Probing the effects of sterile neutrino on the possible ultrahigh energy neutrino signals at a km^2 detector in the framework of 4-flavour neutrino oscillation and unparticle decay

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Introduction

- The existence of a fourth sterile neutrino has been hypothesised since a long time.
- Accelerator and reactor based neutrino experiments (LSND, MINOS, MINOS+, Daya Bay, Bugey, MiniBooNE etc.) give bound on active sterile neutrino mixing angles and Δm^2. considering a 4 (3+1) neutrino framework.
- In this work we consider two scenarios for 4 (3 active + 1 sterile) scheme.
  1. Mass flavour oscillation / flavour suppression.
  2. Oscillation + unparticle decay.
- Apply it for ultrahigh energy (UHE) neutrinos from distant Gamma Ray Bursts (GRBs).
  1. Diffuse GRB neutrino flux for only oscillation / flavour suppression.
  2. Neutrino flux from single GRB (specific baseline length over which neutrino suffers decay) for oscillation + decay.
- Compute the muon/shower yield at a Km^2 detector such as IceCube.

Introduction (Contd...)

- Almost a decade back Georgi proposed the probable existence of a scale invariant sector.
- At a very high energy scale this scale invariance sector and the Standard Model (SM) sector may coexist and the fields of these two sectors can interact via a mediator messenger field of mass scale $M_{\mathcal{U}}$.
- At low energies, the scale invariance of SM is manifestly broken (SM particles have masses).
- The interactions between this scale invariance sector and SM are suppressed (inverse power of $M_{\mathcal{U}}$).
- Georgi observed at low energies scale invariance sector manifests itself by **non-integral number ($d_{\mathcal{U}}$) of massless invisible particles (scaling dimension of scale invariance sector) → Unparticles.**

Introduction (Contd...)

- A prototype model of such scale invariant sector can be obtained from Banks-Zaks theory, where the scale invariance sets in at energy scale $\Lambda_{\mathcal{U}}$.

- We have taken a scalar unparticle operator and scalar interactions with neutrinos that enables a heavy neutrino decay to a lighter neutrino and another unparticle.

Formalism

- **Four and Three Flavour Neutrino Oscillations**

In general the probability for a neutrino $|\nu_\alpha\rangle$ of flavour α to oscillate to a neutrino $|\nu_\beta\rangle$ of flavour β is given by (considering no CP violation in neutrino sector)

$$P_{\nu_\alpha \to \nu_\beta} = \delta_{\alpha\beta} - 4\sum_{j>i} U_{\alpha i} U_{\beta i} U_{\alpha j} U_{\beta j} \sin^2\left(\frac{\pi L}{\lambda_{ij}}\right). \quad \text{........1)}$$

For UHE neutrinos from distant GRBs , the oscillatory part in the probability equation is averaged to half (L is large, $\Delta m^2 L/E \gg 1$) . The probability equation (Eq. (1)) is then reduced to

$$\begin{aligned} P_{\nu_\alpha \to \nu_\beta} &= \delta_{\alpha\beta} - 2\sum_{j>i} U_{\alpha i} U_{\beta i} U_{\alpha j} U_{\beta j} \\ &= \delta_{\alpha\beta} - \sum_i U_{\alpha i} U_{\beta i} \left[\sum_{j \neq i} U_{\alpha j} U_{\beta j}\right] \\ &= \sum_j |U_{\alpha j}|^2 |U_{\beta j}|^2, \end{aligned} \quad \text{.............2)}$$

Considering the present 4-flavour scenario to be the minimal extension of 3-flavour case by a sterile neutrino, the mixing matrix $\tilde{U}$ can be written as

$$\tilde{U} = R_{34}(\theta_{34})R_{24}(\theta_{24})R_{14}(\theta_{14})R_{23}(\theta_{23})R_{13}(\theta_{13})R_{12}(\theta_{12}) \quad .....3)$$

$$\tilde{U}_{(4\times4)} = \begin{pmatrix} c_{14}\mathcal{U}_{e1} & c_{14}\mathcal{U}_{e2} & c_{14}\mathcal{U}_{e3} & s_{14} \\ -s_{14}s_{24}\mathcal{U}_{e1} + c_{24}\mathcal{U}_{\mu1} & -s_{14}s_{24}\mathcal{U}_{e2} + c_{24}\mathcal{U}_{\mu2} & -s_{14}s_{24}\mathcal{U}_{e3} + c_{24}\mathcal{U}_{\mu3} & c_{14}s_{24} \\ \begin{array}{c} -c_{24}s_{14}s_{34}\mathcal{U}_{e1} \\ -s24s34\mathcal{U}_{\mu1} \\ +c_{34}\mathcal{U}_{\tau1} \end{array} & \begin{array}{c} -c_{24}s_{14}s_{34}\mathcal{U}_{e2} \\ -s24s34\mathcal{U}_{\mu2} \\ +c_{34}\mathcal{U}_{\tau2} \end{array} & \begin{array}{c} -c_{24}s_{14}s_{34}\mathcal{U}_{e3} \\ -s24s34\mathcal{U}_{\mu3} \\ +c_{34}\mathcal{U}_{\tau3} \end{array} & c_{14}c_{24}s_{34} \\ \begin{array}{c} -c_{24}c_{34}s_{14}\mathcal{U}_{e1} \\ -s_{24}c_{34}\mathcal{U}_{\mu1} \\ -s_{34}\mathcal{U}_{\tau1} \end{array} & \begin{array}{c} -c_{24}c_{34}s_{14}\mathcal{U}_{e2} \\ -s_{24}c_{34}\mathcal{U}_{\mu2} \\ -s_{34}\mathcal{U}_{\tau2} \end{array} & \begin{array}{c} -c_{24}c_{34}s_{14}\mathcal{U}_{e3} \\ -s_{24}c_{34}\mathcal{U}_{\mu3} \\ -s_{34}\mathcal{U}_{\tau3} \end{array} & c_{14}c_{24}c_{34} \end{pmatrix}, \quad .....4)$$

where $\mathcal{U}_{\alpha i}$ are the matrix elements of 3-flavour neutrino mixing matrix

$$\mathcal{U}_{(3\times3)} = \begin{pmatrix} c_{12}c_{13} & s_{12}s_{13} & s_{13} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13} & c_{12}c_{23} - s_{12}s_{23}s_{13} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13} & -c_{12}s_{23} - s_{12}c_{23}s_{13} & c_{23}c_{13} \end{pmatrix}. \quad ....5)$$

