Connecting LQG and String Theory

From Quantum Geometry to the Nambu-Goto Action

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Outline

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..... only reasonable conclusion!

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Tensor Networks $\equiv$ Spin Networks

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Applications: LQC, BH entropy, ...

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State Space	String Networks	Spin Networks

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States: $\Psi_\Gamma = \psi(g_1, g_2, \ldots, g_n)$

Graph States

$$\Psi_n = \psi(\boldsymbol{x}_1, \ldots, \boldsymbol{x}_n)) \qquad = \int \prod_{i=1}^{n} \left[d^3\boldsymbol{k}_i \right] e^{-i(\boldsymbol{k}_1 \cdot \boldsymbol{x}_1 + \ldots + \boldsymbol{k}_n \cdot \boldsymbol{x}_n)} \times \ldots$$
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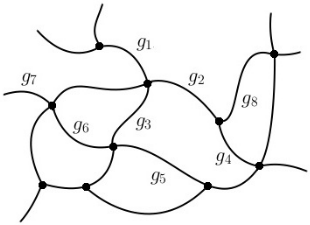
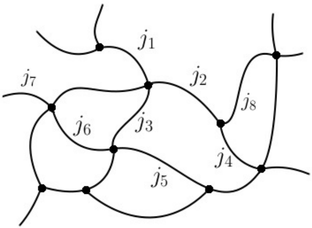
Graph States (contd.)





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Area of 2D surface:

$$A(S) = \int_S d^2x \, \sqrt{h}$$

where: $h_{ab} = e^i_a e^j_b \delta_{ij} \Rightarrow \det(h) = e^i_z e^j_z \delta_{ij}$

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Action on graph state:

$$\frac{\delta}{\delta A^a_i(x)} \Psi(g_1, \ldots, g_k, \ldots, g_n) = n^k_a(x) \tau^i \Psi$$

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$$\hat{A} = \sum_{I=1}^{N} \sqrt{\delta_{jk}\, \hat{e}^j_z\, \hat{e}^k_z} = 8\pi\hbar\gamma G_N \sum_{I=1}^{N} \sqrt{\delta_{jk} \frac{\delta}{\delta A^j_z} \frac{\delta}{\delta A^k_z}}$$

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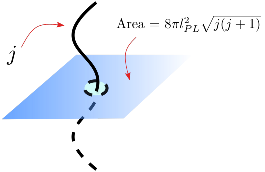

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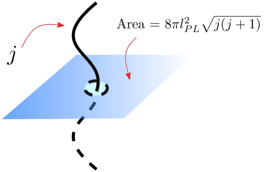

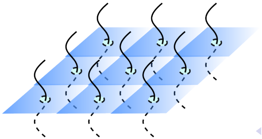

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$$\langle \Psi | \hat{A} | \Psi \rangle \simeq \beta \times \textit{area} \Rightarrow T_{string} = \beta T_{loop}$$

Its a Good Time for Speculation!