GPDs as a (cool) tool to study nucleon structure

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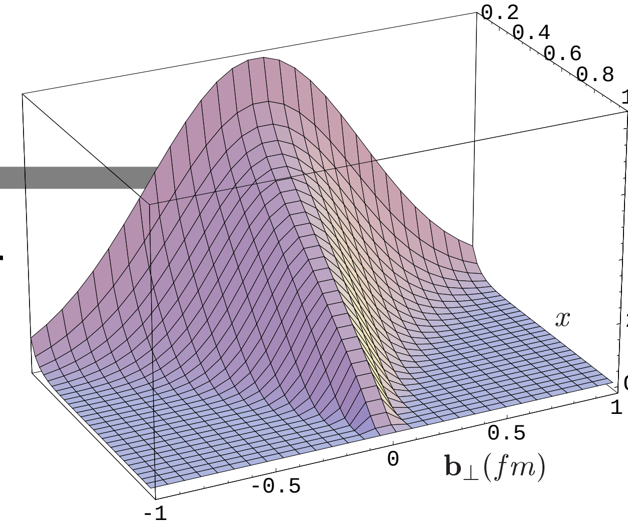
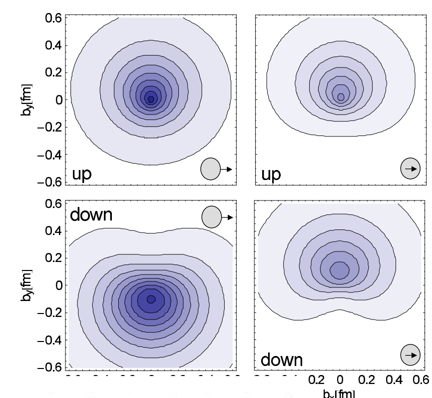
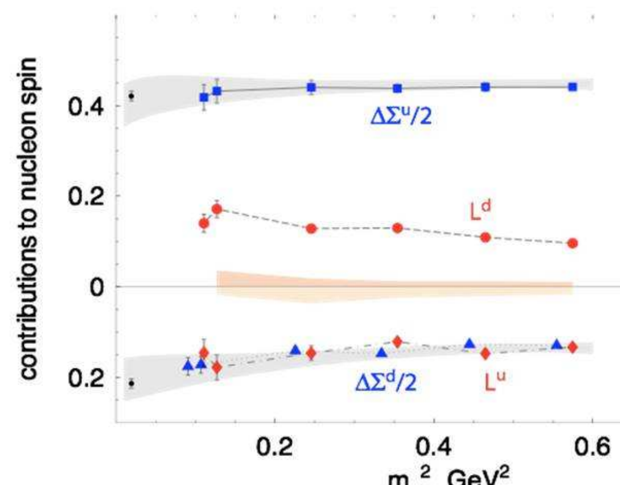
Outline

- Probabilistic interpretation of GPDs as Fourier trafo of impact parameter dependent PDFs
  - $H(x, 0, -\mathbf{\Delta}_\perp^2) \longrightarrow q(x, \mathbf{b}_\perp)$
  - $\tilde{H}(x, 0, -\mathbf{\Delta}_\perp^2) \longrightarrow \Delta q(x, \mathbf{b}_\perp)$
  - $E(x, 0, -\mathbf{\Delta}_\perp^2) \longrightarrow \perp$ distortion of PDFs when the target is $\perp$ polarized
- DVCS $\overset{?}{\rightsquigarrow}$ GPDs
- GPDs for $x = \xi$
- What is orbital angular momentum?
- Summary





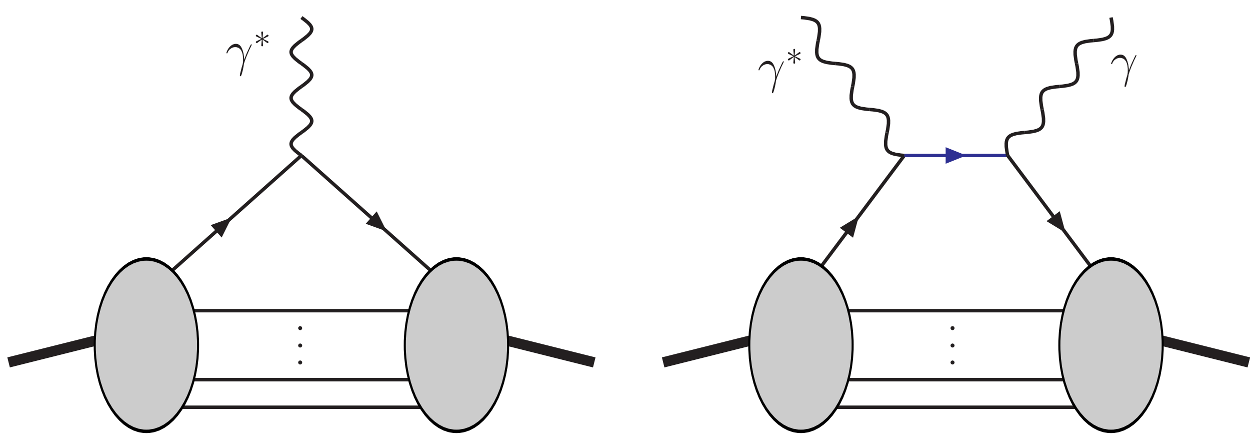


Deeply Virtual Compton Scattering (DVCS)

- virtual Compton scattering: $\gamma^* p \longrightarrow \gamma p$ (actually: $e^- p \longrightarrow e^- \gamma p$)
- 'deeply': $-q_\gamma^2 \gg M_p^2, |t| \longrightarrow$ Compton amplitude dominated by (coherent superposition of) Compton scattering off single quarks

$\hookrightarrow$ only difference between form factor (a) and DVCS amplitude (b) is replacement of photon vertex by two photon vertices connected by quark (energy denominator depends on quark momentum fraction x)

$\hookrightarrow$ DVCS amplitude provides access to momentum-decomposition of form factor = Generalized Parton Distribution (GPDs).



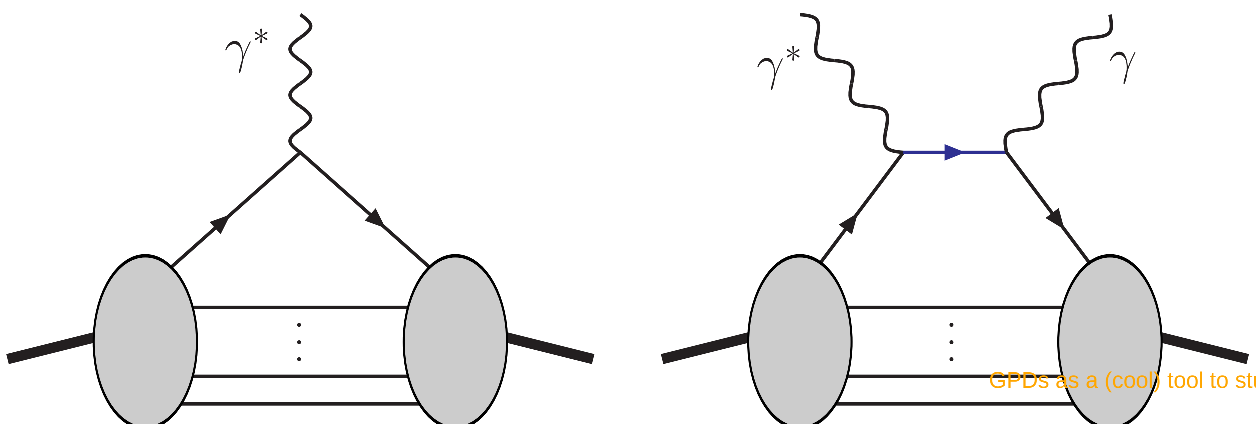
(a) (b)

Generalized Parton Distributions (GPDs)

- GPDs: **decomposition of form factors** at a given value of t, w.r.t. the average momentum fraction $x = \frac{1}{2}(x_i + x_f)$ of the active quark

$$\int dx H_q(x,\xi,t) = F_1^q(t) \qquad \int dx \tilde{H}_q(x,\xi,t) = G_A^q(t)$$
$$\int dx E_q(x,\xi,t) = F_2^q(t) \qquad \int dx \tilde{E}_q(x,\xi,t) = G_P^q(t),$$

  - x_i and x_f are the momentum fractions of the quark before and after the momentum transfer
  - $2\xi = x_f - x_i$
- GPDs can be probed in deeply virtual Compton scattering (DVCS)



Generalized Parton Distributions (GPDs)

- DVCS amplitude

$$\mathcal{A}_{DVCS}(\xi, t) \sim \int_{-1}^{1} \frac{dx}{x - \xi + i\varepsilon} GPD(x, \xi, t)$$

- in the limit of vanishing t and ξ, the nucleon non-helicity-flip GPDs must reduce to the ordinary PDFs:

$$H_q(x, 0, 0) = q(x) \qquad \tilde{H}_q(x, 0, 0) = \Delta q(x).$$

Impact parameter dependent PDFs

- define $\perp$ localized state [D.Soper,PRD**15**, 1141 (1977)]

$$\left|p^+, \mathbf{R}_\perp = \mathbf{0}_\perp, \lambda\right\rangle \equiv \mathcal{N} \int d^2\mathbf{p}_\perp \left|p^+, \mathbf{p}_\perp, \lambda\right\rangle$$

  Note: $\perp$ boosts in IMF form Galilean subgroup $\Rightarrow$ this state has
  $\mathbf{R}_\perp \equiv \frac{1}{P^+} \int dx^- d^2\mathbf{x}_\perp \, \mathbf{x}_\perp T^{++}(x) = \sum_i x_i \mathbf{r}_{i,\perp} = \mathbf{0}_\perp$
  (cf.: working in CM frame in nonrel. physics)

- define impact parameter dependent PDF

$$q(x, \mathbf{b}_\perp) \equiv \int \frac{dx^-}{4\pi} \left\langle p^+, \mathbf{R}_\perp = \mathbf{0}_\perp \right| \bar{q}(-\frac{x^-}{2}, \mathbf{b}_\perp) \gamma^+ q(\frac{x^-}{2}, \mathbf{b}_\perp) \left|p^+, \mathbf{R}_\perp = \mathbf{0}_\perp\right\rangle e^{ixp^+x^-}$$

$$\hookrightarrow \boxed{\begin{aligned} q(x, \mathbf{b}_\perp) &= \int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} e^{i\mathbf{\Delta}_\perp \cdot \mathbf{b}_\perp} H(x, 0, -\mathbf{\Delta}_\perp^2), \\ \Delta q(x, \mathbf{b}_\perp) &= \int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} e^{i\mathbf{\Delta}_\perp \cdot \mathbf{b}_\perp} \tilde{H}(x, 0, -\mathbf{\Delta}_\perp^2), \end{aligned}}$$

Impact parameter dependent PDFs

- No relativistic corrections (Galilean subgroup!)

