Summary and To-Do-List

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a certain lack of input $\Rightarrow$
rather more a personal view than a summary

- GPDs are extremely difficult to determine, why should you care ?
- Status of global fits; D. Müller et al.
- Help from lattice-QCD; Ph. Hägler et al.
- Some issues I personally would like to work on

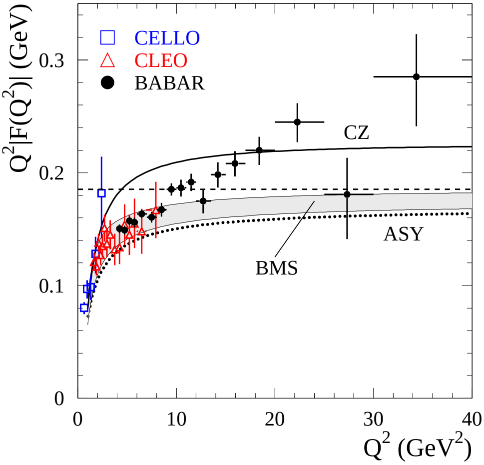
Collinear pQCD is well established, but the physics related to intrinsic transverse momentum / impact parameter is not.
Extremely difficult task, which requires fundamentally new techniques and concepts

Is it worth the effort ?

- reaction phenomenology, e.g. $\gamma^* + \gamma \to \pi^0$
- double hard interactions
- a lattice study of transverse momentum distributions
- single spin asymmetries, Sivers function

BABAR: $\gamma^* + \gamma \to \pi^0$

B. Aubert *et al.* Phys. Rev. D **80** (2009) 052002
[arXiv:0905.4778 [hep-ex]].

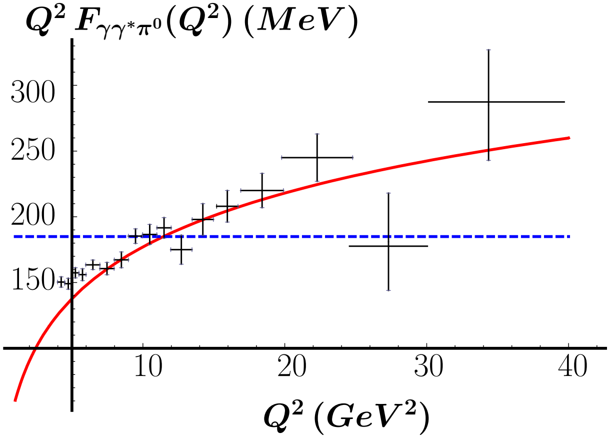


Meanwhile various models, e.g. Radyushkin 0906.0323

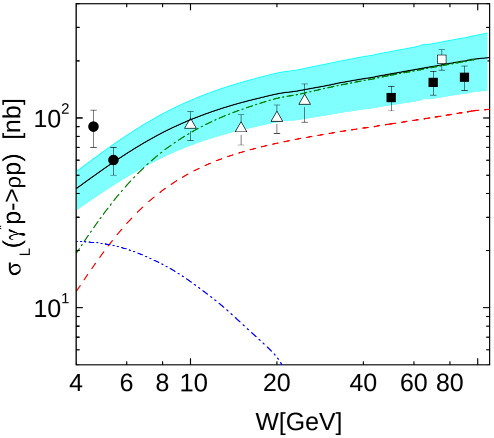
($i.e.$, $\bar{q}q$) contribution

$$
\begin{aligned}
(\epsilon_\perp \times q_\perp) F^{\bar{q}q}_{\gamma^*\gamma\pi^0}(Q^2) &= \frac{1}{4\pi^3\sqrt{3}} \int_0^1 dx \int \frac{(\epsilon_\perp \times (xq_\perp + k_\perp))}{(xq_\perp + k_\perp)^2 - i\epsilon} \\
&\times \Psi(x, k_\perp)\, d^2 k_\perp
\end{aligned}
$$

$$
\begin{aligned}
\Psi(x, k_\perp) &= \frac{4\pi^2 \varphi_\pi(x)}{x\bar{x}\sigma\sqrt{6}} \exp\left(-\frac{k_\perp^2}{2\sigma x\bar{x}}\right) \\
\varphi(x) &= f_\pi
\end{aligned}
$$


