Phenomenological study of two-zero textures in Minimal Extended Seesaw Mechanism

Priyanka Kumar, Mahadev Patgiri

Department of Physics,Cotton University
Assam, India

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Phenomenological study of two - ZERO TEXTURES in Minimal EXTENDED SEESAW MECHANISM

Plan of the talk

- Introduction
- Motivation for zero texture
- Sterile neutrino
- Minimal Extended Seesaw mechanism
- Formalism
- Results
- Conclusion

Introduction

- Standard Model of partile physics $\Rightarrow$ $\underbrace{\nu_{e_L}, \nu_{\mu_L}, \nu_{\tau_L}}_{\text{massless neutrino}}$
- However it has been confirmed that neutrinos undergo a quantum mechanical phenomenon- Neutrino Oscillations. Thereby giving the concept of "neutrino mass".
- Masses and mixings of the three flavours of neutrino $(\nu_e, \nu_\mu, \nu_\tau)$ can be described by a (3×3) complex symmetric Majorana mass matrix M_ν.
- Neutrino mass matrix M_ν is parametrized by a total of 9 parameters.$(m_1, m_2, m_3, \theta_{12}, \theta_{13}, \theta_{23}, \delta, \alpha, \beta)$
- However only 5 parameters have been measured by neutrino oscillation experiments.(3 mixing angles $(\theta_{12}, \theta_{13}, \theta_{23})$ and two mass squared differences $(\Delta m^2_{21}$ and $\Delta m^2_{32}))$.

Motivation for Zero Texture

- Inspite of tremendous progress and painstaking efforts, none of the presently conceivable set of feasible experiments can fully determine the neutrino mass matrix.
- Although there is an increasing information on the numerical values of these parameters, the origin of the leptonic flavor structure is still a mystery.
- Moreover, the smallness of neutrino mass, mass heirarchy, origin of CP violation are some of the striking questions whose answers are still lacking within the Standard Model.

- A phenomenological approach to these questions is to consider some general textures for the Yukawa matrices for quarks and leptons.
- One of the simplest procedure is to consider textures where the mass matrix elements are dependent upon one another, caused by some underlying symmetry.
- This will stand as a solution to account for the experimental constraints by reducing the number of free parameters.
- Also the underlying symmetry of the proposed texture could give some hint on how to extend the SM gauge symmetries.

Zero Textures

- Zeros in the mass matrix elements is the simplest and transparent way of inducing relations among the physical quantities (masses, mixing angles and CP phases) of the mass matrix M_ν and thereby reducing the number of free parameters.
- Fermion mass matrix elements which are negligibly small can be effectively replaced by zeros, which popularly are known as texture zeros [1].
- It is a plausible approach to minimize the number of parameters and thereby restrict the form of the mass matrices.
- One zero in M_ν corresponds to one-zero texture, two zeros in M_ν corresponds to two-zero texture and so on..

Zero Textures

- Zeros in the light neutrino mass matrices can be imposed directly by hand.
- However, zeros in the light neutrino mass matrices can also be imposed via type-I seesaw mechanism -

$$M_\nu = M_D M_R^{-1} M_D^T$$

- Zeros of the Dirac neutrino mass matrix (M_D) and the heavy right-handed Majorana neutrino mass matrix (M_R) may propagate as zeros in the effective neutrino mass matrix (M_ν) through the type-I seesaw mechanism [2].
- So, the study of zero textures of M_D and M_R are more basic than the study of M_ν.

- In the flavor basis, the current neutrino oscillation data disfavours the neutrino mass matrices with three or more zeros.
- Out of 15 possible two-zeros textures in $M_{\nu}^{3\times 3}$, only 7 patterns can withstand the current experimental data [3].
- For one-zero texture with one vanishing neutrino mass it has been observed that out of six possible one-zero textures, four textures survive the experimental constraints with inverted mass ordering ($m_3 = 0$), whereas in normal mass ordering ($m_1 = 0$) all the six textures are ruled out at 3σ range of experimental values [4].
- However, the scenario is different in the (3+1) picture, that is, when *Sterile neutrino is considered.*

Sterile Neutrinos

- Light sterile neutrinos were invoked to explain the results of the LSND experiment which reported anomalies in the anti-neutrino flux corresponding to a mass squared difference $\approx eV^2$ [5].
- Similar departures were confirmed by MiniBooNE experiment [6], Gallium solar neutrino experiment [7], reactor neutrino experiment[8] and cosmological observations[9].
- Recently in May 2018, The MiniBooNE collaboration reported that their data are found to be consistent in both energy and magnitude with the excess of events reported by the LSND experiment, and the significance of the combined LSND and MiniBooNE excesses is 6.1σ [10].

