Probing the effects of sterile neutrino on the possible ultrahigh energy neutrino signals at a km$^2$ detector in the framework of 4-flavour neutrino oscillation and unparticle decay

Madhurima Pandey
Astroparticle Physics and Cosmology Division
Saha Institute of Nuclear Physics, HBNI, Kolkata, India

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IIT, Madras

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Introduction

- The existence of a fourth sterile neutrino has been hypothesised since a long time.
- Accelerator and reactor based neutrino experiments (LSND, MINOS, MINOS+, Daya Bay, Bugey, MiniBooNE etc.) give bound on active sterile neutrino mixing angles and $\Delta m^2$, considering a 4 (3+1) neutrino framework.
- In this work we consider two scenarios for 4 (3 active + 1 sterile) scheme.
  1. Mass flavour oscillation / flavour suppression.
  2. Oscillation + unparticle decay.
- Apply it for ultrahigh energy (UHE) neutrinos from distant Gamma Ray Bursts (GRBs).
  1. Diffuse GRB neutrino flux for only oscillation / flavour suppression.
  2. Neutrino flux from single GRB (specific baseline length over which neutrino suffers decay) for oscillation + decay.
- Compute the muon/shower yield at a Km$^2$ detector such as IceCube.
Introduction (Contd...)

- Almost a decade back Georgi proposed the probable existence of a scale invariant sector.
- At a very high energy scale this scale invariance sector and the Standard Model (SM) sector may coexist and the fields of these two sectors can interact via a mediator messenger field of mass scale $M_U$.
- At low energies, the scale invariance of SM is manifestly broken (SM particles have masses).
- The interactions between this scale invariance sector and SM are suppressed (inverse power of $M_U$).
- Georgi observed at low energies scale invariance sector manifests itself by non-integral number ($d_U$) of massless invisible particles (scaling dimension of scale invariance sector) → Unparticles.
Introduction (Contd...) 

- A prototype model of such scale invariant sector can be obtained from Banks-Zaks theory, where the scale invariance sets in at energy scale $\Lambda_u$.

- We have taken a scalar unparticle operator and scalar interactions with neutrinos that enables a heavy neutrino decay to a lighter neutrino and another unparticle.
Formalism

**Four and Three Flavour Neutrino Oscillations**

In general the probability for a neutrino $|\nu_\alpha\rangle$ of flavour $\alpha$ to oscillate to a neutrino $|\nu_\beta\rangle$ of flavour $\beta$ is given by (considering no CP violation in neutrino sector)

$$P_{\nu_\alpha \rightarrow \nu_\beta} = \delta_{\alpha\beta} - 4\sum_{j>i} U_{\alpha i} U_{\beta i} U_{\alpha j} U_{\beta j} \sin^2 \left( \frac{\pi L}{\lambda_{ij}} \right).$$ ........1)

For UHE neutrinos from distant GRBs, the oscillatory part in the probability equation is averaged to half ($L$ is large, $\Delta m^2 L/E \gg 1$). The probability equation (Eq. (1)) is then reduced to

$$P_{\nu_\alpha \rightarrow \nu_\beta} = \delta_{\alpha\beta} - 2\sum_{j>i} U_{\alpha i} U_{\beta i} U_{\alpha j} U_{\beta j} \left[ \sum_{j\neq i} U_{\alpha j} U_{\beta j} \right]$$

$$= \delta_{\alpha\beta} - \sum_i U_{\alpha i} U_{\beta i} \left[ \sum_{j\neq i} U_{\alpha j} U_{\beta j} \right]$$

$$= \sum_j |U_{\alpha j}|^2 |U_{\beta j}|^2,$$ ...............2)
Considering the present 4-flavour scenario to be the minimal extension of 3-flavour case by a sterile neutrino, the mixing matrix $\tilde{U}$ can be written as

$$\tilde{U} = R_{34}(\theta_{34})R_{24}(\theta_{24})R_{14}(\theta_{14})R_{23}(\theta_{23})R_{13}(\theta_{13})R_{12}(\theta_{12}) \ldots \ldots 3)$$

where $\mathcal{U}_{ai}$ are the matrix elements of 3-flavour neutrino mixing matrix

$$\mathcal{U}_{(3\times3)} = \begin{pmatrix} c_{12}c_{13} & s_{12}s_{13} & s_{13} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13} & c_{12}c_{23} - s_{12}s_{23}s_{13} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13} & -c_{12}s_{23} - s_{12}c_{23}s_{13} & c_{23}c_{13} \end{pmatrix} \ldots \ldots 5)$$
Following Eq. (2), the oscillation probability $P_{\nu_\alpha \rightarrow \nu_\beta}^A$ (where $\alpha, \beta$ denote the flavour indices) for the four-flavour case can now be represented as

$$P_{\nu_\alpha \rightarrow \nu_\beta}^A \equiv \begin{pmatrix} P_{ee} & P_{e\mu} & P_{e\tau} & P_{es} \\ P_{\mu e} & P_{\mu\mu} & P_{\mu\tau} & P_{\mu s} \\ P_{\tau e} & P_{\tau\mu} & P_{\tau\tau} & P_{\tau s} \\ P_{se} & P_{s\mu} & P_{s\tau} & P_{ss} \end{pmatrix} \equiv XX^T,$$

with

$$X = \begin{pmatrix} |\tilde{U}_{e1}|^2 & |\tilde{U}_{e2}|^2 & |\tilde{U}_{e3}|^2 & |\tilde{U}_{e4}|^2 \\ |\tilde{U}_{\mu1}|^2 & |\tilde{U}_{\mu2}|^2 & |\tilde{U}_{\mu3}|^2 & |\tilde{U}_{\mu4}|^2 \\ |\tilde{U}_{\tau1}|^2 & |\tilde{U}_{\tau2}|^2 & |\tilde{U}_{\tau3}|^2 & |\tilde{U}_{\tau4}|^2 \\ |\tilde{U}_{s1}|^2 & |\tilde{U}_{s2}|^2 & |\tilde{U}_{s3}|^2 & |\tilde{U}_{s4}|^2 \end{pmatrix}.$$

