

Effective Field Theory

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VBScan Training event
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Outline

- EFT in VBS
- Unitarisation
- EFT vs resonances
- Positivity constraints (Cen)

SMEFT

$$\mathcal{L}_{\text{Eff}} = \mathcal{L}_{\text{SM}} + \sum_i \frac{C_i^{(6)} O_i^{(6)}}{\Lambda^2} + \mathcal{O}(\Lambda^{-4})$$

- 59(2499) operators at dim-6: [Buchmuller, Wyler Nucl.Phys. B268 \(1986\) 621-653](#)
[Grzadkowski et al arxiv:1008.4884](#)

X^3		φ^6 and $\varphi^4 D^2$		$\psi^2 \varphi^3$	
Q_G	$f^{ABC} G_\mu^{Av} G_\nu^{B\rho} G_\rho^{C\mu}$	Q_φ	$(\varphi^\dagger \varphi)^3$	$Q_{e\varphi}$	$(\varphi^\dagger \varphi)(\bar{l}_p e_\tau \varphi)$
$Q_{\tilde{G}}$	$f^{ABC} \tilde{G}_\mu^{Av} G_\nu^{B\rho} G_\rho^{C\mu}$	$Q_{\varphi\Box}$	$(\varphi^\dagger \varphi)\Box(\varphi^\dagger \varphi)$	$Q_{u\varphi}$	$(\varphi^\dagger \varphi)(\bar{q}_p u_\tau \tilde{\varphi})$
Q_W	$\varepsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$	$Q_{\varphi D}$	$(\varphi^\dagger D^\mu \varphi)^* (\varphi^\dagger D_\mu \varphi)$	$Q_{d\varphi}$	$(\varphi^\dagger \varphi)(\bar{q}_p d_\tau \varphi)$
$Q_{\tilde{W}}$	$\varepsilon^{IJK} \tilde{W}_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$				
$X^2 \varphi^2$		$\psi^2 X \varphi$		$\psi^2 \varphi^2 D$	
$Q_{\varphi G}$	$\varphi^\dagger \varphi G_{\mu\nu}^A G^{A\mu\nu}$	Q_{eW}	$(\bar{l}_p \sigma^{\mu\nu} e_\tau) \tau^I \varphi W_{\mu\nu}^I$	$Q_{\varphi l}^{(1)}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar{l}_p \gamma^\mu l_\tau)$
$Q_{\varphi \tilde{G}}$	$\varphi^\dagger \varphi \tilde{G}_{\mu\nu}^A G^{A\mu\nu}$	Q_{eB}	$(\bar{l}_p \sigma^{\mu\nu} e_\tau) \varphi B_{\mu\nu}$	$Q_{\varphi l}^{(3)}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu^I \varphi)(\bar{l}_p \tau^I \gamma^\mu l_\tau)$
$Q_{\varphi W}$	$\varphi^\dagger \varphi W_{\mu\nu}^I W^{I\mu\nu}$	Q_{uG}	$(\bar{q}_p \sigma^{\mu\nu} T^A u_\tau) \tilde{\varphi} G_{\mu\nu}^A$	$Q_{\varphi e}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar{e}_p \gamma^\mu e_\tau)$
$Q_{\varphi \tilde{W}}$	$\varphi^\dagger \varphi \tilde{W}_{\mu\nu}^I W^{I\mu\nu}$	Q_{uW}	$(\bar{q}_p \sigma^{\mu\nu} u_\tau) \tau^I \tilde{\varphi} W_{\mu\nu}^I$	$Q_{\varphi q}^{(1)}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar{q}_p \gamma^\mu q_\tau)$
$Q_{\varphi B}$	$\varphi^\dagger \varphi B_{\mu\nu} B^{\mu\nu}$	Q_{uB}	$(\bar{q}_p \sigma^{\mu\nu} u_\tau) \tilde{\varphi} B_{\mu\nu}$	$Q_{\varphi q}^{(3)}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu^I \varphi)(\bar{q}_p \tau^I \gamma^\mu q_\tau)$
$Q_{\varphi \tilde{B}}$	$\varphi^\dagger \varphi \tilde{B}_{\mu\nu} B^{\mu\nu}$	Q_{dG}	$(\bar{q}_p \sigma^{\mu\nu} T^A d_\tau) \varphi G_{\mu\nu}^A$	$Q_{\varphi u}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar{u}_p \gamma^\mu u_\tau)$
$Q_{\varphi WB}$	$\varphi^\dagger \tau^I \varphi W_{\mu\nu}^I B^{\mu\nu}$	Q_{dW}	$(\bar{q}_p \sigma^{\mu\nu} d_\tau) \tau^I \varphi W_{\mu\nu}^I$	$Q_{\varphi d}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar{d}_p \gamma^\mu d_\tau)$
$Q_{\varphi \tilde{W}B}$	$\varphi^\dagger \tau^I \varphi \tilde{W}_{\mu\nu}^I B^{\mu\nu}$	Q_{dB}	$(\bar{q}_p \sigma^{\mu\nu} d_\tau) \varphi B_{\mu\nu}$	$Q_{\varphi ud}$	$i(\tilde{\varphi}^\dagger D_\mu \varphi)(\bar{u}_p \gamma^\mu d_\tau)$

$(\bar{L}L)(\bar{L}L)$		$(\bar{R}R)(\bar{R}R)$		$(\bar{L}L)(\bar{R}R)$	
Q_{ll}	$(\bar{l}_p \gamma_\mu l_\tau)(\bar{l}_s \gamma^\mu l_t)$	Q_{ee}	$(\bar{e}_p \gamma_\mu e_\tau)(\bar{e}_s \gamma^\mu e_t)$	Q_{le}	$(\bar{l}_p \gamma_\mu l_\tau)(\bar{e}_s \gamma^\mu e_t)$
$Q_{qq}^{(1)}$	$(\bar{q}_p \gamma_\mu q_\tau)(\bar{q}_s \gamma^\mu q_t)$	Q_{uu}	$(\bar{u}_p \gamma_\mu u_\tau)(\bar{u}_s \gamma^\mu u_t)$	Q_{lu}	$(\bar{l}_p \gamma_\mu l_\tau)(\bar{u}_s \gamma^\mu u_t)$
$Q_{qq}^{(3)}$	$(\bar{q}_p \gamma_\mu \tau^I q_\tau)(\bar{q}_s \gamma^\mu \tau^I q_t)$	Q_{dd}	$(\bar{d}_p \gamma_\mu d_\tau)(\bar{d}_s \gamma^\mu d_t)$	Q_{ld}	$(\bar{l}_p \gamma_\mu l_\tau)(\bar{d}_s \gamma^\mu d_t)$
$Q_{lq}^{(1)}$	$(\bar{l}_p \gamma_\mu l_\tau)(\bar{q}_s \gamma^\mu q_t)$	Q_{eu}	$(\bar{e}_p \gamma_\mu e_\tau)(\bar{u}_s \gamma^\mu u_t)$	Q_{qe}	$(\bar{q}_p \gamma_\mu q_\tau)(\bar{e}_s \gamma^\mu e_t)$
$Q_{lq}^{(3)}$	$(\bar{l}_p \gamma_\mu \tau^I l_\tau)(\bar{q}_s \gamma^\mu \tau^I q_t)$	Q_{ed}	$(\bar{e}_p \gamma_\mu e_\tau)(\bar{d}_s \gamma^\mu d_t)$	$Q_{qu}^{(1)}$	$(\bar{q}_p \gamma_\mu q_\tau)(\bar{u}_s \gamma^\mu u_t)$
		$Q_{ud}^{(1)}$	$(\bar{u}_p \gamma_\mu u_\tau)(\bar{d}_s \gamma^\mu d_t)$	$Q_{qu}^{(8)}$	$(\bar{q}_p \gamma_\mu T^A q_\tau)(\bar{u}_s \gamma^\mu T^A u_t)$
		$Q_{ud}^{(8)}$	$(\bar{u}_p \gamma_\mu T^A u_\tau)(\bar{d}_s \gamma^\mu T^A d_t)$	$Q_{qd}^{(1)}$	$(\bar{q}_p \gamma_\mu q_\tau)(\bar{d}_s \gamma^\mu d_t)$
				$Q_{qd}^{(8)}$	$(\bar{q}_p \gamma_\mu T^A q_\tau)(\bar{d}_s \gamma^\mu T^A d_t)$
$(\bar{L}R)(\bar{R}L)$ and $(\bar{L}R)(\bar{L}R)$		B -violating			
Q_{ledq}	$(\bar{l}_p^j e_\tau)(\bar{d}_s^k q_t^j)$	Q_{duq}	$\varepsilon^{\alpha\beta\gamma} \varepsilon_{jk} [(d_p^\alpha)^T C u_\tau^\beta] [(q_s^{\gamma j})^T C l_t^k]$		
$Q_{quqd}^{(1)}$	$(\bar{q}_p^j u_\tau) \varepsilon_{jk} (\bar{q}_s^k d_t)$	Q_{qqu}	$\varepsilon^{\alpha\beta\gamma} \varepsilon_{jk} [(q_p^{\alpha j})^T C q_\tau^{\beta k}] [(u_s^\gamma)^T C e_t]$		
$Q_{quqd}^{(8)}$	$(\bar{q}_p^j T^A u_\tau) \varepsilon_{jk} (\bar{q}_s^k T^A d_t)$	$Q_{qqq}^{(1)}$	$\varepsilon^{\alpha\beta\gamma} \varepsilon_{jk} \varepsilon_{mn} [(q_p^{\alpha j})^T C q_\tau^{\beta k}] [(q_s^{\gamma m})^T C l_t^n]$		
$Q_{lequ}^{(1)}$	$(\bar{l}_p^j e_\tau) \varepsilon_{jk} (\bar{q}_s^k u_t)$	$Q_{qqq}^{(3)}$	$\varepsilon^{\alpha\beta\gamma} (\tau^I \varepsilon)_{jk} (\tau^I \varepsilon)_{mn} [(q_p^{\alpha j})^T C q_\tau^{\beta k}] [(q_s^{\gamma m})^T C l_t^n]$		
$Q_{lequ}^{(3)}$	$(\bar{l}_p^j \sigma_{\mu\nu} e_\tau) \varepsilon_{jk} (\bar{q}_s^k \sigma^{\mu\nu} u_t)$	Q_{duu}	$\varepsilon^{\alpha\beta\gamma} [(d_p^\alpha)^T C u_\tau^\beta] [(u_s^\gamma)^T C e_t]$		