Following Eq. (2) , the oscillation probability $P^4_{\nu_\alpha \to \nu_\beta}$ (where α, β denote the flavour indices) for the four-flavour case can now be represented as

$$P^4_{\nu_\alpha \to \nu_\beta} \equiv \begin{pmatrix} P_{ee} & P_{e\mu} & P_{e\tau} & P_{es} \\ P_{\mu e} & P_{\mu\mu} & P_{\mu\tau} & P_{\mu s} \\ P_{\tau e} & P_{\tau\mu} & P_{\tau\tau} & P_{\tau s} \\ P_{se} & P_{s\mu} & P_{s\tau} & P_{ss} \end{pmatrix} \equiv XX^T, \quad \ldots 6)$$

with

$$X = \begin{pmatrix} |\tilde{U}_{e1}|^2 & |\tilde{U}_{e2}|^2 & |\tilde{U}_{e3}|^2 & |\tilde{U}_{e4}|^2 \\ |\tilde{U}_{\mu 1}|^2 & |\tilde{U}_{\mu 2}|^2 & |\tilde{U}_{\mu 3}|^2 & |\tilde{U}_{\mu 4}|^2 \\ |\tilde{U}_{\tau 1}|^2 & |\tilde{U}_{\tau 2}|^2 & |\tilde{U}_{\tau 3}|^2 & |\tilde{U}_{\tau 4}|^2 \\ |\tilde{U}_{s1}|^2 & |\tilde{U}_{s2}|^2 & |\tilde{U}_{s3}|^2 & |\tilde{U}_{s4}|^2 \end{pmatrix}. \quad \ldots.. 7)$$

UHE neutrino fluxes from Diffuse GRBs

From the GRBs, the neutrino (antineutrino) flavours are expected to be produced in the ratio $\nu_e : \nu_\mu : \nu_\tau = 1:2:0$ (3 flavour) and $\nu_e : \nu_\mu : \nu_\tau : \nu_s = 1:2:0:0$ (4 flavour).

The standard ratio of intrinsic neutrino flux is

$$\phi_{\nu_e} : \phi_{\nu_\mu} : \phi_{\nu_\tau} : \phi_{\nu_s} = 1:2:0:0. \quad ....8)$$

The isotropic flux (Waxman-Bahcall flux) for ν_μ and $\bar{\nu}_\mu$ estimated by summing over all the sources is given as

$$\mathcal{F}(E_\nu) = \frac{dN_{\nu_\mu + \bar{\nu}_\mu}}{dE_\nu} = \mathcal{N}\left(\frac{E_\nu}{1\ \mathrm{GeV}}\right)^{-n} \mathrm{cm}^{-2}\ \mathrm{s}^{-1}\ \mathrm{sr}^{-1}\ \mathrm{GeV}^{-1}. \quad ......9)$$

In the above,

$$\mathcal{N} = 4.0 \times 10^{-13}, \qquad n = 1 \quad \text{for } E_\nu < 10^5\ \mathrm{GeV},$$
$$\mathcal{N} = 4.0 \times 10^{-8}, \qquad n = 2 \quad \text{for } E_\nu > 10^5\ \mathrm{GeV}.$$

Therefore the fluxes of the corresponding flavours (which are the same for both neutrinos and antineutrinos since no CP violation is considered in the neutrino sector) can be expressed as

$$\frac{dN_{\nu_\mu}}{dE_\nu} = \phi_{\nu_\mu} = \frac{dN_{\bar{\nu}_\mu}}{dE_\nu} = \phi_{\bar{\nu}_\mu} = 0.5\mathcal{F}(E_\nu),$$
$$\frac{dN_{\nu_e}}{dE_\nu} = \phi_{\nu_e} = \frac{dN_{\bar{\nu}_e}}{dE_\nu} = \phi_{\bar{\nu}_e} = 0.25\mathcal{F}(E_\nu). \quad ...10)$$

The fluxes of neutrino flavours for four and three flavour cases, on reaching the earth will respectively be

$$
\begin{aligned}
F^4_{\nu_e} &= P^4_{\nu_e\to\nu_e}\phi_{\nu_e} + P^4_{\nu_\mu\to\nu_e}\phi_{\nu_\mu},\\
F^4_{\nu_\mu} &= P^4_{\nu_\mu\to\nu_\mu}\phi_{\nu_\mu} + P^4_{\nu_e\to\nu_\mu}\phi_{\nu_e},\\
F^4_{\nu_\tau} &= P^4_{\nu_e\to\nu_\tau}\phi_{\nu_e} + P^4_{\nu_\mu\to\nu_\tau}\phi_{\nu_\mu},\\
F^4_{\nu_s} &= P^4_{\nu_e\to\nu_s}\phi_{\nu_e} + P^4_{\nu_\mu\to\nu_s}\phi_{\nu_\mu},
\end{aligned}
$$

.....11)

and

$$
\begin{aligned}
F^3_{\nu_e} &= P^3_{\nu_e\to\nu_e}\phi_{\nu_e} + P^3_{\nu_\mu\to\nu_e}\phi_{\nu_\mu},\\
F^3_{\nu_\mu} &= P^3_{\nu_\mu\to\nu_\mu}\phi_{\nu_\mu} + P^3_{\nu_e\to\nu_\mu}\phi_{\nu_e},\\
F^3_{\nu_\tau} &= P^3_{\nu_e\to\nu_\tau}\phi_{\nu_e} + P^3_{\nu_\mu\to\nu_\tau}\phi_{\nu_\mu}.
\end{aligned}
$$

..........12)

Now with Eqs. (2, 8), Eq. (11) can be rewritten as

$$
\begin{pmatrix} F^4_{\nu_e} \\ F^4_{\nu_\mu} \\ F^4_{\nu_\tau} \\ F^4_{\nu_s} \end{pmatrix} = \begin{pmatrix} |\tilde{U}_{e1}|^2 & |\tilde{U}_{e2}|^2 & |\tilde{U}_{e3}|^2 & |\tilde{U}_{e4}|^2 \\ |\tilde{U}_{\mu 1}|^2 & |\tilde{U}_{\mu 2}|^2 & |\tilde{U}_{\mu 3}|^2 & |\tilde{U}_{\mu 4}|^2 \\ |\tilde{U}_{\tau 1}|^2 & |\tilde{U}_{\tau 2}|^2 & |\tilde{U}_{\tau 3}|^2 & |\tilde{U}_{\tau 4}|^2 \\ |\tilde{U}_{s1}|^2 & |\tilde{U}_{s2}|^2 & |\tilde{U}_{s3}|^2 & |\tilde{U}_{s4}|^2 \end{pmatrix}
$$

$$
\times \begin{pmatrix} |\tilde{U}_{e1}|^2 & |\tilde{U}_{\mu 1}|^2 & |\tilde{U}_{\tau 1}|^2 & |\tilde{U}_{s1}|^2 \\ |\tilde{U}_{e2}|^2 & |\tilde{U}_{\mu 2}|^2 & |\tilde{U}_{\tau 2}|^2 & |\tilde{U}_{s2}|^2 \\ |\tilde{U}_{e3}|^2 & |\tilde{U}_{\mu 3}|^2 & |\tilde{U}_{\tau 3}|^2 & |\tilde{U}_{s3}|^2 \\ |\tilde{U}_{e4}|^2 & |\tilde{U}_{\mu 4}|^2 & |\tilde{U}_{\tau 4}|^2 & |\tilde{U}_{s4}|^2 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 0 \\ 0 \end{pmatrix} \phi_{\nu_e}.
$$