$Q^2\,F_{\gamma\gamma^*\pi^0}(Q^2)\ (MeV)$
300
250
200
150
10
20
30
40
$Q^2\ (GeV^2)$

Peter Kroll:

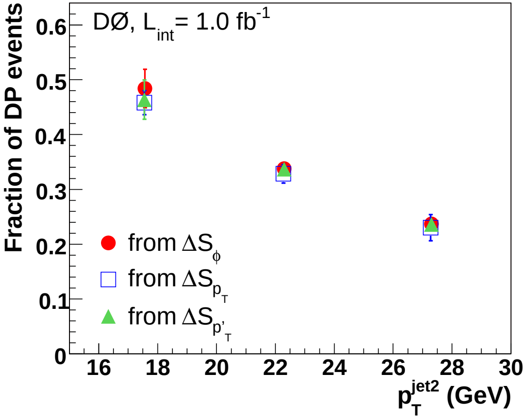


$Q^2 = 3.8\,\mathrm{GeV}^2$ glue+sea, glue, valence +interf.
data: H1 (open), ZEUS (filled squares), E665 (triangles), HERMES (circles)

double hard interactions

Already at Tevatron double-hard reactions are very relevant

D0 analyzed γ +3 jet relative 2 jet to get the fraction of double hard reactions in $p + \bar{p}$ at $\sqrt{s} = 1.96$ TeV



M. Diehl and AS tbp: the double DY amplitude

$$
\begin{aligned}
F(x_i, \vec{k}_i, \vec{y}_1) &= \int \frac{dz_1^- d^2\vec{z}_1}{(2\pi)^3} e^{\mathrm{i}x_1 z_1^- p^+ - \mathrm{i}\vec{z}_1\cdot\vec{k}_1} \\
&\times \int \frac{dz_2^- d^2\vec{z}_2}{(2\pi)^3} e^{\mathrm{i}x_2 z_2^- p^+ - \mathrm{i}\vec{z}_2\cdot\vec{k}_2} \\
&\times 2p^+ \int dy_1^- \langle p|\bar{q}(-z_2/2)\Gamma_2 q(z_2/2) \\
&\times \sum_X |X\rangle\langle X| \\
&\times \bar{q}(y_1 - z_2/2)\Gamma_2 q(y_1 + z_2/2)|p\rangle
\end{aligned}
$$

If $X = p$ dominates one gets

$$F(x_i, \vec{k}_i, \vec{y}_1) \sim \int d^2\vec{b}\ \ f(x_2, \vec{b}) f(x_1, \vec{b} + \vec{y}_1)$$

$$f(x, \vec{b}) = \int \frac{d^2\Delta_\perp}{(2\pi)^2}\, e^{-\mathrm{i}\vec{\Delta}\cdot\vec{b}}\, H(x, 0, -\Delta_\perp^2)$$

But there is no good argument to assume this

$\Rightarrow$ systematic uncertainties of the order of the double GPD contribution.

possible consequences: If ALICE finds an unexplained difference between A+A and p+p or p+A:
QGP physics or EMC effect for double hard reactions ?

$\Rightarrow$ One has to calculate the double GPD contribution and show that it is small.

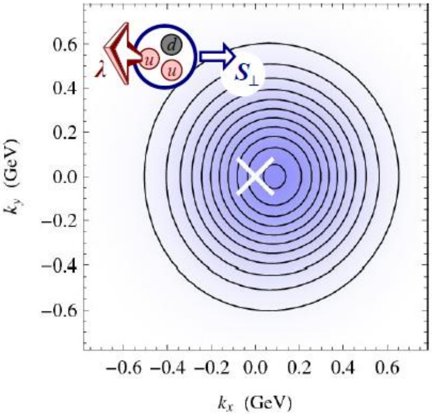
tmdPDFs: transverse momentum dependent PDFs

$$
\begin{aligned}
\Phi_q^{[\Gamma]}(x, \vec{k}_{perp}; P, S; \mathcal{C}) &= \int \frac{d(\ell \cdot P)}{2\pi} e^{\mathrm{i}x\ell\cdot P} \int \frac{d^2\vec{\ell}_\perp}{(2\pi)^2} e^{\mathrm{i}\vec{\ell}_\perp \cdot \vec{k}_\perp} \\
&\times \frac{1}{2} \langle P, S | \bar{q}(\ell) \Gamma U[\mathcal{C}_\ell] q(0) | P, S \rangle \Big|_{\ell^+ = 0} \\
U[\mathcal{C}_\ell] &= \mathcal{P} \exp\Big(-\mathrm{i} \int_{\mathcal{C}_\ell} d\xi^\mu A_\mu(\xi) \Big)
\end{aligned}
$$

each choice for C_ℓ defines a different tmdPDF. Usually the experimental situations determines which to take. What is still missing is an exact formalism to relate them.