4-fermion operators

First lecture recap

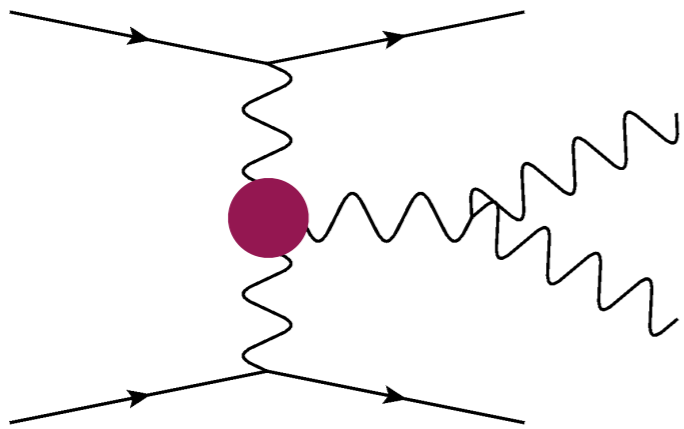
- SMEFT:
 - only SM fields
 - respecting SM symmetries ✓
 - valid below scale Λ
- Gauge invariant ✓
- Higher-order corrections: renormalisable order by order in $1/\Lambda$

$$\mathcal{O}(\alpha_s) + \mathcal{O}\left(\frac{1}{\Lambda^2}\right) + \mathcal{O}\left(\frac{\alpha_s}{\Lambda^2}\right) + \dots$$

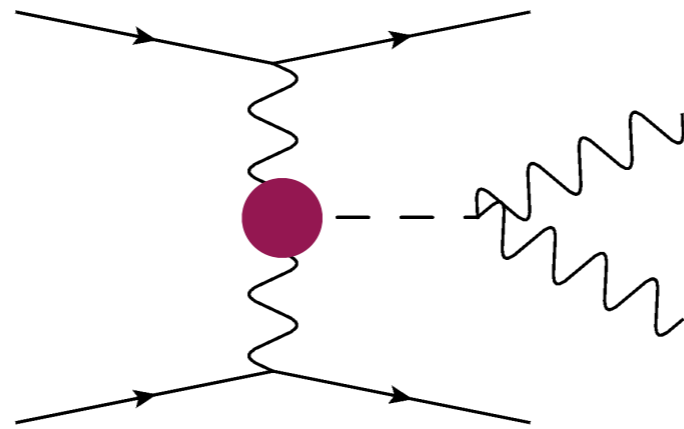
- Complete description ✓
- Model Independent ✓

EFT in VBS

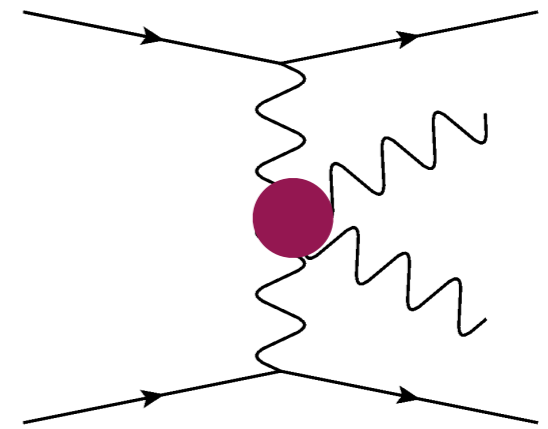
VBS is interesting as it probes:



TGC



Higgs couplings



QGC

Connection to dibosons, Higgs physics, EWPO etc

dim-6 operators in VBS

$$\mathcal{O}_{WWW} = \text{Tr}[W_{\mu\nu} W^{\nu\rho} W_{\rho}^{\mu}]$$

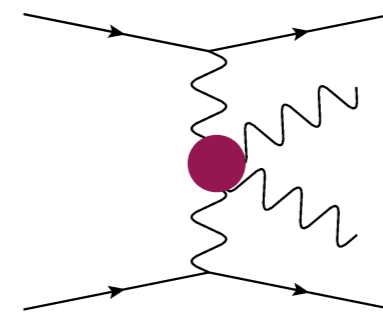
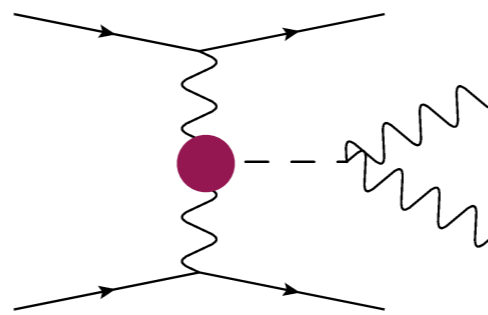
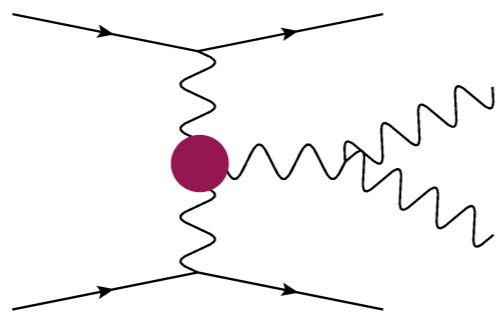
$$\mathcal{O}_W = (D_{\mu}\Phi)^{\dagger} W^{\mu\nu} (D_{\nu}\Phi)$$

$$\mathcal{O}_B = (D_{\mu}\Phi)^{\dagger} B^{\mu\nu} (D_{\nu}\Phi)$$

$$\mathcal{O}_{\partial\Phi} = \partial_{\mu}(\Phi^{\dagger}\Phi) \partial^{\mu}(\Phi^{\dagger}\Phi)$$


$$\mathcal{O}_{\Phi W} = (\Phi^{\dagger}\Phi) \text{Tr}[W^{\mu\nu} W_{\mu\nu}]$$

$$\mathcal{O}_{\Phi B} = (\Phi^{\dagger}\Phi) B^{\mu\nu} B_{\mu\nu}$$




How do these operators change the VBS process:


	ZWW	AWW	HWW	HZZ	HZA	HAA	WWWW	ZZWW	ZAWW	AAWW
\mathcal{O}_{WWW}	✓	✓					✓	✓	✓	✓
\mathcal{O}_W	✓	✓	✓	✓	✓		✓	✓	✓	
\mathcal{O}_B	✓	✓		✓	✓					
$\mathcal{O}_{\partial\Phi}$			✓	✓	✓					
$\mathcal{O}_{\Phi W}$			✓	✓	✓	✓				
$\mathcal{O}_{\Phi B}$				✓	✓	✓				



