# Phenomenological Experience with hard Meson Electroproduction 

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Outline:

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## Electroproduction of mesons

rigorous proof of collinear factorization for $Q^{2} \rightarrow \infty$ (Radyushkin (96); Collins et al (97))
hard subprocesses
$\gamma^{*} g \rightarrow V g, \gamma^{*} q \rightarrow V, P q$
and GPDs and meson w.f. (encode the soft physics)

dominant transition $\gamma_{L}^{*} \rightarrow V_{L}, P \quad$ other transitions power suppressed

## Transverse photon polarization matters

vector-meson electroproduction $R=\sigma_{L} / \sigma_{T} \quad$ (HERA $W \simeq 80 \mathrm{GeV}$ )
$\gamma_{T}^{*} \rightarrow V_{T}$ transitions substantial power corr. and/or higher twist needed

various moments of $\pi^{+}$cross section measured with trans. pol. target $\sin \phi_{s}$ moment very large does not seem to vanish for $t^{\prime} \rightarrow 0$ $A_{U T}^{\sin \phi_{S}} \propto \operatorname{Im}\left[M_{0+, 0+}^{*} M_{0-,++}\right]$ requires n -f. ampl. $\mathcal{M}_{0-,++}$
$\gamma_{T}^{*} \rightarrow P$ transitions substantial ( $H_{T}$, transversity)


HERMES (09)
$Q^{2} \simeq 2.5 \mathrm{GeV}^{2}, W=3.99 \mathrm{GeV}$

## Corrections to the l.-t. amplitudes?

vector-meson electroproduction:
predictions for $\sigma_{L}$ exceed data by a large factor (HERA $W=75 \mathrm{GeV} \rho$ ) power corr. and/or higher orders of pQCD?
(Diehl-Kugler 07, Ivanov 07)

$\pi^{+}$electroproduction:
contribution from pion exchange requires $\pi$ elm form factor (measured there) lead. twist only about a third of exp. value fails with cross section by order of magnitude
 additional contributions required
assump. of dominance of l.-t. contr. to LL ampl. at low $Q^{2}$ has no justification (large power corr. not implausible: see $\pi^{0} \gamma$ form factor
BaBar data up to $\simeq 40 \mathrm{GeV}^{2}$ cannot be understood without them)

## The $\gamma^{*} p \rightarrow V B$ amplitudes

need to go beyond coll. factorization $\Longrightarrow k_{\perp}$ factorization (mod. pert. appr.) (based on work by Ellis-Furmanski-Petronzio, Collins-Soper, Sterman et al) consider large $Q^{2}, W$ and small $t$;
kinematics fixes skewness: $\xi \simeq \frac{x_{\mathrm{Bj}}}{2-x_{\mathrm{Bj}}}\left[1+m_{V}^{2} / Q^{2}\right] \simeq x_{\mathrm{Bj}} / 2+$ m.m.c.

$$
\begin{aligned}
& \mathcal{M}_{\mu+, \mu+}(V)=\frac{e_{0}}{2}\left\{\sum_{a} e_{a} \mathcal{C}_{V}^{a a}\left\langle H_{\mathrm{eff}}^{g}\right\rangle_{V \mu}+\sum_{a b} \mathcal{C}_{V}^{a b}\left\langle H_{\mathrm{eff}}^{a b}\right\rangle_{V \mu}\right\}, \\
& \mathcal{M}_{\mu-, \mu+}(V)=-\frac{e_{0}}{2} \frac{\sqrt{-t}}{M+m}\left\{\sum_{a} e_{a} \mathcal{C}_{V}^{a a}\left\langle E^{g}\right\rangle_{V \mu}+\sum_{a b} \mathcal{C}_{V}^{a b}\left\langle E^{a b}\right\rangle_{V \mu}\right\},
\end{aligned}
$$

$\mathcal{C}_{V}^{a b}$ flavor factors, $\quad M(m)$ mass of $B(p), \quad H_{\text {eff }}=H-\xi^{2} /\left(1-\xi^{2}\right) E$ contributions from $\widetilde{H}$ to T-T amplitude not shown electroproduction with unpolarized protons at small $\xi$ :
$E$ not much larger than $H$ (see below) $\Longrightarrow H_{\text {eff }} \rightarrow H$ for small $\xi$ $\left|M_{\mu-, \mu+}\right|^{2} \propto t / m^{2}$ neglected $\quad \Longrightarrow$ probes $H \quad$ (exception $\rho^{+}$)
trans. polarized target:
polarized beam and target:
probes $\operatorname{Im}\left[\langle E\rangle^{*}\langle H\rangle\right]$ interference probes $\operatorname{Re}\left[\langle H\rangle^{*}\langle\widetilde{H}\rangle\right]$ interference

