Vector Mesons Fragmentation- A Brief Review

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Outline of Talk

- Introduction
  1. Vector Meson Nonet
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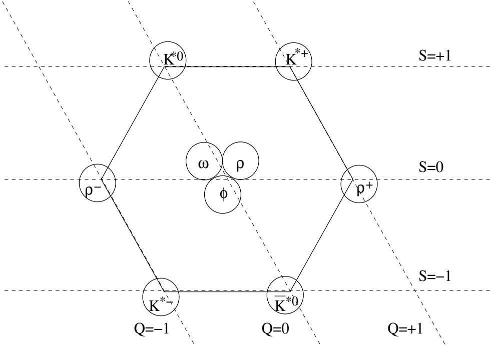
- Model - Broken SU(3) Symmetry
- Data Analysis and Results
  - e^-- e^+ process at Next-to-Leading Order (NLO)
  - $p\,p$ process at NLO
- Conclusion

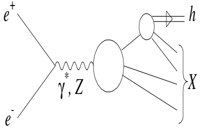
Mesons : SU(3) Symmetry

- According to the spin and parity, mesons (quark-antiquark pair) are classified as follows:
  - Pseudoscalar Meson - spin 0 odd parity
  - Vector Meson - spin 1 odd parity
    $(\rho(\rho^+, \rho^-, \rho^0)$, $K^*(K^{*+}, K^{*-}, K^{*0}, \bar{K^{*0}})$, ω and $\phi)$



Fragmentation Process

- Quarks and antiquarks cannot be seen individually due to their confinement property.
- The quark-antiquark pair obtained in various ($e^+ e^-$, p p) processes are observed only through the hadrons they produce. This is known to be *hadronisation or fragmentation process*.



- These fragmentation processes are characterised by fragmentation functions. QCD which explains this process at the quark level cannot explain the origin of these fragmentation functions. However, QCD can explain their scale (Q^2) dependence.
- A comparison with experimental data allows the fragmentation functions to be determined at a starting scale.
- A model is needed to determine these fragmentation functions from the observed event rates!

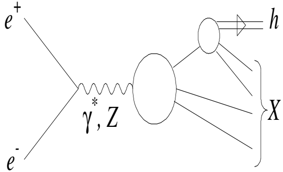
Model - SU(3) Symmetry

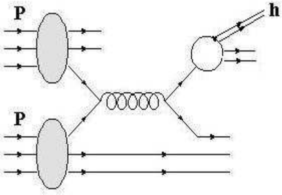
- Each meson has seven fragmentation functions associated with its production: $D_q(x, Q^2), D_{\bar{q}}(x, Q^2); q = u, d, s$, and $D_g(x, Q^2)$.
- So we have to predict 56 (8×7) unknown fragmentation functions. It is quite hard.
- A simple model for a light quark (u, d, s) to fragment into a vector meson is proposed using SU(3) symmetry which is a good description of octet hadrons.
- Symmetries like *isospin invariance and charge conjugation* reduces the functions further: *three* fragmentation functions valence(V), sea(γ) and gluon fragmentation function.
- The symmetry is broken due to the strange quark s which is massive (~ 95 MeV$/c^2$) and strangeness suppression parameter is accounted.
- ω and ϕ, mixtures of octet-singlet mixing, can be explained with the inclusion of very few parameters like mixing angle θ, etc..
- No new fragmentation function will be introduced for the singlet sector which shows the power of the model.

Quark fragmentation functions in terms of valence (V) and sea (γ) functions.

fragmenting quark		K^{*+}	fragmenting quark		K^{*0}
u	:	$V+2\gamma$	u	:	2γ
d	:	2γ	d	:	$V+2\gamma$
s	:	2γ	s	:	2γ
fragmenting quark		ω/ϕ	fragmenting quark		ρ^0
u	:	$\frac{1}{6}V+2\gamma$	u	:	$\frac{1}{2}V+2\gamma$
d	:	$\frac{1}{6}V+2\gamma$	d	:	$\frac{1}{2}V+2\gamma$
s	:	$\frac{4}{6}V+2\gamma$	s	:	2γ
fragmenting quark		ρ^+	fragmenting quark		ρ^-
u	:	$V+2\gamma$	u	:	2γ
d	:	2γ	d	:	$V+2\gamma$
s	:	2γ	s	:	2γ
fragmenting quark		$\overline{K^{*0}}$	fragmenting quark		K^{*-}
u	:	2γ	u	:	2γ
d	:	2γ	d	:	2γ
s	:	$V+2\gamma$	s	:	$V+2\gamma$