TGC

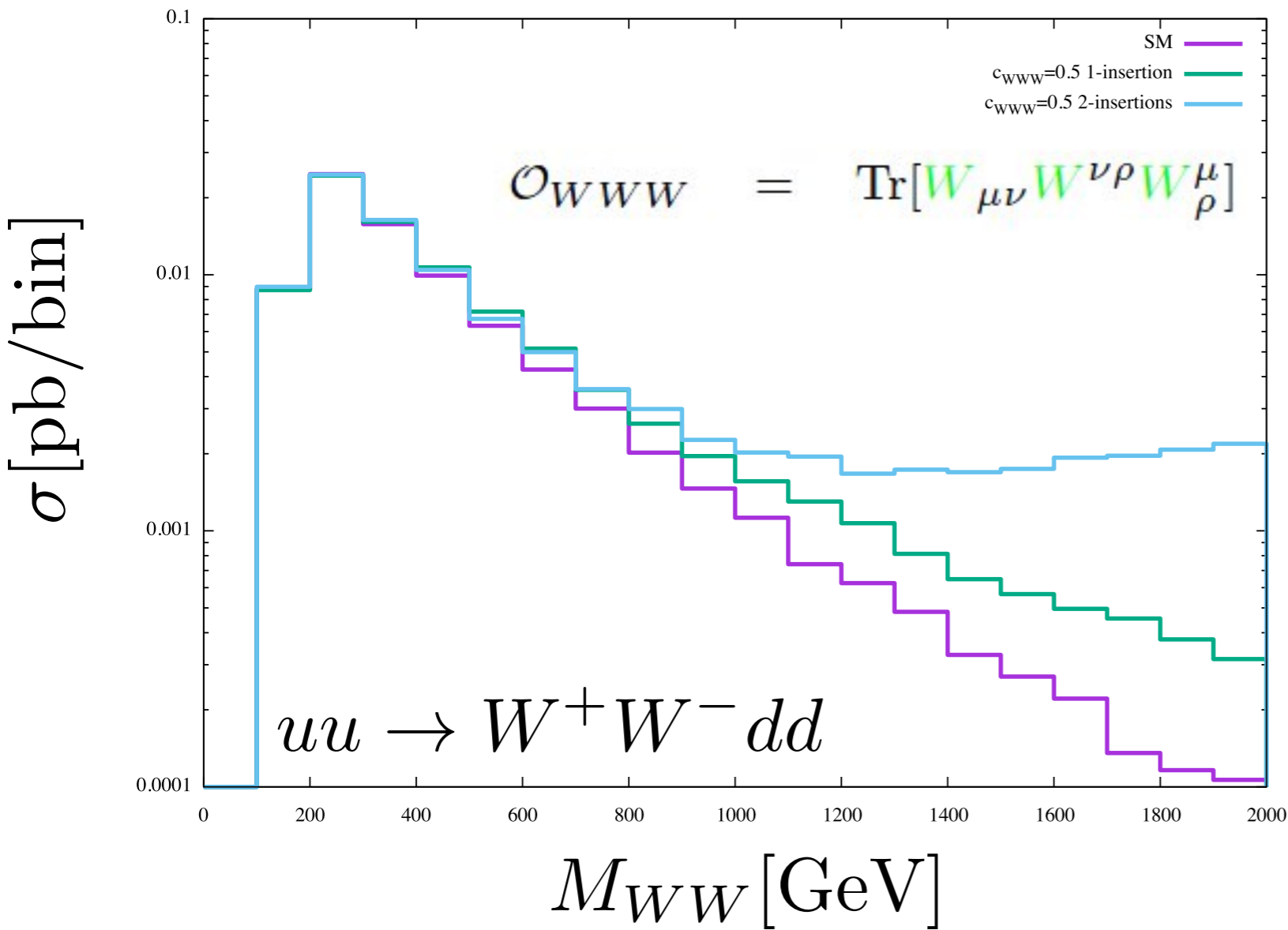


Higgs

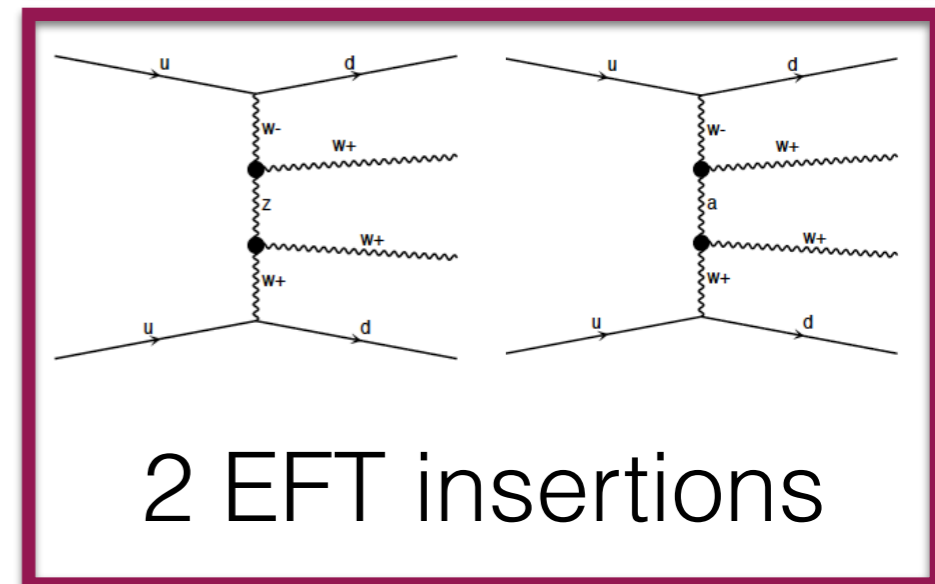
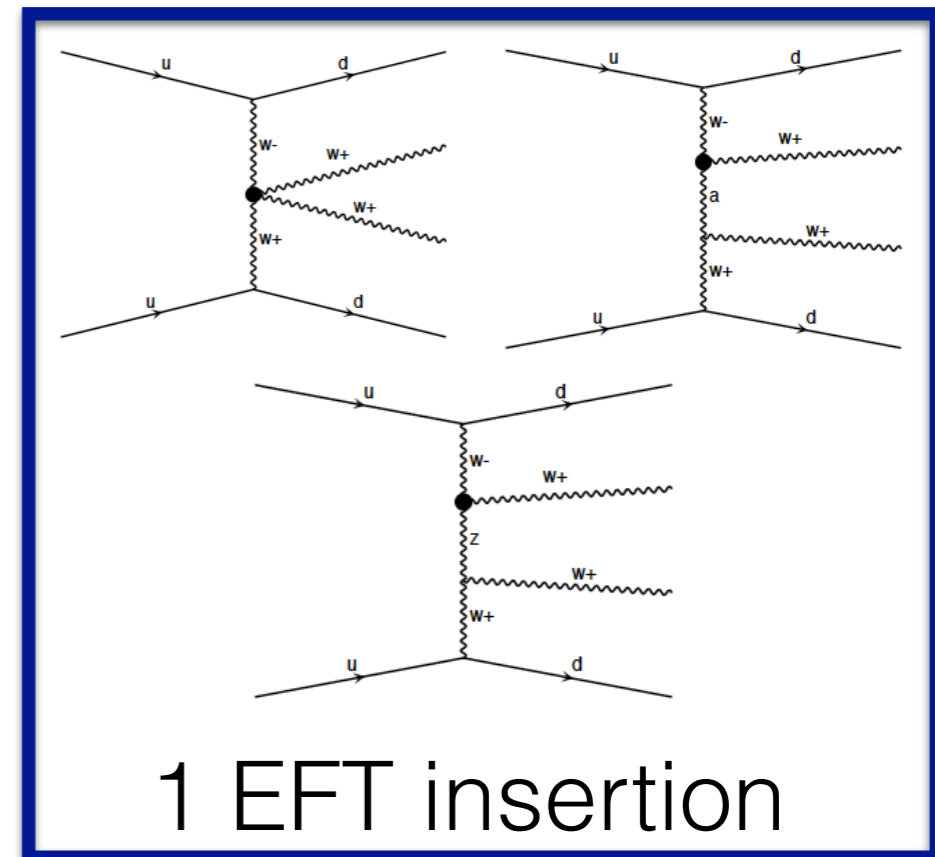


QGC

How does a dim-6 operator affect VBS?



Multiple insertions can be relevant for large couplings



Going beyond dim-6: Why also dim-8 in VBS?

- At dimension-6 TGC and QGC are correlated
- Dimension-6 operators do not allow a fully generic parametrisation of the helicities of the 4 gauge bosons
- New physics may appear at tree-level for the dimension-8 operators compared to one-loop level for dimension-6 operators
- There are UV theories which predict dominance of dim-8 operators
- Dimension-6 operators entering VBS can be probed more precisely in other processes: VV and Higgs production, QGC are unique to VBS

dim-8 operators in VBS

Longitudinal

$$O_{S,0} = [(D_\mu \Phi)^\dagger D_\nu \Phi] \times [(D^\mu \Phi)^\dagger D^\nu \Phi]$$

$$O_{S,1} = [(D_\mu \Phi)^\dagger D^\mu \Phi] \times [(D_\nu \Phi)^\dagger D^\nu \Phi]$$

$$O_{S,2} = [(D_\mu \Phi)^\dagger D_\nu \Phi] \times [(D^\nu \Phi)^\dagger D^\mu \Phi]$$

Mixed

$$O_{M,0} = \text{Tr} \left[\hat{W}_{\mu\nu} \hat{W}^{\mu\nu} \right] \times [(D_\beta \Phi)^\dagger D^\beta \Phi]$$

$$O_{M,1} = \text{Tr} \left[\hat{W}_{\mu\nu} \hat{W}^{\nu\beta} \right] \times [(D_\beta \Phi)^\dagger D^\mu \Phi]$$

$$O_{M,2} = \left[\hat{B}_{\mu\nu} \hat{B}^{\mu\nu} \right] \times [(D_\beta \Phi)^\dagger D^\beta \Phi]$$

$$O_{M,3} = \left[\hat{B}_{\mu\nu} \hat{B}^{\nu\beta} \right] \times [(D_\beta \Phi)^\dagger D^\mu \Phi]$$

$$O_{M,4} = [(D_\mu \Phi)^\dagger \hat{W}_{\beta\nu} D^\mu \Phi] \times \hat{B}^{\beta\nu}$$

$$O_{M,5} = \frac{1}{2} \left[(D_\mu \Phi)^\dagger \hat{W}_{\beta\nu} D^\nu \Phi \right] \times \hat{B}^{\beta\mu} + h.c.$$

$$O_{M,7} = [(D_\mu \Phi)^\dagger \hat{W}_{\beta\nu} \hat{W}^{\beta\mu} D^\nu \Phi]$$

Transverse

$$O_{T,0} = \text{Tr} \left[\hat{W}_{\mu\nu} \hat{W}^{\mu\nu} \right] \times \text{Tr} \left[\hat{W}_{\alpha\beta} \hat{W}^{\alpha\beta} \right]$$

$$O_{T,1} = \text{Tr} \left[\hat{W}_{\alpha\nu} \hat{W}^{\mu\beta} \right] \times \text{Tr} \left[\hat{W}_{\mu\beta} \hat{W}^{\alpha\nu} \right]$$

$$O_{T,2} = \text{Tr} \left[\hat{W}_{\alpha\mu} \hat{W}^{\mu\beta} \right] \times \text{Tr} \left[\hat{W}_{\beta\nu} \hat{W}^{\nu\alpha} \right]$$

$$O_{T,5} = \text{Tr} \left[\hat{W}_{\mu\nu} \hat{W}^{\mu\nu} \right] \times \hat{B}_{\alpha\beta} \hat{B}^{\alpha\beta}$$

$$O_{T,6} = \text{Tr} \left[\hat{W}_{\alpha\nu} \hat{W}^{\mu\beta} \right] \times \hat{B}_{\mu\beta} \hat{B}^{\alpha\nu}$$

$$O_{T,7} = \text{Tr} \left[\hat{W}_{\alpha\mu} \hat{W}^{\mu\beta} \right] \times \hat{B}_{\beta\nu} \hat{B}^{\nu\alpha}$$

$$O_{T,8} = \hat{B}_{\mu\nu} \hat{B}^{\mu\nu} \times \hat{B}_{\alpha\beta} \hat{B}^{\alpha\beta}$$