## Subprocess amplitudes

$F=H, E \quad \lambda$ parton helicities
$\langle F\rangle_{V \mu}^{a b(g)}=\sum_{\lambda} \int d \bar{x} \mathcal{H}_{\mu \lambda, \mu \lambda}^{V a b(g)}\left(\bar{x}, \xi, Q^{2}, t=0\right) F^{a b(g)}(\bar{x}, \xi, t)$
$F^{a a}=F^{a}, \quad F^{a b}=F^{a}-F^{b} \quad(a \neq b)$ (with flavor symmetry)


Sudakov factor (Sterman et al)
$S \propto \ln \frac{\ln \left(\tau Q / \sqrt{2} \Lambda_{\mathrm{QCD}}\right)}{-\ln \left(b \Lambda_{\mathrm{QCD}}\right)}+\mathrm{NLL}$ $\hat{\mathcal{F}} \mathrm{FT}$ of hard scattering kernel
e.g. FT of $\propto e_{a} /\left[k_{\perp}^{2}+\tau(\bar{x}+\xi) Q^{2} /(2 \xi)\right]$ regularizes also TT amplitude
in collinear appr:
TT: $\quad \int_{0}^{1} d \tau \frac{\Phi_{V}(\tau)}{\tau^{2}} \propto \int_{0}^{1} \frac{d \tau}{\tau}$
IR singular

## Double distributions

integral representation ( $\mathrm{i}=$ valence, sea quarks, gluons)

$$
H_{i}(\bar{x}, \xi, t)=\int_{-1}^{1} d \beta \int_{-1+|\beta|}^{1-|\beta|} d \alpha \delta(\beta+\xi \alpha-\bar{x}) f_{i}(\beta, \alpha, t)+D_{i} \Theta\left(\xi^{2}-\bar{x}^{2}\right)
$$

$f_{i}$ double distributions Mueller et al (94), Radyushkin (99) advantage - polynomiality automatically satisfied $D_{i}(\bar{x}, t)(i=$ gluon, sea) additional free function, support $-\xi<\bar{x}<\xi$ parameterization of $f_{i}$ in terms of forward limits and Regge-like $t$ dependence (forw. limit of $H$ : PDFs - reduction formula respected)

$$
\begin{aligned}
& f_{i}(\beta, \alpha, t)=h_{i}(\beta) \exp \left[\left(b_{i}+\alpha_{i}^{\prime} \ln (1 / \beta)\right) t\right] \frac{\Gamma\left(2 n_{i}+2\right)}{2^{2 n_{i}+1} \Gamma^{2}\left(n_{i}+1\right)} \frac{\left[(1-|\beta|)^{2}-\alpha^{2}\right]^{n_{i}}}{(1-|\beta|)^{2 n_{i}+1}} \\
& h(\beta)=q(\beta), \beta g(\beta) \quad \text { (properly continued to }-1<\beta<0 \text { ) } \\
& n_{g}=n_{\text {sea }}=2, \quad \alpha_{g}^{\prime}=\alpha_{\text {sea }}^{\prime}=0.15 \mathrm{GeV}^{-2} \text { (sea - gluon mixing under evolution) } \\
& n_{\text {val }}=1, \quad \alpha_{v}^{\prime}=0.9 \mathrm{GeV}^{-2}
\end{aligned}
$$

few free parameters $\left(b_{i}\right)$
if forw. limit unknown (e.g. $E$ ): $h_{i} \sim \beta^{\alpha_{R_{i}}(0)}(1-\beta)^{\alpha_{i}} \quad$ more parameters

## Numerical results for cross sections

$H$ constructed from CTEQ6 PDFs through the double distr. ansatz ( $D=0$, sum rules and positivity bounds checked numerically)

Gaussian wave fcts for the mesons $\quad \Psi_{V j}\left(\tau, \mathbf{k}_{\perp}\right) \propto \exp \left[-a_{V j}^{2} \mathbf{k}_{\perp}^{2} /(\tau \bar{\tau})\right]$ (MPA: 'Gegenbauer filter' - higher Gegenbauer terms strongly suppressed at low $Q^{2}$ )

L an T different, free parameters $-a_{L, T}^{V}\left(\right.$ transverse size $\left.\left\langle k_{\perp}^{2}\right\rangle^{1 / 2} \propto 1 / a_{L, T}^{V}\right)$
meson wf. provides effects of order $\left\langle k_{\perp}^{2}\right\rangle / Q^{2} \quad$ separation of both GPDs mainly influence the $\xi\left(x_{B j}\right)$ dependence effects possible
fit to all data from HERMES, COMPASS, E665, H1, ZEUS cover large range of kinematics $\quad Q^{2} \simeq 3-100 \mathrm{GeV}^{2} \quad W \simeq 5-180 \mathrm{GeV}$ main features of $H$ fairly well fixed

