p_4)^2}$$

Three very different pole structures contributing to the same matrix element.

Let's cut the problem in piece

$$|M_T|^2 = \frac{|M_1|^2 + |M_2|^2}{|M_1|^2 + |M_2|^2} |M_T|^2$$

$$\int |M_{tot}|^2 = \int \frac{\sum_i |M_i|^2}{\sum_j |M_j|^2} |M_{tot}|^2 = \sum_i \int \frac{|M_i|^2}{\sum_j |M_j|^2} |M_{tot}|^2$$

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Key Idea

- One diagram is manageable
- All other peaks taken care of by denominator sum

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≈ 1

Key Idea

- One diagram is manageable
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N Integral

- Errors add in quadrature so no extra cost
- Parallel in nature

$$\int |M_{tot}|^2 = \int \frac{\sum_i |M_i|^2}{\sum_j |M_j|^2} |M_{tot}|^2 = \sum_i \int \frac{|M_i|^2}{\sum_j |M_j|^2} |M_{tot}|^2$$

P1_qq_wpwm

s= 725.73 ± 2.07 (pb)

Graph	Cross-Section ↓	Error	Events (K)	Unwgt	Luminosity
G2.2	377.6	1.67	142.285	7941.0	21
G3	239	1.16	220.04	10856.0	45.5
G1	109.1	0.378	70.88	3793.0	34.8

P1_gg_wpwm

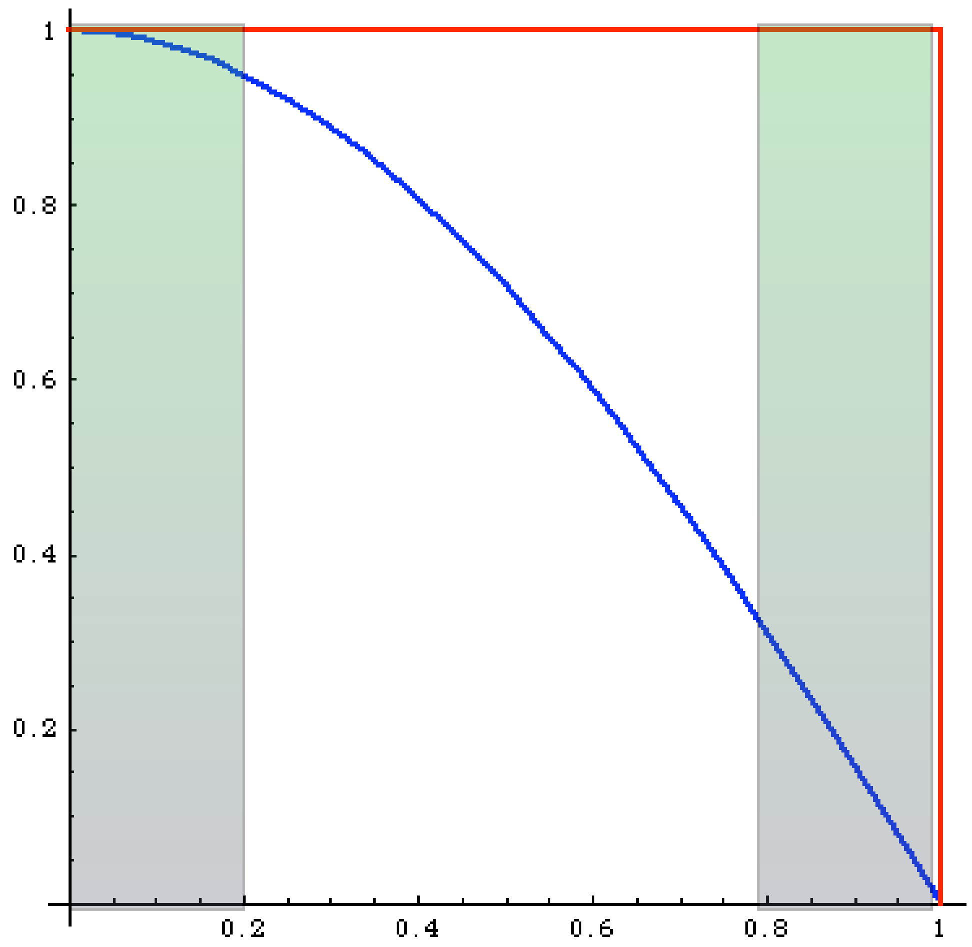
s= 20.714 ± 0.332 (pb)

Graph	Cross-Section ↓	Error	Events (K)	Unwgt	Luminosity
G1.2	20.71	0.332	7.01	373.0	18

term of the above sum.

each term might not be gauge invariant

Event generation

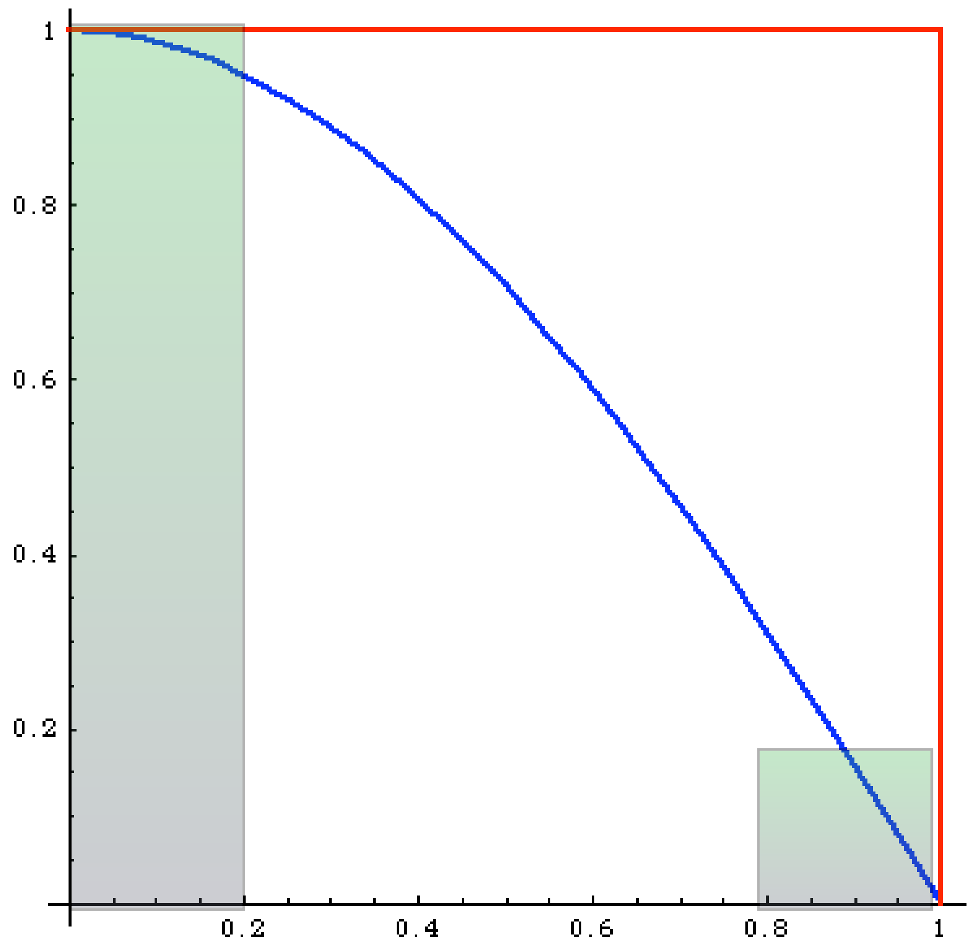


What's the difference between weighted and unweighted?

Weighted:

Same # of events in areas of phase space with very different probabilities: events must have different weights

Event generation



What's the difference between weighted and unweighted?

Unweighted:

events is proportional to the probability of areas of phase space:
events have all the same weight ("unweighted")

Events distributed as in nature

Event generation

$$\int f(x)dx = \frac{1}{N}\sum_{i=1}^{N} f(x_i)$$

Event generation

$$\int f(x)dx = \frac{1}{N}\sum_{i=1}^{N} f(x_i) = \frac{1}{N}\sum_{i=1}^{N} \frac{f(x_i)}{max(f)} max(f)$$

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Number between 0 and 1 (assuming positive function)
-> re-interpret as the probability to keep the events

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For each events throw the dice and see if we keep or reject the events

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$$\int f(x)dx = \frac{1}{N}\sum_{i=1}^{N} f(x_i) = \frac{1}{N}\sum_{i=1}^{N} P(x_i) max(f) \simeq \frac{max(f)}{N}\sum_{i=1}^{n} 1$$

Event generation

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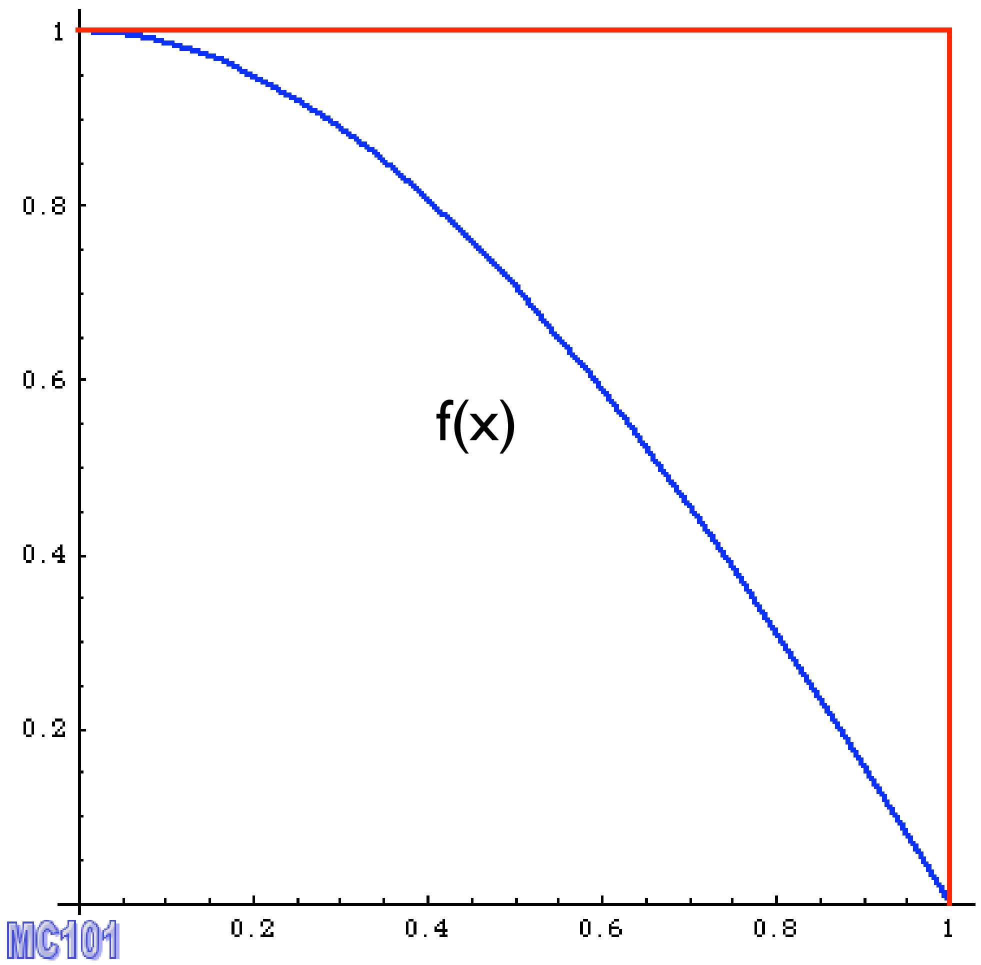

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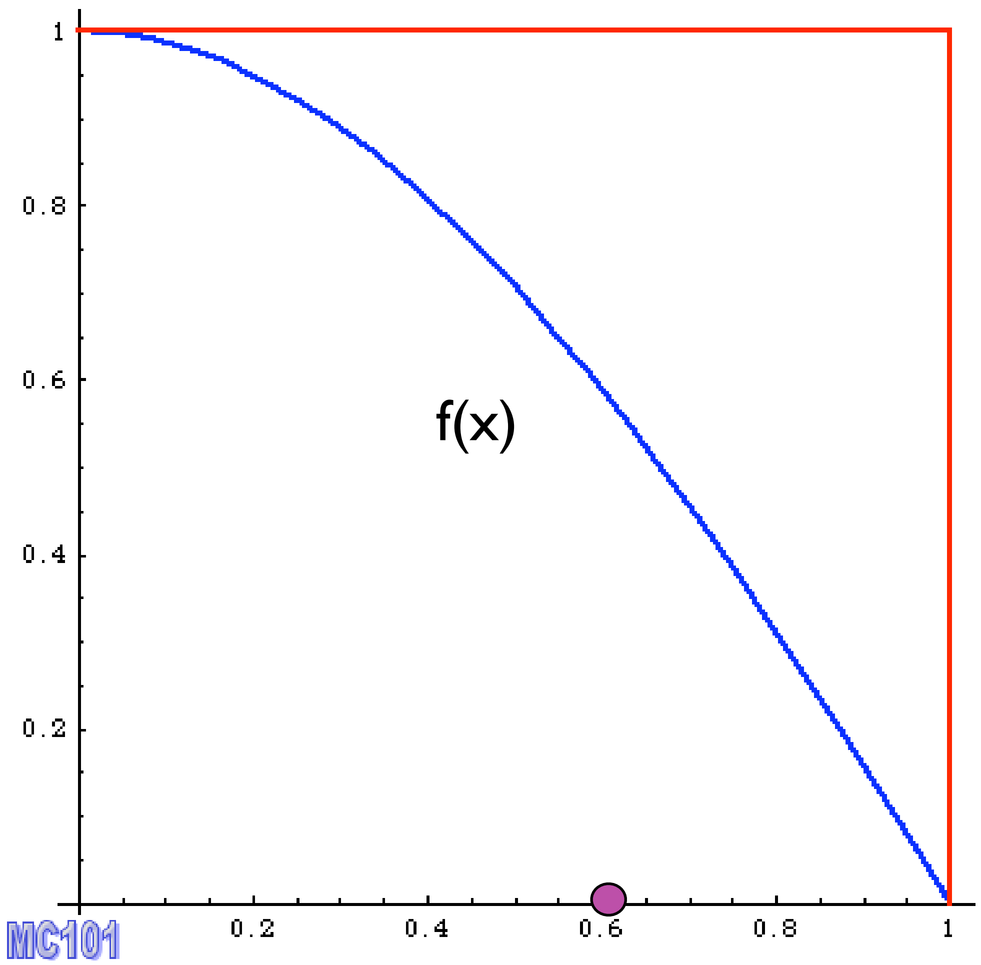

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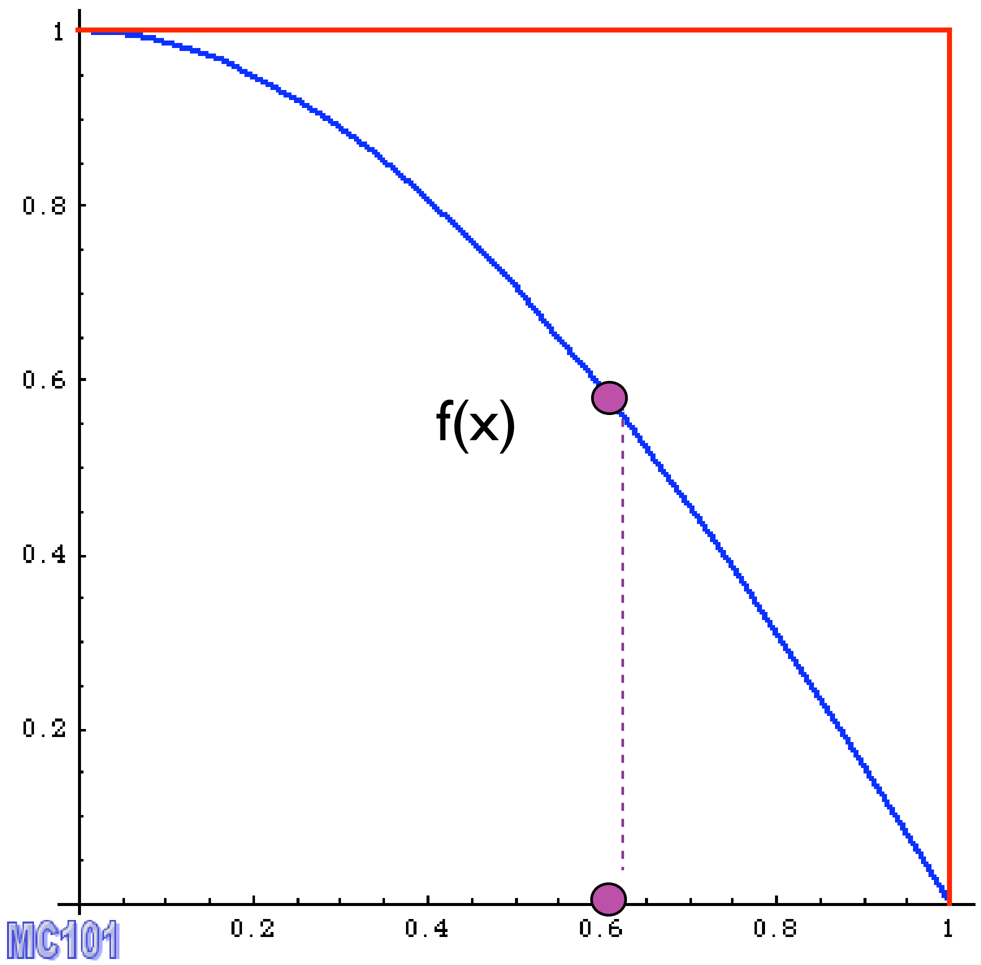

1. pick x_i
2. calculate $f(x_i)$

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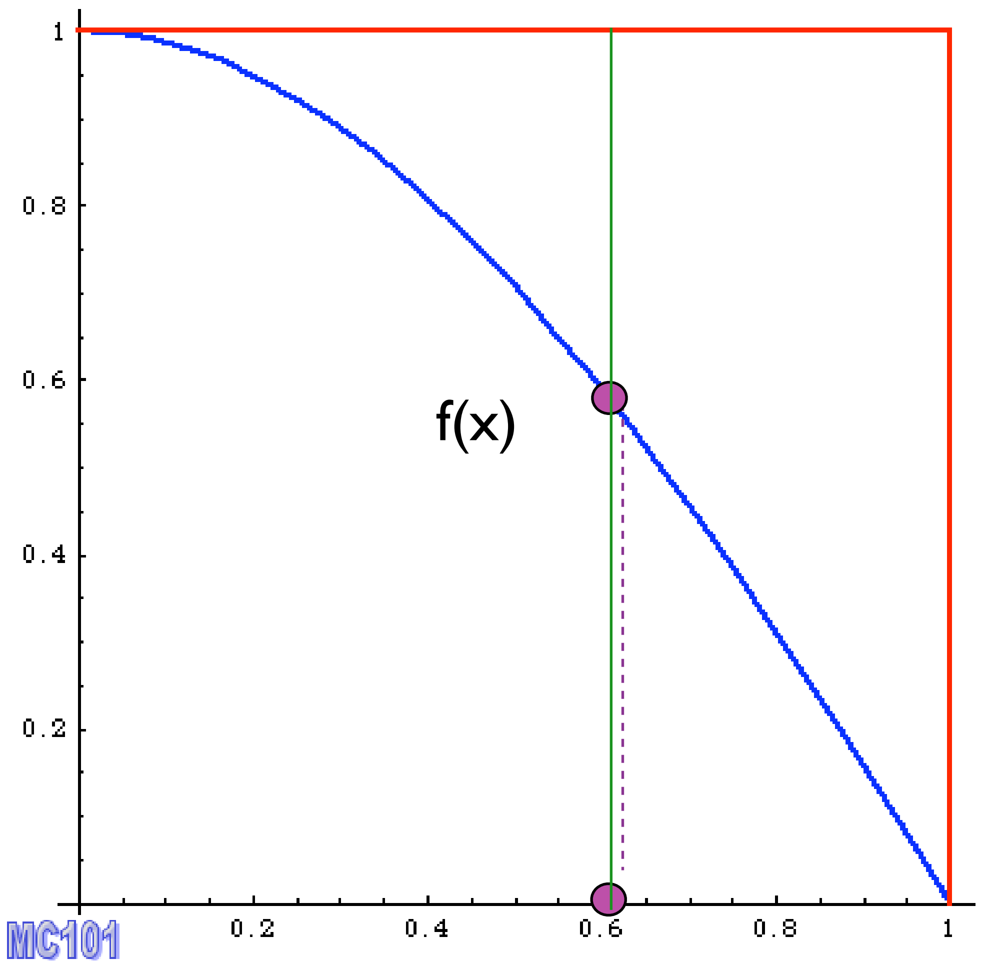

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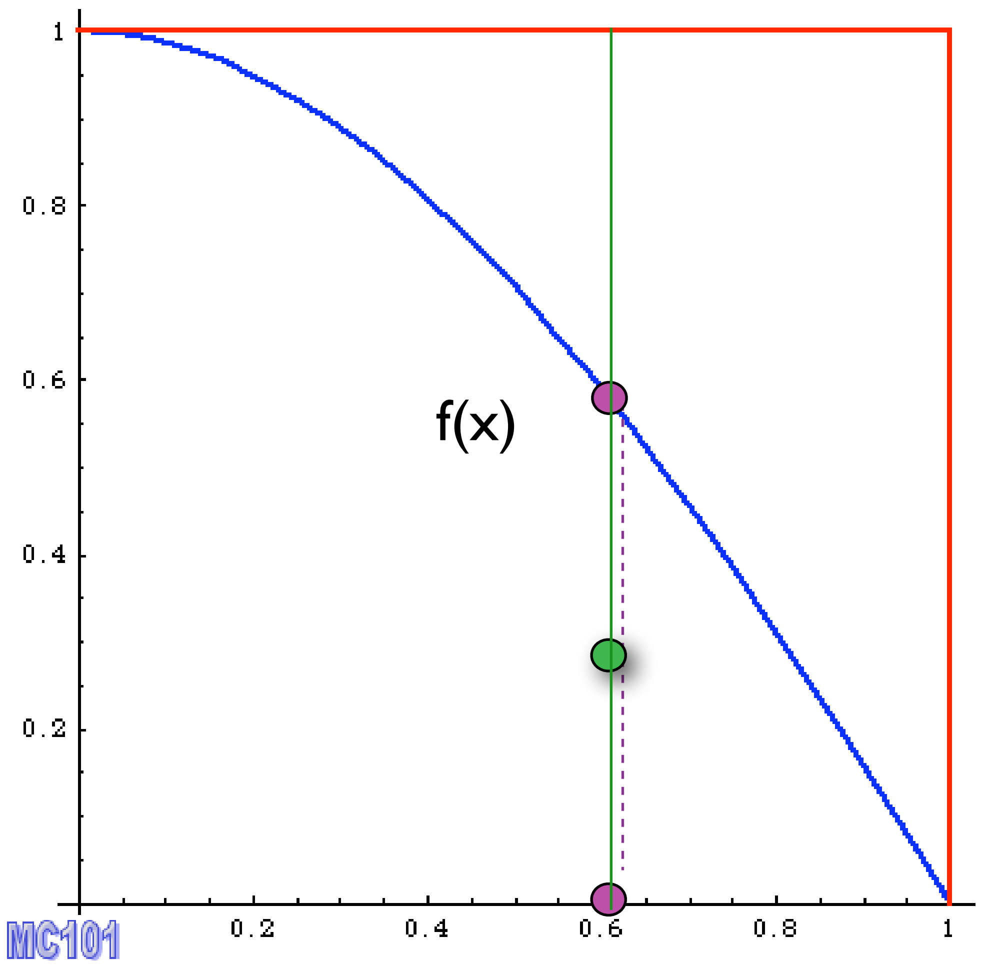