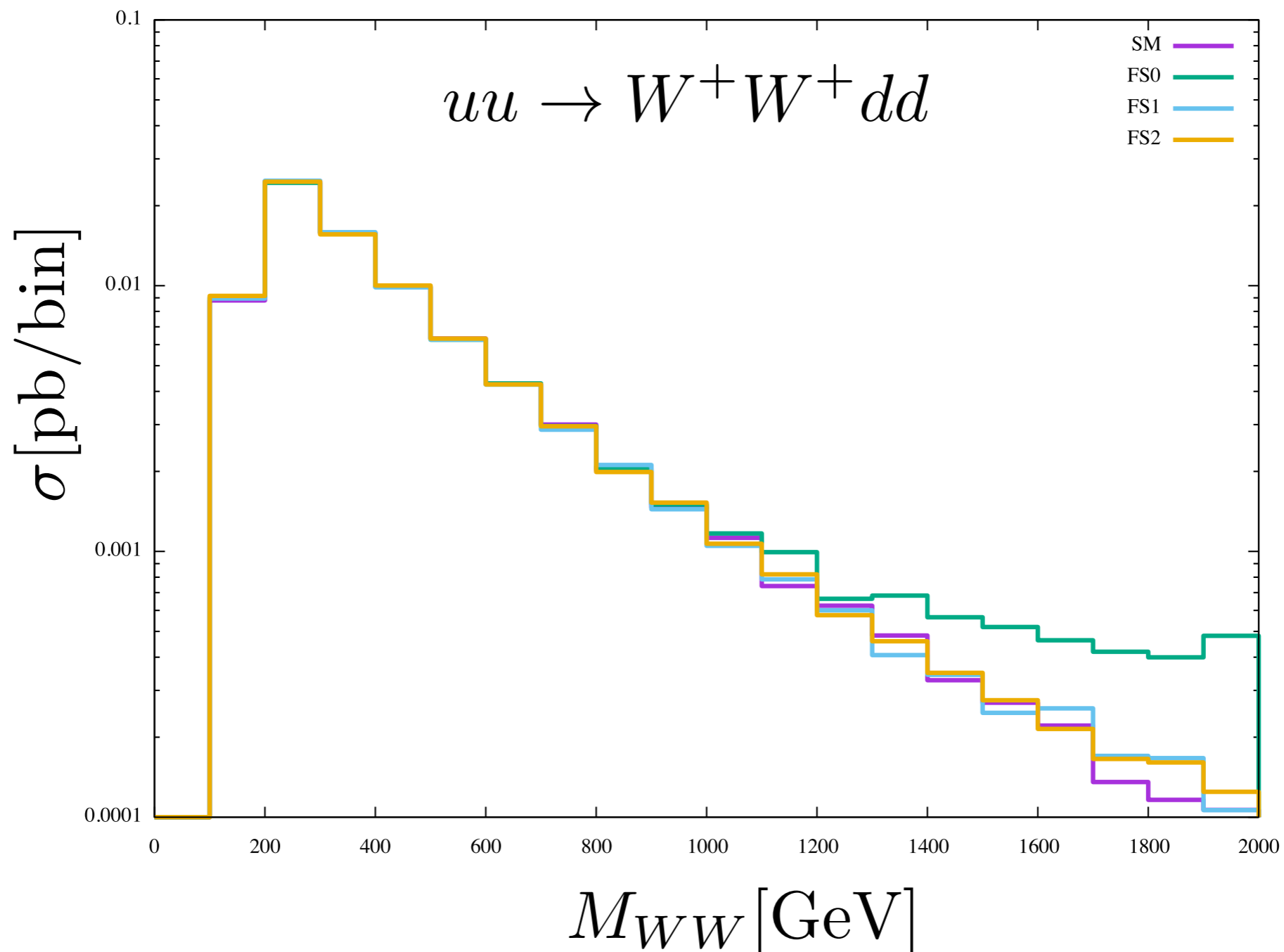
$$O_{T,9} = \hat{B}_{\alpha\mu} \hat{B}^{\mu\beta} \times \hat{B}_{\beta\nu} \hat{B}^{\nu\alpha},$$

How do these operators change the VBS process:

	WWWW	WWZZ	ZZZZ	WWAZ	WWAA	ZZZA	ZZAA	ZAAA	AAAA
$O_{S,0/1}$	✓	✓	✓						
$O_{M,0/1/6/7}$	✓	✓	✓	✓	✓	✓	✓		
$O_{M,2/3/4/5}$		✓	✓	✓	✓	✓	✓		
$O_{T,0/1/2}$	✓	✓	✓	✓	✓	✓	✓	✓	✓
$O_{T,5/6/7}$		✓	✓	✓	✓	✓	✓	✓	✓
$O_{T,8/9}$			✓			✓	✓	✓	✓

