Global GPD fits (restricted to DVCS)

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K. Kumerički, DM, K. Passek-Kumerički (KMP-K), hep-ph/0703179
GPD fits at NLO and NNLO of H1/ZEUS data

KMP-K, 0805.0152 [hep-ph]
constructive critics on ad hoc GPD model approach [lot of good news]
first applications of dispersion integral approach

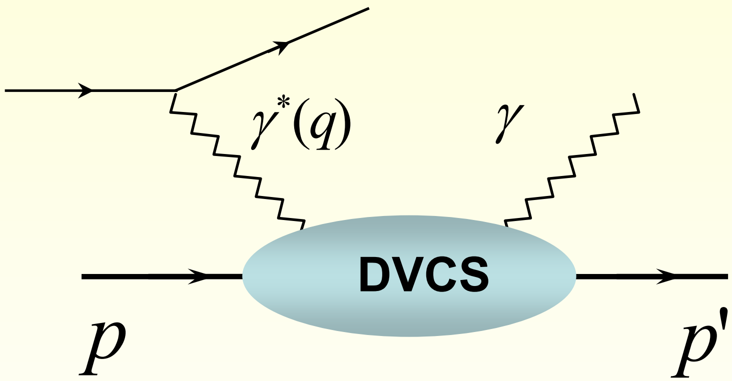
KMP-K, 0807.0159 [hep-ph]; KM 0904.0458 [hep-ph]
flexible GPD model for small *x* and fits of H1/ZEUS data
dispersion integral fits of HERMES and JLAB data

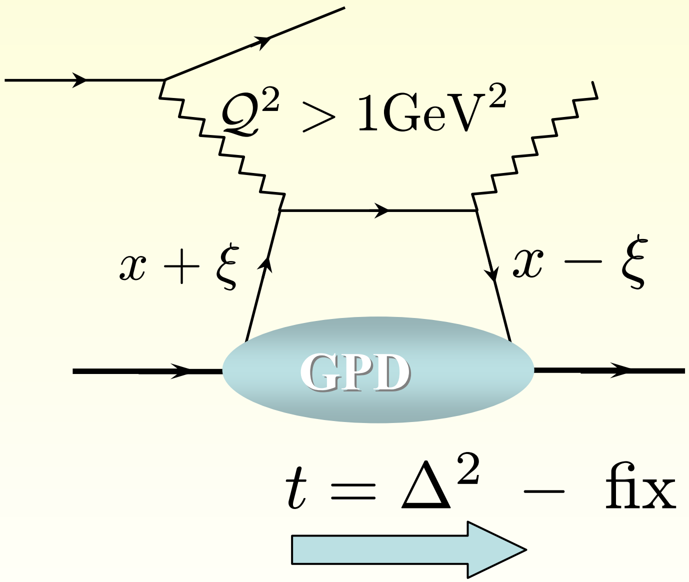
GPDs embed non-perturbative physics

GPDs appear in various hard exclusive processes,

e.g., hard electroproduction of photons (DVCS)

[DM et. al (90/94)
Radyushkin (96)
Ji (96)]





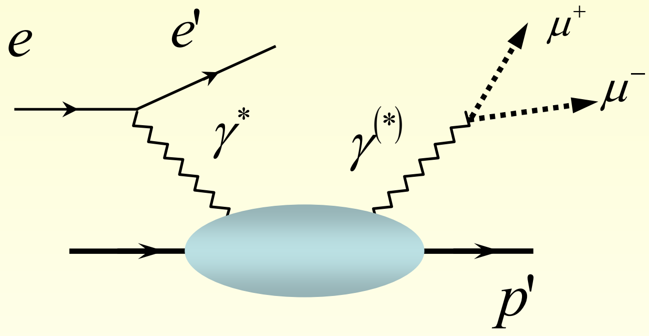
$$\mathcal{F}(\xi, \mathcal{Q}^2, t) = \int_{-1}^{1} dx\, C(x, \xi, \alpha_s(\mu), \mathcal{Q}/\mu) F(x, \xi, t, \mu) + O(\tfrac{1}{\mathcal{Q}^2})$$

CFF	**hard scattering part**	**GPD**	**higher twist**
Compton form factor observable	perturbation theory (our conventions/microscope)	universal (conventional)	depends on approximation

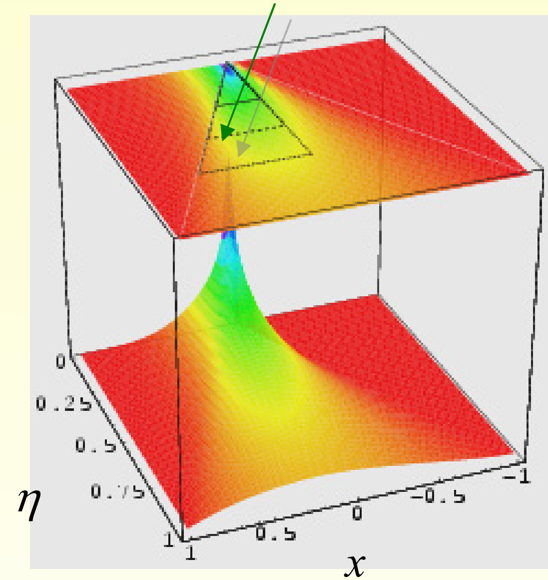
GPD related hard exclusive processes

- Deeply virtual Compton scattering (clean probe)

$$ep \to e'p'\gamma$$
$$ep \to e'p'\mu^+\mu^-$$
$$\gamma p \to p'e^+e^-$$



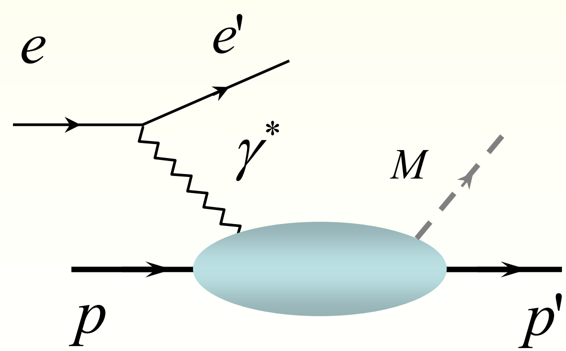
scanned area of the surface as a functions of lepton energy



$ep \to e'\,p'\,\mu^+\mu^-$

- Hard exclusive meson production (flavor filter)

$$ep \to e'p'\pi$$
$$ep \to e'p'\rho$$
$$ep \to e'n\pi^+$$
$$ep \to e'n\rho^+$$

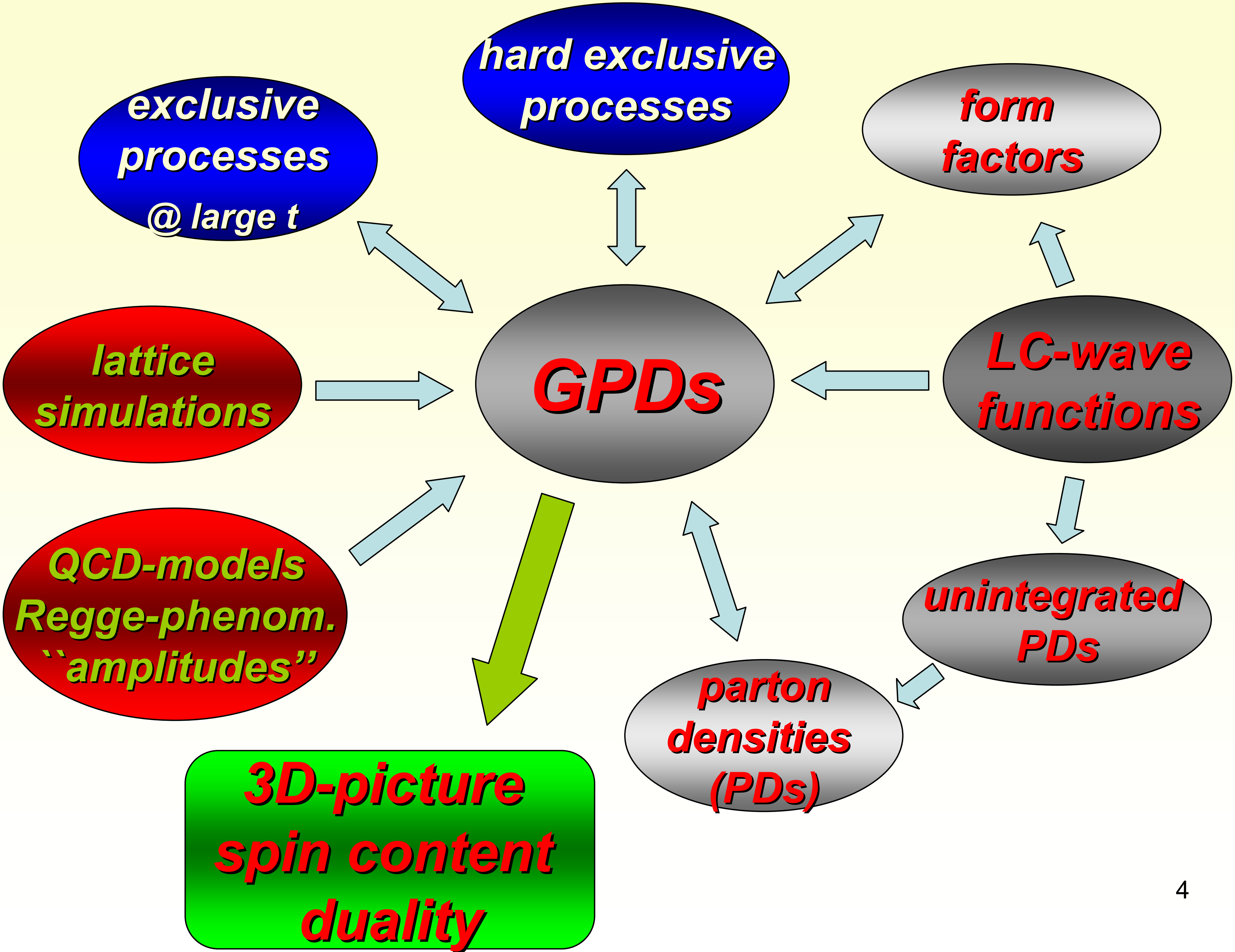


<u>twist-two observables:</u>

cross sections

transverse target spin asymmetries

- etc.


hard exclusive processes
exclusive processes @ large t
form factors
lattice simulations
GPDs
LC-wave functions
QCD-models Regge-phenom. ``amplitudes”
unintegrated PDs
parton densities (PDs)
3D-picture spin content duality

Can one `measure' GPDs?