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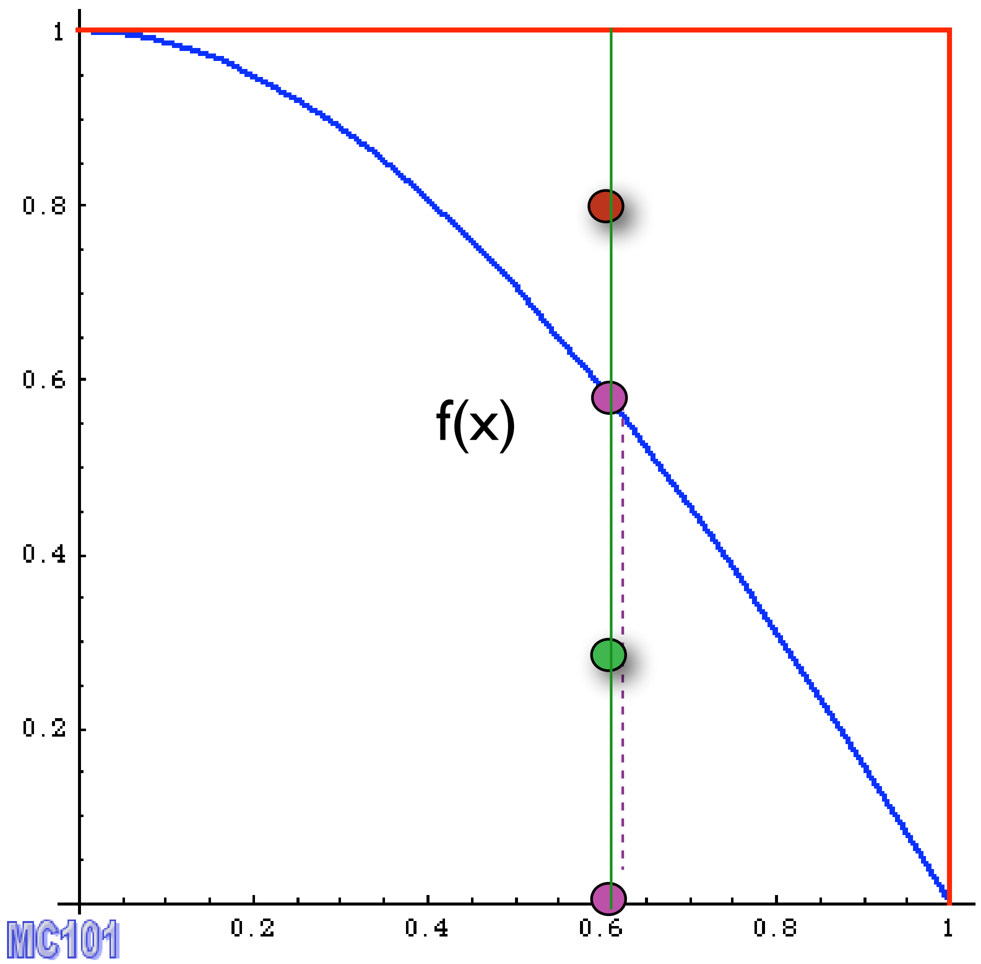

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Event generation

MC integrator

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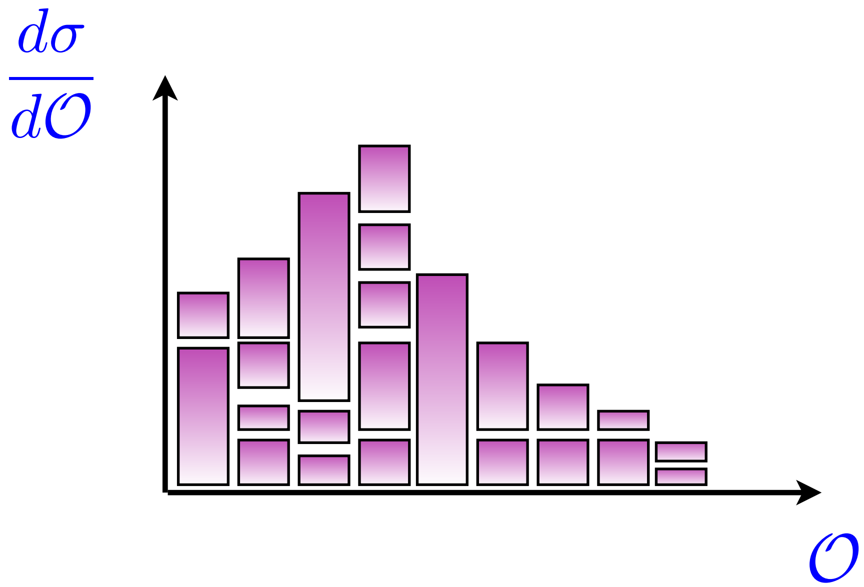

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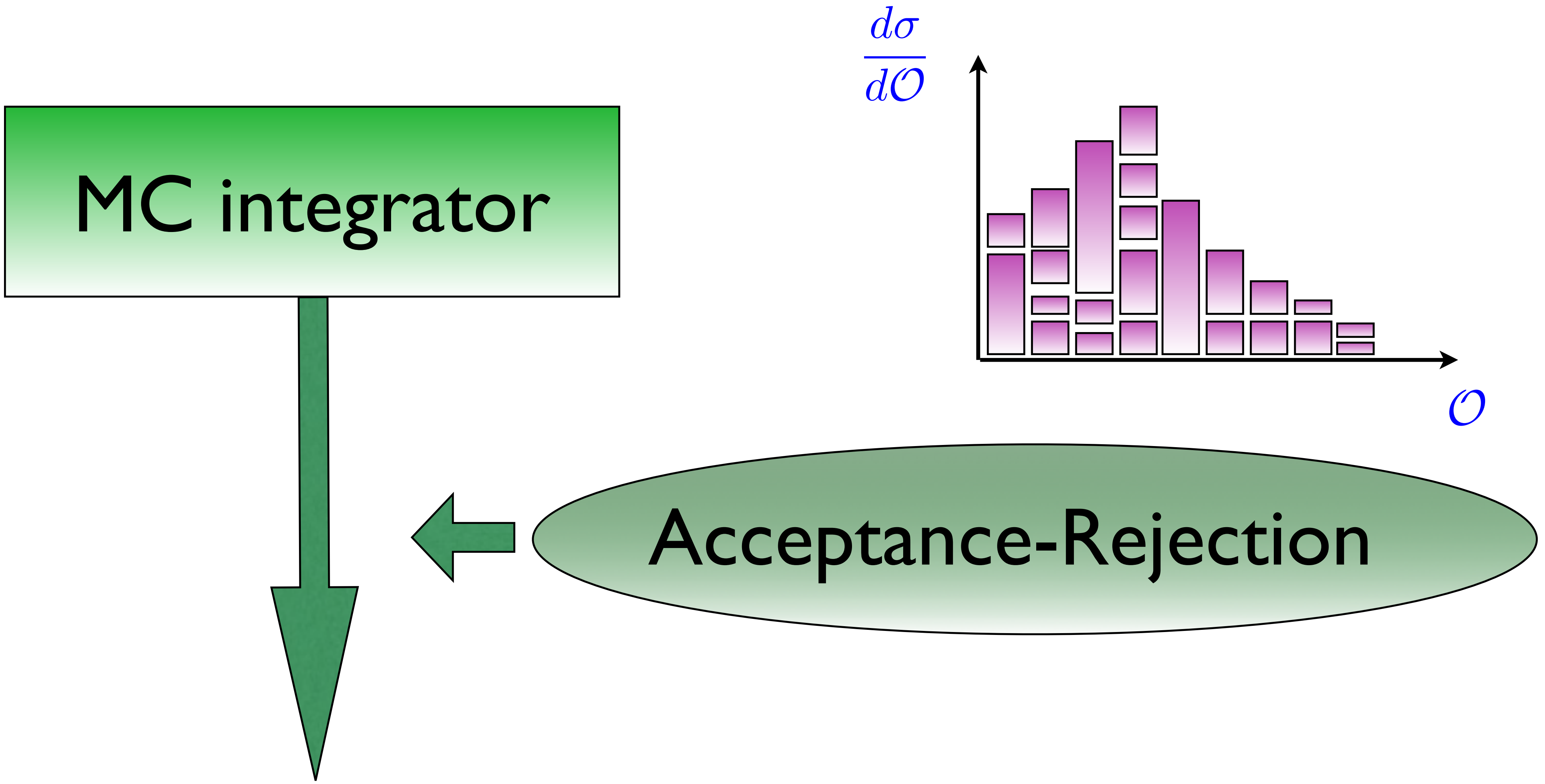

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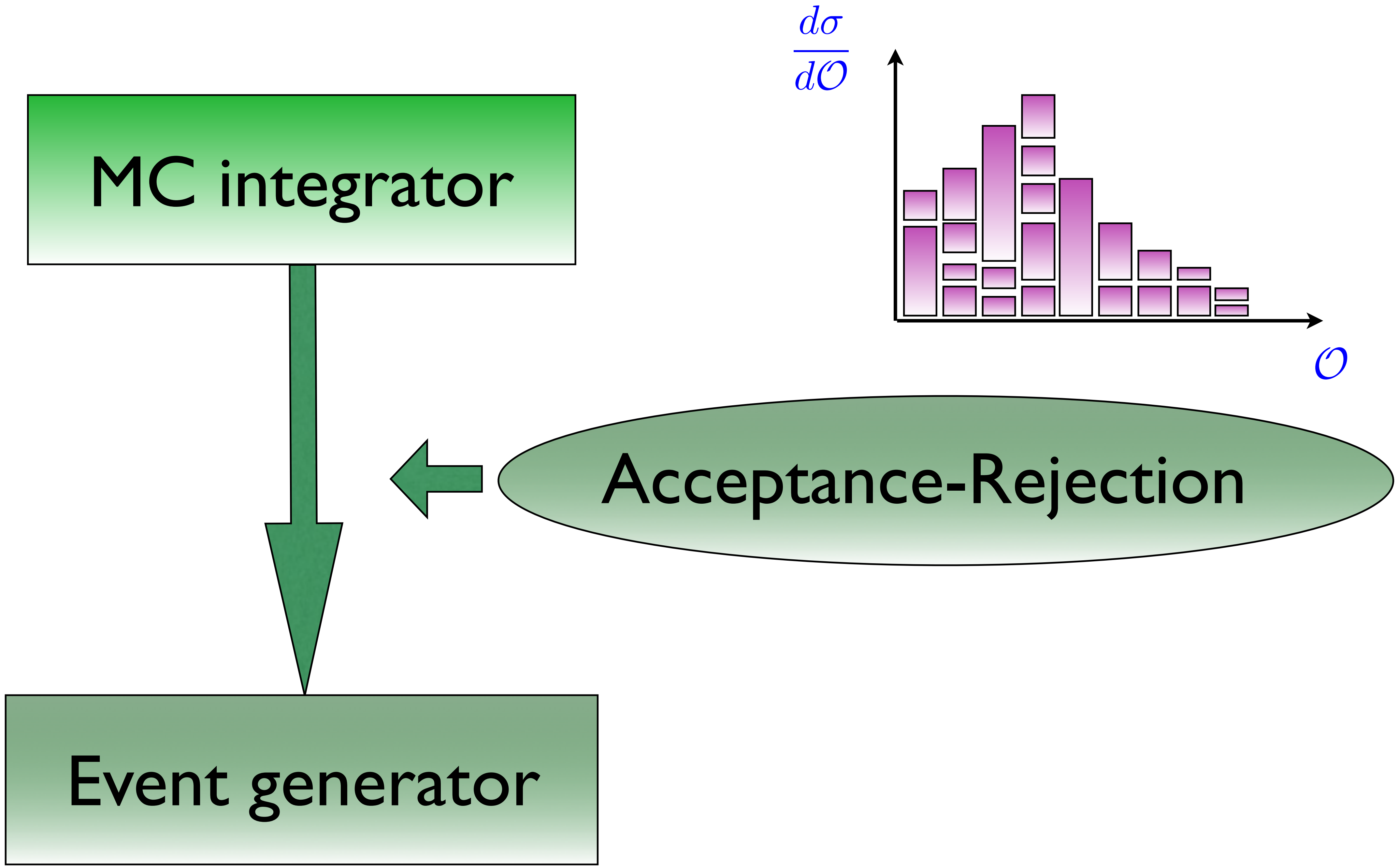

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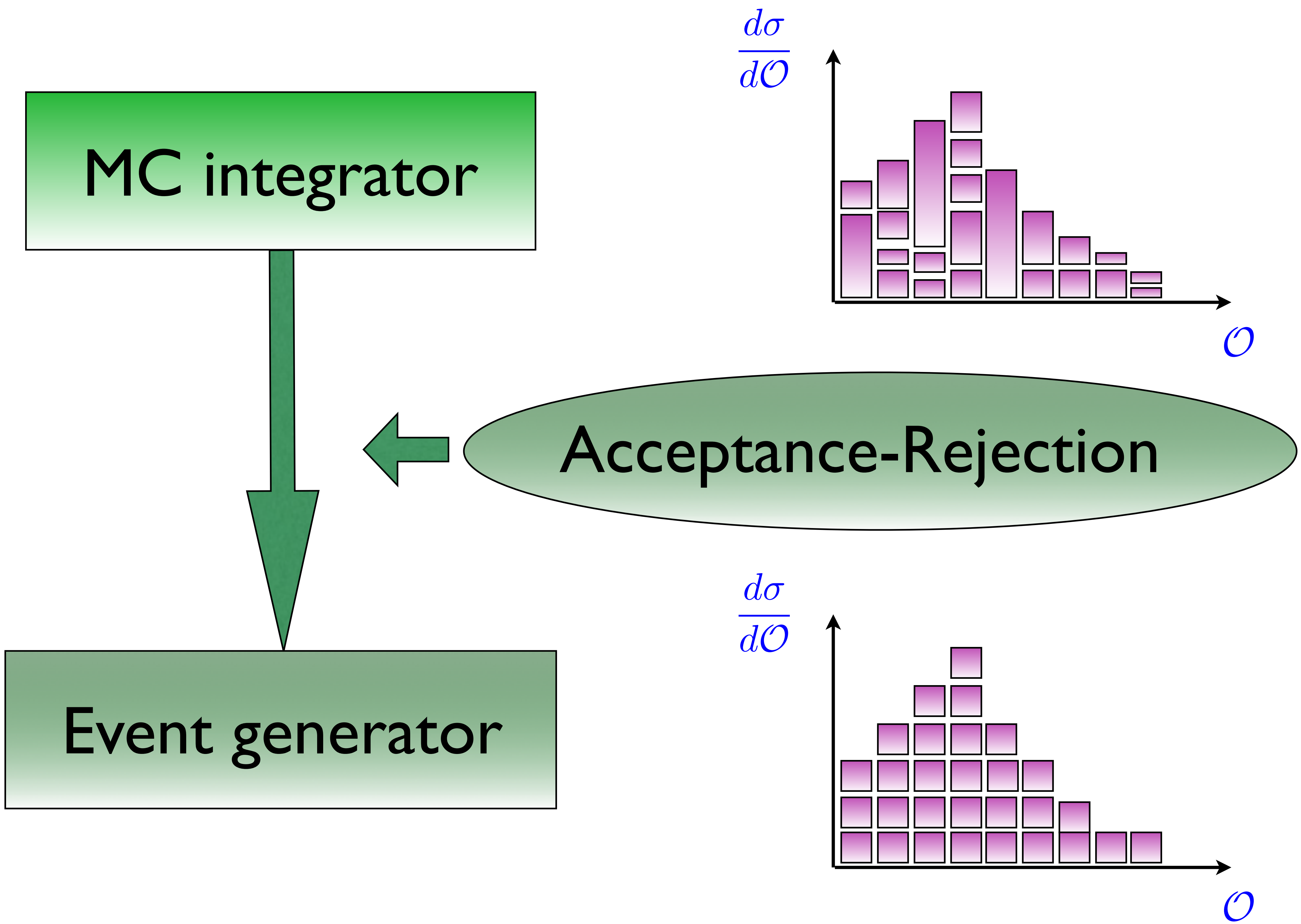

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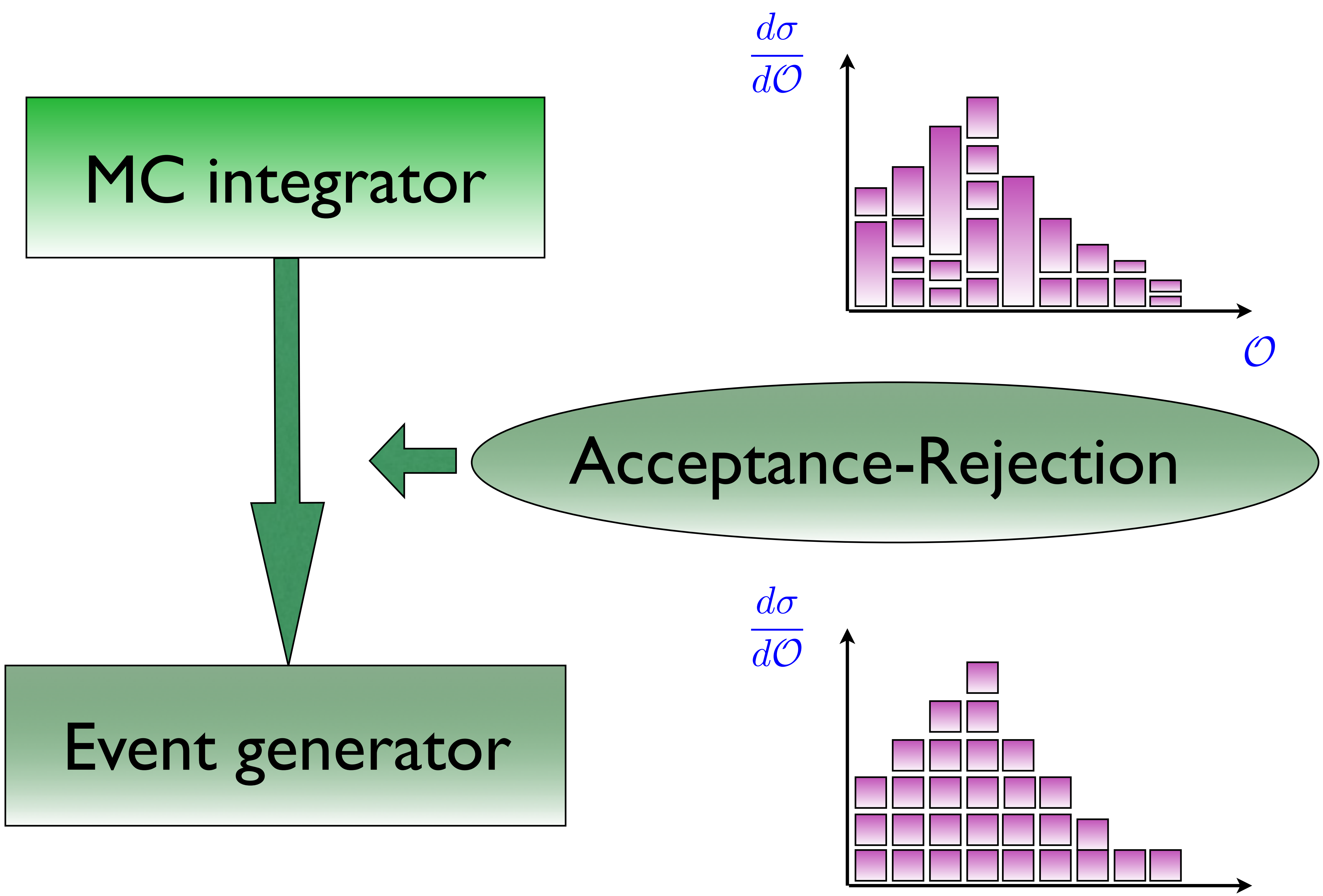

☞ **This is possible only if f(x)<∞ AND has definite sign!**

Monte-Carlo Summary

Bad Point

- Slow Convergence (especially in low number of Dimension)
- Need to know the function
  - Impact on cut

Monte-Carlo Summary

Bad Point

- Slow Convergence (especially in low number of Dimension)
- Need to know the function
  - Impact on cut

Good Point

- Complex area of Integration
- Easy Error estimate
- quick estimation of the integral
- Possibility to have unweighted events

What to remember

- Analytical computation can be slower than numerical method
- Any BSM model are supported (at LO)
- Phase Space integration are slow
  - need knowledge of the function
  - cuts can be problematic
- Event generation are from free.

Plan

Lecture I

- Overview of Monte-Carlo
- Details of the computation
    - Evaluation of matrix-element
    - Phase-Space integration

Lecture II

- Narrow-width
- Basic of matching/merging
- Basic of NLO computation
- Overview of MG5aMC

Plan

Lecture II

- Narrow-width
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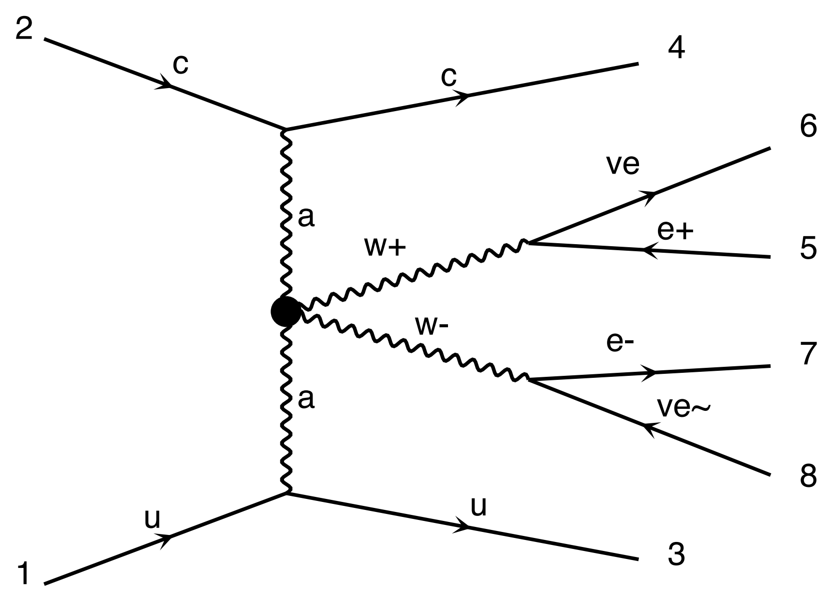
Decay

Resonant Diagram



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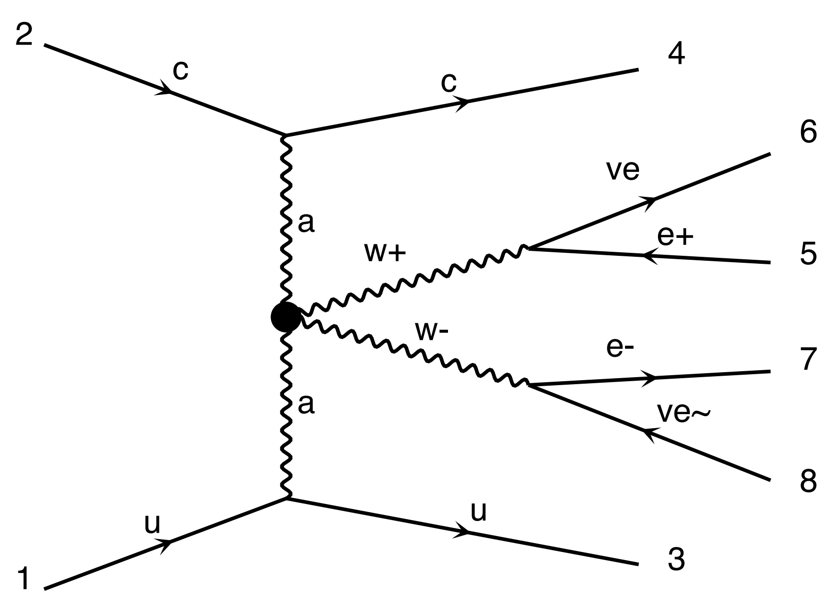
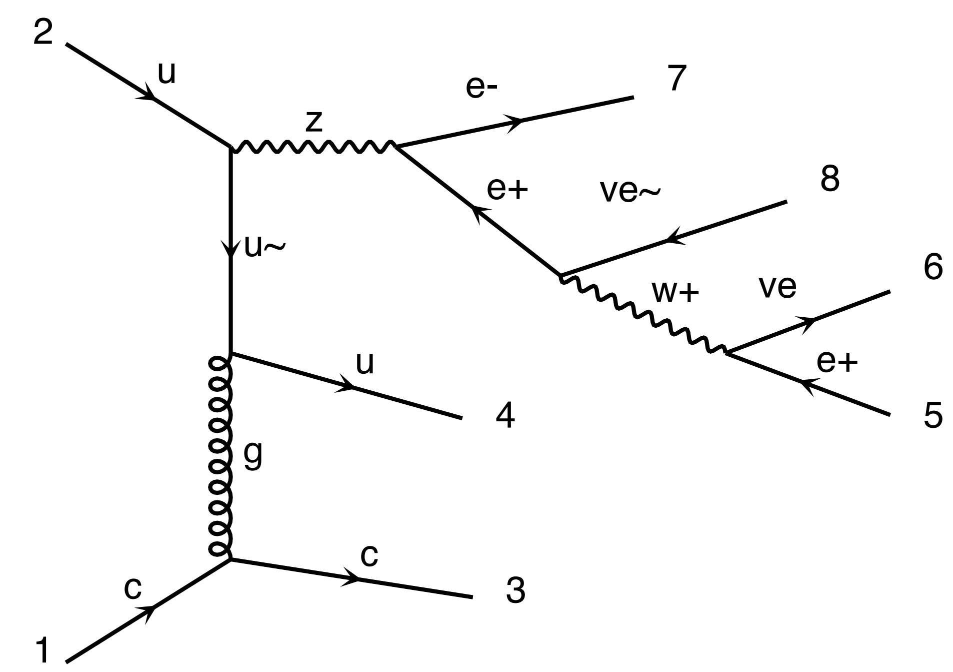