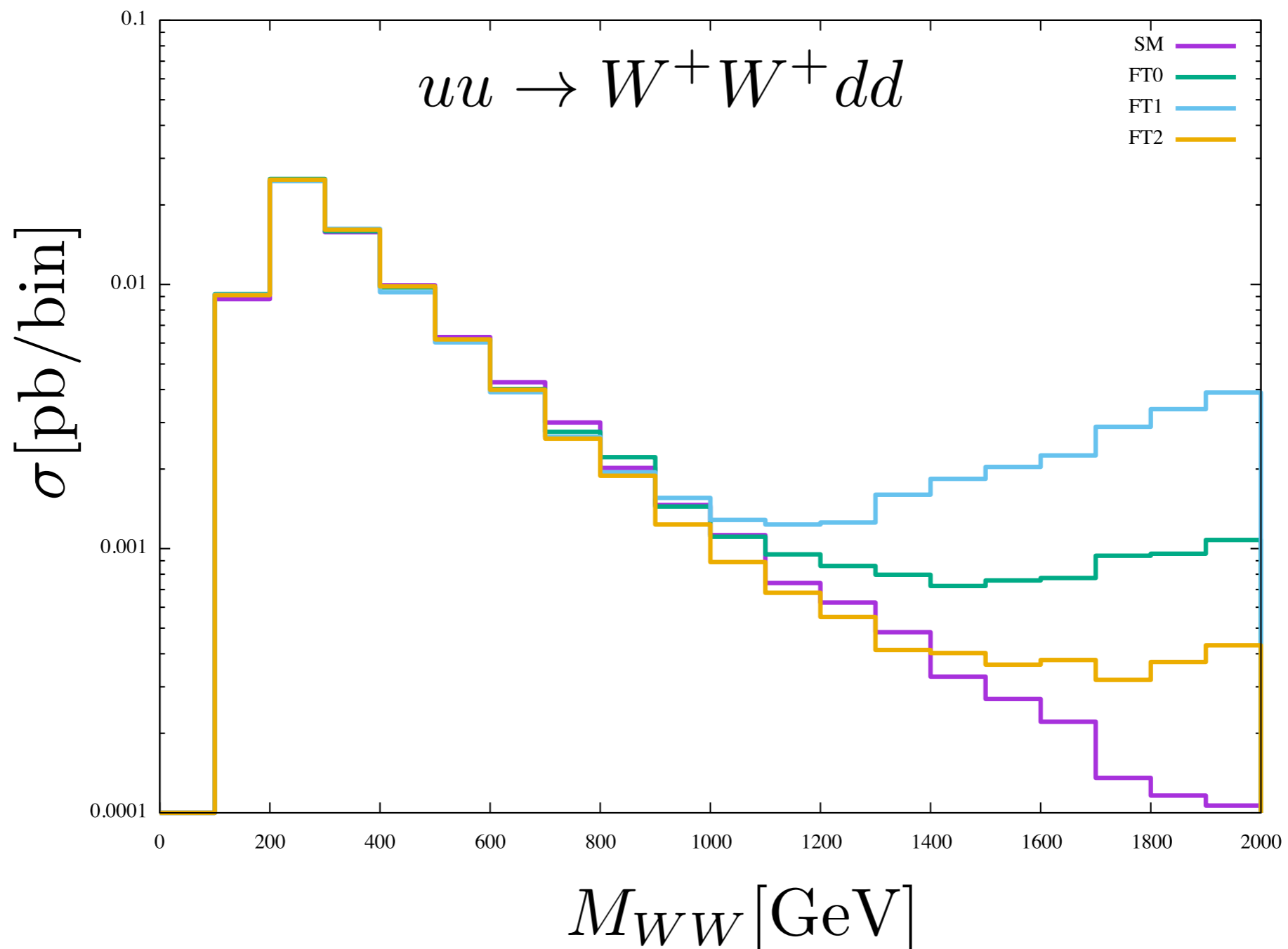
Example: same sign W

Distributions for dim-8: Longitudinal operators



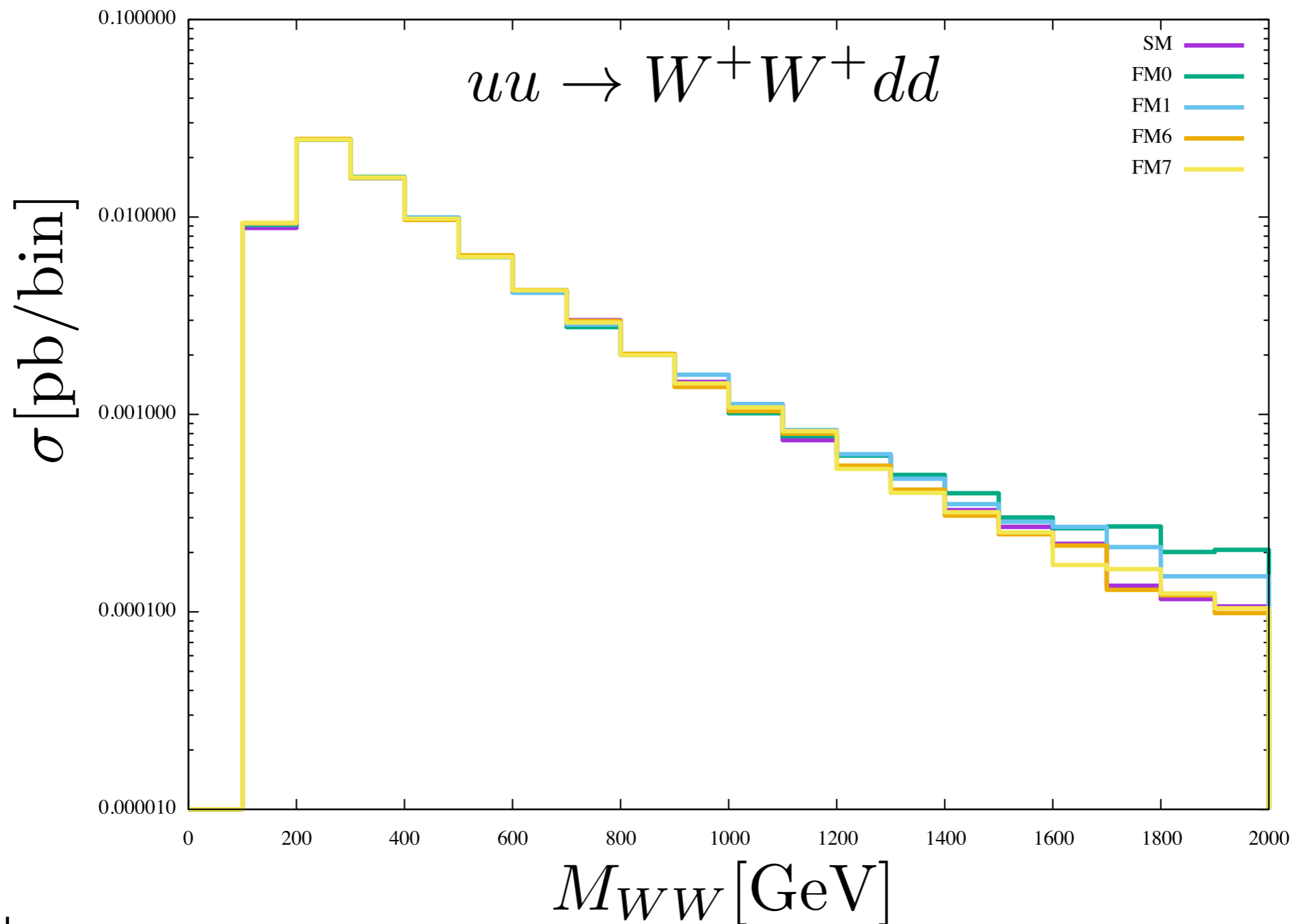
Example: same sign W

Distributions for dim-8: Transverse operators

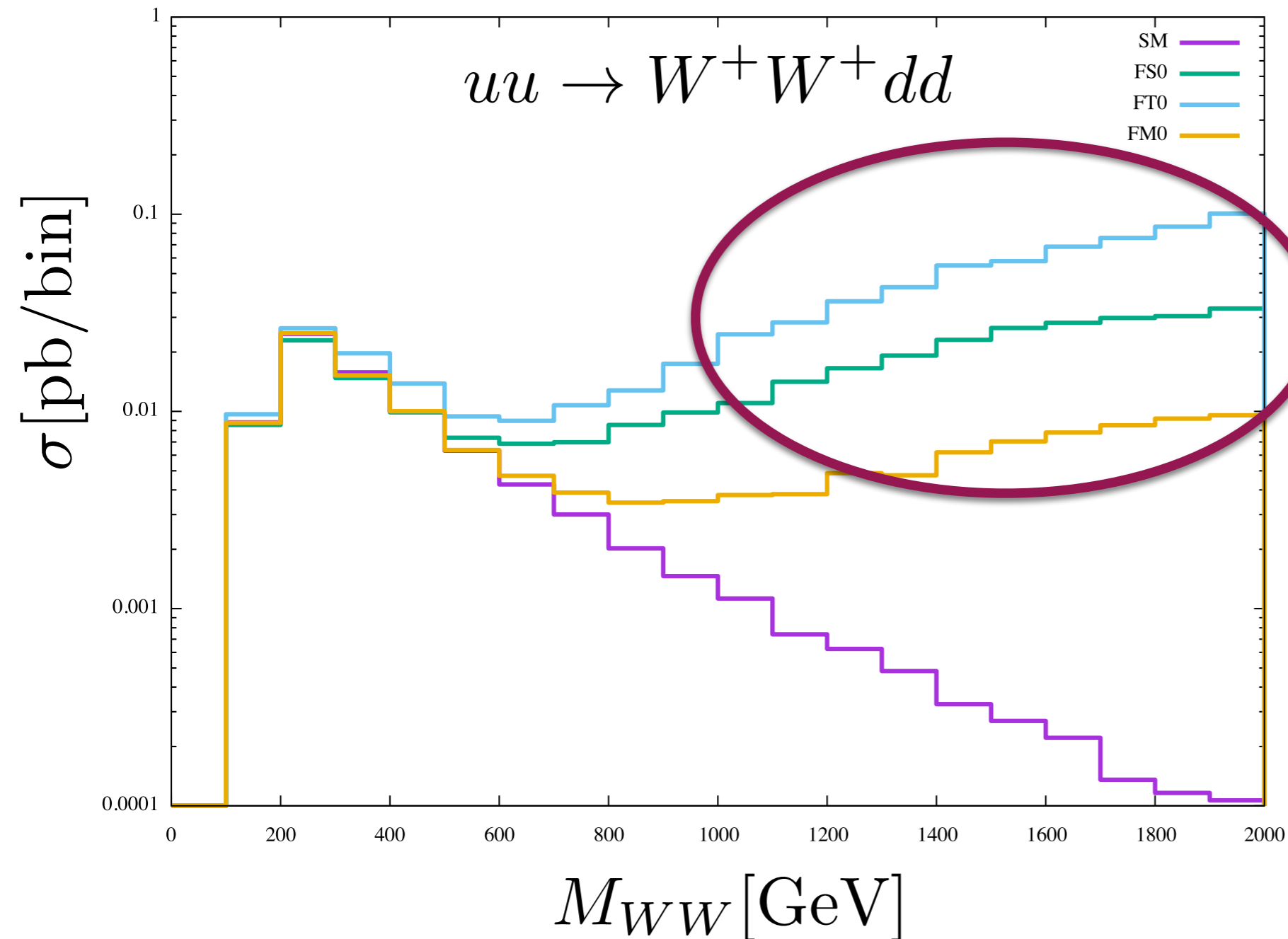


Example: same sign W

Distributions for dim-8: Mixed operators



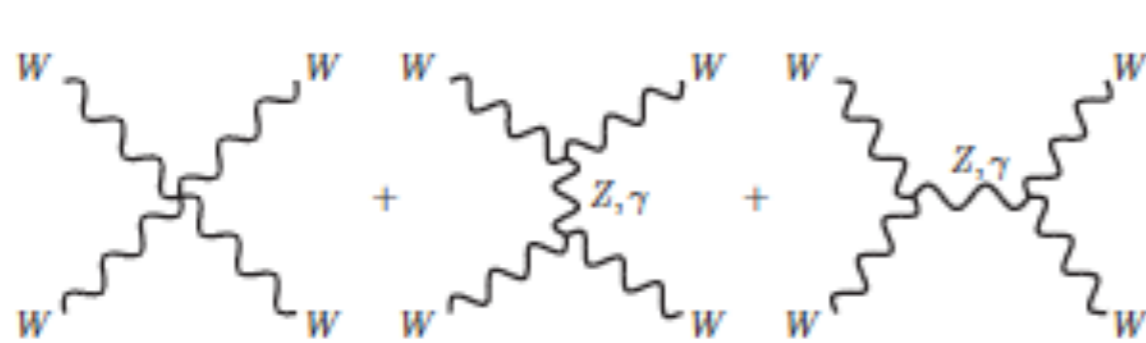
What happens for larger values of the coefficients?



Unphysical high-energy behaviour
Amplitude rises with the energy

Unitarity

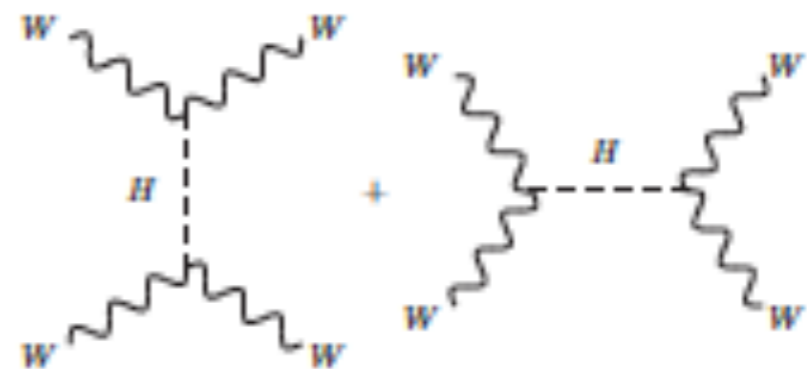
The special role of the Higgs:



$$a_0 \sim \frac{E^2}{v^2}$$

amplitude
growing with energy

+



$$a_0 \sim \frac{E^2}{v^2} - \frac{E^2}{v^2} \sim \frac{m_H^2}{v^2}$$

Probability < 1



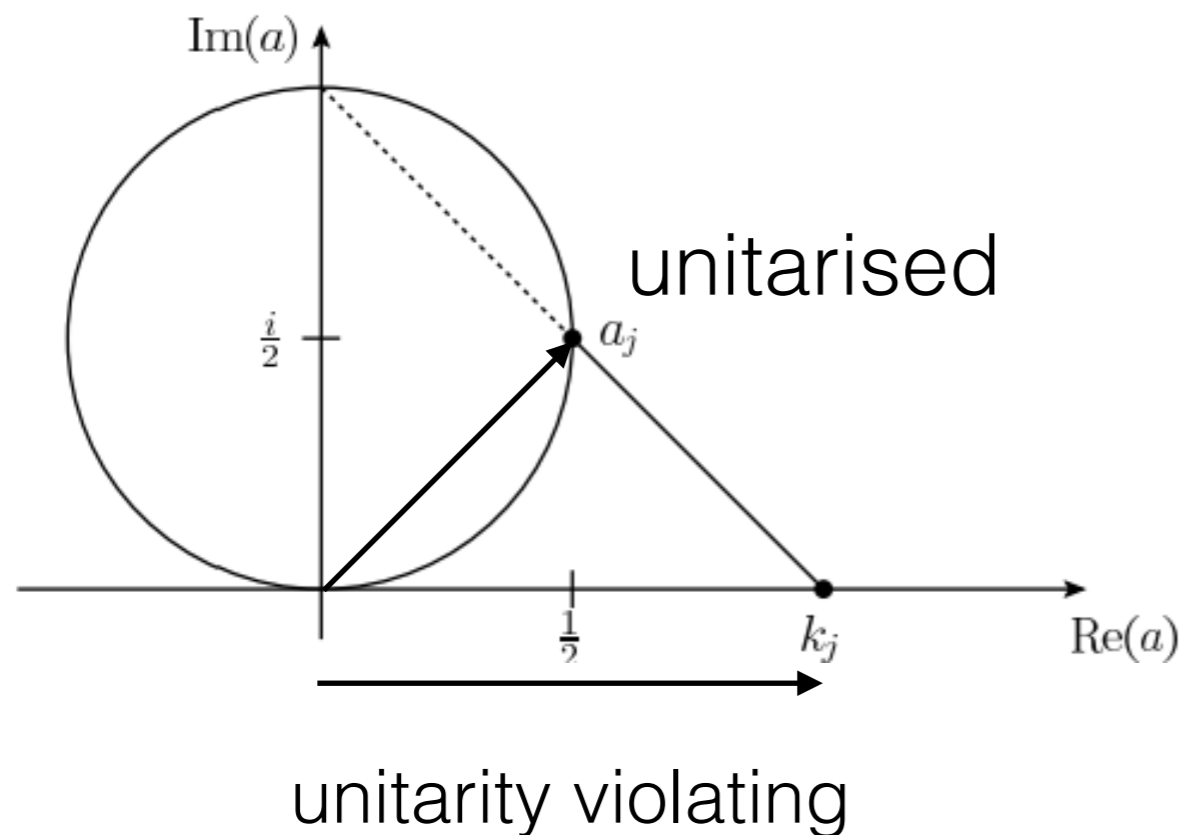
Amplitude in the SM is finite (due to Higgs exchange)

Modification of the TGC and QGC can lead to unitarity violating behaviour

What can we do about this?

