Lattice calculations of GPDs

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excellence cluster universe

Overview

- Physics
- What we can / cannot do on the lattice
- x-moments of GPDs: Our A, B, C
- Impact on the nucleon spin sum rule
- Impact on the transverse size
- Conclusions, open questions, and perspectives

Physics: Concentrate on:

Decomposition of the nucleon spin

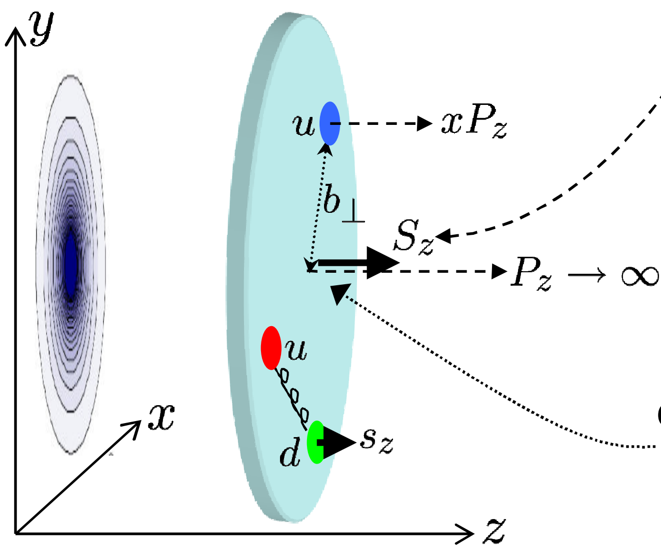
Ji's nucleon spin sum rule (X. Ji, PRL 1997)

$$\frac{1}{2} = S_z = \int_{-1}^{1} dx\, x \left\{ H(x,\xi,t{=}0) + E(x,\xi,t{=}0) \right\}$$
$$= \sum_q J_q + J_g = \sum_q \frac{1}{2}\Delta\Sigma_q + \sum_q L_q + J_g$$

$$L_q \equiv J_q - \frac{1}{2}\Delta\Sigma_q$$
$$(L_g \equiv J_g - \Delta G)$$

everything is:
-gauge-invariant
-scale & scheme dependent
-measurable

Tomography; Transverse size



center of momentum
$R_\perp = \sum x_i r_{\perp i}$

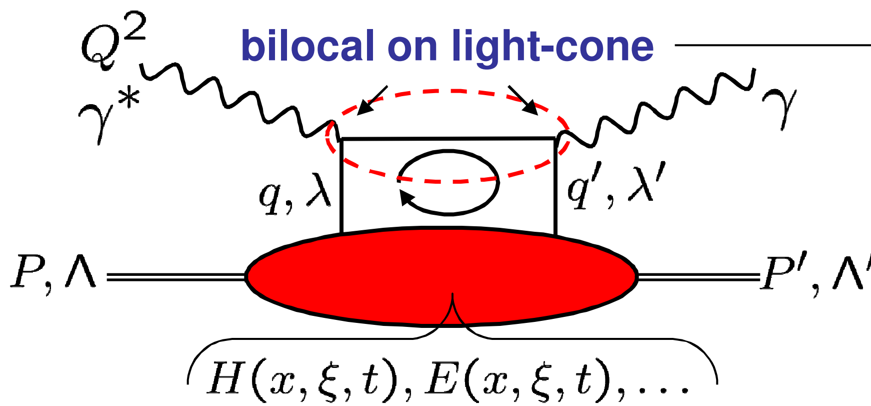
$$H(x, b_\perp) \overset{\text{FT}}{\leftrightarrow} H(x, \xi{=}0, t)$$

$$b_\perp \leftrightarrow -\Delta_\perp \widehat{=} |t|^{\frac{1}{2}}$$

Burkardt PRD 2000

What we can / cannot do on the lattice (presently)

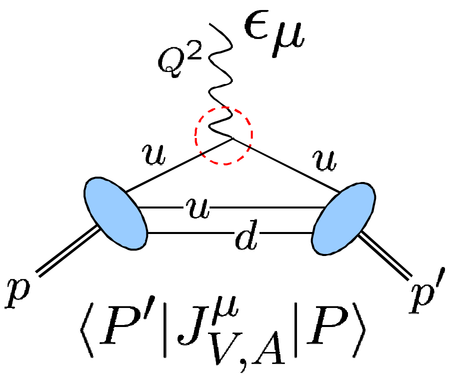
DVCS – factorization
[Collins `99,`01]



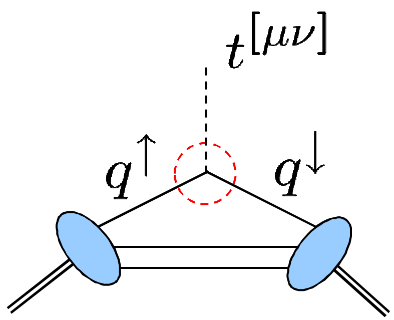
first exploratory studies of *non-local* couplings on the lattice related to TMDs [Musch, PhH, Schäfer, Negele]

for the time being: local couplings

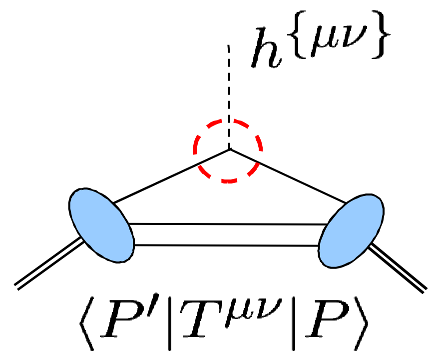
spin-1 (axial-)vector coupling



spin-1 helicity-flip coupling



spin-2 *gravitonal* coupling



+ higher-spin couplings

...

flavor decomposition; quark/hadron polarizations; range of momentum transfers straightforward

Generalized form factors and basic sumrules

$$\langle P' | \overline{q}(0) \Gamma D^{\mu_1} D^{\mu_2} \ldots D^{\mu_n} q(0) | P \rangle = \overline{U}(P', S') \left(a_\Gamma^{\mu_1 \mu_2 \cdots} A_\Gamma(t) + b_\Gamma^{\mu_1 \mu_2 \cdots} B_\Gamma(t) + \ldots \right) U(P, S)$$