In B. Musch, P. Hägler, AS, and J. Negele, Europhys. Lett. **88** (2009) 61001 [arXiv:0908.1283] we used a direct link.

$\Rightarrow$ probability distributions in momentum space

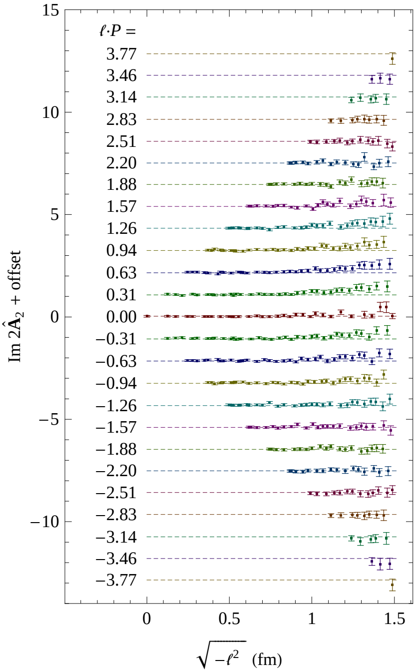


density of up quarks with positive helicity in a nucleon polarized in transverse x-direction; $\langle k_x \rangle = 67(5)$ MeV. Not the Fourier transformation of $f(x, \vec{b})$.

$$\left\langle P,S\left|\bar{q}(\ell)\gamma_\mu\mathcal{U}(\ell)q(0)\right|P,S\right\rangle = 4\tilde{A}_2(\ell^2,\ell\cdot P)P_\mu + 4\mathrm{i}{m_N}^2\tilde{A}_3(\ell^2,\ell\cdot P)\ell_\mu\ell\cdot S$$

factorization of transverse and longitudinal momentum distribution would imply

$$\hat{\mathbf{A}}_2(\ell^2,\ell\cdot P) = \tilde{A}_2(\ell^2,\ell\cdot P)/\mathrm{Re}\tilde{A}_2(\ell^2,0) \stackrel{?}{=} \hat{\mathbf{A}}_2(\ell\cdot P)$$



$\ell \cdot P =$
3.77
3.46
3.14
2.83
2.51
2.20
1.88
1.57
1.26
0.94
0.63
0.31
0.00
−0.31
−0.63
−0.94
−1.26
−1.57
−1.88
−2.20
−2.51
−2.83
−3.14
−3.46
−3.77
$\mathrm{Im}\ 2\hat{\mathbf{A}}_2$ + offset
$\sqrt{-\ell^2}$ (fm)

Conclusions

- Understanding non-collinear QCD is a major task
- Progress is needed to understand: double-hard reactions, SSAs, exclusive reaction in general, at scales up to tens of GeV^2.
- GPDs are the only known rigorous and undisputed approach. They do not answer all questions, but most questions cannot be answered without them.

Towards a global GPD fit

D. Müller, K. Kumericki, K. Passek-Kumericki

Note: the variable η for $\gamma^*(q_1) + N(p_1) \to \gamma^*(q_2) + N(p_2)$:

$$q = (q_1 + q_2)/2 \quad , \quad q^2 = -Q^2 \quad , \quad p = p_1 + p_2 \quad , \quad \Delta = p_1 - p_2$$

$$\xi = \frac{Q^2}{p \cdot q} \quad , \quad \eta = \frac{\Delta \cdot q}{p \cdot q}$$

For LO DVCS $\eta = \xi$.