Sterile Neutrino

- Again in August 2018, the ANITA experiment reported that they observed two unusual upgoing air shower events, which are consistent with the τ-lepton decay origin but are in contradiction with the standard neutrino-matter interaction models [11].
- The evidence of such experimental anomalies is a strong indication of existence of an additional light sterile neutrino.

Minimal Extended Seesaw Mechanism

- Seesaw mechanism in the three neutrino scenario has played a major role for the theoretical understanding of the smallness of neutrino masses.
- In the (3+1) framework a similar approach have been made for generating an eV scale sterile neutrino by extending the type-I seesaw mechanism.
- In this model, popularly known as the Minimal Extended (type-I)Seesaw mechanism(MES), one singlet fermion 'S' is added along with three right-handed neutrinos [12].
- This results into a naturally occurring eV scale sterile neutrino without requiring to impose any tiny Yukawa term or mass scales.

- The Lagrangian representing the neutrino mass term takes the form

$$-\mathcal{L}_m = \bar{\nu}_L M_D \nu_R + \bar{S}^c M_S \nu_R + \frac{1}{2}\bar{\nu}_R^c M_R \nu_R + h.c. \qquad (1)$$

- Here M_S is a (1×3) row matrix.
- Considering $M_R \gg M_S > M_D$, M_ν takes the 4×4 form as

$$M_\nu^{4\times 4} = -\begin{pmatrix} M_D M_R^{-1} M_D^T & M_D M_R^{-1} M_S^T \\ M_S (M_R^{-1})^T M_D^T & M_S M_R^{-1} M_S^T \end{pmatrix} \qquad (2)$$

- This $M_\nu^{4\times4}$ is a square matrix with four light eigenstate corresponding to three active neutrinos and one sterile neutrino. The mass matrix $M_\nu^{4\times4}$ can have at most rank 3 since

$$\begin{aligned}\det(M_\nu^{4\times4}) &= \det(M_D M_R^{-1} M_D^T)\det[-M_S M_R^{-1} M_S^T \\ &\quad + M_S M_R^{-1} M_D^T (M_D M_R^{-1} M_D^T)^{-1} M_D M_R^{-1} M_S^T] \\ &= \det(M_D M_R^{-1} M_D^T)\det[M_S(M_R^{-1} - M_R^{-1})M_S^T] \\ &= 0\end{aligned} \tag{3}$$

where both M_D and M_R are considered to be non-singular.

- Thus at least one of the active neutrino mass states remains as massless.

Sterile Neutrino

- In the flavor basis the(4×4) Majorana neutrino mass matrix can be expressed as

$$M_\nu^{4\times 4} = V M_\nu^{diag} V^T = \begin{pmatrix} m_{ee} & m_{e\mu} & m_{e\tau} & m_{es} \\ m_{e\mu} & m_{\mu\mu} & m_{\mu\tau} & m_{\mu s} \\ m_{e\tau} & m_{\mu\tau} & m_{\tau\tau} & m_{\tau s} \\ m_{es} & m_{\mu s} & m_{\tau s} & m_{ss} \end{pmatrix} \qquad (4)$$

where V corresponds to the (4×4) PMNS lepton mixing matrix and $M_\nu^{diag} = diag(m_1, m_2, m_3, m_4)$.