.13)

Similarly for 3-flavour scenario, we can write Eq. (12) as

$$\begin{pmatrix} F^3_{\nu_e} \\ F^3_{\nu_\mu} \\ F^3_{\nu_\tau} \end{pmatrix} = \begin{pmatrix} |\mathcal{U}_{e1}|^2 & |\mathcal{U}_{e2}|^2 & |\mathcal{U}_{e3}|^2 \\ |\mathcal{U}_{\mu1}|^2 & |\mathcal{U}_{\mu2}|^2 & |\mathcal{U}_{\mu3}|^2 \\ |\mathcal{U}_{\tau1}|^2 & |\mathcal{U}_{\tau2}|^2 & |\mathcal{U}_{\tau3}|^2 \end{pmatrix} \begin{pmatrix} |\mathcal{U}_{e1}|^2 & |\mathcal{U}_{\mu1}|^2 & |\mathcal{U}_{\tau1}|^2 \\ |\mathcal{U}_{e2}|^2 & |\mathcal{U}_{\mu2}|^2 & |\mathcal{U}_{\tau2}|^2 \\ |\mathcal{U}_{e3}|^2 & |\mathcal{U}_{\mu3}|^2 & |\mathcal{U}_{\tau3}|^2 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} \phi_{\nu_e}. \quad \ldots 14)$$

Eq. (13) then follows that

$$\begin{aligned}
F^4_{\nu_e} &= [|\tilde{U}_{e1}|^2(1+|\tilde{U}_{\mu1}|^2-|\tilde{U}_{\tau1}|^2-|\tilde{U}_{s1}|^2)+|\tilde{U}_{e2}|^2(1+|\tilde{U}_{\mu2}|^2-|\tilde{U}_{\tau2}|^2-|\tilde{U}_{s2}|^2) \\
&\quad +|\tilde{U}_{e3}|^2(1+|\tilde{U}_{\mu3}|^2-|\tilde{U}_{\tau3}|^2-|\tilde{U}_{s3}|^2)+|\tilde{U}_{e4}|^2(1+|\tilde{U}_{\mu4}|^2-|\tilde{U}_{\tau4}|^2-|\tilde{U}_{s4}|^2)]\phi_{\nu_e}, \\
F^4_{\nu_\mu} &= [|\tilde{U}_{\mu1}|^2(1+|\tilde{U}_{\mu1}|^2-|\tilde{U}_{\tau1}|^2-|\tilde{U}_{s1}|^2)+|\tilde{U}_{\mu2}|^2(1+|\tilde{U}_{\mu2}|^2-|\tilde{U}_{\tau2}|^2-|\tilde{U}_{s2}|^2) \\
&\quad +|\tilde{U}_{\mu3}|^2(1+|\tilde{U}_{\mu3}|^2-|\tilde{U}_{\tau3}|^2-|\tilde{U}_{s3}|^2)+|\tilde{U}_{\mu4}|^2(1+|\tilde{U}_{\mu4}|^2-|\tilde{U}_{\tau4}|^2-|\tilde{U}_{s4}|^2)]\phi_{\nu_e}, \\
F^4_{\nu_\tau} &= [|\tilde{U}_{\tau1}|^2(1+|\tilde{U}_{\mu1}|^2-|\tilde{U}_{\tau1}|^2-|\tilde{U}_{s1}|^2)+|\tilde{U}_{\tau2}|^2(1+|\tilde{U}_{\mu2}|^2-|\tilde{U}_{\tau2}|^2-|\tilde{U}_{s2}|^2) \\
&\quad +|\tilde{U}_{\tau3}|^2(1+|\tilde{U}_{\mu3}|^2-|\tilde{U}_{\tau3}|^2-|\tilde{U}_{s3}|^2)+|\tilde{U}_{\tau4}|^2(1+|\tilde{U}_{\mu4}|^2-|\tilde{U}_{\tau4}|^2-|\tilde{U}_{s4}|^2)]\phi_{\nu_e}, \quad \ldots 15) \\
F^4_{\nu_s} &= [|\tilde{U}_{s1}|^2(1+|\tilde{U}_{\mu1}|^2-|\tilde{U}_{\tau1}|^2-|\tilde{U}_{s1}|^2)+|\tilde{U}_{s2}|^2(1+|\tilde{U}_{\mu2}|^2-|\tilde{U}_{\tau2}|^2-|\tilde{U}_{s2}|^2) \\
&\quad +|\tilde{U}_{s3}|^2(1+|\tilde{U}_{\mu3}|^2-|\tilde{U}_{\tau3}|^2-|\tilde{U}_{s3}|^2)+|\tilde{U}_{s4}|^2(1+|\tilde{U}_{\mu4}|^2-|\tilde{U}_{\tau4}|^2-|\tilde{U}_{s4}|^2)]\phi_{\nu_e}.
\end{aligned}$$

Eq. (14) can be written as

$$\begin{aligned}
F^3_{\nu_e} &= [|\mathcal{U}_{e1}|^2(1+|\mathcal{U}_{\mu1}|^2-|\mathcal{U}_{\tau1}|^2)+|\mathcal{U}_{e2}|^2(1+|\mathcal{U}_{\mu2}|^2-|\mathcal{U}_{\tau2}|^2)+|\mathcal{U}_{e3}|^2(1+|\mathcal{U}_{\mu3}|^2-|\mathcal{U}_{\tau3}|^2)]\phi_{\nu_e}, \\
F^3_{\nu_\mu} &= [|\mathcal{U}_{\mu1}|^2(1+|\mathcal{U}_{\mu1}|^2-|\mathcal{U}_{\tau1}|^2)+|\mathcal{U}_{\mu2}|^2(1+|\mathcal{U}_{\mu2}|^2-|\mathcal{U}_{\tau2}|^2)+|\mathcal{U}_{\mu3}|^2(1+|\mathcal{U}_{\mu3}|^2-|\mathcal{U}_{\tau3}|^2)]\phi_{\nu_e}, \quad \ldots 16) \\
F^3_{\nu_\tau} &= [|\mathcal{U}_{\tau1}|^2(1+|\mathcal{U}_{\mu1}|^2-|\mathcal{U}_{\tau1}|^2)+|\mathcal{U}_{\tau2}|^2(1+|\mathcal{U}_{\mu2}|^2-|\mathcal{U}_{\tau2}|^2)+|\mathcal{U}_{\tau3}|^2(1+|\mathcal{U}_{\mu3}|^2-|\mathcal{U}_{\tau3}|^2)]\phi_{\nu_e}.
\end{aligned}$$

Formalism

Unparticle decay of GRB neutrinos

- We consider a decay phenomenon, where neutrino having mass eigenstate ν_j decays to the invisible unparticle $(\mathcal{U})$ and another light neutrino with mass eigenstate ν_i.