Ji&Lebed PRD 2000
Ph.H. PLB 2004

relation to (moments of) GPDs

$$\int_{-1}^{1} dx x^{n-1} H(x, \xi, \Delta^2 = t) = H^n(\xi, t) = \sum_{i=0,\mathrm{even}}^{n-1} (-2\xi)^i A_{ni}(t) + (-2\xi)^n C_{n0}(t) |_{n\ \mathrm{even}}$$

$$\int_{-1}^{1} dx x^{n-1} E(x, \xi, \Delta^2 = t) = E^n(\xi, t) = \sum_{i=0,\mathrm{even}}^{n-1} (-2\xi)^i B_{ni}(t) - (-2\xi)^n C_{n0}(t) |_{n\ \mathrm{even}}$$

n_{max}~4

full x(⊠)-dependence cannot be reconstructed model-independently

quark, anti-quark contributions cannot be separated

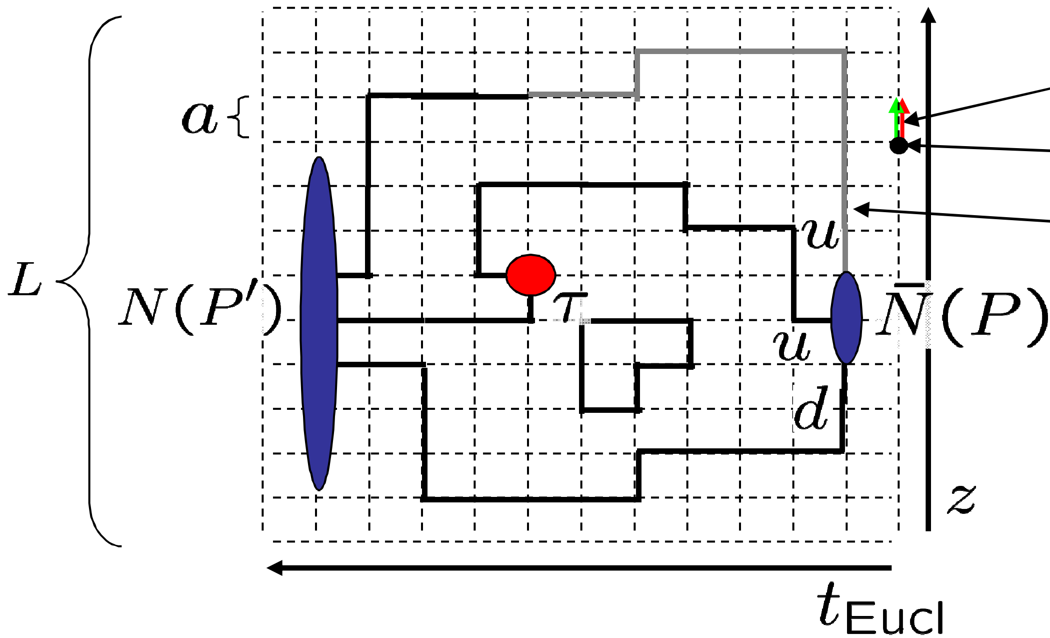
n=1 ★ Dirac&Pauli FFs

$$F_1(Q^2 = -t) = A_{10}(t)$$
$$F_2(Q^2 = -t) = B_{10}(t)$$

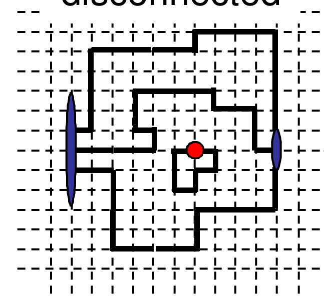
n=2 ★ FFs of the EM-tensor

$$A_{20}(t), B_{20}(t), C_{20}(t)$$

Lattice QCD calculations of hadron structure



disconnected



hard, but can be done

recent progress by Collins, Bali, Schäfer; Kentucky group

u-d is safe

(red dot) = local vector-, axialvector-, *quark spin flip-,* (spin-2) *graviton-, „spin-n"* coupling

$$C_{3pt}(P',P,\tau) \leftrightarrow e^{-E'(T-\tau)-E\tau}\langle P',\Lambda'|\mathcal{O}|P,\Lambda\rangle \propto g_A, \Delta\Sigma, F_1(t), F_2(t), \langle x\rangle, A_{20}(t), \ldots$$

$t = \Delta^2 (\hat{=} q^2)$

$$\langle q_2\bar{q}_1\rangle \propto \int DADqd\bar{q}\, e^{iS[q,\bar{q},A]} \to \left[\int DU e^{-S[U]} \det D[U]\right] D^{-1}_{1\to 2}[U] \approx \frac{1}{N}\sum_{i=1}^{N} D^{-1}_{1\to 2}[U_i]$$

MC integration

Lattice QCD calculations of hadron structure

systematic „ab initio"-approach, but

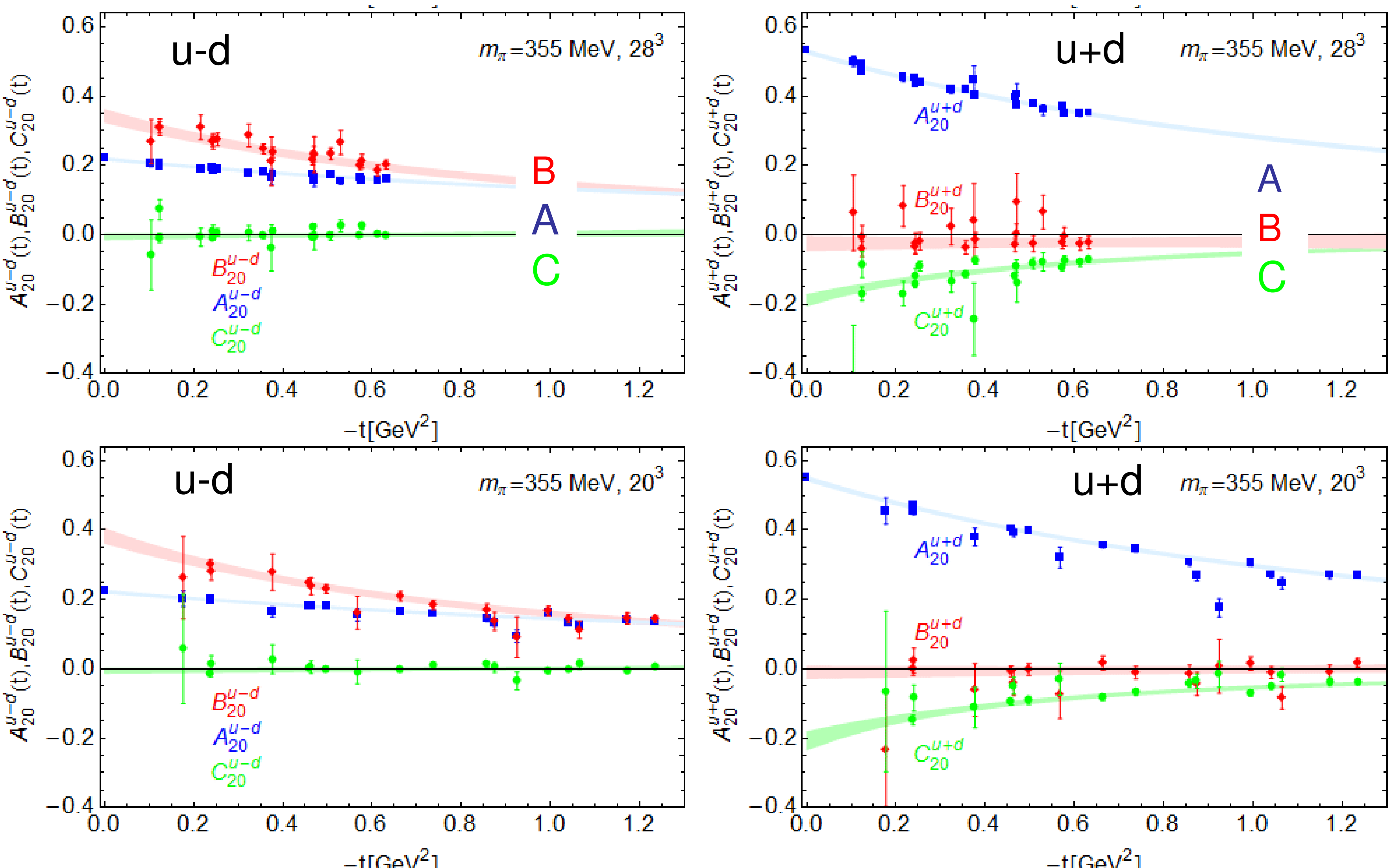
- statistical errors from MC integration
- discretization and finite volume errors/effects
- contaminations from excited states
- large quark masses $m_\pi (\propto \sqrt{m_q}) \gtrapprox 300\ \text{MeV}$
- large minimal non-zero momenta $p_\text{min} = \frac{2\pi}{aL} \approx 300\ \text{MeV}$