One needs GPDs ($\mathcal{R}e$ and $\mathcal{I}m$ part) for the whole region $-1 < x, \eta < 1$.
central region: $-\eta < x < \eta$; outer region: $\eta < x$

Input:

- DVCS LO gives only information on the line $x = \xi = \eta$
- evolution is sensitive to outer region

$$\mu^2 \frac{d}{d\mu^2} H(x, x, t, \mu^2) = \int_0^1 \frac{dy}{x} V(1, x/y, \alpha_s(\mu^2)) H(y, x, \mu^2)$$

- dispersion relations allow to relate Coulomb form factors

$$\mathcal{R}e\ \mathcal{F}(\xi, t, Q^2) = \frac{1}{\pi} \mathrm{PV} \int_0^1 d\xi' \left(\frac{1}{\xi - \xi'} \mp \frac{1}{\xi + \xi'} \right) \mathcal{I}m\ \mathcal{F}(\xi', t, Q^2) + \mathcal{C}(t, Q^2)$$

- Formally one can uniquely continue GPDs from the inner region to the outer region (if they are known as analytic functions).

- polynomiality, e.g.:

$$\int_0^1 dx\, x^2\, H(x,\xi,t) \;=\; A_{30}(t) \;+\; (2\xi)^2 A_{32}(t)$$

Thus GPDs are strongly constraint in the whole x, η range by their values for $x = \eta$. The practical value of this fact depends crucially on the size of error bars.

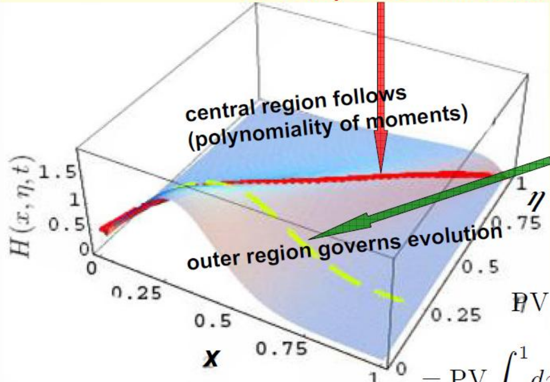
One needs the $\mathcal{R}e$ and $\mathcal{I}m$ part and precise data in a large x, η, Q^2 region.

Modeling & Evolution

outer region governs the evolution at the cross-over trajectory

$$\mu^2 \frac{d}{d\mu^2} H(x,x,t,\mu^2) = \int_x^1 \frac{dy}{x} V(1, x/y, \alpha_s(\mu)) H(y,x,\mu^2)$$

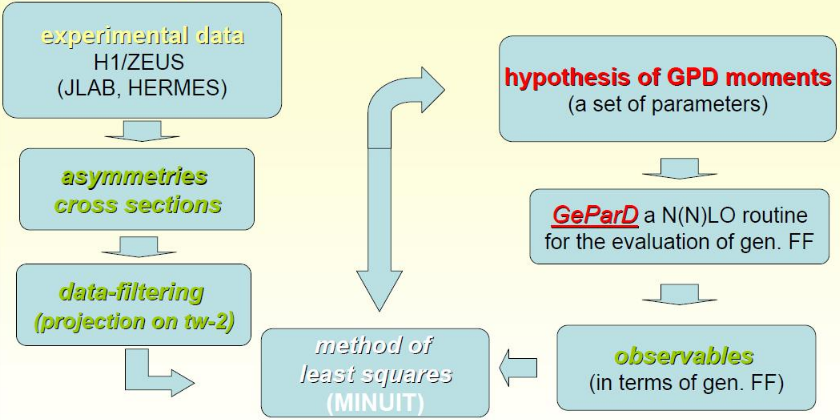
GPD at $\eta = x$ is `measurable' (LO)



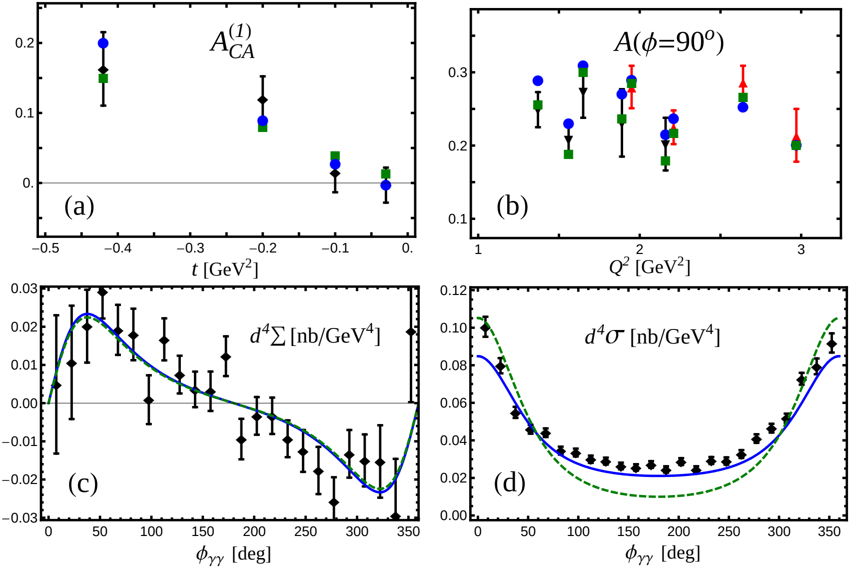
net contribution of outer + central region is governed by a sum rule:

$$\mathrm{PV}\int_0^1 dx\, \frac{2x}{\eta^2 - x^2} H^-(x,\eta,t)$$
$$= \mathrm{PV}\int_0^1 dx\, \frac{2x}{\eta^2 - x^2} H^-(x,x,t) + \frac{1}{2} C(t)$$

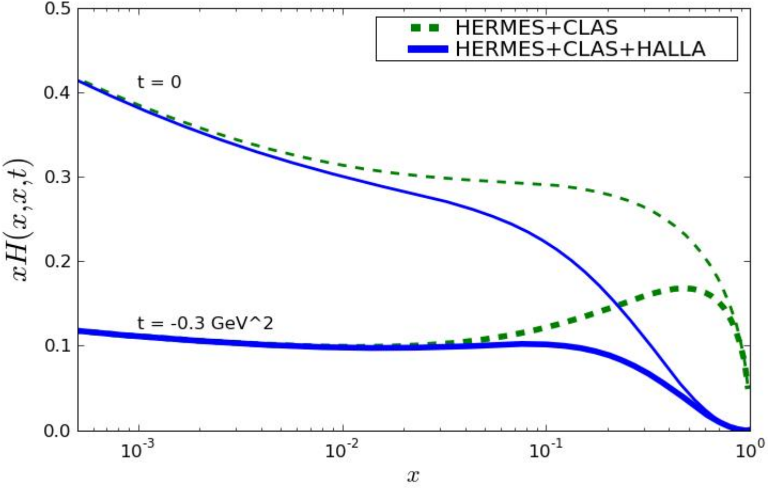
Ready for flexible GPD model fits?



global DVCS fit from 04/2009 in LO: (a) BCA, HERMES (b) BSA CLAS (c) $\Delta\sigma$ for HALL A (d) σ for HALL A



Present data does not yet determine $H(x, x, t)$, not to speak of $H(x, \eta, t, Q^2)$ and the other GPDs ($\chi^2/dof \leq 1$).
Large x region will be explored by JLAB@11 GeV.



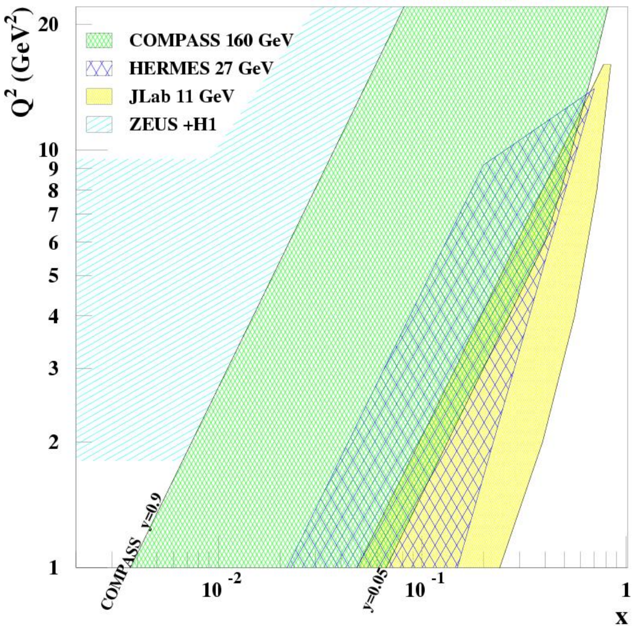
specific advantages of the COMPASS experiment

- Q^2, x range complementary to JLab
- BCSA

$$\frac{d\sigma^{+\downarrow} - d\sigma^{-\uparrow}}{d\sigma^{+\downarrow} + d\sigma^{-\uparrow}}$$