- **CFF** given as **GPD convolution**:

$$\mathcal{H}(\xi,t,\mathcal{Q}^2) \stackrel{\rm LO}{=} \int_{-1}^{1} dx \left(\frac{1}{\xi - x - i\epsilon} - \frac{1}{\xi + x - i\epsilon}\right) H(x,\eta=\xi,t,\mathcal{Q}^2)$$

$$\stackrel{\rm LO}{=} i\pi H^{-}(x=\xi,\eta=\xi,t,\mathcal{Q}^2) + \mathrm{PV}\int_0^1 dx \frac{2x}{\xi^2 - x^2} H^{-}(x,\eta=\xi,t,\mathcal{Q}^2)$$

- *H(x,x,t,$\mathcal{Q}^2$)* viewed as **"spectral function"** (*s*-channel cut):

$$H^{-}(x,x,t,Q^2) \equiv H(x,x,t,Q^2) - H(-x,x,t,Q^2) \stackrel{\rm LO}{=} \frac{1}{\pi}\Im\mathrm{m}\mathcal{F}(\xi=x,t,Q^2)$$

[Frankfurt et al (97)
Chen (97)
Terayev (05)
KMP-K (07)
Diehl, Ivanov (07)]

- **CFFs** satisfy `**dispersion relations**'
(not the physical ones, threshold ξ_0 set to 0)

$$\Re\mathrm{e}\mathcal{F}(\xi,t,Q^2) = \frac{1}{\pi}\mathrm{PV}\int_0^1 d\xi' \left(\frac{1}{\xi-\xi'} \mp \frac{1}{\xi+\xi'}\right)\Im\mathrm{m}\mathcal{F}(\xi',t,Q^2) + \mathcal{C}(t,Q^2)$$

[Terayev (05)]

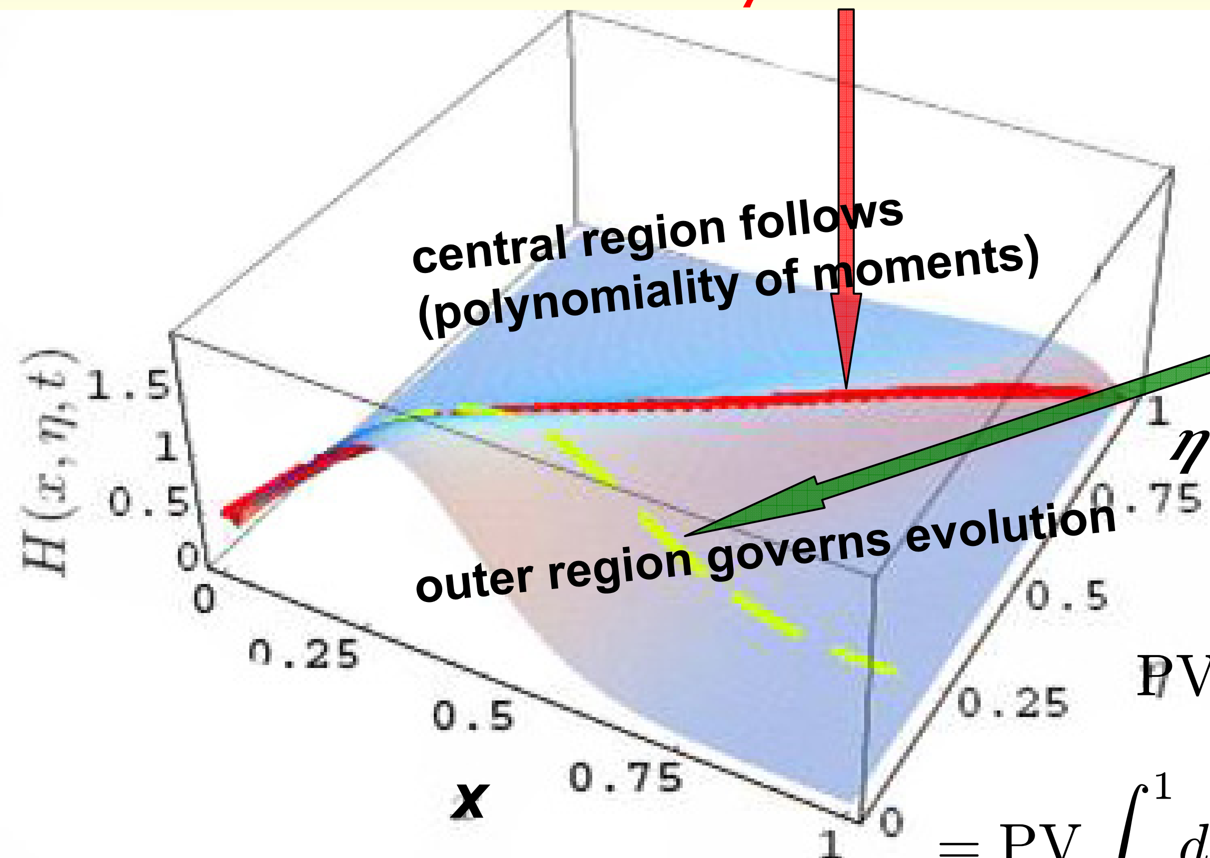
access to the **GPD** on the **cross-over line** $\eta = x$ (at LO)

Modeling & Evolution

outer region governs the evolution at the cross-over trajectory

$$\mu^2 \frac{d}{d\mu^2} H(x,x,t,\mu^2) = \int_x^1 \frac{dy}{x} V(1, x/y, \alpha_s(\mu)) H(y,x,\mu^2)$$

GPD at $\eta = x$ is `measurable' (LO)

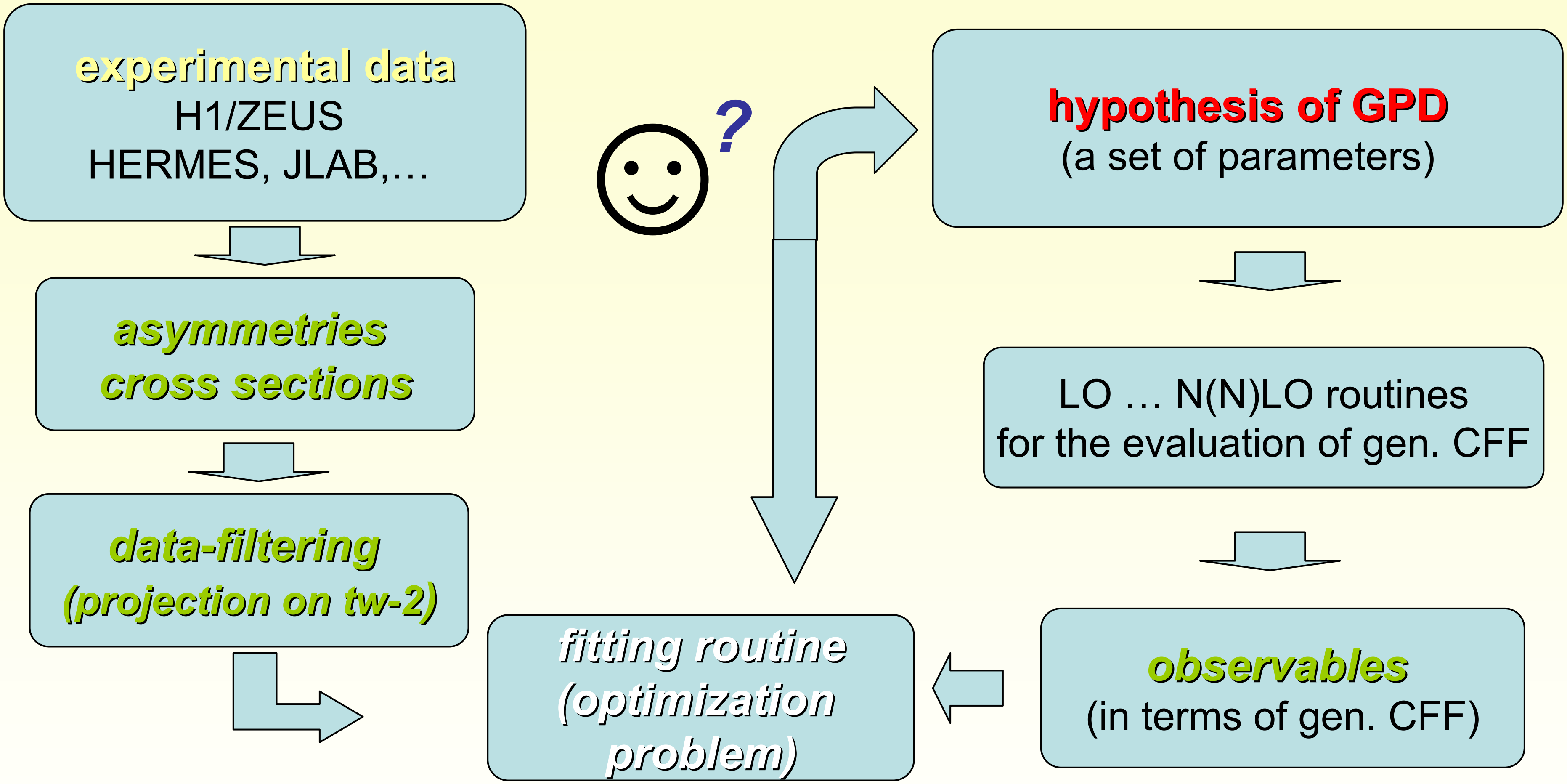


net contribution of outer + central region is governed by a sum rule:

$$\mathrm{PV}\int_0^1 dx\, \frac{2x}{\eta^2 - x^2} H^-(x,\eta,t)$$

$$= \mathrm{PV}\int_0^1 dx\, \frac{2x}{\eta^2 - x^2} H^-(x,x,t) + C(t)$$

The fitting problem



- many different observables (formed from cross sections)
- complex theoretical formulae, many modeling possibilities (many parameters)
- GPDs depends on form factors and PDFs, too (known only to a certain extend)
- i.e., to pin down GPDs one might fit to FF, structure functions, and exclusive data

Data set for unpolarized proton target

- H1/ZEUS 98 [σ, $d\sigma/dt$] +1x6 [BCA(φ)] $\langle\langle x \rangle\rangle \approx 10^{-3}$, $\langle|t|\rangle \leq 0.8$ GeV^2 $\langle\langle Q^2 \rangle\rangle \approx 8$ GeV^2
- HERMES(02) 12+3 [BSA, $sin(\varphi)$]
- HERMES(08) 12x2 [BCA, $cos(0\ \varphi)$, $cos(\varphi)$] $0.05 \leq \langle x \rangle \leq 0.2$, $\langle|t|\rangle \leq 0.4$ GeV^2
  12x2 [$cos(2\ \varphi)$, $cos(3\ \varphi)$] $\langle\langle Q^2 \rangle\rangle \approx 2.5$ GeV^2
- HERMES(09) not included new BSA and BCA data
- CLAS(07) 12x12 [BSA(φ)] $0.14 \leq \langle x \rangle \leq 0.35$, $\langle|t|\rangle \leq 0.3$ GeV^2
  40x12 [BSA(φ)] (large |t| or bad sta.) $\langle\langle Q^2 \rangle\rangle \approx 1.8$ GeV^2
- HALL A(06) 12x24 [$\Delta\sigma(\varphi)$] $\langle x \rangle = 0.36$, $\langle|t|\rangle \leq 0.33$ GeV^2
  3x24 [$\sigma(\varphi)$] $\langle\langle Q^2 \rangle\rangle \approx 1.8$ GeV^2

How to analyze φ dependence?

