Scale of Non-Commutativity: A “Reality" check

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Outline

What are NC spaces?

$[x, y] = \iota\theta$

- Non-Commutative Spaces (henceforth referred to as ’NC spaces’) are spaces in which coordinates do not commute among themselves,
  $[x_i, x_j] = \iota\theta_{ij}$
- θ_{ij} is a constant, real, anti-symmetric matrix. Captures the intrinsic “unknowability" of space.
- For dimensional considerations, $\theta_{ij} = \theta \times f_{ij}$, f_{ij} is $\mathcal{O}(1)$, θ is $[L^2]$, value of θ defines the scale of non-commutativity.
- Estimate of $\theta \longrightarrow$ in this presentation.

Why study them?

- Arise in String Theory,
  “(Examining) open strings ending on D-branes in the presence of a (non-zero constant) B field...B dependence of the effective action is completely described by making spacetime noncommutative." - (Seiberg N. and Witten E., 1999 [1])
- Even in the case of a general quantum theory, Planck length gives an estimate of “knowability" of space.
- Early universe scenarios when distances were comparable to Planck length.

Moyal star product

Ordering important! Type of ordering decides form of star product.
Converts from non-commutative to commutative quantities.

$$(f \star g)(x) = f(x) e^{\iota \theta^{\mu\nu} \overleftarrow{\partial_\mu} \overrightarrow{\partial_\nu}} g(x) \tag{1}$$

Simple prescription : “Replace all ordinary multiplication with star products"

Illustration - Hydrogen atom

In non-relativistic quantum mechanics

Based on (Chaichian, M et al. 2001 [2]) and (Ghoderao, P.S. and Ramadevi, P., 2017 [3]). Consider the hydrogen atom problem as a *fixed* proton in the centre and an electron surrounding it.

$$V = -\frac{1}{\sqrt{x_i' x_i'}} \tag{2}$$

There exists a linear transform which brings back the usual Heisenberg commutation relations - Canonical Formulation of NCQM[1]

$$x_i' \to x_i - \frac{\theta_{ij}}{2\hbar} p_j; \qquad p_i' \to p_i \tag{3}$$

[1] This the same transformation we expect from Moyal product if exponential is considered as a translation operator.

Potential is modified to:

$$V = -1/\sqrt{(\hat{x}_i - \sum_{j=1}^{3} \frac{\theta_{ij}}{2\hbar}\hat{p}_j)(\hat{x}_i - \sum_{k=1}^{3} \frac{\theta_{ik}}{2\hbar}\hat{p}_k)} \tag{4}$$

Performing a binomial expansion keeping terms upto order θ,

$$V = -\frac{1}{r} - \frac{1}{4\hbar r^3}(\vec{L} \cdot \vec{\Theta}) \tag{5}$$

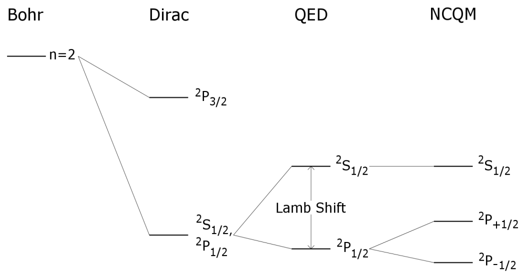
here, $\vec{\Theta} = (\theta_x, \theta_y, \theta_z)$
$\longrightarrow$ Perturbation term to the $1/r$ potential.
First order perturbation theory :

$$\Delta E = \langle n, j, j_z| - \frac{\theta_z \hat{L}_z}{4\hbar r^3} |n, j, j_z\rangle \tag{6}$$

$$= \frac{\theta_z j_z}{4}\left(1 \mp \frac{1}{2l+1}\right)\left(\frac{1}{n^3 l(l+1/2)(l+1)}\right) \tag{7}$$

Note: j_z splits the Lamb shift into two lines.

The NC spectrum for hydrogen?



Lamb shift measurement can potentially reveal the NC scale.
Caveat: This illustration has assumed that proton is stationary.

NC hydrogen spectrum

- Basic quantum mechanics problem,

$$H = \frac{p^2}{2\mu} - \frac{1}{r} \tag{8}$$

- Relative coordinates : $r_i = x_i{}^{electron} - x_i{}^{proton}$
- θ for opposite charges has to be of opposite sign (Sheikh-Jabbari M.M. 2000 [4]).
- Relative coordinates do not have any non-commutativity! $[r_i, r_j] = 0$

$\Longrightarrow$ No NC correction.

- Proton is a composite particle!
- Modify the potential as (Chaichian, M et al. 2004 [5])

$$V = V_{u_1} + V_{u_2} + V_d \tag{9}$$

$$= \sum_{i=u_1,u_2,d} -\frac{e^2 Q_i}{r_i} \tag{10}$$

$\Longrightarrow$ Simple modification of potential which gives NC correction.

- Proton must be considered as composite operator in field theory and not simply as three separate particles with different charge to modify the EM potential.

$\Longrightarrow$ We must investigate NCQCD.

$\diamondsuit$ We simplify to NCQED, but assuming the same bound structure.