$\hookrightarrow$ corrolary: interpretation of 2d-FT of $F_1(Q^2)$ as charge density in transverse plane also free from relativistic corrections

- $q(x, \mathbf{b}_\perp)$ has probabilistic interpretation as number density ($\Delta q(x, \mathbf{b}_\perp)$ as difference of number densities)
- $\xi = 0$ <u>essential</u> for probabilistic interpretation

$$\left\langle p^{+\prime}, 0_\perp \right| b^\dagger(x, \mathbf{b}_\perp) b(x, \mathbf{b}_\perp) \left| p^+, 0_\perp \right\rangle \sim \left| b(x, \mathbf{b}_\perp) \rangle |p^+, 0_\perp \right|^2$$

works only for $p^+ = p^{+\prime}$

- Reference point for IPDs is transverse center of (longitudinal) momentum $\mathbf{R}_\perp \equiv \sum_i x_i \mathbf{r}_{i,\perp}$

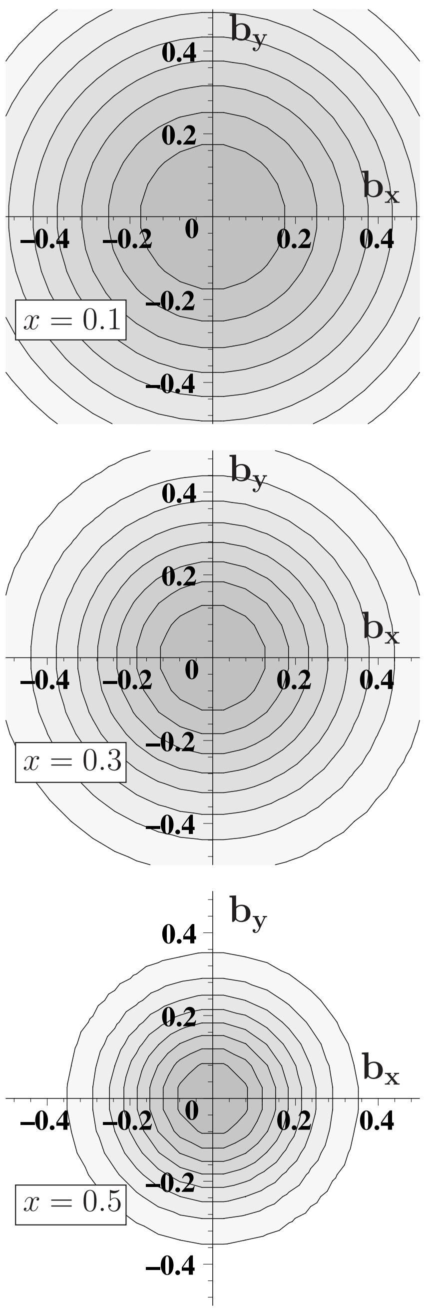
$\hookrightarrow$ for $x \to 1$, active quark 'becomes' COM, and $q(x, \mathbf{b}_\perp)$ must become very narrow (δ-function like)

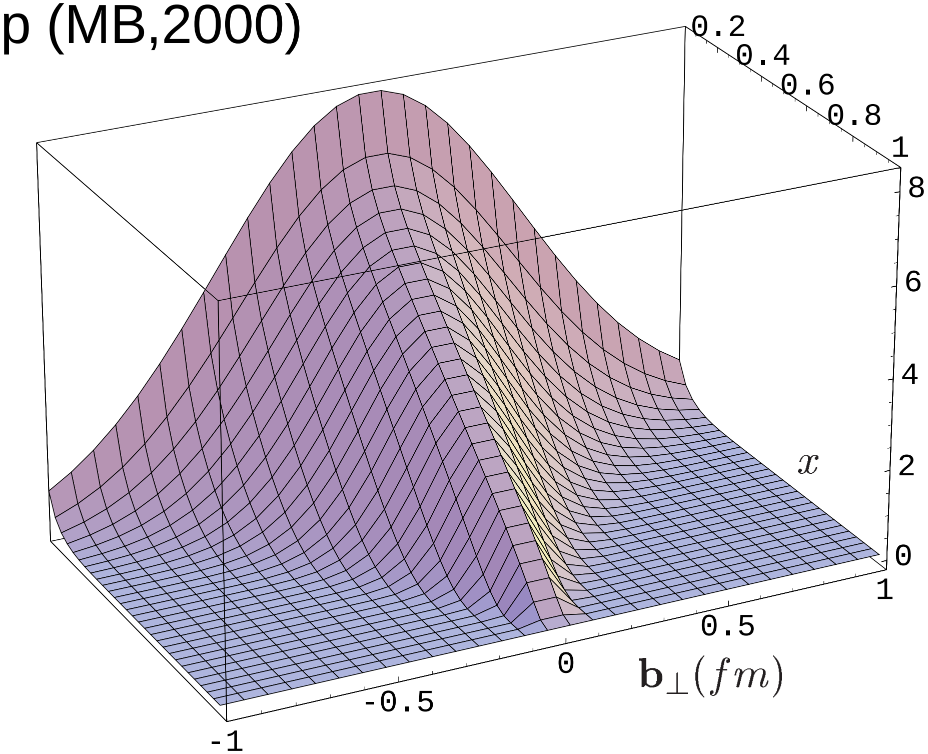
$\hookrightarrow$ $H(x, 0, -\mathbf{\Delta}_\perp^2)$ must become $\mathbf{\Delta}_\perp$ indep. as $x \to 1$ (MB, 2000)

$\hookrightarrow$ consistent with lattice results for first few moments

unpolarized p (MB,2000)

$q(x, \mathbf{b}_\perp)$ for unpol. p

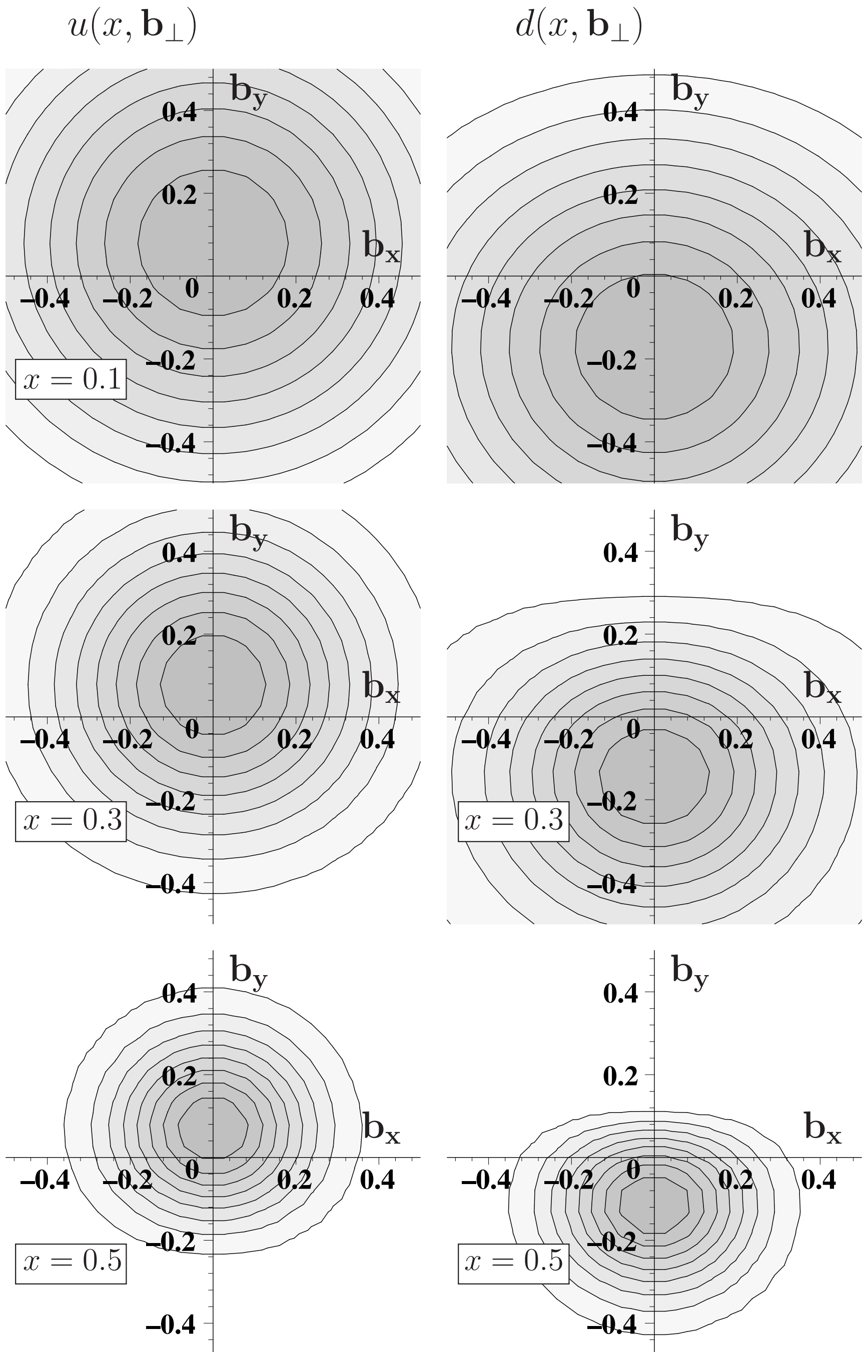


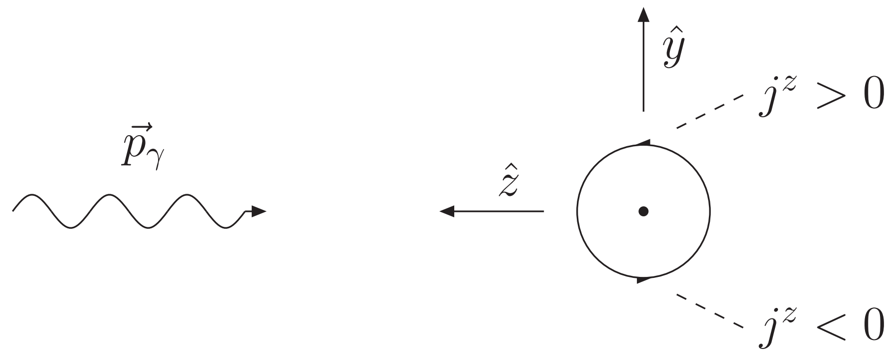


$x =$ momentum fraction of the quark

$\vec{b} = \perp$ position of the quark

p polarized in $+\hat{x}$ direction (MB,2003)



