Positivity constraints on QGC operators

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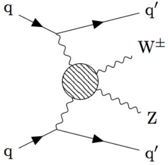
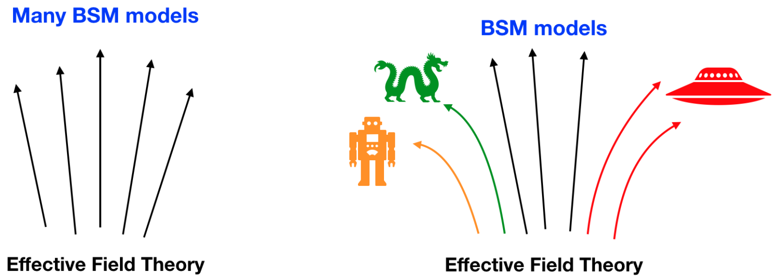
QGC: Quartic Gauge-boson Couplings



- What else do we know about them? $\Downarrow$

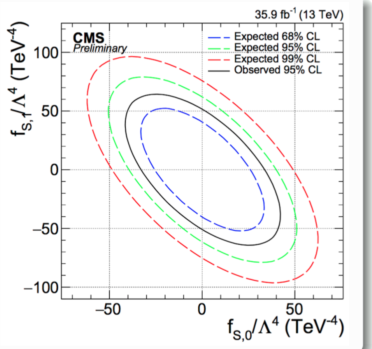
$$
\begin{aligned}
O_{S,0} &= [(D_\mu\Phi)^\dagger D_\nu\Phi] \times [(D^\mu\Phi)^\dagger D^\nu\Phi] \\
O_{S,1} &= [(D_\mu\Phi)^\dagger D^\mu\Phi] \times [(D_\nu\Phi)^\dagger D^\nu\Phi] \\
O_{S,2} &= [(D_\mu\Phi)^\dagger D_\nu\Phi] \times [(D^\nu\Phi)^\dagger D^\mu\Phi] \\
O_{M,0} &= \mathrm{Tr}\left[\hat{W}_{\mu\nu}\hat{W}^{\mu\nu}\right] \times \left[(D_\beta\Phi)^\dagger D^\beta\Phi\right] \\
O_{M,1} &= \mathrm{Tr}\left[\hat{W}_{\mu\nu}\hat{W}^{\nu\beta}\right] \times \left[(D_\beta\Phi)^\dagger D^\mu\Phi\right] \\
O_{M,2} &= \left[\hat{B}_{\mu\nu}\hat{B}^{\mu\nu}\right] \times \left[(D_\beta\Phi)^\dagger D^\beta\Phi\right] \\
O_{M,3} &= \left[\hat{B}_{\mu\nu}\hat{B}^{\nu\beta}\right] \times \left[(D_\beta\Phi)^\dagger D^\mu\Phi\right] \\
O_{M,4} &= \left[(D_\mu\Phi)^\dagger \hat{W}_{\beta\nu} D^\mu\Phi\right] \times \hat{B}^{\beta\nu} \\
O_{M,5} &= \left[(D_\mu\Phi)^\dagger \hat{W}_{\beta\nu} D^\nu\Phi\right] \times \hat{B}^{\beta\mu} (+h.c.) \\
O_{M,7} &= \left[(D_\mu\Phi)^\dagger \hat{W}_{\beta\nu}\hat{W}^{\beta\mu} D^\nu\Phi\right]
\end{aligned}
$$

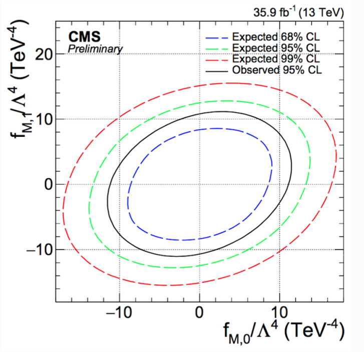
$$
\begin{aligned}
O_{T,0} &= \mathrm{Tr}\left[\hat{W}_{\mu\nu}\hat{W}^{\mu\nu}\right] \times \mathrm{Tr}\left[\hat{W}_{\alpha\beta}\hat{W}^{\alpha\beta}\right] \\
O_{T,1} &= \mathrm{Tr}\left[\hat{W}_{\alpha\nu}\hat{W}^{\mu\beta}\right] \times \mathrm{Tr}\left[\hat{W}_{\mu\beta}\hat{W}^{\alpha\nu}\right] \\
O_{T,2} &= \mathrm{Tr}\left[\hat{W}_{\alpha\mu}\hat{W}^{\mu\beta}\right] \times \mathrm{Tr}\left[\hat{W}_{\beta\nu}\hat{W}^{\nu\alpha}\right] \\
O_{T,5} &= \mathrm{Tr}\left[\hat{W}_{\mu\nu}\hat{W}^{\mu\nu}\right] \times \hat{B}_{\alpha\beta}\hat{B}^{\alpha\beta} \\
O_{T,6} &= \mathrm{Tr}\left[\hat{W}_{\alpha\nu}\hat{W}^{\mu\beta}\right] \times \hat{B}_{\mu\beta}\hat{B}^{\alpha\nu} \\
O_{T,7} &= \mathrm{Tr}\left[\hat{W}_{\alpha\mu}\hat{W}^{\mu\beta}\right] \times \hat{B}_{\beta\nu}\hat{B}^{\nu\alpha} \\
O_{T,8} &= \hat{B}_{\mu\nu}\hat{B}^{\mu\nu} \times \hat{B}_{\alpha\beta}\hat{B}^{\alpha\beta} \\
O_{T,9} &= \hat{B}_{\alpha\mu}\hat{B}^{\mu\beta} \times \hat{B}_{\beta\nu}\hat{B}^{\nu\alpha},
\end{aligned}
$$