approximations can be continuously improved

limited by computational and **human resources**

A, B, C

LHPC n_f=2+1 mixed; arXiv:1001.3620 (updating PRD 2008, 0810.1933)

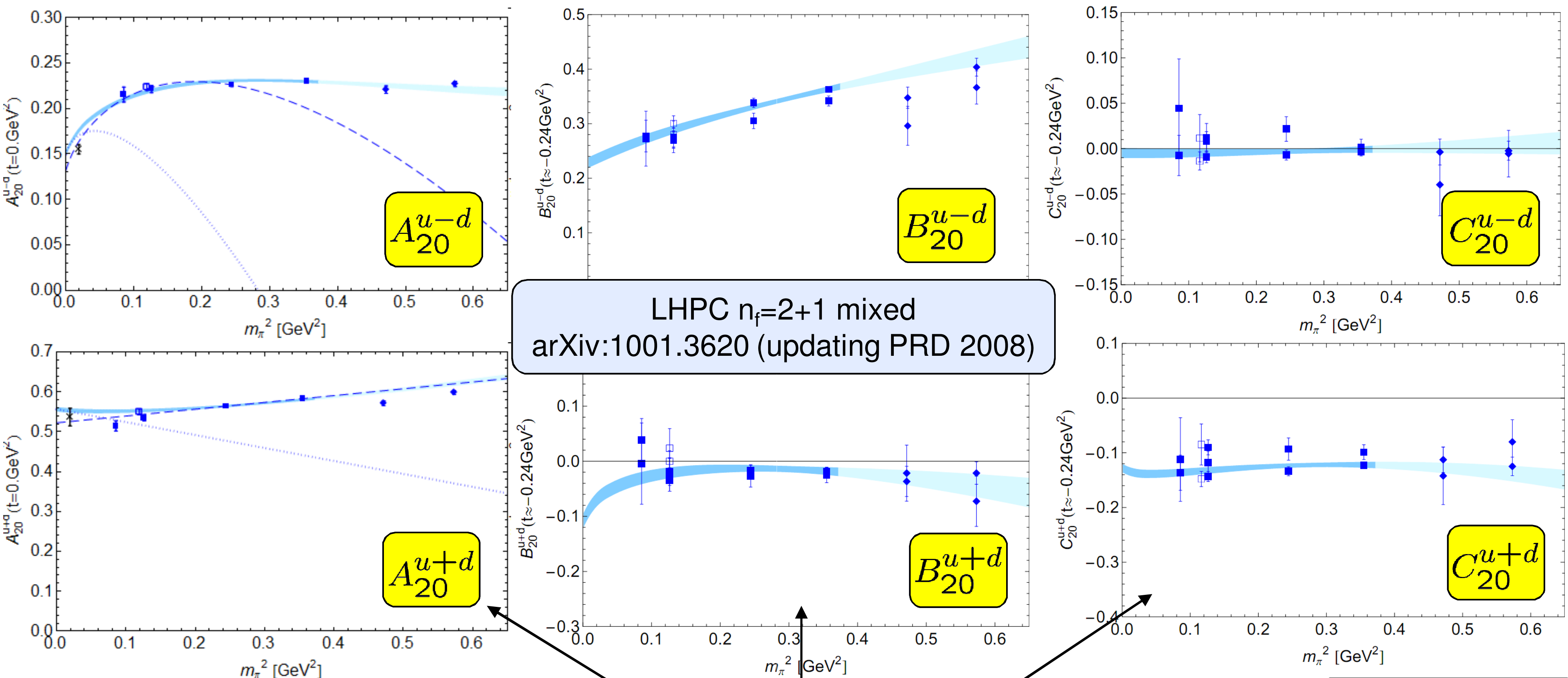


$\overline{MS}$ at 4 GeV^2

disconnected contributions are not included↔ only u-d is „exact"

Chiral extrapolations of A,B,C

global simultaneous fits of A, B, C with common parameter <x>
+ 8 additional free parameters/LECs, to >80 lattice data points in each case (u-d and u+d)



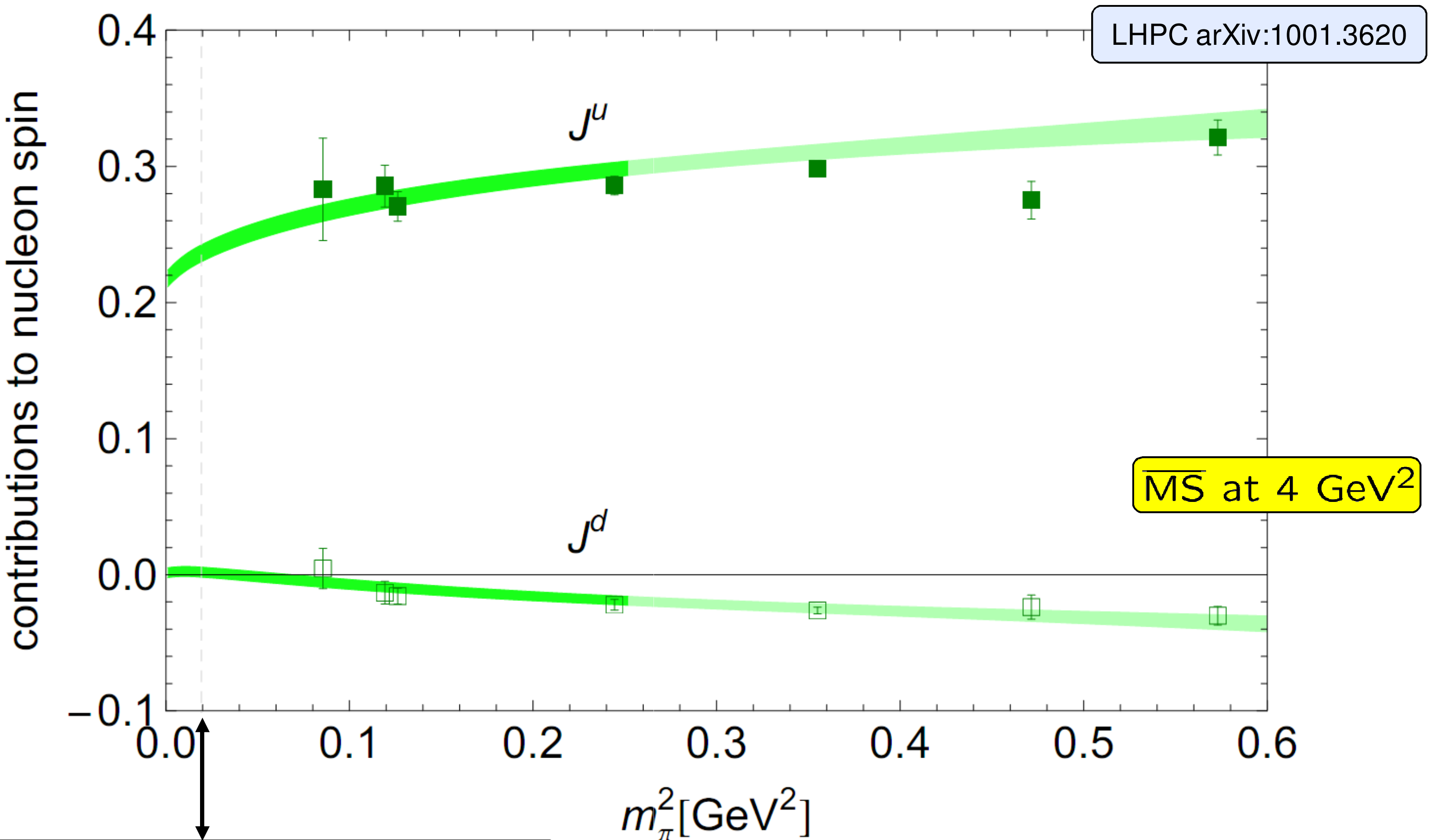
LHPC n_f=2+1 mixed
arXiv:1001.3620 (updating PRD 2008)