Non Resonant Diagram



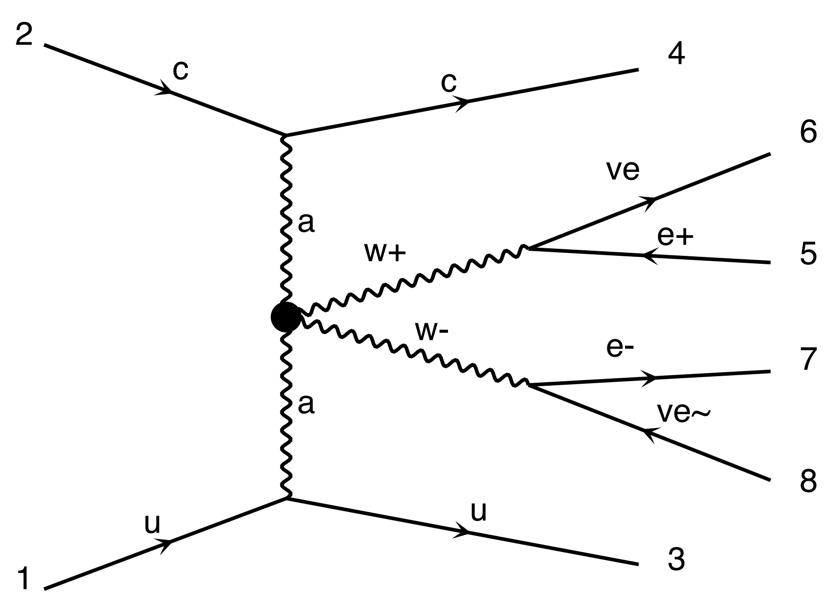
Problem

- Process complicated to have the full process

  ➡Including off-shell contribution

Decay

Resonant Diagram



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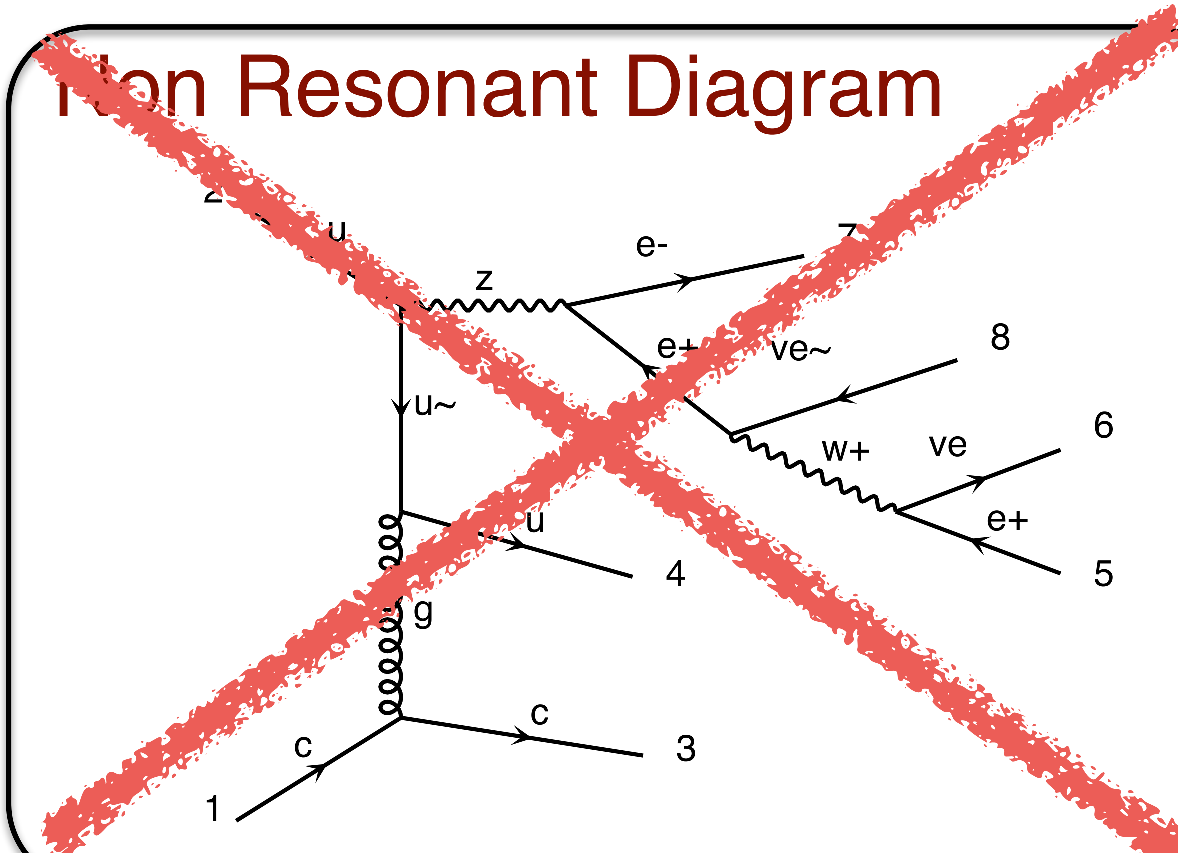

Problem

- Process complicated to have the full process

  ➡Including off-shell contribution

Solution

- Only keep on-shell contribution

Narrow-Width Approx.

Theory

$$\int dq^2 \left| \frac{1}{q^2 - M^2 - iM\Gamma} \right|^2 \approx \frac{\pi}{M\Gamma} \, \delta(q^2 - M^2)$$

$$\sigma_{full} = \sigma_{prod} * (BR + \mathcal{O}(\frac{\Gamma}{M}))$$

Comment

Narrow-Width Approx.

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Comment

- This is an Approximation!
- This force the particle to be on-shell!
  - Recover by re-introducing the Breit-wigner up-to a cut-off

Decay chain

- p p > t t~ w+, (t > w+ b, w+ > l+ vl), \
  (t~ > w- b~, w- > j j), \
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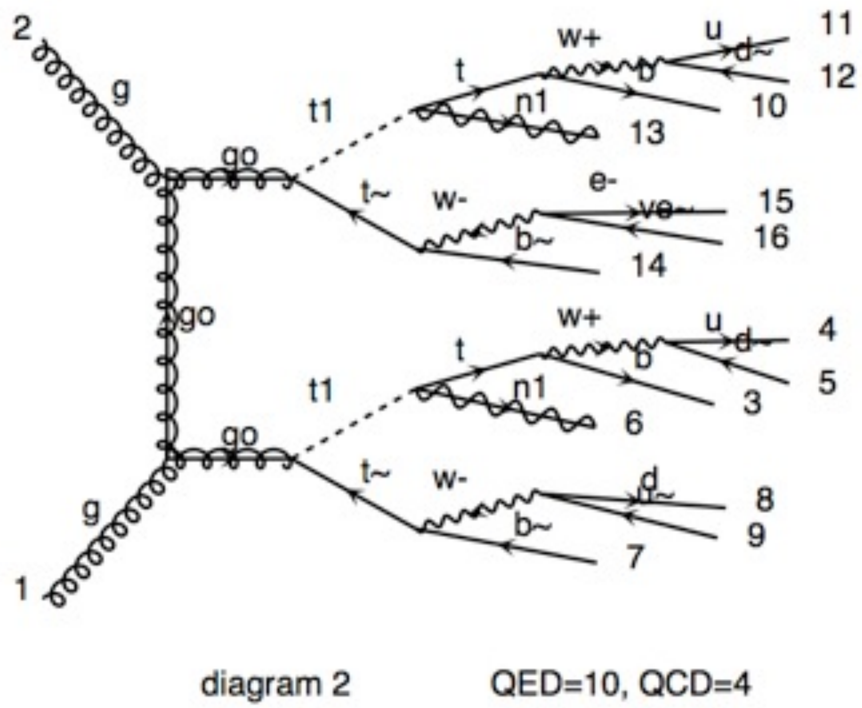

very long
decay chains possible to simulate
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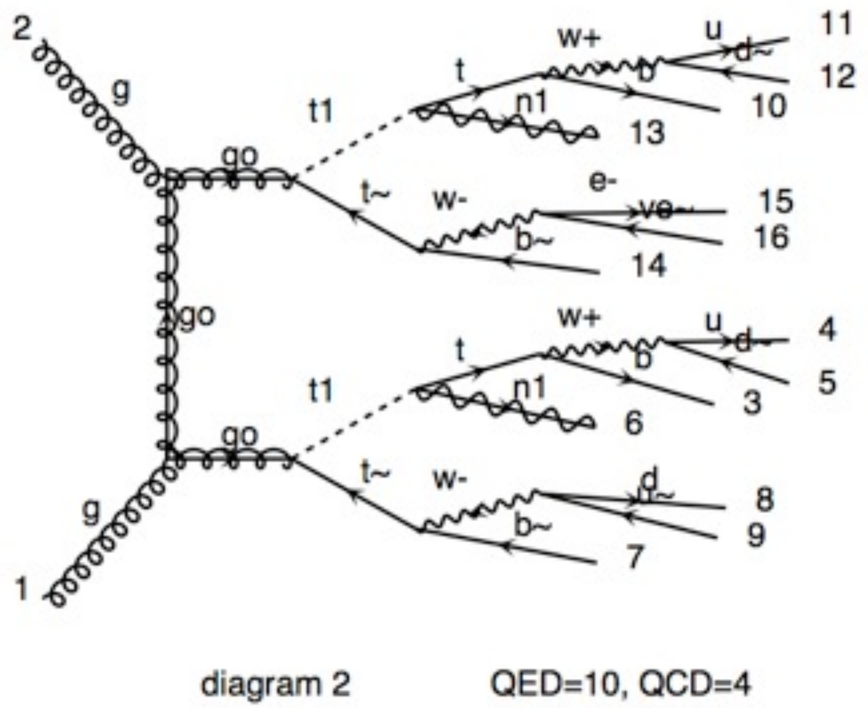


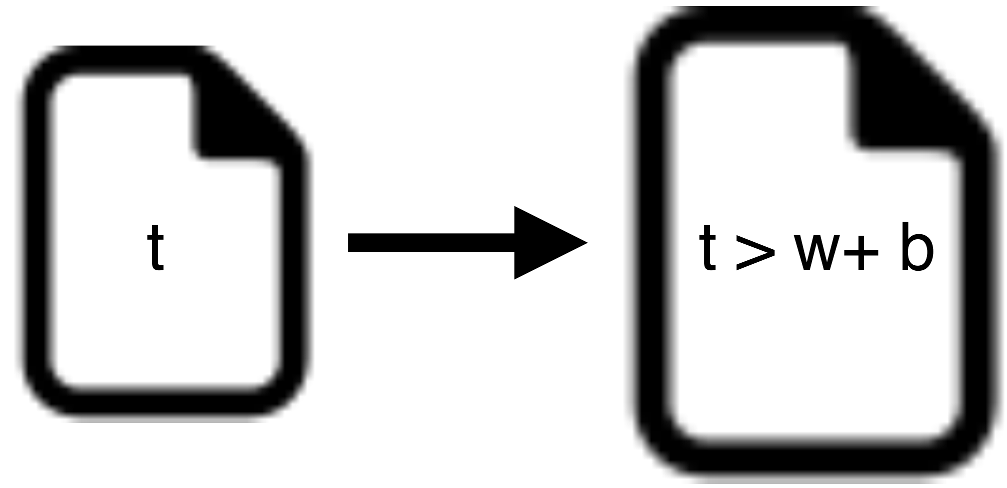
very long
decay chains possible to simulate
directly in MadGraph!

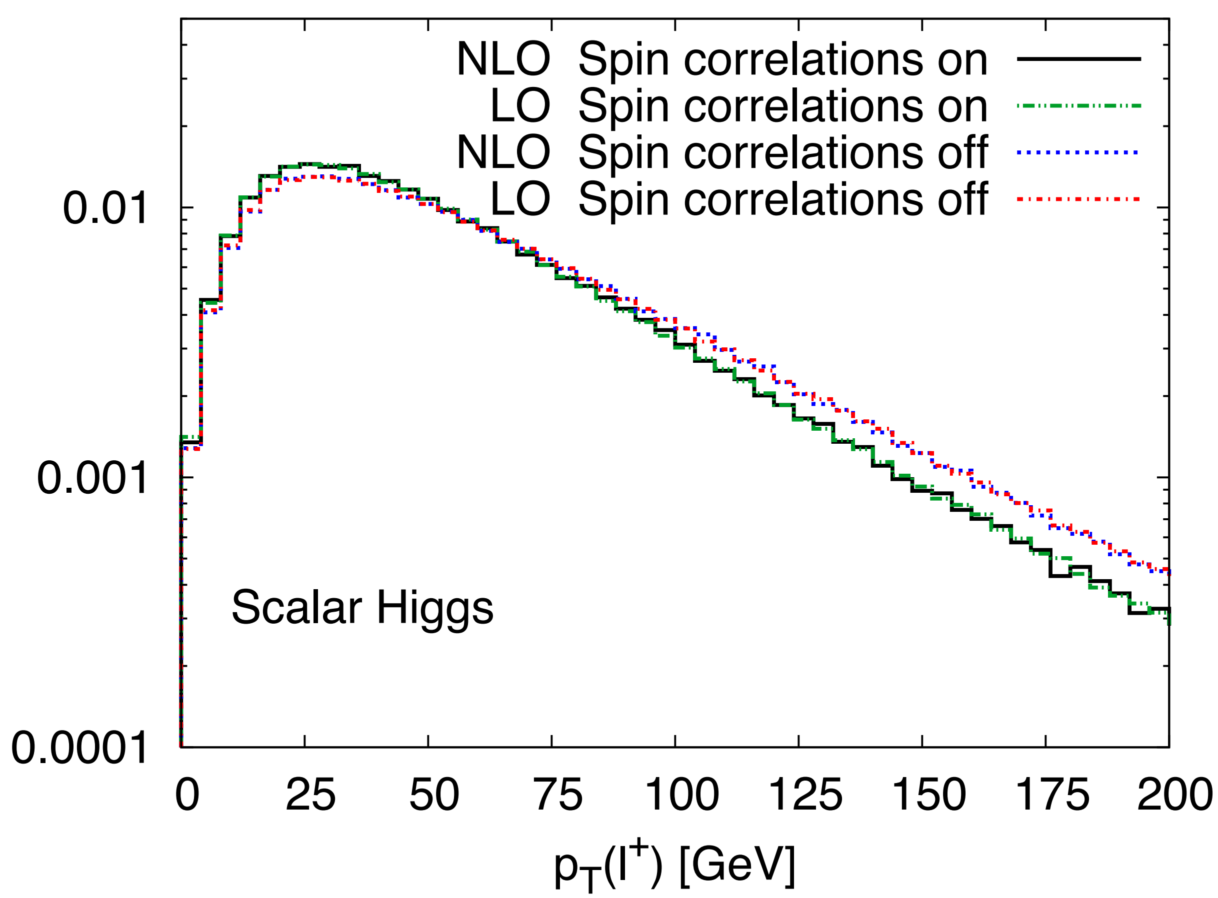
- Full spin-correlation
- Off-shell effects (up to cut-off)
- NWA not used for the cross-section

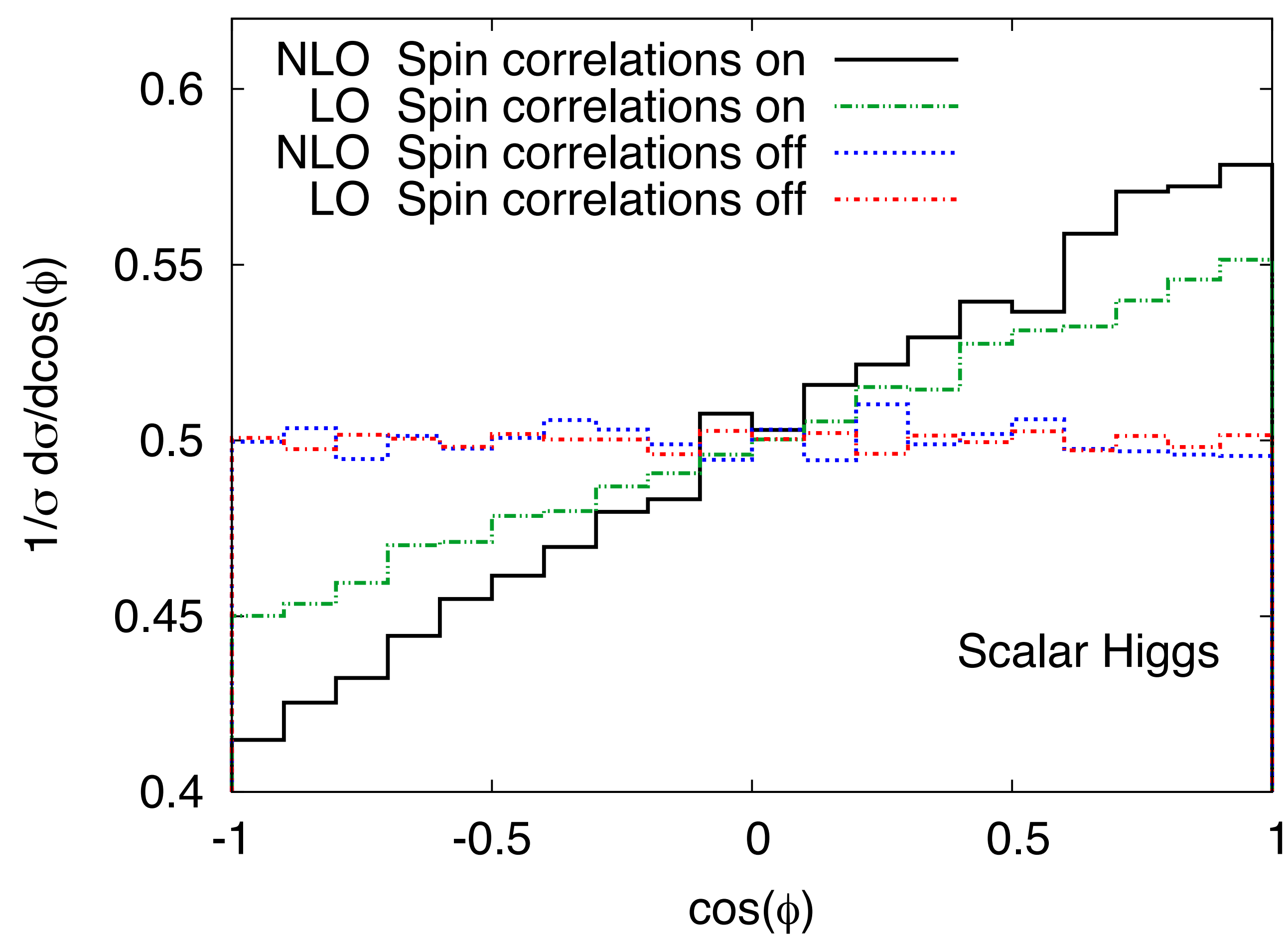
MadSpin

Decay as post-processing

- Independently of event generation
- But same accuracy (spin-correlation)
- Use NWA for cross-section



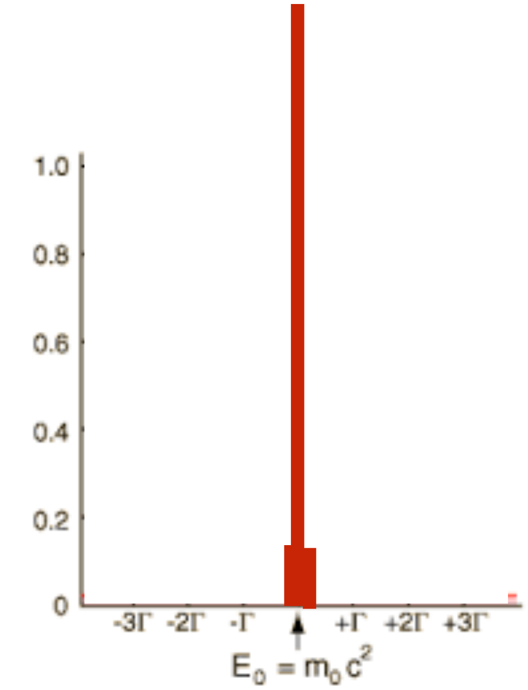




Very small width

$\Gamma < 10^{-8} M$

- Slows down the code
- Can lead to numerical instability



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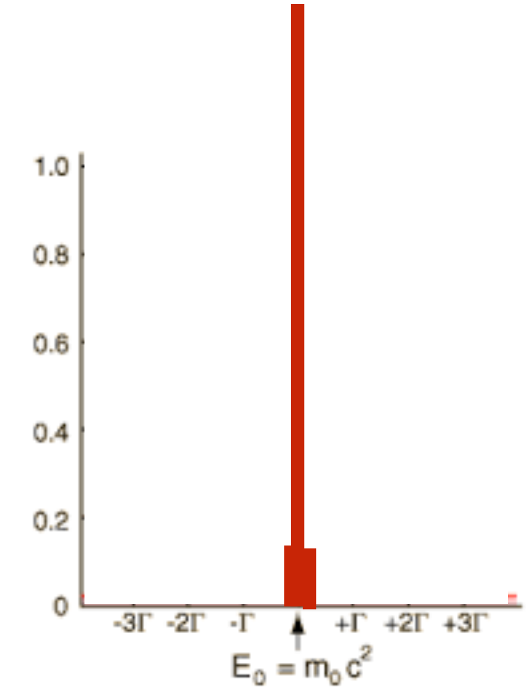
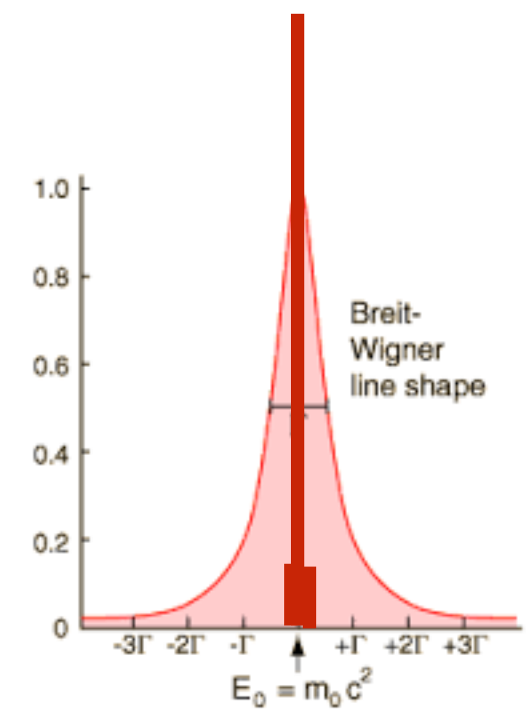


Solution

- Use a Fake-Width for the evaluation of the matrix-element
- Correct cross-section according to NWA formula $\frac{\Gamma_{fake}}{\Gamma_{true}}$



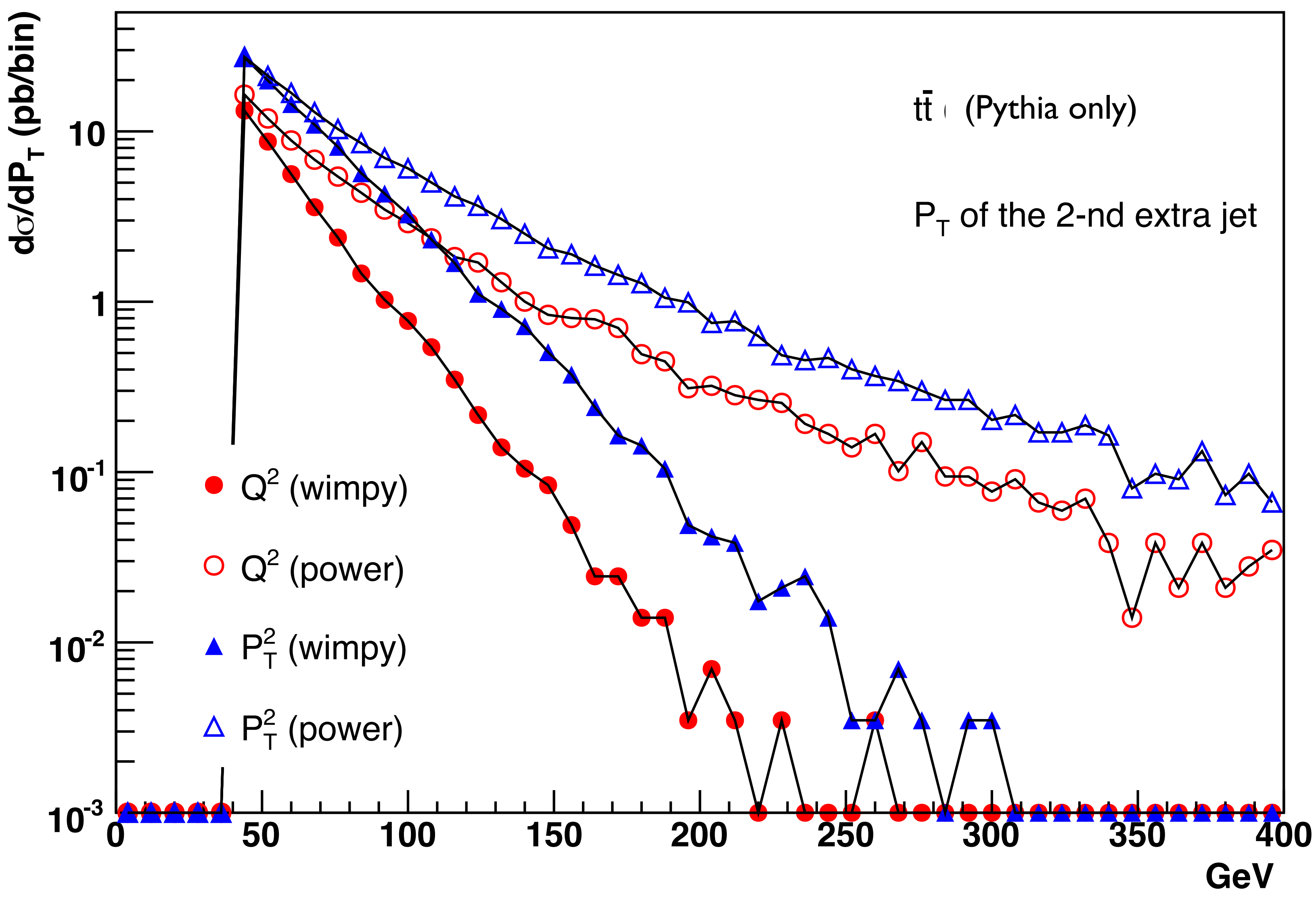
Plan

Lecture II

- Narrow-width
- Basic of matching/merging
- Basic of NLO computation
- Overview of MG5aMC