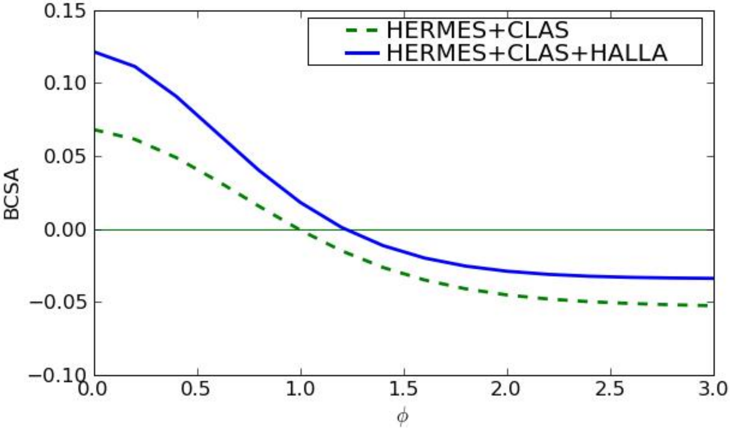
  is sensitive to $\mathcal{R}e\ H$.

- the transverse BCSA is sensitive to $\mathcal{E}$.

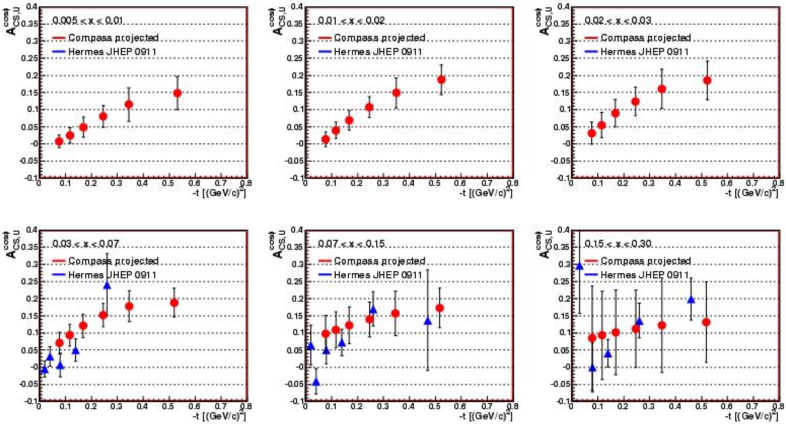


COMPASS 160 GeV
HERMES 27 GeV
JLab 11 GeV
ZEUS +H1
Q^2 (GeV^2)
20
10
9
8
7
6
5
4
3
2
1
COMPASS y=0.9
y=0.05
10^{-2}
10^{-1}
1
x

predicted BCSA asymmetry for COMPASS



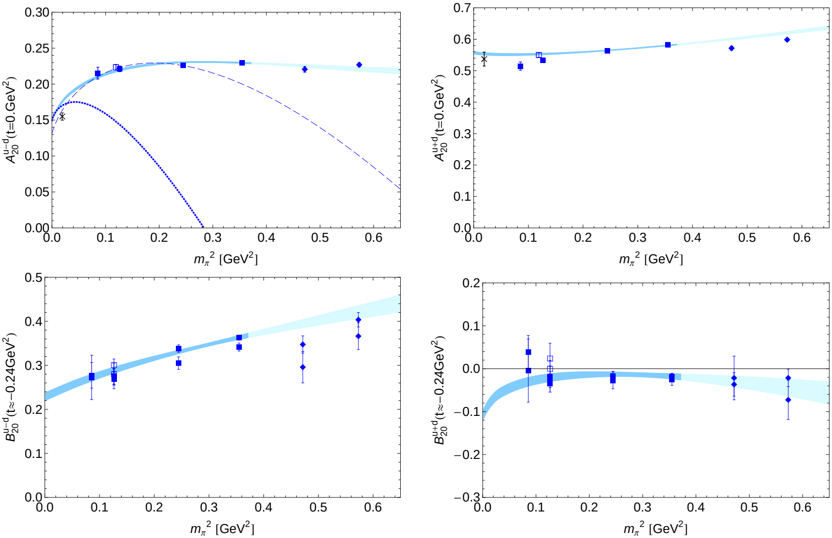
comparison with recent HERMES data (not yet in the global fit)

