Heavy quark transport coefficients in a non-trivial Polyakov loop background

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Summary
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- Hadrons containing heavy quarks (open mesons) are considered as important probe of QGP.
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• Heavy quarks do not thermalise with bulk and make multiple collisions with the light quarks and gluons.
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• Interactions of heavy quarks in the QGP reflect in the transverse momentum ($p_T$) spectra of open mesons, most notably their elliptic flow, $v_2$.

• Heavy quarks do not thermalise with bulk and make multiple collisions with the light quarks and gluons.

• Charm and bottom quarks are heavy.
• $m_Q \gg T$

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Small momentum transfer

• Momentum transfer from medium is small compared to its momentum in the thermal medium, $p_{th}^2 \sim m_Q T \gg T^2$. 
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Fokker-Plank equation
• Small momentum transfer leads to Brownian motion which can be described by Fokker Plank equation.
Heavy quark in the plasma of light quarks

- Boltzmann equation for heavy quark interacting with the light quarks and gluons
Heavy quark in the plasma of light quarks

- Boltzmann equation for heavy quark interacting with the light quarks and gluons

Boltzmann Equation

\[
\left[ \frac{\partial}{\partial t} + \frac{\vec{p}}{E} \cdot \frac{\partial}{\partial \vec{x}} + \vec{F} \cdot \frac{\partial}{\partial \vec{p}} \right] f(\vec{x}, \vec{p}, t) = \left[ \frac{\partial f}{\partial t} \right]_{\text{col}}
\]
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\]

\[
\left[ \frac{\partial f}{\partial t} \right]_{col} = \int d^3 k \left[ \omega(p + k, k) f(p + k) - \omega(p, k) f(p) \right] \tag{2}
\]

collision rate \( \omega(p, k) \)

\[
\omega(p, k) = g \int \frac{d^3 q}{(2\pi)^3} f(q) v_{rel} \sigma_{p,q\rightarrow p-k,q+k} \tag{3}
\]
• Assuming the scattering process to be dominated by small momentum transfer

\[ \omega(p + k, k)f(p + k) \approx \omega(p, k)f(p) + k \frac{\partial}{\partial p}(\omega p) + \frac{1}{2} k_i k_j \frac{\partial^2}{\partial p_i \partial p_j}(\omega p). \]  
\[ (4) \]
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(4)

• Boltzmann equation reduces to Fokker-Plank equation i.e,

\[ \left[ \frac{\partial f}{\partial t} \right]_{col} = \frac{\partial}{\partial p_i} \left[ A_i(p)f + \frac{\partial}{\partial p_i}[B_{ij}(p)f] \right] \]  

(5)

where the kernels \( A_i \) and \( B_{ij} \) stands for drag and diffusion coefficients respectively.
Drag

- If $p$ and $p'$ are initial and final momenta of heavy quark then $A_i(p)$ is written as

$$A_i(p) = \frac{1}{2E_p} \int \frac{d^3q}{(2\pi)^3E_q} \int \frac{d^3p'}{(2\pi)^3E_{p'}} \int \frac{d^3q'}{(2\pi)^3E_{q'}}$$

$$\times \frac{1}{g_{HQ}} \sum |M|^2 (2\pi)^4 \delta^4(p + q - p' - q') f(q)$$

$$(1 + f(q'))((p - p')_i) \equiv \langle \langle (p - p') \rangle \rangle$$

(6)

where $|M|^2$ is scattering amplitude squared for the processes $l/g(q), HQ(p) \rightarrow l/g(q'), HQ(p').$
• $B_{ij}(p)$ is given as

$$B_{ij}(p) = \frac{1}{2E_p} \int \frac{d^3q}{(2\pi)^3 E_q} \int \frac{d^3p'}{(2\pi)^3 E_{p'}} \int \frac{d^3q'}{(2\pi)^3 E_{q'}}$$

$$\times \frac{1}{g_{HQ}} \sum |M|^2 (2\pi)^4 \delta^4(p + q - p' - q') f(q)$$

$$\times (1 + f(q')) \left[ \frac{1}{2} (p - p')_i (p - p')_j \right]$$

$$\equiv \langle \langle (p - p')_i (p - p')_j \rangle \rangle$$

(7)
Non-perturbative effects

- $|M|^2$ is evaluated in a quasi-particle approach where thermal quark and gluon masses are required.
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- Effect of Polyakov loop in drag and diffusion coefficients is manifested through thermal masses of quark and gluon.
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- $|\mathcal{M}|^2$ is evaluated in a quasi-particle approach where thermal quark and gluon masses are required.

- Effect of Polyakov loop in drag and diffusion coefficients is manifested through thermal masses of quark and gluon.

- The value of Polyakov loop ($T_c \approx 170$ MeV, $I(T_c) \approx 0.5$) and constituent quark masses ($m(T_c) = 134$) is taken from PQM model.
• Background gauge field, $A_0^{ab} = \frac{1}{g} Q_a \delta_{ab}$
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• Polyakov loop is,

$$l = \frac{1}{N} \sum_{1}^{3} e^{iQ^{a} \frac{T}{T}} = \frac{1}{3} (1 + 2 \cos(2\pi q))$$

(8)
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• For $SU(3)$, $Q^{a} = 2\pi T(q, 0, -q)$

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$$l = \frac{1}{N} \sum_{1}^{3} e^{iQ^{a}T} = \frac{1}{3}(1 + 2\cos(2\pi q)) \quad (8)$$

• Quark and gluon distribution functions

$$\tilde{f}_{a}(E) = \frac{1}{e^{\beta(E-iQ^{a})} + 1}$$

$$f_{ab}(E) = \frac{1}{e^{\beta(E-i(Q^{a}-Q^{b}))} - 1}$$
Diagrams for debye and thermal mass

Figure: Gluon self energy in double line notation

Figure: Fermion self energy in double line notation
• Debye mass and quark thermal mass

**Masses**

\[
M_D^2 = \sum_{a=1}^{3} \frac{M_a^2}{3}
\]

\[
M_a^2 = \frac{g^2}{3} \left[ \sum_{e=1}^{N} A(Q_{ae}) - N_f \tilde{A}(Q^a) \right]
\]

\[
m_a^2 = \frac{g^2}{24} \sum_{c=1}^{3} \left( A(Q_{ac}) - \tilde{A}(Q_c) - \frac{1}{3} (A(0) - \tilde{A}(Q_a)) \right)
\]

\[
A(Q) = \frac{3}{\pi^2} \int_{0}^{\infty} kdk \left( \frac{1}{e^{\beta(E-iQ)} - 1} + \frac{1}{e^{\beta(E+iQ)} - 1} \right)
\]

\[
\tilde{A}(Q) = A(Q + \pi T)
\]
Thermal masses:

Figure: Left panel: Quark thermal mass. Right panel: Debye screening mass
Drag coefficient:

**Figure**: Left panel: Drag coefficient for different value of $p$. Right panel: Drag coefficient for different value of $T$. 

Drag and Diffusion of heavy quark
Diffusion coefficient:

Figure: Left panel: Diffusion coefficient for different value of $p$. Right panel: Diffusion coefficient for different value of $T$. 
Non-perturbative effects are important at low $T$.

In the quark thermal mass and gluon Debye mass the non-perturbative effects are important below temperature 300 MeV for transition temperature $T_c = 174$ MeV.

Heavy quark drag coefficient has almost constant dependence of temperature.

Heavy quark Diffusion coefficient increase with increase in temperature.
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THANK YOU
• A comparison \(^1\)

**Figure:** Drag in different models