PS alone vs matched samples

In the soft-collinear approximation of Parton Shower MCs, parameters are used to tune the result ⇒ Large variation in results (small prediction power)



Matrix Elements vs. Parton Showers

Matrix Elements vs. Parton Showers

ME

1. Fixed order calculation
2. Computationally expensive
3. Limited number of particles
4. Valid when partons are hard and well separated
5. Quantum interference correct
6. Needed for multi-jet description

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Shower MC

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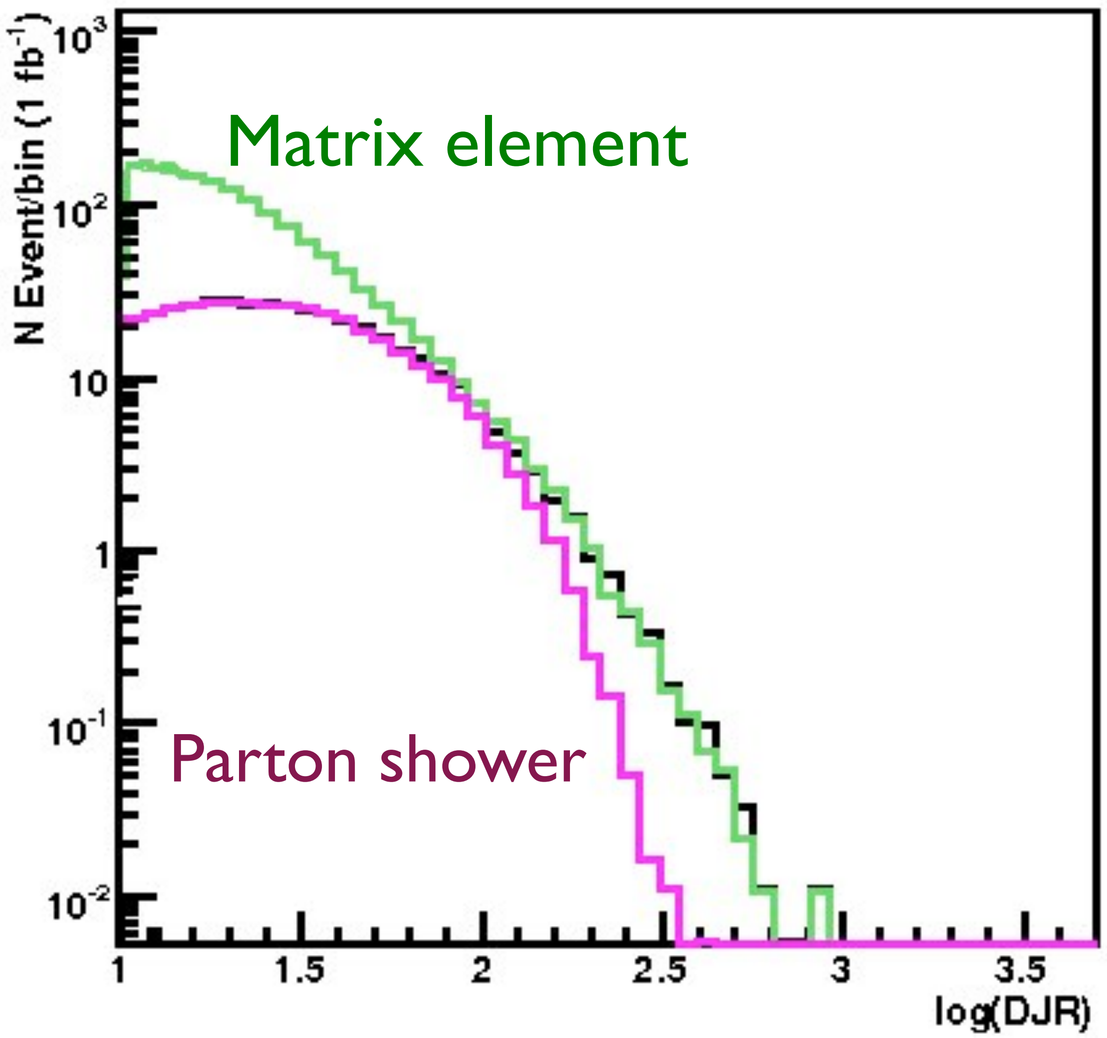
Approaches are complementary: merge them!

Difficulty: avoid double counting, ensure smooth distributions

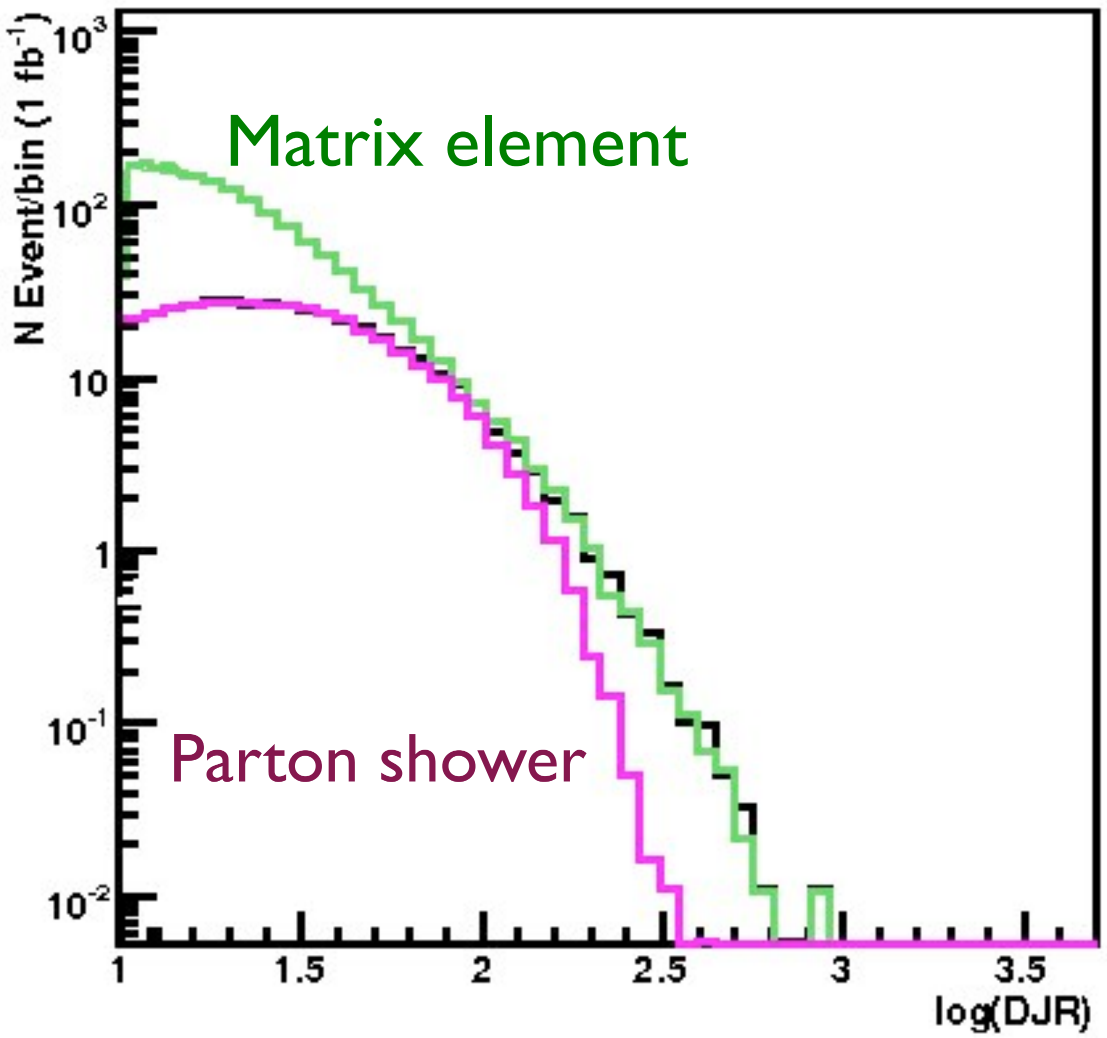
Goal for ME-PS merging/matching



2nd QCD radiation jet in top pair production at the LHC, using MadGraph + Pythia

Goal for ME-PS merging/matching

- Regularization of matrix element divergence



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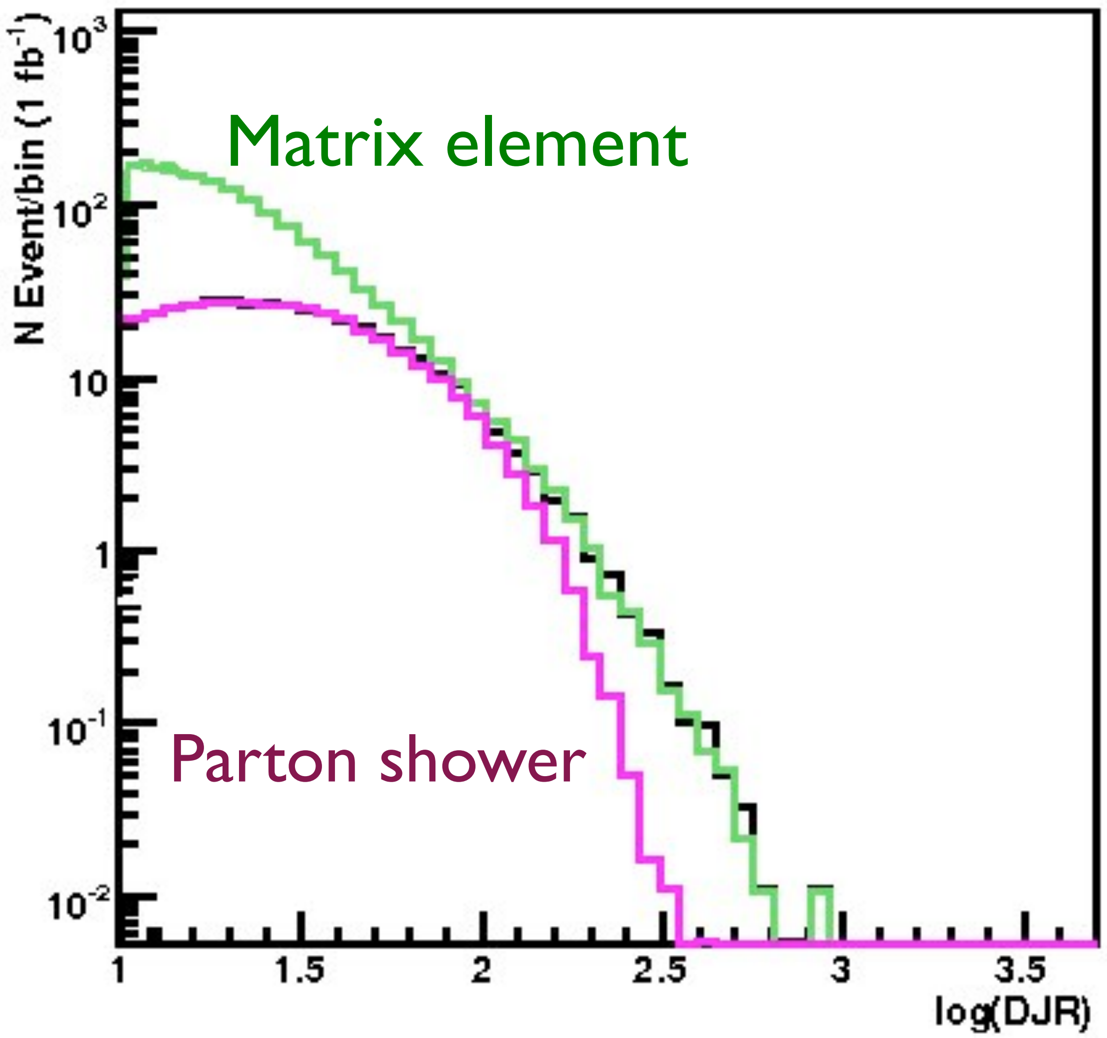

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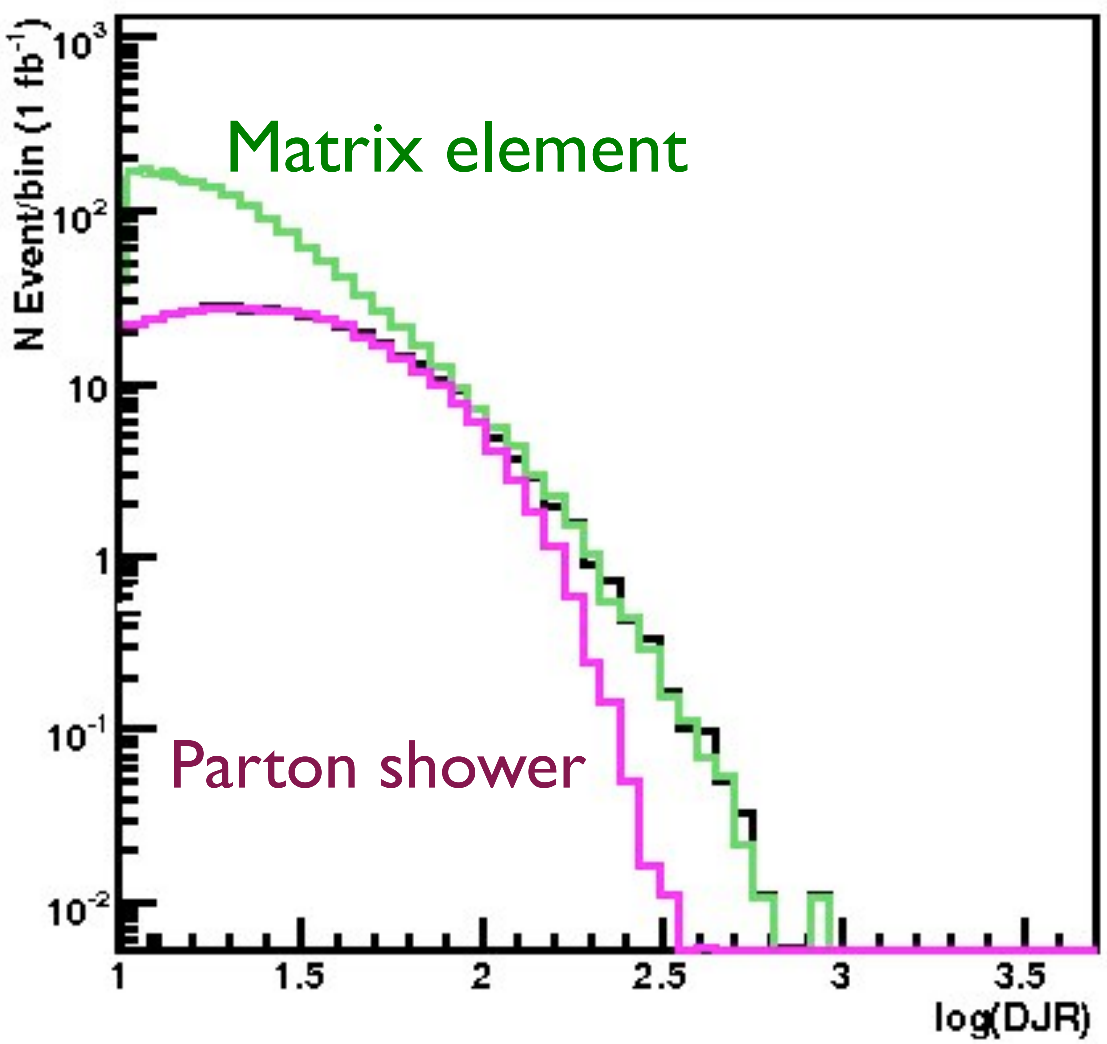

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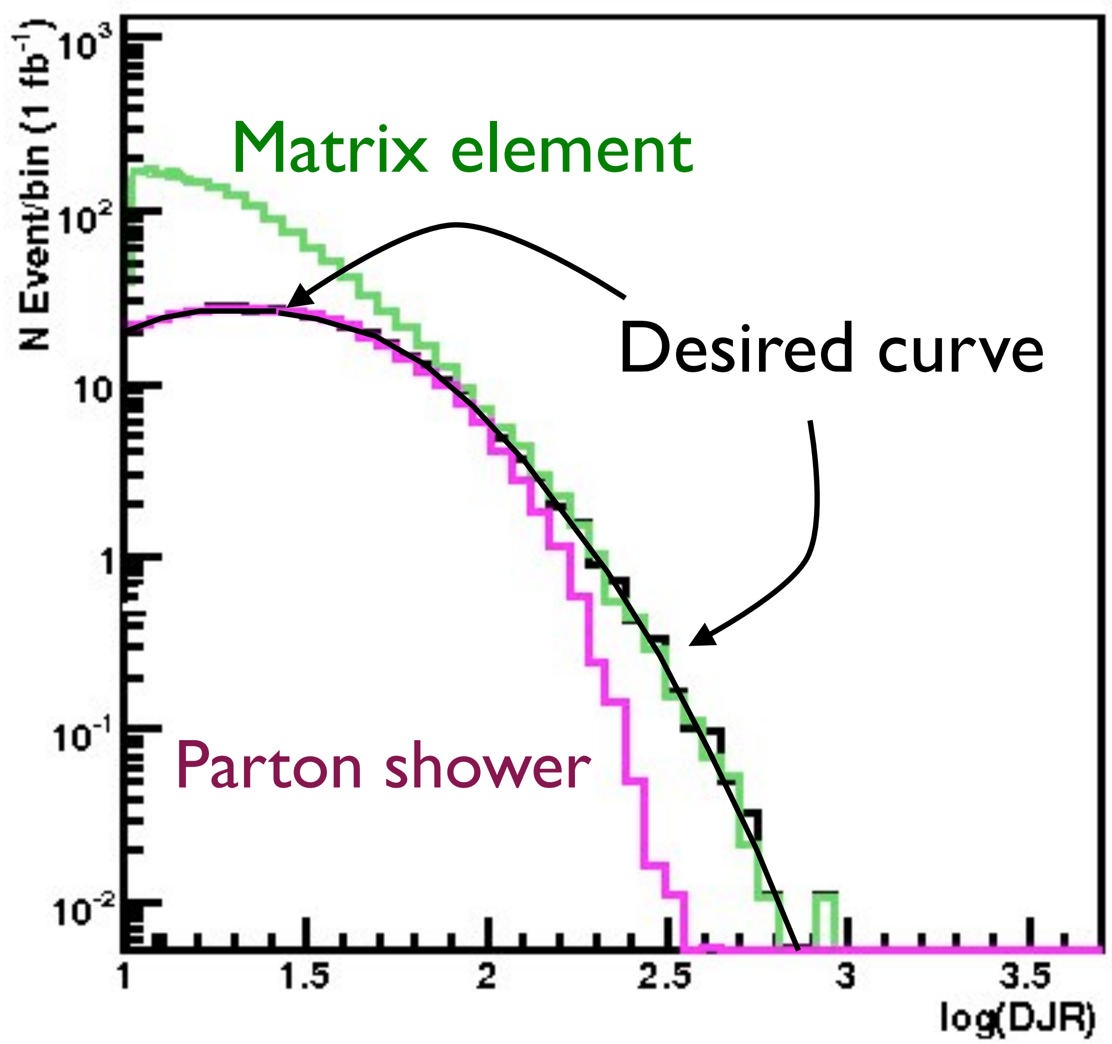


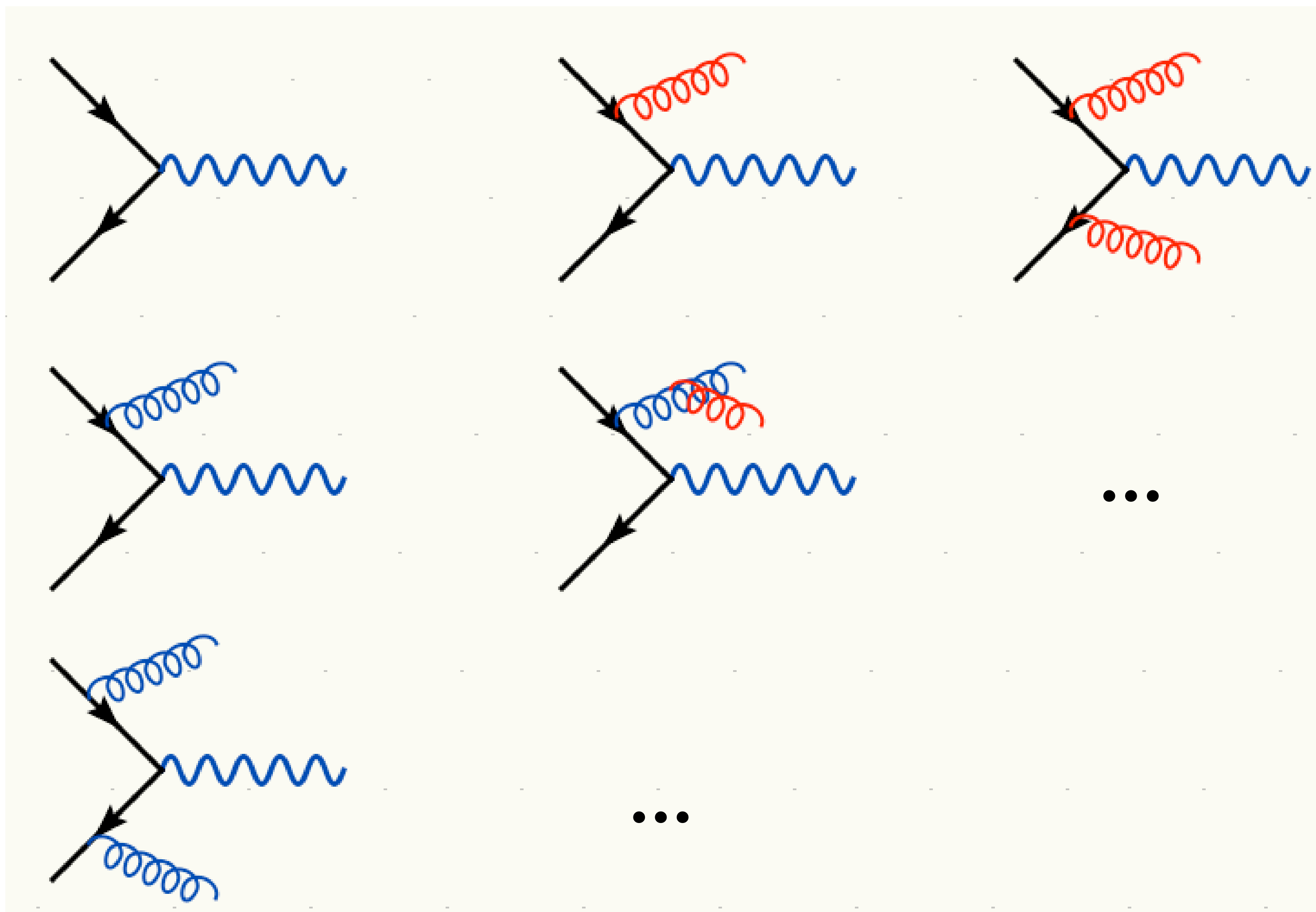
2nd QCD radiation jet in top pair production at the LHC, using MadGraph + Pythia

Merging ME with PS

[Mangano]
[Catani, Krauss, Kuhn, Webber]
[Lönnblad]