..6)

..7)
UHE neutrino fluxes from Diffuse GRBs

From the GRBs, the neutrino (antineutrino) flavours are expected to be produced in the ratio $\nu_e : \nu_\mu : \nu_\tau = 1:2:0$ (3 flavour) and $\nu_e : \nu_\mu : \nu_\tau : \nu_s = 1:2:0:0$ (4 flavour).

The standard ratio of intrinsic neutrino flux is

$$\phi_{\nu_e} : \phi_{\nu_\mu} : \phi_{\nu_\tau} : \phi_{\nu_s} = 1:2:0:0.$$  

The isotropic flux (Waxman-Bahcall flux) for $\nu_\mu$ and $\bar{\nu}_\mu$ estimated by summing over all the sources is given as

$$F(E_\nu) = \frac{dN_{\nu_\mu + \bar{\nu}_\mu}}{dE_\nu} = N \left( \frac{E_\nu}{1 \text{ GeV}} \right)^{-n} \text{cm}^{-2} \text{s}^{-1} \text{sr}^{-1} \text{GeV}^{-1}.$$  

In the above,

$$N = 4.0 \times 10^{-13}, \quad n = 1 \quad \text{for} \quad E_\nu < 10^5 \text{ GeV},$$

$$N = 4.0 \times 10^{-8}, \quad n = 2 \quad \text{for} \quad E_\nu > 10^5 \text{ GeV}.$$

Therefore the fluxes of the corresponding flavours (which are the same for both neutrinos and antineutrinos since no CP violation is considered in the neutrino sector) can be expressed as

$$\frac{dN_{\nu_\mu}}{dE_\nu} = \phi_{\nu_\mu} = \frac{dN_{\bar{\nu}_\mu}}{dE_\nu} = \phi_{\bar{\nu}_\mu} = 0.5F(E_\nu),$$

$$\frac{dN_{\nu_e}}{dE_\nu} = \phi_{\nu_e} = \frac{dN_{\bar{\nu}_e}}{dE_\nu} = \phi_{\bar{\nu}_e} = 0.25F(E_\nu).$$
The fluxes of neutrino flavours for four and three flavour cases, on reaching the earth will respectively be

\[
\begin{align*}
F_{\nu_e}^4 &= P_{\nu_e \rightarrow \nu_e}^4 \phi_{\nu_e} + P_{\nu_e \rightarrow \nu_\mu}^4 \phi_{\nu_\mu}, \\
F_{\nu_\mu}^4 &= P_{\nu_\mu \rightarrow \nu_\mu}^4 \phi_{\nu_\mu} + P_{\nu_\mu \rightarrow \nu_e}^4 \phi_{\nu_e}, \\
F_{\nu_e}^4 &= P_{\nu_e \rightarrow \nu_e}^4 \phi_{\nu_e} + P_{\nu_e \rightarrow \nu_\tau}^4 \phi_{\nu_\tau}, \\
F_{\nu_\tau}^4 &= P_{\nu_\tau \rightarrow \nu_\tau}^4 \phi_{\nu_\tau} + P_{\nu_\tau \rightarrow \nu_e}^4 \phi_{\nu_e}.
\end{align*}
\]

\[\ldots..11)\]

and

\[
\begin{align*}
F_{\nu_e}^3 &= P_{\nu_e \rightarrow \nu_e}^3 \phi_{\nu_e} + P_{\nu_e \rightarrow \nu_\mu}^3 \phi_{\nu_\mu}, \\
F_{\nu_\mu}^3 &= P_{\nu_\mu \rightarrow \nu_\mu}^3 \phi_{\nu_\mu} + P_{\nu_\mu \rightarrow \nu_e}^3 \phi_{\nu_e}, \\
F_{\nu_e}^3 &= P_{\nu_e \rightarrow \nu_e}^3 \phi_{\nu_e} + P_{\nu_e \rightarrow \nu_\tau}^3 \phi_{\nu_\tau}, \\
F_{\nu_\tau}^3 &= P_{\nu_\tau \rightarrow \nu_\tau}^3 \phi_{\nu_\tau} + P_{\nu_\tau \rightarrow \nu_e}^3 \phi_{\nu_e}.
\end{align*}
\]

\[\ldots..12)\]