## Results on cross sections


$Q^{2}=3.8 \mathrm{GeV}^{2}$,
glue+sea, glue, valence +interf.
gluons (+ sea) dominant for COMPASS kinematics
data: H1 (open), ZEUS (filled squares), E665 (triangles), HERMES (circles)


COMPASS data on $\rho$ and $\phi$ may verify dominance of gluons (+ sea)

## What do we know about $E_{v}$ ?

analysis of Pauli FF for proton and neutron at $\xi=0$ Diehl et al (04):

$$
F_{2}^{p(n)}=e_{u(d)} \int_{0}^{1} d x E_{v}^{u}(x, \xi=0, t)+e_{d(u)} \int_{0}^{1} d x E_{v}^{d}(x, \xi=0, t)
$$

ansatz for small $-t: E_{v}^{a}=e_{v}^{a}(x) \exp \left\{t\left(\alpha_{v}^{\prime} \ln (1 / x)+b_{a}^{e}\right)\right\}$ forward limit: $e_{v}^{a}=N_{a} x^{-\alpha_{v}(0)}(1-x)^{\beta_{v}^{a}}$ (analogously to PDFs)
$N_{a}$ fixed from $\kappa_{a}=\int_{0}^{1} d x E_{v}^{a}(x, \xi=0, t=0)$
fitting FF data provides: $\beta_{v}^{u}=4, \beta_{v}^{d}=5.6$ (other powers not excluded in 04 analysis) new JLab data on $G_{E, M}^{n}$ up to $3.5(5.0) \mathrm{GeV}^{2}$ favor $\beta_{v}^{u}<\beta_{v}^{d}$ Input to double distribution ansatz


## $E$ for gluons and sea quarks

sum rule (Ji's s.r. and momentum s.r. of DIS) at $t=\xi=0$

$$
\int_{0}^{1} d x x e_{g}(x)=e_{20}^{g}=-\sum e_{20}^{a_{v}}-2 \sum e_{20}^{\bar{a}}
$$

valence term very small, in particular if $\beta_{v}^{u} \leq \beta_{v}^{d}$
$\Rightarrow$ gluon and sea quark moments cancel each other almost completely
positivity bound forbids large sea $\Rightarrow$ gluon small too
parameterization (flavor symm. sea for $E$ assumed)

$$
e^{i}=N_{i} x^{-\alpha_{g}(0)}(1-x)^{\beta^{i}}
$$

and Regge-like $t$ dependence:

$$
\propto \exp \left\{t\left(\alpha_{i}^{\prime} \ln (1 / x)+b_{i}^{e}\right)\right\}
$$

input to double distribution ansatz for $E$

## Variants of $E$ and Ji's sum rule

$$
\left\langle J^{a}\right\rangle=\frac{1}{2}\left[q_{20}^{a}+e_{20}^{a}\right] \quad\left\langle J^{g}\right\rangle=\frac{1}{2}\left[g_{20}+e_{20}^{g}\right]
$$

$(\xi=0) \quad\langle J\rangle$ means average value of three component of $J$

| var. | $\beta_{\text {val }}^{u}$ | $\beta_{\text {val }}^{d}$ | $\beta^{g}$ | $\beta^{s}$ | $N_{g}$ | $N_{s}$ | $J^{u}$ | $J^{d}$ | $J^{s}$ | $J^{g}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 4 | 5.6 | - | - | 0.000 | 0.000 | 0.250 | 0.020 | 0.015 | 0.214 |
| 2 | 4 | 5.6 | 6 | 7 | -0.873 | 0.155 | 0.276 | 0.046 | 0.041 | 0.132 |
| 3 | 4 | 5.6 | 6 | 7 | 0.776 | -0.155 | 0.225 | -0.005 | -0.011 | 0.286 |
| 4 | 10 | 5 | 7 | - | 0.523 | 0.000 | 0.209 | 0.013 | 0.015 | 0.257 |

$J^{i}$ quoted at scale $4 \mathrm{GeV}^{2}$ (spread indicates uncertainties of present knowledge)
$\sum J^{i}=1 / 2$, the spin of the proton
characteristic, stable pattern: for all variants $J^{u}$ and $J^{g}$ are large, others small

## Results for $A_{U T}(V)$

data: HERMES (08)

$W=5 \mathrm{GeV} \quad Q^{2}=3 \mathrm{GeV}^{2}$
variant $1,2,3,4$

COMPASS prel.