only quark line connected contributions

$\overline{\text{MS}}$ at 4 GeV^2

Quark angular momentum

$J_q = \frac{1}{2}\left(A_{20}^q(0) + B_{20}^q(0)\right)$ — from covariant BChPT extrapolations



$J^u = 0.236(6) \approx 47\%$ of $1/2$

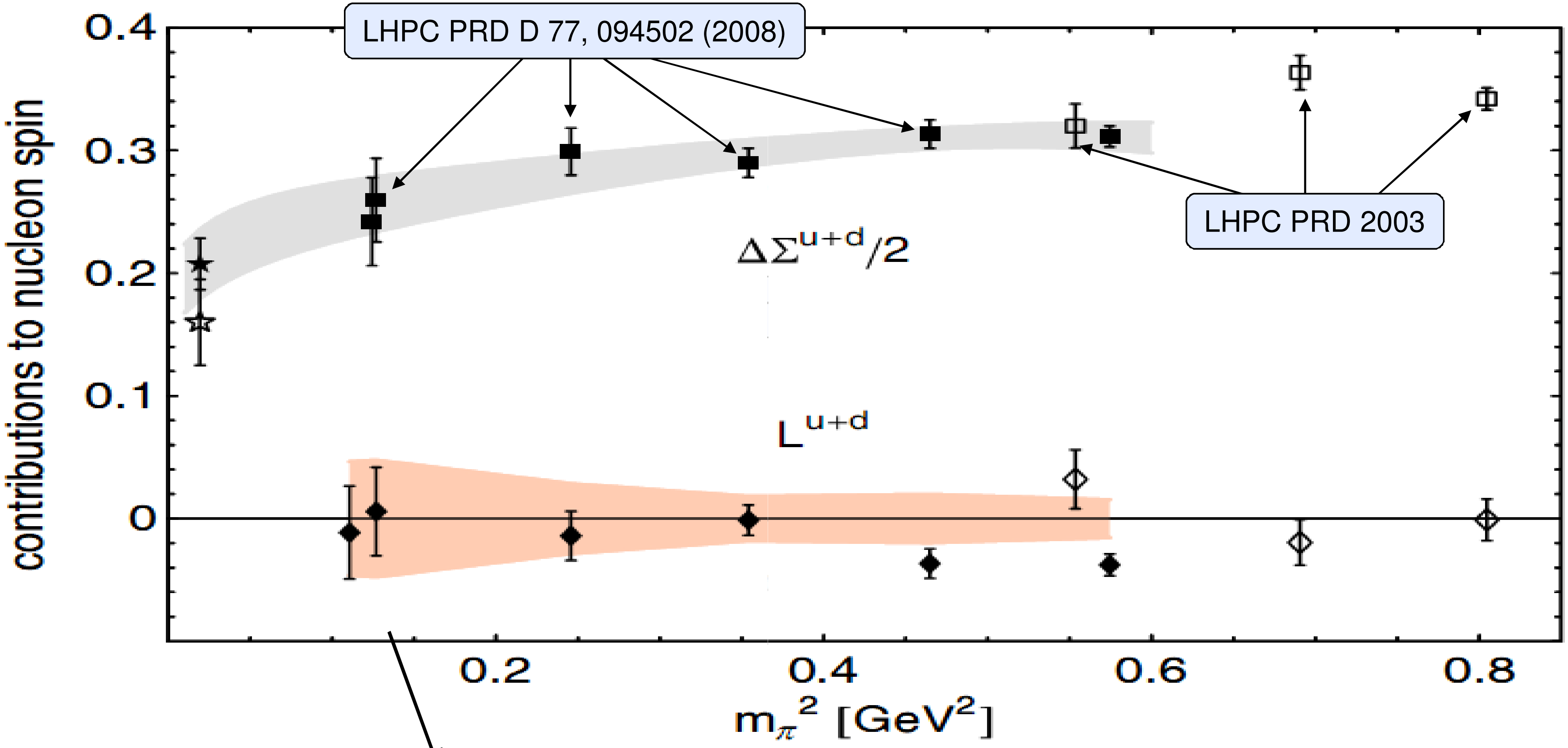
$J^d = 0.0018(37) \approx 1\%$ of $1/2$

$J^{u+d} \approx 0.238 \pm 0.008 \approx 48\%$ of $1/2$

Nucleon spin structure and spin sum rule

$J_q = \frac{1}{2}\left(A^q_{20}(0) + B^q_{20}(0)\right)$ — $L_q \equiv J_q - \Delta\Sigma_q/2$