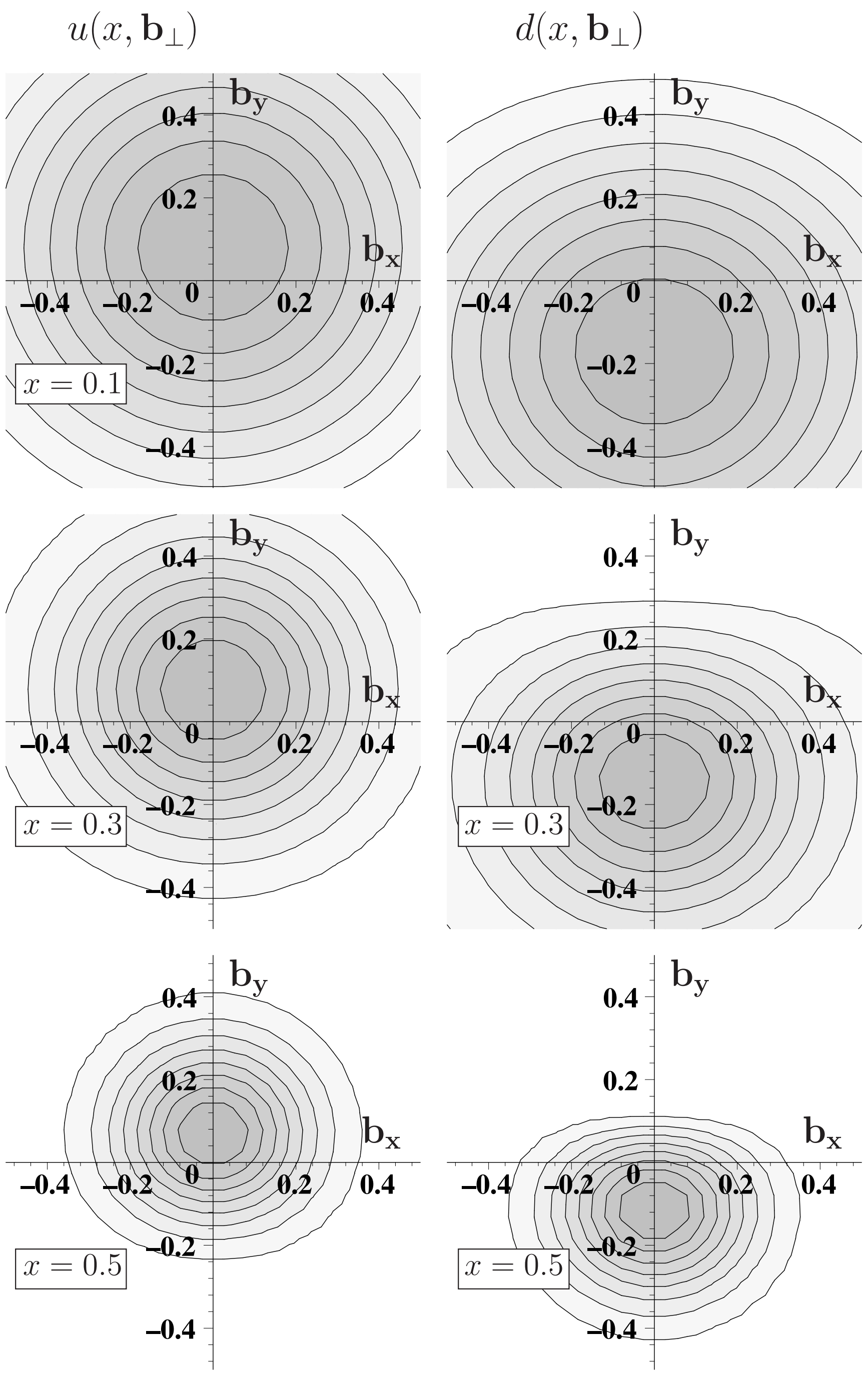


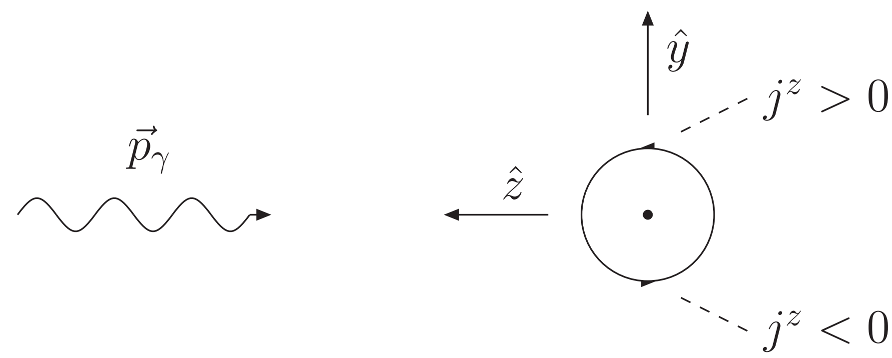
- photon interacts more strongly with quark currents that point in direction opposite to photon momentum

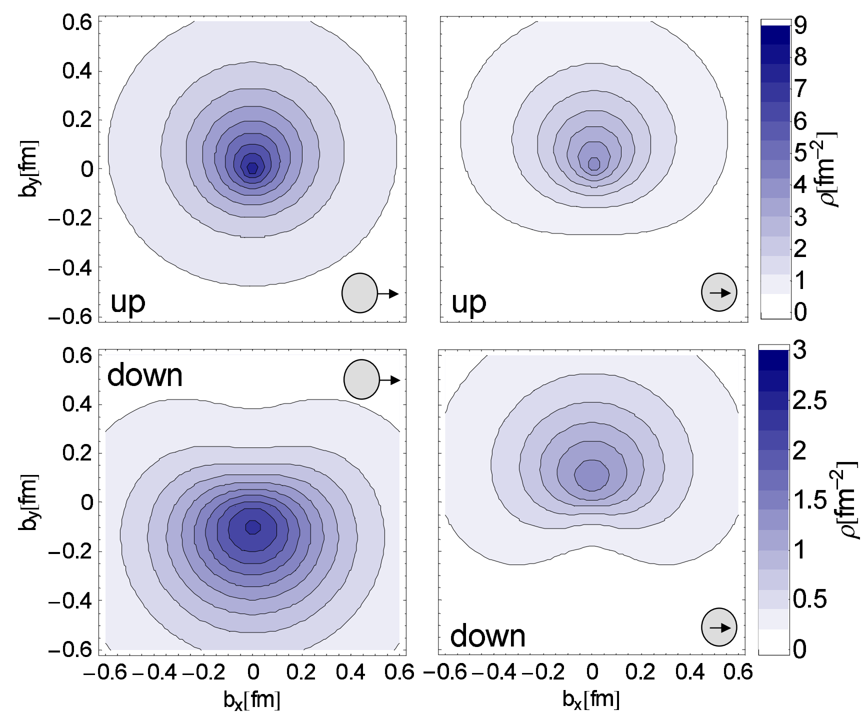
$\hookrightarrow$ sideways shift of quark distributions

- sign & magnitude of shift (model-independently) predicted to be related to the proton/neutron anomalous magnetic moment!

p polarized in $+\hat{x}$ direction







lattice results (QCDSF)

Accessing GPDs in DVCS

- $\mathcal{A}_{DVCS}(\xi,t) \longrightarrow \int_{-1}^{1} dx \frac{GPD^{(+)}(x,\xi,t)}{x-\xi+i\varepsilon}$
  - ξ longitudinal mometum transfer on the target $\xi = \frac{p^{+\prime}-p^{+}}{p^{+\prime}+p^{+}}$
  - x (average) momentum fraction of the active quark $x = \frac{k^{+\prime}+p^{+}}{p^{+\prime}+p^{+}}$
- $\Im\mathcal{A}_{DVCS}(\xi,t) \longrightarrow GPD^{(+)}(\xi,\xi,t)$
  - only sensitive to 'diagonal' $x=\xi$
  - limited ξ range

$$-t = \frac{4\xi^2 M^2 + \mathbf{\Delta}_\perp^2}{1-\xi^2}$$

$\hookrightarrow$ $-t_{min} = \frac{4\xi^2 M^2}{1-\xi^2}$ or ξ_{max} for given value of $-t$

- $\Re\mathcal{A}_{DVCS}(\xi,t) \longrightarrow \int_{-1}^{1} dx \frac{GPD^{(+)}(x,\xi,t)}{x-\xi}$ probes GPDs off the diagonal, but ...