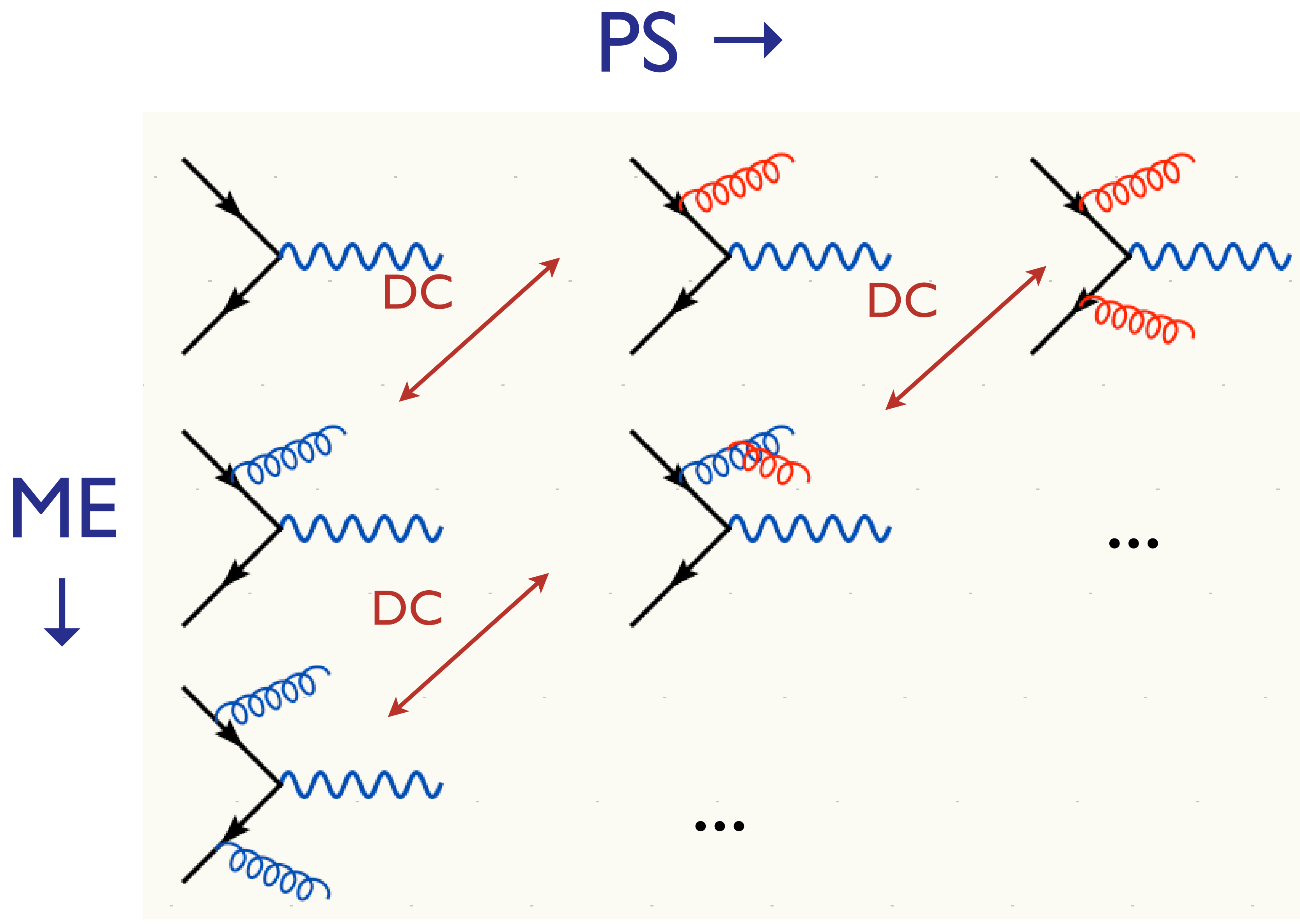
PS →

ME ↓



Merging ME with PS

[Mangano]
[Catani, Krauss, Kuhn, Webber]
[Lönnblad]



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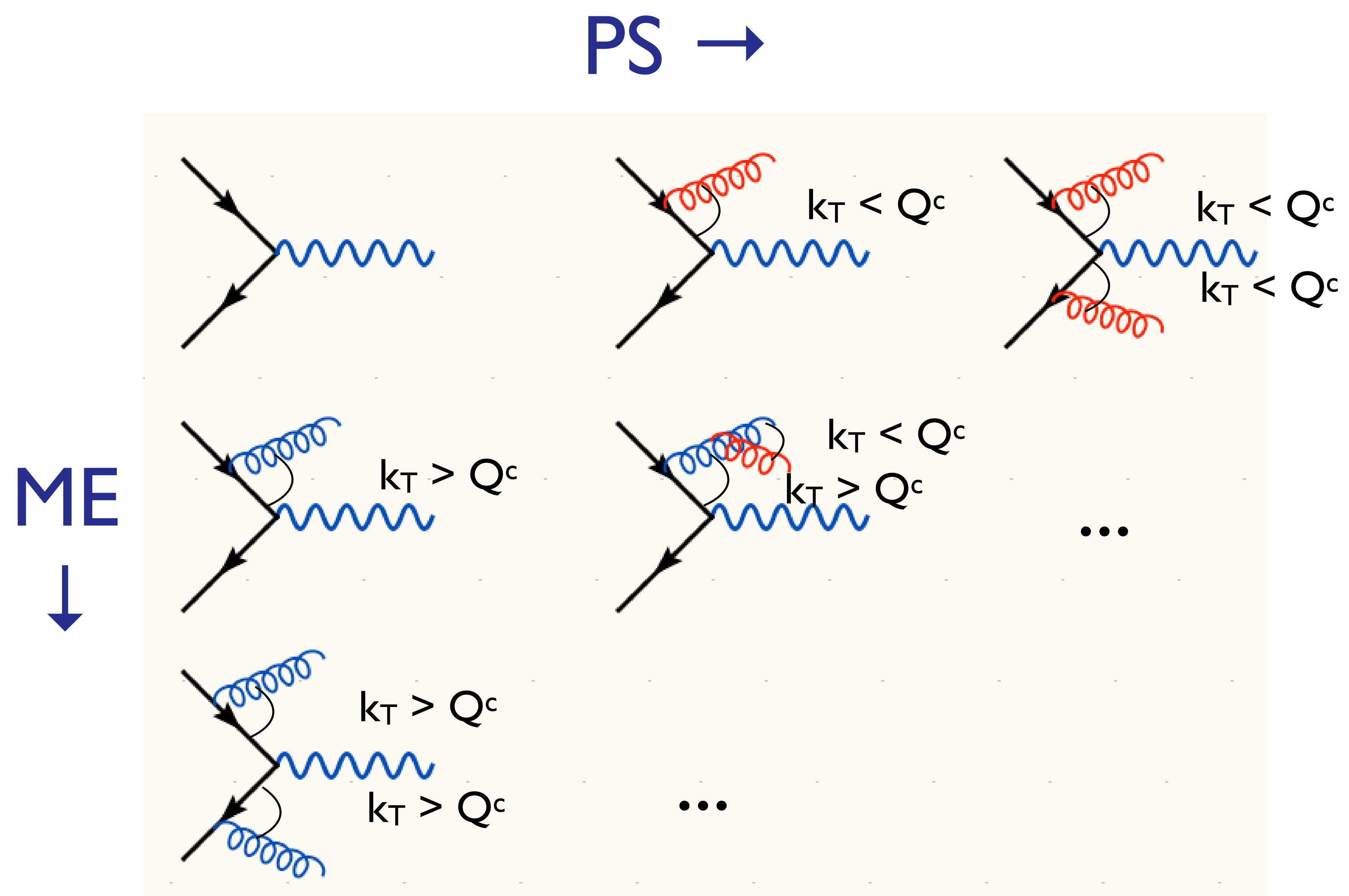

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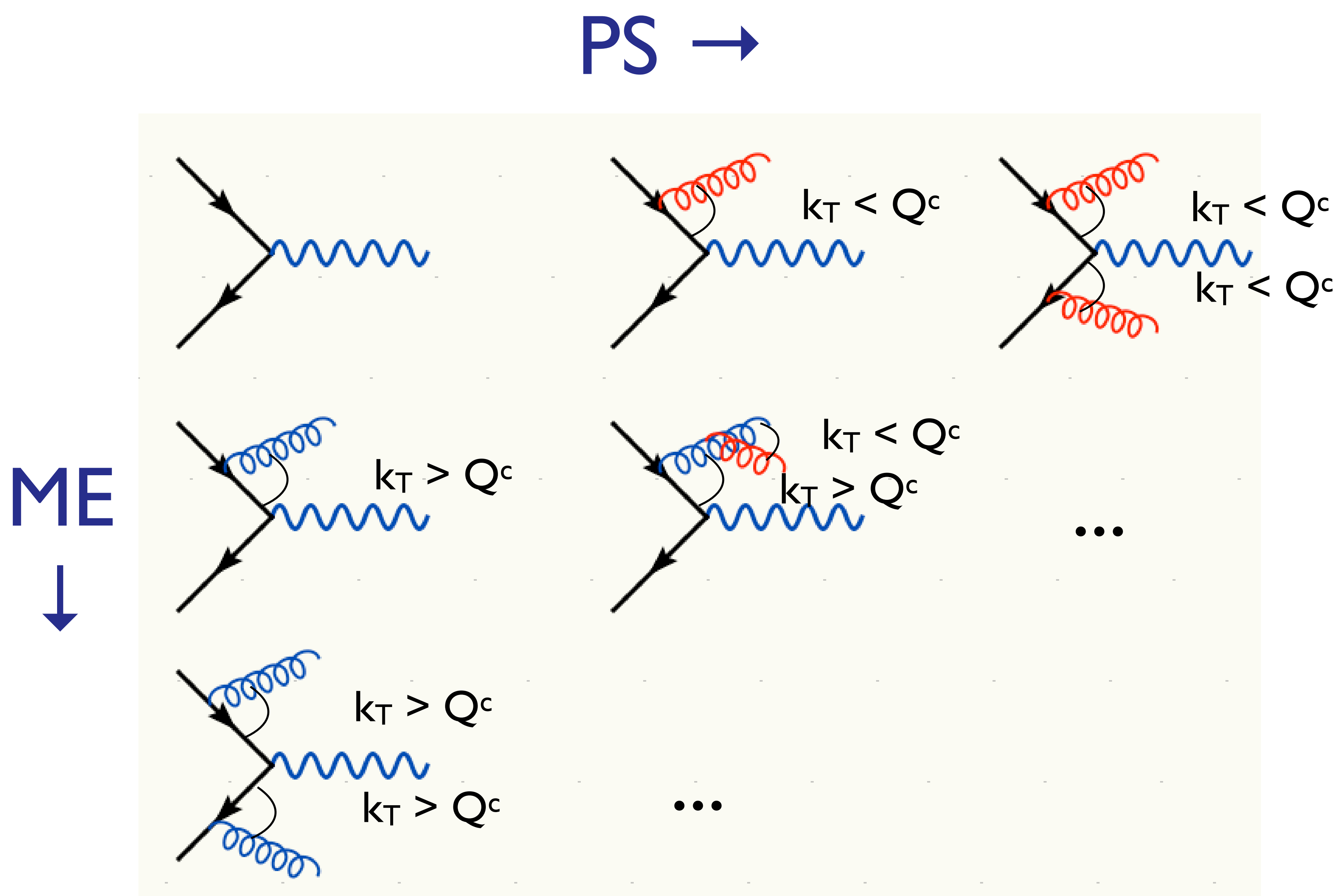


Double counting between ME and PS easily avoided using phase space cut between the two: PS below cutoff, ME above cutoff.

Merging ME with PS

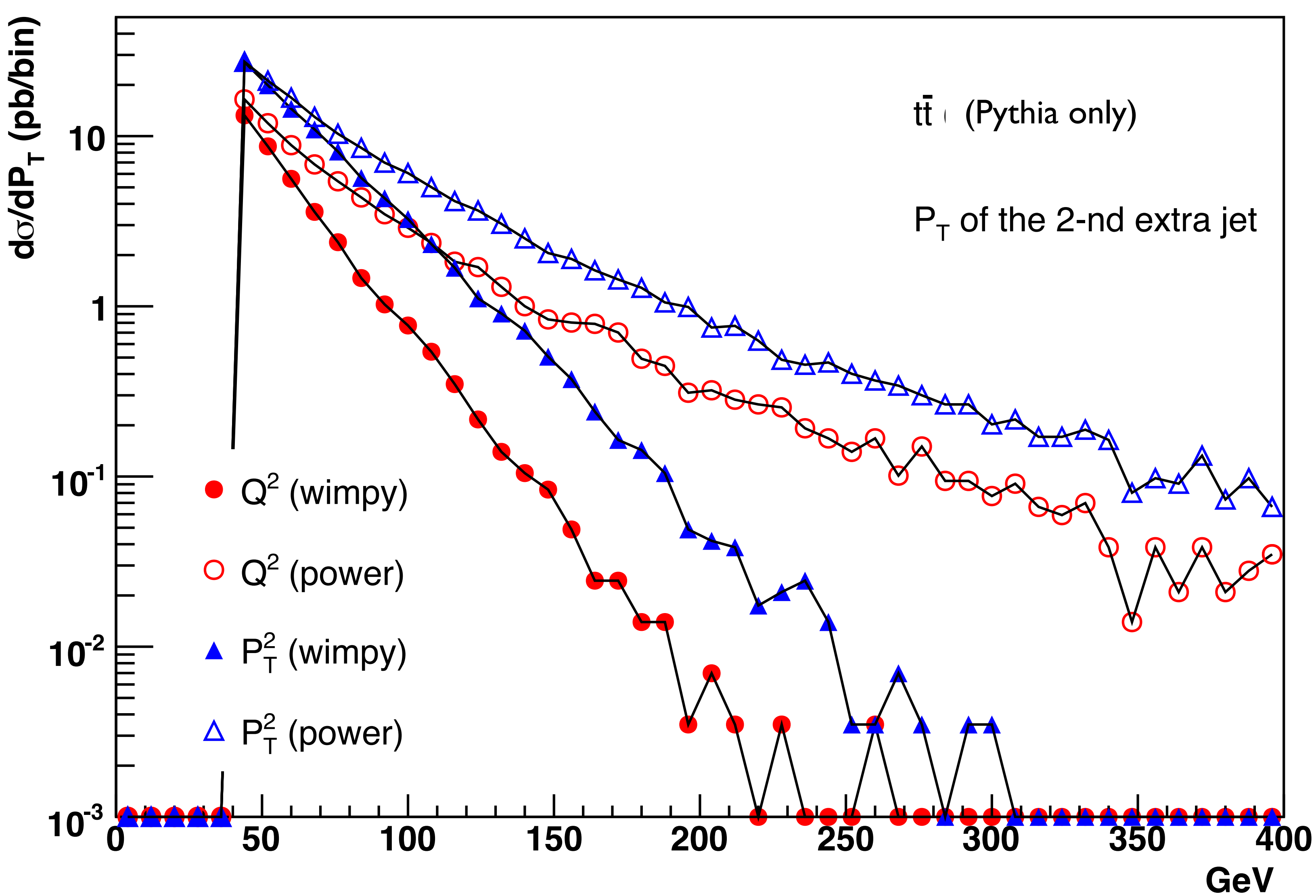
- So double counting problem easily solved, but what about getting smooth distributions that are independent of the precise value of Q^c?
- Below cutoff, distribution is given by PS
  - need to make ME look like PS near cutoff
- Correct the Matrix-Element to include
  - ➡ Interaction by interaction scale
- Need to include the Probability of no-emission

Matching schemes

- We still haven't specified how to apply the Sudakov reweighting to the matrix element
- Three general schemes available in the literature:
  - ➡ CKKW scheme [Catani,Krauss,Kuhn,Webber 2001; Krauss 2002]
  - ➡ Lönnblad scheme (or CKKW-L) [Lönnblad 2002]
  - ➡ MLM scheme [Mangano *unpublished* 2002; Mangano et al. 2007]

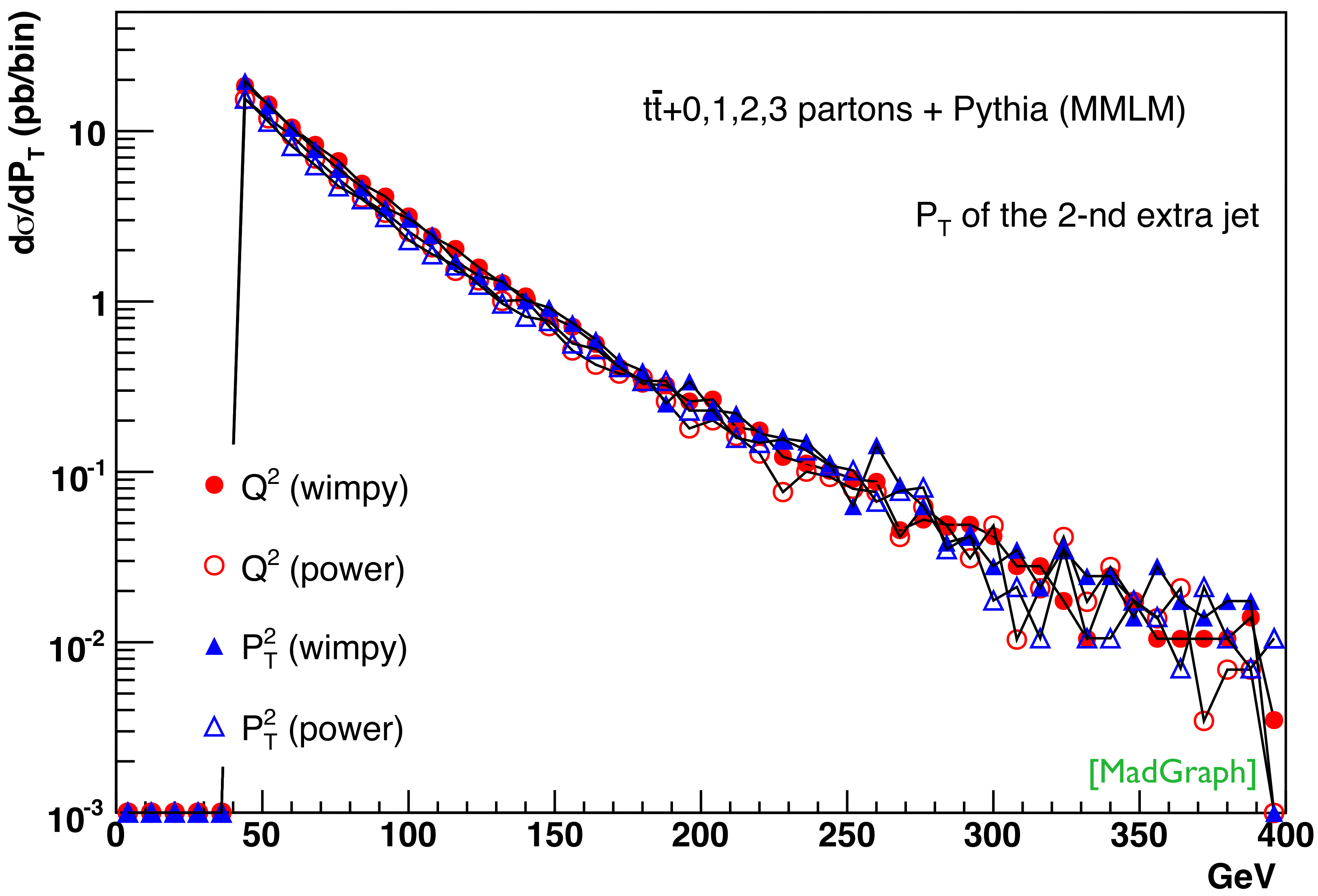
PS alone vs matched samples

In the soft-collinear approximation of Parton Shower MCs, parameters are used to tune the result ⇒ Large variation in results (small prediction power)



PS alone vs ME matching

In a matched sample these differences are irrelevant since the behaviour at high pt is dominated by the matrix element.



Plan

Lecture II

- Narrow-width
- Basic of matching/merging
- Basic of NLO computation
- Overview of MG5aMC

Perturbative expansion

$d\hat{\sigma}_{ab\to X}(\hat{s},\mu_F,\mu_R)$ **Parton-level cross section**

- The parton-level cross section can be computed as a series in perturbation theory, using the coupling constant as an expansion parameter, schematically:

$$\hat{\sigma} = \sigma^{\mathrm{Born}}\left(1 + \frac{\alpha_s}{2\pi}\sigma^{(1)} + \left(\frac{\alpha_s}{2\pi}\right)^2\sigma^{(2)} + \left(\frac{\alpha_s}{2\pi}\right)^3\sigma^{(3)} + \dots\right)$$

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LO predictions

Perturbative expansion

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NLO corrections

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LO predictions

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- Including higher corrections improves predictions and reduces theoretical uncertainties

Improved predictions

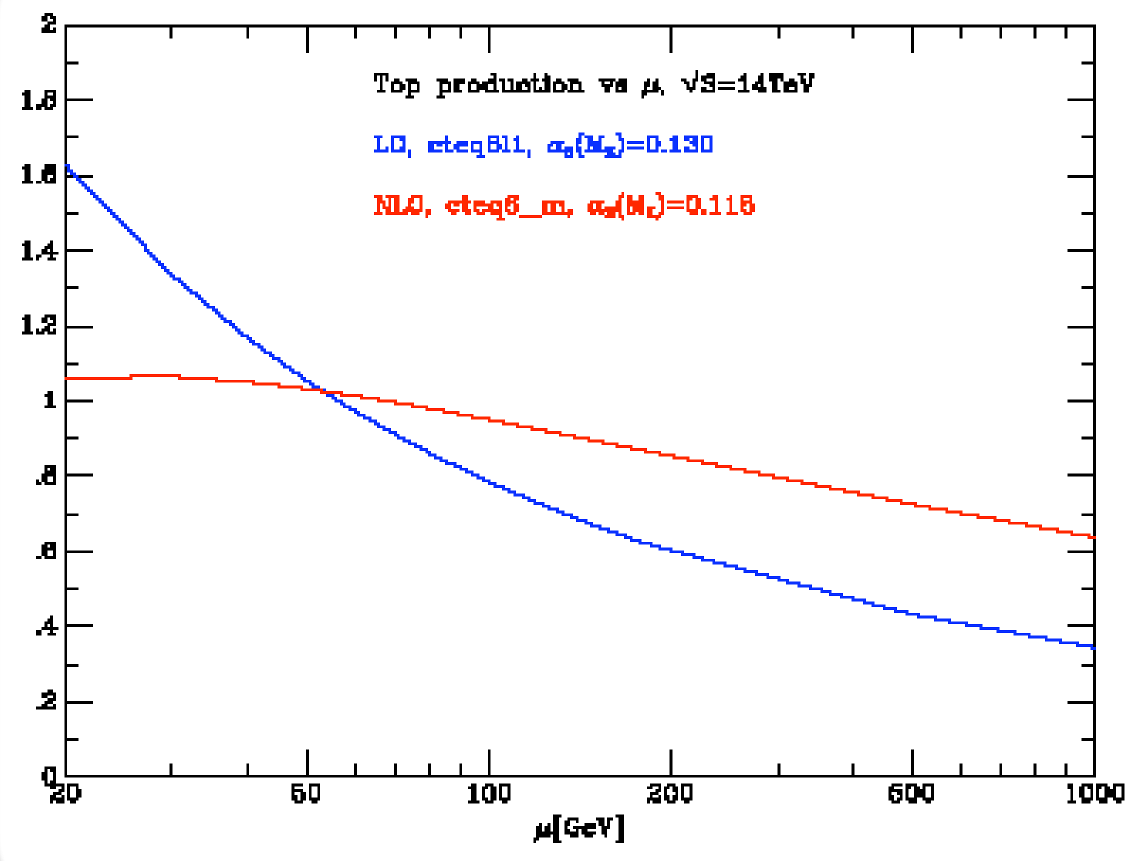
$$d\sigma = \sum_{a,b} \int dx_1 dx_2 \;\; f_a(x_1, \mu_F) f_b(x_2, \mu_F) \, d\hat{\sigma}_{ab\to X}(\hat{s}, \mu_F, \mu_R)$$

$$\hat{\sigma} = \sigma^{\mathrm{Born}} \left(1 + \frac{\alpha_s}{2\pi} \sigma^{(1)} + \left(\frac{\alpha_s}{2\pi}\right)^2 \sigma^{(2)} + \left(\frac{\alpha_s}{2\pi}\right)^3 \sigma^{(3)} + \dots \right)$$

- Leading Order predictions can depend strongly on the renormalization and factorization scales
- Including higher order corrections reduces the dependence on these scales

Going NLO

- At NLO the dependence on the renormalization and factorization scales is reduced
  - First order where scale dependence in the running coupling and the PDFs is compensated for via the loop corrections: **first reliable estimate of the total cross section**
  - Better description of final state: impact of extra radiation included (e.g. jets can have substructure)
  - Opening of additional initial state partonic channels