- fit within assumed functional form **[CLAS(07)]**
- fit with respect to dominant and higher harmonics **[HERMES(08)]**
- utilize Fourier analyze (with or without additional weight) **[BMK(01)]**

⇒ equivalent results for CLAS data with small stat. errors

DVCS fits for H1 and ZEUS data

DVCS cross section measured at small $x_{\rm Bj} \approx 2\xi = \frac{2\mathcal{Q}^2}{2W^2+\mathcal{Q}^2}$

$$40\text{GeV} \lesssim W \lesssim 150\text{GeV}, \quad 2\text{GeV}^2 \lesssim \mathcal{Q}^2 \lesssim 80\text{GeV}^2, \quad |t| \lesssim 0.8\text{GeV}^2$$

predicted by

$$\frac{d\sigma}{dt}(W,t,\mathcal{Q}^2) \approx \frac{4\pi\alpha^2}{\mathcal{Q}^4}\frac{W^2\xi^2}{W^2+\mathcal{Q}^2}\left[|\mathcal{H}|^2 - \frac{\Delta^2}{4M_{\rm p}^2}|\mathcal{E}|^2 + \left|\widetilde{\mathcal{H}}\right|^2\right](\xi,t,\mathcal{Q}^2)\Big|_{\xi=\frac{\mathcal{Q}^2}{2W^2+\mathcal{Q}^2}}$$

suppressed contributions *<<0.05>>* relative *O(ξ)*

- LO data could not be described before **2008**
- NLO works with ad hoc GPD models **[Freund, McDermott (02)]**

  results strongly depend on employed PDF parameterization

⇨ ***do a simultaneous fit to DIS and DVCS*** **[KMP-K (07)]**

➡ ***use flexible GPD models in a two-step fit*** **[KMP-K (08)]**

effective functional form at small x:

PDFs: $$q^{\rm sea}(\xi,\mathcal{Q}) = n(\mathcal{Q})\xi^{-\alpha(\mathcal{Q})}, \quad \alpha \sim 1, \quad F^{\rm sea}(0) = 1$$

GPDs: $$H = r(\eta/x=1,\mathcal{Q})F^{\rm sea}(t)\xi^{\alpha'(t,\mathcal{Q})}q^{\rm sea}(\xi,\mathcal{Q})$$

skewness ***transverse distribution***

? $E(\xi,\xi,t,\mathcal{Q})$

neglected in "standard" Regge phenomenology

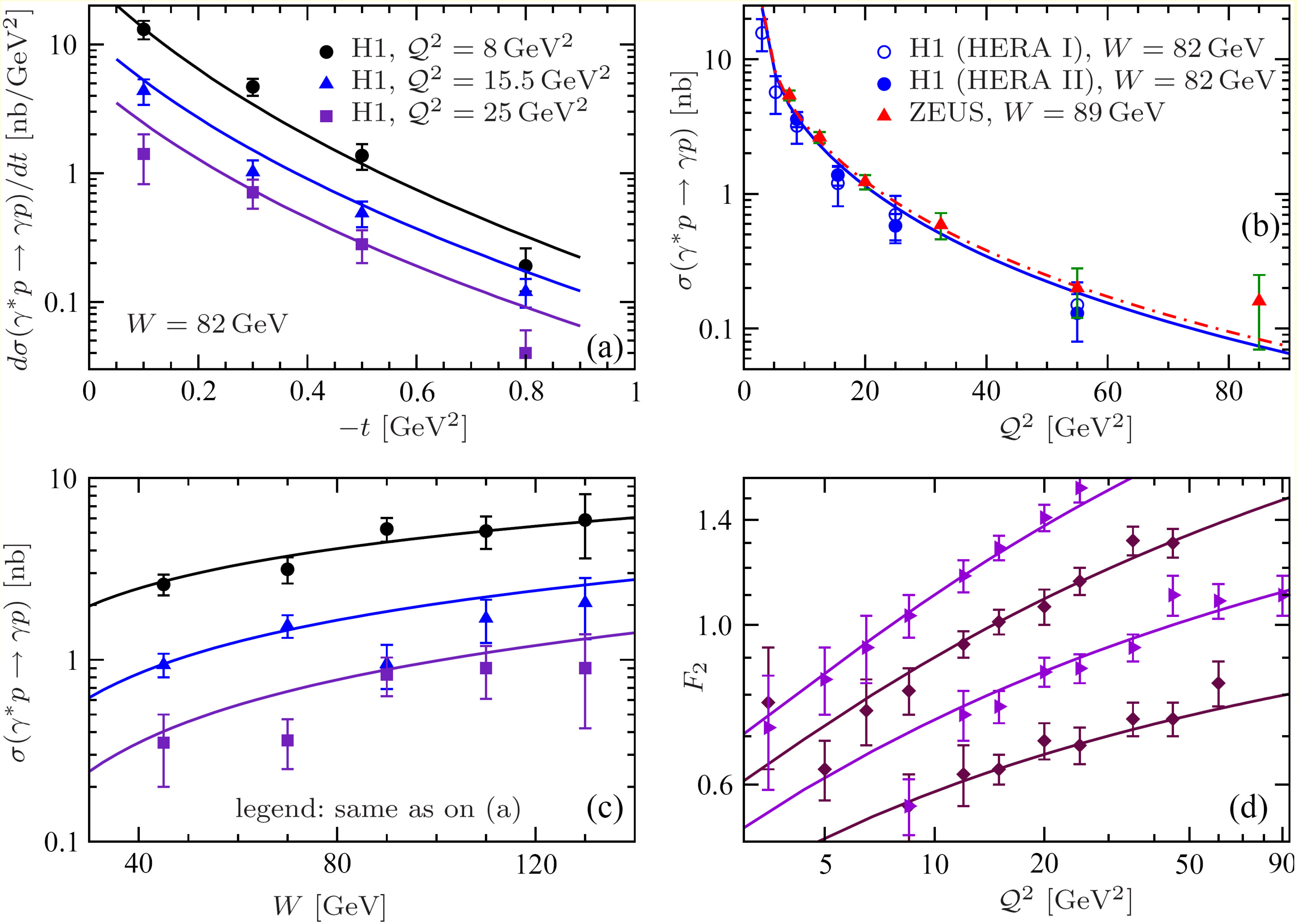
chromo-magnetic "pomeron" might be sizeable (instantons)

pQCD suggests pomeron intercept

qualitative understanding of *E* is needed (not only forJi`s spin sum rule)

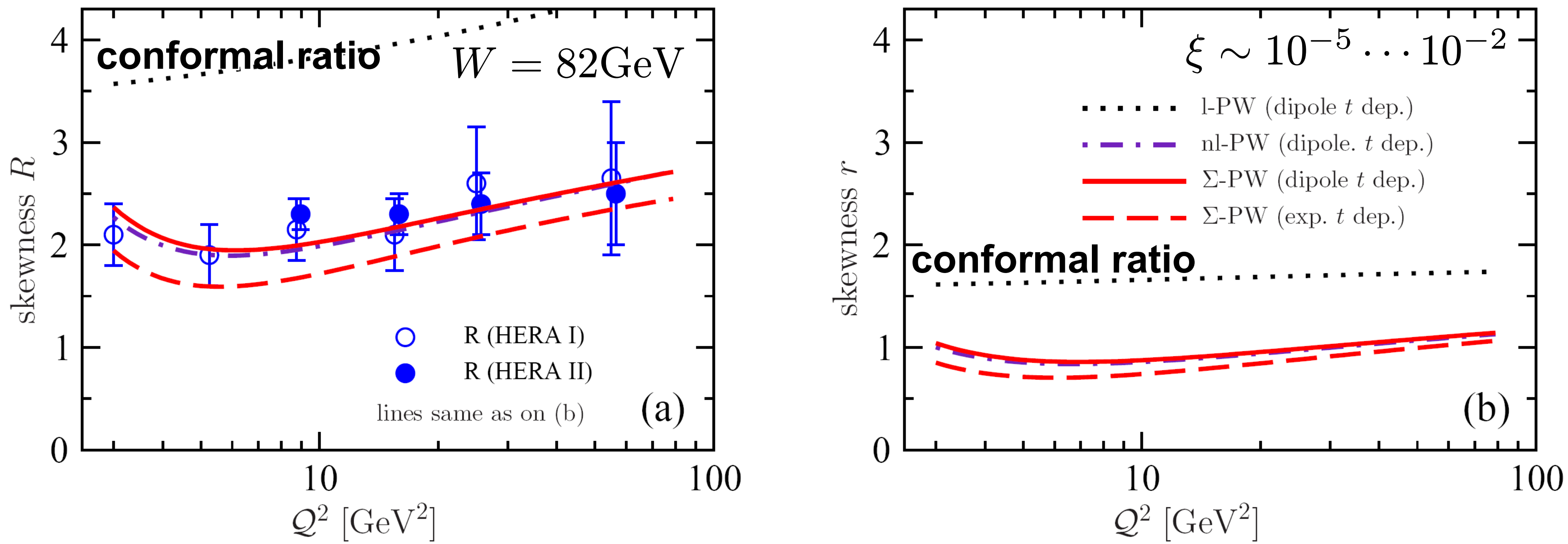
$$B = \int_0^1 dx\, x E(x,\eta,t,\mathcal{Q})$$

good DVCS fits at LO, NLO, and NNLO with flexible GPD ansatz



quark skewness ratio from DVCS fits @ LO

$$R = \frac{\Im\mathrm{m} A_{\mathrm{DVCS}}}{\Im\mathrm{m} A_{\mathrm{DIS}}} \stackrel{LO}{=} \frac{H(\xi,\xi)}{H(2\xi,0)} \approx 2^{\alpha} r \qquad r = \frac{H(\xi,\xi)}{H(\xi,0)}$$

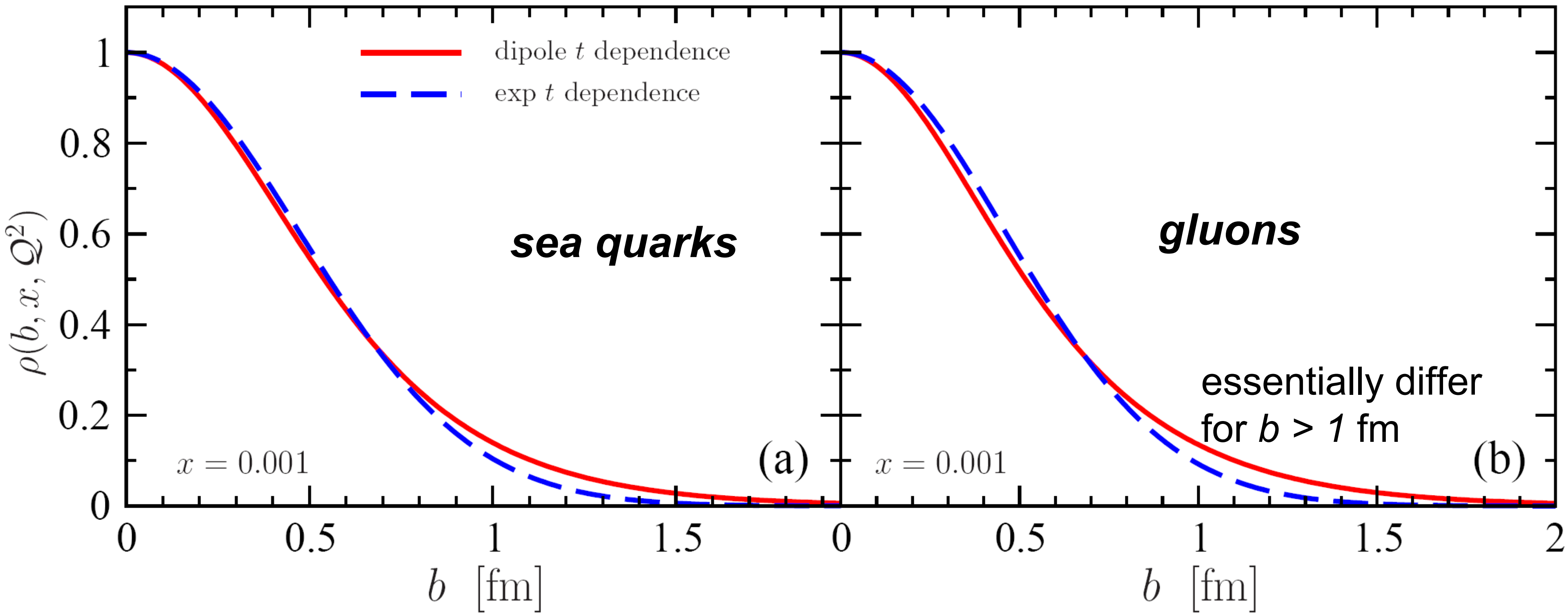


- @LO the conformal ratio is ruled out for sea quark GPD
- a generically zero-skewness effect over a large Q^2 lever arm
- scaling violation consistent with pQCD prediction
- this zero-skewness effect is non-trivial to realize in conformal space (SO(3) sibling poles are required)