Now with Eqs. (2, 8), Eq. (11) can be rewritten as

\[
\begin{pmatrix}
F_{\nu_e}^4 \\
F_{\nu_\mu}^4 \\
F_{\nu_e}^4 \\
F_{\nu_\tau}^4
\end{pmatrix} = 
\begin{pmatrix}
|\bar{U}_{e1}|^2 & |\bar{U}_{e2}|^2 & |\bar{U}_{e3}|^2 & |\bar{U}_{e4}|^2 \\
|\bar{U}_{\mu1}|^2 & |\bar{U}_{\mu2}|^2 & |\bar{U}_{\mu3}|^2 & |\bar{U}_{\mu4}|^2 \\
|\bar{U}_{\tau1}|^2 & |\bar{U}_{\tau2}|^2 & |\bar{U}_{\tau3}|^2 & |\bar{U}_{\tau4}|^2 \\
|\bar{U}_{s1}|^2 & |\bar{U}_{s2}|^2 & |\bar{U}_{s3}|^2 & |\bar{U}_{s4}|^2
\end{pmatrix}
\begin{pmatrix}
1 \\
2 \\
1 \\
0 \\
0
\end{pmatrix} \phi_{\nu_e}.
\]

\[.13)\]
Similarly for 3-flavour scenario, we can write Eq. (12) as

\[
\begin{pmatrix}
F^3_{\nu_e} \\
F^3_{\nu_\mu} \\
F^3_{\nu_\tau}
\end{pmatrix} = 
\begin{pmatrix}
|U_{e1}|^2 & |U_{e2}|^2 & |U_{e3}|^2 \\
|U_{\mu1}|^2 & |U_{\mu2}|^2 & |U_{\mu3}|^2 \\
|U_{\tau1}|^2 & |U_{\tau2}|^2 & |U_{\tau3}|^2
\end{pmatrix} \begin{pmatrix}
|U_{e1}|^2 & |U_{\mu1}|^2 & |U_{\tau1}|^2 \\
|U_{e2}|^2 & |U_{\mu2}|^2 & |U_{\tau2}|^2 \\
|U_{e3}|^2 & |U_{\mu3}|^2 & |U_{\tau3}|^2
\end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} \phi_{\nu_e} \cdot \text{...14)}
\]

Eq. (13) then follows that

\[
F^4_{\nu_e} = |\bar{U}_{e1}|^2(1 + |\bar{U}_{\mu1}|^2 - |\bar{U}_{\tau1}|^2 - |\bar{U}_{s1}|^2) + |\bar{U}_{e2}|^2(1 + |\bar{U}_{\mu2}|^2 - |\bar{U}_{\tau2}|^2 - |\bar{U}_{s2}|^2) \\
\quad + |\bar{U}_{e3}|^2(1 + |\bar{U}_{\mu3}|^2 - |\bar{U}_{\tau3}|^2 - |\bar{U}_{s3}|^2) + |\bar{U}_{e4}|^2(1 + |\bar{U}_{\mu4}|^2 - |\bar{U}_{\tau4}|^2 - |\bar{U}_{s4}|^2) \phi_{\nu_e}, \quad \text{...15)}
\]

\[
F^4_{\nu_\mu} = |\bar{U}_{\mu1}|^2(1 + |\bar{U}_{\mu1}|^2 - |\bar{U}_{\tau1}|^2 - |\bar{U}_{s1}|^2) + |\bar{U}_{\mu2}|^2(1 + |\bar{U}_{\mu2}|^2 - |\bar{U}_{\tau2}|^2 - |\bar{U}_{s2}|^2) \\
\quad + |\bar{U}_{\mu3}|^2(1 + |\bar{U}_{\mu3}|^2 - |\bar{U}_{\tau3}|^2 - |\bar{U}_{s3}|^2) + |\bar{U}_{\mu4}|^2(1 + |\bar{U}_{\mu4}|^2 - |\bar{U}_{\tau4}|^2 - |\bar{U}_{s4}|^2) \phi_{\nu_e},
\]

\[
F^4_{\nu_\tau} = |\bar{U}_{\tau1}|^2(1 + |\bar{U}_{\mu1}|^2 - |\bar{U}_{\tau1}|^2 - |\bar{U}_{s1}|^2) + |\bar{U}_{\tau2}|^2(1 + |\bar{U}_{\mu2}|^2 - |\bar{U}_{\tau2}|^2 - |\bar{U}_{s2}|^2) \\
\quad + |\bar{U}_{\tau3}|^2(1 + |\bar{U}_{\mu3}|^2 - |\bar{U}_{\tau3}|^2 - |\bar{U}_{s3}|^2) + |\bar{U}_{\tau4}|^2(1 + |\bar{U}_{\mu4}|^2 - |\bar{U}_{\tau4}|^2 - |\bar{U}_{s4}|^2) \phi_{\nu_e}, \quad \text{...15)}
\]

Eq. (14) can be written as

\[
F^3_{\nu_e} = |U_{e1}|^2(1 + |U_{\mu1}|^2 - |U_{\tau1}|^2) + |U_{e2}|^2(1 + |U_{\mu2}|^2 - |U_{\tau2}|^2) + |U_{e3}|^2(1 + |U_{\mu3}|^2 - |U_{\tau3}|^2) \phi_{\nu_e},
\]

\[
F^3_{\nu_\mu} = |U_{\mu1}|^2(1 + |U_{\mu1}|^2 - |U_{\tau1}|^2) + |U_{\mu2}|^2(1 + |U_{\mu2}|^2 - |U_{\tau2}|^2) + |U_{\mu3}|^2(1 + |U_{\mu3}|^2 - |U_{\tau3}|^2) \phi_{\nu_e}, \quad \text{...16)}
\]

\[
F^3_{\nu_\tau} = |U_{\tau1}|^2(1 + |U_{\mu1}|^2 - |U_{\tau1}|^2) + |U_{\tau2}|^2(1 + |U_{\mu2}|^2 - |U_{\tau2}|^2) + |U_{\tau3}|^2(1 + |U_{\mu3}|^2 - |U_{\tau3}|^2) \phi_{\nu_e}.
\]
Formalism