$$\nu_j \to \mathcal{U} + \nu_i \quad ....17)$$

- The effective Lagrangian for the above mentioned process takes the following form in the low energy regime

$$L_s = \frac{\lambda_\nu^{\alpha\beta}}{\Lambda_{\mathcal{U}}^{d_{\mathcal{U}}-1}} \bar{\nu}_\alpha \nu_\beta \mathcal{O}_{\mathcal{U}} \quad ....18)$$

- The most relevant quantity for the decay process is the total decay rate Γ_j or equivalently the lifetime of neutrino $\tau_{\mathcal{U}} = 1/\Gamma_j$. The lifetime $\tau_{\mathcal{U}}$ can be expressed as

$$\frac{\tau_{\mathcal{U}}}{m_j} = \frac{16\pi^2 d_{\mathcal{U}}(d_{\mathcal{U}}^2 - 1)}{A_d|\lambda_\nu^{ij}|^2} \left(\frac{\Lambda_{\mathcal{U}}^2}{m_j^2}\right)^{d_{\mathcal{U}}-1} \frac{1}{m_j^2} \quad .... 19)$$

where m_j is the mass of the decaying neutrino.

Formalism (Contd.)

- The normalization constant in Eq. (18) is defined as

$$A_d = \frac{16\pi^{5/2}}{(2\pi)^{2d_{\mathcal{U}}}} \frac{\Gamma(d_{\mathcal{U}} + 1/2)}{\Gamma(d_{\mathcal{U}} - 1)\Gamma(2d_{\mathcal{U}})} \quad ....20)$$

- The flux for a neutrino $|\nu_\alpha\rangle$ of flavour α on reaching the Earth from GRB after undergoing the unparticle decay along the baseline length (L) is given as

$$\phi_{\nu_\alpha}(E) = \sum_i \sum_\beta \phi^s_{\nu_\beta} |U_{\beta i}|^2 |U_{\alpha i}|^2 exp(-4\pi L/(\lambda_d)_i) \ . \quad ....21)$$

$U_{\alpha i}$, $U_{\beta i}$ - PMNS matrix elements

- The decay length $((\lambda_d)_i)$ in the Eq. (21) can be expressed as $(\lambda_d)_i = 4\pi \frac{E_\nu}{\alpha_i} = 2.5 \text{ Km} \frac{E_\nu}{\text{GeV}} \frac{\text{ev}^2}{\alpha_i}$ ..22)
where $[\alpha_i (= m_i/\tau)]$

Formalism (Contd.) (GRB neutrino flux)

In the absence of decay or oscillation the neutrino spectrum on reaching the Earth from a GRB at redshift z takes the form

$$\frac{dN_\nu}{dE_\nu} = \frac{dN_\nu}{dE_\nu^s}\frac{1}{4\pi L^2(z)}(1+z) \qquad ...23)$$

In the absence of CP violation $\mathcal{F}(E_\nu^s) = \frac{dN_\nu}{dE_\nu^s} = \frac{dN_{\nu+\bar{\nu}}}{dE_\nu^s}$. The spectra for neutrinos will be $0.5\mathcal{F}(E_\nu^s)$.

Now the neutrinos produced in the GRB process in the proportion

$$\nu_e : \nu_\mu : \nu_\tau : \nu_s \; = \; 1:2:0:0 \qquad ....24)$$

Therefore

$$\phi_{\nu_e}^s = \frac{1}{6}\mathcal{F}(E_\nu^s) \; , \phi_{\nu_\mu}^s = \frac{2}{6}\mathcal{F}(E_\nu^s) = 2\phi_{\nu_e}^s \; , \phi_{\nu_\tau}^s = 0 \; , \phi_{\nu_s}^s = 0 \qquad .....25)$$

where $\phi_{\nu_e}^s$, $\phi_{\nu_\mu}^s$, $\phi_{\nu_\tau}^s$ and $\phi_{\nu_s}^s$ are the fluxes of ν_e, ν_μ, ν_τ and ν_s at source respectively.

Formalism (Contd.)

Applying the equation Eq. (21) and by considering the condition that the lightest mass state $|\nu_1\rangle$ is stable we can write the flux of neutrino flavours for 4 flavour cases on reaching the Earth as

$$\begin{aligned}
\phi_{\nu_e}^{\text{detector}} &= ([|\tilde{U}_{e1}|^2(1+|\tilde{U}_{\mu 1}|^2-|\tilde{U}_{\tau 1}|^2-|\tilde{U}_{s1}|^2) \\
&+|\tilde{U}_{e2}|^2(1+|\tilde{U}_{\mu 2}|^2-|\tilde{U}_{\tau 2}|^2-|\tilde{U}_{s2}|^2)\exp(-4\pi L(z)/(\lambda_d)_2) \\
&+|\tilde{U}_{e3}|^2(1+|\tilde{U}_{\mu 3}|^2-|\tilde{U}_{\tau 3}|^2-|\tilde{U}_{s3}|^2)\exp(-4\pi L(z)/(\lambda_d)_3) \\
&+|\tilde{U}_{e4}|^2(1+|\tilde{U}_{\mu 4}|^2-|\tilde{U}_{\tau 4}|^2-|\tilde{U}_{s4}|^2)\exp(-4\pi L(z)/(\lambda_d)_4)]\phi_{\nu_e}^s) \\
\phi_{\nu_\mu}^{\text{detector}} &= ([|\tilde{U}_{\mu 1}|^2(1+|\tilde{U}_{\mu 1}|^2-|\tilde{U}_{\tau 1}|^2-|\tilde{U}_{s1}|^2) \\
&+|\tilde{U}_{\mu 2}|^2(1+|\tilde{U}_{\mu 2}|^2-|\tilde{U}_{\tau 2}|^2-|\tilde{U}_{s2}|^2)\exp(-4\pi L(z)/(\lambda_d)_2) \\
&+|\tilde{U}_{\mu 3}|^2(1+|\tilde{U}_{\mu 3}|^2-|\tilde{U}_{\tau 3}|^2-|\tilde{U}_{s3}|^2)\exp(-4\pi L(z)/(\lambda_d)_3) \\
&+|\tilde{U}_{\mu 4}|^2(1+|\tilde{U}_{\mu 4}|^2-|\tilde{U}_{\tau 4}|^2-|\tilde{U}_{s4}|^2)\exp(-4\pi L(z)/(\lambda_d)_4)]\phi_{\nu_e}^s) \quad \ldots.26) \\
\phi_{\nu_\tau}^{\text{detector}} &= ([|\tilde{U}_{\tau 1}|^2(1+|\tilde{U}_{\mu 1}|^2-|\tilde{U}_{\tau 1}|^2-|\tilde{U}_{s1}|^2) \\
&+|\tilde{U}_{\tau 2}|^2(1+|\tilde{U}_{\mu 2}|^2-|\tilde{U}_{\tau 2}|^2-|\tilde{U}_{s2}|^2)\exp(-4\pi L(z)/(\lambda_d)_2) \\
&+|\tilde{U}_{\tau 3}|^2(1+|\tilde{U}_{\mu 3}|^2-|\tilde{U}_{\tau 3}|^2-|\tilde{U}_{s3}|^2)\exp(-4\pi L(z)/(\lambda_d)_3) \\
&+|\tilde{U}_{\tau 4}|^2(1+|\tilde{U}_{\mu 4}|^2-|\tilde{U}_{\tau 4}|^2-|\tilde{U}_{s4}|^2)\exp(-4\pi L(z)/(\lambda_d)_4)]\phi_{\nu_e}^s) \\
\phi_{\nu_s}^{\text{detector}} &= ([|\tilde{U}_{s1}|^2(1+|\tilde{U}_{\mu 1}|^2-|\tilde{U}_{\tau 1}|^2-|\tilde{U}_{s1}|^2) \\
&+|\tilde{U}_{s2}|^2(1+|\tilde{U}_{\mu 2}|^2-|\tilde{U}_{\tau 2}|^2-|\tilde{U}_{s2}|^2)\exp(-4\pi L(z)/(\lambda_d)_2) \\
&+|\tilde{U}_{s3}|^2(1+|\tilde{U}_{\mu 3}|^2-|\tilde{U}_{\tau 3}|^2-|\tilde{U}_{s3}|^2)\exp(-4\pi L(z)/(\lambda_d)_3) \\
&+|\tilde{U}_{s4}|^2(1+|\tilde{U}_{\mu 4}|^2-|\tilde{U}_{\tau 4}|^2-|\tilde{U}_{s4}|^2)\exp(-4\pi L(z)/(\lambda_d)_4)]\phi_{\nu_e}^s)
\end{aligned}$$