- CFF *H* posses ``pomeron behavior'' $\xi^{-\alpha(Q) - \alpha'(Q)t}$
  - ✓ ***α* increases with growing** *Q^2*
  - ✓ ***α'* decreases growing** *Q^2*
- *t*-dependence: exponential shrinkage is disfavored *($\alpha' \approx 0$)*

  dipole shrinkage is visible *($\alpha' \approx 0.15$ at Q^2=4 GeV^2)*
- (normalized) profile functions

$$\rho \propto \int d^2\vec{\Delta}_\perp \, e^{i\vec{b}\cdot\vec{\Delta}_\perp} H(x, 0, t = -\vec{\Delta}_\perp^2)$$



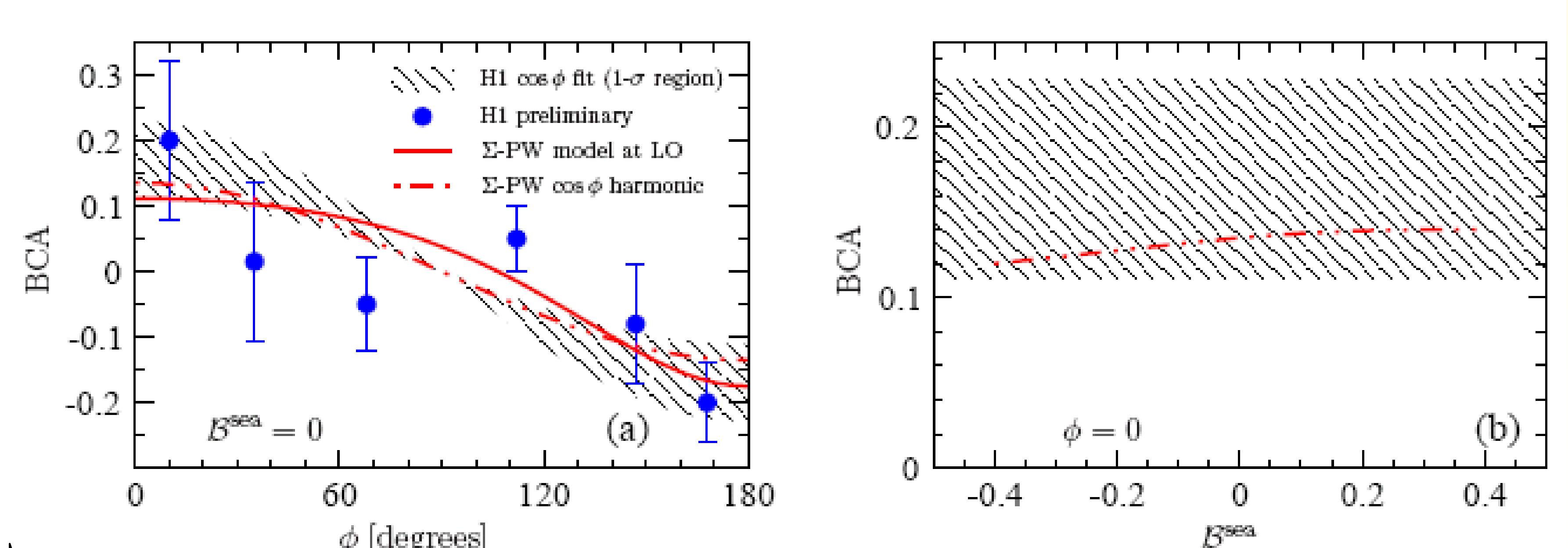
Beam charge asymmetry

$$BCA = \frac{d\sigma_{e^+} - d\sigma_{e^-}}{d\sigma_{e^+} + d\sigma_{e^-}} = \frac{\mathcal{T}_{\text{Interference}}}{|\mathcal{T}_{\text{BH}}|^2 + |\mathcal{T}_{\text{DVCS}}|^2}$$

$$\propto F_1(t)\Re\mathrm{e}\mathcal{H} + \frac{|t|}{4M^2} F_2(t)\Re\mathrm{e}\mathcal{E}$$

the unknown in Ji's nucleon spin sum rule

- set $E_{\text{sea}} \propto H_{\text{sea}}$, use *anomalous gravitomagnetic moment* $B_{\text{sea}} = \int_0^1 dx\, x E_{\text{sea}}$ as parameter



unfortunately, H1 data do not allow to access B_{sea}

Dispersion relation fits to unpolarized DVCS

- model of GPD *H(x,x,t) within DD motivated ansatz at* Q^2=2 GeV^2

fixed: *PDF normalization* — *eff. Reage pole* — *large t-counting rules*

$$H(x,x,t) = \frac{n\,r\,2^\alpha}{1+x}\left(\frac{2x}{1+x}\right)^{-\alpha(t)}\left(\frac{1-x}{1+x}\right)^{b}\frac{1}{\left(1-\frac{1-x}{1+x}\frac{t}{M^2}\right)^{p}}.$$

free: *r-ratio at small x* — *large x-behavior* — *p-pole mass*

sea quarks (taken from LO fits)

$$n = 0.68,\quad r = 1,\quad \alpha(t) = 1.13 + 0.15t/\mathrm{GeV}^2,\quad m^2 = 0.5\mathrm{GeV}^2,\quad p = 2$$

valence quarks

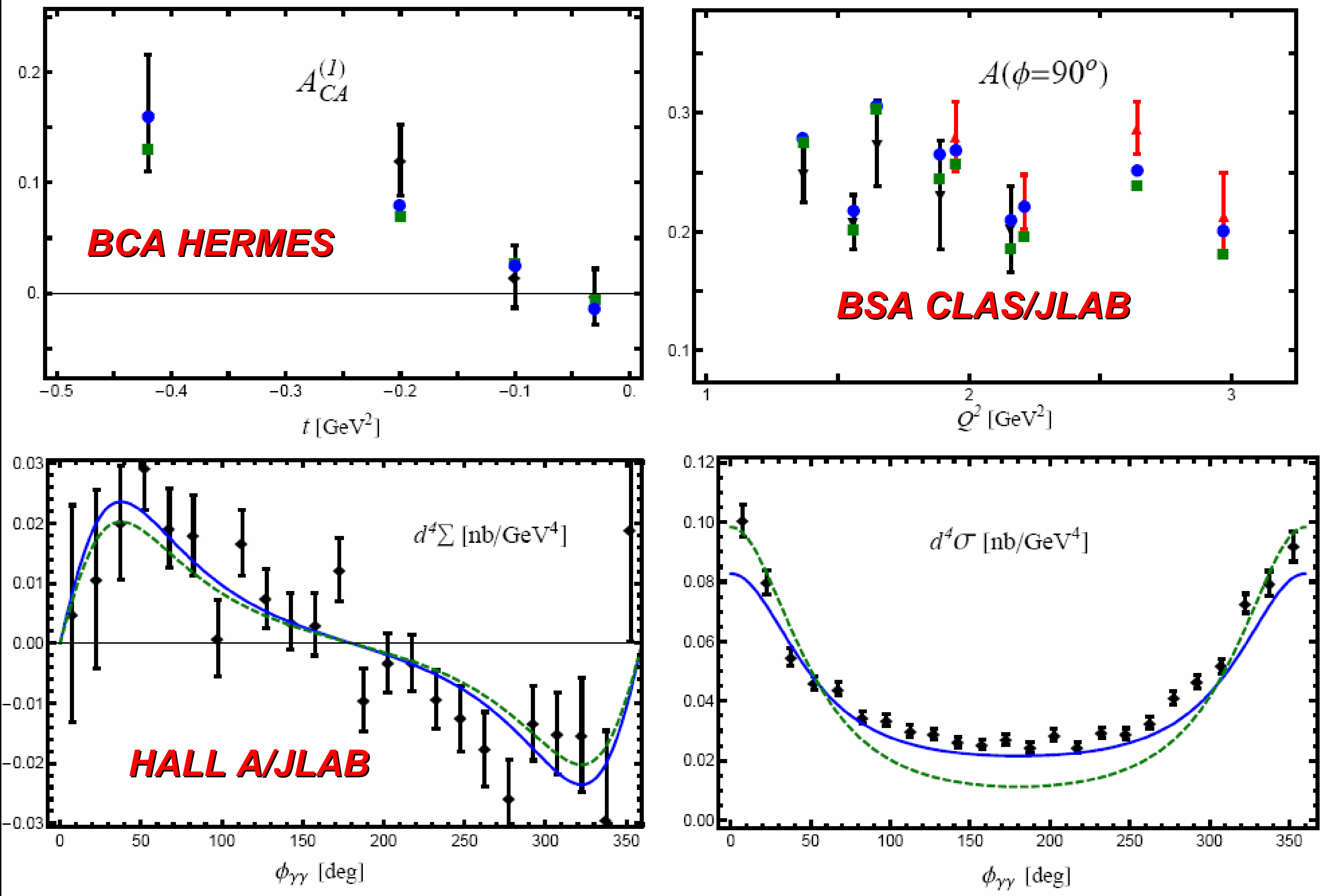
$$n = 1.0,\quad \alpha(t) = 0.43 + 0.85t/\mathrm{GeV}^2,\quad p = 1$$

flexible parameterization of subtraction constant $\mathcal{D}(t) = \frac{-C}{(1-t/M_c^2)^2}$

+ pion-pole contribution

36 + 4 data points quality of ***global fit*** is good $\chi^2/\mathrm{d.o.f.} \approx 1$