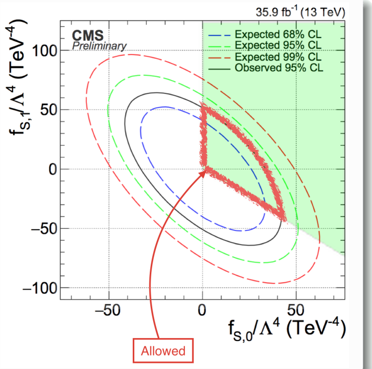
- In a bottom-up approach, we could be "too much" model-independent.
- "Positivity constraints" give us some hints.
  - In particular, the actual BSM parameter space is only $\sim 2\%$ of what you naively expect from EFT operators.

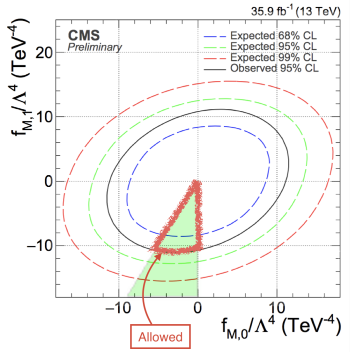
CMS-PAS-SMP-18-001





CMS-PAS-SMP-18-001 + Positivity





Outline

What it is

What are "positivity constraints":

- A linear combination of coefs. $(F_{S,0}, F_{S,1}, F_{S,2}, \cdots)$ must be positive.

$$
\begin{aligned}
&8a_3^2b_3^2t_W^4\left(F_{S,0}+F_{S,1}+F_{S,2}\right)+\left[a_3^2\left(b_1^2+b_2^2\right)\right.\\
&\left.+\left(a_1^2+a_2^2\right)b_3^2\right]t_W^2\left(-t_W^4F_{M,3}+t_W^2F_{M,5}-2F_{M,1}+F_{M,7}\right)\\
&+\left[\left(a_1b_1+a_2b_2\right)^2+\left(a_1^2+a_2^2\right)\left(b_1^2+b_2^2\right)\right]\left(2t_W^8F_{T,9}\right.\\
&\left.+4t_W^4F_{T,7}+8F_{T,2}\right)+8\left(a_1b_1+a_2b_2\right)^2\left[t_W^4\left(t_W^4F_{T,8}\right.\right.\\
&\left.\left.+2F_{T,5}+2F_{T,6}\right)+4F_{T,0}+4F_{T,1}\right]\geq 0
\end{aligned}
$$

- t_W is the Weinberg angle. a_i, b_i are free (complex) parameters.

What it is

What are "positivity constraints":

- A linear combination of coefs. $(F_{S,0}, F_{S,1}, F_{S,2}, \cdots)$ must be positive.

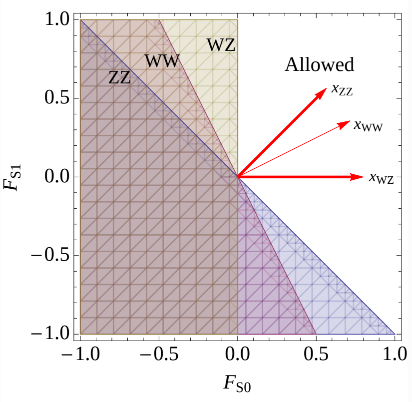
- Or equivalently, consider a vector $\vec{c} = (F_{S,0}, F_{S,1}, F_{S,2}, \cdots)$. Positivity says that $\vec{c}$ has to be positive upon projection on a certain direction $\vec{x}_i$, i.e.

$$\vec{c} \cdot \vec{x}_i \geq 0$$

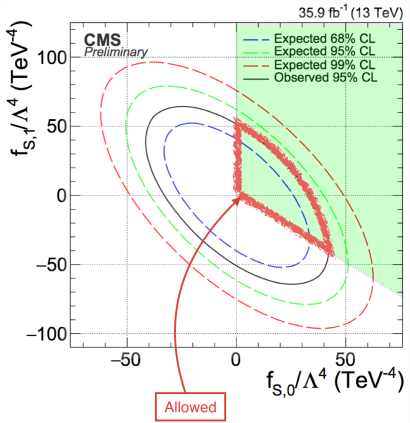
- $\vec{x}_i$ come from the requirements that the VBS amplitudes (WW, ZZ, ... with polarisation $\vec{a}, \vec{b}$) satisfy the fundamental principles of QFT (analyticity, unitarity, etc.), i.e. we have $\vec{x}_{WW}(\vec{a}, \vec{b})$, $\vec{x}_{ZZ}(\vec{a}, \vec{b})$, $\vec{x}_{WZ}(\vec{a}, \vec{b})$, ...