$\overline{\text{MS}}$ at 4 GeV^2

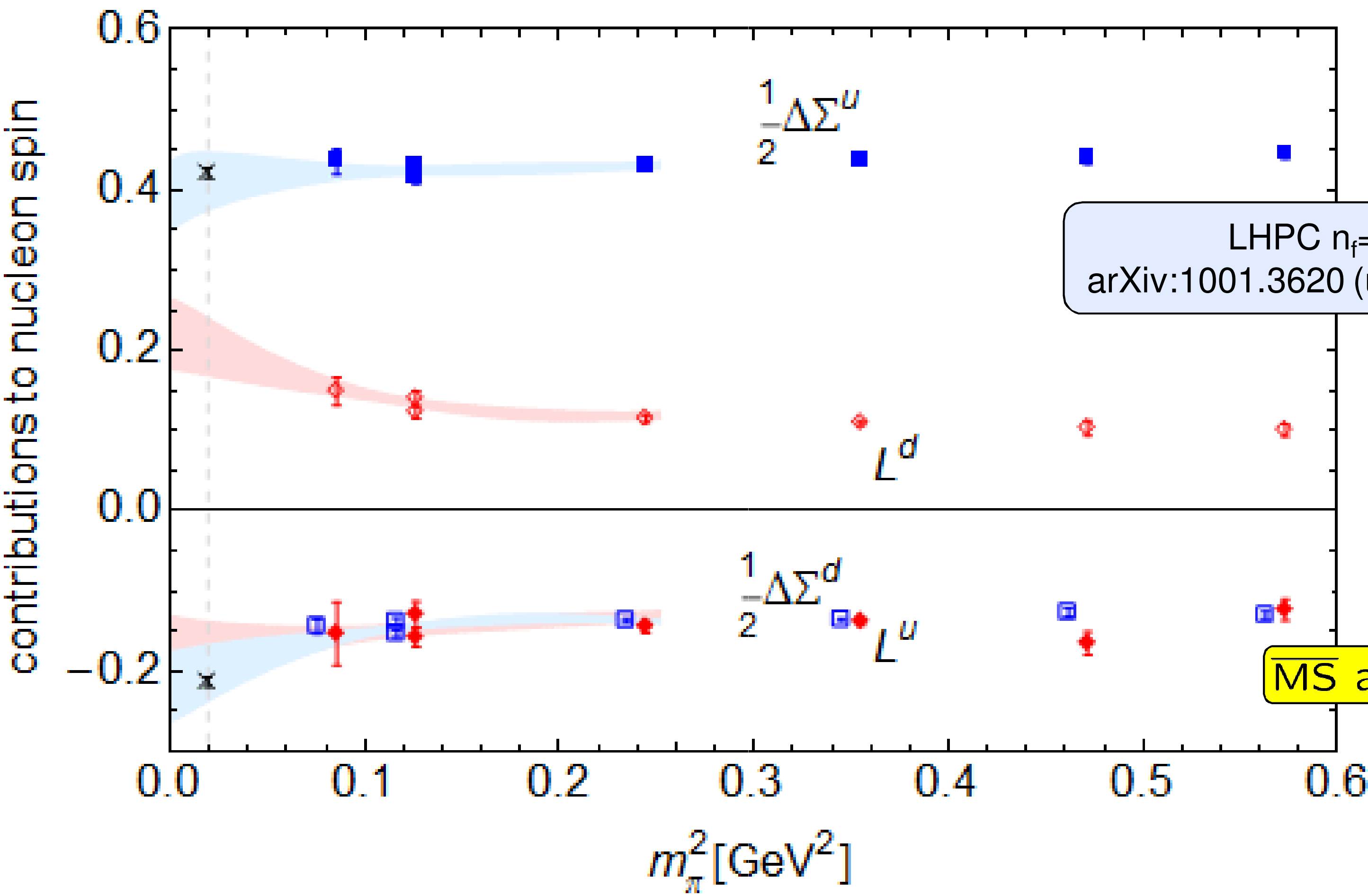


$\Delta\Sigma^{u+d}/2 \approx 0.21 \pm 0.006 \approx 42\%$ of $1/2$

$L^{u+d} \approx 0.030 \pm 0.012 \approx 6\%$ of $1/2$

→ $J^{u+d} \approx 0.238 \pm 0.008 \approx 48\%$ of $1/2$

Nucleon spin structure and spin sum rule



LHPC n_f=2+1 mixed
arXiv:1001.3620 (updating PRD 2008)

$\overline{\text{MS}}$ at 4 GeV^2

$J^u \approx 0.236 \pm 0.006 \,\hat{=}\, 48\%$ of 1/2
$J^d \approx 0.002 \pm 0.004$

$L^d \approx -L^u \approx 0.185 \pm 0.06 \approx 36\%$ of 1/2
$L^{u+d} \approx 0.030 \pm 0.012 \approx 6\%$ of 1/2

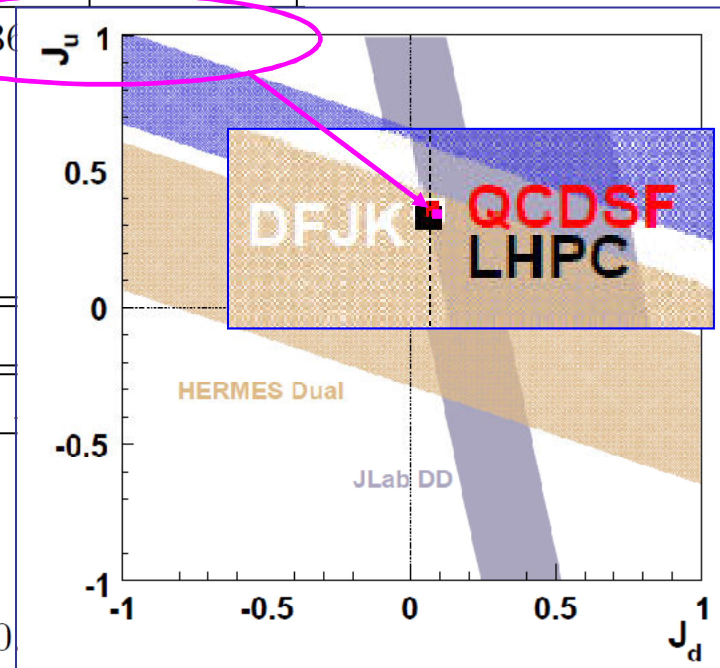
pioneering lattice calculations by Gadiyak, Ji and Jung in 2001

$(\kappa^{u+d} = 3\kappa^{p+n} = -0.36)$

Contributions to the nucleon spin

$\overline{\text{MS}}$ at 4 GeV^2

	J^{u-d}	J^{u+d}	J^u	J^d
BChPT	0.234(6)	0.238(8)	0.23[illegible]	[illegible]
HBChPT		0.264(6)		
HBChPT + Δ		0.226(22)		
mixed ChPT				
experiment				



	$g_A = \Delta\Sigma^{u-d}$	$\Delta\Sigma^{u+d}/2$	$\Delta\Sigma^u/2$	$\Delta\Sigma^d/2$				
BChPT								
HBChPT		0.208(10)						
HBChPT + Δ	1.21(17)							
mixed ChPT			0.411(36)	−0.203(35)	−0[illegible]			[illegible])
experiment	1.2670(35)	0.208(9)	0.421(6)	−0.214(6)				

$$\frac{1}{2} \overset{*}{\approx} 0.238(8)_{[J^{u+d}]} + J_g = 0.210(6)_{[\Delta\Sigma^{u+d}/2]} + 0.030(12)_{[L^{u+d}]} + J_g$$