Polynomiality & the D-term

- Lorentz invariance $\Rightarrow$ polynomiality (n odd)

$$\int_0^1 dx x^n GPD^{(+)}(x,\xi,t) = B_{n0}(t) + B_{n2}(t)\xi^2 + B_{n4}(t)\xi^4 + .. + B_{n,n+1}(t)\xi^{n+1}$$

Consider in the following only charge-even GPDs, e.g. $H^{(+)}(x,\xi,t) \equiv H(x,\xi,t) - H(-x,\xi,t)$ but drop superscript $^{(+)}$

$\hookrightarrow$ Polynomiality highly constrains possible functional form of GPDs and plays crucial role in 'deconvolution' of the DVCS amplitude

- original 'double distribution' representation for GPDs (Radyushkin) manifestly satisfied polynomiality, but without $B_{n,n+1}$-term
- 'D-term' (Polyakov & Weiss) added to allow for highest power of ξ

$$H(x,\xi,t) = H_{DD}(x,\xi,t) + \Theta(\xi^2 - x^2) D\left(\frac{x}{\xi}, t\right)$$

Polynomiality & the D-term

- 'D-term' (Polyakov & Weiss) added to allow for highest power of ξ

$$H(x,\xi,t) = H_{DD}(x,\xi,t) + \Theta(\xi^2 - x^2) D\left(\frac{x}{\xi}, t\right)$$

- D-term contributes only to real part of DVCS amplitude, with 'D-form factor' $\Delta(t) = \int_0^1 dz \frac{D(z,t)}{1-z}$

$$\Re\mathcal{A}(\xi,t) = \int_0^1 dx \frac{H_{DD}^+(x,\xi,t)}{x-\xi} + \Delta(t)$$

- For fixed x, contribution of D-term to $H(x,\xi,t)$ disappears as $\xi \to 0$, but $\delta(x)$-like contribution to Compton Amplitude

$$\lim_{\xi\to 0} \frac{H(x,\xi,t)}{x-\xi} = \frac{H_{DD}(x,0,t)}{x} + \delta(x)\Delta(t)$$

- More recently (Anikin & Teryaev): Δ arises as subtraction-constant in dispersion relation for DVCS amplitude

$\mathcal{A}(\xi,t) \longleftrightarrow GPD^{(+)}(\xi,\xi,t),\ \Delta(t)$

- (Anikin & Teryaev): Δ arises as subtraction-constant in dispersion relation for DVCS amplitude

$$\Re\mathcal{A}(\nu,t) = \frac{\nu^2}{\pi}\int_0^\infty \frac{d\nu'^2}{\nu'^2}\frac{\Im\mathcal{A}(\nu',t)}{\nu'^2-\nu^2} + \Delta(t)$$

- In combination with LO factorization ($\mathcal{A} = \int_{-1}^1 dx \frac{H(x,\xi,t)}{x-\xi+i\varepsilon}$)

$$\Re\mathcal{A}(\xi,t) = \int_{-1}^1 dx \frac{H(x,\xi,t)}{x-\xi} = \int_{-1}^1 dx \frac{H(x,x,t)}{x-\xi} + \Delta(t)$$

- earlier derived from polynomiality (Goeke,Polyakov,Vanderhaeghen)

$\hookrightarrow$ Possible to ‘condense’ information contained in $\mathcal{A}_{DVCS}$ (fixed Q^2, assuming leading twist factorization) into $GPD(x,x,t)$ & $\Delta(t)$

$$\mathcal{A}(\xi,t) \leftrightarrow \begin{cases} GPD(\xi,\xi,t) \\ \Delta(t) \end{cases}$$

$\mathcal{A}(\xi,t) \longleftrightarrow GPD(\xi,\xi,t),\ \Delta(t)$

- $\Re\mathcal{A}(\xi,t) = \int_{-1}^{1} dx \frac{H(x,\xi,t)}{x-\xi}$ probes GPDs for $x \neq \xi$, but new information
  - using polynomiality/dispersion relation, DVCS information on GPDs (fixed Q^2) can be 'projected back' onto diagonal plus D-term!

  $\hookrightarrow$ better to fit parameterizations for $GPD(x,x,t)$ plus $\Delta(t)$ to $\mathcal{A}_{DVCS}$ rather than parameterizations for $GPD(x,\xi,t)$?
- even after 'projecting back' onto $GPD(x,x,t)$, $\Re\mathcal{A}(\xi,t)$ still provides new (not in $\Im\mathcal{A}$) info on GPDs:
  - D-form factor
  - constraints from $\int dx \frac{GPD(x,x,t)}{x-\xi}$ on $GPD(\xi,\xi,t)$ in kinematically inaccessible range $-t \leq -t_0 \equiv \frac{4M^2\xi^2}{1-\xi^2}$
- good news for model builders: as long as a model fits $\Im\mathcal{A}(\xi,t)$, it should also do well for $\Re\mathcal{A}(\xi,t)$, provided
  - model has polynomility
  - allows for a D-form factor

$\mathcal{A}(\xi,t) \longleftrightarrow GPD(\xi,\xi,t),\ \Delta(t)$

- trivial solution:

$$H_D D(x,\xi,t) \equiv H(x,x,t)$$

plus suitable $\Delta(t)$ will
  - fit DVCS data
  - satisfy polynomiality (trivially!)

Application of $\int_{-1}^1 dx \frac{H(x,\xi,t)}{x-\xi} = \int_{-1}^1 dx \frac{H(x,x,t)}{x-\xi} + \Delta(t)$

- take $\xi \to 0$ (should exist for $-t$ sufficiently large)

$$\int_{-1}^1 dx \frac{H^{(+)}(x,0,t)}{x} = \int_{-1}^1 dx \frac{H^{(+)}(x,x,t)}{x} + \Delta(t)$$

$\hookrightarrow$ DVCS allows access to same generalized form factor $\int_{-1}^1 dx \frac{H^{(+)}(x,0,t)}{x}$ also available in WACS (wide angle Compton scattering), but t does not have to be of order Q^2

$\hookrightarrow$ after flavor separation, $\frac{1}{F_1(t)} \int_{-1}^1 dx \frac{H^{(+)}(x,0,t)}{x}$ at large t provides information about the 'typical x' that dominates large t form factor

GPDs for $x = \xi$

examples for interesting physics that can be extracted from GPDs:

- impact parameter dependent PDFs
  $q(x, \mathbf{b}_\perp) = \int \frac{d^2\boldsymbol{\Delta}_\perp}{(2\pi)^2} e^{i\boldsymbol{\Delta}_\perp \cdot \mathbf{b}_\perp} H(x, 0, -\boldsymbol{\Delta}_\perp^2)$

$\hookrightarrow$ ξ needs to be zero

- Ji: $\langle \vec{J}^q \rangle_{\vec{S}} = \vec{S} \int dx\, x \left[H(x, \xi, 0) + E(x, \xi, 0) \right]$

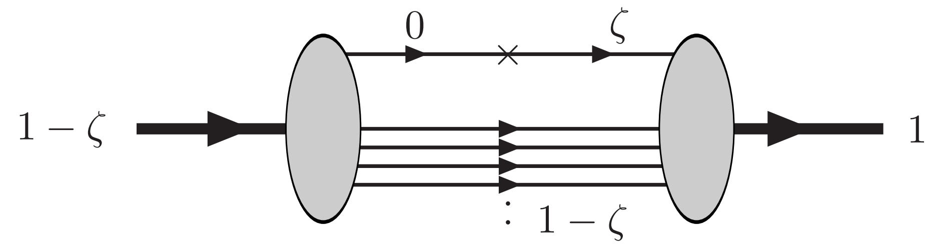
$\hookrightarrow$ ξ can be arbitrary but fixed value

DVCS experiments provide information about:

- $GPDs(\xi, \xi, t)$ directly from imaginary part of DVCS amplitude
- $\int \frac{dx}{x \pm \xi} GPDs(x, \xi, t)$ from real part, which is probably dominated by vicinity $x \approx \xi$
- additional constraints from PDFs, form factors, positivity, polynomiality, evolution, ...

- until GPDs have been globally gegenbauered to the point where $x - \xi$ dependence has been disentangled, what can we learn from $GPDs(\xi, \xi, t)$?

Overlap Representation for GPDs $(x > \zeta)$



$$GPD(x,\zeta,t) = \sum_{n,\lambda_i}(1-\zeta)^{1-\frac{n}{2}}\int\prod_{i=1}^{n}\frac{\mathrm{d}x_i\mathrm{d}\mathbf{k}_{\perp,i}}{16\pi^3}16\pi^3\delta\left(1-\sum_{j=1}^{n}x_j\right)\delta\left(\sum_{j=1}^{n}\mathbf{k}_{\perp j}\right)\delta(x-x_1)$$

$$\times\psi^{s'}_{(n)}(x'_i,\mathbf{k}'_{\perp i},\lambda_i)^*\psi^{s}_{(n)}(x_i,\mathbf{k}_{\perp i},\lambda_i),$$

- $GPD(x,\zeta,t) = \frac{\sqrt{1-\zeta}}{1-\frac{\zeta}{2}}H(x,\zeta,t) - \frac{\zeta^2}{4\left(1-\frac{\zeta}{2}\right)\sqrt{1-\zeta}}E(x,\zeta,t)$, for $s' = s$
- $GPD(x,\zeta,t) = \frac{1}{\sqrt{1-\zeta}}\frac{\Delta^1 - i\Delta^2}{2M}E(x,\zeta,t)$, for $s' =\uparrow$ and $s =\downarrow$
- $\mathbf{\Delta}$ is the transverse momentum transfer.
- $x'_1 = \frac{x_1-\zeta}{1-\zeta}$ and $\mathbf{k}'_{\perp 1} = \mathbf{k}_{\perp 1} - \frac{1-x_1}{1-\zeta}\mathbf{\Delta}_\perp$ for the active quark, and
- $x'_i = \frac{x_i}{1-\zeta}$ and $\mathbf{k}'_{\perp i} = \mathbf{k}_{\perp i} + \frac{x_i}{1-\zeta}\mathbf{\Delta}_\perp$ for the spectators $i = 2, ..., n$.