Implications on EXP results

$\vec{x}_{WW,ZZ,WZ}$ and positivity bounds



Combined with measurements



The approach

- First established in [A. Adams et al. JHEP '06]: dispersion relation + optical theorem, forward 2-to-2 scattering.
- Non-forward generalization: [C. de Rham et al. Phys.Rev.D '17], [C. de Rham et al. JHEP '18]
- Application in collider pheno:
  - *ZZ* and $Z\gamma$: *[B. Bellazzini and F. Riva '18]*
  - Implications in Higgs physics under ceratin assumptions: *[I. Low et al. '09]* *[A. Falkowski et al. '12]*
- In general the approach has strong implication on SMEFT dim-8 operators, which are important for the interpretation of VBS, so we should understand the constraints.

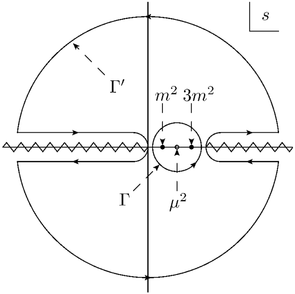
Analytic dispersion relation

- As an simplified version: consider the forward scattering ($t = 0$) of two identical particles with mass m, with possible heavy new physics.

  (see [C. Cheung and G. N. Remmen '16] for a quick overview)

- If the UV completion exists, the amplitude $M(s, t = 0)$
  - is analytic and
  - satisfies Froissart unitarity bound $M(s, 0) \leq \mathcal{O}(s \ln^2 s)$.

Analytic dispersion relation



- Consider the contour integral:

$$f = \frac{1}{2\pi i}\oint_\Gamma ds \frac{M(s,0)}{(s-\mu^2)^3}$$

- Deform Γ to Γ′ and notice that boundary contribution vanishes due to Froissart bound:

$$f = \frac{1}{2\pi i}\oint_\Gamma ds \frac{M(s,0)}{(s-\mu^2)^3} = \frac{1}{2\pi i}\left(\int_{-\infty}^{0} + \int_{4m^2}^{\infty}\right) ds \frac{\mathrm{Disc}M(s,0)}{(s-\mu^2)^3}$$

i.e. sum of residues at low energy =

discontinuity along +real axis + discontinuity along -real axis

- Note that BSM (above Λ) enters the discontinuity, as poles (tree level) or branch cuts (heavy loops).

Derivation of positivity

- discontinuity along real axis must positive, because of optical theorem (disc. = xsec >0) (plus crossing symmetry for $s < 0$)
- $\Rightarrow$ sum of residues at low energy is positive.

We started with the amplitude in the full theory, but have reached a conclusion that only involves low energy, which can be computed in SMEFT:

$$\boxed{\text{sum of residues at low energy}} = \frac{\mathrm{d}^2 M(s,0)}{\mathrm{d}s^2} = \sum_i c_i^{(8)} x_i + \sum_{i,j} c_i^{(6)} c_j^{(6)} y_{i,j} > 0$$

- Conclusion: the above positivity condition must be satisfied, if
  - SMEFT has a UV completion, that satisfies unitarity, Lorentz symmetry, is analytic.
  - At low energy, the SMEFT is valid and tree level calculation is a good approximation, which anyway need to be assumed in a real measurement.
  - Potential contaminations from higher dim operators, SM loops, EFT loops and so on. Interpret with care.

Dim-6 contributions can be removed

$$\sum_i c_i^{(8)} x_i \geq -\sum_{i,j} c_i^{(6)} c_j^{(6)} y_{i,j}$$

- In general, we expect dim-6 to be better constrained by other processes.
- But in any case, dim-6 doesn't matter, because by explicit calculation the RHS is positive.
- E.g. from WZ scattering:

$$\text{R.H.S} \propto a_3^2 b_3^2 \left[e^2 C_{DW} - s_W^2 c_W^2 C_{\varphi D} - 4 s_W^3 c_W C_{\varphi WB}\right]^2 + 36(a_1 b_1 + a_2 b_2)^2 e^2 s_W^2 c_W^2 C_W^2$$

- and from WW:

$$\text{R.H.S} \propto a_3^2 b_3^2 s_W^2 \left(e^2 C_{DB} + c_W^2 C_{\varphi D}\right)^2 + e^2 c_W^2 \left[6(a_1 b_1 + a_2 b_2) s_W C_W + a_3 b_3 e C_{DW}\right]^2$$

$$\boxed{\sum_i c_i^{(8)} x_i \geq -\sum_{i,j} c_i^{(6)} c_j^{(6)} y_{i,j} \geq 0}$$

or simply: $\boxed{\vec{c} \cdot \vec{x}_i \geq 0}$

Polarisation

- Polarisation matters. Consider $V_1 V_2 \to V_1 V_2$

$$\begin{aligned} V_1: &\quad \vec{a} = (a_1, a_2, a_3) \\ V_2: &\quad \vec{b} = (b_1, b_2, b_3) \end{aligned}$$