Unparticle decay of GRB neutrinos

- We consider a decay phenomenon, where neutrino having mass eigenstate $\nu_j$ decays to the invisible unparticle ($U$) and another light neutrino with mass eigenstate $\nu_i$.

\[ \nu_j \rightarrow U + \nu_i \quad \ldots \ldots 17 \]

- The effective Lagrangian for the above mentioned process takes the following form in the low energy regime

\[ L_s = \frac{\chi_{\nu}^{\alpha \beta}}{\Lambda_{\nu}^{d\nu - 1}} \bar{\nu}_{\alpha} \nu_{\beta} \mathcal{O}_U \quad \ldots \ldots 18 \]

- The most relevant quantity for the decay process is the total decay rate $\Gamma_j$ or equivalently the lifetime of neutrino $\tau_u = 1/\Gamma_j$. The lifetime $\tau_u$ can be expressed as

\[ \frac{\tau_u}{m_j} = \frac{16 \pi^2 d_u (d_u^2 - 1)}{A_d |\chi_u^{ij}|^2} \left( \frac{\Lambda_{\nu}^{d\nu}}{m_j^2} \right)^{d_u - 1} \frac{1}{m_j^2} \quad \ldots \ldots 19 \]

where $m_j$ is the mass of the decaying neutrino.
Formalism (Contd.)

- The normalization constant in Eq. (18) is defined as

\[ A_d = \frac{16\pi^{5/2}}{(2\pi)^{2d_\alpha}} \frac{\Gamma(d_\alpha + 1/2)}{\Gamma(d_\alpha - 1)\Gamma(2d_\alpha)} \]  

- The flux for a neutrino $|\nu_\alpha\rangle$ of flavour $\alpha$ on reaching the Earth from GRB after undergoing the unparticle decay along the baseline length ($L$) is given as

\[ \phi_{\nu_\alpha}(E) = \sum_i \sum_\beta \phi_{\nu_\beta}^i |U_{\beta i}|^2 |U_{\alpha i}|^2 \exp(-4\pi L/(\lambda_d)_i) \]  

$U_{\alpha i}, U_{\beta i}$ - PMNS matrix elements

- The decay length $(\lambda_d)_i$ in the Eq. (21) can be expressed as

\[ (\lambda_d)_i = 4\pi \frac{E_\nu}{E_i} = 2.5 \text{ Km} \frac{E_\nu}{\text{GeV}} \frac{\text{ev}^2}{\alpha_i} \]  

where $[\alpha_i(= m_i/\tau)]$
Formalism (Contd.) (GRB neutrino flux)

In the absence of decay or oscillation the neutrino spectrum on reaching the Earth from a GRB at redshift $z$ takes the form

$$\frac{dN_\nu}{dE_\nu} = \frac{dN_\nu}{dE^s_\nu} \frac{1}{4\pi L^2(z)(1+z)}$$

...23)

In the absence of CP violation $\mathcal{F}(E^s_\nu) = \frac{dN_\nu}{dE^s_\nu} = \frac{dN_{\nu+\bar{\nu}}}{dE^s_\nu}$. The spectra for neutrinos will be $0.5\mathcal{F}(E^s_\nu)$.

Now the neutrinos produced in the GRB process in the proportion

$$\nu_e : \nu_\mu : \nu_\tau : \nu_s = 1 : 2 : 0 : 0$$

....24)

Therefore

$$\phi^{s}_{\nu_e} = \frac{1}{6}\mathcal{F}(E^s_\nu), \phi^{s}_{\nu_\mu} = \frac{2}{6}\mathcal{F}(E^s_\nu) = 2\phi^{s}_{\nu_e}, \phi^{s}_{\nu_\tau} = 0, \phi^{s}_{\nu_s} = 0$$

.....25)

where $\phi^{s}_{\nu_e}, \phi^{s}_{\nu_\mu}, \phi^{s}_{\nu_\tau}$ and $\phi^{s}_{\nu_s}$ are the fluxes of $\nu_e, \nu_\mu, \nu_\tau$ and $\nu_s$ at source respectively.
Formalism (Contd.)