Detection of UHE neutrinos from a diffuse GRB

The secondary muon yields from the GRB neutrinos can be detected in a detector of unit area above a threshold energy $E_{\rm thr}$ is given by

$$S = \int_{E_{\rm thr}}^{E_{\nu\max}} dE_\nu \frac{dN_\nu}{dE_\nu} P_{\rm shadow}(E_\nu) P_\mu(E_\nu, E_{\rm thr}). \quad .....27)$$

$P_{\rm shadow}(E_\nu)$ - Probability that a neutrino reaches the detector travelling through Earth matter

$$P_{\rm shadow}(E_\nu) = \frac{1}{2\pi} \int_{-1}^{0} d\cos\theta_z \int d\phi \exp[-z(\theta_z)/L_{\rm int}(E_\nu)], \quad .... 28)$$

The interaction length $L_{\rm int} = \dfrac{1}{\sigma^{\rm tot}(E_\nu) N_A}.$ ...... 29)

The effective path length $z(\theta_z) = \int \rho(r(\theta_z, l)) dl.$ ..... 30)

Probability of neutrino induced muon to reach the detector $P_\mu(E_\nu, E_{\rm thr}) = N_A \sigma^{\rm cc}(E_\nu) \langle R(E_\mu; E_{\rm thr})\rangle$ ....31)

where the average muon range in the rock $\langle R(E_\mu; E_{\rm thr})\rangle$ is given by

$$\langle R(E_\mu; E_{\rm thr})\rangle = \frac{1}{\sigma_{\rm CC}} \int_0^{(1-E_{\rm thr}/E_\nu)} dy R(E_\nu(1-y); E_{\rm thr}) \times \frac{d\sigma_{\rm CC}(E_\nu, y)}{dy}. \quad ..... 32)$$

$$R(E_\mu; E_{\rm thr}) = \int_{E_{\rm thr}}^{E_\mu} \frac{dE_\mu}{\langle dE_\mu/dX \rangle} \simeq \frac{1}{\xi} \ln\left(\frac{\alpha + \xi E_\mu}{\alpha + \xi E_{\rm thr}}\right). \quad ......33)$$

The average energy loss of muon with energy E_μ is given as

$$\left\langle \frac{dE_\mu}{dX} \right\rangle = -\alpha - \xi E_\mu. \quad .... 34)$$

$$\alpha = 2.033 + 0.077 \ln[E_\mu(\mathrm{GeV})] \times 10^3 \ \mathrm{GeV\,cm^2\,gm^{-1}},$$

$$\xi = 2.033 + 0.077 \ln[E_\mu(\mathrm{GeV})] \times 10^{-6} \ \mathrm{GeV\,cm^2\,gm^{-1}} \quad ...\ 35)$$

for $E_\mu \lesssim 10^6$ GeV and otherwise $\alpha = 2.033 \times 10^{-3} \ \mathrm{GeV\,cm^2\,gm^{-1}}$,

$$\xi = 3.9 \times 10^{-6} \ \mathrm{GeV\,cm^2\,gm^{-1}}.$$

.... 36)

$\frac{dN_\nu}{dE_\nu}$ in Eq. (27) is replaced by $F^4_{\nu_\mu}$ from Eq. (15) and by $F^3_{\nu_\mu}$ from Eq. (16) for the four-flavour and Three-flavour scenarios respectively.

- Detection of UHE neutrinos from a single GRB

$$P_{\rm shadow} = exp[-X(\theta_z)/l_{int}(E_\nu)] \ , \quad ...37)$$

In the case of detecting muon events at a 1 km² detector such as IceCube the flux $\frac{dN_\nu}{dE_\nu}$ in Eq. (27) is replaced by $\phi^4_{\nu_\mu}$ in Eq. (26).

Calculations and Results

Diffused Neutrino Flux

For the purpose of our analysis, we have considered a ratio R between the muon and the shower events, which is defined as

$$R = \frac{T_\mu}{T_{\rm sh}}, \quad ......38)$$

Where

$$\begin{aligned} T_\mu &= S(\text{for } \nu_\mu) + S(\text{for } \nu_\tau), \\ T_{\rm sh} &= S_{\rm sh}(\text{for } \nu_e \text{ CC interaction}) \\ &\quad + S_{\rm sh}(\text{for } \nu_e \text{ NC interaction}) \\ &\quad + S_{\rm sh}(\text{for } \nu_\mu \text{NC interaction}) \\ &\quad + S_{\rm sh}(\text{for } \nu_\tau \text{ NC interaction}), \end{aligned} \quad ....39)$$

The event rate of muons (S) and the same for the shower ($S_{\rm sh}$) are expressed as

$$S = \int_{E_{\rm thr}}^{E_{\nu_{\rm max}}} dE_\nu \frac{dN_\nu}{dE_\nu} P_{\rm shadow}(E_\nu) P_\mu(E_\nu, E_{\rm thr}). \quad ....40)$$

$$S_{\rm sh} = V \int_{E_{\rm thr}}^{E_{\nu_{\rm max}}} dE_\nu \frac{dN_\nu}{dE_\nu} P_{\rm shadow}(E_\nu) \int dy \frac{1}{\sigma^i} \frac{d\sigma^i}{dy} P_{\rm int}(E_\nu, y). \quad .....41)$$

- For the isotropic fluxes

θ_{14}	θ_{24}	θ_{34}	R_4 (in 4f)	R_3 (in 3f)
3°	5°	20°	9.48	1.80
4°	6°	15°	9.68	1.80

Table 1: Comparison of the muon to shower ratio for a diffused GRB neutrino flux for the 4 flavour (3+1) case compared with the same for 3 flavour case for two sets of active sterile neutrino mixing angles.