- As a result, $ZZ \to ZZ$ gives the following constraint:

$$\begin{aligned} &8a_3^2 b_3^2 t_W^4 (F_{S,0} + F_{S,1} + F_{S,2}) + \left[a_3^2 \left(b_1^2 + b_2^2\right)\right. \\ &\left. + \left(a_1^2 + a_2^2\right) b_3^2\right] t_W^2 \left(-t_W^4 F_{M,3} + t_W^2 F_{M,5} - 2F_{M,1} + F_{M,7}\right) \\ &+ \left[(a_1 b_1 + a_2 b_2)^2 + \left(a_1^2 + a_2^2\right)\left(b_1^2 + b_2^2\right)\right] \left(2t_W^8 F_{T,9}\right. \\ &\left. +4t_W^4 F_{T,7} + 8F_{T,2}\right) + 8\,(a_1 b_1 + a_2 b_2)^2 \left[t_W^4 \left(t_W^4 F_{T,8}\right.\right. \\ &\left.\left. +2F_{T,5} + 2F_{T,6}\right) + 4F_{T,0} + 4F_{T,1}\right] \geq 0 \end{aligned}$$

- Depending on $\vec{a}$, $\vec{b}$, there is a infinite number of constraints from ZZ …
- Other constraints from $W^\pm Z$, $W^\pm W^\pm$, $W^\pm W^\mp$, $W^\pm \gamma$, $Z\gamma$, $\gamma\gamma$.

Outline

1D limits

- Consider one operator at a time:

$f_{S,0}$	$f_{S,1}$	$f_{S,2}$	$f_{M,0}$	$f_{M,1}$	$f_{M,2}$	$f_{M,3}$	$f_{M,4}$	$f_{M,5}$
+	+	+	✗	−	✗	−	✗	✗

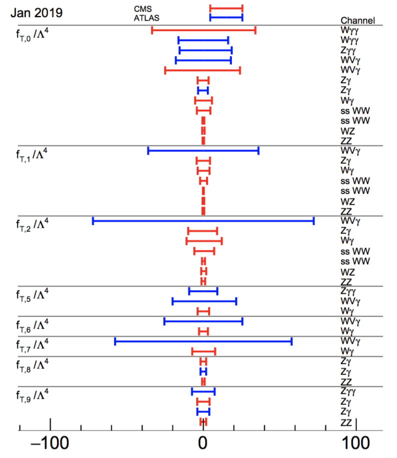
$f_{M,7}$	$f_{T,0}$	$f_{T,1}$	$f_{T,2}$	$f_{T,5}$	$f_{T,6}$	$f_{T,7}$	$f_{T,8}$	$f_{T,9}$
+	+	+	+	✗	+	✗	+	+

+: positive −: negative ✗: forbidden

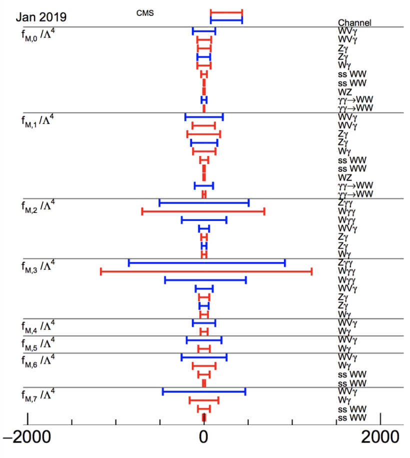
- Note there are coefficients that are not individually allowed.

1D limits: EXP

Individual limits on transversal coefficients



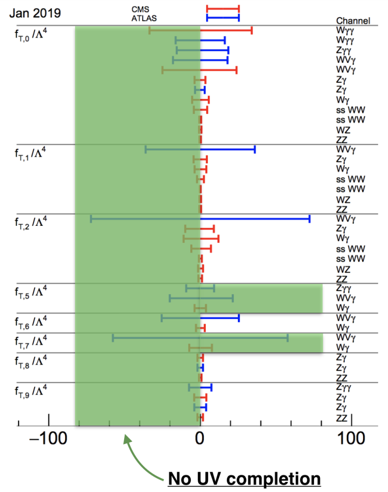
Individual limits on mixed coefficients



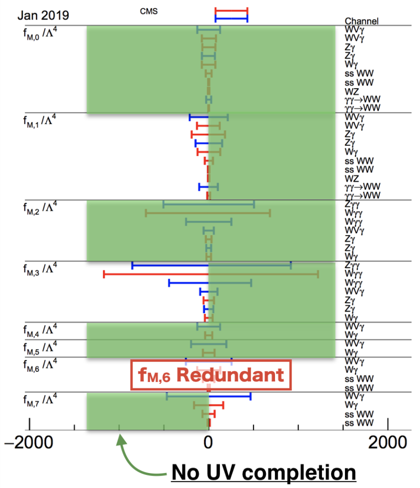
https://twiki.cern.ch/twiki/bin/view/CMSPublic/PhysicsResultsSMPaTGC#aQGC_Results