Applying the equation Eq. (21) and by considering the condition that the lightest mass state $|\nu_1\rangle$ is stable we can write the flux of neutrino flavours for 4 flavour cases on reaching the Earth as

\[
\begin{align*}
\phi_{\nu_e}^{\text{detector}} &= (\langle |\bar{U}_{e1}|^2 |1 + |\bar{U}_{\mu_1}|^2 - |\bar{U}_{\tau_1}|^2 - |\bar{U}_{s1}|^2 \rangle \\
&\quad + |\bar{U}_{e2}|^2 (1 + |\bar{U}_{\mu_2}|^2 - |\bar{U}_{\tau_2}|^2 - |\bar{U}_{s2}|^2) \exp(-4\pi L(z)/(\lambda_d)_2) \\
&\quad + |\bar{U}_{e3}|^2 (1 + |\bar{U}_{\mu_3}|^2 - |\bar{U}_{\tau_3}|^2 - |\bar{U}_{s3}|^2) \exp(-4\pi L(z)/(\lambda_d)_3) \\
&\quad + |\bar{U}_{e4}|^2 (1 + |\bar{U}_{\mu_4}|^2 - |\bar{U}_{\tau_4}|^2 - |\bar{U}_{s4}|^2) \exp(-4\pi L(z)/(\lambda_d)_4) \phi_{\nu_e}^{\text{source}}) \\
\phi_{\nu_{\mu}}^{\text{detector}} &= (\langle |\bar{U}_{\mu_1}|^2 (1 + |\bar{U}_{\mu_1}|^2 - |\bar{U}_{\tau_1}|^2 - |\bar{U}_{s1}|^2 \rangle \\
&\quad + |\bar{U}_{\mu_2}|^2 (1 + |\bar{U}_{\mu_2}|^2 - |\bar{U}_{\tau_2}|^2 - |\bar{U}_{s2}|^2) \exp(-4\pi L(z)/(\lambda_d)_2) \\
&\quad + |\bar{U}_{\mu_3}|^2 (1 + |\bar{U}_{\mu_3}|^2 - |\bar{U}_{\tau_3}|^2 - |\bar{U}_{s3}|^2) \exp(-4\pi L(z)/(\lambda_d)_3) \\
&\quad + |\bar{U}_{\mu_4}|^2 (1 + |\bar{U}_{\mu_4}|^2 - |\bar{U}_{\tau_4}|^2 - |\bar{U}_{s4}|^2) \exp(-4\pi L(z)/(\lambda_d)_4) \phi_{\nu_{\mu}}^{\text{source}}) \\
\phi_{\nu_{\tau}}^{\text{detector}} &= (\langle |\bar{U}_{\tau_1}|^2 (1 + |\bar{U}_{\mu_1}|^2 - |\bar{U}_{\tau_1}|^2 - |\bar{U}_{s1}|^2 \rangle \\
&\quad + |\bar{U}_{\tau_2}|^2 (1 + |\bar{U}_{\mu_2}|^2 - |\bar{U}_{\tau_2}|^2 - |\bar{U}_{s2}|^2) \exp(-4\pi L(z)/(\lambda_d)_2) \\
&\quad + |\bar{U}_{\tau_3}|^2 (1 + |\bar{U}_{\mu_3}|^2 - |\bar{U}_{\tau_3}|^2 - |\bar{U}_{s3}|^2) \exp(-4\pi L(z)/(\lambda_d)_3) \\
&\quad + |\bar{U}_{\tau_4}|^2 (1 + |\bar{U}_{\mu_4}|^2 - |\bar{U}_{\tau_4}|^2 - |\bar{U}_{s4}|^2) \exp(-4\pi L(z)/(\lambda_d)_4) \phi_{\nu_{\tau}}^{\text{source}}) \\
\phi_{\nu_{s}}^{\text{detector}} &= (\langle |\bar{U}_{s_1}|^2 (1 + |\bar{U}_{\mu_1}|^2 - |\bar{U}_{\tau_1}|^2 - |\bar{U}_{s1}|^2 \rangle \\
&\quad + |\bar{U}_{s_2}|^2 (1 + |\bar{U}_{\mu_2}|^2 - |\bar{U}_{\tau_2}|^2 - |\bar{U}_{s2}|^2) \exp(-4\pi L(z)/(\lambda_d)_2) \\
&\quad + |\bar{U}_{s_3}|^2 (1 + |\bar{U}_{\mu_3}|^2 - |\bar{U}_{\tau_3}|^2 - |\bar{U}_{s3}|^2) \exp(-4\pi L(z)/(\lambda_d)_3) \\
&\quad + |\bar{U}_{s_4}|^2 (1 + |\bar{U}_{\mu_4}|^2 - |\bar{U}_{\tau_4}|^2 - |\bar{U}_{s4}|^2) \exp(-4\pi L(z)/(\lambda_d)_4) \phi_{\nu_{s}}^{\text{source}})
\end{align*}
\]
Detection of UHE neutrinos from a diffuse GRB

The secondary muon yields from the GRB neutrinos can be detected in a detector of unit area above a threshold energy $E_{\text{thr}}$ is given by

$$S = \int_{E_{\text{thr}}}^{E_{\text{max}}} dE_{\nu} \frac{dN_{\nu}}{dE_{\nu}} P_{\text{shadow}}(E_{\nu}) P_{\mu}(E_{\nu}, E_{\text{thr}}).$$  

(27)

$P_{\text{shadow}}(E_{\nu})$ - Probability that a neutrino reaches the detector travelling through Earth matter

$$P_{\text{shadow}}(E_{\nu}) = \frac{1}{2\pi} \int_{-1}^{0} d\cos\theta_z \int d\phi \exp[-z(\theta_z)/L_{\text{int}}(E_{\nu})].$$  

(28)

The interaction length

$$L_{\text{int}} = \frac{1}{\sigma_{\text{tot}}(E_{\nu}) N_A}.$$  

(29)

The effective path length

$$z(\theta_z) = \int \rho(r(\theta_z, l)) dl.$$  

(30)