Process Involved



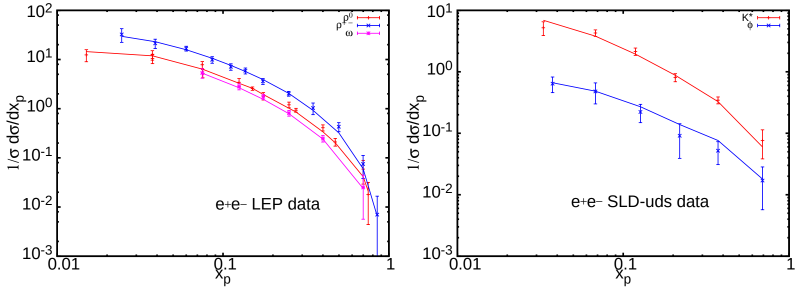


$\mathcal{O}(1)(D_q^h) + \mathcal{O}(\alpha_s)(D_q^h + D_g^h)$ $\quad$ $\mathcal{O}(\alpha_s^2)(D_q^h + D_g^h) + \mathcal{O}(\alpha_s^3)(D_q^h + D_g^h)$

- where $\alpha_s(Q^2)$ is the strong coupling constant runs with the energy Q^2 scale of the process.

Fits to $e^+ e^-$ hadroproduction data

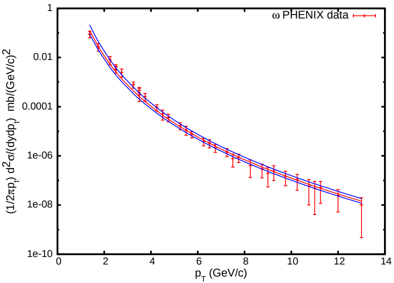
- Cross section behaviour as a function of x_p for vector meson fragmentation in e^+e^- collisions.
- The solid lines are the best fits resulting from the present model.

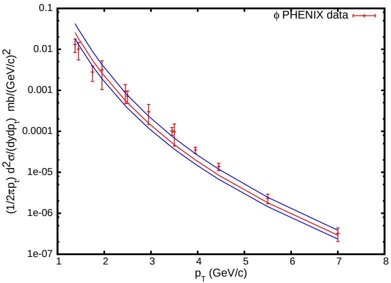


Fits to $p\,p$ hadroproduction data

Scale (p_T) dependence-RHIC

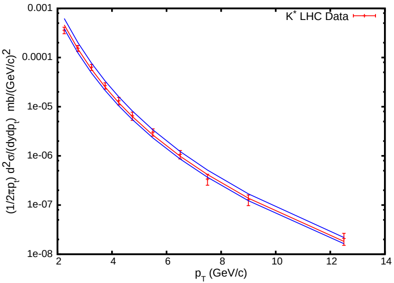
- Cross section as a function of p_T for ω (L) and ϕ (R) meson hadroproduction in $p\,p$ collisions at $\sqrt{s} = 200$ GeV and $|y| \leq 0.35$.
- Bands show the scale uncertainty on changing $Q^2 = p_T^2$ over a range $p_T^2/2$ (upper curve) $\leq Q^2 \leq 2p_T^2$ (lower curve) for all the three scales.

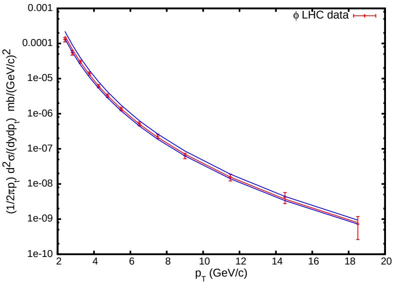




Scale (p_T) dependence-LHC

- Cross section as a function of p_T for K* (L) and ϕ (R) meson hadroproduction in $p\,p$ collisions at $\sqrt{s} = 2.76$ TeV and $|y| \le 0.5$.
- Bands show the scale uncertainty on changing $Q^2 = p_T^2$ over a range $p_T^2/2$ (upper curve) $\le Q^2 \le 2p_T^2$ (lower curve) for all the three scales.





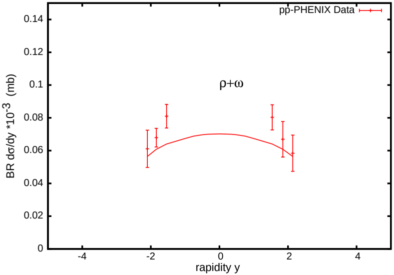
Various Input Parameters at NLO

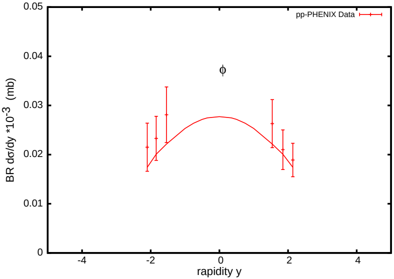
Best fit values of the parameters defining the input valence, sea quark, gluon fragmentation functions and other mixing parameters at the starting scale of $Q_0^2 = 1.5$ GeV^2, with their error bars.

