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Constraining light DM fermions from relic density, SN1987A cooling and the role of Tsallis statistics

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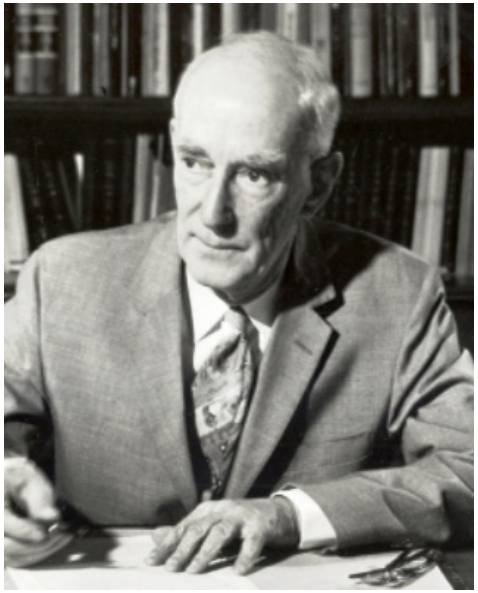
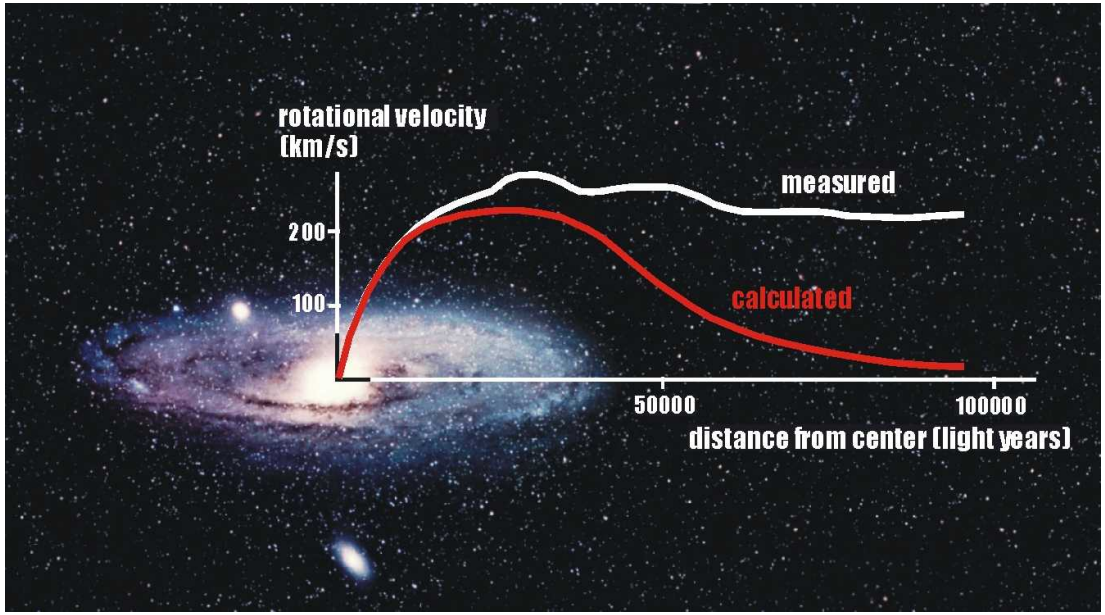
Historical Events and Evidences of Dark Matter



In 1932 **Jan Hendrik Oort** analyzed the motion of the stars in the neighbourhoods of our solar system. From the estimated gravitational potential he found out that the known stars are moving faster than expected.

In 1933 **Fritz Zwicky** estimated the average mass of the galaxies within Coma cluster as **160** times larger than expected from their luminosity. Also the orbital velocities of member galaxies were **10** times larger than expected from the total mass of the Coma cluster.