Probability of neutrino induced muon to reach the detector

$$P_{\mu}(E_{\nu}, E_{\text{thr}}) = N_A \sigma_{\text{cc}}(E_{\nu}) \langle R(E_{\mu}; E_{\text{thr}}) \rangle.$$  

(31)

where the average muon range in the rock $\langle R(E_{\mu}; E_{\text{thr}}) \rangle$ is given by

$$\langle R(E_{\mu}; E_{\text{thr}}) \rangle = \frac{1}{\sigma_{\text{cc}}} \int_{0}^{(1-E_{\text{thr}}/E_{\nu})} dy R(E_{\nu}(1-y); E_{\text{thr}}) \times \frac{d\sigma_{\text{cc}}(E_{\nu}, y)}{dy}.$$  

(32)
The average energy loss of muon with energy $E_\mu$ is given as

$$\langle \frac{dE_\mu}{dX} \rangle = -\alpha - \xi E_\mu. \quad \ldots \quad 34)$$

\[
\alpha = 2.033 + 0.077 \ln[E_\mu(\text{GeV})] \times 10^3 \ \text{GeV cm}^2 \text{gm}^{-1}, \\
\xi = 2.033 + 0.077 \ln[E_\mu(\text{GeV})] \times 10^{-6} \ \text{GeV cm}^2 \text{gm}^{-1} \quad \ldots \quad 35)
\]

for $E_\mu \lesssim 10^6 \ \text{GeV}$ and otherwise

\[
\alpha = 2.033 \times 10^{-3} \ \text{GeV cm}^2 \text{gm}^{-1}, \\
\xi = 3.9 \times 10^{-6} \ \text{GeV cm}^2 \text{gm}^{-1}. \quad \ldots \quad 36)
\]

In Eq. (27) is replaced by $F_{\nu_\mu}^A$ from Eq. (15) and by $F_{\nu_\mu}^3$ from Eq. (16) for the four-flavour and Three-flavour scenarios respectively.

- Detection of UHE neutrinos from a single GRB

$$P_{\text{shadow}} = \exp[-X(\theta_z)/l_{\text{int}}(E_\nu)], \quad \ldots \quad 37)$$

In the case of detecting muon events at a 1 km$^2$ detector such as IceCube the flux $\frac{dN_\nu}{dE_\nu}$ in Eq. (27) is replaced by $\phi_{\nu_\mu}^4$ in Eq. (26).
Calculations and Results

Diffused Neutrino Flux

For the purpose of our analysis, we have considered a ratio $R$ between the muon and the shower events, which is defined as

$$R = \frac{T_\mu}{T_{sh}} \quad \text{......38) }$$

Where

$$T_\mu = S(\text{for } \nu_\mu) + S(\text{for } \nu_\tau) ,$$

$$T_{sh} = S_{sh}(\text{for } \nu_e \text{ CC interaction}) + S_{sh}(\text{for } \nu_e \text{ NC interaction}) + S_{sh}(\text{for } \nu_\mu \text{ NC interaction}) + S_{sh}(\text{for } \nu_\tau \text{ NC interaction}), \quad \text{......39) }$$

The event rate of muons ($S$) and the same for the shower ($S_{sh}$) are expressed as

$$S = \int_{E_{thr}}^{E_{\text{max}}} dE_\nu \frac{dN_\nu}{dE_\nu} P_{\text{shadow}}(E_\nu) P_\mu(E_\nu, E_{thr}). \quad \text{......40) }$$

$$S_{sh} = V \int_{E_{thr}}^{E_{\text{max}}} dE_\nu \frac{dN_\nu}{dE_\nu} P_{\text{shadow}}(E_\nu) \int dy \frac{1}{\sigma^i} \frac{d\sigma^i}{dy} P_{\text{int}}(E_\nu, y). \quad \text{......41) }$$
For the isotropic fluxes

<table>
<thead>
<tr>
<th>$\theta_{14}$</th>
<th>$\theta_{24}$</th>
<th>$\theta_{34}$</th>
<th>$R_4$ (in 4f)</th>
<th>$R_3$ (in 3f)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3°</td>
<td>5°</td>
<td>20°</td>
<td>9.48</td>
<td>1.80</td>
</tr>
<tr>
<td>4°</td>
<td>6°</td>
<td>15°</td>
<td>9.68</td>
<td>1.80</td>
</tr>
</tbody>
</table>

Table 1: Comparison of the muon to shower ratio for a diffused GRB neutrino flux for the 4 flavour (3+1) case compared with the same for 3 flavour case for two sets of active sterile neutrino mixing angles.

Figure 1: Variation of $R_4$ with $\theta_{24}$ and $\theta_{34}$ for (a) $\theta_{14} = 1°$ and (b) $\theta_{14} = 4°$. 
From the analysis of the high-energy starting events (HESE) data (the IceCube Collaboration), they calculated a best fit power law for the neutrino flux as

$$E^2 \phi(E) = 2.46 \pm 0.8 \times 10^{-8} \left( \frac{E}{100 \text{ TeV}} \right)^{-0.92} \text{ GeV cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1}.$$ \[42\]

For a one component fit the neutrino flux \( \phi(E) \sim E^{-\gamma} \), with the index \( \gamma = 2.92^{+0.33}_{-0.29} \). We have computed \( R_4 \), \( R_3 \) for this flux as well and the energy range 60 TeV is to be considered for such calculations.