par	Cent. Val.	Error Bars
$V\vert\ a$	2.42	0.30
b	2.24	0.21
c	2.71	0.13
d	2.43	0.59
e	1.17	0.78
$\gamma\vert a$	0.32	0.01
b	-0.73	0.03
c	3.53	0.13
d	0.70	0.14
e	0.42	0.26
$D_g\vert\ a$	2.43	0.07
b	0.94	0.05
c	2.68	0.03
d	-0.18	0.04
e	1.04	0.07

par	Cent. Val.	Error Bars
λ	0.097	0.013
θ	39.5	1.4
f_{sea}^{ω}	0.99	0.10(*)
f_{sea}^{ϕ}	λ^2	const
$f_1^u(\omega)$	0.000	–
$f_1^s(\phi)$	7.48	1.75
$f_g^{K^*}$	0.42	0.02
f_g^{ω}	0.90	0.02
f_g^{ϕ}	0.22	0.01

Fits to the rapidity dependence of $p\,p$ data





- ρ and ω falls slower with rapidity from central to forward regions and are barely consistent with the data while in the case of ϕ hadroproduction, the model fits well with the data.
- In fact, reproducing this rapidity dependence was the biggest constraint in determining the model parameter fits.

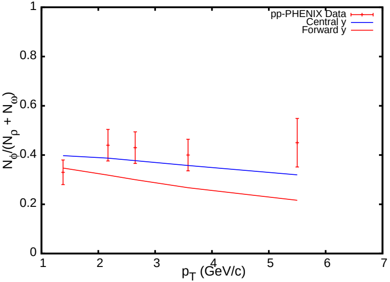
Events ratio

The event ratio is given by,

$$\frac{N_\phi}{(N_\omega + N_\rho)} = \frac{BR(\phi \to \mu\mu)\sigma_\phi}{(BR(\omega \to \mu\mu)\sigma_\omega + BR(\rho \to \mu\mu)\sigma_\rho)} ,$$

for $1.22 \leq p_T \leq 7$ GeV/c, central ($|y| \leq 0.35$) and forward rapidity ($1.2 \leq |y| \leq 2.2$) regions.

•The ratio determined as 0.40 (central), and 0.30 (forward) is in agreement with the data (0.390 ± 0.021(stat) ± 0.035 (sys)).



χ^2 values

- χ^2 values obtained from best-fits to ρ, K*, ω and ϕ hadroproduction from $e^+ e^-$ LEP, SLD data, and ω and ϕ hadroproduction for central rapidity.

Data Set	No. of data points	χ^2
Total $e^+ e^-$	44	17.91
ρ^0	14	7.56
ρ^{+-}	12	3.05
K*	6	3.65
ω	6	1.02
ϕ	6	2.63
Total $p\,p$(RHIC+LHC)	70	64.93
ω(RHIC)	33	16.89
ϕ(RHIC)	13	33.62
K^*(LHC)	11	16.89
ϕ(LHC)	13	33.62

Conclusion

- Vector meson fragmentation has been studied for the first time in both $e^+ e^-$ and $p\,p$ collisions at NLO.
- The SU(3) model with three light flavours uses universal functions, the valence $V(x, Q^2)$, sea $\gamma(x, Q^2)$ quark fragmentation functions and a gluon fragmentation function $D_g(x, Q^2)$.
- No new fragmentation function or additional parameters are introduced in order to explain the $p\,p$ hadroproduction data.
- The fragmentation of ϕ meson for $p\,p$ collisions (RHIC) data will be useful to understand, its production in nucleus-nucleus collisions as a signal in QGP studies.
- A table of quark and gluon fragmentaion functions for all vector mesons and a sample fortran code to generate the fragmentation functions are available at,
  http://www.imsc.res.in/ indu/Fragfn .