$\overline{\text{MS}}$ at 4 GeV^2

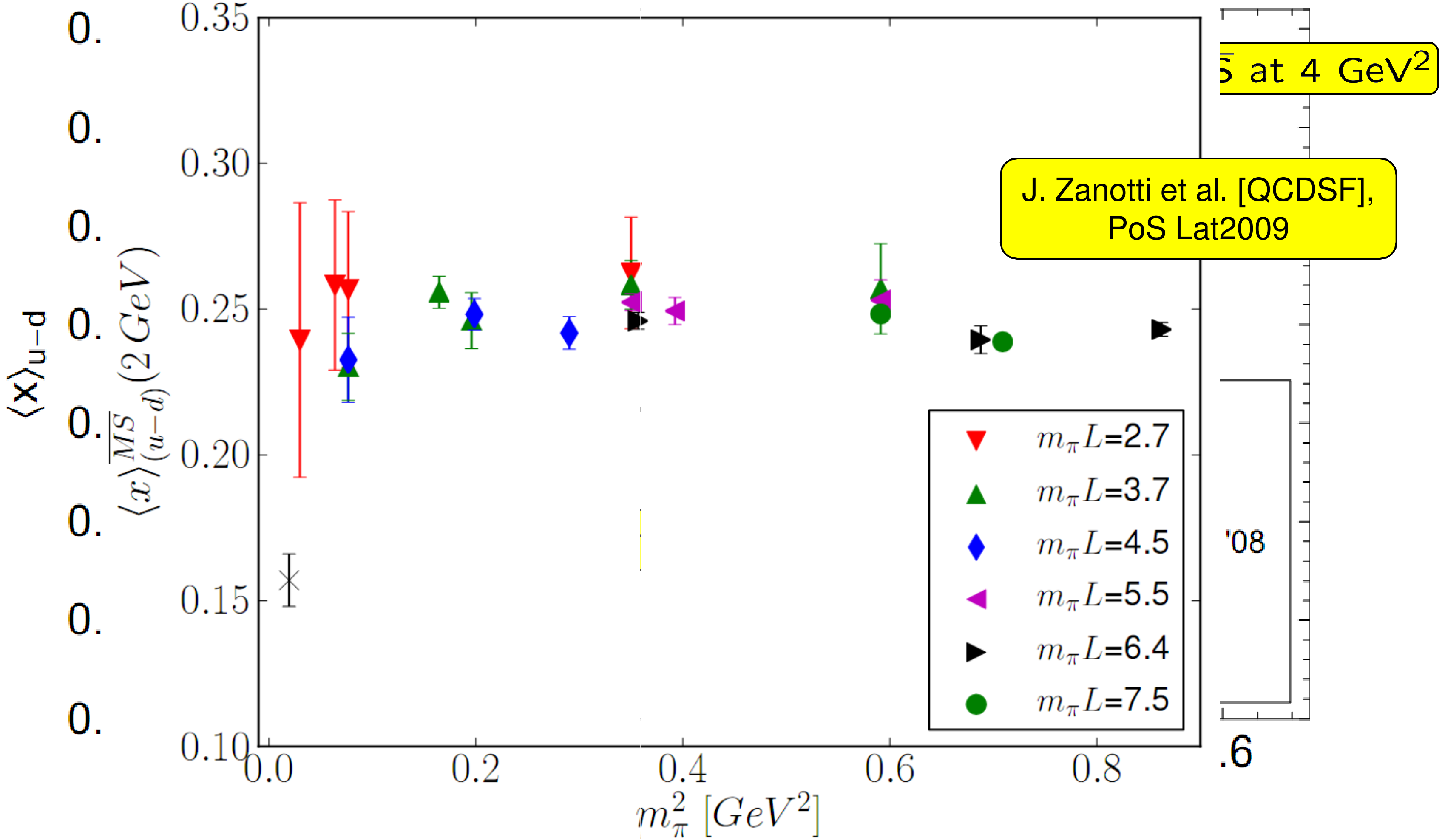
compares well with study by Goloskokov&Kroll 2008

* [non-singlet, connected only]

Looks great, but ...

momentum fraction of quarks in the nucleon

$$\langle P|\overline{q}\gamma^{\{\mu}D^{\nu\}}q|P\rangle = \overline{U}(P)\gamma^{\{\mu}P^{\nu\}}U(P)\langle x\rangle \longleftrightarrow \langle x\rangle = A_{20}(0) = \int_{-1}^{+1} dx\, x\, q(x) = \langle x\rangle_q + \langle x\rangle_{\bar{q}}$$



Transverse size of the nucleon – basic observations

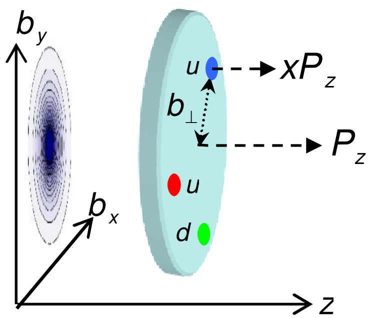
„transverse size"

$$\langle r_\perp^2\rangle(x) = \frac{\int d^2b_\perp b_\perp^2 H(x,b_\perp)}{\int d^2b_\perp H(x,b_\perp)}$$

$H(x,b_\perp) = \mathrm{FT}\left[H(x,\xi=0,\Delta_\perp^2=-t)\right]$

Burkardt PRD 2000;...

distance to COM



x-dependent; defined at ⊠=0 ⇔ x_B=0

cannot be inferred *directly* *(model-independently)* from DVCS cross section, i.e. the x_B-dependence of slope in t