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<thead>
<tr>
<th>( \theta_{14} )</th>
<th>( \theta_{24} )</th>
<th>( \theta_{34} )</th>
<th>( R_4 ) (in 4f)</th>
<th>( R_3 ) (in 3f)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3°</td>
<td>5°</td>
<td>20°</td>
<td>2.01</td>
<td>0.55</td>
</tr>
<tr>
<td>4°</td>
<td>6°</td>
<td>15°</td>
<td>2.04</td>
<td>0.55</td>
</tr>
</tbody>
</table>

Table 2: Same as Table 1, but here we consider the diffuse flux of UHE neutrinos obtained from the recent analysis of the IceCube (HESE) data.

Figure 1: Variation of \( R_4 \) with \( \theta_{24} \) and \( \theta_{34} \) for (a) \( \theta_{14} = 1^\circ \) and (b) \( \theta_{14} = 4^\circ \) (the recent IceCube HESE data).
Oscillation+Decay (with single GRB flux)

Figure 2: The Variations of the neutrino decay life time $\tau/m$ with the unparticle dimension $d_u$ are shown for four different values (0.1, 0.01, 0.001, 0.0001) of couplings $\lambda^u_\nu$. 
Figure 2: The variations of the neutrino induced upward going muons per year from the GRB with (a) different values of $d_U$ for four different fixed values of $\lambda_{ij}^\nu$ as well as for the mass flavour case (no decay case), (b) different values of $\lambda_{ij}^\nu$ for four different fixed values of the unparticle dimension $d_U$ (1.1, 1.2, 1.3, 1.4) and in addition for no decay case.
Table 3: The ratio $R$ of muon yields at IceCube for UHE neutrinos from a GRB with and without decay in a four neutrino framework for two sets of values of decay parameters namely $d_\mathcal{U}$, $\chi_{ij}^\nu$.

<table>
<thead>
<tr>
<th>$\theta_{14}$</th>
<th>$\theta_{24}$</th>
<th>$\theta_{34}$</th>
<th>$d_\mathcal{U}$</th>
<th>$\chi_{ij}^\nu$</th>
<th>$R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3^\circ$</td>
<td>$5^\circ$</td>
<td>$20^\circ$</td>
<td>1.2</td>
<td>$10^{-4}$</td>
<td>0.9</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1.3</td>
<td>$10^{-2}$</td>
<td>0.6</td>
</tr>
</tbody>
</table>
Summary

The ratio of muon events to the shower events was calculated for both the three-flavour and four-flavour cases, which are denoted as $R_3$ and $R_4$. For this purpose, we have considered two sets of UHE neutrino fluxes –

1. theoretical flux for diffuse isotropic UHE neutrinos from GRBs given by Waxman and Bahcall.
2. the analysis of the recent IceCube HESE data.

The muon to shower ratio can be 6-8 times larger for 3+1 scenario when compared to that for normal three active neutrino formalism if the Waxman-Bahcall flux is considered, and $R_4$ can be 3-4 times greater than $R_3$ using the flux given by the IceCube (HESE data) analysis. Therefore, the present analysis has shown that any excess of such events detected in a 1 Km2 detector over that predicted for three-neutrino mixing can clearly indicate the presence of active-sterile neutrino mixing.

For the case of oscillation + unparticle decay we demonstrated how the effect of unparticle decay of neutrino affect the detection of muon yield for UHE neutrinos from a single GRB in 4-flavour scheme.
THANK YOU
BACKUP SLIDES
Table 4: Same as Table 3 but for the flavour ratio of active neutrinos in 4-flavour framework.

<table>
<thead>
<tr>
<th>$\theta_{14}$</th>
<th>$\theta_{24}$</th>
<th>$\theta_{34}$</th>
<th>$dU$</th>
<th>$\lambda_{ij}^{ij}$</th>
<th>$\phi_{\nu_e} : \phi_{\nu_\mu} : \phi_{\nu_\tau}$ (with decay and oscillation)</th>
<th>$\phi_{\nu_e} : \phi_{\nu_\mu} : \phi_{\nu_\tau}$ (no decay (only oscillation))</th>
</tr>
</thead>
<tbody>
<tr>
<td>3°</td>
<td>5°</td>
<td>20°</td>
<td>1.2</td>
<td>$10^{-4}$</td>
<td>0.349 : 0.621 : 0.489</td>
<td>0.449 : 0.979 : 0.834</td>
</tr>
<tr>
<td>3°</td>
<td>5°</td>
<td>20°</td>
<td>1.3</td>
<td>$10^{-2}$</td>
<td>0.344 : 0.265 : $9.5 \times 10^{-2}$</td>
<td>0.449 : 0.979 : 0.834</td>
</tr>
</tbody>
</table>
Formalism