GPDs in $\perp$ position space $(n = 2)$

$$GPD(x,\zeta,t) = \sum_{\lambda_i} \int \frac{\mathrm{d}\mathbf{k}_{\perp,1}}{16\pi^3} \psi^{s'}(x_1', \mathbf{k}_{\perp 1}', \lambda_i)^* \psi^s(x_1, \mathbf{k}_{\perp 1}, \lambda_i),$$

- $x_1' = \frac{x_1 - \zeta}{1-\zeta}$ and $\mathbf{k}_{\perp 1}' = \mathbf{k}_{\perp 1} - \frac{1-x_1}{1-\zeta}\mathbf{\Delta}_\perp$ for the active quark
- spectator momentum constrained by momentum conservation: $x_2 = 1 - x_1$ and $\mathbf{k}_{\perp 2} = -\mathbf{k}_{\perp 1}$

Diagonalize by Fourier transform

- $\tilde{\psi}^s(x, \mathbf{r}_\perp) = \int \frac{d^2\mathbf{k}_\perp}{2\pi} \psi^s(x, \mathbf{k}_\perp) e^{i\mathbf{k}_\perp \cdot \mathbf{r}_\perp}$
- $\mathbf{r}_\perp$ is the $\perp$ distance between active quark and spectator

$\hookrightarrow$ $GPD(x,\zeta,t) \propto \int d^2\mathbf{r}_\perp \tilde{\psi}^*(x', \mathbf{r}_\perp)\tilde{\psi}^*(x', \mathbf{r}_\perp) e^{-i\frac{1-x}{1-\zeta}\mathbf{r}_\perp \cdot \mathbf{\Delta}_\perp}$

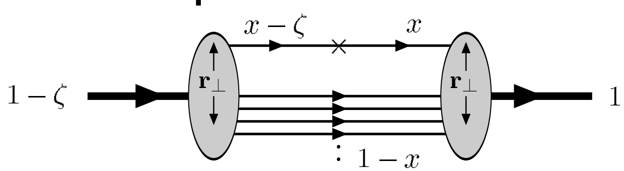
GPDs in $\perp$ position space (general case)

- repeating the same steps in the general case ($n \geq 3$) yields.......

$$GPD(x,\zeta,t) = \sum_n (1-\zeta)^{1-\frac{n}{2}} \int \prod_{i=1}^{n} \frac{d^2\mathbf{r}_{\perp i}}{2\pi} \tilde{\psi}_{(n)}(x_i', \mathbf{r}_{\perp i})^* \tilde{\psi}^s_{(n)}(x_i, \mathbf{r}_{\perp i}) e^{-i\frac{1-x}{1-\zeta}(\mathbf{r}_{\perp 1} - \mathbf{R}_{\perp s})\cdot\mathbf{\Delta}_\perp}$$

- $\mathbf{R}_{\perp s}$ is the center of momentum of the spectators.

$\hookrightarrow$ FT of GPD w.r.t. $\mathbf{\Delta}_\perp$ gives overlap when active quark and spectators are distance $\frac{1-x}{1-\zeta}\mathbf{r}_\perp$ apart



GPDs in $\perp$ position space (general case)

- general case: $\Delta_\perp$ conjugate to $\frac{1-x}{1-\zeta}\mathbf{r}_\perp$
- special case: $\zeta = 0 \quad \Rightarrow \quad \frac{1-x}{1-\zeta}\mathbf{r}_\perp = (1-x)\mathbf{r}_\perp = \mathbf{b}_\perp =$ distance between active quark and center of momentum of hadron.
- special case: $x = \zeta \quad \Rightarrow \quad \frac{1-x}{1-\zeta}\mathbf{r}_\perp = \mathbf{r}_\perp$

$\hookrightarrow$ for $x = \zeta$, the variable that is (Fourier) conjugate to $\mathbf{\Delta}_\perp$ is the distance between the active quark and the center of momentum of the spectators $\mathbf{r}_\perp$

- unlike the $\mathbf{b}_\perp$ distribution, which must become point-like for $x \to 1$, the $\mathbf{r}_\perp$-distribution does **not** have to become narrow for $x \to 1$
- Note: the t-slope still has to go to zero as $\zeta \to 1$, as

$$-t = \frac{\zeta^2 M^2 + \mathbf{\Delta}_\perp{}^2}{1-\zeta}$$

$\hookrightarrow$ t-slope B and $\mathbf{\Delta}_\perp^2$-slope $B_\perp$ related via $B = (1-\zeta)B_\perp$

Motivation

- polarized DIS: only $\sim 30\%$ of the proton spin due to quark spins

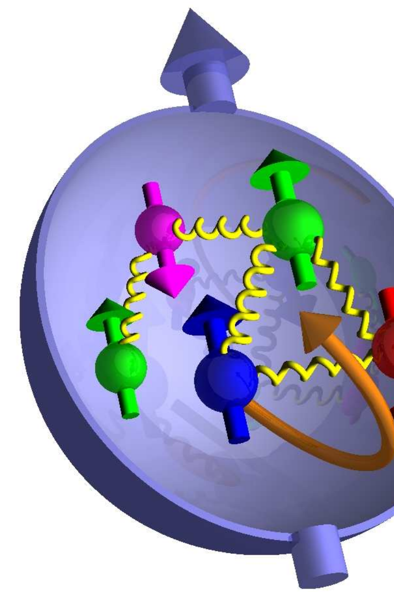
$\hookrightarrow$ 'spin crisis' $\longrightarrow$ 'spin puzzle', because $\Delta\Sigma$ much smaller than the quark model result $\Delta\Sigma = 1$

$\hookrightarrow$ quest for the remaining 70%

  - quark orbital angular momentum (OAM)
  - gluon spin
  - gluon OAM

$\hookrightarrow$ How are the above quantities defined?

$\hookrightarrow$ How can the above quantities be measured



example: angular momentum in QED

- consider, for simplicity, first QED without electrons:

$$\vec{J} = \int d^3r\, \vec{x} \times \left(\vec{E} \times \vec{B}\right) = \int d^3r\, \vec{x} \times \left[\vec{E} \times \left(\vec{\nabla} \times \vec{A}\right)\right]$$

- use $\vec{E} \times \left(\vec{\nabla} \times \vec{A}\right) = E^j \vec{\nabla} A^j - \left(\vec{E} \cdot \vec{\nabla}\right) \vec{A}$ and integrate 2^{nd} term by parts

$$\hookrightarrow \vec{J} = \int d^3r \left[E^j \left(\vec{x} \times \vec{\nabla}\right) A^j + \left(\vec{x} \times \vec{A}\right) \vec{\nabla} \cdot \vec{E} + \vec{E} \times \vec{A}\right]$$

- drop 2^{nd} term (eq. of motion $\vec{\nabla} \cdot \vec{E} = 0$), yielding $\vec{J} = \vec{L} + \vec{S}$ with

$$\vec{L} = \int d^3r\, E^j \left(\vec{x} \times \vec{\nabla}\right) A^j \qquad \vec{S} = \int d^3r\, \vec{E} \times \vec{A}$$

- note: $\vec{L}$ and $\vec{S}$ not separately gauge invariant

example: angular momentum in QED with electrons

$$
\begin{aligned}
\vec{J}_\gamma &= \int d^3r\, \vec{r} \times \left(\vec{E} \times \vec{B}\right) = \int d^3r\, \vec{x} \times \left[\vec{E} \times \left(\vec{\nabla} \times \vec{A}\right)\right] \\
&= \int d^3r \left[E^j \left(\vec{r} \times \vec{\nabla}\right) A^j - \vec{r} \times (\vec{E} \cdot \vec{\nabla})\vec{A}\right] \\
&= \int d^3r \left[E^j \left(\vec{r} \times \vec{\nabla}\right) A^j + \left(\vec{r} \times \vec{A}\right) \vec{\nabla} \cdot \vec{E} + \vec{E} \times \vec{A}\right]
\end{aligned}
$$

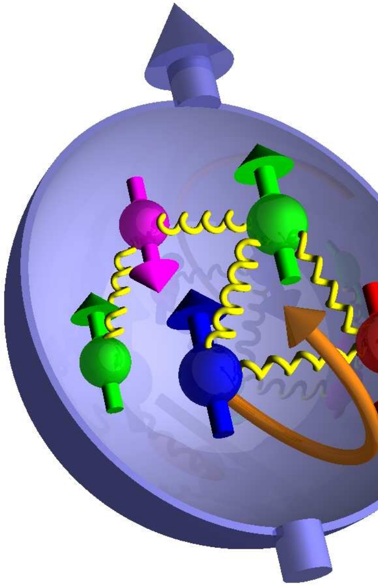
- replace 2^{nd} term (eq. of motion $\vec{\nabla} \cdot \vec{E} = ej^0 = e\psi^\dagger\psi$), yielding

$$
\vec{J}_\gamma = \int d^3r \left[\psi^\dagger \vec{r} \times e\vec{A}\psi + E^j \left(\vec{x} \times \vec{\nabla}\right) A^j + \vec{E} \times \vec{A}\right]
$$

- $\psi^\dagger \vec{r} \times e\vec{A}\psi$ cancels similar term in electron OAM $\psi^\dagger \vec{r} \times (\vec{p} - e\vec{A})\psi$

$\hookrightarrow$ decomposing $\vec{J}_\gamma$ into spin and orbital also shuffles angular momentum from photons to electrons!

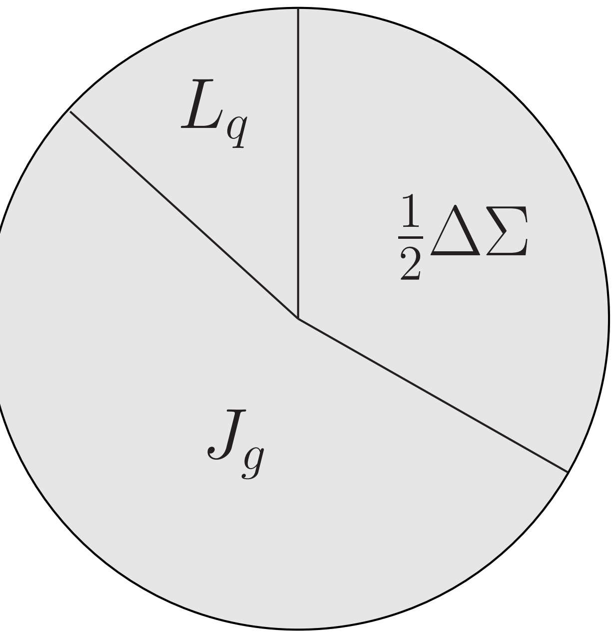
Outline



- Ji decomposition
- Jaffe decomposition
- recent lattice results (Ji decomposition)
- model/QED illustrations for Ji v. Jaffe
- Chen-Goldman decomposition