Galaxy Rotation Curve



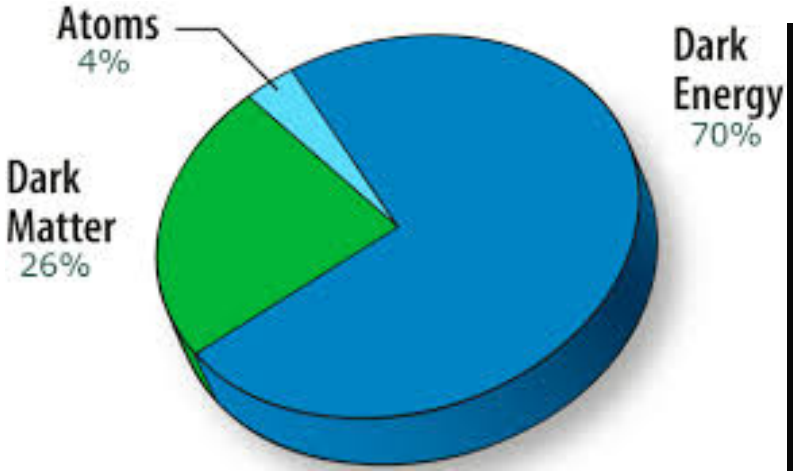
In 1975 **Vera Rubin** infered that most of the stars in the outer region of the spiral galaxies orbit roughly with the equal velocity.

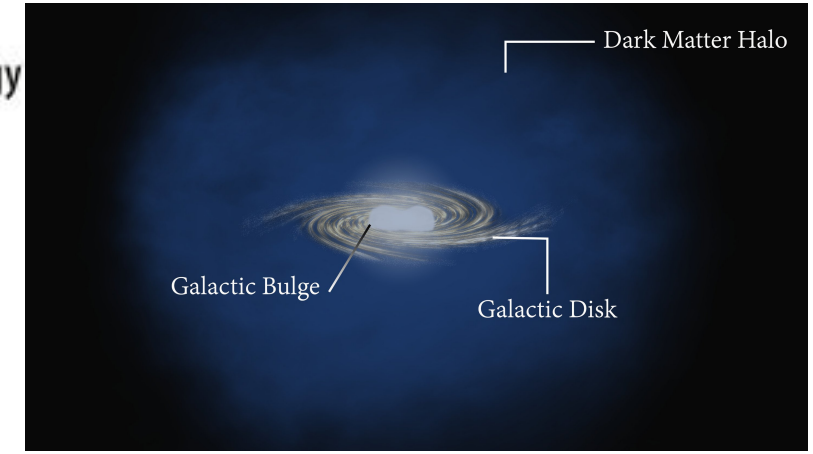
$$v = \sqrt{\frac{GM(r)}{r}} \quad \text{and} \quad \mathrm{v(r)} \approx \text{ constant} \quad \Longrightarrow \quad \mathrm{M(r)} \propto \mathrm{r}$$

Evidences for the existence of DM include gravitational lensing, bullet cluster.

Present Understanding

- Structure formation of the universe: Luminous matter ($\sim 4\%$) is not sufficient, non-luminous matter (dubbed as dark matter $\sim 26\%$) is required.





- They are dark as they can't absorb, emit, reflect light! They does not interact with the electromagnetic force like SM particles.
- Their interaction with the SM particles is very weak whereas they are highly massive (WIMPs).