cannot be computed *directly* in lattice QCD

DVCS (LO) ⟷ $H(\xi,\xi,t)$

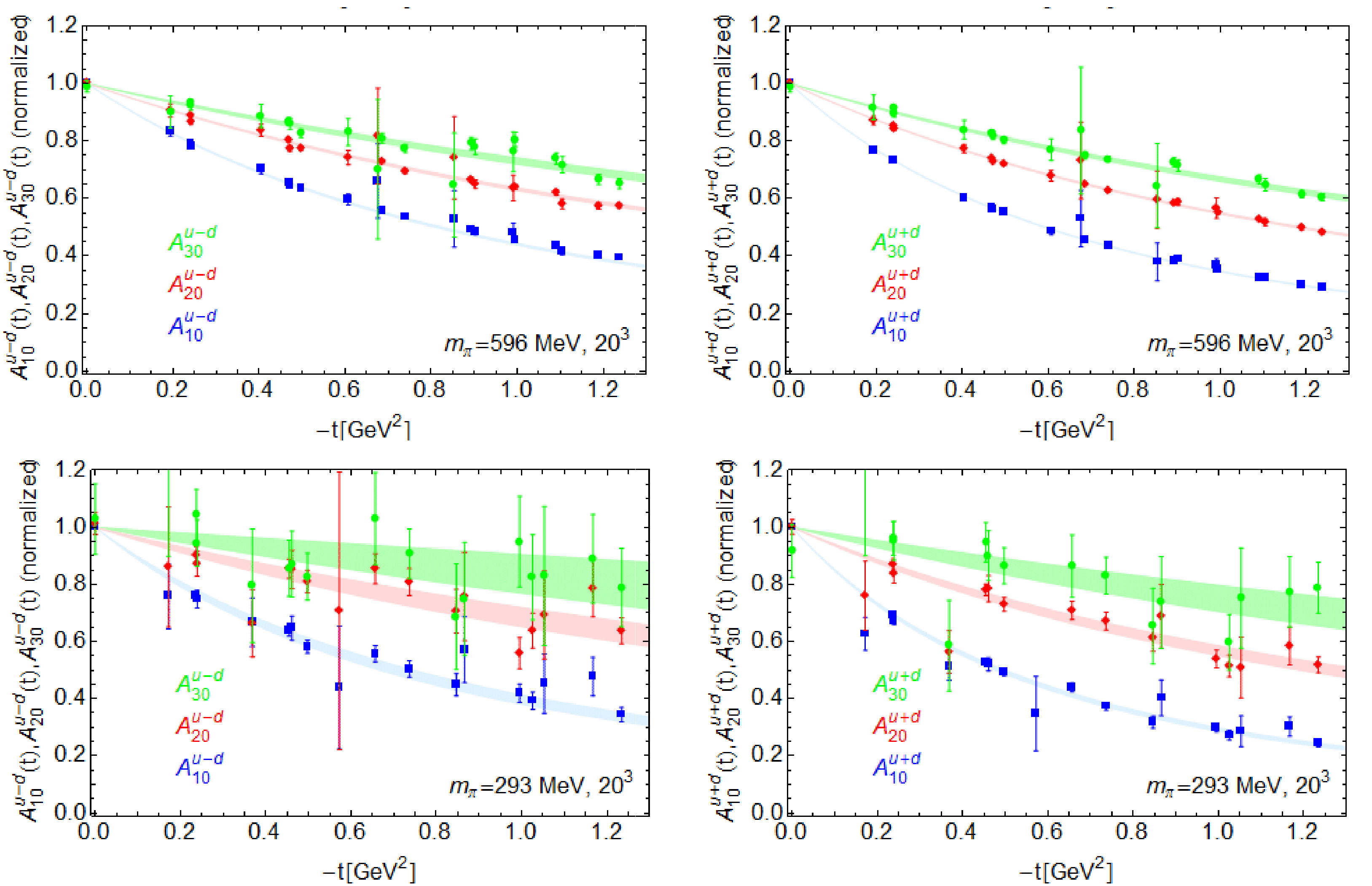
local matrix elements ⇔ x-moments

same for $\dfrac{\langle r_\perp^2\rangle(x)}{1-x}$ distance to spectators

correlations in x and t

LHPC n_f=2+1 mixed; arXiv:1001.3620
(updating PRD 2008, 0810.1933)

$$\bar{x} \to 1 \Leftrightarrow n \to \infty$$

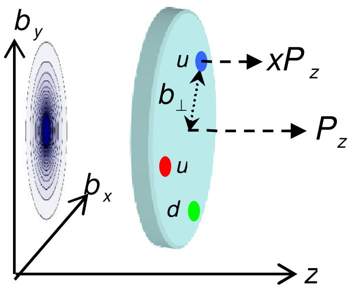


Generalized mean square radii of the nucleon

correlations in x and $b_\perp$

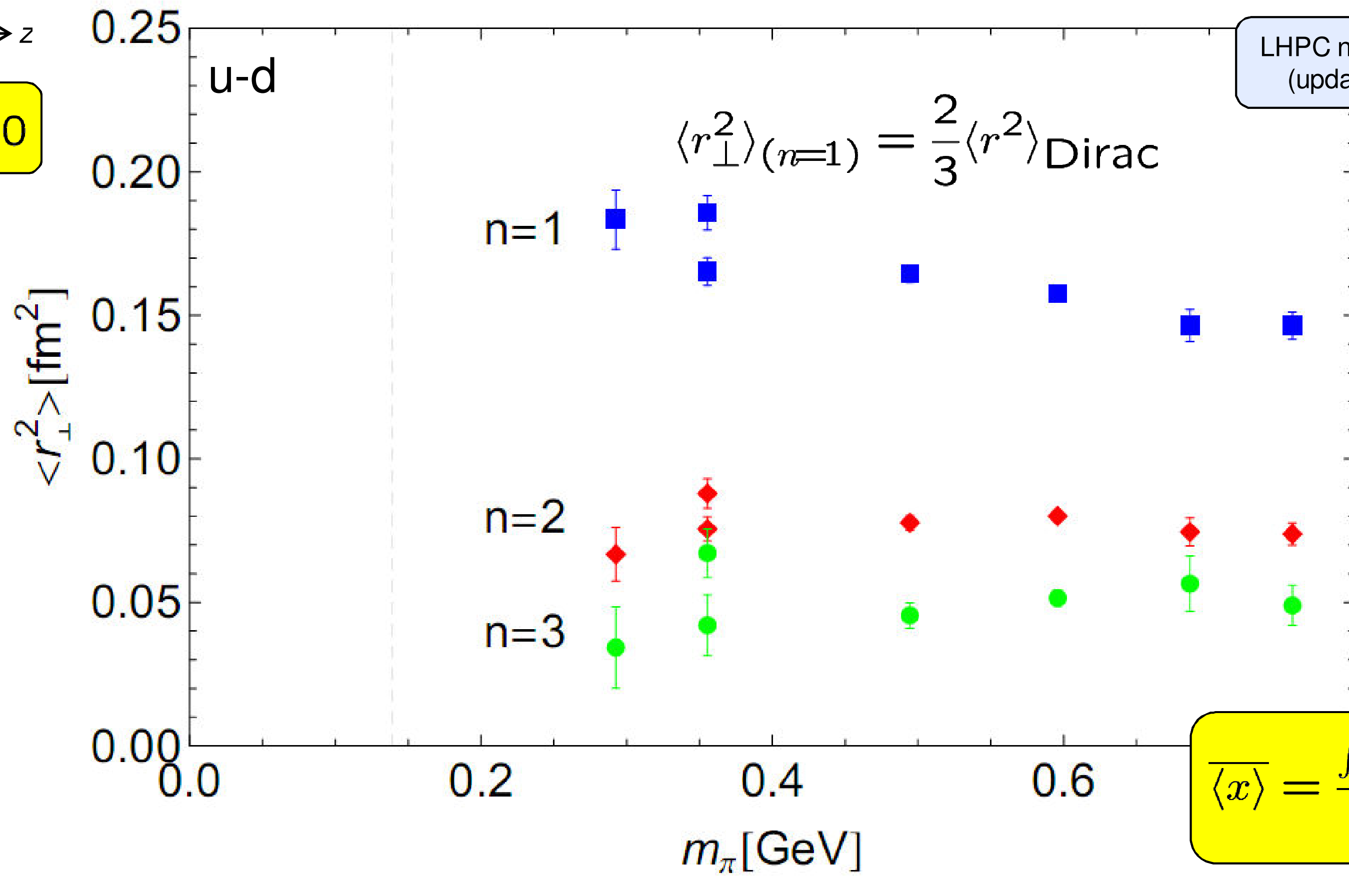
LHPC PRL 93 (2004)

$$\langle r_\perp^2\rangle_n = \frac{\int d^2b\_ b_\perp^2 \int dx x^{n-1} H(x,b_\perp)}{\int d^2b_\perp \int dx x^{n-1} H(x,b_\perp)} = \left(\frac{-4}{A_{n0}(t)}\frac{d}{dt}A_{n0}(t)\right)_{t=0}$$



$$\langle r_\perp^2\rangle_n \xrightarrow{n\to\infty} 0$$

LHPC n_f=2+1 mixed; arXiv:1001.3620 (updating PRD 2008, 0810.1933)