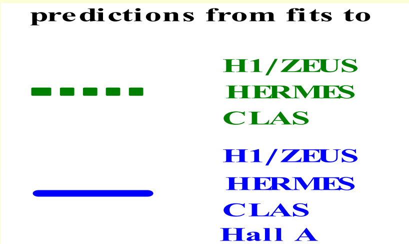
Global GPD fit example: HERMES & JLAB

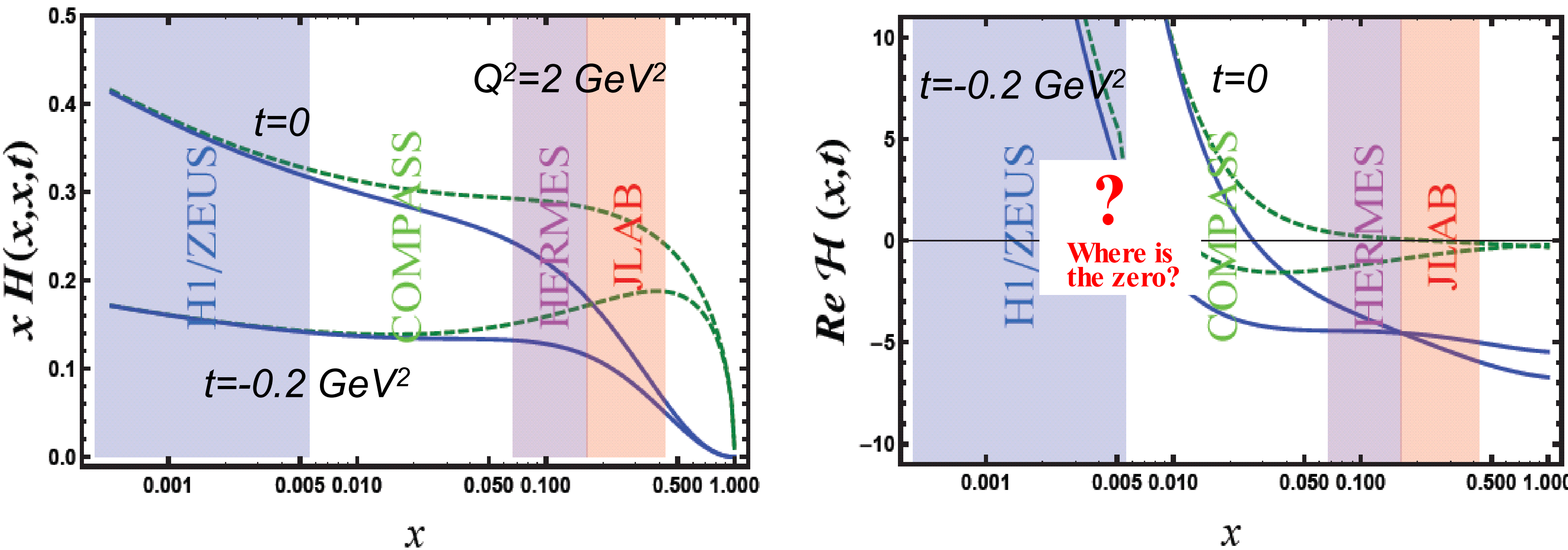


A qualitative interpretation of first global fits

$\widetilde{H}(x,x,t)$ is **two × bigger** as $H(x,x,t)$ in valence region (sounds wrong)

- ➢ ansatz is to improve (or to reinterpret)
- ➢ longitudinal polarized proton data will help to pin down $\mathcal{H}$
- ➢ real part of $\mathcal{H}$ is ***crucial*** to reveal *H(x,x,t)*





- real part of $\mathcal{H}$ has a zero: ***Can it be revealed by COMPASS?***

✓ large negative value of $\mathcal{R}e\mathcal{H}$*@x=1* arises from substraction constant (so-called ***D-term***, **[Goeke, Polyakov, Vanderhaeghen (01); lattice (≥ 07)]**)

DVCS observables at COMPASS (unpolarized target)

$$d\sigma^{\downarrow +}(\phi),\ \ d\sigma^{\uparrow -}(\phi),\ \ \ d\sigma^{\downarrow +}(\phi) \pm d\sigma^{\downarrow +}(\pm\phi),\ \ \ d\sigma^{\downarrow +}(\phi) \pm d\sigma^{\uparrow -}(\pm\phi),$$

- revealing real part of $\mathcal{H}$

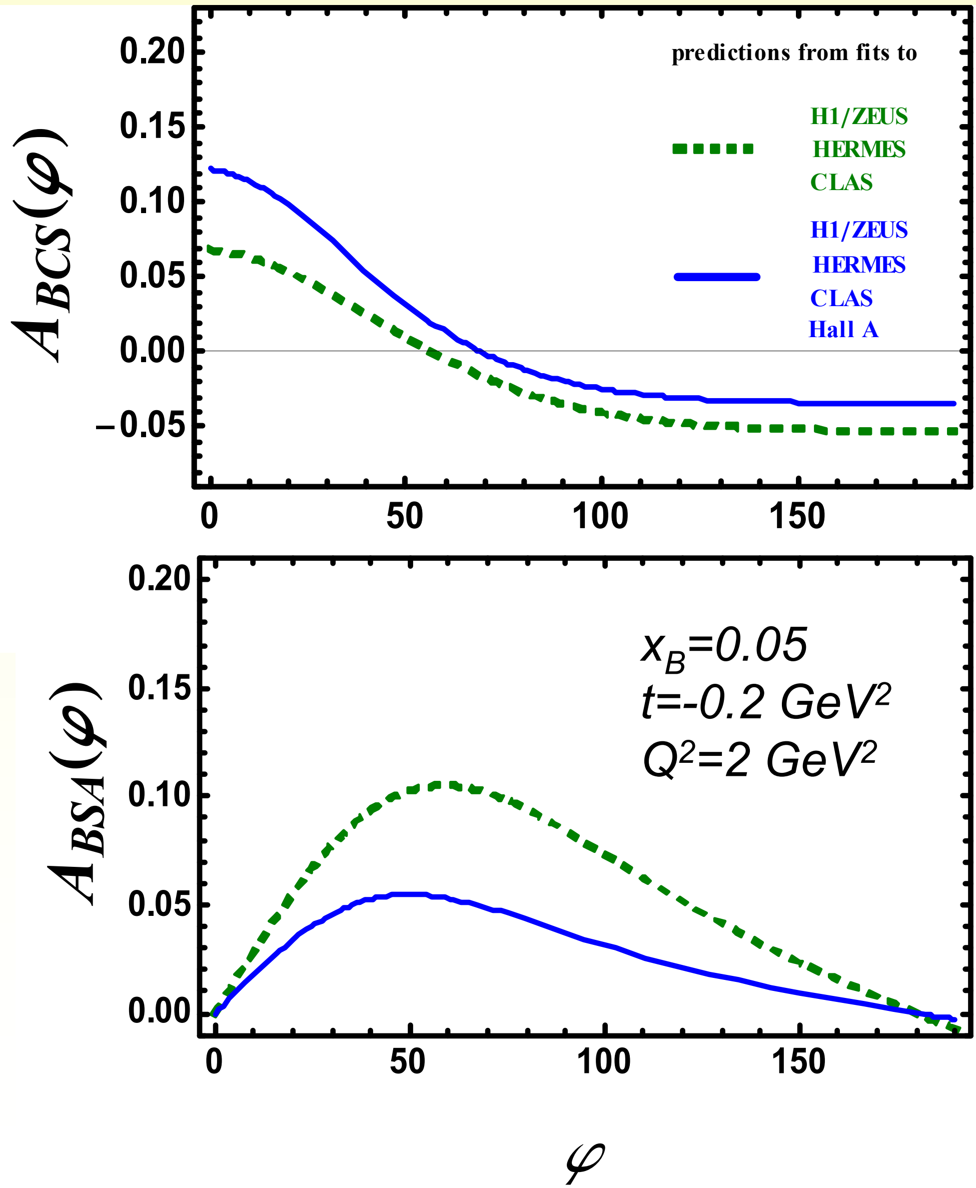
$$d\sigma^{\downarrow +}(\phi) - d\sigma^{\uparrow -}(\phi)$$

$$A_{\rm BCS} = \frac{d\sigma^{\downarrow +}(\phi) - d\sigma^{\uparrow -}(\phi)}{d\sigma^{\downarrow +}(\phi) + d\sigma^{\uparrow -}(\phi)}$$

- revealing imaginary part of $\mathcal{H}$

$$d\sigma^{\downarrow +}(\phi) - d\sigma^{\downarrow +}(-\phi)$$

$$A_{\rm BSA} = \frac{d\sigma^{\downarrow +}(\phi) - d\sigma^{\downarrow +}(-\phi)}{d\sigma^{\downarrow +}(\phi) + d\sigma^{\downarrow +}(-\phi)}$$



A GPD fit agenda

(a personal view)

decomposition of twist-two CFFs *(with and without twist-two dominance hypothesis)*

- *dispersion integral fits (least square method) on the full set of fixed target data*
- *data filtering is crucial to get rid of the twist-two dominance hypothesis*

GPD model fits *(based on the least square method)*

- *fully flexible GPD model in conformal moment space up to NNLO*
- *(reggeized) spectator quark models up (to NLO)*
  (given in double distribution representation, positivity constraints are implemented)
- *holographic GPD models, such as Radyushkin double distribution model*

neural networks *(representing and extracting CFFs or GPDs)*

- *to get rid of theoretical biases*
- *error propagation/estimates*

Towards realistic GPD (TMD) models

$$\mathcal{L} = \bar{\psi}\left(i\partial\!\!\!/ - m\right)\psi - \frac{1}{2}\phi\left(\partial^2 + \lambda^2\right)\phi + g\bar{\psi}\psi\phi$$

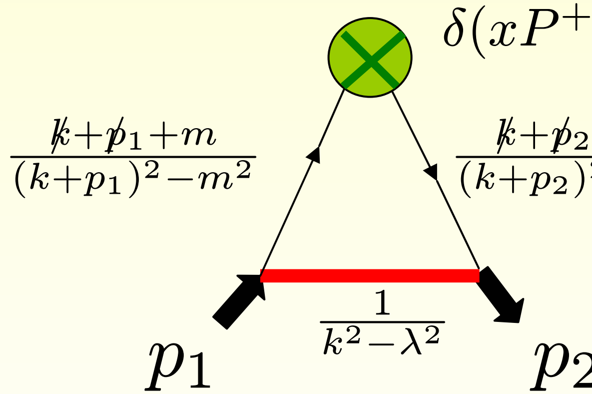
struck spin-*1/2* quark

collective scalar diquark spectator

coupling knows about spin

Diagrammatic approach:

via covariant time ordered perturbation theory



many studies of spectator quark models

[HWANG, DM (07)]

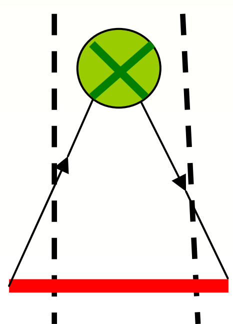
LC- Hamiltonian approach

$$k^\mu \rightarrow (k^+, k^-, \mathbf{k}_\perp),\ k^\pm = k^0 \pm k^3,\ \mathbf{k}_\perp = (k^1, k^2).$$

integrate out minus component to find LCWF

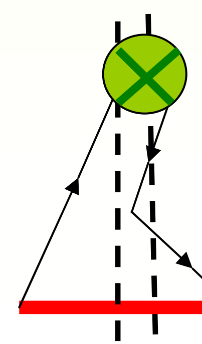
parton number conserved LCWF

(outer region)