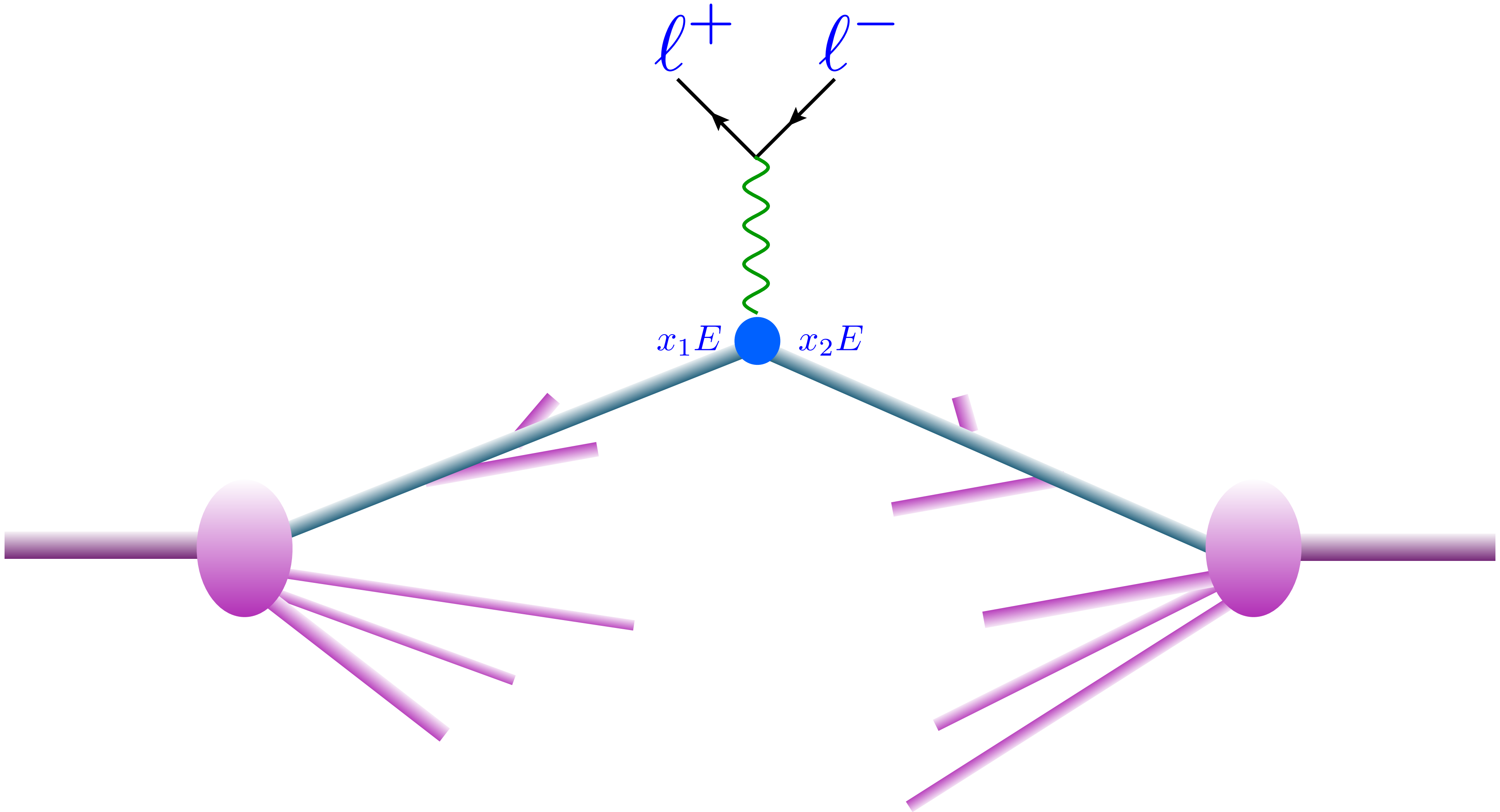
NLO corrections

- **NLO corrections have three parts:**
  - The Born contribution, i.e. the Leading order.
  - Virtual (or Loop) corrections: formed by an amplitude with a closed loop of particles interfered with the Born amplitudes
  - Real emission corrections: formed by amplitudes with one extra parton compared to the Born process
- **Both Virtual and Real emission have one power of α_s extra compared to the Born process**

$$\sigma^{\mathrm{NLO}} = \int_m d\sigma^B + \int_m d\sigma^V + \int_{m+1} d\sigma^R$$

NLO predictions

- As an example, consider Drell-Yan Z/γ* production



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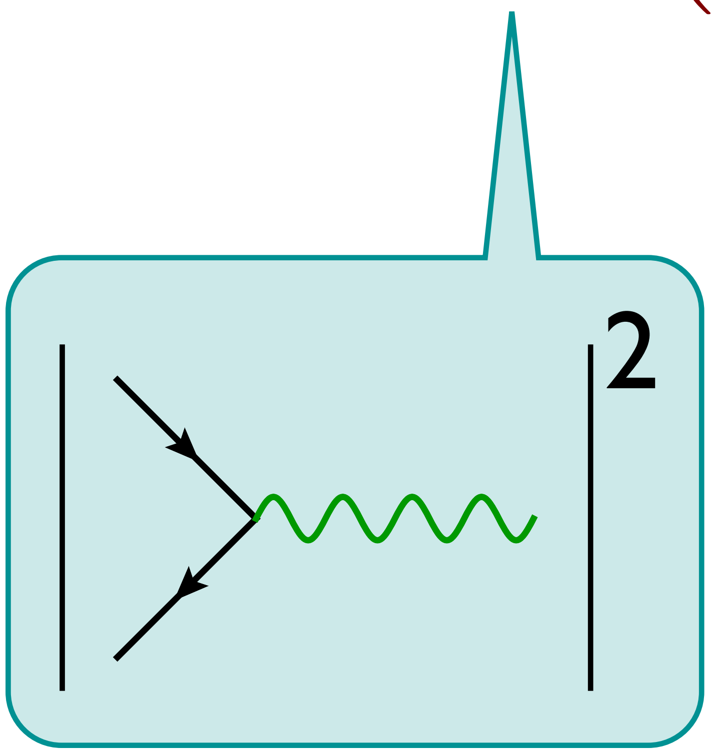

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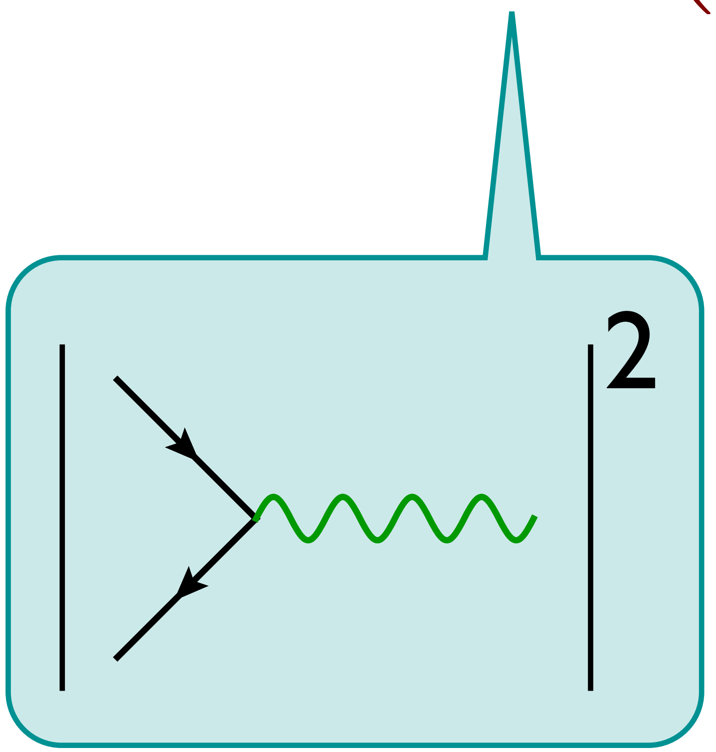

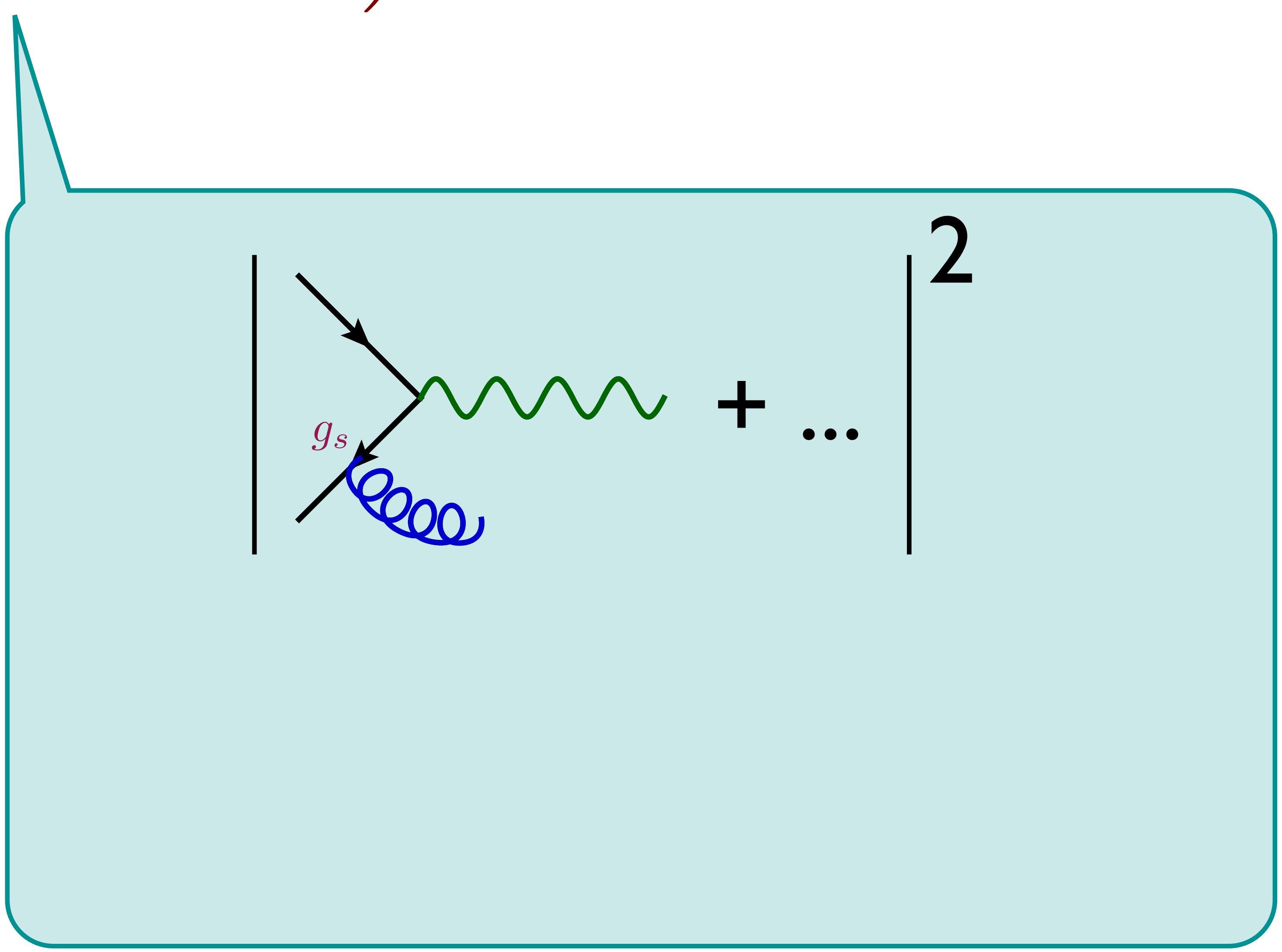

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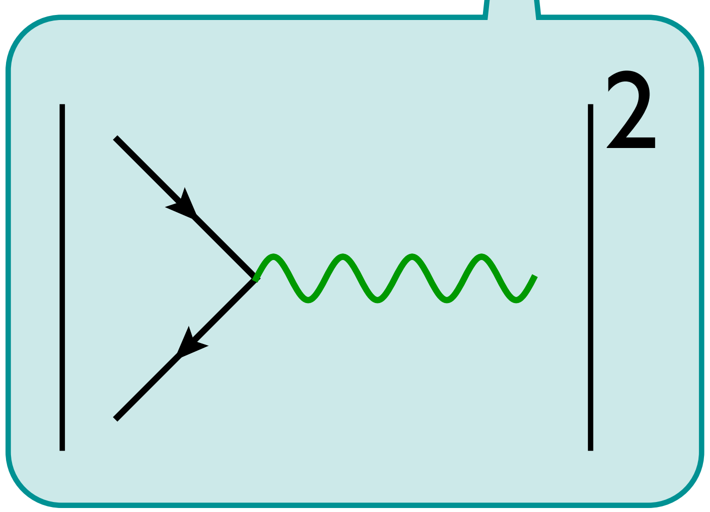

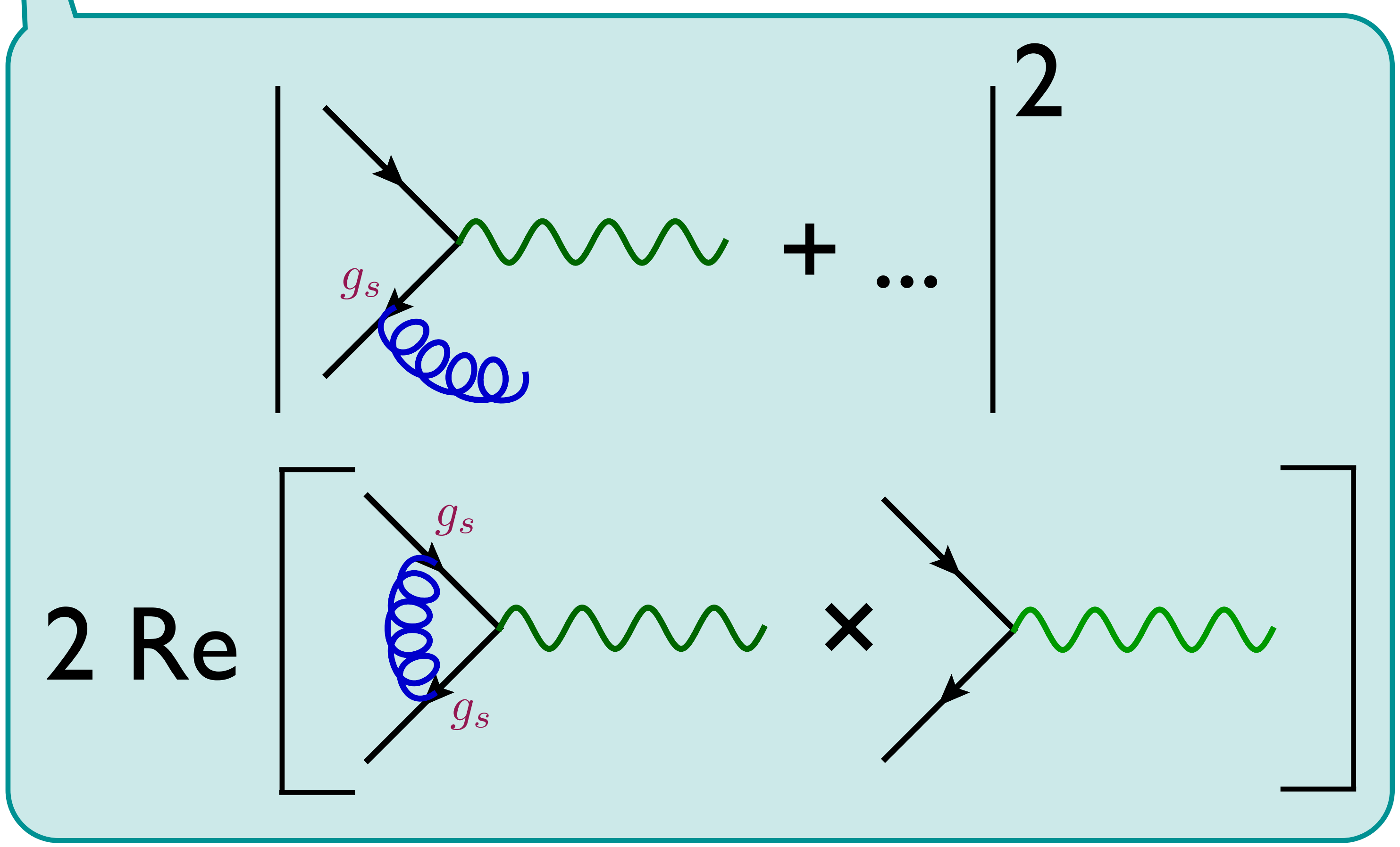

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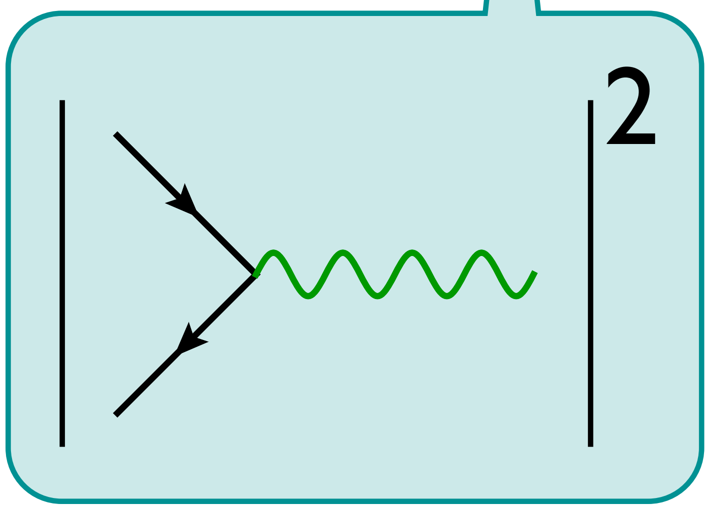
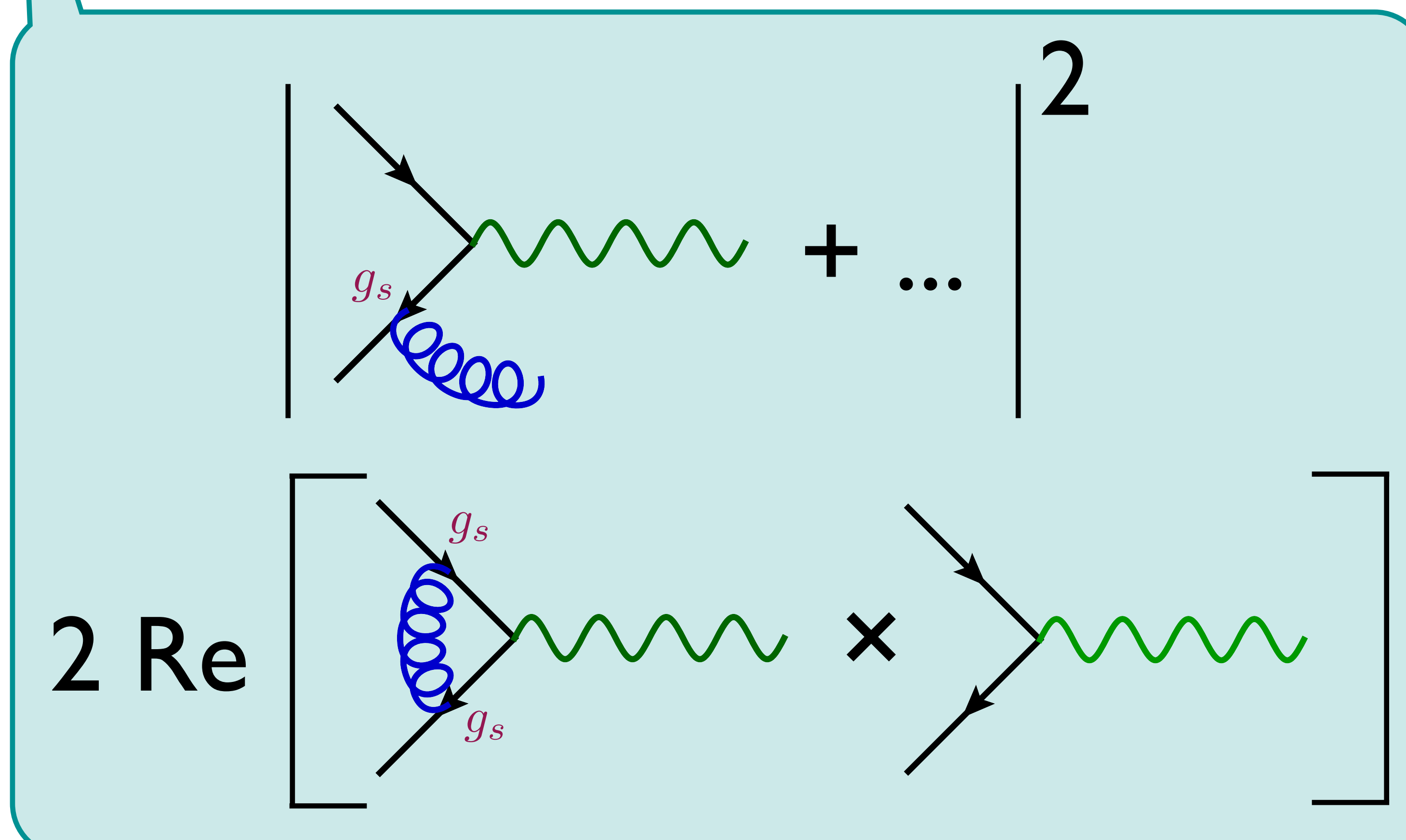
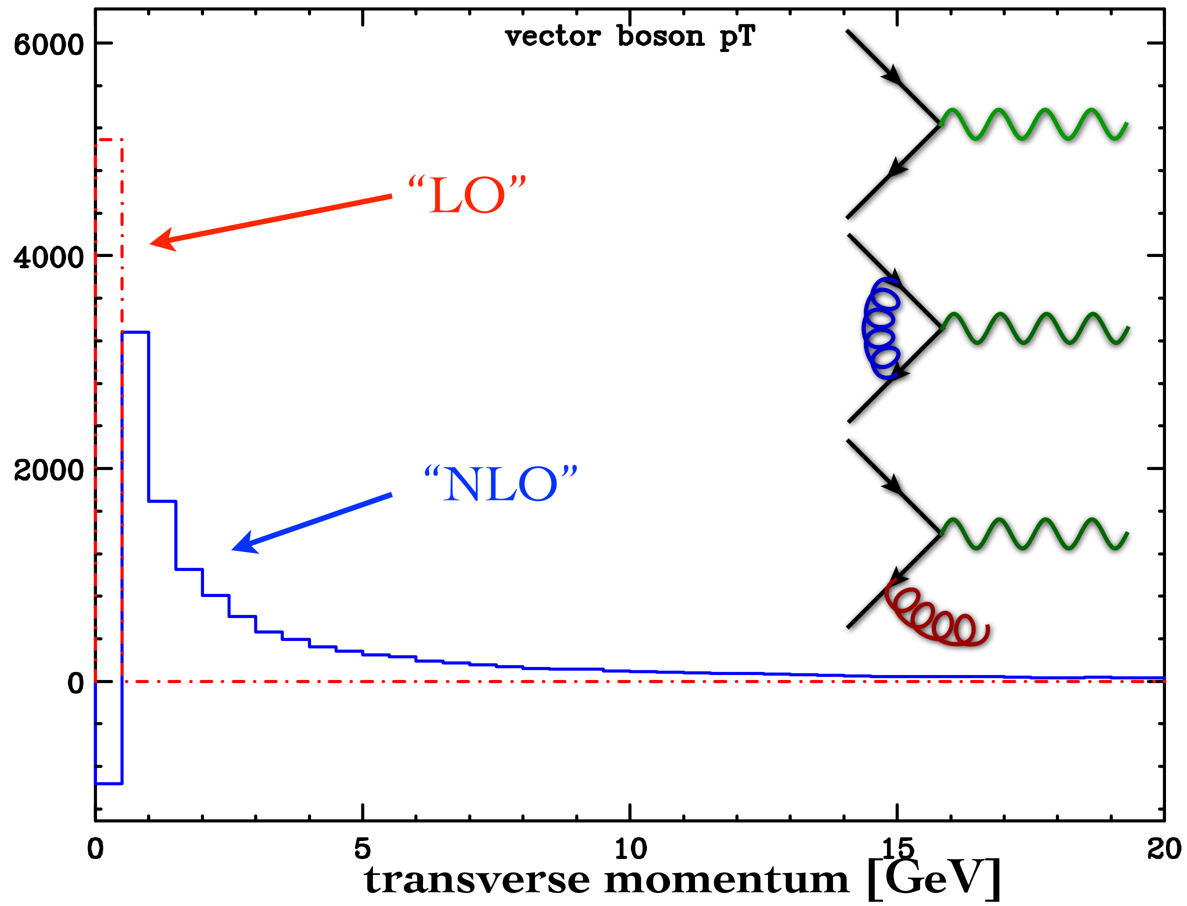