- Considering the parametrization[13] of the 4×4 PMNS lepton mixing matrix as

$$V = UP \tag{5}$$

where

$$U = (R_{34}\tilde{R}_{24}\tilde{R}_{14})(R_{23}\tilde{R}_{13})R_{12} \tag{6}$$

and

$$P = diag(1, e^{-i\alpha/2}, e^{-i(\beta/2-\delta_{13})}, e^{-i(\gamma/2-\delta_{14})}) \tag{7}$$

where α, β, γ are the Majorana phases and $R_{ij}/\tilde{R}_{ij}$ are the rotation matrix in the ij flavor space, e.g.

$$R_{34} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & c_{34} & s_{34} \\ 0 & 0 & -s_{34} & c_{34} \end{pmatrix}, \quad \tilde{R}_{24} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & c_{24} & 0 & s_{24}e^{-i\delta_{24}} \\ 0 & 0 & 1 & 0 \\ 0 & -s_{24}e^{i\delta_{24}} & 0 & c_{24} \end{pmatrix} \tag{8}$$

$c_{ij} = \cos\theta_{ij}$, $s_{ij} = \sin\theta_{ij}$.

Two-zero textures in $M_\nu^{4\times4}$

Viable two zero textures [14] of rank 3. Here 'X' indicates the elements with non-zero entries.

A_1	A_2	B_3	B_4
$\begin{pmatrix} 0 & 0 & X & X \\ 0 & X & X & X \\ X & X & X & X \\ X & X & X & X \end{pmatrix}$	$\begin{pmatrix} 0 & X & 0 & X \\ X & X & X & X \\ 0 & X & X & X \\ X & X & X & X \end{pmatrix}$	$\begin{pmatrix} X & 0 & X & X \\ 0 & 0 & X & X \\ X & X & X & X \\ X & X & X & X \end{pmatrix}$	$\begin{pmatrix} X & X & 0 & X \\ X & X & X & X \\ 0 & X & 0 & X \\ X & X & X & X \end{pmatrix}$
C	D_1	D_2	E_1
$\begin{pmatrix} X & X & X & X \\ X & 0 & X & X \\ X & X & 0 & X \\ X & X & X & X \end{pmatrix}$	$\begin{pmatrix} X & X & X & X \\ X & 0 & 0 & X \\ X & 0 & X & X \\ X & X & X & X \end{pmatrix}$	$\begin{pmatrix} X & X & X & X \\ X & X & 0 & X \\ X & 0 & 0 & X \\ X & X & X & X \end{pmatrix}$	$\begin{pmatrix} 0 & X & X & X \\ X & 0 & X & X \\ X & X & X & X \\ X & X & X & X \end{pmatrix}$
E_2	F_1	F_2	F_3
$\begin{pmatrix} 0 & X & X & X \\ X & X & X & X \\ X & X & 0 & X \\ X & X & X & X \end{pmatrix}$	$\begin{pmatrix} X & 0 & 0 & X \\ 0 & X & X & X \\ 0 & X & X & X \\ X & X & X & X \end{pmatrix}$	$\begin{pmatrix} X & 0 & X & X \\ 0 & X & 0 & X \\ X & 0 & X & X \\ X & X & X & X \end{pmatrix}$	$\begin{pmatrix} X & X & 0 & X \\ X & X & 0 & X \\ 0 & 0 & X & X \\ X & X & X & X \end{pmatrix}$

Zeros of M_D, M_R and M_S

- The zeros of $M_\nu^{4\times4}$ in MES result from the zeros of M_D, M_R and M_S.

$$M_\nu^{4\times4} = -\begin{pmatrix} M_D M_R^{-1} M_D^T & M_D M_R^{-1} M_S^T \\ M_S (M_R^{-1})^T M_D^T & M_S M_R^{-1} M_S^T \end{pmatrix} \tag{9}$$

- We consider the (5+4) scheme, that is, 5 zeros in M_D and 4 zeros in M_R.
- The Dirac neutrino mass matrix M_D being in general non symmetric, has 9 independent entries in its 3×3 form:

$$M_D = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & l \end{pmatrix} \tag{10}$$

- Therefore, there are ${}^9C_5 = 126$ possible 5-zero textures of M_D.
- A non-singular symmetric right handed Majorana mass matrix M_R having 6 independent entries can accomodate a maximum of four zeros.
- There are 15 possible 4 zero textures of M_R, out of which only three of them are non-singular.

$$M_R^a = \begin{pmatrix} 0 & B & 0 \\ B & 0 & 0 \\ 0 & 0 & F \end{pmatrix}, M_R^b = \begin{pmatrix} 0 & 0 & C \\ 0 & D & 0 \\ C & 0 & 0 \end{pmatrix}, M_R^c = \begin{pmatrix} A & 0 & 0 \\ 0 & 0 & E \\ 0 & E & 0 \end{pmatrix} \tag{11}$$