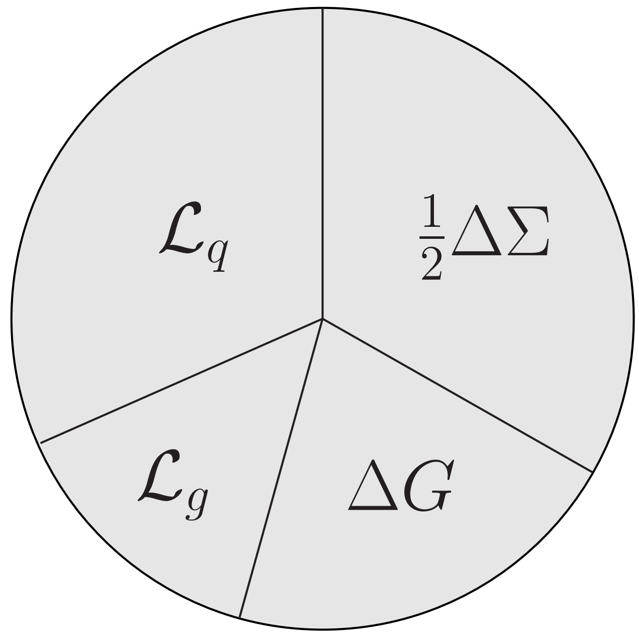
The nucleon spin pizza(s)

Ji



'pizza tre stagioni'

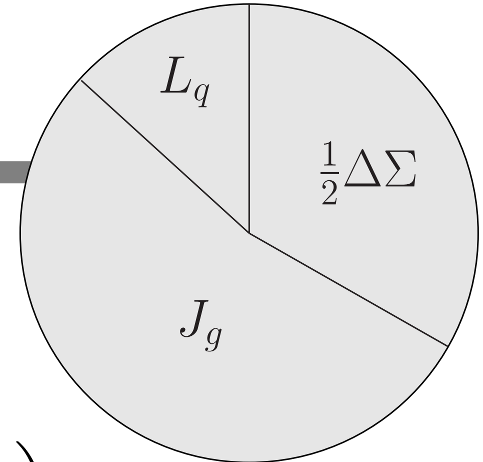
Jaffe & Manohar



'pizza quattro stagioni'

- only $\frac{1}{2}\Delta\Sigma \equiv \frac{1}{2}\sum_q \Delta q$ common to both decompositions!

Ji-decomposition



- Ji (1997)

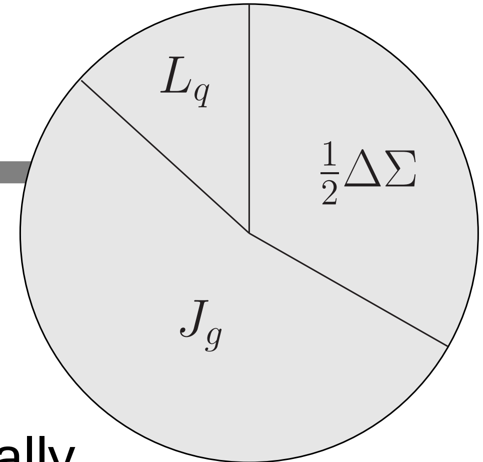
$$\frac{1}{2} = \sum_q J_q + J_g = \sum_q \left(\frac{1}{2}\Delta q + L_q \right) + J_g$$

with ($P^\mu = (M, 0, 0, 1)$, $S^\mu = (0, 0, 0, 1)$)

$$\begin{aligned}
\frac{1}{2}\Delta q &= \frac{1}{2}\int d^3x \, \langle P, S| \, q^\dagger(\vec{x})\Sigma^3 q(\vec{x}) \, |P, S\rangle \qquad \Sigma^3 = i\gamma^1\gamma^2 \\
L_q &= \int d^3x \, \langle P, S| \, q^\dagger(\vec{x}) \left(\vec{x} \times i\vec{D}\right)^3 q(\vec{x}) \, |P, S\rangle \\
J_g &= \int d^3x \, \langle P, S| \left[\vec{x} \times \left(\vec{E} \times \vec{B}\right)\right]^3 |P, S\rangle
\end{aligned}$$

- $i\vec{D} = i\vec{\partial} - g\vec{A}$

Ji-decomposition



- $\vec{J} = \sum_q \frac{1}{2} q^\dagger \vec{\Sigma} q + q^\dagger \left(\vec{r} \times i\vec{D} \right) q + \vec{r} \times \left(\vec{E} \times \vec{B} \right)$
  applies to each vector component of nucleon angular momentum, but Ji-decomposition usually applied only to $\hat{z}$ component where at least quark spin has parton interpretation as difference between number densities
- Δq from polarized DIS
- $J_q = \frac{1}{2}\Delta q + L_q$ from exp/lattice (GPDs)
- L_q in principle independently defined as matrix elements of $q^\dagger \left(\vec{r} \times i\vec{D} \right) q$, but in practice easier by subtraction $L_q = J_q - \frac{1}{2}\Delta q$
- J_g in principle accessible through gluon GPDs, but in practice easier by subtraction $J_g = \frac{1}{2} - J_q$
- Ji makes no further decomposition of J_g into intrinsic (spin) and extrinsic (OAM) piece

L_q for proton from Ji-relation (lattice)

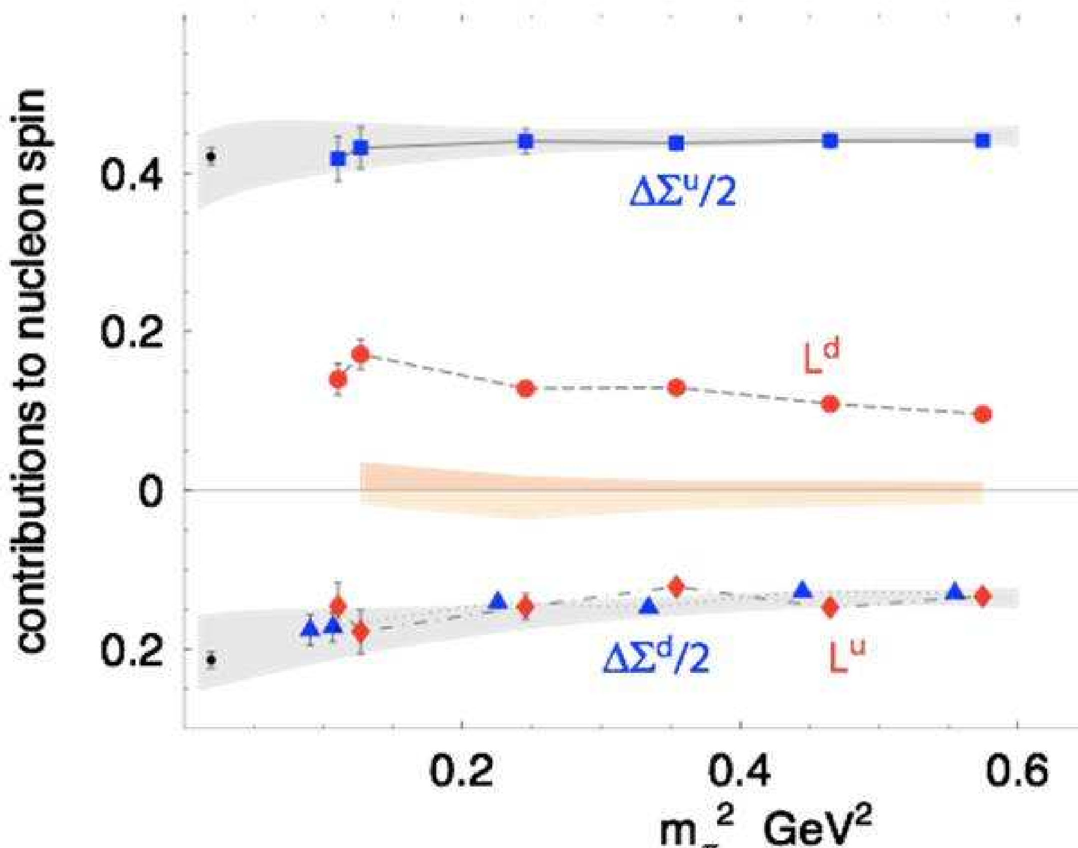
- lattice QCD $\Rightarrow$ moments of GPDs (LHPC; QCDSF)

$\hookrightarrow$ insert in Ji-relation

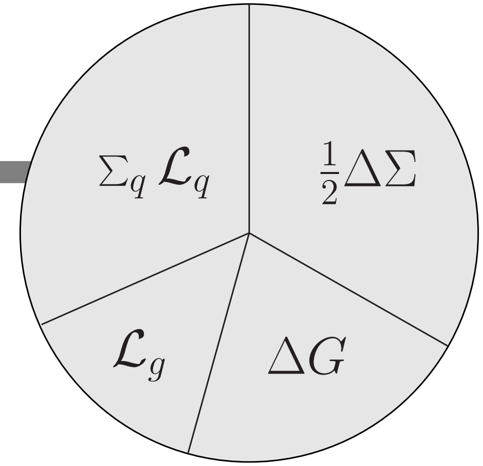
$$\langle J_q^i \rangle = S^i \int dx \left[H_q(x,0) + E_q(x,0) \right] x .$$

$\hookrightarrow$ $L_q^z = J_q^z - \frac{1}{2}\Delta q$

- L_u, L_d both large!
- present calcs. show $L_u + L_d \approx 0$, but
  - disconnected diagrams ..?
  - m_π^2 extrapolation
  - parton interpret. of L_q...