- **Unparticle decay of GRB neutrinos**
  - We consider a decay phenomenon, where neutrino having mass eigenstate $\nu_j$ decays to the invisible unparticle (U) and another light neutrino with mass eigenstate $\nu_i$.
    \[ \nu_j \rightarrow U + \nu_i \quad \text{(4)} \]
  - The effective lagrangian for the above mentioned process takes the following form in the low energy regime
    \[ L_s = \frac{\lambda_{\nu}^{\alpha \beta}}{\Lambda_{U}^{\phi_{U}-1}} \bar{\nu}_\alpha \nu_\beta \mathcal{O}_U \quad \text{(5)} \]
    where $\alpha, \beta = e, \mu, \tau, s$ - flavour indices, $d_U$ - the scaling dimension of the scalar unparticle operator $\mathcal{O}_U$, $\Lambda_U$ - the dimension transmutation scale at which the scale invariance sets in, $\lambda_{\nu}^{\alpha \beta}$ - the relevant coupling constant.
  - The neutrino and flavour eigenstates are related through
    \[ |\nu_i\rangle = \sum_{\alpha} U_{\alpha i}^* |\nu_\alpha\rangle \quad \text{(6)} \]
    where $U_{\alpha i}$ - elements of the PMNS mixing matrix.
  - In the mass basis the interaction between neutrinos and the unparticles can be written as
    \[ \lambda_{\nu}^{ij} \bar{\nu}_i \nu_j \mathcal{O}_U / \Lambda_{U}^{d_U-1} \]
    where $\lambda_{\nu}^{ij}$ is the coupling constant in the mass eigenstate $i, j$.
    \[ \lambda_{\nu}^{ij} \quad \text{can be expressed as} \quad \lambda_{\nu}^{ij} = \sum_{\alpha, \beta} U_{\alpha i}^* \lambda_{\nu}^{\alpha \beta} U_{\beta j} \quad \text{(7)} \]
Formalism

Detection of UHE neutrinos from a single GRB:

The secondary muon yields from the GRB neutrinos can be detected in a detector of unit area above a threshold energy $E_{th}$ is given by

$$S = \int_{E_{th}}^{E_{\mu,max}} dE_{\nu} \frac{dN_{\nu}}{dE_{\nu}} P_{\text{shadow}}(E_{\nu}) P_{\mu}(E_{\nu}, E_{th})$$

where $P_{\text{shadow}}(E_{\nu})$ represents the probability that a neutrino reaches the terrestrial detector such as IceCube being unabsorbed by the Earth, which takes the form

$$P_{\text{shadow}} = \exp[-X(\theta_{z})/L_{\text{int}}(E_{\nu})]$$

where energy dependent neutrino-nucleon interaction length $L_{\text{int}}(E_{\nu})$ is given by

$$L_{\text{int}}(E_{\nu}) = \frac{1}{\sigma_{\nu n}(E_{\nu}) N_{A}}$$

The effective path length $X(\theta_{z})$ can be written as

$$X(\theta_{z}) = \int \rho(r(\theta_{z}, l)) dl$$

The probability $P_{\mu}(E_{\nu}, E_{th})$ that a neutrino induced muon reaching the detector with an energy above $E_{th}$ can be written as

$$P_{\mu}(E_{\nu}, E_{th}) = N_{A} \sigma_{cc}(E_{\nu}) \langle R(E_{\mu}; E_{th}) \rangle$$

where the average muon range in the rock $\langle R(E_{\mu}; E_{th}) \rangle$ is given by

$$\langle R(E_{\mu}; E_{th}) \rangle = \frac{1}{\sigma_{cc}} \int_{0}^{(1-E_{th}/E_{\nu})} dy R(E_{\nu}(1-y); E_{th}) \times \frac{d\sigma_{cc}(E_{\nu}; y)}{dy}$$
Formalism

The muon range $R(E_{\mu}; E_{th})$ can be expressed as

$$R(E_{\mu}, E_{th}) = \int_{E_{th}}^{E_{\mu}} \frac{dE_{\mu}}{\langle dE_{\mu}/dX \rangle} \simeq \frac{1}{\beta} \ln \left( \frac{\alpha + \beta E_{\mu}}{\alpha + \beta E_{th}} \right) \quad \text{.....19)}$$

The average energy loss of muon with energy $E_{\mu}$ is given as

$$\langle \frac{dE_{\mu}}{dX} \rangle = -\alpha - \beta E_{\mu} \quad \text{......... 20)}$$

The values of the constants $\alpha$ and $\beta$ in Eq. (20), which we have considered in our calculations are

$$\alpha = 2.033 + 0.077 \ln[E_{\mu}(\text{GeV})] \times 10^3 \, \text{GeV cm}^2 \, \text{gm}^{-1},$$

$$\beta = 2.033 + 0.077 \ln[E_{\mu}(\text{GeV})] \times 10^{-6} \, \text{GeV cm}^2 \, \text{gm}^{-1}, \quad \text{... 21)}$$

for $E_{\mu} \leq 10^6 \, \text{GeV}$ and otherwise

$$\alpha = 2.033 \times 10^{-3} \, \text{GeV cm}^2 \, \text{gm}^{-1}$$

$$\beta = 3.9 \times 10^{-6} \, \text{GeV cm}^2 \, \text{gm}^{-1}. \quad \text{......... 22)}$$

In the case of detecting muon events at a 1 km$^2$ detector such as IceCube the flux $\frac{dN_{\nu}}{dE_{\nu}}$ in Eq. (14) is replaced by $\phi_{\nu_{\mu}}^4$ in Eq. (13).
Figure 3: Variations of the neutrino induced muons per year from the GRB with different redshifts ($z$) for three different values of $\lambda_{\nu}^{ij}$ as well as for no decay case at a fixed zenith angle ($\theta_z = 160^\circ$).