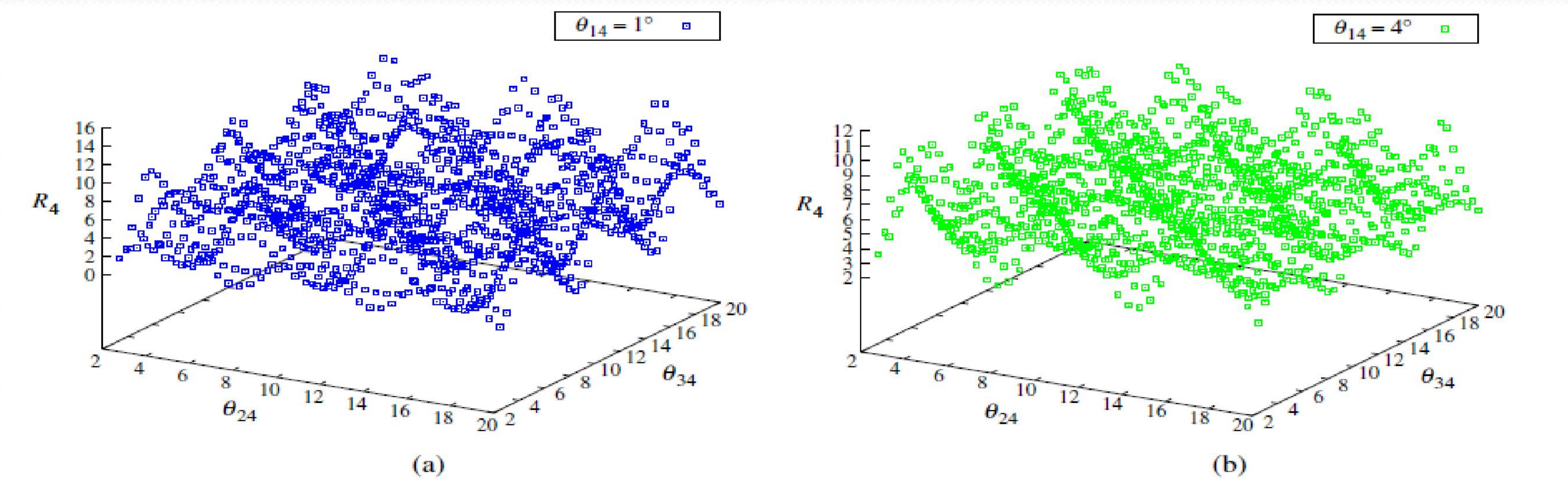


Figure 1: . Variation of R_4 with θ_{24} and θ_{34} for (a) θ_{14} = 1° and (b) θ_{14} = 4°.

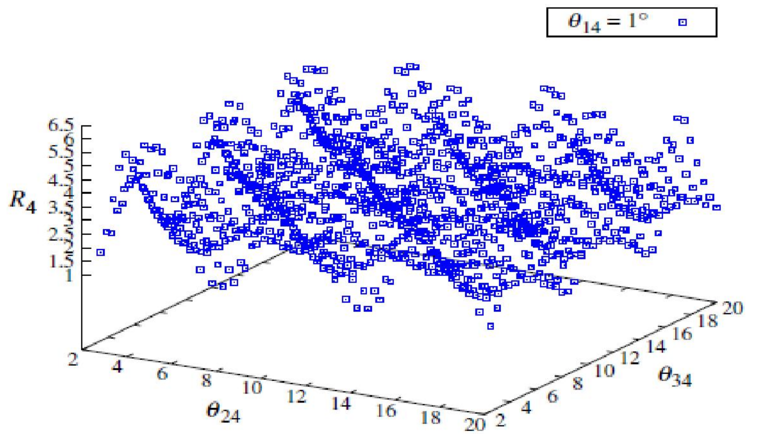
From the analysis of the high-energy starting events (HESE) data (the IceCube Collaboration), they calculated a best fit power law for the neutrino flux as

$$E^2\phi(E) = 2.46 \pm 0.8 \times 10^{-8} \left(\frac{E}{100\ \mathrm{TeV}}\right)^{-0.92}\ \mathrm{GeV\,cm^{-2}\,s^{-1}\,sr^{-1}} \quad \ldots\ldots 42)$$

For a one component fit the neutrino flux $\phi(E) \sim E^{-\gamma}$, with the index $\gamma = 2.92^{+0.33}_{-0.29}$. We have computed R_4, R_3 for this flux as well and the energy range 60 TeV is to be considered for such calculations.

θ_{14}	θ_{24}	θ_{34}	R_4 (in 4f)	R_3 (in 3f)
3°	5°	20°	2.01	0.55
4°	6°	15°	2.04	0.55

Table 2: Same as Table 1, but here we consider the diffuse flux of UHE neutrinos obtained from the recent analysis of the IceCube (HESE) data.



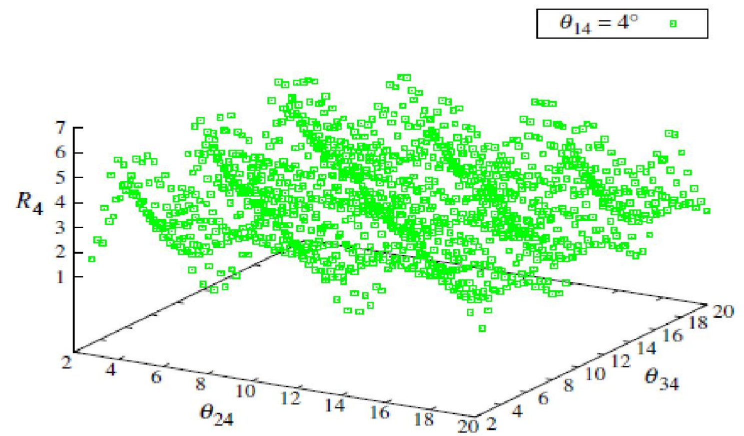


Figure 1: Variation of R_4 with θ_{24} and θ_{34} for (a) θ_{14} = 1° and (b) θ_{14} = 4° (the recent IceCube HESE data).