Jaffe/Manohar decomposition



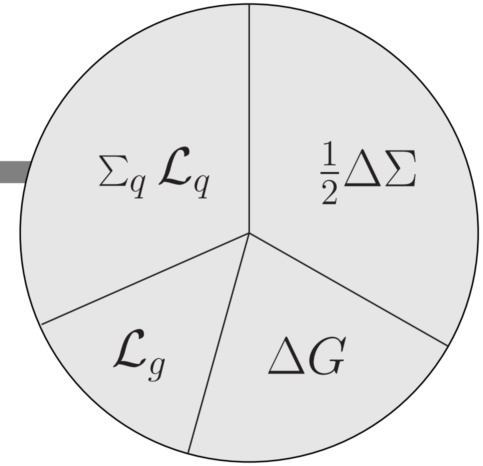
- in light-cone framework & light-cone gauge $A^+ = 0$ one finds for $J^z = \int dx^- d^2\mathbf{r}_\perp M^{+xy}$

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \sum_q \mathcal{L}_q + \Delta G + \mathcal{L}_g$$

where ($\gamma^+ = \gamma^0 + \gamma^z$)

$$\begin{aligned}
\mathcal{L}_q &= \int d^3r \, \langle P,S| \, \bar{q}(\vec{r})\gamma^+ \left(\vec{r} \times i\vec{\partial}\right)^z q(\vec{r}) \, |P,S\rangle \\
\Delta G &= \varepsilon^{+-ij} \int d^3r \, \langle P,S| \, \mathrm{Tr} F^{+i} A^j \, |P,S\rangle \\
\mathcal{L}_g &= 2 \int d^3r \, \langle P,S| \, \mathrm{Tr} F^{+j} \left(\vec{x} \times i\vec{\partial}\right)^z A^j \, |P,S\rangle
\end{aligned}$$

Jaffe/Manohar decomposition



$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \sum_q \mathcal{L}_q + \Delta G + \mathcal{L}_g$$

- $\Delta\Sigma = \sum_q \Delta q$ from polarized DIS (or lattice)
- ΔG from $\vec{p}\overleftarrow{p}$ or polarized DIS (evolution)

$\hookrightarrow$ ΔG gauge invariant, but local operator only in light-cone gauge

- $\int dx x^n \Delta G(x)$ for $n \geq 1$ *can* be described by manifestly gauge inv. local op. ($\longrightarrow$ lattice)
- $\mathcal{L}_q$, $\mathcal{L}_g$ independently defined, but
  - no exp. identified to access them
  - not accessible on lattice, since nonlocal except when $A^+ = 0$
- parton net OAM $\mathcal{L} = \mathcal{L}_g + \sum_q \mathcal{L}_q$ by subtr. $\mathcal{L} = \frac{1}{2} - \frac{1}{2}\Delta\Sigma - \Delta G$
- in general, $\mathcal{L}_q \neq L_q$ $\qquad$ $\mathcal{L}_g + \Delta G \neq J_g$
- makes no sense to 'mix' Ji and JM decompositions, e.g. $J_g - \Delta G$ has no fundamental connection to OAM

$L_q \neq \mathcal{L}_q$

- L_q matrix element of

$$q^\dagger \left[\vec{r} \times \left(i\vec{\partial} - g\vec{A}\right)\right]^z q = \bar{q}\gamma^0 \left[\vec{r} \times \left(i\vec{\partial} - g\vec{A}\right)\right]^z q$$

- $\mathcal{L}_q^z$ matrix element of $(\gamma^+ = \gamma^0 + \gamma^z)$

$$\bar{q}\gamma^+ \left[\vec{r} \times i\vec{\partial}\right]^z q\Big|_{A^+=0}$$

- (for $\vec{p} = 0$) matrix element of $\bar{q}\gamma^z \left[\vec{r} \times \left(i\vec{\partial} - g\vec{A}\right)\right]^z q$ vanishes (parity!)

$\hookrightarrow$ L_q identical to matrix element of $\bar{q}\gamma^+ \left[\vec{r} \times \left(i\vec{\partial} - g\vec{A}\right)\right]^z q$ (nucleon at rest)

$\hookrightarrow$ even in light-cone gauge, L_q^z and $\mathcal{L}_q^z$ still differ by matrix element of $q^\dagger \left(\vec{r} \times g\vec{A}\right)^z q\Big|_{A^+=0} = q^\dagger \left(xgA^y - ygA^x\right) q\Big|_{A^+=0}$

Summary part 1:

- Ji: $J^z = \frac{1}{2}\Delta\Sigma + \sum_q L_q + J_g$
- Jaffe: $J^z = \frac{1}{2}\Delta\Sigma + \sum_q \mathcal{L}_q + \Delta G + \mathcal{L}_g$
- ΔG can be defined without reference to gauge (and hence gauge invariantly) as the quantity that enters the evolution equations and/or $\vec{p}\overleftarrow{p}$

$\hookrightarrow$ represented by simple (i.e. local) operator only in LC gauge and corresponds to the operator that one would naturally identify with 'spin' only in that gauge

- in general $L_q \neq \mathcal{L}_q$ or $J_g \neq \Delta G + \mathcal{L}_g$, but
- how significant is the difference between L_q and $\mathcal{L}_q$, etc. ?

OAM in scalar diquark model

[M.Burkardt + H.BC, PRD **79**, 071501 (2009)]

- toy model for nucleon where nucleon (mass M) splits into quark (mass m) and scalar 'diquark' (mass λ)

$\hookrightarrow$ light-cone wave function for quark-diquark Fock component

$$\psi^{\uparrow}_{+\frac{1}{2}}(x,\mathbf{k}_\perp) = \left(M+\frac{m}{x}\right)\phi \qquad \psi^{\uparrow}_{-\frac{1}{2}} = -\frac{k^1+ik^2}{x}\phi$$

with $\phi = \frac{c/\sqrt{1-x}}{M^2-\frac{\mathbf{k}_\perp^2+m^2}{x}-\frac{\mathbf{k}_\perp^2+\lambda^2}{1-x}}$.

- quark OAM according to JM: $\mathcal{L}_q = \int_0^1 dx \int \frac{d^2\mathbf{k}_\perp}{16\pi^3}(1-x)\left|\psi^{\uparrow}_{-\frac{1}{2}}\right|^2$
- quark OAM according to Ji: $L_q = \frac{1}{2}\int_0^1 dx\, x\left[q(x)+E(x,0,0)\right]-\frac{1}{2}\Delta q$

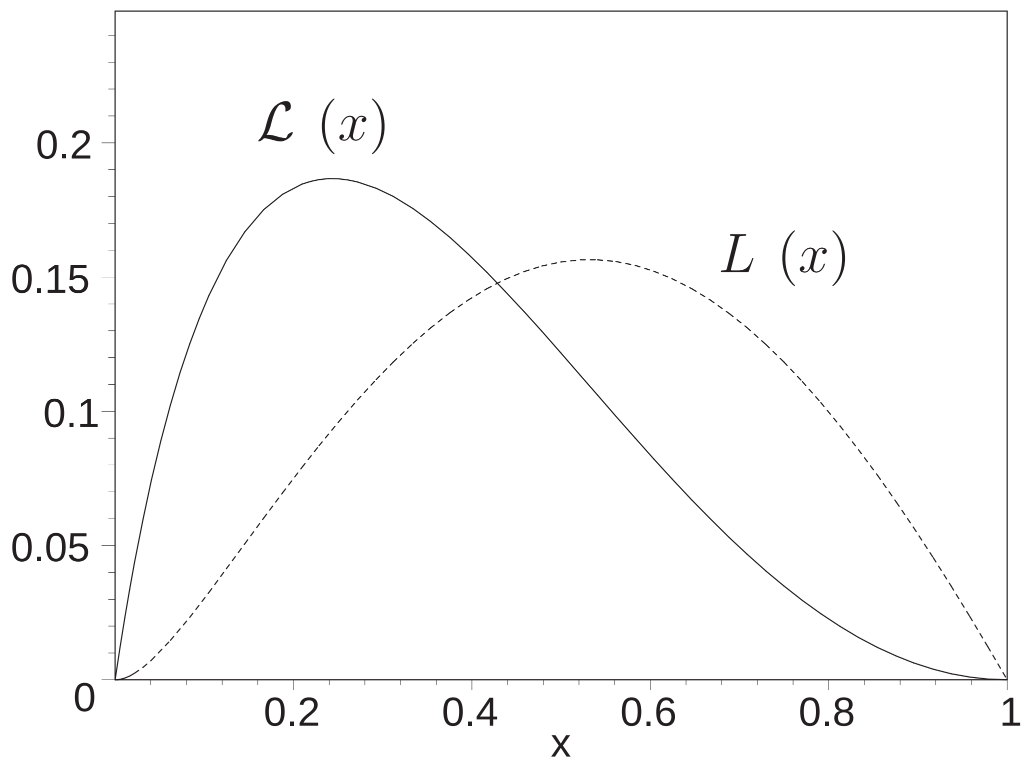
$\rightsquigarrow$ (using Lorentz inv. regularization, such as Pauli Villars subtraction) both give identical result, i.e. $L_q = \mathcal{L}_q$

- not surprising since scalar diquark model is <u>not</u> a gauge theory

OAM in scalar diquark model

- But, even though $L_q = \mathcal{L}_q$ in this non-gauge theory

$$\mathcal{L}_q(x) \equiv \int \frac{d^2\mathbf{k}_\perp}{16\pi^3}(1-x)\left|\psi^{\uparrow}_{-\frac{1}{2}}\right|^2 \neq \frac{1}{2}\left\{x\left[q(x)+E(x,0,0)\right]-\Delta q(x)\right\} \equiv L_q(x)$$



$\hookrightarrow$ ‘unintegrated Ji-relation’ does <u>not</u> yield x-distribution of OAM

OAM in QED

- light-cone wave function in $e\gamma$ Fock component

$$\Psi^{\uparrow}_{+\frac{1}{2}+1}(x,\mathbf{k}_\perp) = \sqrt{2}\frac{k^1 - ik^2}{x(1-x)}\phi \qquad \Psi^{\uparrow}_{+\frac{1}{2}-1}(x,\mathbf{k}_\perp) = -\sqrt{2}\frac{k^1 + ik^2}{1-x}$$

$$\Psi^{\uparrow}_{-\frac{1}{2}+1}(x,\mathbf{k}_\perp) = \sqrt{2}\left(\frac{m}{x} - m\right)\phi \qquad \Psi^{\uparrow}_{-\frac{1}{2}-1}(x,\mathbf{k}_\perp) = 0$$

- OAM of e^- according to Jaffe/Manohar
$$\mathcal{L}_e = \int_0^1 dx \int d^2\mathbf{k}_\perp (1-x)\left[\left|\Psi^{\uparrow}_{+\frac{1}{2}-1}(x,\mathbf{k}_\perp)\right|^2 - \left|\Psi^{\uparrow}_{+\frac{1}{2}+1}(x,\mathbf{k}_\perp)\right|^2\right]$$
- e^- OAM according to Ji $L_e = \frac{1}{2}\int_0^1 dx\, x\,[q(x) + E(x,0,0)] - \frac{1}{2}\Delta q$