Classification of Dark Matter

Dark Matter can be classified on the basis of [1]

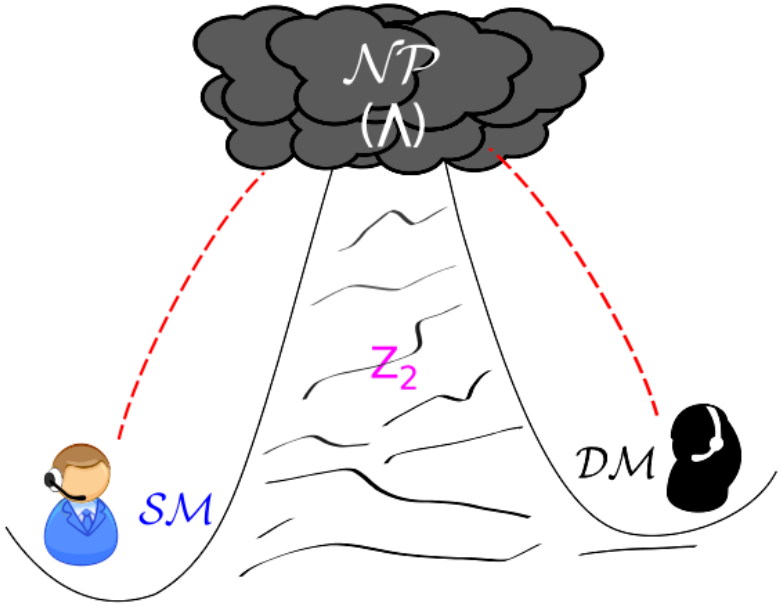
- **Thermal History:** Thermal DM are produced via the collision of cosmic plasma in radiation dominated area, whereas, non-thermal DM particles are produced by decay of massive particles.
- **On the basis of particle type:**
  - (i) Scalar
  - (ii) Vector
  - (iii) Fermion
- **From mass and speed:**
  - (a) Hot dark matter candidates are of low mass ($\sim$ keV) and move with relativistic speeds.
  - (b) Cold dark matter candidates are non-relativistic and made up of heavier particles (mass $\sim$ MeV and more).
  - (c) Warm dark matter candidates (sterile neutrinos) are intermediate of these two.

[1] Debasish Majumdar, *DARK MATTER: An Introduction*, CRC Press (2014).

Effective Model Approach

Model described by the effective Lagrangian

$$\mathcal{L}_{eff} = \mathcal{L}_{SM} + \sum_i \frac{f_i^{(5)}}{\Lambda} O_i^{(5)} + \sum_i \frac{f_i^{(6)}}{\Lambda^2} O_i^{(6)} + \cdots$$

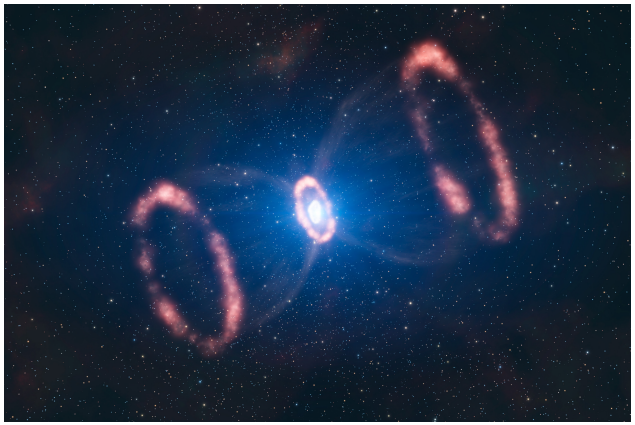


The following symmetry should hold for the Lagrangian

- For scalar and fermionic DM: $SU(3)_c \times SU(2)_L \times U(1)_Y \times Z_2$
- For vector DM: $SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_\chi$

Supernova SN1987A:

- The supernova explosion caused by the death of the star **Sanduleak** $-69^0 \quad 202$
- The progenitor was a B3 Supergiant of mass $20M_\odot$, luminosity $10^5 L_\odot$ and temperature 16000 K.



- The energy released in SN1987A explosion is enormous: it is the gravitational binding energy E_g of the proto-neutron star (of mass M_{PNS}), given by

$$E_g = \frac{3G_N M_{PNS}^2}{5R_{NS}} \sim 3.0 \times 10^{53} \text{ erg.}$$

Here $M_{PNS} = 1.5M_\odot$, $R_{NS} = 10$ Km and G_N is the Newton's gravitational constant.