Gauge transform

Finite gauge transformation (Hayakawa, 1999 [6]):

$$U_\star(\lambda(x)) \star \psi(x) = e_\star^{\iota\lambda} \star \psi = (1 + \iota\lambda + \frac{(\iota)^2}{2!}\lambda \star \lambda + ...) \star \psi(x) \tag{11}$$

This is unitary and admits inverse $U_\star^{-1} = e_\star^{-\iota\lambda}$.

It can be shown that the field tensor then is given by

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - \iota g[A_\mu, A_\nu]_\star \tag{12}$$

Note coupling constant between photon fields, g.

Fermionic Part

Covariant derivative is,

$$D_\mu = \partial_\mu \psi - \iota n A_\mu \star \psi \tag{13}$$

where n is the charge.
Choose proper “fundamental" representation as (Sheikh-Jabbari, M.M. 2000 [4]),

$$\psi' = U^n \star \psi \tag{14}$$

Applying infinitesimal transformation on both sides, we first showed that

$$n = 0 \text{ or } n = g \tag{15}$$

Composite Particles

Consider a composite proton of same fractionally charged quarks but in NC space :

$$\psi_p = \psi_u \star \tilde{\psi}_u \star \psi_d \tag{16}$$

Under gauge transformation we expect,

$$\psi'_p = (U^{2/3} \star \psi_u) \star (U^{2/3} \star \tilde{\psi}_u) \star (U^{-1/3} \star \psi_d) = U \star \psi_p \tag{17}$$

Under infinitesimal transform, LHS:

$$\psi_u \star \tilde{\psi}_u \star \psi_d + \frac{2\iota}{3}(\psi_u \star \lambda + \lambda \star \psi_u) \star (\tilde{\psi}_u \star \psi_d) - \frac{\iota}{3}(\psi_u \star \tilde{\psi}_u \star \lambda \star \psi_d) \tag{18}$$

but RHS:

$$\psi_u \star \tilde{\psi}_u \star \psi_d + \iota(\lambda \star \psi_u \star \tilde{\psi}_u \star \psi_d) \tag{19}$$

Our Attempts to resolve the problem

- Take a linear combination of the quark fields, $\psi_i = a_i\psi_u + b_i\tilde{\psi}_u + c_i\psi_d$. Unsuccessful to obtain a covariant composite with the requisite charge of $+1$.
- Considering $\tilde{\psi}_u = \psi_u$, cannot change the order in star product, hence it also does not work.
- Integrating the left and right sides of the transform equations and using the cyclic property of star product also does not help to reconcile the transformed composite.

Conclusion

Star product characteristics do not permit a gauge-invariant composite.

"Composite particles are forbidden in NC spaces." Realistic ??

Tolerating compositeness

Albeit approximately

- Examining star product definition,

$$\lambda \star \psi - \psi \star \lambda = \iota\theta^{\mu\nu}\partial_\mu\lambda\, \partial_\nu\psi + \mathcal{O}(\theta^3) \longrightarrow 0 \qquad (20)$$

- By uncertainty principle, $\partial \sim \frac{1}{r}$
- Above definition implies,

$$\boxed{\theta << r^2} \qquad (21)$$

  For a particular composite particle to be allowed, the NC scale must lie well below its radius.
- Since $r_{\text{proton}} \sim 10^{-15} m$, $\sqrt{\theta}$ must be still lower.

Fixing a scale

$n = \pm g, 0$ and $\sqrt{\theta} << r$

Quarks and leptons have charges 0, 1, $\frac{2}{3}$ and $-\frac{1}{3}$.
$\implies$ Such charges not permitted in NC spaces : Only $\pm g$ and 0!
$\therefore$ NC spaces possible ONLY IF quarks and leptons are composite.

LHC search for quark and lepton compositeness,
$\Lambda = 10 - 25 TeV$. (Review of Particle Physics, 2018 [9])
$\implies$ Length scale of $\approx 7.9 - 19.7 \times 10^{-21} m$.
Therefore for realistic NC space, $\sqrt{\theta}$ must be even lower :

Thus we find the NC scale to be $\sqrt{\theta} << 10^{-20} m$.

Possible Preon constructions

Possible preon models for illustrative purposes: An already existing elaborate model,

- *Rishon Model* $\longrightarrow$ Three new “fundamental" particles of charge +1/3, -1/3, 0 (Harari H. and Seiberg N. 1982 [10])
- Or another with two new “fundamental" particles of charge +1/6 and -1/6.
- Note: Charge quantisation needs to be established by suitable tweaking of constituents of quarks and leptons in any generalised model.

Conclusion and Outlook

- Found new results which gauge covariance forces on NC space
  - Fermionic charge must exactly equal the coupling constant between photon fields, $n = \pm g, 0$
  - Composite operators are allowed only if the NC scale is much smaller than radius of composite particle.
- In order for NC spaces to be a physical reality, quarks and leptons ought to possess substructure.
- Derived a limit on NC scale, $\sqrt{\theta} << 2 \times 10^{-20} m$ which can be improved at future experiments

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