Connecting LQG and String Theory
From Quantum Geometry to the Nambu-Goto Action

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1. Motivations
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3. LQG & Quantized Geometry
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1. Motivations
2. Hints from AdS/CFT
3. LQG & Quantized Geometry
4. Nambu-Goto Action from LQG
LQG + String Theory = Why Bother?

Possibilities:

Both are wrong
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..... only reasonable conclusion!
Hints from AdS/CFT

Holography: ’t Hooft (’93), Susskind (’94)
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Tensor Networks ≡ Spin Networks
Background independence is non-negotiable.
Loop Quantum Gravity

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Casts Gravity as gauge theory of connections and tetrads.
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Canonical quantization a-la ADM.
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Spin Networks - exact solutions of diffeo & Gauss constraints.
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Central result - Quantum Geometry, discrete areas and volumes.
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Central result - Quantum Geometry, discrete areas and volumes.

Applications: LQC, BH entropy, ...
String Theory

QFT of extended objects.
Hints from AdS/CFT

String Theory

QFT of extended objects.

Explains Regge trajectories.

Scalar field: $\Phi(x)$, "Dilaton" - determines string coupling strength

Two index antisymmetric field: $B_{\mu \nu}(x)$, "Kalb-Ramond" - sources gauge fields

Two index, symmetric traceless field: $h_{\mu \nu}(x)$, "Graviton" - sources gravity...
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Particle spectrum including gauge fields and graviton.
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Projecting out unphysical string modes we find:

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AdS/CFT
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Phase space: triads $e_i^a$, $su(2)$ connection $A_a^i$
LQG Outline

Phase space: triads $e_i^a$, $su(2)$ connection $A_a^i$

Satisfy P.B.: $\{A_a^i, e_j^b\} = \kappa \delta_a^b \delta_j^i$
Phase space: triads $e^a_i$, $su(2)$ connection $A^i_a$

Satisfy P.B.: $\left\{ A^i_a, e^b_j \right\} = \kappa \delta^b_a \delta^i_j$

Gauge invariant variables:
Phase space: triads $e_i^a$, $su(2)$ connection $A^i_a$

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Gauge invariant variables:

Holonomies: $g_\gamma[A] = \mathcal{P} \exp \left\{-i \int_\gamma A^i_a(x) \tau_i n^a dx \right\}$
Phase space: triads $e_i^a$, $\mathfrak{su}(2)$ connection $A_a^i$

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Smeared triads: $P_{(Sf)} = \int_{Sf_i(x)} e_a^i e_b^k \epsilon_{ijk} dx^a dx^b$
Phase space: triads $e_i^a$, su(2) connection $A_a^i$

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Smeared triads: $P_{(S_f)} = \int S f_i(x) e_a^i e_b^k \epsilon^{ijk} dx^a dx^b$

States: $\Psi_\Gamma = \psi(g_1, g_2, \ldots, g_n)$
Graph States

\[ \Psi_n = \psi(x_1, \ldots, x_n) = \int \prod_{i=1}^{n} \left[ d^3 k_i \right] e^{-i(k_1 \cdot x_1 + \ldots + k_n \cdot x_n)} \times \ldots \times \tilde{\Psi}(k_1, \ldots, k_n) \]
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\[ \Psi_{\Gamma} = \psi(g_1, g_2, \ldots, g_n) \]

\[ = \sum_{i=1}^{n} \sum_{j_i=1/2}^{\infty} D^{j_1}(g_1)_{a_1 b_1} \ldots D^{j_n}(g_n)_{a_n b_n} \times \ldots \]

\[ \ldots \times \tilde{\Psi}^{a_1 b_1 \ldots a_n b_n}(j_1, \ldots, j_n) \]
Graph States (contd.)
Area Operator

Area of 2D surface:

\[ A(S) = \int_S d^2x \sqrt{h} \]

where: \( h_{ab} = e^i_a e^j_b \delta_{ij} \Rightarrow \det(h) = e^i_z e^j_z \delta_{ij} \)
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\( e^a_i \) is canonically conjugate to the connection \( A^i_a \), from which:

\[ \hat{e}^a_i = -i\hbar \kappa \frac{\delta}{\delta A^i_a} \]
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Action of “momentum” operator on holonomy:

\[ \frac{\delta}{\delta A^i_a} g_{\gamma} [A] = n_a(x) \tau^i g_{\gamma} [A] \]
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Action on graph state:

\[ \frac{\delta}{\delta A_i^a(x)} \Psi(g_1, \ldots, g_k, \ldots, g_n) = n_a^k(x) \tau^i \Psi \]
Area Operator (contd.)

Area Operator:

\[ \hat{A} = \sum_{I=1}^{N} \sqrt{\delta_{jk} \hat{e}_{z}^{j} \hat{e}_{z}^{k}} = 8\pi\hbar \gamma G_{N} \sum_{I=1}^{N} \sqrt{\delta_{jk} \frac{\delta}{\delta A_{z}^{j}} \frac{\delta}{\delta A_{z}^{k}}} \]
Area Operator (contd.)

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\]

\[
\hat{A}_S \Psi_\Gamma = 8\pi\gamma l_{PL}^2 \sum_k \sqrt{j_k (j_k + 1)} \Psi_\Gamma
\]
Area Operator (contd.)

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\[ \hat{A}_S \Psi_\Gamma = 8\pi\gamma l_{PL}^2 \sum_k \sqrt{j_k(j_k+1)} \Psi_\Gamma \]

Area = \(8\pi l_{PL}^2 \sqrt{j(j+1)}\)
Area Operator (contd.)

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String Action from Area

Nambu-Goto action:

$$S_{NG} = -T \int d\tau \, d\sigma \sqrt{-\text{det}(h_{AB})}$$
String Action from Area

Nambu-Goto action:

\[ S_{NG} = -T \int d\tau \ d\sigma \sqrt{-\text{det}(h_{AB})} \]

Conjecture:

\[ S_{NG} \propto \langle \psi | \hat{A} | \psi \rangle \]
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String Field Theory action:

\[ S_{SFT} = \langle \Psi | Q | \Psi \rangle \]

where \( Q \) is a BRST operator. What is the relation between \( Q \) and \( \hat{A} \)?
String Action from Area

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Immirizzi parameter and string tension:
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Immirizi parameter and string tension:

\[ S_{NG} \sim \text{tension} \times \text{area} \]

\[ \langle \Psi | \hat{A} | \Psi \rangle \sim \beta \times \text{area} \Rightarrow T_{\text{string}} = \beta T_{\text{loop}} \]