- 1) EFT interpretation not valid: Apply a hard energy cut above which the EFT interpretation is not meaningful
- 2) Use unitarisation to enforce unitarity for the amplitudes

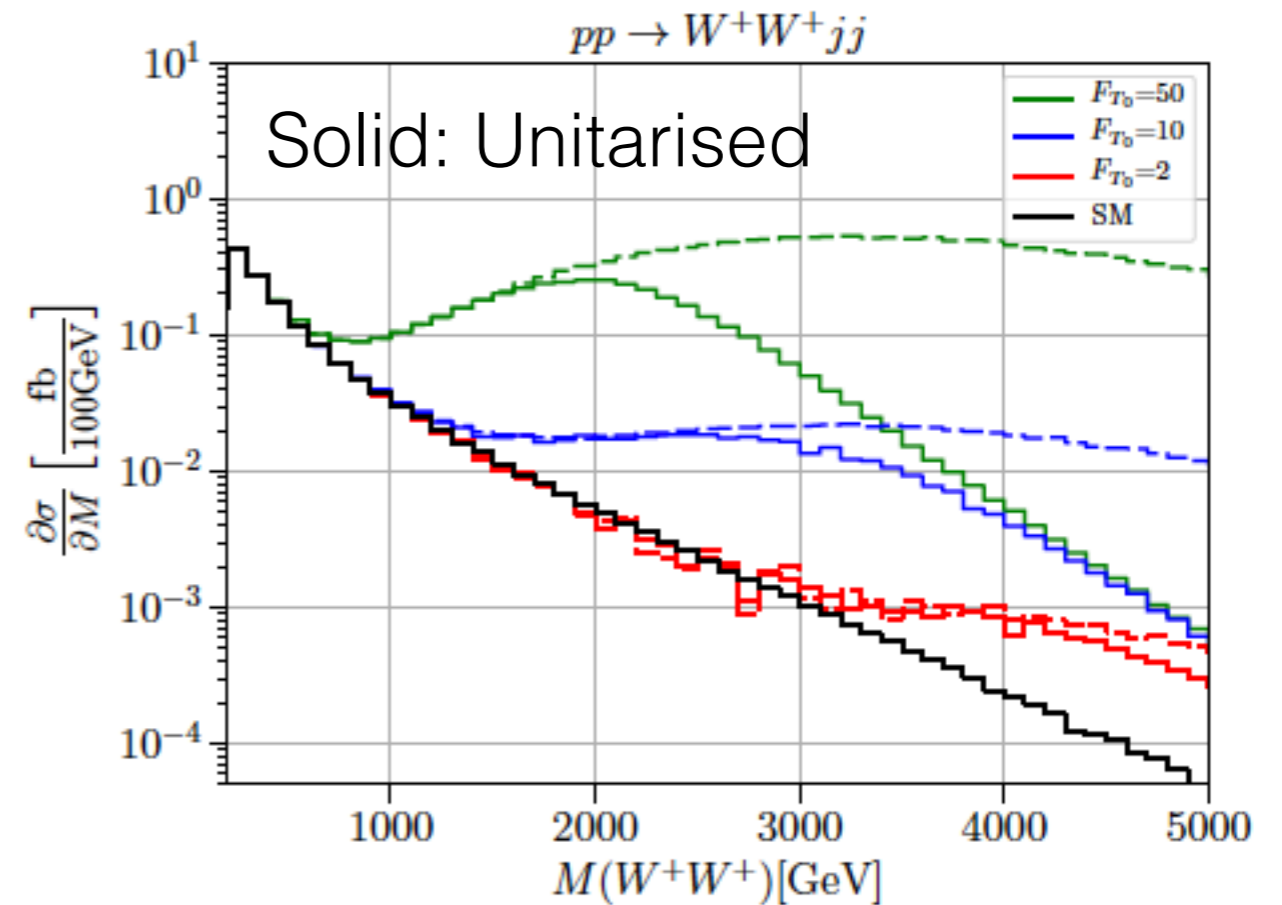
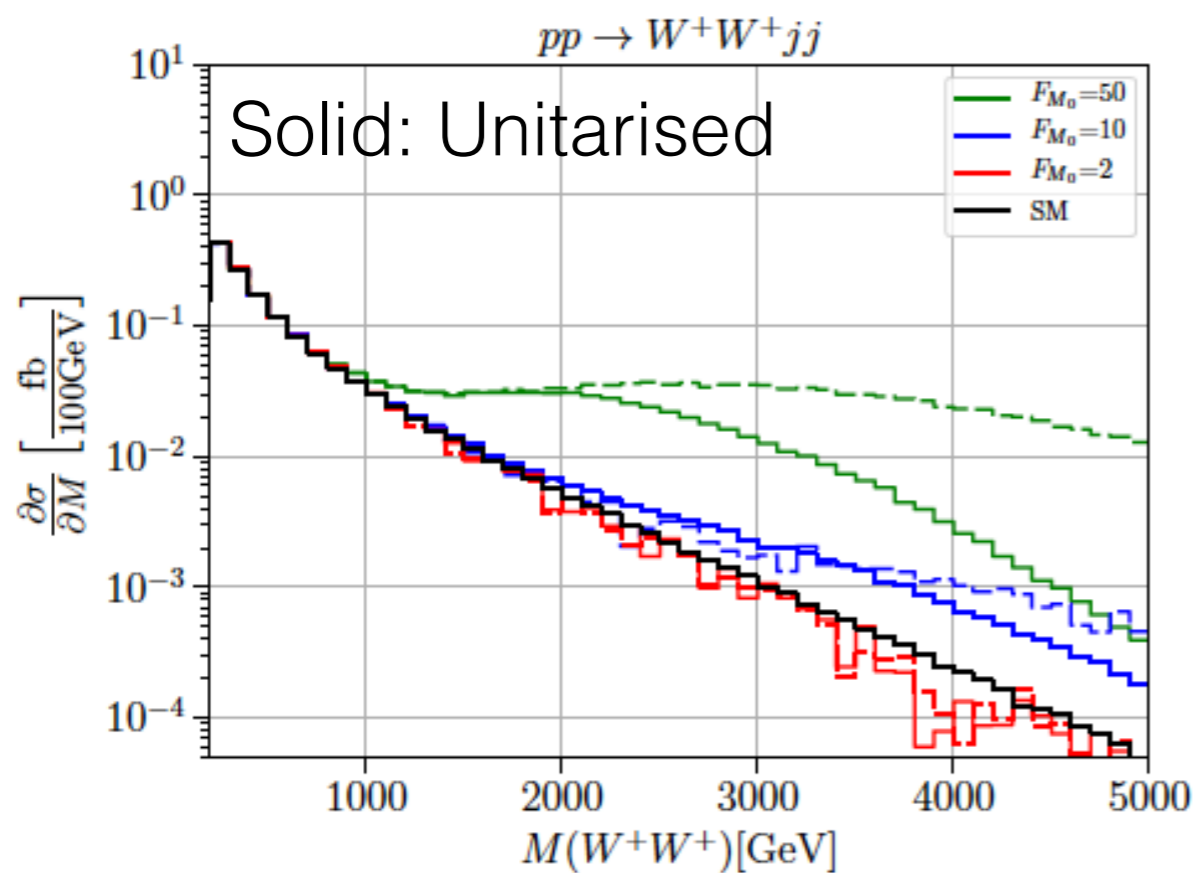
A simple example: K-matrix Unitarisation



One can project the unitarity-violating amplitude back to the Argand circle

Various unitarisation methods:
Alboteanu et al arXiv:0806.4145
Kilian et al arXiv:1408.6207
Perez et al arXiv:1807.02707

Does unitarisation work?