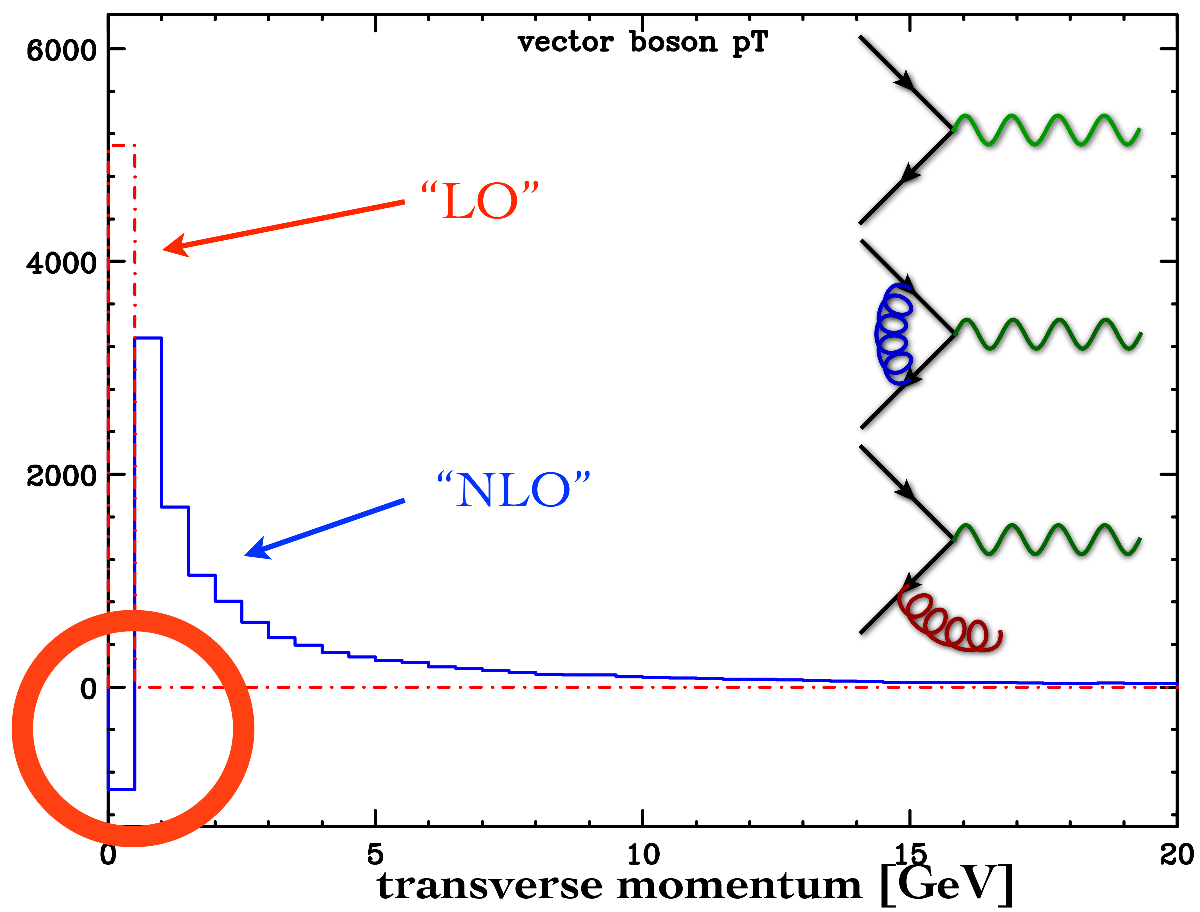


Not definite positive

Fixed Order calculations



Fixed Order calculations



Negative contribution of the 0-bin

Infrared safe observables

- For an observable to be calculable in fixed-order perturbation theory, the observable should be infrared safe, i.e., it should be insensitive to the emission of soft or collinear partons.

- In particular, if p_i is a momentum occurring in the definition of an observable, it most be invariant under the branching

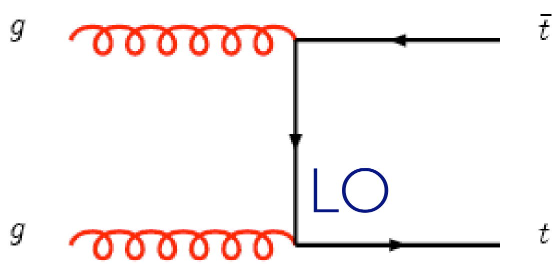
$$p_i \longrightarrow p_j + p_k,$$

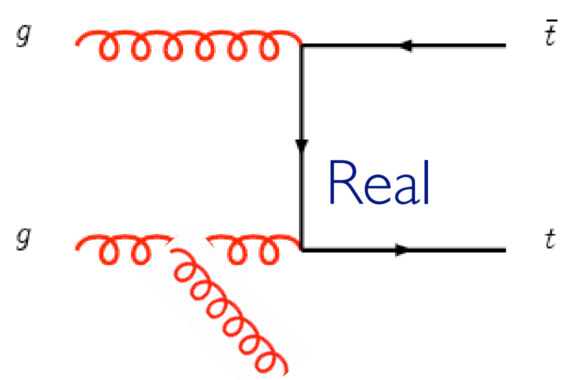
whenever p_j and p_k are collinear or one of them is soft.

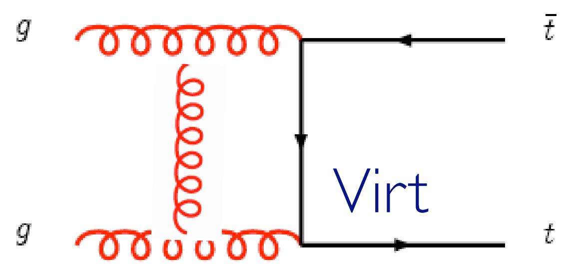
- Examples
    - "The number of gluons" produced in a collision is not an infrared safe observable
    - "The number of hard jets defined using the k_T algorithm with a transverse momentum above 40 GeV," produced in a collision is an infrared safe observable

NLO...?

- Are all (IR-safe) observables that we can compute using a NLO code correctly described at NLO? Suppose we have a NLO code for pp $\longrightarrow$ ttbar







- Total cross section
- Transverse momentum of the top quark
- Transverse momentum of the top-antitop pair
- Transverse momentum of the jet
- Top-antitop invariant mass
- Azimuthal distance between the top and anti-top

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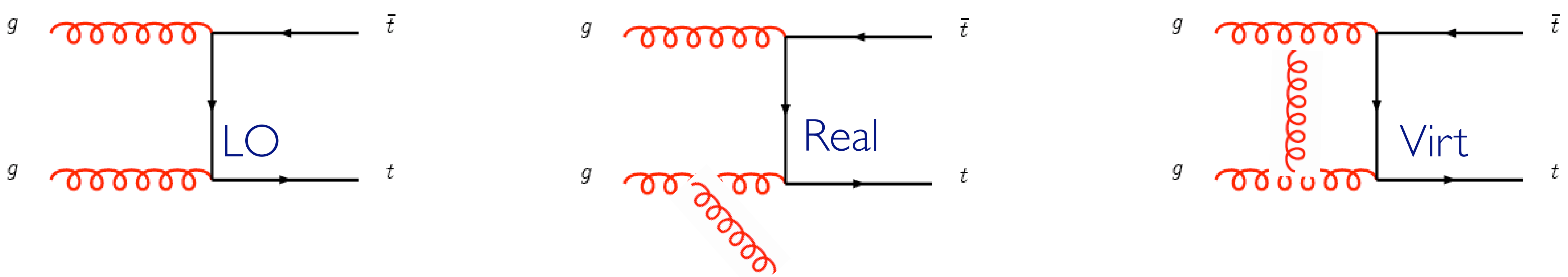

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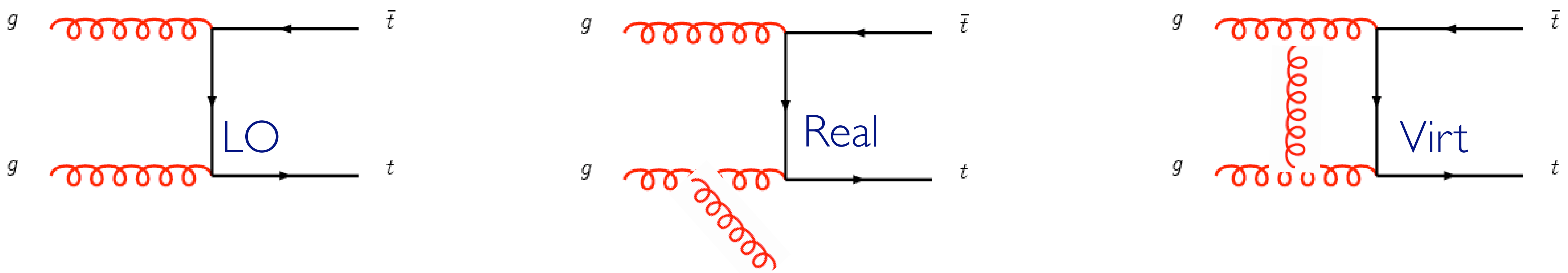

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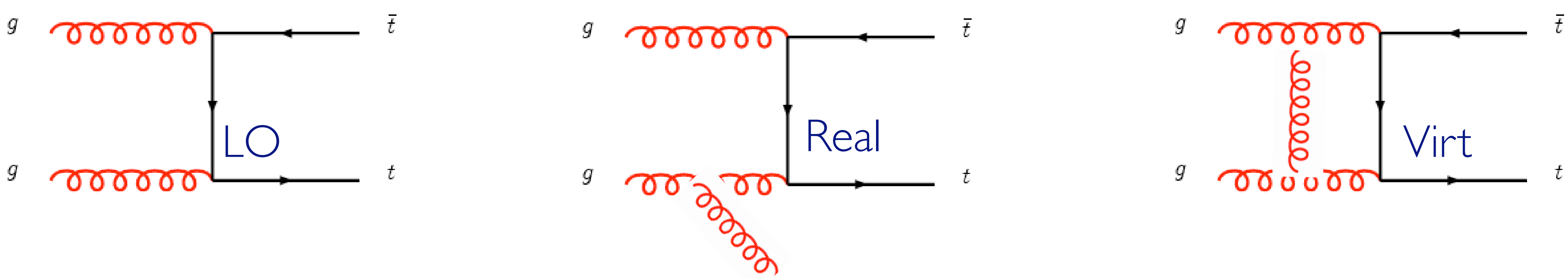

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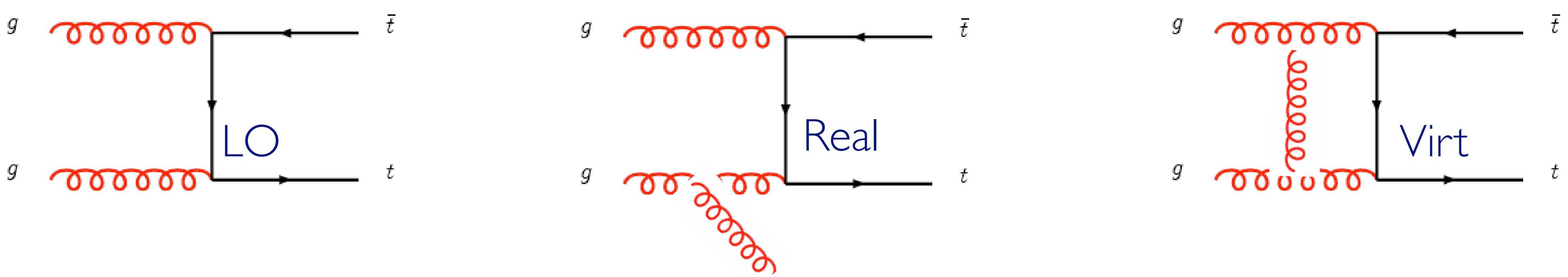

NLO?

- Total cross section ✔
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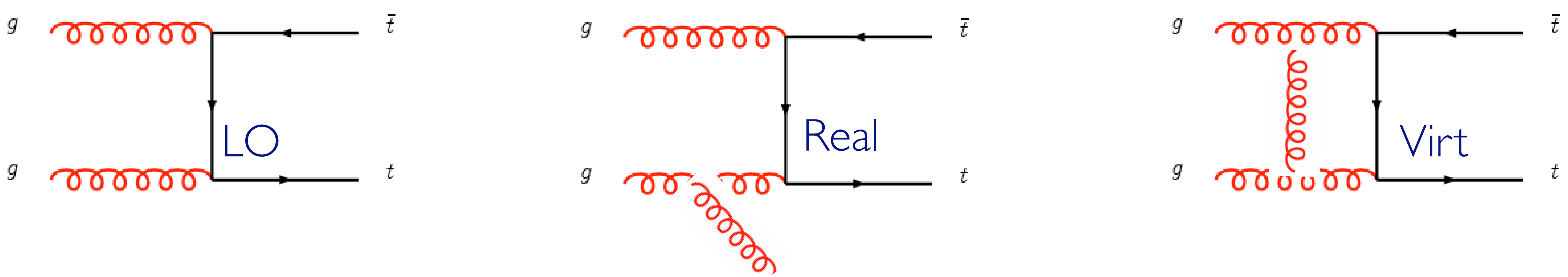

NLO?

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- Transverse momentum of the top quark ✔
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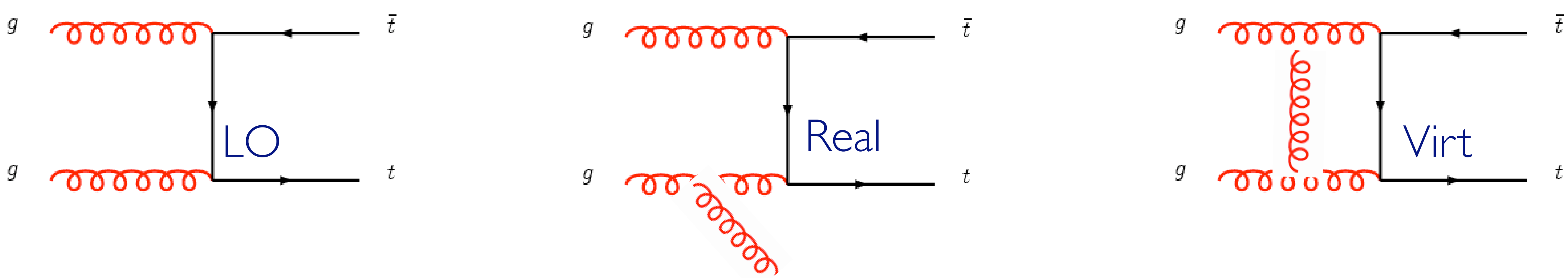

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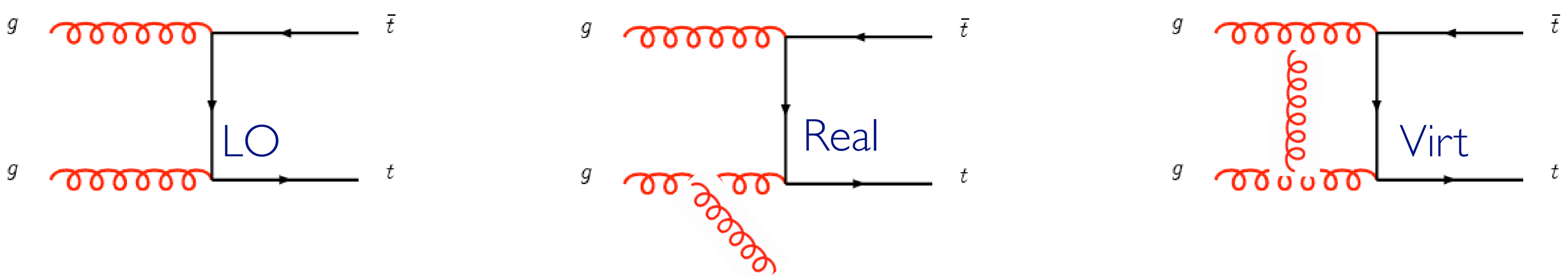

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NLO+PS matching

ME	Shower MC
1. Fixed order calculation 2. Computationally expensive 3. Limited number of particles 4. Valid when partons are hard and well separated 5. Quantum interference correct 6. Needed for multi-jet description	1. Resums logs to all orders 2. Computationally cheap 3. No limit on particle multiplicity 4. Valid when partons are collinear and/or soft 5. Partial interference through angular ordering 6. Needed for hadronization

Approaches are complementary: merge them!

Difficulty: avoid double counting, ensure smooth distributions

NLO+PS matching

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Shower MC

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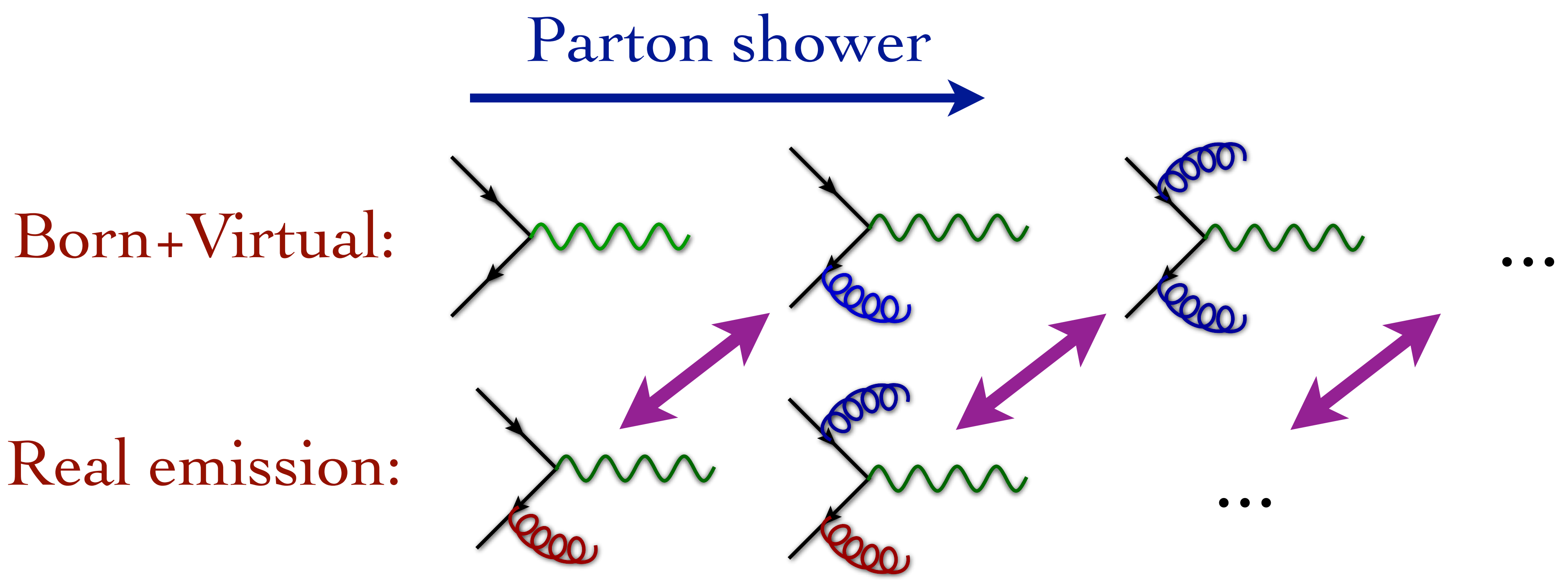
No longer true at NLO!

Approaches are complementary: merge them!

Difficulty: avoid double counting, ensure smooth distributions

Matching NLO

- At NLO one faces even more severe double-counting issues:



- And also part of the virtual contribution is double counted through the definition of the Sudakov factor Δ

MC@NLO procedure

[Frixione & Webber (2002)]

- To remove the double counting, we can add and subtract the same term to the *m* and *m*+1 body configurations

$$\frac{d\sigma_{\mathrm{NLOwPS}}}{dO} = \left[d\Phi_m \left(B + \int_{\mathrm{loop}} V + \int d\Phi_1 MC\right)\right] I_{\mathrm{MC}}^{(m)}(O)$$

$$+ \left[d\Phi_{m+1}(R - MC)\right] I_{\mathrm{MC}}^{(m+1)}(O)$$

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- This MC term depends of the Parton-shower (since it removes the exact double counting)

$$MC = \left|\frac{\partial\left(t^{MC}, z^{MC}, \phi\right)}{\partial\Phi_1}\right| \frac{1}{t^{MC}} \frac{\alpha_s}{2\pi} \frac{1}{2\pi} P\left(z^{MC}\right) \mathcal{B}$$

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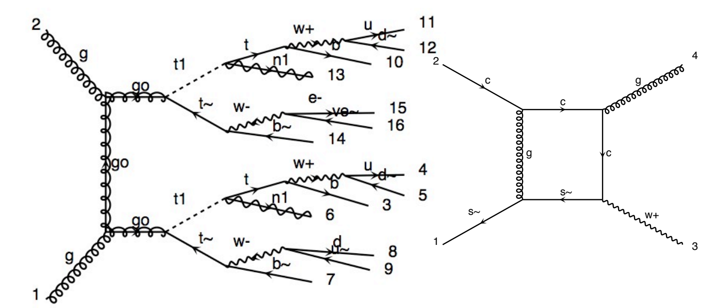
Type of generation

	Tree (B)SM	NLO (QCD) (SM)	NLO (QCD) (BSM)	NLO (EW) (SM)	Loop Induced (B)SM
Fix Order	✓	✓	✓	✓	✓



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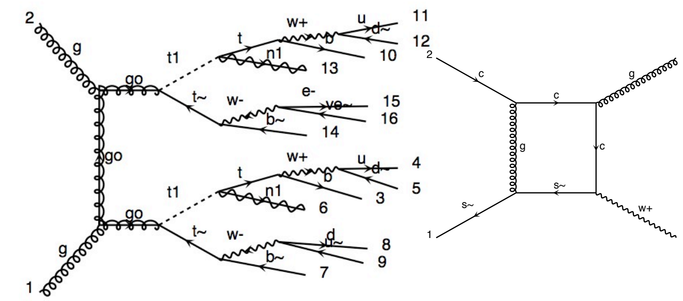
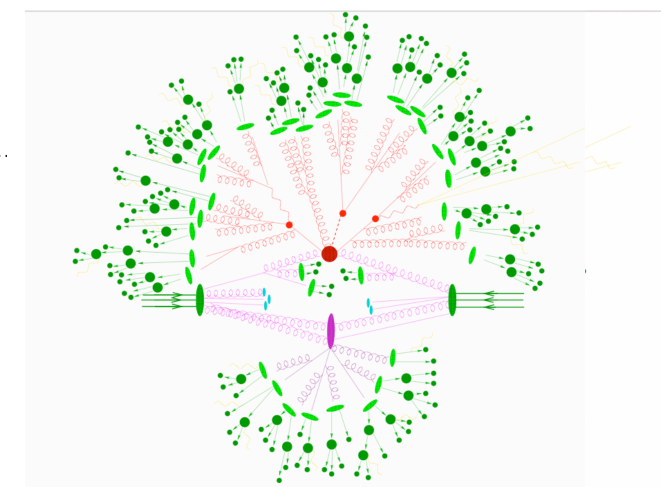
+Parton Shower	✓	✓	✓	✗	✓