Supernova SN1987A energy loss:

- Neutrino takes away 99% of the energy 3.0×10^{53} erg. released in SN1987A explosion. The emitted neutrino energy $E_\nu = \frac{E_g}{6} = 0.5 \times 10^{53}$ erg.
- The observed neutrino luminosity in the detector (IMB and Kamiokande) is $L_\nu = 10^{53}$ erg. So $\frac{L_\nu}{6} \sim 10^{52}$ erg.
- Remaining 1% by other channels $\rightarrow$ New Physics channels!!

Raffelt's criteria on energy loss rate for any new physics channel:

If any energy-loss rate of any new(physics) channel

$$\dot{\epsilon_x} > 10^{19} \text{ erg g}^{-1} \text{ s}^{-1}$$

then it will remove sufficient energy from the explosion to invalidate the current understanding of Type-II supernovae neutrino signal [a].

[a] G. G. Raffelt, *Star as laboratories for fundamental physics*, University of Chicago Press, Chicago (1996).

q-generalized distribution function for fermions

For a system with equilibrium temperature T,

$$p_i = \frac{e^{-\beta\epsilon_i}}{\sum_{i=1}^{W} e^{-\beta\epsilon_i}}$$

If the temperature is fluctuating around an average value and the system is in an out-of-equilibrium quasi-stationary state, the Boltzmann-Gibbs entropy can be generalized to a non-extensive q-deformed entropy [2]

$$S_q = k\sum_{i=1}^{W} p_i \ln_q \frac{1}{p_i} \equiv \frac{k\left(1-\sum_{i=1}^{W} p_i^q\right)}{q-1},$$

- In the limit $q \to 1$, it coincides with the Boltzmann-Gibbs entropy, i.e. $S_{q=1} = S_{\rm BG}$.
- In the framework of Tsallis statistics these special non-equilibrium states can be described with different q and the equilibrium statistical mechanics can be treated as a special case for $q = 1.0$.

[2] C. Tsallis, *Introduction to Nonextensive Statistical Mechanics: Approaching a Complex World*, Springer-Verlag, New York (2001).

q-generalized distribution function for fermions

Extremizing S_q subject to constraints yields a generalized canonical ensemble where the probability to observe a microstate of energy ϵ_i is given by

$$p_i \;=\; e_q^{-\beta\epsilon_i} \;\equiv\; \frac{1}{[1+(q-1)\beta\epsilon_i]^{\frac{1}{q-1}}}\,,$$

The q-deformed probability distribution function may be driven by the non-equilibrium situation with local temperature fluctuation over a region of space, which can be accounted for by defining a χ^2 distribution of the form [3]

$$f(\beta) \;=\; \frac{1}{\Gamma\left(\frac{n}{2}\right)}\left(\frac{n}{2\beta_0}\right)^{n/2}\beta^{\frac{n}{2}-1}\,\exp\left(-\frac{n\beta}{2\beta_0}\right)\,,$$

where n is the degree of the distribution, i.e. the number of independent Gaussian random variables $X_i,\; i=1,....,n$ and $\beta=\sum_{i=1}^{n}X_i^2$ is the fluctuating inverse temperature, with the average value [4]

$$\beta_0 \;\equiv\; \langle\beta\rangle \;=\; n\langle X_i^2\rangle \;=\; \int_0^\infty d\beta\,\beta f(\beta)\,.$$

[3] G. Kaniadakis, Phys. Rev. E **66**, 056125 (2002).
[4] C. Beck and E. G. D. Cohen, Physica A **322**, 267 (2003).

q-generalized distribution function for fermions

Taking into account the local temperature fluctuation, integrating over all β, we find the q-generalized Maxwell-Boltzmann distribution

$$\mathcal{P}(E) \equiv \frac{1}{Z}B(E) = \frac{1}{Z}\int_0^\infty d\beta\, e^{-\beta E} f(\beta) = \frac{1}{[1+(q-1)bE]^{\frac{1}{q-1}}} \equiv e_q^{-\beta_0 E},$$