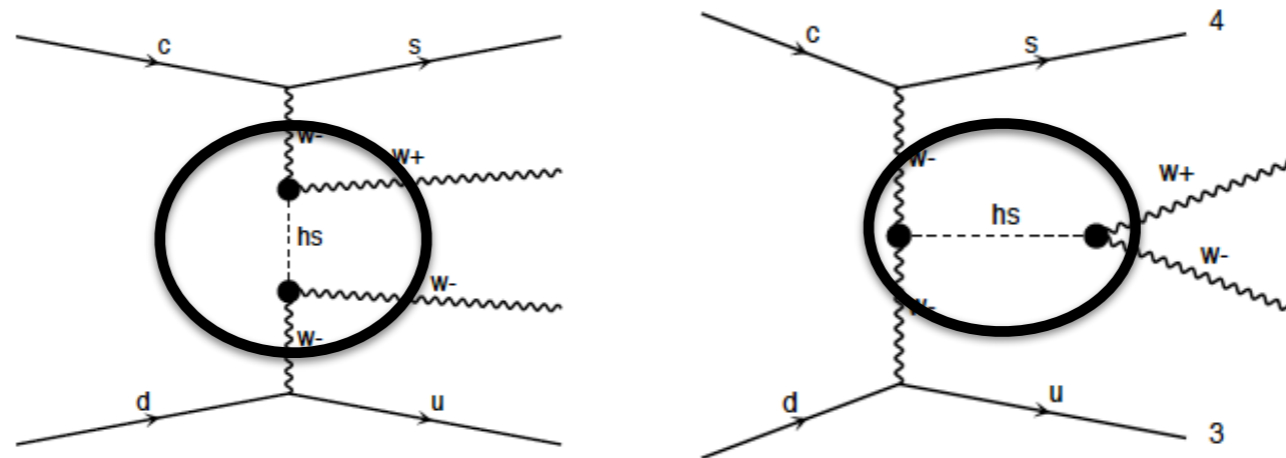


Brass et al arXiv:1807.02512

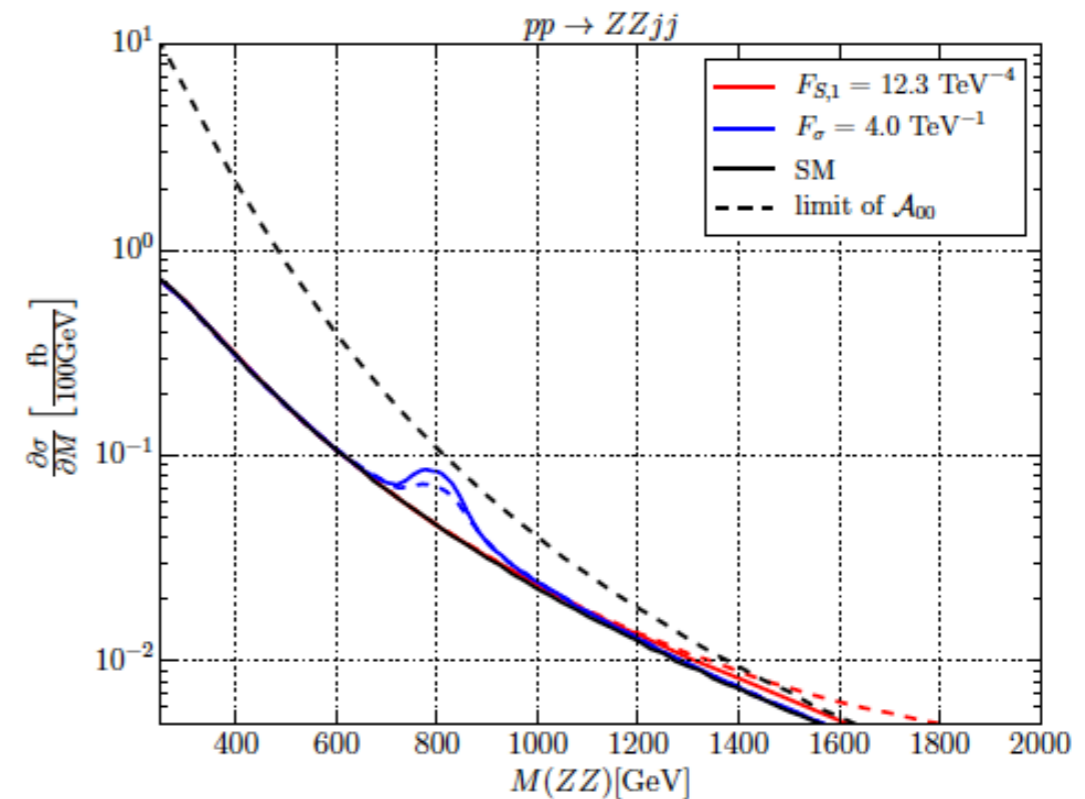
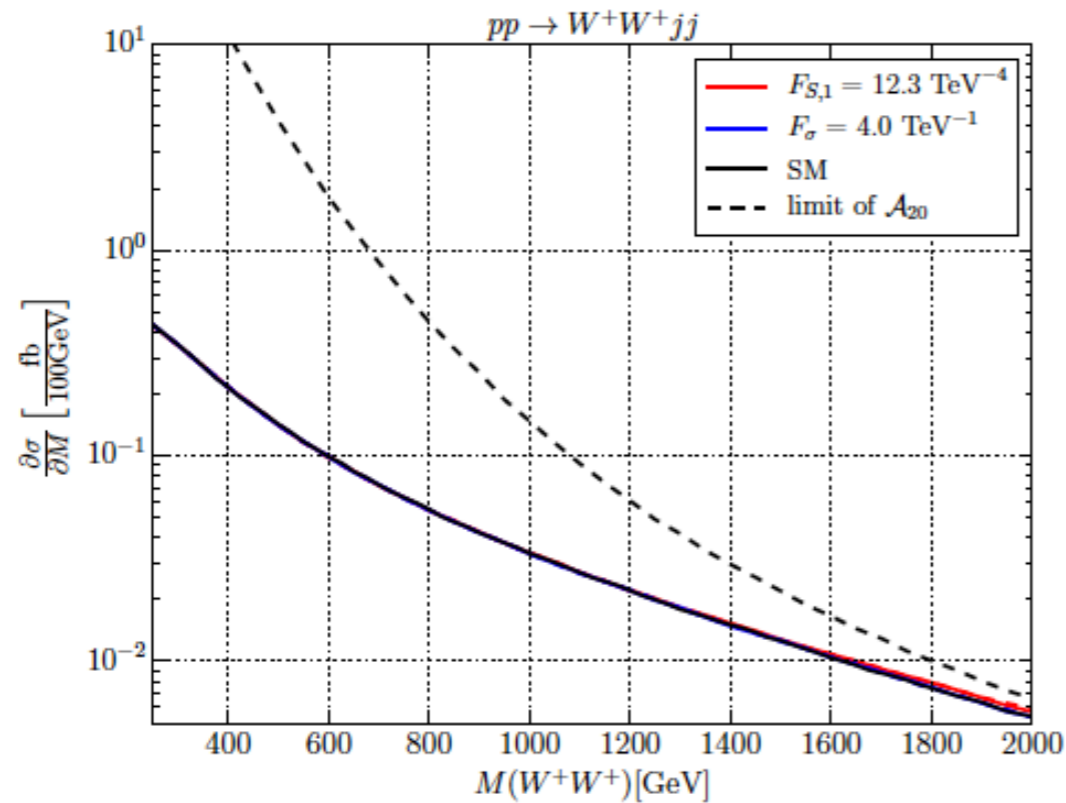
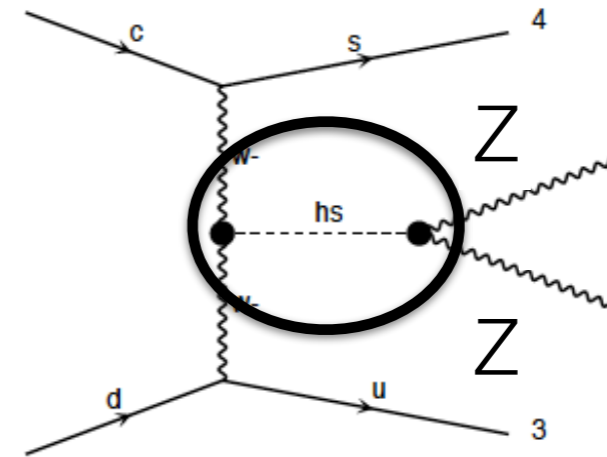
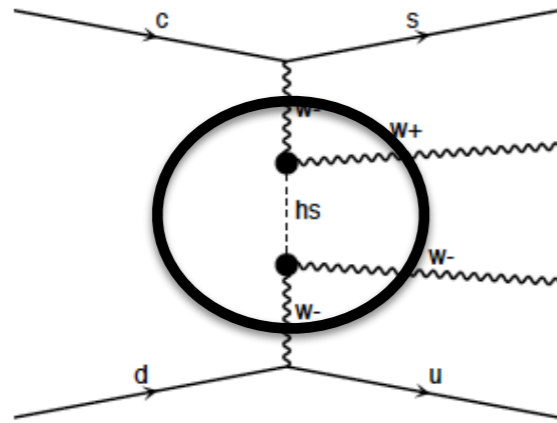
Unitarisation cures the unphysical predictions in the high energy region but it is not straightforward to connect any EFT interpretation of the results to UV physics

EFT or Resonances in VBS?

- EFT applies in the absence of light states
- Any model with resonances coupling to gauge bosons will affect VBS
- Depending on the channel the resonance will be in the s-channel, t-channel or both:

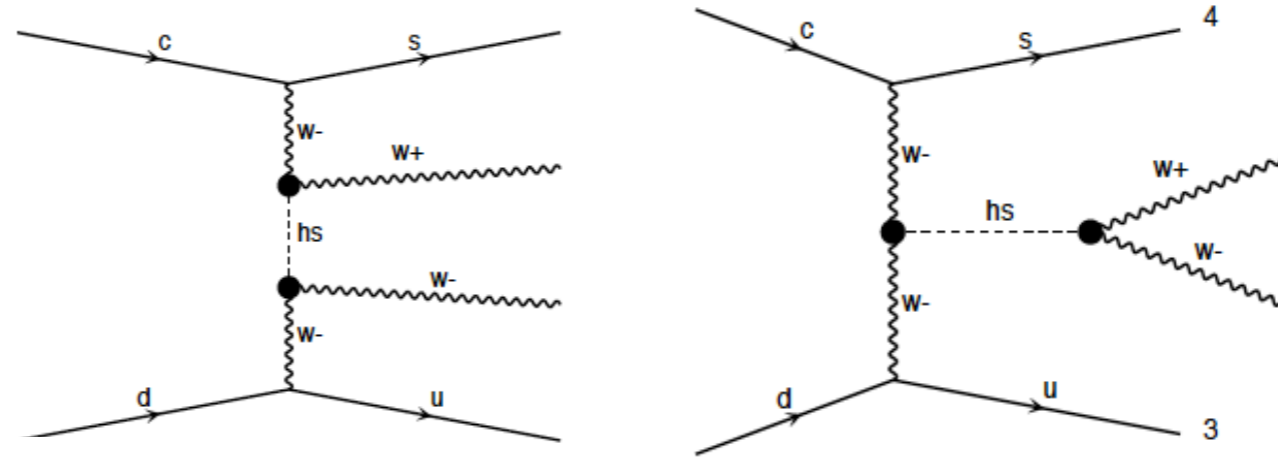


EFT or Resonances in VBS?



