A study in the non-canonical domain of Goldstone inflation

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Motivation

- Precision era in cosmology → Theoretically motivated models of cosmological inflation are under stringent constraints.

- Natural inflation $V(\phi) = \Lambda^4(1 + \cos(\phi/f))$ is at verge of getting strongly disfavoured by Planck 2018.

- Sub-Planckian breaking scale does not give enough e-folds.

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1Croon et al., arXiv:1503.08097

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- Natural inflation $V(\phi) = \Lambda^4(1 + \cos(\phi/f))$ is at verge of getting strongly disfavoured by Planck 2018.

- Sub-Planckian breaking scale does not give enough e-folds.

- Natural inflation is a limiting case of a much more generalised Lagrangian: Goldstone inflation\(^1\).

\[
V(\phi) = \Lambda^4(C_\Lambda + \alpha \cos(\phi/f) + \beta \sin^2(\phi/f)).
\]

- Fine tuning needed to fit canonical Goldstone inflation with 2015 Planck.

- Motivations to study non-canonical regime comes from string theory.

- General Lagrangian:

\[
\mathcal{L} = K(X, \phi) - V(\phi), \quad X \equiv \frac{1}{2} \partial_\mu \phi \partial^\mu \phi
\]

\[
K(X, \phi) = K_{nc}(\phi)K_{kin}(X).
\]

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(SINP,HBN)
Case 1: $K_{nc}$ switched on, $K_{kin}(X) = X$

- Choice: $K_{nc}(\phi) = V(\phi)/\Lambda^4$.

\[
\ddot{\phi} + 3H\dot{\phi} + \frac{K_{nc,\phi}}{2K_{nc}} \dot{\phi}^2 + \frac{V,\phi}{K_{nc}} = 0
\]

\[
\epsilon_V = \frac{M_{Pl}^2}{2K_{nc}} \left( \frac{V,\phi}{V} \right)^2
\]

\[
\eta_V = \frac{M_{Pl}^2}{V} \left( \frac{V,\phi K_{nc,\phi}}{K_{nc}} - \frac{V,\phi}{2K_{nc}^2} \right)
\]

\[
N = \frac{1}{M_{Pl}} \int_{\phi_i}^{\phi_e} V \frac{V}{V,\phi \sqrt{K_{nc}}} d\phi
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$$N = \frac{1}{M_{Pl}} \int_{\phi_i}^{\phi_e} \frac{V}{V,\phi \sqrt{K_{nc}}} d\phi$$

- Non-canonical kinetic term provides friction.

$$r = 16\epsilon_V.$$
Case 2: $K_{kin}(X)$ switched on, $K_{nc}(\phi) = 1$

- Choice: $K_{kin}(X) \equiv K_0 X^n$.

$$\ddot{\phi} + \frac{3H}{2n-1} \dot{\phi} + \frac{V,\phi}{(2n^2 - n)K_0 X^{n-1}} = 0$$

$$\epsilon_V = \frac{1}{2} \alpha_{kin}(n) \left( \frac{V,\phi}{V(3n-1)} \right)^\frac{1}{2n-1}$$

$$\eta_V = \alpha_{kin}(n) \left( \frac{V(2n-1)}{Vn V(2n-2) \phi_{,\phi}} \right)^\frac{1}{2n-1}$$

$$N = \int_{\phi_e}^{\phi_i} \frac{1}{\alpha_{kin}(n)} \sqrt{V} \left( \frac{\sqrt{V}}{V,\phi} \right)^\frac{1}{2n-1} d\phi$$

$$\alpha_{kin}(n) = \left( \frac{6^{n-1}}{n K_0} \right)^\frac{1}{2n-1}$$

Dependence of $\epsilon_V$, $\eta_V$ and $N$ on $K_0, \alpha$ and $\Lambda$.

For NI with kinetic term: $\epsilon_V \propto \frac{1}{(K_0 \alpha \Lambda^4)^{1/3}}$. 
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\[
c_s = \frac{1}{\sqrt{2n-1}}, \ r = 16 c_s \epsilon_V.
\]

\[
n = 2
\]
Comparison of Canonical and Non-canonical regime

- $C_\Lambda = 1, \beta/\alpha = 0.5$
- Data combinations used:
  1. Planck TT, TE, EE + lowE + lensing data (2018)
  2. Planck TT, TE, EE + lowE (2018) + lensing + BK14 + BAO.
Varying $\beta$ (Case 1)

- $C_\Lambda = 1$,
- $\beta/\alpha = 0, 0.25, 0.5, 0.75$ in red, cyan, magenta and green.
- Data combinations used:
  1. Planck TT,TE,EE+lowE+lensing data (2018)
  2. Planck TT,TE,EE+lowE (2018)+lensing+BK14+BAO.
Varying $\beta$ (Case 2)

- $C_\Lambda = 1$,
- $\beta/\alpha = 0, 0.2, 0.5$ in red, magenta and blue.
- Lowest point is for $f = 0.5M_{Pl}$.
- Data combinations used:
  1. Planck TT,TE,EE+lowE+lensing data (2018)
  2. Planck TT,TE,EE+lowE (2018)+lensing+BK14+BAO.
- Inside 96% confidence limit of Planck 2018 (dataset 2 above) for sub-Planckian $f$. 

![Graph showing tensor-to-scalar ratio and primordial tilt values]
\( C_\Lambda = 1, f = 5 M_{Pl}. \)

\( \beta/\alpha = 0, 0.2, 0.5 \) in red, magenta and blue.

Canonical, Non-canonical and Kinetic (L2R).

Data combinations used:
1. Planck TT,TE,EE+lowE+lensing data (2018)
2. Planck TT,TE,EE+lowE (2018)+lensing+BK14+BAO.
Discussions

- Implementation of Goldstone inflation in the non-canonical regime drives the tensor-to-scalar ratio $r$ to a lower value for Case 1.
- For Case 1, inflation at sub-Planckian breaking scales corresponds to points outside the CMB observations by Planck 2018.
- For Case 2, $r$ stays at high values and sub-Planckian breaking scale corresponds to points in $n_s$-$r$ plane that are within $2\sigma$ of current observations. Future detection of early universe tensor modes can easily verify.
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- Implementation of Goldstone inflation in the non-canonical regime drives the tensor-to-scalar ratio $r$ to a lower value for Case 1.
- For Case 1, inflation at sub-Planckian breaking scales corresponds to points outside the CMB observations by Planck 2018.
- For Case 2, $r$ stays at high values and sub-Planckian breaking scale corresponds to points in $n_s-r$ plane that are within $2\sigma$ of current observations. → Future detection of early universe tensor modes can easily verify.
Red: $f = 5M_{Pl}$, Blue: $f = 0.5M_{Pl}$.