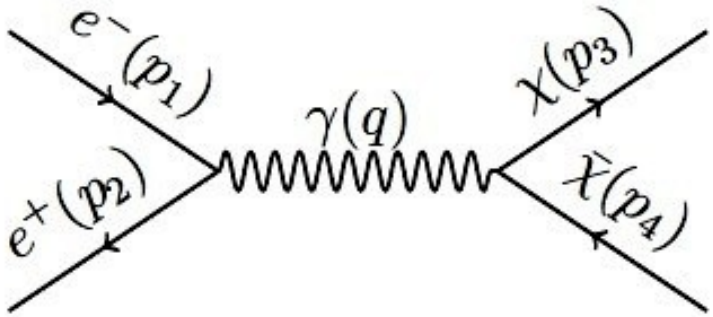
$b = \frac{\beta_0}{4-3q}$ and the normalization constant $Z = \int_0^\infty B(E)dE$.
The average occupation number of any particle within this q-deformed statistics formalism is given by [5]

$$f(\beta, E) = \frac{1}{[1+(q-1)bE]^{\frac{1}{q-1}} \pm 1} \equiv \frac{1}{e_q^{\beta_0 E} \pm 1},$$

$e_q^{\beta_0 E}$ is the effective Boltzmann factor. In $q \to 1$ limit, $e_q^{\beta_0 E}$ reduces to the usual Boltzmann factor $e^{-bE} = e^{-\beta_0 E}$.

[5] C. Beck, Eur. Phys. J. A **40**,267 (2009).

Interaction via Effective Coupling : [6]



The effective Lagrangian for the vertex $\gamma \to \chi\bar{\chi}$

$$\mathcal{L}_{DM} = -\frac{i}{2}\bar{\chi}\sigma_{\mu\nu}(\mu_\chi + \gamma_5 d_\chi)\chi F^{\mu\nu}$$

The cross-section for the process:

$$\sigma(e^-e^+ \xrightarrow{\gamma} \chi\bar{\chi}) = \frac{\alpha}{6s} \cdot \sqrt{1 - \frac{4m_\chi^2}{s}} \cdot \left[\mu_\chi^2(s + 8m_\chi^2) + d_\chi^2(s - 4m_\chi^2)\right]$$

[6] A. Guha and P. K. Das, JHEP **1806**, 139 (2018); A. Guha, Selvaganapathy J. and P. K. Das, Phys. Rev. D **95**, no. 1, 015001 (2017).

Energy loss rate:

$$\dot{\varepsilon}_{e^-e^+\to\chi\overline{\chi}} = \frac{1}{\rho_{SN}}\langle n_{e^-} n_{e^+} \sigma_{e^-e^+\to\chi\overline{\chi}} V_{rel} E_{C.M.}\rangle$$
$$= \frac{1}{\rho_{SN}}\frac{1}{\pi^4}\int_{m_\chi}^{\infty}\int_{m_\chi}^{\infty} dE_1 dE_2 \frac{E_1 E_2 (E_1+E_2)^3}{2D_1 D_2}\sigma_{e^-e^+\to\chi\overline{\chi}}$$

$$\dot{\varepsilon}_{e^-e^+\to\chi\overline{\chi}} = \frac{\alpha T^7}{12\pi^4\rho_{SN}}\int_{\frac{m_\chi}{T}}^{\infty} dx_1 \int_{\frac{m_\chi}{T}}^{\infty} dx_2 \frac{x_1\ x_2\ (x_1+x_2)}{\left[\left(1+\frac{b}{\tau}(Tx_1-\mu_{e^-})\right)^\tau+1\right]}\frac{\mathcal{F}}{\left[\left(1+\frac{b}{\tau}(Tx_2-\mu_{e^+})\right)^\tau+1\right]}$$

with

$$\mathcal{F} = \sqrt{1-\frac{4m_\chi^2}{T^2(x_1+x_2)^2}}\cdot\left[\mu_\chi^2\left\{(x_1+x_2)^2+\frac{8m_\chi^2}{T^2}\right\}+d_\chi^2\left\{(x_1+x_2)^2-\frac{4m_\chi^2}{T^2}\right\}\right], \tau = \frac{1}{q-1}$$