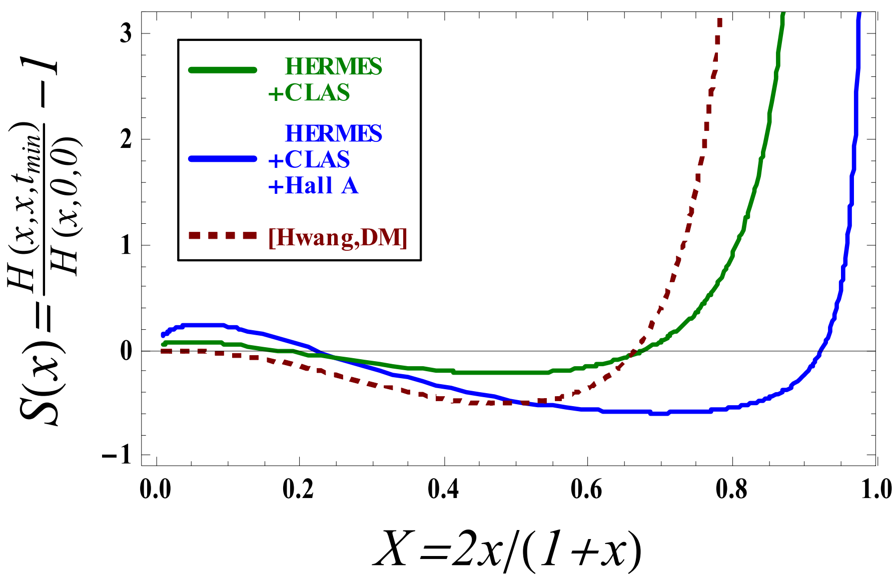
parton number violating LCWF

(central region)



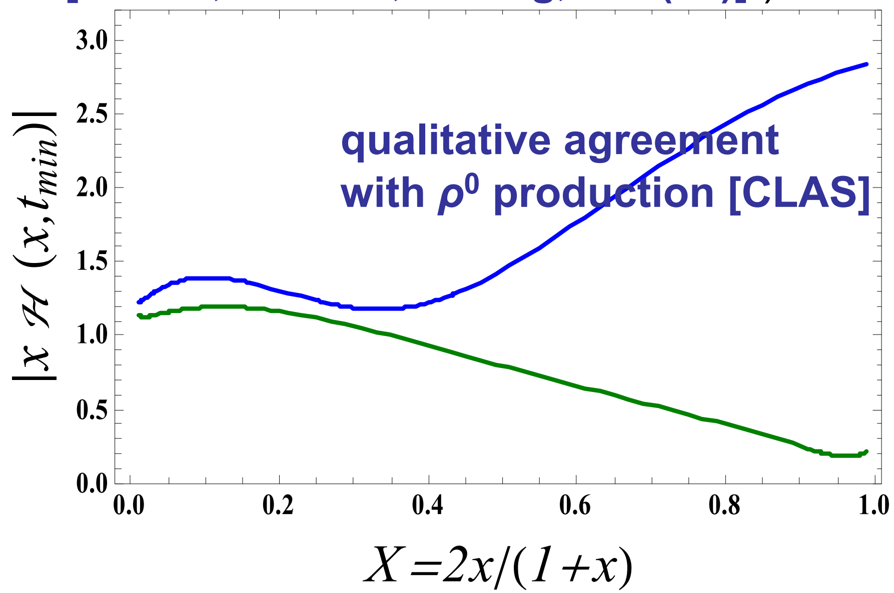
✓ ***skewness effect*** :

(as expected: **[Hwang, DM (07)]**



✓ ***enhanced*** $|\mathcal{H}|$ @ large $X = x_{Bj}$:

[Guidal, Morrow; Hwang, DM (07)])



➢ ***femto-photography*** **[Pire, Ralston (01)]**

$$b_\perp^{\text{pseudo}} = \sqrt{4 \frac{d}{dt} \ln H(x,x,t)}\Big|_{t=0}$$

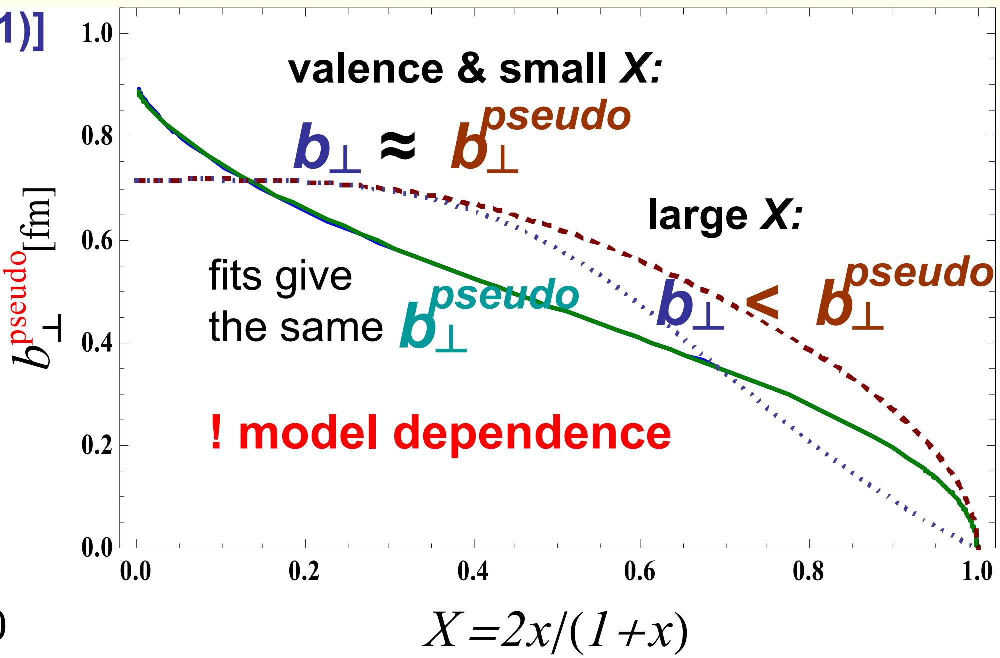
viewed as transverse `pseudo' width **[DM]**

amplitude interpretation **[Diehl (02)]**

(distance of struck quark to spectator system)

recall of transverse width **[Burkardt (02)]**

$$b_\perp = \sqrt{4 \frac{d}{dt} \ln H(x,0,t)}\Big|_{t=0}$$



Summary

hard exclusive leptoproduction

- possesses a rich structure, allowing to access various CFFs/GPDs
- it is elaborated in NLO and offers a new insight in QCD
- DVCS is widely considered as a theoretical clean process
- they address our partonic/QCD understanding

GPDs are intricate and (thus) a promising tool

- COMPASS DVCS physics is complementary to, e.g., JLAB 12@GeV
- unpolarized target: revealing the real and imaginary part of H
- needed to reveal *t*- and skewness dependence of H GPD
- transversally polarized target: allows to access E (expected to be sizeable)

tools/technology for next generation of global fits are required:

to quantify the partonic picture and to get a better QCD understanding

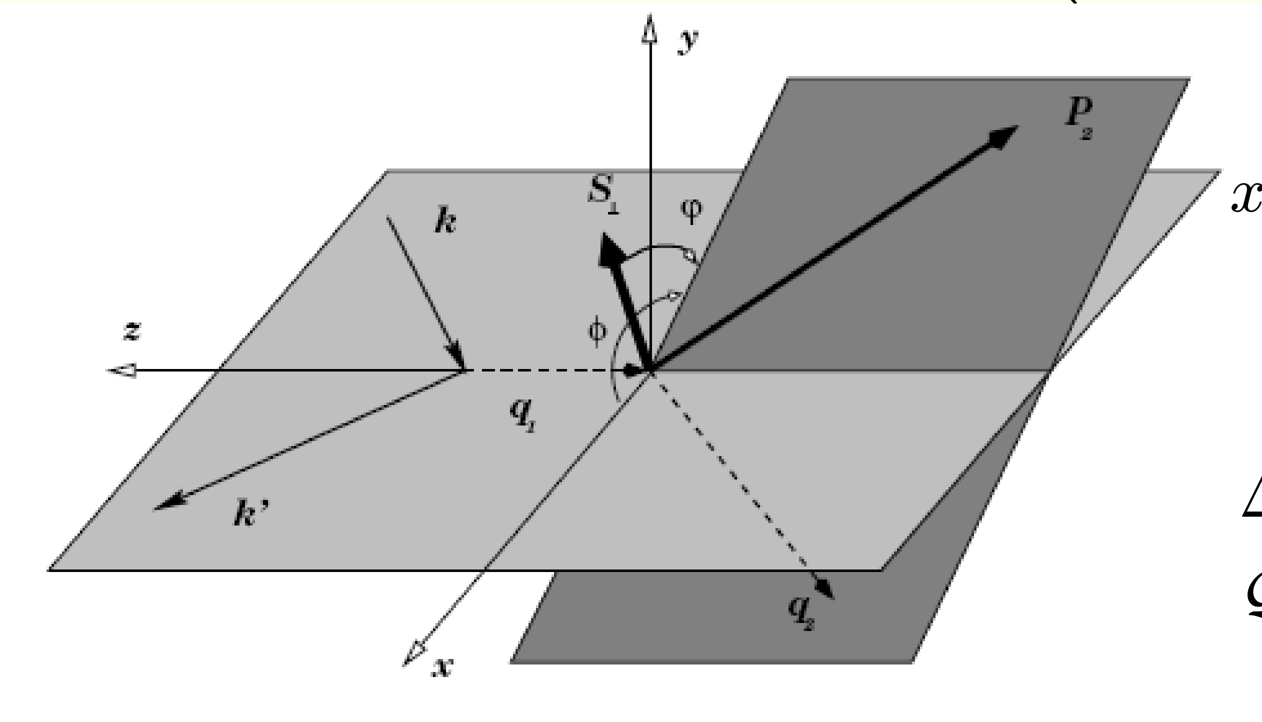
Back up slides are coming

Photon leptoproduction $e^{\pm}N \to e^{\pm}N\gamma$

measured by **H1, ZEUS, HERMES, CLAS, HALL A** collaborations

planed at **COMPASS, JLAB@12GeV**, perhaps at ?? EIC,

$$\frac{d\sigma}{dx_{\mathrm{Bj}}dyd|\Delta^2|d\phi d\varphi} = \frac{\alpha^3 x_{\mathrm{Bj}} y}{16\,\pi^2\,\mathcal{Q}^2}\left(1+\frac{4M^2x_{Bj}^2}{\mathcal{Q}^2}\right)^{-1/2}\left|\frac{\mathcal{T}}{e^3}\right|^2,$$