Oscillation+Decay (with single GRB flux)

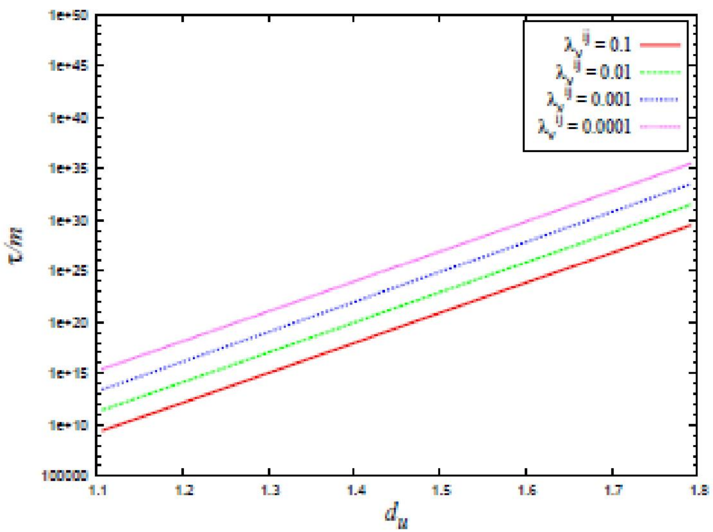


Figure 2: The Variations of the neutrino decay life time (τ/m) with the unparticle dimension $(d_{\mathcal{U}})$ are shown for four different values (0.1, 0.01, 0.001, 0.0001) of couplings λ_ν^{ij}.

Oscillation+Decay Contd…

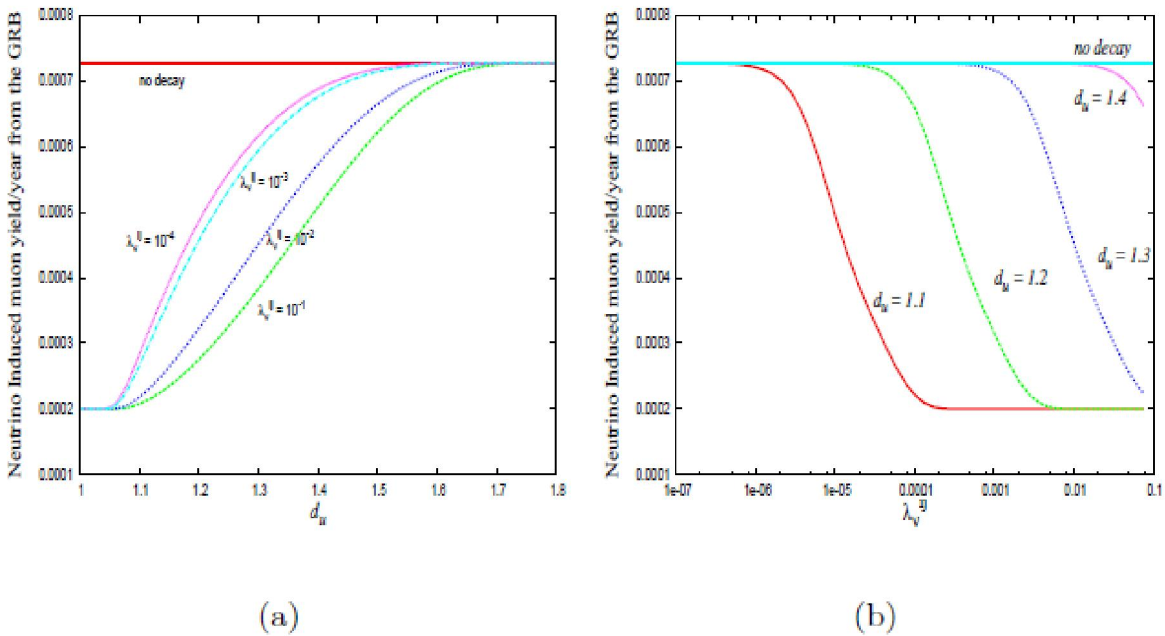


Figure 2: The variations of the neutrino induced upward going muons per year from the GRB with (a) different values of $d_{\mathcal{U}}$ for four different fixed values of λ_ν^{ij} as well as for the mass flavour case (no decay case), (b) different values of λ_ν^{ij} for four different fixed values of the unparticle dimension $d_{\mathcal{U}}$ (1.1, 1.2, 1.3, 1.4) and in addition for no decay case.

Oscillation+Decay Contd...

θ_{14}	θ_{24}	θ_{34}	$d_{\mathcal{U}}$	λ_ν^{ij}	R
3°	5°	20°	1.2	10^{-4}	0.9
			1.3	10^{-2}	0.6

Table 3: The ratio R of muon yields at IceCube for UHE neutrinos from a GRB with and without decay in a four neutrino framework for two sets of values of decay parameters namely $d_{\mathcal{U}}$, λ_ν^{ij}.

Summary

- The ratio of muon events to the shower events was calculated for both the three-flavour and four-flavour cases, which are denoted as R_3 and R_4. For this purpose, we have considered two sets of UHE neutrino fluxes –
  1. theoretical flux for diffuse isotropic UHE neutrinos from GRBs given by Waxman and Bahcall.
  2. the analysis of the recent IceCube HESE data.

- The muon to shower ratio can be 6-8 times larger for 3+1 scenario when compared to that for normal three active neutrino formalism if the Waxman-Bahcall flux is considered, and R_4 can be 3 -4 times greater than R_3 using the flux given by the IceCube (HESE data) analysis. Therefore, the present analysis has shown that any excess of such events detected in a 1 Km2 detector over that predicted for three-neutrino mixing can clearly indicate the presence of active-sterile neutrino mixing.

- For the case of oscillation + unparticle decay we demonstrated how the effect of unparticle decay of neutrino affect the detection of muon yield for UHE neutrinos from a single GRB in 4-flavour scheme.

THANK YOU

BACKUP SLIDES

Oscillation+Decay Contd…

θ_{14}	θ_{24}	θ_{34}	$d_{\mathcal{U}}$	λ_ν^{ij}	$\phi_{\nu_e} : \phi_{\nu_\mu} : \phi_{\nu_\tau}$ (with decay and oscillation)	$\phi_{\nu_e} : \phi_{\nu_\mu} : \phi_{\nu_\tau}$ no decay (only oscillation)
3°	5°	20°	1.2	10^{-4}	$0.349 : 0.621 : 0.489$	$0.449 : 0.979 : 0.834$
			1.3	10^{-2}	$0.344 : 0.265 : 9.5\times10^{-2}$	

Table 4 : Same as Table 3 but for the flavour ratio of active neutrinos in 4-flavour framework.

Formalism

Unparticle decay of GRB neutrinos

- We consider a decay phenomenon, where neutrino having mass eigenstate ν_j decays to the invisible unparticle $(\mathcal{U})$ and another light neutrino with mass eigenstate ν_i.

$$\nu_j \to \mathcal{U} + \nu_i \quad ....4)$$

- The effective lagrangian for the above mentioned process takes the following form in the low energy regime

$$L_s = \frac{\lambda_\nu^{\alpha\beta}}{\Lambda_{\mathcal{U}}^{d_{\mathcal{U}}-1}} \bar{\nu}_\alpha \nu_\beta \mathcal{O}_{\mathcal{U}} \quad ....5)$$

where $\alpha, \beta = e, \mu, \tau, s$ - flavour indices, $d_{\mathcal{U}}$ - the scaling dimension of the scalar unparticle operator $\mathcal{O}_{\mathcal{U}}$ $\Lambda_{\mathcal{U}}$ - the dimension transmutation scale at which the scale invariance sets in, $\lambda_\nu^{\alpha\beta}$ - the relevant coupling constant.