1D limits: EXP+positivity

Transversal coefficients, positivity



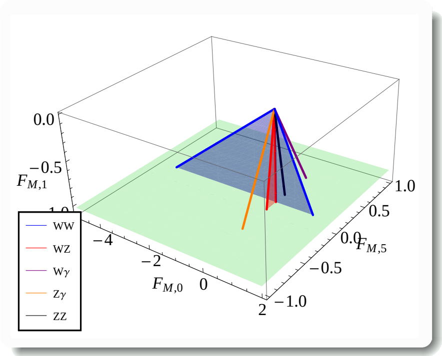
Mixed coefficients, positivity

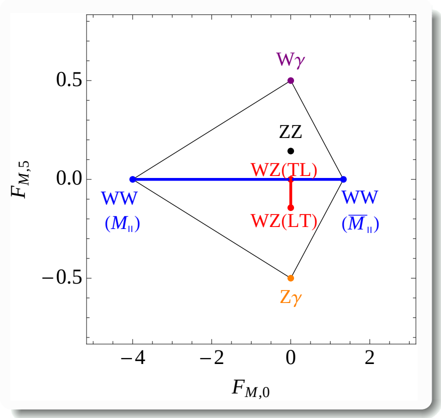


Going global: the pyramid case

As a first 3D example, consider $F_{M,0}$, $F_{M,1}$ and $F_{M,5}$.

- Remember we have $\vec{c} \cdot \vec{x}(\vec{a}, \vec{b}) > 0$.
- **$\vec{x}$ inside a pyramid formed by other $\vec{x}_i$ does not give new info!**





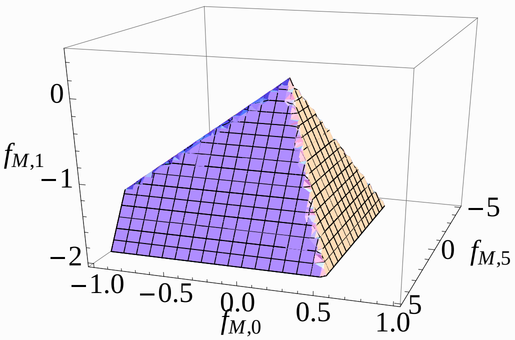
The pyramid case

- Allowed region is given by

$$
\begin{aligned}
-2F_{M1} + F_{M5} &\geq 0, \\
-2F_{M1} - F_{M5} &\geq 0, \\
-4F_{M0} - F_{M1} &\geq 0, \\
4F_{M0} - 3F_{M1} &\geq 0.
\end{aligned}
$$

- In principle same approach applies for higher-D case: *the problem is equivalent to finding a* ***D-1 dimensional convex hull****.*
- Caveat: boundaries can be curves.

3D allowed region given by a pyramid



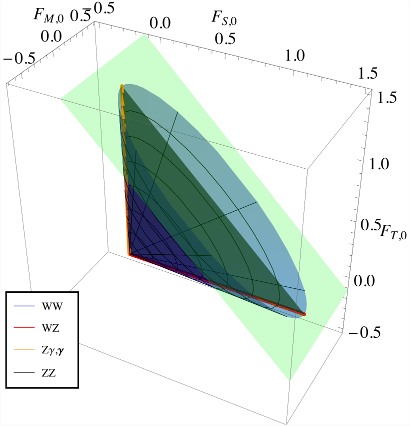
Recall:

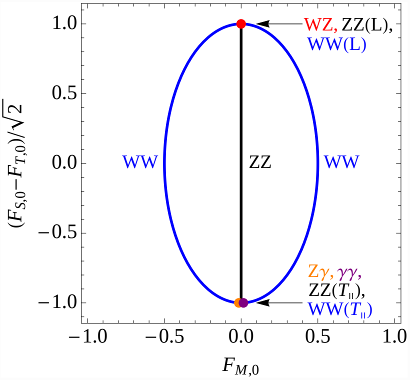
$f_{M,0}$	$f_{M,1}$	$f_{M,2}$	$f_{M,3}$	$f_{M,4}$	$f_{M,5}$
✗	–	✗	–	✗	✗

The cone case

Consider $F_{S,0}$, $F_{M,0}$ and $F_{T,0}$.

- Possible $\vec{x}$ directions form a cone, pointing to the positive (F_{S0}, F_{T0}) direction



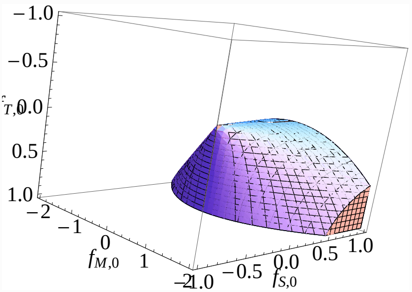


The cone case

- In this case positivity carves out a cone instead of a pyramid.
- The solution is

$$F_{S0} > 0$$
$$F_{T0} > 0$$
$$8F_{S0}F_{T0} > F_{M0}^2$$

- Note that F_{S0} and F_{T0} are no longer decoupled for $F_{M0} > 0$.