Matching example

A scalar resonance:



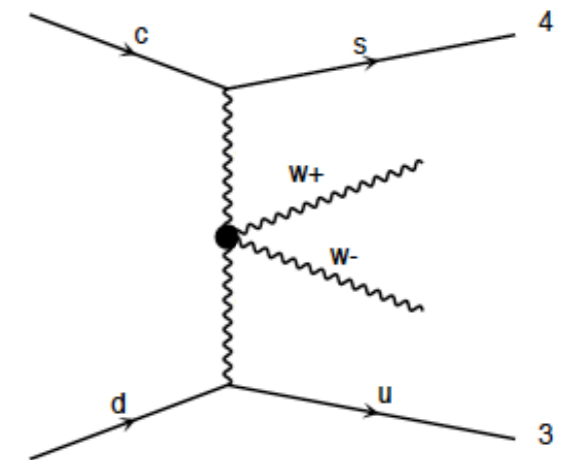
$$\mathcal{L}_{kin} = \frac{1}{2} D_\mu \mathcal{S} D^\mu \mathcal{S} - \frac{1}{2} M_S^2 \mathcal{S}^2 \quad \mathcal{L}_{int} = \mathcal{S} \mathcal{J}_S$$

$$\mathcal{J}_S = a_H (D_\mu \phi)^\dagger D^\mu \phi + a_W W_{\mu\nu}^i W^{i\mu\nu} + a_B B_{\mu\nu} B^{\mu\nu}$$

If $M \gg E$ we can match

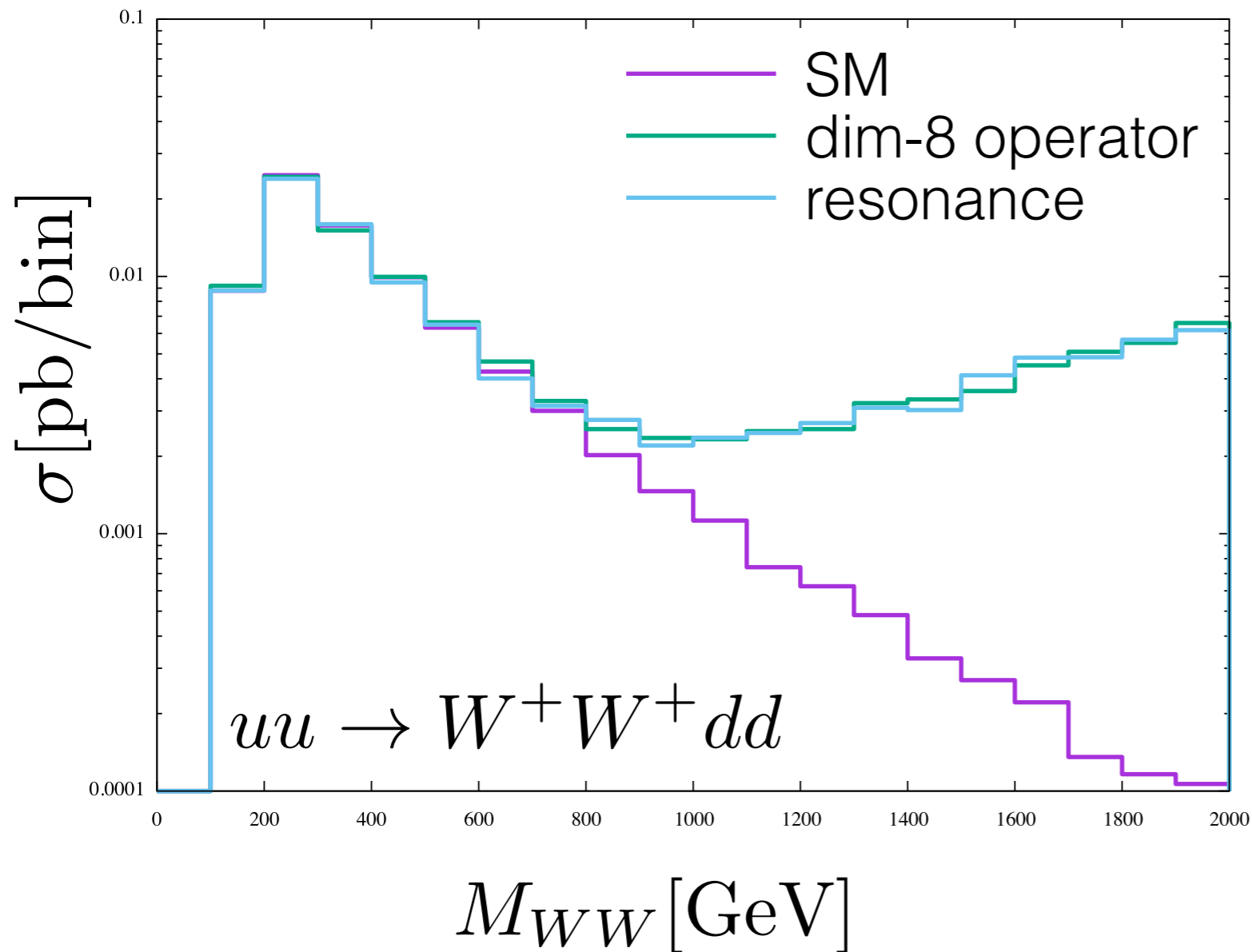
Matching
conditions

$$\begin{aligned} F_{S,1} &= \frac{a_H^2}{2M_S^2} \Lambda^4 \\ F_{T,0} &= \frac{\bar{a}_W^2}{2M_S^2} \Lambda^4 \\ F_{T,5} &= \frac{\bar{a}_W \bar{a}_B}{M_S^2} \Lambda^4 \\ F_{T,8} &= \frac{\bar{a}_B^2}{2M_S^2} \Lambda^4 \\ F_{M,0} &= \frac{a_H \bar{a}_W}{M_S^2} \Lambda^4 \\ F_{M,2} &= \frac{a_H \bar{a}_B}{M_S^2} \Lambda^4 \end{aligned}$$



$$\begin{aligned} \bar{a}_W &= -2g^{-2} a_W \\ \bar{a}_B &= -4g'^{-2} a_B \end{aligned}$$

Does the EFT work?

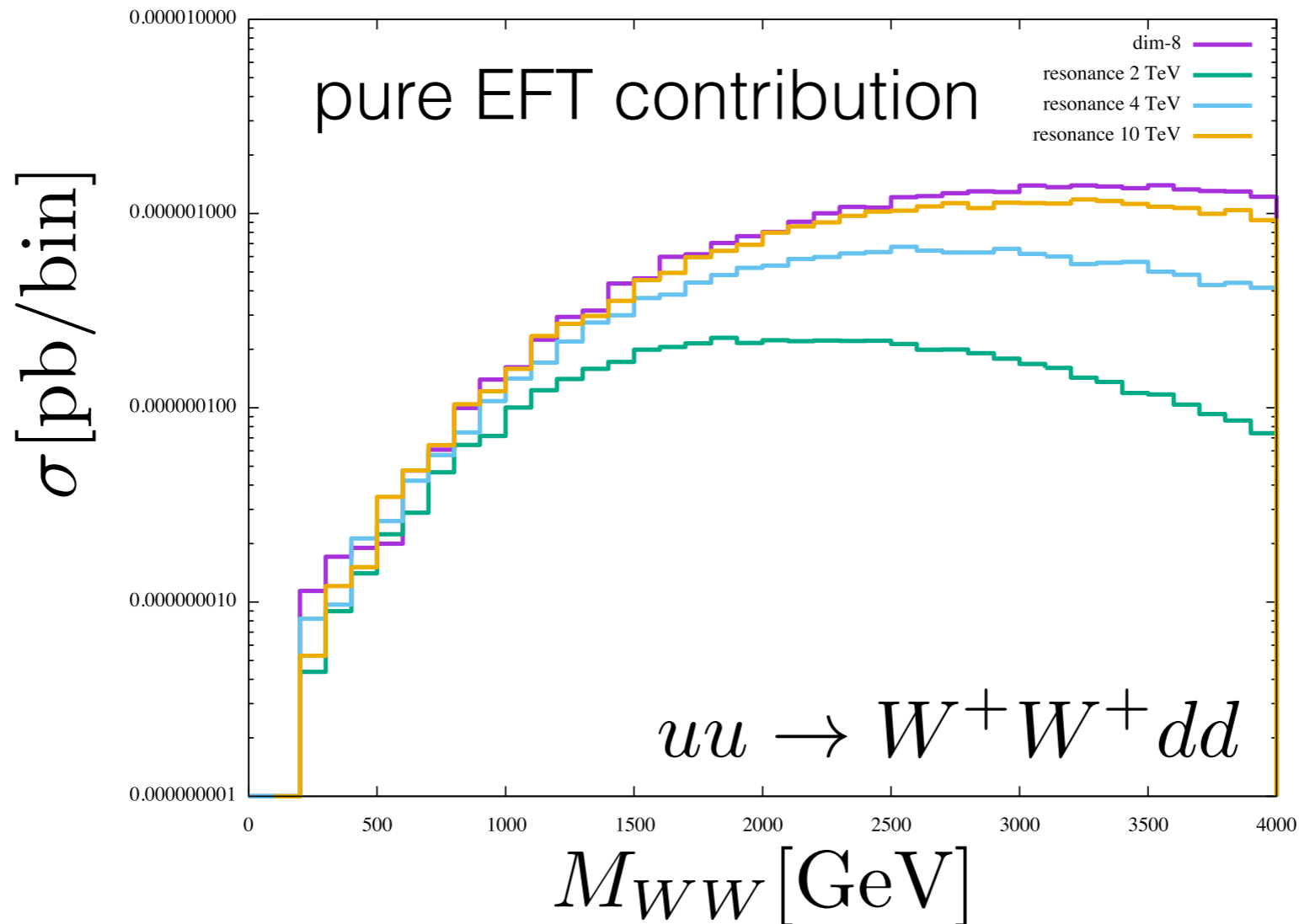


Resonance mass: 20 TeV
Energy probed \ll 20 TeV
EFT description is valid



Good agreement between
EFT and resonance plots

When does the EFT fail?



For light resonances the EFT is not reliable
As we approach the mass of the resonance the EFT
becomes unreliable

Thank you for your attention