- Possible one-zero texture of M_S are given by Eq. (12)

$$M_S^{(1)} = \begin{pmatrix} 0 & s_2 & s_3 \end{pmatrix}, \quad M_S^{(2)} = \begin{pmatrix} s_1 & 0 & s_3 \end{pmatrix} \quad M_S^{(3)} = \begin{pmatrix} s_1 & s_2 & 0 \end{pmatrix} \tag{12}$$

Realization of two zero textures

- For realization of our textures we excluded all those textures of M_D with row-zero, column-zero or block zeros - as they lead to singular forms of M_D.
- We find that two zero textures of M_S are not allowed for realization of our textures.

Texture A_1

- The following combination of M_R, M_D and M_S

$$M_R = \begin{pmatrix} A & 0 & 0 \\ 0 & 0 & E \\ 0 & E & 0 \end{pmatrix}, M_D = \begin{pmatrix} 0 & b & 0 \\ d & 0 & 0 \\ g & 0 & l \end{pmatrix}, M_S^{(2)} = \begin{pmatrix} s_1 & 0 & s_3 \end{pmatrix} \tag{13}$$

leads to the folowing form of $M_\nu^{4\times4}$

$$M_\nu^{(4\times4)} = \begin{pmatrix} 0 & 0 & \frac{bl}{E} & \frac{bs_3}{E} \\ 0 & \frac{d^2}{A} & \frac{dg}{A} & \frac{ds_1}{A} \\ \frac{bl}{E} & \frac{dg}{A} & \frac{g^2}{A} & \frac{gs_1}{A} \\ \frac{bs_3}{E} & \frac{ds_1}{A} & \frac{gs_1}{A} & \frac{s_1^2}{A} \end{pmatrix} \tag{14}$$

which is the texture A_1.

- $M_\nu^{4\times4}$ leads to the following correlations

$$\frac{m_{\mu\mu}}{m_{\mu\tau}} = \frac{m_{\mu\tau}}{m_{\tau\tau}} = \frac{m_{\mu s}}{m_{\tau s}} = \sqrt{\frac{m_{\mu\mu}}{m_{\tau\tau}}} \tag{15}$$

$$\frac{m_{\mu\tau}}{m_{\mu s}} = \frac{m_{\tau\tau}}{m_{ss}} = \frac{m_{\tau s}}{m_{ss}} = \sqrt{\frac{m_{\tau\tau}}{m_{ss}}} \tag{16}$$

$$\frac{m_{\mu\mu}}{m_{\mu s}} = \frac{m_{\mu\tau}}{m_{\tau s}} = \frac{m_{\mu s}}{m_{ss}} = \sqrt{\frac{m_{\mu\mu}}{m_{ss}}} \tag{17}$$

- To check the survivability of a texture, we examine the correlations under recent neutrino oscillation data.
- Textures which survives the experimental constraints are considered to be Allowed Textures.

Experimental constraints

Parameter	Best Fit	3σ Range
$\Delta m^2_{21}[10^{-5} eV^2]$	7.37	6.93-7.97
$\Delta m^2_{31}[10^{-3} eV^2]$(NH) $\Delta m^2_{31}[10^{-3} eV^2]$(IH)	2.50 2.46	2.37-2.63 2.33-2.60
$sin\theta^2_{12}/10^{-1}$	2.97	2.50-3.54
$sin\theta^2_{13}/10^{-2}$(NH) $sin\theta^2_{13}/10^{-2}$(IH)	2.14 2.18	1.85-2.46 1.86-2.48
$sin\theta^2_{23}/10^{-1}$(NH) $sin\theta^2_{23}/10^{-1}$(IH)	4.37 5.69	3.79-6.16 3.83-6.37
δ_{13}/π(NH) δ_{13}/π(IH)	1.35 1.32	0-2 0-2
$\Delta m^2_{LSND}(\Delta m^2_{41}$ or $\Delta m^2_{43})eV^2)$ $\sin\theta^2_{14}$ $\sin\theta^2_{24}$ $sin\theta_{34}$	1.63 0.027 0.013 -	0.87-2.04 0.012-0.047 0.005-0.03 < 0.5

Table : 1: The latest best-fit and 3σ ranges of active ν oscillation parameters from [15]. The current constraints on sterile neutrino parameters are from the global analysis [16].