General solution: linear inequalities in S and M space

Following the above idea, one can solve for the entire 18-D space and arrive at a description of the allowed parameter space, for any and all $\vec{a}$, $\vec{b}$ values (complex).

$$M_{S,ij}F_{S,j} > 0$$
$$M_{M,ij}F_{M,j} > 0$$

$$M_S = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix} \qquad M_M = \begin{pmatrix} 0 & -2c_W^4 & 0 & -s_W^4 & 0 & s_W^2 - s_W^4 & c_W^4 \\ 0 & -2c_W^4 & 0 & -s_W^4 & 0 & -c_W^2 s_W^2 & c_W^4 \\ 0 & -2 & 0 & 0 & 0 & 0 & 1 \\ 0 & -2 & 0 & -1 & 0 & 1 & 1 \\ 0 & -2 & 0 & -1 & 0 & -1 & 1 \end{pmatrix}$$

General solution: linear inequalities in T space

$$M_{T,ij}F_{T,j} > 0$$

$$M_T = \begin{pmatrix}
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 2 & 1 & 0 & 0 & 0 & 0 & 0 \\
2 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\
8 & 12 & 5 & 0 & 0 & 0 & 0 & 0 \\
8c_W^8 & 8c_W^8 & 4c_W^8 & 4c_W^4 s_W^4 & 4c_W^4 s_W^4 & 2c_W^4 s_W^4 & 2s_W^8 & s_W^8 \\
0 & 0 & 4c_W^8 & 0 & 0 & 2c_W^4 s_W^4 & 0 & s_W^8 \\
0 & 0 & 4c_W^4 & 0 & 0 & s_W^4 & 0 & 0 \\
0 & 16c_W^4 & 4c_W^4 & 0 & 4s_W^4 & s_W^4 & 0 & 0 \\
0 & 0 & 4 & 0 & 0 & 1 & 0 & 0 \\
0 & 16 & 4 & 0 & 4 & 1 & 0 & 0 \\
0 & 0 & 8c_W^4 & 0 & 0 & \left(c_W^2 - s_W^2\right)^2 & 0 & 2s_W^4 \\
32c_W^4 & 32c_W^4 & 16c_W^4 & -16c_W^2 s_W^2 & 4\left(c_W^2 - s_W^2\right)^2 & 1 - 8s_W^2 c_W^2 & 8s_W^4 & 4s_W^4 \\
0 & 0 & 4 & 0 & 0 & 2 & 0 & 1 \\
8 & 8 & 4 & 4 & 4 & 2 & 2 & 1
\end{pmatrix}$$

General solution: higher order ones

$\boxed{WW,\ M_{+-}\&\bar{M}_{+-}}$

$$32(2F_{S,0}+F_{S,1}+F_{S,2})(2F_{T,0}+F_{T,1}+F_{T,2}) - \max(0,4F_{M,0}+F_{M,1},-4F_{M,0}+3F_{M,1}-2F_{M,7})^2>0$$

$\boxed{WW,\ M_{||}\&\bar{M}_{||}}$

$$8(2F_{S,0}+F_{S,1}+F_{S,2})(8F_{T,0}+12F_{T,1}+5F_{T,2}) - \max(0,4F_{M,0}+F_{M,1},-4F_{M,0}+3F_{M,1}-2F_{M,7})^2>0$$

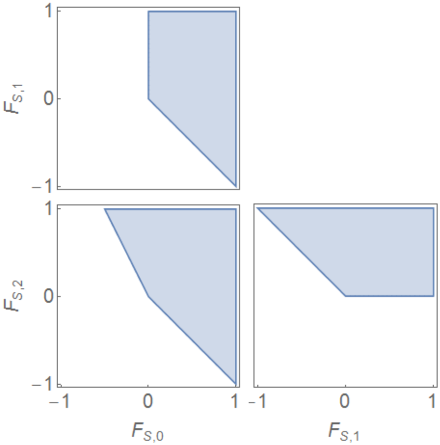
$\boxed{ZZ,\ M_l\&\bar{M}_l}$

$$8(F_{S,0}+F_{S,1}+F_{S,2})\left[4c_W^8(2F_{T,0}+2F_{T,1}+F_{T,2})+2c_W^4s_W^4(2F_{T,5}+2F_{T,6}+F_{T,7}) +s_W^8(2F_{T,8}+F_{T,9})\right] - \max\left[0,2\left(2c_W^4F_{M,0}+F_{M,2}s_W^4-F_{M,4}s_W^4+F_{M,4}s_W^2\right), -c_W^4(4F_{M,0}-2F_{M,1}+F_{M,7})-2c_W^2F_{M,4}s_W^2-s_W^4(2F_{M,2}-F_{M,3})-F_{M,5}\left(s_W^2-s_W^4\right)\right]^2>0$$