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Thank You

Backup Slides

DGLAP Evolution Equations

- The DGLAP evolution equations, named after the people who introduced them —Dokshitser[1], Gribov and Lipatov[2], Altarelli and Parisi[3]— form the baseline to understand the fragmentation functions, independent of processes involved.
- The general form of the DGLAP equation, a $(2N_f + 1)$ dimensional matrix equation in the space of quarks and gluons for the fragmentation process is given by:

$$t\frac{\partial}{\partial t}\begin{pmatrix} D^h_{q_j}(x,t) \\ D^h_g(x,t) \end{pmatrix} = \frac{\alpha_s(t)}{2\pi}\sum_{q_i,\overline{q}_i}\int_x^1 \frac{dz}{z}\begin{pmatrix} \mathcal{P}_{q_j q_i}(z,\alpha_s(t)) & \mathcal{P}_{g q_i}(z,\alpha_s(t)) \\ \mathcal{P}_{q_j g}(z,\alpha_s(t)) & \mathcal{P}_{gg}(z,\alpha_s(t)) \end{pmatrix} \times \begin{pmatrix} D^h_{q_j}(z,t) \\ D^h_g(z,t) \end{pmatrix} .$$

Here the subscripts i,j run over different flavours.

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Quark fragmentation functions in terms of α, β and γ SU(3) functions

Let q^i be a quark triplet and M^i_j a member of the pseudo-scalar meson octet, where $i, j, k = 1, 2, 3$. **Case 1 :** X being triplet, X^i: The invariant amplitude for a process

$$q^i \rightarrow V^i_j + X^j;$$

is $q_i V^i_j X^j$, where X^i and q^i are normalised. It is to be remembered throughout the calculation that the indices of the amplitude should contract to tensor of rank zero since cross section is a scalar quantity. Hence in this case, $X^i = q^i$, M^i_j are the elements of the pseudo-scalar meson matrix . For example, the rate for $u \rightarrow \rho^+ + X$ is $\alpha|q_1 V^1_2 X^2|^2$ which is equal to α. Here α is defined as the relevant fragmentation function for the process. In a similar way, the rates for d and s quarks that fragments into ρ^+ meson can be done and for all the three light quarks into other pseudo-scalar mesons as well.
Likewise, the other two SU(3) probability functions β and γ for $X(=\overline{6}, 15)$ are also calculated (see Appendix E of the Thesis).

Charge Factors of Cross Section at LO of $e^+ e^-$

The charge factors c_q are associated with the quark q_i with flavour i, written in terms of the electromagnetic charge e_i, vector and axial vector electroweak couplings, $v_i = T_{3i} - 2e_i \sin^2\theta_{\rm w}$ and $a_i = T_{3i}$, and similarly those of electrons, v_e and a_e as

$$\begin{aligned}
c_q &= c_q^V + c_q^A \,, \\
c_q^V &= \frac{4\pi\alpha^2}{s}[e_q^2 + 2e_q v_e v_q\,\rho_1(s) + (v_e^2 + a_e^2)v_q^2\,\rho_2(s)] \,, \\
c_q^A &= \frac{4\pi\alpha^2}{s}(v_e^2 + a_e^2)\; a_q^2\; \rho_2(s) \,, \\
\rho_1(s) &= \frac{1}{4\sin^2\theta_{\rm w}\cos^2\theta_{\rm w}}\;\frac{s(m_Z^2 - s)}{(m_Z^2 - s)^2 + m_Z^2\Gamma_Z^2} \,, \\
\rho_2(s) &= \frac{1}{(4\sin^2\theta_{\rm w}\cos^2\theta_{\rm w})^2}\;\frac{s^2}{(m_Z^2 - s)^2 + m_Z^2\Gamma_Z^2} \,.
\end{aligned}$$

For values of e_i, charge of the particle, a_i, the third component of weak isospin, and v_i with weak mixing angle $\theta_{\rm w}$ see appendix of thesis. Γ_Z and $m_{\rm Z}$ are the decay width and mass of the Z-intermediate gauge boson for high energy scale.

Co-efficients of Cross Section at LO of $e^+ e^-$

- The singlet D_0 and the non-singlet D_3, D_8, D_{15} and D_{24} combinations are:

$$
\begin{aligned}
D_0 &= D_u + D_d + D_s + D_c + D_b + \text{anti-quarks} \\
D_3 &= D_u - D_d + \text{ anti-quarks} \\
D_8 &= D_u + D_d - 2D_s + \text{ anti-quarks} \\
D_{15} &= D_u + D_d + D_s - 3D_c + \text{ anti-quarks} \\
D_{24} &= D_u + D_d + D_s + D_c - 4D_b + \text{ anti-quarks}
\end{aligned}
$$

- The corresponding co-efficients involved in the cross section equation are

$$
\begin{aligned}
a_0 &= (c_u + c_d + c_s + c_c + c_b)/5 \ , \\
a_3 &= (c_u - c_d)/2 \ , \\
a_8 &= (c_u + c_d - 2c_s)/6 \ , \\
a_{15} &= (c_u + c_d + c_s - 3c_c)/12 \ , \\
a_{24} &= (c_u + c_d + c_s + c_c - 4c_b)/20 \ , \qquad (1)
\end{aligned}
$$