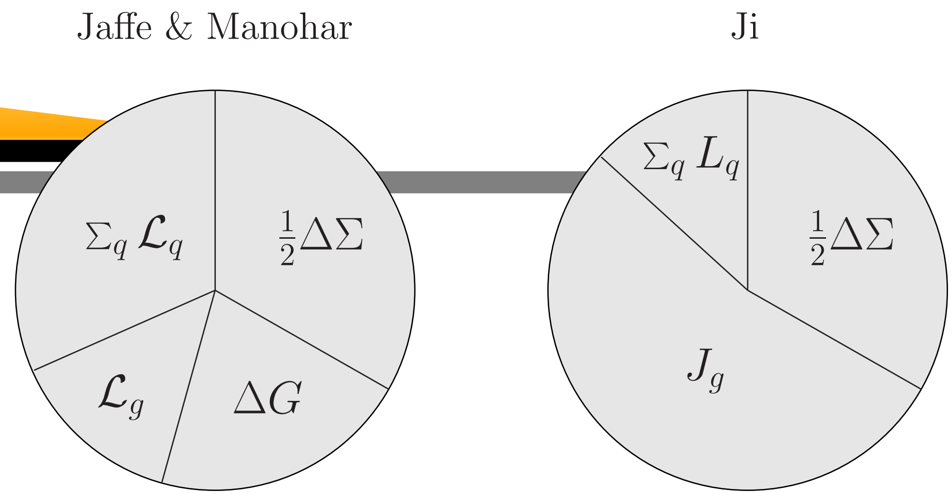
$\rightsquigarrow$ $\mathcal{L}_e = L_e + \frac{\alpha}{4\pi} \neq L_e$

- Likewise, computing J_γ from photon GPD, and $\Delta\gamma$ and $\mathcal{L}_\gamma$ from light-cone wave functions and <u>defining</u> $\hat{L}_\gamma \equiv J_\gamma - \Delta\gamma$ yields
$\hat{L}_\gamma = \mathcal{L}_\gamma + \frac{\alpha}{4\pi} \neq \mathcal{L}_\gamma$
- $\frac{\alpha}{4\pi}$ appears to be small, but here $\mathcal{L}_e$, L_e are all of $\mathcal{O}(\frac{\alpha}{\pi})$

OAM in QCD

- $\hookrightarrow$ 1-loop QCD: $\mathcal{L}_q - L_q = \frac{\alpha_s}{3\pi}$ (for $j_z = +\frac{1}{2}$)
- recall (lattice QCD): $L_u \approx -.15$; $L_d \approx +.15$
- QCD evolution yields negative correction to L_u and positive correction to L_d
- $\hookrightarrow$ evolution suggested (A.W.Thomas) to explain apparent discrepancy between quark models (low Q^2) and lattice results ($Q^2 \sim 4GeV^2$)
- above result suggests that $\mathcal{L}_u > L_u$ and $\mathcal{L}_d < L_d$
- additional contribution (with same sign) from vector potential due to spectators (MB, to be published)
- $\hookrightarrow$ possible that lattice result consistent with $\mathcal{L}_u > \mathcal{L}_d$

Summary part 2



- inclusive $\overrightarrow{e}\overleftarrow{p}$ / $\overrightarrow{p}\overleftarrow{p}$ provide access to
  - quark spin $\frac{1}{2}\Delta q$
  - gluon spin ΔG
  - parton grand total OAM $\mathcal{L} \equiv \mathcal{L}_g + \sum_q \mathcal{L}_q = \frac{1}{2} - \Delta G - \frac{1}{2}\sum_q \Delta q$
- DVCS & polarized DIS and/or lattice provide access to
  - quark spin $\frac{1}{2}\Delta q$
  - J_q & $L_q = J_q - \frac{1}{2}\Delta q$
  - $J_g = \frac{1}{2} - \sum_q J_q$
- $J_g - \Delta G$ does <u>not</u> yield gluon OAM $\mathcal{L}_g$
- $L_q - \mathcal{L}_q = \mathcal{O}(0.1 * \alpha_s)$ for $\mathcal{O}(\alpha_s)$ dressed quark

pizza tre e mezzo stagioni

- Chen, Goldman et al.: integrate by parts in J_g only for term involving $\mathbf{A}_{phys}$, where

$$\mathbf{A} = \mathbf{A}_{pure} + \mathbf{A}_{phys} \qquad \text{with} \qquad \nabla \cdot \mathbf{A}_{phys} = 0 \quad \nabla \times \mathbf{A}_{pure} = 0$$

- $\frac{1}{2} = \sum_q J_q + J_g = \sum_q \left(\frac{1}{2}\Delta q + L_q'\right) + S_g' + L_g'$ with Δq as in JM/Ji

$$\begin{aligned}
L_q' &= \int d^3x \, \langle P,S| \, q^\dagger(\vec{x}) \left(\vec{x} \times i\vec{D}_{pure}\right)^3 q(\vec{x}) \, |P,S\rangle \\
S_g' &= \int d^3x \, \langle P,S| \left(\vec{E} \times \vec{A}_{phys}\right)^3 |P,S\rangle \\
L_g' &= \int d^3x \, \langle P,S| \, E^i \left(\vec{x} \times \vec{\nabla}\right)^3 A^i_{phys} \, |P,S\rangle
\end{aligned}$$

- $i\vec{D}_{pure} = i\vec{\partial} - g\vec{A}_{pure}$
- only $\frac{1}{2}\Delta q$ accessible experimentally

example: angular momentum in QED

- consider now, QED with electrons:

$$\vec{J}_\gamma = \int d^3r\,\vec{x} \times \left(\vec{E} \times \vec{B}\right) = \int d^3r\,\vec{x} \times \left[\vec{E} \times \left(\vec{\nabla} \times \vec{A}\right)\right]$$

- integrate by parts

$$\vec{J} = \int d^3r \left[E^j \left(\vec{x} \times \vec{\nabla}\right) A^j + \left(\vec{x} \times \vec{A}\right) \vec{\nabla} \cdot \vec{E} + \vec{E} \times \vec{A}\right]$$

- replace 2^{nd} term (eq. of motion $\vec{\nabla} \cdot \vec{E} = ej^0 = e\psi^\dagger\psi$), yielding

$$\vec{J}_\gamma = \int d^3r \left[\psi^\dagger \vec{r} \times e\vec{A}\psi + E^j \left(\vec{x} \times \vec{\nabla}\right) A^j + \vec{E} \times \vec{A}\right]$$

- $\psi^\dagger \vec{r} \times e\vec{A}\psi$ cancels similar term in electron OAM $\psi^\dagger \vec{r} \times (\vec{p} - e\vec{A})\psi$

$\hookrightarrow$ decomposing $\vec{J}_\gamma$ into spin and orbital also shuffles angular momentum from photons to electrons!

pizza tre e mezzo stagioni

- Chen, Goldman et al.: integrate by parts in J_g only for term involving $\mathbf{A}_{pure}$, where

$$\mathbf{A} = \mathbf{A}_{pure} + \mathbf{A}_{phys} \qquad \text{with} \qquad \nabla \cdot \mathbf{A}_{phys} = 0 \quad \nabla \times \mathbf{A}_{pure} = 0$$

B.L.T. pizza ?

- Bakker, Leader, Trueman:
- JM only applies for $\mathbf{s} = \hat{\mathbf{p}}$ (helicity sum rule)
- Ji applies to any component, but parton interpretation only for S_z
- For $\mathbf{p} \neq 0$, Ji only applies to helicity
- 'sum rule' $\mathbf{s} \perp \hat{\mathbf{p}}$

$$\frac{1}{2} = \frac{1}{2} \sum_{a \in q, \bar{q}} \int dx h_1^a(x) + \sum_{a \in q, \bar{q}, g} \langle L_{s_T}^a \rangle$$

where $L_{s_T}^a$ component of $\mathbf{L}^a$ along $\mathbf{s}_T$

- note: $\sum_{a \in q, \bar{q}} \int dx h_1^a(x)$ not tensor charge (latter is: '$q - \bar{q}$')
- $\mathbf{L}^a \sim \psi^\dagger \mathbf{k} \times \nabla_k \psi$
- distinction between transversity and transverse spin obscure in two-component formalism used

B.L.T. pizza ?

- 'B.L.T. sum rule' $\mathbf{s} \perp \hat{\mathbf{p}}$
  $\frac{1}{2} = \frac{1}{2}\sum_{a\in q,\bar{q}}\int dx h_1^a(x) + \sum_{a\in q,\bar{q},s}\langle L^a_{s_T}\rangle$
- should already be suspicious as $T^{\mu\nu}$ is chirally even ($m_q = 0$) and so should $\vec{J}$...
- $\langle L^a_{s_T}\rangle$ not accessible experimentally, i.e. B.L.T. not experimentally falsifyable, but
- studies (diquark model) under way to test B.L.T. ...

(Grand) Summary

- GPDs $\stackrel{FT}{\longleftrightarrow}$ IPDs (impact parameter dependent PDFs)
- $E(x,0,-\mathbf{\Delta}_\perp^2) \longrightarrow \perp$ deformation of PDFs for $\perp$ polarized target
- DVCS at fixed $Q^2 \leftrightarrow GPDs(\xi,\xi,t), \Delta(t)$
- Fourier transform of GPDs w.r.t. $\mathbf{\Delta}_\perp$ provides dependence of overlap matrix element on $\frac{1-x}{1-\zeta}\mathbf{r}_\perp$ where $\mathbf{r}_\perp$ is **separation between active quark and the COM of spectators**

$\hookrightarrow$ for $x=\zeta$, variable conjugate to $\mathbf{\Delta}_\perp$ is $\mathbf{r}_\perp$
(note: t-slope $=(1-\zeta)\times$ $\Delta_\perp^2$-slope)

- $\frac{1}{2}-\frac{1}{2}\sum_q \int dx x\left[H_q(x,\xi,0)+E_q(x,\xi,0)\right]-\Delta G \neq \mathcal{L}_g$