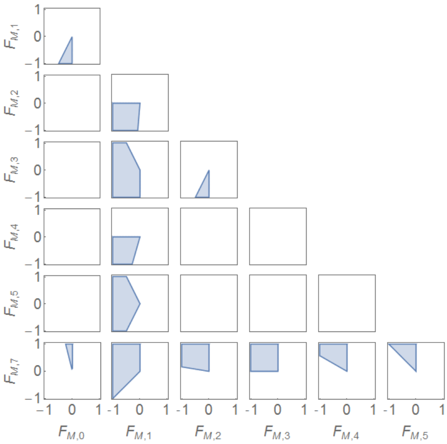
$\boxed{WZ,\ M_l'\&\bar{M}_l'}$

$$16(F_{S,0}+F_{S,2})\left[4c_W^4(4F_{T,1}+F_{T,2})+s_W^4(4F_{T,6}+F_{T,7})\right]-\max\Big[0,-2c_W^2F_{M,7} -2\sqrt{(2F_{M,1}-F_{M,7})\left(c_W^4(2F_{M,1}-F_{M,7})+c_W^2F_{M,5}s_W^2+F_{M,3}s_W^4\right)}-4F_{M,4}s_W^2-F_{M,5}s_W^2, 2c_W^2F_{M,7}-2\sqrt{(2F_{M,1}-F_{M,7})\left(c_W^4(2F_{M,1}-F_{M,7})+c_W^2F_{M,5}s_W^2+F_{M,3}s_W^4\right)} +4F_{M,4}s_W^2+F_{M,5}s_W^2\Big]^2>0$$

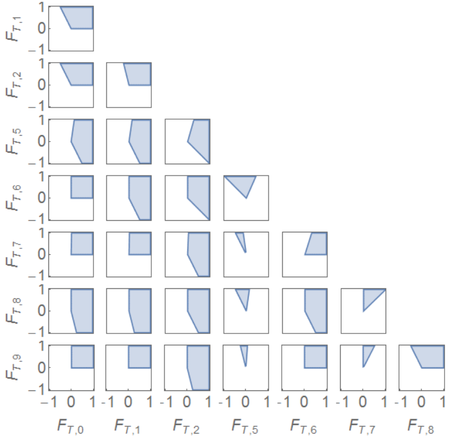
Two-parameter cases



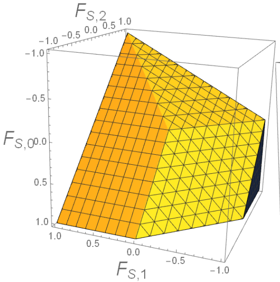
Two-parameter cases

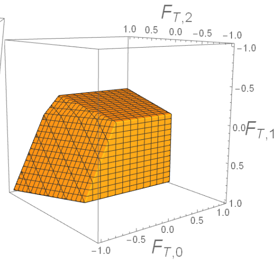


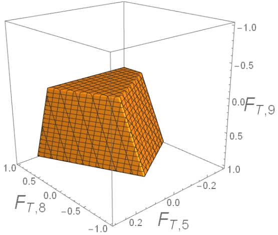
Two-parameter cases

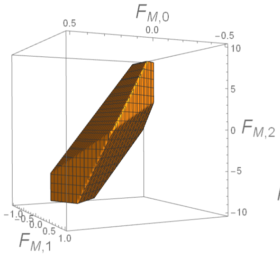


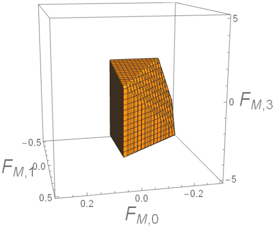
Tree-parameter cases



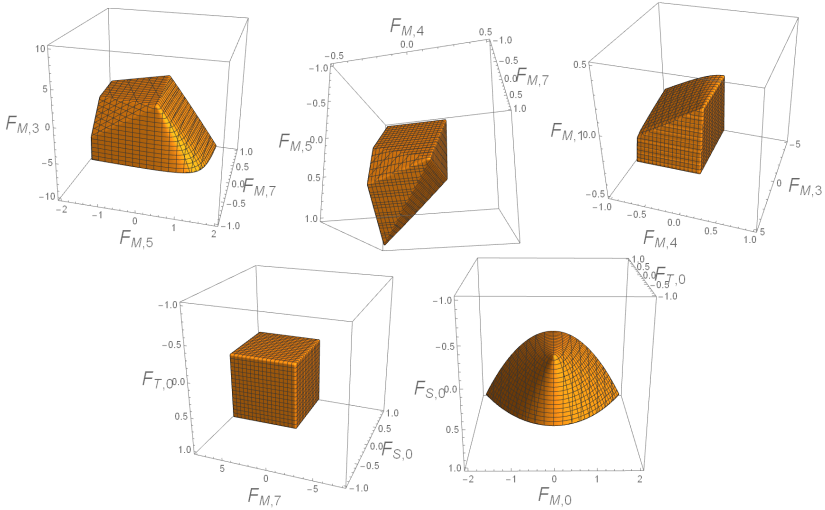








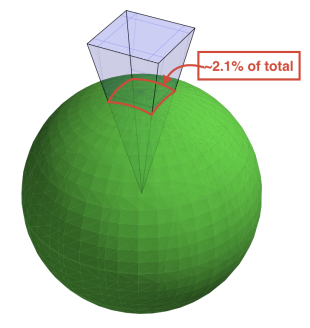
Tree-parameter cases



Volume in full parameter space

When all 18 parameters are turned on, how much of the parameter space is excluded by positivity?