$W=8 \mathrm{GeV} \quad Q^{2}=2 \mathrm{GeV}^{2}$
variant 1
negative value of $A_{U T}$ favored, variant 4 disfavored
$A_{U T}(\phi) \simeq 0$ in agreement with prel. HERMES data

## Results continued


variant 1 for $\omega, \rho^{+}, K^{* 0}$
$t$ dependence controlled by trivial factor $\sqrt{-t^{\prime}}$
except for $\rho^{+}$: since $H_{v}^{u}-H_{v}^{d}$ small and $E_{v}^{u}-E_{v}^{d}$ large
$E$ non-negligible in cross section, contribution from helicity flip ampl. $\propto t^{\prime}$
more data on $\rho^{0}, \omega, \phi$ from HERMES and COMPASS will come
will likely improve knowledge of $E$ for valence quarks

## What did (can) we learn about GPDs from DME?

What is probed by experiment:
imaginary parts $\propto$ GPDs at $\xi \simeq x \quad+\mathcal{O}\left(<k_{\perp}^{2}>/ Q^{2}\right)$
real parts - convolutions, dominated by $x$ near $\xi \quad$ (see also disp. rel.)
$\xi \simeq 10^{-3}$ HERA
$\simeq 10^{-2}$ COMPASS
$\simeq 10^{-1}$ HERMES
$\simeq 0.1-0.4 \mathrm{JLab}$
$x \geq 0.6$ not probed
large $x$ region important

As compared to DVCS:
disadvantage: need for GPDs and meson wave functions
advantages: allows for flavor separation (mesons select their valence quarks
$J / \Psi: \quad$ gluon from the proton to lead. twist accuracy)
$\phi$ : gluon + sea
$\rho^{0}, \omega: \quad$ gluon + sea + valence
$\rho^{+}, \pi^{+}: \quad$ valence

$$
\pi^{+}: \widetilde{H}, \widetilde{E}
$$

## Valence quark GPDs



|  | $H$ | $E$ |  |
| :---: | :---: | :--- | ---: |
| $u_{v}$ | 2 | $\kappa_{u}=1.67$ | 0.93 |
| $d_{v}$ | 1 | $\kappa_{d}=-2.03$ | -0.34 |

lowest moments fix signs and rel. sizes if GPDs have no nodes and similar $t$ dependence

## Summary

- MPA accounts for most features of DME including power corrections for $W \gtrsim 4 \mathrm{GeV} \quad Q^{2} \gtrsim 3 \mathrm{GeV}^{2} \quad$ and small $t$ unsettled: rel. phase between $L L$ and $T T$ too small, see $\operatorname{Re} r_{10}^{5}$ not yet calculated for vector mesons: $T \rightarrow L$ ampl., e.g. seen in $r_{00}^{5}$ (twist-3 as for $\pi^{+}, \mathcal{M}_{0-++}, H_{T}$ )
- DD ansatz for GPDs (parameterized by forward limits, Regge-like $t$ dep. and meson DAs as weight fcts) seems to be flexible enough; analytic properties may be a useful additional constraint (Müller) detailed comparison of different sets of GPDs is needed
- DVCS: not analysed by us (lack of man power)
we want to 'finish' meson production first from our GPDs we can work out DVCS to NLO for COMPASS kinematics in princple


## $\rho^{0}$ and $\phi$ cross sections



at $Q^{2}=4(3.8) \mathrm{GeV}^{2} \quad \operatorname{E665}(\Delta), \operatorname{HERMES}(\bullet), \operatorname{CORNELL}(\boldsymbol{\Delta})$
ZEUS (■), H1 (■), CLAS (○)
Goloskokov-K (09)
$\omega, \rho^{+}$very large at small $W$ too
double distribution model too simple for valence quarks for large $\xi$ ? breakdown of handbag physics? Lacking nucleon resonances? (Mueller) implications for DVCS?

## Cross sections for $\omega, \rho^{+}$and $K^{* 0}$



results for HERMES and COMPASS ( $W=5,10 \mathrm{GeV}$ ) $\rho^{0}, \omega, \rho^{+}, K^{* 0}$ $W$ dependence controlled by Regge behaviour $\sigma \propto W^{4(\alpha(0)-1)}$ at fixed $Q^{2}$ $\rho^{0}, \omega: \alpha-1=\delta \simeq 0.1 \quad$ diffractive (gluon + sea)
$\rho^{+} \quad: \alpha-1 \simeq-0.5 \quad K^{* 0}$ intermediate
valence quark contributions die out quickly with increasing $W$ at small $\xi$

$A_{L L}$ at $W=5(10) \mathrm{GeV}$ probes $\operatorname{Re}\left[\left\langle\widetilde{H}>^{*}<H\right\rangle\right]$
data: HERMES (circles), COMPASS (diamond)