$\overline{\langle x\rangle} \approx 0.2$

$\overline{\langle x\rangle} \approx 0.25$

$\overline{\langle x\rangle} \approx 0.4$

$$\overline{\langle x\rangle} = \frac{\int_{-1}^{1} dx x (x^{n-1} f(x))}{\int_{-1}^{1} dx x^{n-1} f(x)}$$

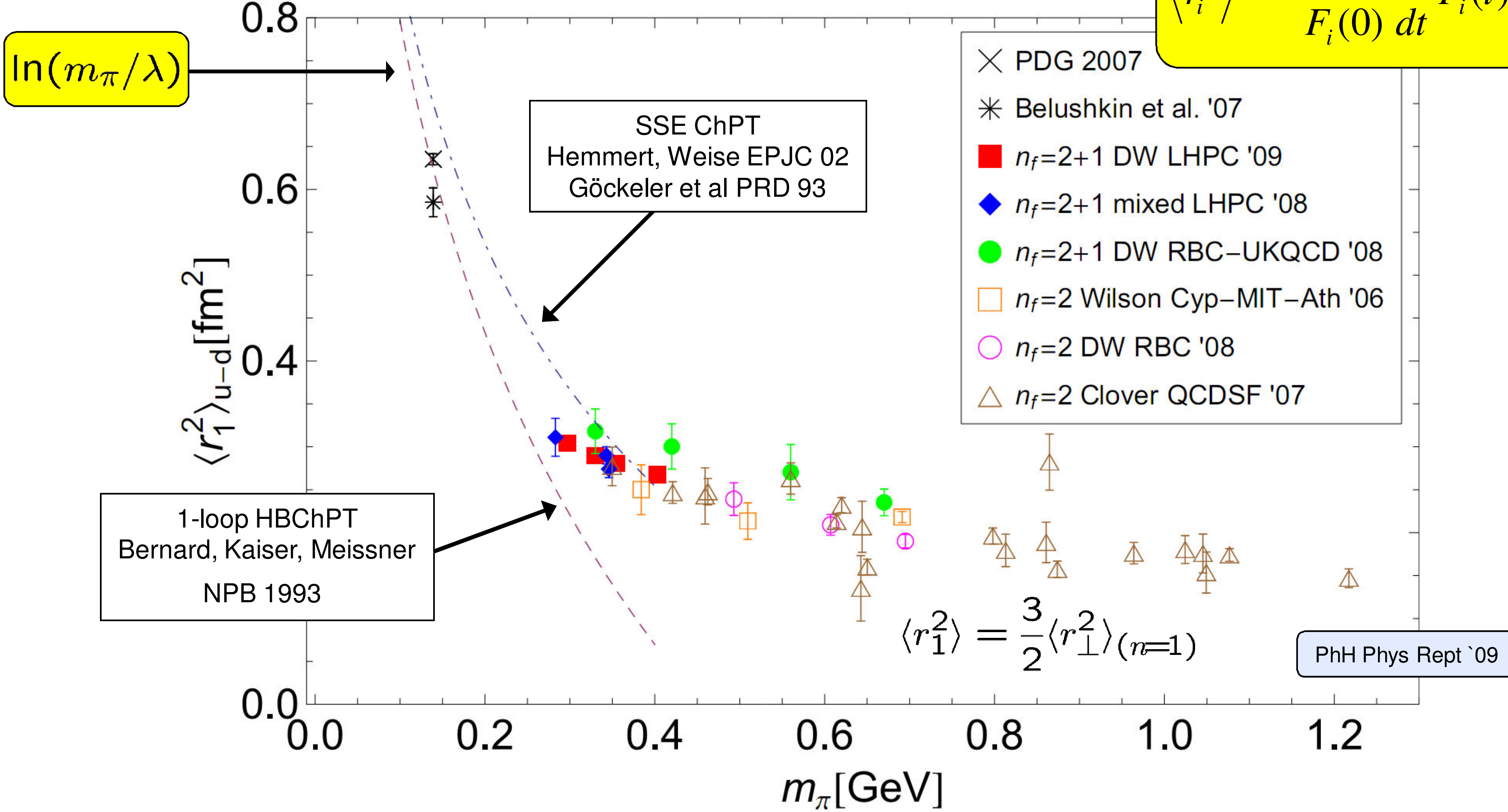
strong correlations in x and $b_\perp$ ⟷ no factorization of GPDs in x and t

Pion mass dependence : Dirac mean square radius

Dirac and Pauli FFs

$$\langle P'|\overline{q}\gamma_\mu q|P\rangle = \overline{U}(P')\Big\{\gamma_\mu F_1(t) + i\frac{\sigma_{\mu\nu}\Delta^\nu}{2m_N}F_2(t)\Big\}U(P)$$

$$\left\langle r_i^2\right\rangle = -\frac{6}{F_i(0)}\frac{d}{dt}F_i(t)\Big|_{t=0}$$



Pion mass dependence of generalized radii for n>1

$$\langle r_\perp^2 \rangle_{(n>1)} = \left(\frac{-4}{A_{n0}(t)} \frac{d}{dt} A_{n0}(t) \right)_{t=0} \approx \frac{\text{const}}{\langle x^{n-1} \rangle (m_\pi)}$$

Diehl, Manashov, Schäfer
EPJ A31 (2007)

no (logarithmic) divergence in m_π for n>1

strong effect seen for n=1,...,3 will most likely persist when $m_\pi \to m_\pi^{phys}$

in any case

$$\frac{\langle r_\perp^2 \rangle^{u-d}_{(n=1)}}{\langle r_\perp^2 \rangle^{u-d}_{(n>1)}} \xrightarrow{m_\pi \to 0} \infty$$

Conclusions, open questions, and perspectives I

- remarkable progress in lattice studies of lowest moments of GPDs
  - small statistical errors at m□~300...350 MeV for many observables
    - time to address systematic effects and uncertainties on the lattice
      - disconnected contributions
      - gluon contributions
      - renormalization
      - (dipole, chiral,...) extrapolations in t; ptbcs

- most chiral extrapolations still not quantitatively reliable
  - need high statistics lattice results for m□⊙250 MeV — global, simultaneous chiral fits

Conclusions, open questions, and perspectives II

lattice calculations and phenomenological/experimental studies of GPDs are mostly *complementary*

We observe strong correlations in x and t based on, e.g., generalized radii

Transverse size strongly reduced as larger x are approached

Precise relevant region in x?

What about correlations in ⊠,x_B and t ?

Interpretation?

Lattice results on spin sum rule, quark OAM are exciting and surprising; observe many cancellations

strong motivation for further phenomenological and experimental studies

Should lattice results be used to constrain GPD-models/parametrizations?

as always, I am indebted to my collaborators

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References: LHPC PRD 77, 094502 (2008)
LHPC arXiv:1001.3620; QCDSF 0912.0167