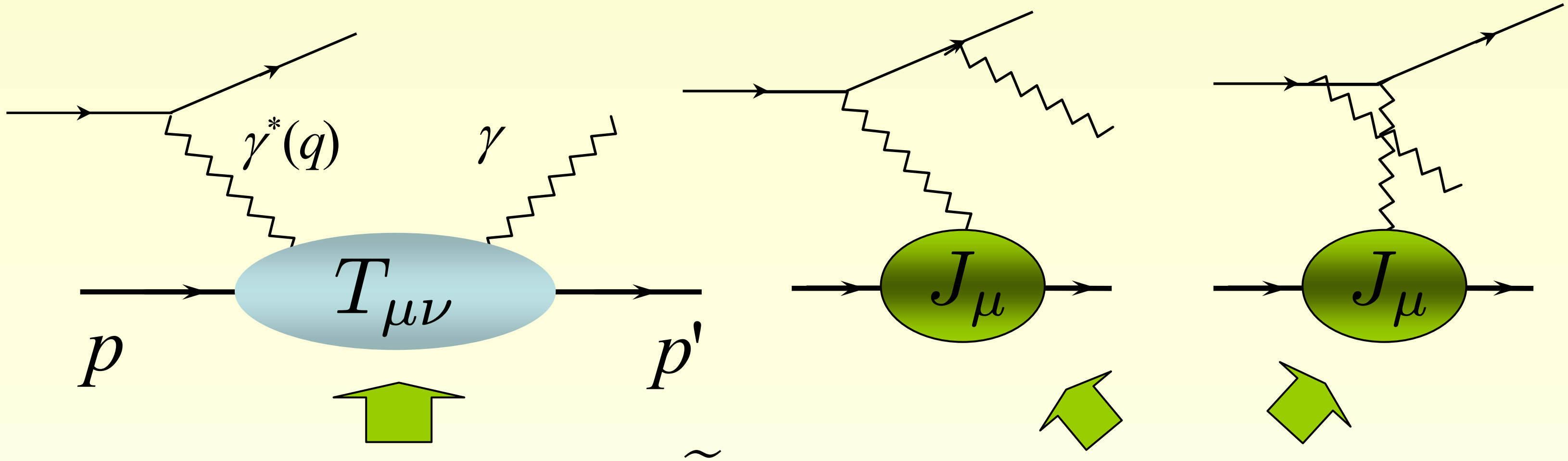
$$x_{\mathrm{Bj}} = \frac{\mathcal{Q}^2}{2P_1\cdot q_1} \approx \frac{2\xi}{1+\xi},$$

$$y = \frac{P_1\cdot q_1}{P_1\cdot k},$$

$$\Delta^2 = t \quad (\text{fixed, small}),$$

$$\mathcal{Q}^2 = -q_1^2 \quad (> 1\mathrm{GeV}^2),$$

interference of *DVCS* and *Bethe-Heitler* processes



12 Compton form factors $\mathcal{H}, \mathcal{E}, \widetilde{\mathcal{H}} \cdots$ (helicity amplitudes)

elastic form factors F_1, F_2

$$|\mathcal{T}_{\rm BH}|^2 = \frac{e^6(1+\epsilon^2)^{-2}}{x_{\rm Bj}^2 y^2 \Delta^2 \mathcal{P}_1(\phi)\mathcal{P}_2(\phi)} \left\{ c_0^{\rm BH} + \sum_{n=1}^{2} c_n^{\rm BH} \cos(n\phi) \right\},$$

exactly known (LO, QED)

$$|\mathcal{T}_{\rm DVCS}|^2 = \frac{e^6}{y^2 \mathcal{Q}^2} \left\{ c_0^{\rm DVCS} + \sum_{n=1}^{2} \left[c_n^{\rm DVCS} \cos(n\phi) + s_n^{\rm DVCS} \sin(n\phi) \right] \right\},$$

harmonics ⇕ 1:1 helicity ampl.

$$\mathcal{I} = \frac{\pm e^6}{x_{\rm Bj} y^3 \Delta^2 \mathcal{P}_1(\phi)\mathcal{P}_2(\phi)} \left\{ c_0^{\mathcal{I}} + \sum_{n=1}^{3} \left[c_n^{\mathcal{I}} \cos(n\phi) + s_n^{\mathcal{I}} \sin(n\phi) \right] \right\}.$$

harmonics ⇕ 1:1 helicity ampl.

relations among **harmonics** and **GPDs** are based on $1/\mathcal{Q}$ expansion:
(all harmonics are expressed by twist-2 and -3 GPDs) **[Diehl et. al (97) Belitsky, DM, Kirchner (01)]**

$$\begin{Bmatrix} c_1 \\ s_1 \end{Bmatrix}^{\mathcal{I}} \propto \frac{\Delta}{\mathcal{Q}} \text{tw-2(GPDs)} + O(1/\mathcal{Q}^3), \qquad c_0^{\mathcal{I}} \propto \frac{\Delta^2}{\mathcal{Q}^2} \text{tw-2(GPDs)} + O(1/\mathcal{Q}^4),$$

$$\begin{Bmatrix} c_2 \\ s_2 \end{Bmatrix}^{\mathcal{I}} \propto \frac{\Delta^2}{\mathcal{Q}^2} \text{ tw-3(GPDs)} + O(1/\mathcal{Q}^4), \qquad \begin{Bmatrix} c_3 \\ s_3 \end{Bmatrix}^{\mathcal{I}} \propto \frac{\Delta \alpha_s}{\mathcal{Q}} (\text{tw-2})^{\mathrm{T}} + O(1/\mathcal{Q}^3),$$

$$c_0^{\mathrm{CS}} \propto (\text{tw-2})^2, \qquad \begin{Bmatrix} c_1 \\ s_1 \end{Bmatrix}^{\mathrm{CS}} \propto \frac{\Delta}{Q} (\text{tw-2})(\text{tw-3}), \qquad \begin{Bmatrix} c_2 \\ s_2 \end{Bmatrix}^{\mathrm{CS}} \propto \alpha_s (\text{tw-2})(\text{tw-2})^{\mathrm{GT}}$$

setting up the **perturbative framework:** **[Belitsky, DM (97); Mankiewicz et. al (97); Ji,Osborne (98)]**

- ✓ ***twist-two*** coefficient functions at ***next-to-leading*** order
- ✓ evolution kernels at ***next-to-leading*** order **[Belitsky, DM, Freund (01)]**
- ✓ ***next-to-next-to-leading*** order in a specific conformal subtraction scheme **[KMP-K & Schaefer 06]**
- ✓ ***twist-three*** including quark-gluon-quark correlation at LO **[Anikin,Teryaev, Pire (00); Belitsky DM (00); Kivel et. al]**
- ✓ partial ***twist-three*** sector at ***next-to-leading*** order **[Kivel, Mankiewicz (03)]**
- ✓ \`target mass corrections' (not well understood) **[Belitsky DM (01)]**

Overview: GPD representations

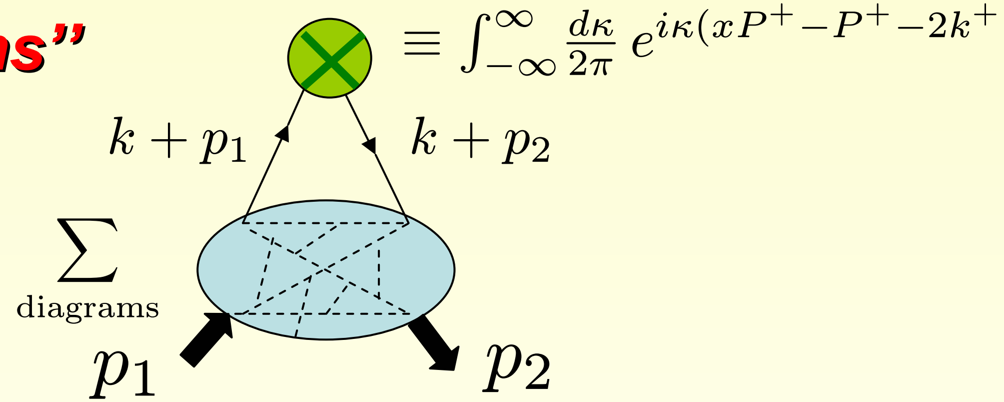
``light-ray spectral functions''

diagrammatic α-representation

DM, Robaschik, Geyer, Dittes, Hořejši (88 (92) 94)

called ***double distributions***

A. Radyushkin (96)



light cone wave function overlap

(Hamiltonian approach in light-cone quantization)

Diehl, Feldmann, Jakob, Kroll (98,00)

Diehl, Brodsky, Hwang (00)

SL(2,R) (conformal) expansion

(series of local operators)

Radyushkin (97); Belitsky, Geyer, DM, Schäfer (97); DM, Schäfer (05); ….

one version is called Shuvaev transformation, used in `dual' (*t*-channel) GPD parameterization

Shuvaev (99,02); Noritzsch (00) Polyakov (02,07)

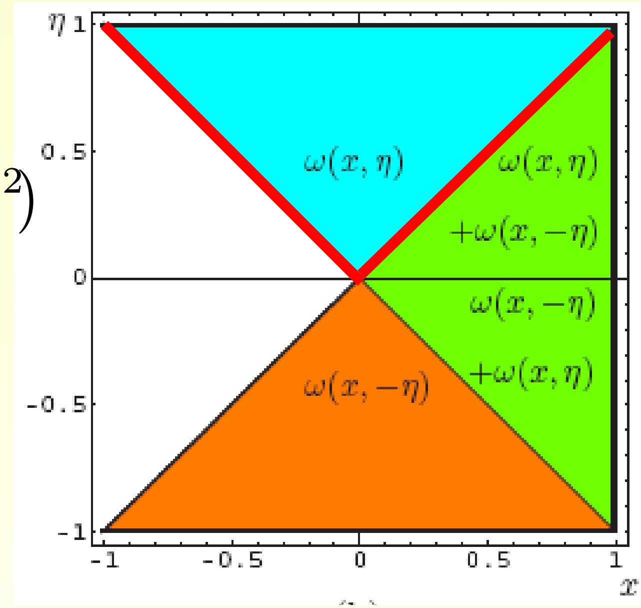
each representation has its own ***advantages***, however, they are ***equivalent*** **(clearly spelled out in [Hwang, DM 07])**

A partonic duality interpretation

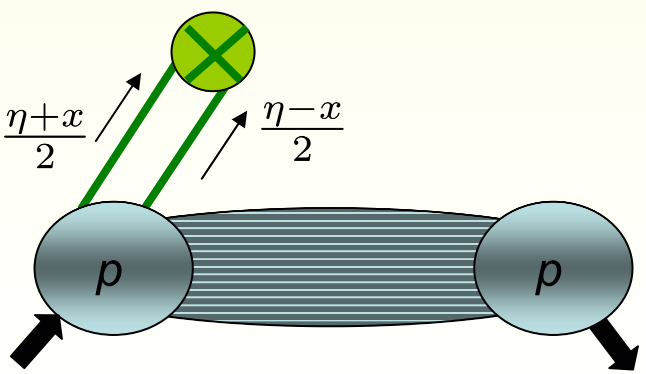
quark GPD (anti-quark $x \to -x$):

$$F = \theta(-\eta \le x \le 1)\, \omega\big(x, \eta, \Delta^2\big) + \theta(\eta \le x \le 1)\, \omega\big(x, -\eta, \Delta^2\big)$$

$$\omega\left(x, \eta, \Delta^2\right) = \frac{1}{\eta} \int_0^{\frac{x+\eta}{1+\eta}} dy\, x^p f\left(y, (x-y)/\eta, \Delta^2\right)$$