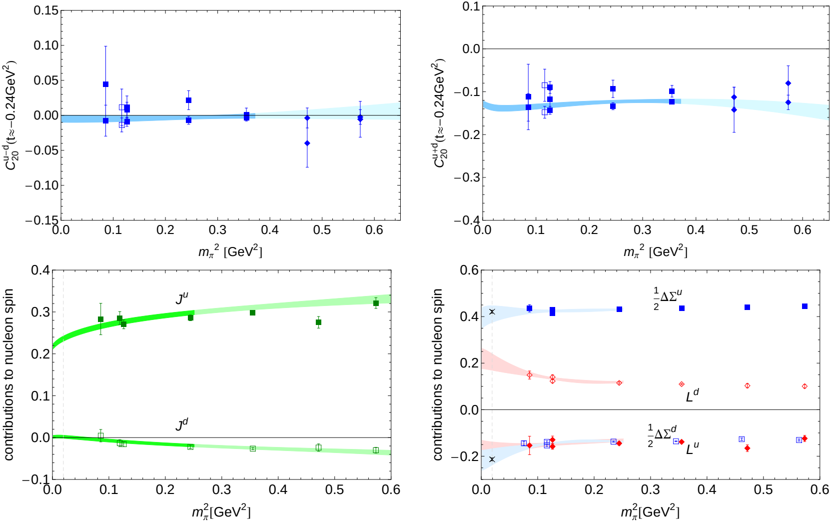


Help from the Lattice

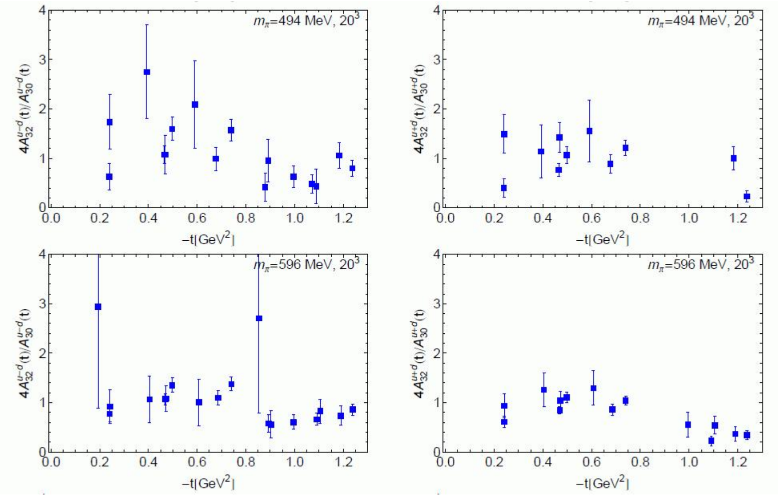
P. Hägler: new results from LHPC arXiv:1001.3620

$$\begin{aligned} H^n(\xi,t) &\equiv \int_{-1}^{1} dx\, x^{n-1} H(x,\xi,t) \\ H^{n=1}(\xi,t) &= A_{10}(t) \\ H^{n=2}(\xi,t) &= A_{20}(t) + (2\xi)^2 C_{20}(t) \\ H^{n=3}(\xi,t) &= A_{30}(t) + (2\xi)^2 A_{32}(t) \end{aligned}$$


$A_{20}^{u-d}(t{=}0.\mathrm{GeV}^2)$
$A_{20}^{u+d}(t{=}0.\mathrm{GeV}^2)$
$B_{20}^{u-d}(t{\approx}-0.24\mathrm{GeV}^2)$
$B_{20}^{u+d}(t{\approx}-0.24\mathrm{GeV}^2)$
m_π^2 [GeV^2]