- $\sin\theta_{34}$ is constrained from an upper bound < 0.5. But we have taken its lower limit as 0.
- The correlations obtained in each texture are plotted against $sin\theta_{34}$ having the range from $(0 - 0.5)$.
- The Dirac CP phases $(\delta_{13}, \delta_{14}, \delta_{24})$ and Majorana phases (α, β, γ) are kept unconstrained.

Survivability of a texture

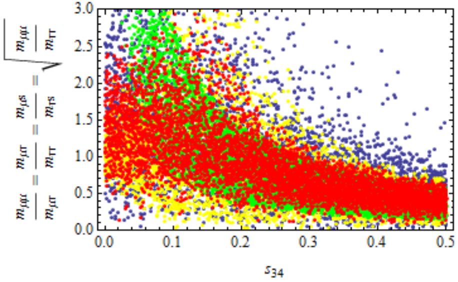

Figure : (1). Correlation plot for texture $A_1(\frac{m_{\mu\mu}}{m_{\mu\tau}}; \frac{m_{\mu\tau}}{m_{\tau\tau}}; \frac{m_{\mu s}}{m_{\tau s}}; \sqrt{\frac{m_{\mu\mu}}{m_{\tau\tau}}})$.

Overlapping exists except for $\sin\theta_{34} \approx (0 - 0.02)$.

Survivability of a texture

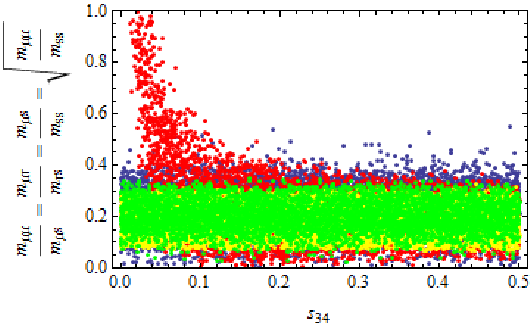

Figure : (2). Correlation plot for texture $A_1(\frac{m_{\mu\mu}}{m_{\mu s}}; \frac{m_{\mu\tau}}{m_{\tau s}}; \frac{m_{\mu s}}{m_{ss}}; \sqrt{\frac{m_{\mu\mu}}{m_{ss}}})$.

Overlapping exists except for $\sin\theta_{34} \approx (0 - 0.02)$.

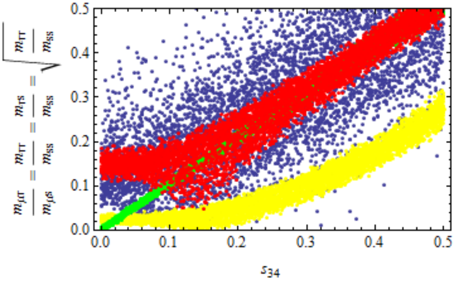

Figure : (3). Correlation plot for texture A_1 ($\frac{m_{\mu\tau}}{m_{\mu s}}$; $\frac{m_{\tau\tau}}{m_{ss}}$; $\frac{m_{\tau s}}{m_{ss}}$; $\sqrt{\frac{m_{\tau\tau}}{m_{ss}}}$).

Overlapping ceases!!!

Texture is NOT allowed

- B_3 has zeros in its $e-\mu$ and $\mu-\mu$ entries of the mass matrix $M_\nu^{4\times4}$.
- Texture B_3 allows both the mass patterns: normal heirarchy(NH) and inverted heirarchy(IH).
- The combination of M_D, M_R and M_S yeilds the correlations

$$M_R = \begin{pmatrix} A & 0 & 0 \\ 0 & 0 & E \\ 0 & E & 0 \end{pmatrix}, M_D = \begin{pmatrix} a & 0 & 0 \\ 0 & e & 0 \\ g & 0 & l \end{pmatrix}, M_S^{(2)} = \begin{pmatrix} s_1 & 0 & s_3 \end{pmatrix} \tag{18}$$

$$\frac{m_{ee}}{m_{e\tau}} = \frac{m_{e\tau}}{m_{\tau\tau}} = \sqrt{\frac{m_{ee}}{m_{\tau\tau}}} \tag{19}$$

$$\frac{m_{\tau\tau}}{m_{\tau s}} = \frac{m_{e\tau}}{m_{es}} = \sqrt{\frac{m_{\tau\tau}}{m_{ss}}} \tag{20}$$