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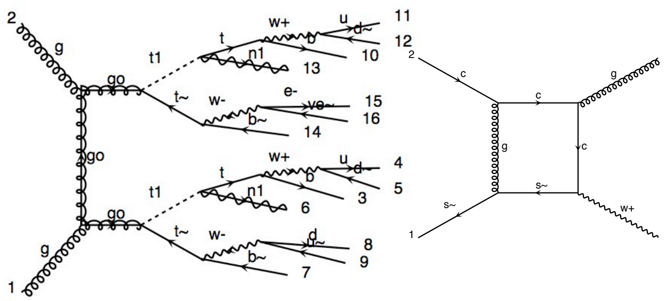
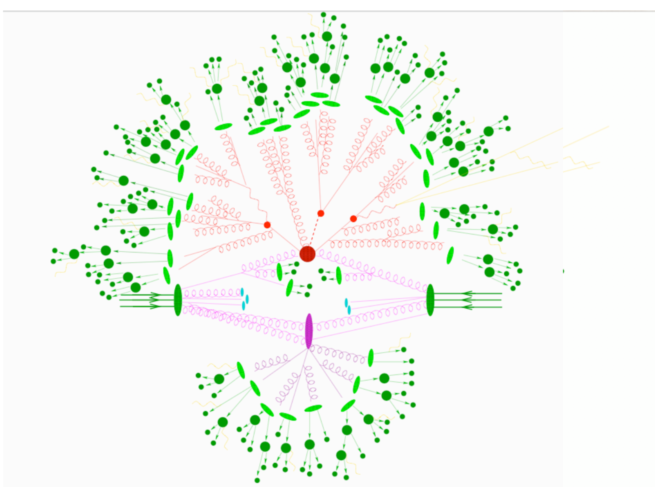
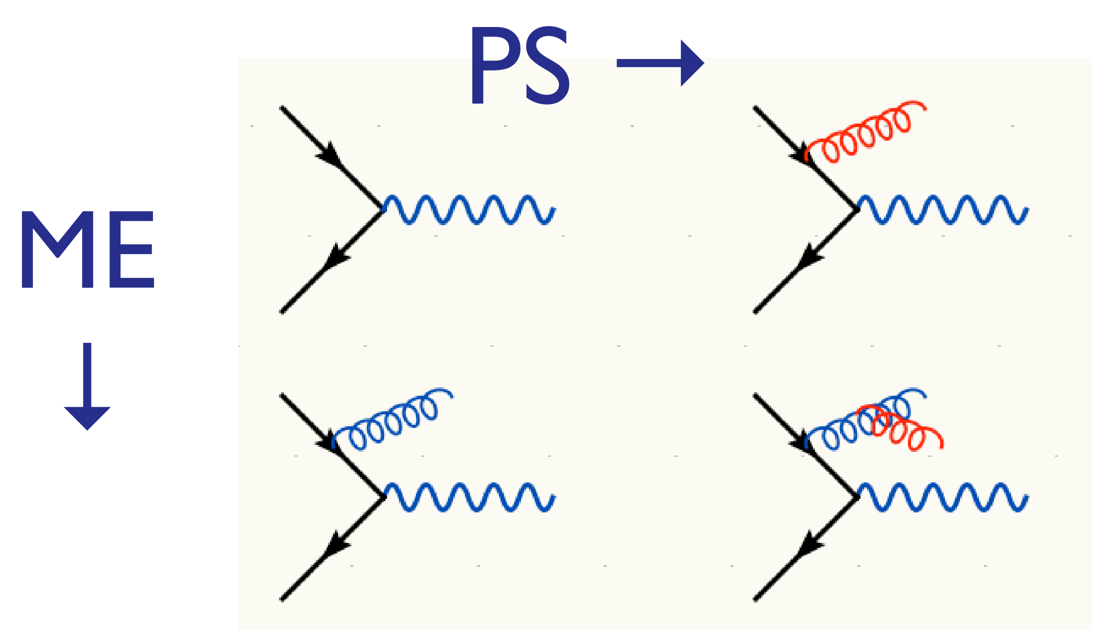
+Parton Shower	✓	✓	✓	✗	✓
Merged Sample	✓	✓	?	✗	✓







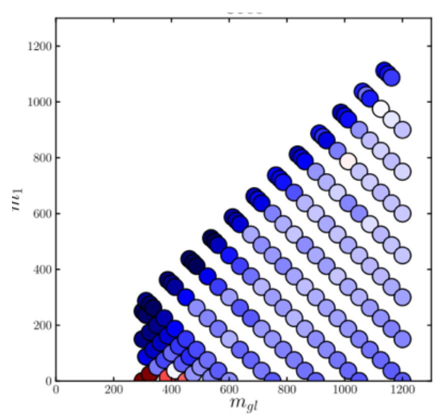
LO Feature

LO Feature

Auto-Width

$$\Gamma = ?$$

Parameter scan

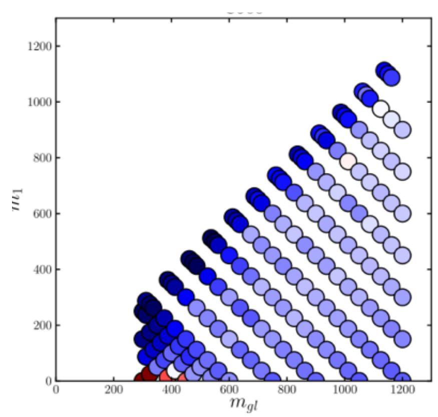


LO Feature

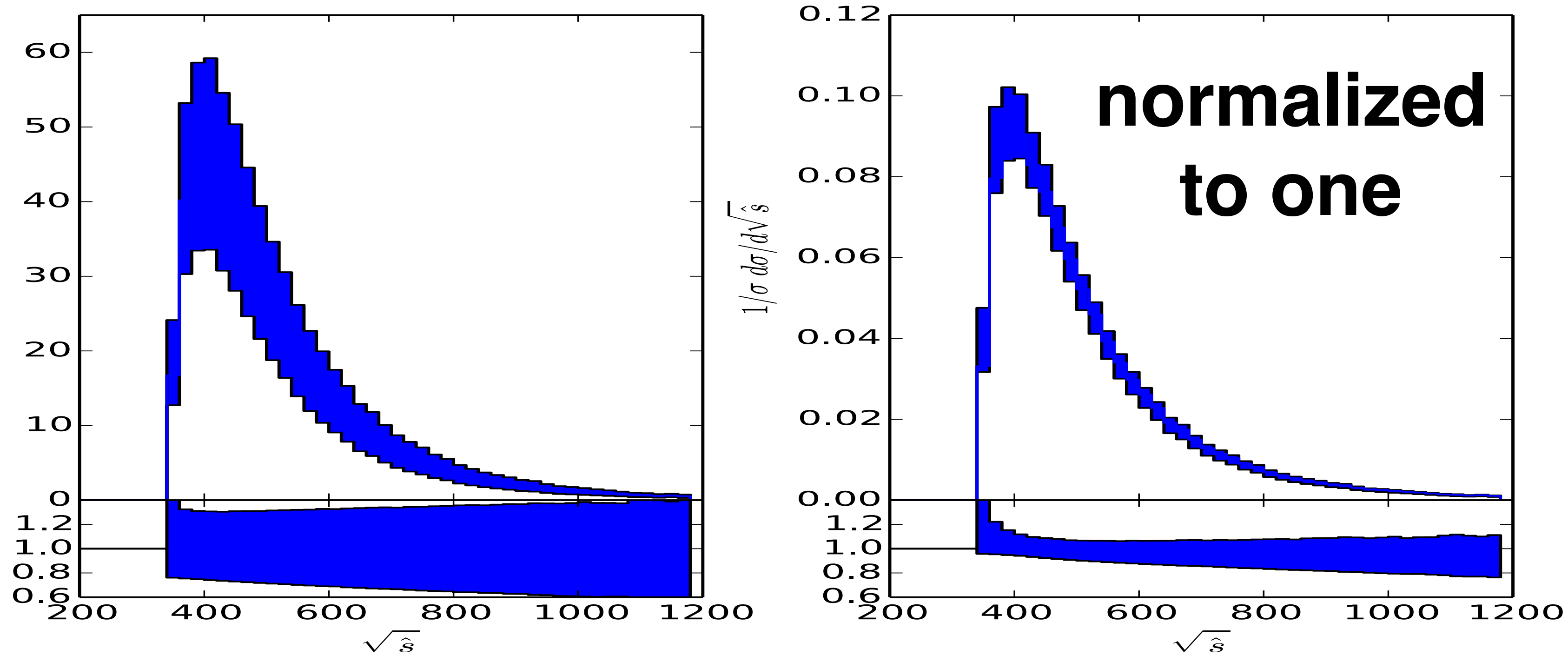
Auto-Width

$$\Gamma = ?$$

Parameter scan



Systematics



BSM re-weighting

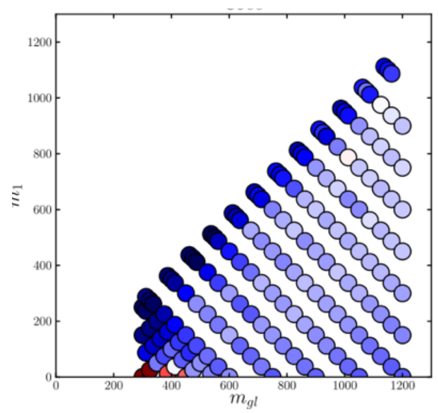
$$|M_{new}|^2 / |M_{old}|^2$$

LO Feature

Auto-Width

$$\Gamma = ?$$

Parameter scan



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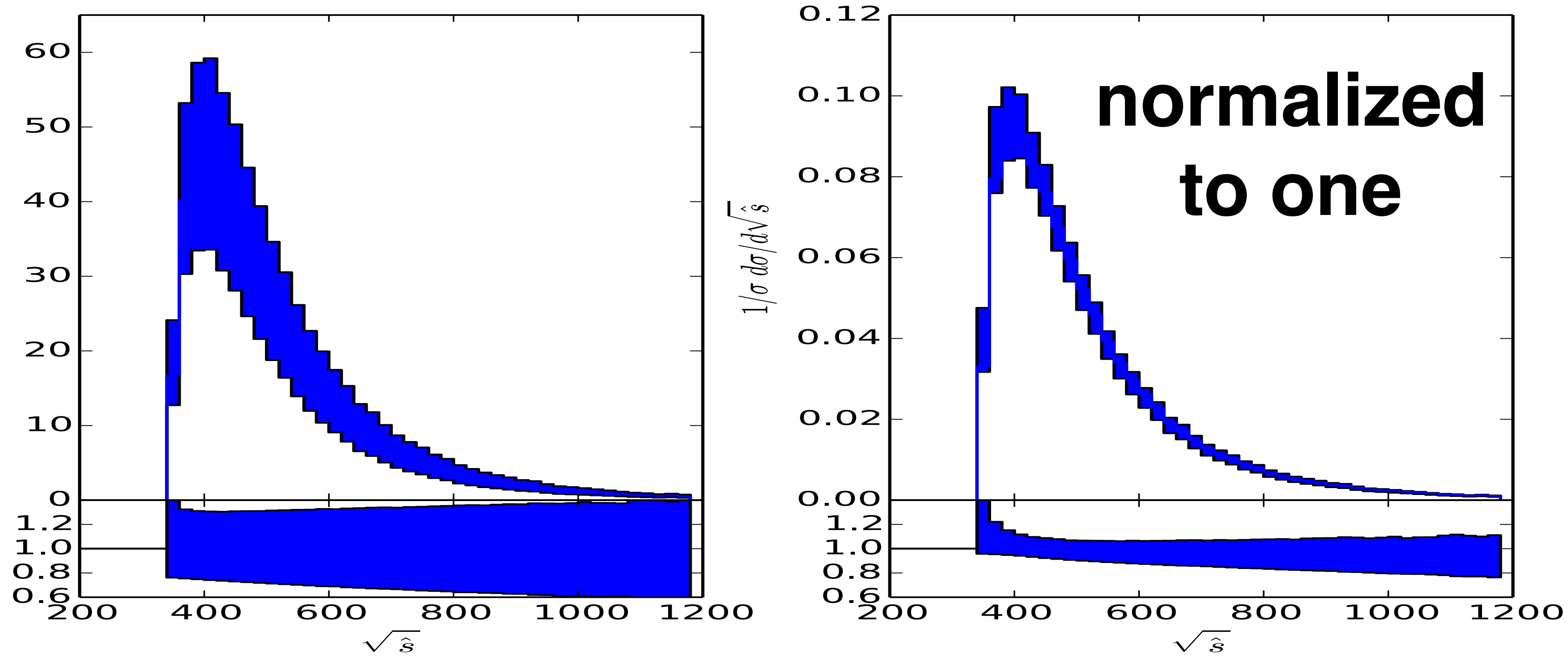
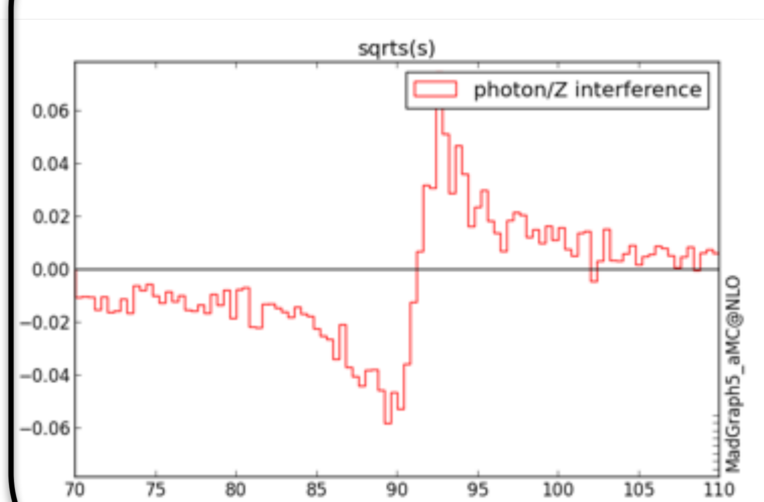


Interference



BSM re-weighting

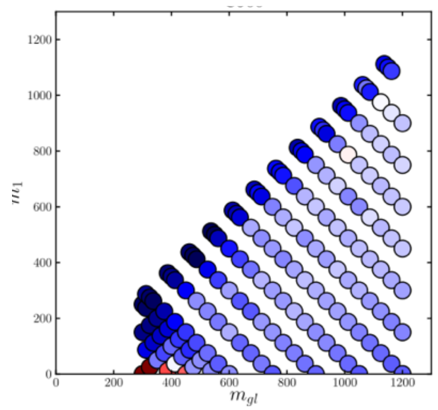
$$|M_{new}|^2 / |M_{old}|^2$$

LO Feature

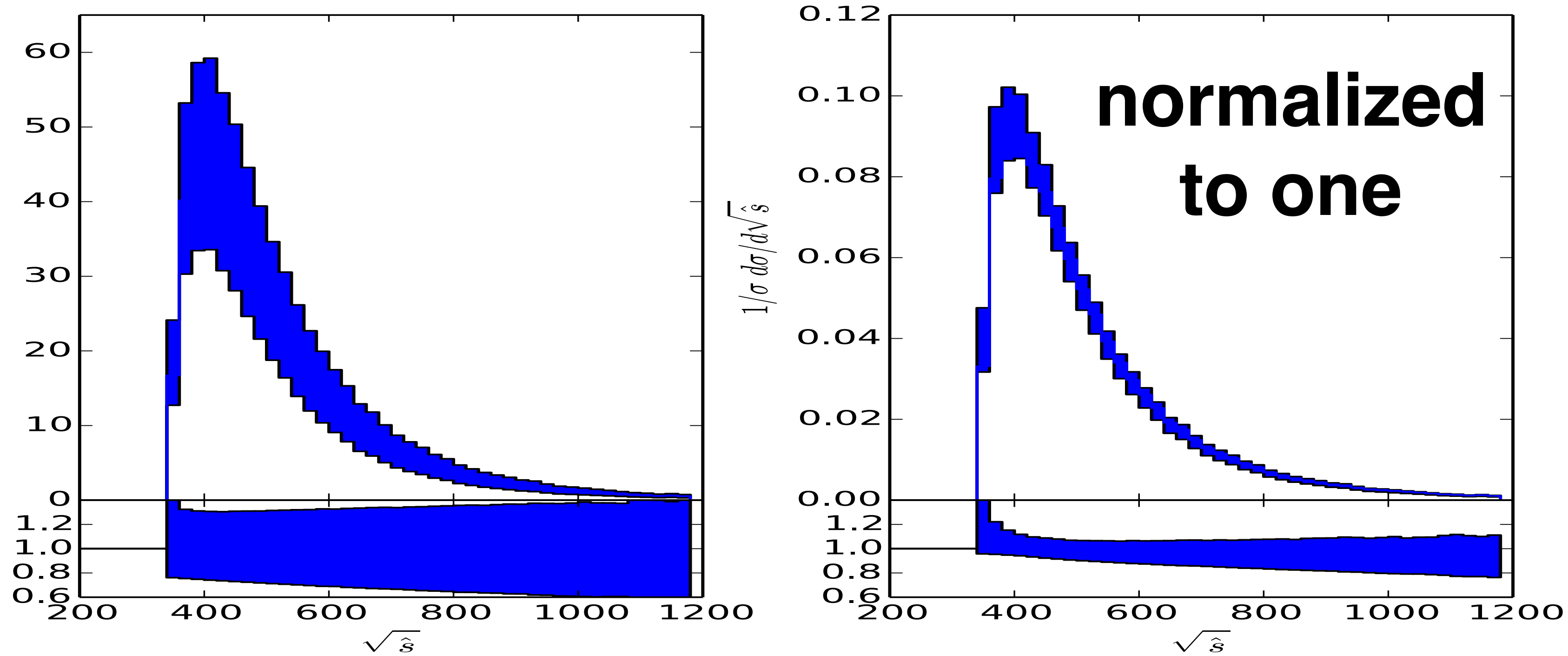
Auto-Width

$$\Gamma = ?$$

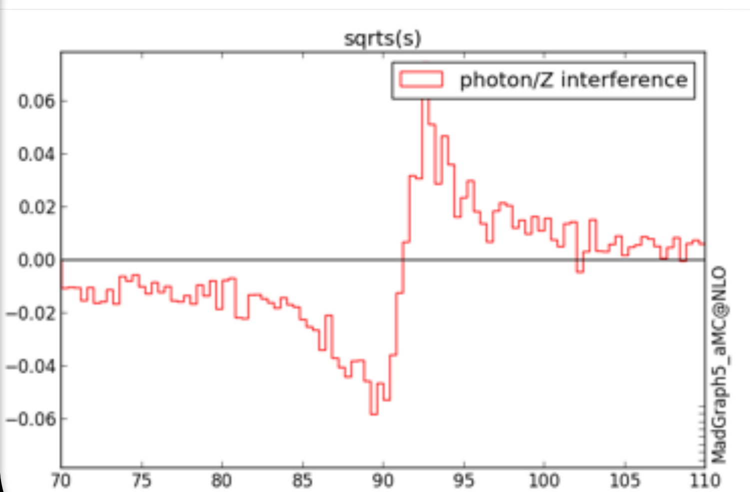
Parameter scan



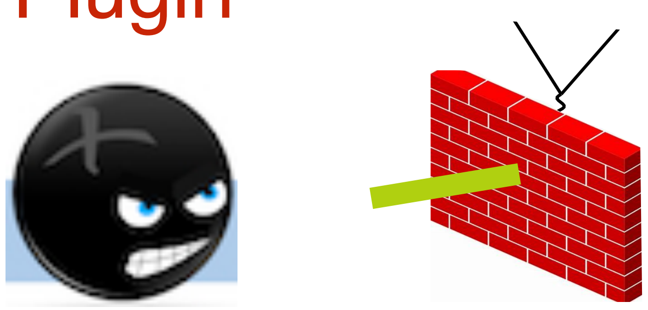
Systematics



Interference



Plugin



Interface

MadAnalysis 5

BSM re-weighting

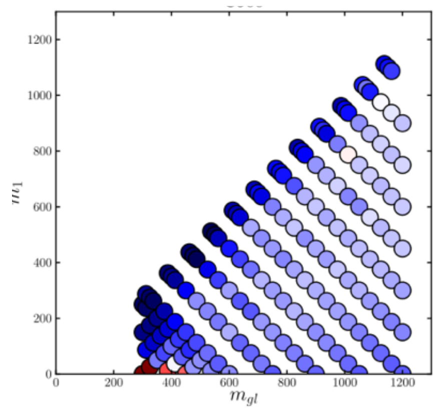
$$|M_{new}|^2 / |M_{old}|^2$$

LO Feature

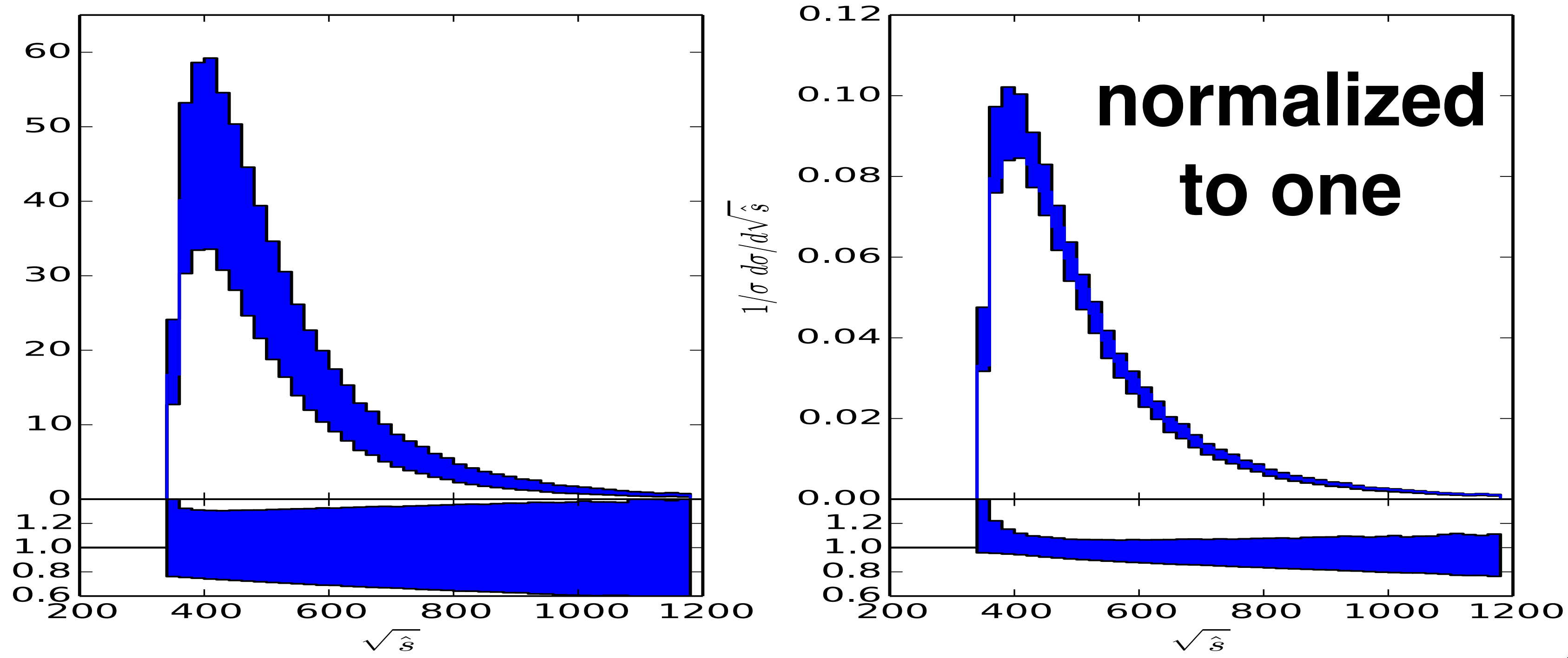
Auto-Width

$$\Gamma = ?$$

Parameter scan



Systematics



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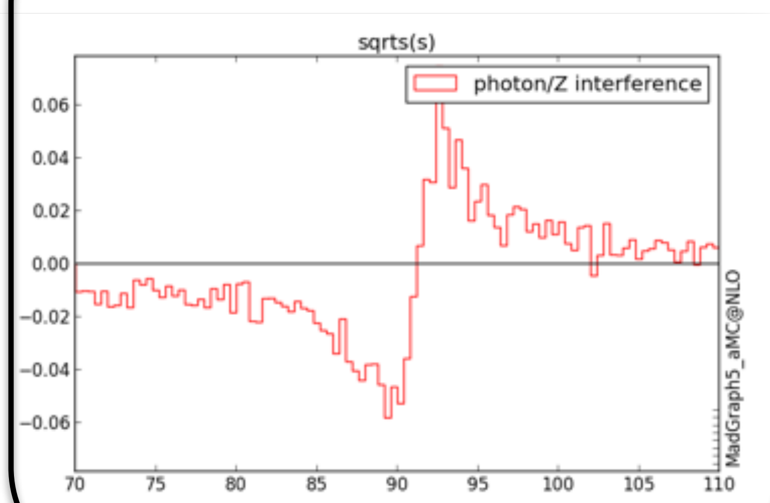
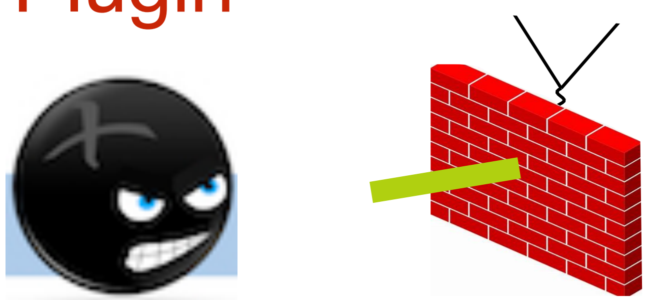
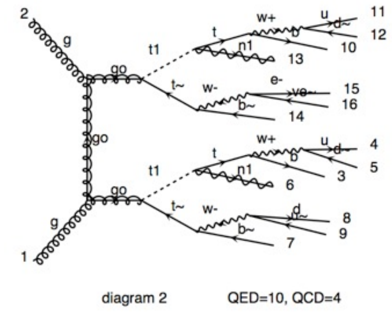


Plugin



Narrow-width



Interface

BSM re-weighting

$$|M_{new}|^2 / |M_{old}|^2$$

What to remember

- LO provides shapes
- NLO reduces uncertainty on the total cross-section
    - Not all observables are NLO accurate
- Can merge sample to increase accuracy on some observables
- MG5aMC provides all those simulations
- Need that you understand the hypothesis