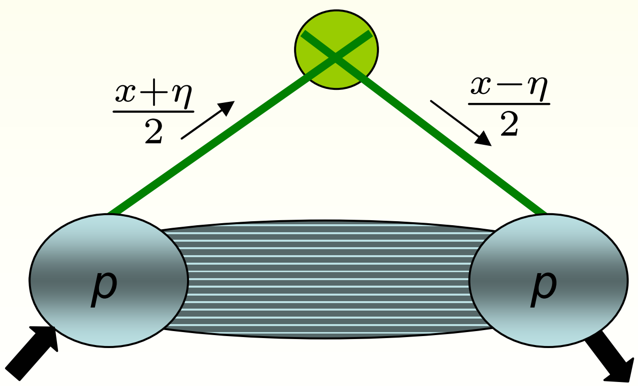


dual interpretation on partonic level:



support extension
is unique [DM et al. 92]

ambiguous (*D*-term)
[DM, A. Schäfer (05)
KMP-K (07)]



central region $-\eta < x < \eta$

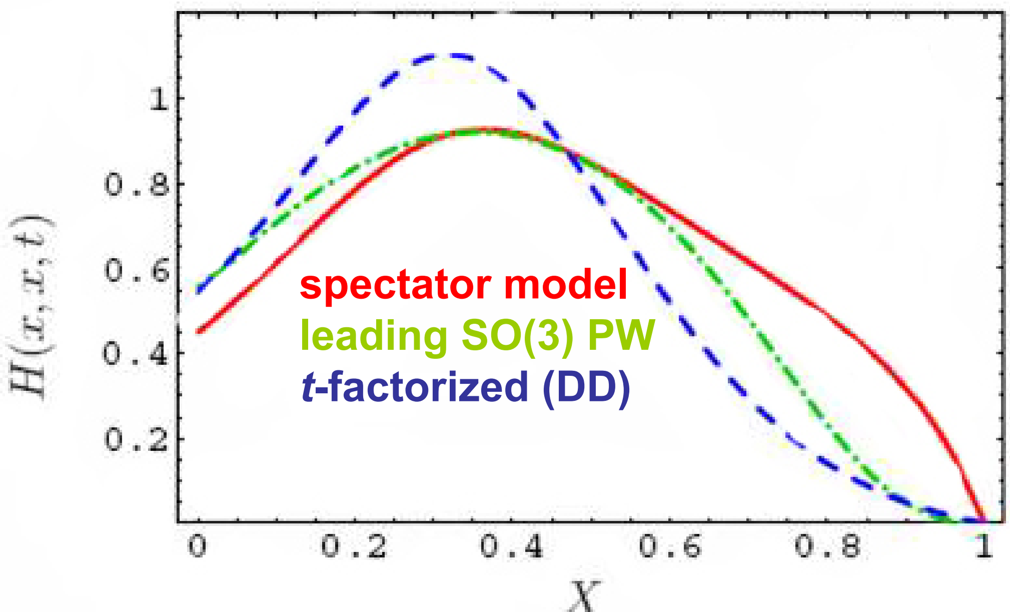
mesonic exchange in *t*-channel

outer region $\eta < x$

partonic exchange in *s*-channel

(partonic) `quantum' numbers in GPD representations

name	's-channel' variable	't-channel' variable
GPD	PMF x	PMF ratio η
DD	PMF y	PMF z
CPWE	conformal spin $j+2$	PMF ratio η
'forward-like' CPWE	forward-like PMF z	PMF ratio η
Mellin-Barnes CPWE	conformal spin $j+2$	PMF ratio η
'dual' CPWE	forward-like PMF z	$\rho = j+2-J$
'dual' Mellin-Barnes CPWE	conformal spin $j+2$	t-channel AM J
SO(3)-PWE	PMF x	t-channel AM J



? ***about representation is not so essential***

should be replaced by

How a GPD looks like on its cross-over trajectory ?

SL(2,R) representations for GPDs

- support is a consequence of Poincaré invariance (polynomiality)

$$H_j(\eta,t,\mu^2) = \int_{-1}^{1} dx\, c_j(x,\eta) H(x,\eta,t,\mu^2)\,, \qquad c_j(x,\eta) = \eta^j C_j^{3/2}(x/\eta)$$

- conformal moments evolve autonomous (to LO and beyond in a special scheme)

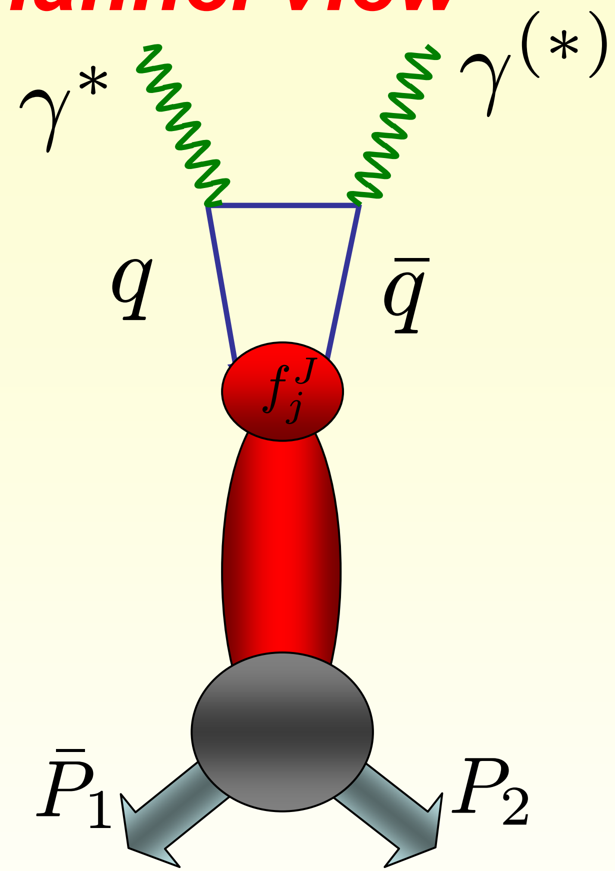
$$\mu \frac{d}{d\mu} H_j(\eta,t,\mu^2) = -\frac{\alpha_s(\mu)}{2\pi}\gamma_j^{(0)} H_j(\eta,t,\mu^2)$$

- inverse relation is given as series of mathematical distributions:

$$H(x,\eta,t) = \sum_{j=0}^{\infty} (-1)^j p_j(x,\eta) H_j(\eta,t)\,,\ p_j(x,\eta) \propto \theta(|x| \le \eta)\frac{\eta^2 - x^2}{\eta^{j+3}} C_j^{3/2}(-x/\eta)$$

- various ways of resummation were proposed:
  - smearing method [Radyushkin (97); Geyer, Belitsky, DM., Niedermeier, Schäfer (97/99)]
  - mapping to a kind of forward PDFs [A. Shuvaev (99), J. Noritzsch (00)]
  - dual parameterization (a mixture of both) [M. Polyakov, A. Shuvaev (02)]
  - based on conformal light-ray operators [Balitsky, Braun (89); Kivel, Mankewicz (99)]
  - ***Mellin-Barnes integral*** [DM, Schäfer (05); A. Manashov, M. Kirch, A. Schäfer (05)]

GPD ansatz at small x from t-channel view



- at short distance a quark/anti-quark state is produced, labeled by ***conformal spin*** *j*+2
- they form an intermediate mesonic state with total angular momentum ***J*** strength of ***coupling*** is $f_j^J,\ J \leq j+1$
- mesons propagate with $\frac{1}{m^2(J)-t} \propto \frac{1}{J-\alpha(t)}$
- decaying into a nucleon anti-nucleon pair with given angular momentum *J*, described by an ***impact form factor***

$$F_j^J(t) = \frac{f_j^J}{J-\alpha(t)} \frac{1}{(1-\frac{t}{M^2(J)})^p}$$

! GPD *E* is zero if chiral symmetry holds (partial waves are Gegenbauer polynomials with index 3/2)

D-term arises from the SO(3) partial wave *J=j+1 (j* → *-1)*

Can the skewness function be constrained from lattice ?

- relation among measurable and GPD Mellin moments at $\eta=0$:

$$\int_0^1 d\xi\, \xi^j \Im\mathrm{m}\mathcal{F}(\xi,t,\mathcal{Q}^2) \overset{\mathrm{LO}}{=} \pi f_j(t,\mathcal{Q}^2)\left[1+\delta_j(t,\mathcal{Q}^2)\right]$$

- deviation factors: $\delta_j(t,\mu^2) = \dfrac{\int_0^1 dx\, x^j S(x,t,\mu^2) F(x,\eta=0,t,\mu^2)}{\int_0^1 dx\, x^j F(x,\eta=0,t,\mu^2)}$

are given by a series of operator expectation values with increasing spin *j+n+1*

$$\delta_j(t,\mu^2) = \sum_{\substack{n=2\\ \text{even}}}^{\infty} \frac{f^{(n)}_{j+n}(t,\mu^2)}{f_j(t,\mu^2)}\,, \qquad f^{(n)}_j(t,\mu^2) = \frac{1}{n!}\frac{d^n}{d\eta^n} f_j(\eta,t,\mu^2)\Big|_{\eta=0}$$

- lattice can evaluate *j=0,1,2,(3)*, i.e., *n=2:* $\delta_0(t,\mu^2 = 4\,\mathrm{GeV}^2) \approx 0.2+?$ thanks to **Ph. Hägler**

- ? wrong expectation from evolution: $\delta_j \sim \dfrac{2^{j+1}\Gamma(5/2+j)}{\Gamma(3/2)\Gamma(3+j)} - 1$

the analog small *x* prediction is ruled out $\delta_0 \sim 0.5 \qquad \delta_1 \sim 1.5$

[Shuvaev et al. (99)]

Strategies to analyze DVCS data

GPD model approach:

ad hoc modeling: **(first decade)**

- **VGG** code [**Goeke et. al (01)** based on **Radyuskin's** DDA]
- BKM model [**Belitsky, Kirchner, DM (01)** based on RDDA]
- `aligned jet' model [**Freund, McDermott, Strikman (02)**]
- **Kroll/Goloskokov (05)** based on RDDA [not utilized for DVCS]
- `dual' model [**Polyakov,Shuvaev 02;Guzey,Teckentrup 06;Polyakov 07**]
- " -- " [**KMP-K (07)** in MBs-representation]
- Bernstein polynomials [**Liuti et. al (07)**]

dynamical models: not applied **[Radyuskin et.al (02); Tiburzi et.al (04); Hwang DM (07)]…**

flexible models: (for fits)

- any representation by including ***unconstrained*** degrees of freedom
- **KMP-K (07/08)** for H1/ZEUS in MBs-representation

What is the physical content of ***`invisible'*** (***unconstrained***) degrees of freedom?

Extracting CFFs from data: real and imaginary part

0. analytic formulae [BMK 01]

i. (almost) without modeling **[Guidal, Moutarde (08-10)]**

ii. dispersion integral fits **[KMP-K (08),KM (08/09)]**

iii. flexible GPD modeling **[KM (08/09)]**