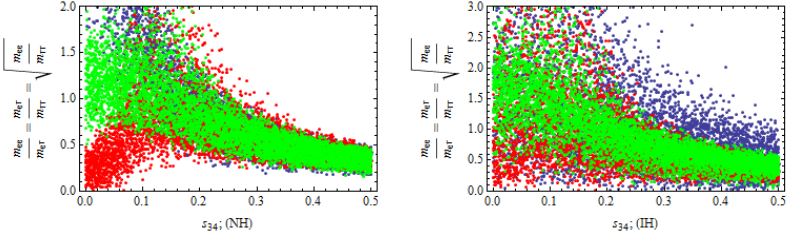

Figure : (4). Correlation plot for texture B_3 for both NH(left) and IH(right) ($\frac{m_{ee}}{m_{e\tau}}$; $\frac{m_{e\tau}}{m_{\tau\tau}}$; $\sqrt{\frac{m_{ee}}{m_{\tau\tau}}}$).

- For NH, the texture disfavours the region $\sin\theta_{34} \leq 0.08$.
- For IH, the texture is allowed for all ranges of $\sin\theta_{34}$.

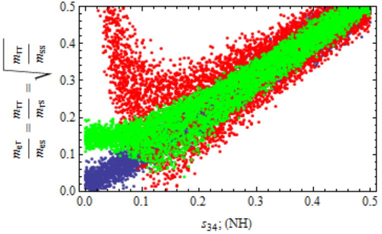

Figure : (5). Correlation plot for texture B_3 for both NH(left) and IH(right) ($\frac{m_{e\tau}}{m_{es}}$; $\frac{m_{\tau\tau}}{m_{\tau s}}$; $\sqrt{\frac{m_{\tau\tau}}{m_{ss}}}$).

- For NH, the texture disfavours the region $\sin\theta_{34} \leq 0.08$.
- For IH, the texture disfavours the region $\sin\theta_{34} \leq 0.04$.

Correlation plots for both the correlation for texture B_3 shows that-
$\Rightarrow$ For NH the texture is not allowed for $\sin\theta_{34} \leq 0.08$.
$\Rightarrow$ For IH the texture is not allowed for $\sin\theta_{34} \leq 0.04$

- Similar procedure has been adopted for examining the viability of all the textures.
- Texture $A_1 \rightarrow$ "Not Allowed"
- $A_2 \rightarrow$ Allowed for $\sin\theta_{34} \geq 0.08$
- $B_3 \rightarrow$ Allowed for $\sin\theta_{34} \geq 0.08$ (NH) and $\sin\theta_{34} \geq 0.04$ (IH).
- B_4, D_2 Allowed for all ranges of $\sin\theta_{34} = (0 - 0.5)$
- D_1 Allowed for $\sin\theta_{34} \geq 0.08$ (NH) and $\sin\theta_{34} \geq 0.06$ (IH)
- The textures C, E_1, E_2, F_1, F_2, F_3 cannot be realized in the (5+4) scheme in the context of MES mechansim.

Conclusion

- Out of 12 two-zero textures of rank 3, only 6 textures can be realized in the context of MES mechanism in the (5+4) scenario.
- On realizing the textures we arrive at certain constrained relations - correlations.
- Viability of the textures have been checked by plotting their respective correlations against $\sin\theta_{34}$ keeping the CP phases unconstrained.
- It has been found that out of the 6 textures $(A_1, A_2, B_3, B_4, D_1, D_2)$, recent neutrino oscillation data disfavours one of the texture - A_1.
- Texture B_4 and D_2 are allowed for all ranges of $\sin\theta_{34}$
- Texture A_2, B_3, D_1 disfavours lower values of $\sin\theta_{34}$.
- In our work, the CP phases are kept unconstrained. However, it will be interesting to see the dynamics of the textures when CP phases are constrained to some ranges or constrained to some definite values.

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Thank you!