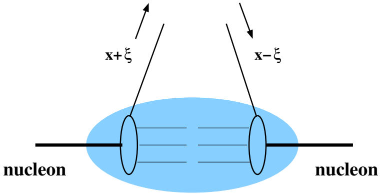


$C_{20}^{u-d}(t\approx-0.24\text{GeV}^2)$
m_π^2 [GeV^2]
$C_{20}^{u+d}(t\approx-0.24\text{GeV}^2)$
m_π^2 [GeV^2]
contributions to nucleon spin
J^u
J^d
m_π^2[GeV^2]
contributions to nucleon spin
$\frac{1}{2}\Delta\Sigma^u$
L^d
$\frac{1}{2}\Delta\Sigma^d$
L^u
m_π^2[GeV^2]



m_π=494 MeV, 20^3
$4A_{32}^{u-d}(t)/A_{30}^{u-d}(t)$
$-t$[GeV^2]
m_π=494 MeV, 20^3
$4A_{32}^{u+d}(t)/A_{30}^{u+d}(t)$
$-t$[GeV^2]
m_π=596 MeV, 20^3
$4A_{32}^{u-d}(t)/A_{30}^{u-d}(t)$
$-t$[GeV^2]
m_π=596 MeV, 20^3
$4A_{32}^{u+d}(t)/A_{30}^{u+d}(t)$
$-t$[GeV^2]

the interpretation of $4A_{32}/A_{30} \sim 1$ is still unclear



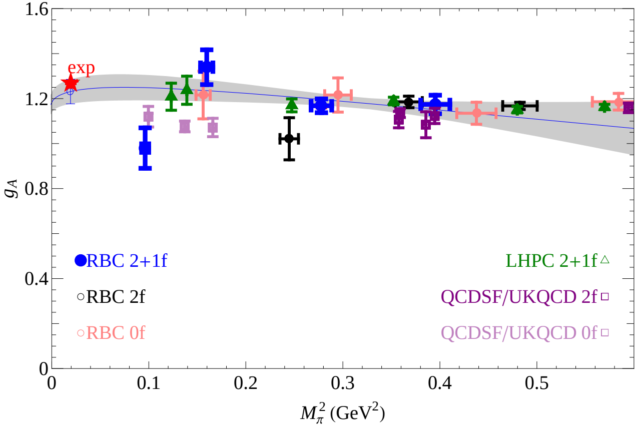
momentum fractions in the in and outgoing states:

$$y_i = \frac{x+\xi}{1+\xi} \quad , \quad y_f = \frac{x-\xi}{1-\xi}$$

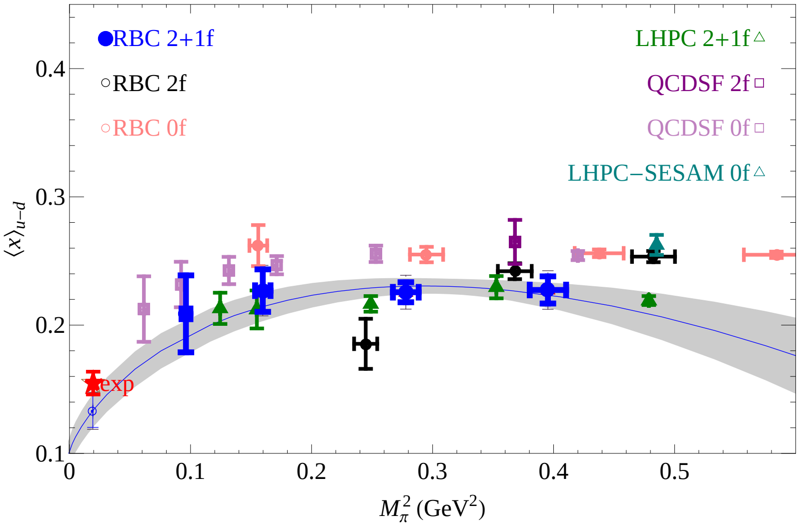
assumption: $\psi(x, k_\perp) \sim (1-x)^{3/2}\, f(k_\perp)$ for large x
$\Rightarrow \psi_f^\dagger \psi_i \sim (1-y_1)^{3/2}(1-y_2)^{3/2} = (1-x)^3[1 + 1.5\xi^2 + ...]$

comparison of different lattice actions ⇒ systematic error

H. W. Lin, AIP Conf. Proc. **1149** (2009) 552 [arXiv:0903.4080]



the importance of ChPT



Our next steps

- Improve the analysis of transverse position/momentum dependencies and keep up with other people's work in this field
- Improve the global fit of Mueller, Kumericki and Passek-Kumericki
  - Update LO fit
  - finish code for NLO
  - include exclusive meson production
- Improve lattice results. Results from QCDSF (and other groups) would give a realistic estimate for the systematic errors.
- improve ChPT for moments of GPDs (finite volume effects)