- The neutrino and flavour eigenstates are related through $|\nu_i\rangle = \sum_\alpha U_{\alpha i}^* |\nu_\alpha\rangle$ ....6)
  $U_{\alpha i}$ - elements of the PMNS mixing matrix.
- In the mass basis the interaction between neutrinos and the unparticles can be written as $\lambda_\nu^{ij} \bar{\nu}_i \nu_j \mathcal{O}_{\mathcal{U}} / \Lambda_{\mathcal{U}}^{d_{\mathcal{U}}-1}$, where λ_ν^{ij} is the coupling constant in the mass eigenstate i, j.

λ_ν^{ij} can be expressed as $\lambda_\nu^{ij} = \sum_{\alpha,\beta} U_{\alpha i}^* \lambda_\nu^{\alpha\beta} U_{\beta j}$ ....7)

Formalism

- Detection of UHE neutrinos from a single GRB :-

The secondary muon yields from the GRB neutrinos can be detected in a detector of unit area above a threshold energy $E_{\rm th}$ is given by

$$S = \int_{E_{\rm th}}^{E_{\nu\rm max}} dE_\nu \frac{dN_\nu}{dE_\nu} P_{\rm shadow}(E_\nu) P_\mu(E_\nu, E_{\rm th}) \quad ,....14)$$

where $P_{\rm shadow}(E_\nu)$ represents the probability that a neutrino reaches the terrestrial detector such as IceCube being unabsorbed by the Earth, which takes the form

$$P_{\rm shadow} = exp[-X(\theta_z)/l_{int}(E_\nu)] \quad , ... 15)$$

where energy dependent neutrino-nucleon interaction length $L_{\rm int}(E_\nu)$ is given by

$$L_{\rm int}(E_\nu) = \frac{1}{\sigma_{\rm tot}(E_\nu) N_A} \quad ...... 16)$$

The effective path length $X(\theta_z)\ ({\rm gm/cm^2})$ can be written as

$$X(\theta_z) = \int \rho(r(\theta_z, l)) dl \ . \quad ..... 17)$$

The probability $P_\mu(E_\nu, E_{\rm th})$ that a neutrino induced muon reaching the detector with an energy above $E_{\rm th}$ can be written as

$$P_\mu(E_\nu, E_{\rm th}) = N_A \sigma_{cc}(E_\nu) \langle R(E_\mu; E_{\rm th}) \rangle \quad , ...18)$$

where the average muon range in the rock $\langle R(E_\mu; E_{\rm th}) \rangle$ is given by

$$\langle R(E_\mu; E_{\rm th}) \rangle = \frac{1}{\sigma_{\rm CC}} \int_0^{(1-E_{\rm th}/E_\nu)} dy R(E_\nu(1-y); E_{\rm th}) \times \frac{d\sigma_{\rm CC}(E_\nu, y)}{dy} \quad ..... 19)$$

Formalism

The muon range $R(E_\mu; E_{\rm th})$ can be expressed as

$$R(E_\mu, E_{\rm th}) = \int_{E_{\rm th}}^{E_\mu} \frac{dE_\mu}{\langle dE_\mu/dX \rangle} \simeq \frac{1}{\beta} \ln\left(\frac{\alpha + \beta E_\mu}{\alpha + \beta E_{\rm th}}\right) \quad \text{......19)}$$

The average energy loss of muon with energy E_μ is given as

$$\left\langle \frac{dE_\mu}{dX} \right\rangle = -\alpha - \beta E_\mu \ . \quad \text{........ 20)}$$

The values of the constants α and β in Eq. (20), which we have considered in our calculations are

$$\begin{aligned} \alpha &= 2.033 + 0.077\ \ln[E_\mu(\text{GeV})] \times 10^3\ \text{GeV cm}^2\ \text{gm}^{-1}\ , \\ \beta &= 2.033 + 0.077\ \ln[E_\mu(\text{GeV})] \times 10^{-6}\ \text{GeV cm}^2\ \text{gm}^{-1}\ , \end{aligned} \quad \text{... 21)}$$

for $E_\mu \leq 10^6$ GeV | and otherwise

$$\begin{aligned} \alpha &= 2.033 \times 10^{-3}\ \text{GeV cm}^2\ \text{gm}^{-1} \\ \beta &= 3.9 \times 10^{-6}\ \text{GeV cm}^2\ \text{gm}^{-1}\ . \end{aligned} \quad \text{....... 22)}$$

In the case of detecting muon events at a 1 km² detector such as IceCube the flux $\frac{dN_\nu}{dE_\nu}$ in Eq. (14) is Replaced by $\phi^4_{\nu_\mu}$ in Eq. (13).

Calculations and Results

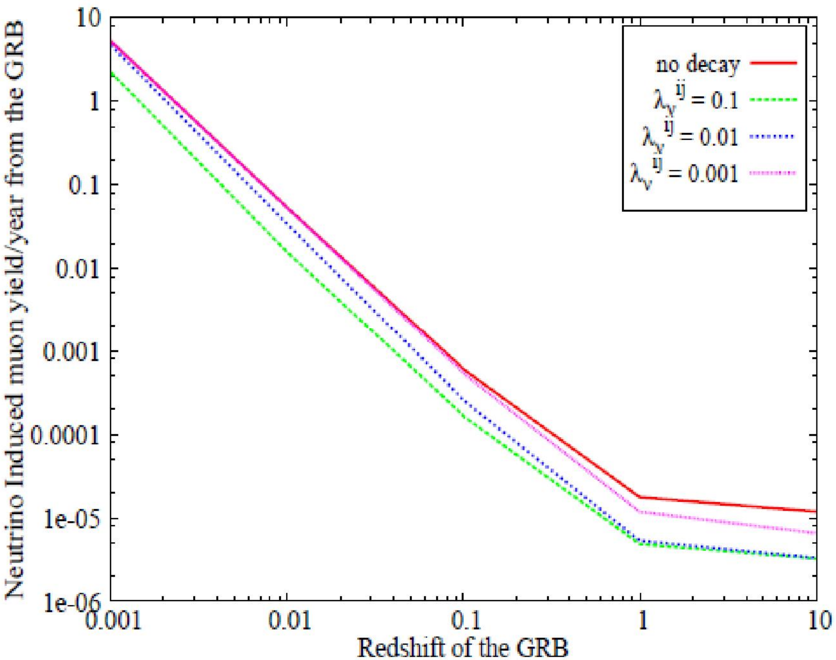


Figure 3: Variations of the neutrino induced muons per year from the GRB with different redshifts (z) for three different values of λ_ν^{ij} as well as for no decay case at a fixed zenith angle $(\theta_z = 160^\circ)$.