- Randomly generate points on a 18D sphere, uniformly distributed, and count how many of the them fall within constraints for all polarizations.
- We find that **only** $\sim 2.1\%$ **parameter space** is left (allowing complex polarisation vectors)



Outline

Conclusion

- Dim-8 aQGC operator coefficients satisfy a set of positivity constraints, if they are generated by a UV completion.
- They have strong implication, e.g. 18D parameter space reduced to 2.1%, independent of experimental precision.
- The shape of the allowed parameter space shows interesting structure.

Thank you!

Backups

Example: simplified model

Consider the simplified model in [Brass, Fleper, Kilian, Reuter, Sekulla '18]

In the present paper, we do not refer to a specific scenario. We construct a simplified model with transverse couplings of a generic heavy resonance σ. The effective Lagrangian takes the following form,

$$\mathcal{L}_\sigma = -\frac{1}{2}\sigma(m_\sigma^2 - \partial^2)\sigma + \sigma(J_{\sigma\|} + J_{\sigma\perp}) \tag{19a}$$

$$J_{\sigma\|} = F_{\sigma H}\,\mathrm{tr}\left[(\mathbf{D}_\mu\mathbf{H})^\dagger(\mathbf{D}^\mu\mathbf{H})\right] \tag{19b}$$

$$J_{\sigma\perp} = g^2 F_{W\sigma}\sigma\,\mathrm{tr}\left[\mathbf{W}_{\mu\nu}\mathbf{W}^{\mu\nu}\right] + g'^2 F_{B\sigma}\sigma\,\mathrm{tr}\left[\mathbf{B}_{\mu\nu}\mathbf{B}^{\mu\nu}\right] \tag{19c}$$

with three independent coupling parameters.

In the low-energy limit, the scalar resonance can be integrated out, and we obtain the SMEFT Lagrangian with the following nonzero coefficients of the dimension-8 operators at leading order:

$$F_{S_0} = F_{\sigma H}^2/2m_\sigma^2 \tag{20a}$$

$$F_{M_0} = -F_{\sigma H}F_{\sigma W}/m_\sigma^2 \tag{20b}$$

$$F_{M_2} = -F_{\sigma H}F_{\sigma B}/m_\sigma^2 \tag{20c}$$

$$F_{T_0} = F_{\sigma W}^2/2m_\sigma^2 \tag{20d}$$

$$F_{T_5} = F_{\sigma W}F_{\sigma B}/m_\sigma^2 \tag{20e}$$

$$F_{T_8} = F_{\sigma B}^2/2m_\sigma^2. \tag{20f}$$

Example: simplified model

If we plug in the dim-8 coefficients into our positivity constraints, we see:

$$
\begin{aligned}
ZZ &: (a_1b_1+a_2b_2)^2\left(s_W^4F_{\sigma B}+2c_W^4F_{\sigma W}\right)^2+a_3^2b_3^2s_W^4c_W^4e^{-4}F_{\sigma H}^2>0\\
W^\pm Z &: a_3^2b_3^2F_{\sigma H}^2>0\\
W^\pm W^\pm &: (a_1b_1+a_2b_2)^2F_{\sigma W}^2+\left[(a_1b_1+a_2b_2)F_{\sigma W}+a_3b_3s_W^2e^{-2}F_{\sigma H}\right]^2>0\\
W^\pm W^\mp &: (a_1b_1+a_2b_2)^2F_{\sigma W}^2+\left[(a_1b_1+a_2b_2)F_{\sigma W}-a_3b_3s_W^2e^{-2}F_{\sigma H}\right]^2>0\\
ZA &: (a_1b_1+a_2b_2)^2\left[s_W^2F_{\sigma B}-2c_W^2F_{\sigma W}\right]^2>0\\
WA &: \text{none}\\
AA &: (a_1b_1+a_2b_2)^2\left(F_{\sigma B}+2F_{\sigma W}\right)^2>0
\end{aligned}
$$

*up to factors of 2 that can be absorbed in the definitions of $F_{\sigma X}$

All inequalities are satisfied, as they are all sum of squares.

- In a top-down approach, positivity is automatically true, in different models, different ways — by asking for positivity, we are not restricting the UV models.
- In a bottom-up approach, we can derive the same constraints, but without using model details, and therefore we restrict the parameter space without losing model-independence.

“Unitarity”

- It is well-known that unitarity violation can be a problem in SMEFT.
  - In VBS, unitarization techniques are needed. E.g. [Perez, Sekulla, Zeppenfeld '18] [Brass, Fleper, Kilian, Reuter, Sekulla '18]
  - However, here unitarity problem concerns only the prediction of the SMEFT, and only signals the breakdown of EFT.
- Our bounds are derived from a different information, i.e. the Froissart unitarity bound. This unitarity refers to the behaviour of the UV theory at large energy.
  - This is then connected to the IR (EFT) of the theory by the dispersion relation

i.e. Unitarity in UV (full theory) $\Rightarrow$ Positivy in IR (EFT)