In undeformed scenario

$$\dot{\varepsilon}_{e^-e^+\to\chi\overline{\chi}} = \frac{\alpha T^7}{12\pi^4\rho_{SN}}\int_{\frac{m_\chi}{T}}^{\infty} dx_1 \int_{\frac{m_\chi}{T}}^{\infty} dx_2 \frac{x_1(x_1+x_2)}{\exp\left(x_1-\frac{\mu_{e^-}}{T}\right)+1}\frac{x_2}{\exp\left(x_2-\frac{\mu_{e^+}}{T}\right)+1}\mathcal{F}$$

Bounds on Λ using $\dot{\varepsilon}_{e^-e^+\to\chi\overline{\chi}} \leq 10^{19}\ \mathrm{erg\ g^{-1}\ s^{-1}}$ and $\mu_\chi \sim \mathrm{d}_\chi \sim \frac{1}{\Lambda}$

Free Streaming Bound and Optical Depth Criteria

Mean free path of the DM fermions

$$\lambda_\chi = \frac{1}{n_e \cdot \sigma_{e\chi \to e\chi}}$$

- n_e = number density of colliding electrons inside SN $= 8.7 \times 10^{43}\ \mathrm{m}^{-3}$.
- $\sigma_{e\chi \to e\chi}$ is related to $\sigma_{ee \to \chi\chi}$ via crossing symmetry.

The optical depth criteria: [7] Emergent particles originate near and above the layer at which optical depth $= 2/3$

$$\int_{r_0}^{R_c} \frac{dr}{\lambda_\chi} \leq \frac{2}{3}$$

to investigate whether the DM fermion produced at a depth r_0 free streams out of the SN or gets trapped inside the SN. We set $r_0 = 0.9 R_c$ in our analysis, where R_c ($\simeq 10$ km), the radius of the supernova core (proto-neutron star).

[7] Shapiro and Teukolsky, *Black Holes, White Dwarfs and Neutron Stars: The Physics of Compact Objects*, Wiley (1983).

Constraining DM fermions from relic density

- After freeze-out(when the particle species fall out of thermal equilibrium) the microscopic evolution of the phase space distribution of a particle species is governed by Boltzmann equation.
- The Boltzmann equation in FRW cosmology [8]

$$\frac{dY}{dx} = \frac{1}{3H}\frac{ds}{dx}\,\langle\sigma v_{rel}\rangle\,\left(Y^2 - Y_{eq}^2\right)$$

With $Y = \frac{n}{s},\ x = \frac{m}{T}$($s, m, T$ denotes the entropy density, mass of the particle species and temperature respectively).
The thermal averaged crosssection times relative velocity

$$\langle\sigma v_{rel}\rangle = \frac{\int \sigma v_{rel}\ e^{-E_1/T}\ e^{-E_2/T}\ d^3p_1\ d^3p_2}{\int e^{-E_1/T}\ e^{-E_2/T}\ d^3p_1\ d^3p_2}$$

FRW cosmology $H = \left(\frac{8}{3}\pi G\rho\right)^{\frac{1}{2}}$
The expressions for the density ρ and entropy density s

$$\rho = g_{eff}(T)\frac{\pi^2}{30}T^4 \quad ; \quad s = h_{eff}(T)\frac{2\pi^2}{45}T^3$$

[8] Kolb and Turner, *The Early universe*, Addison-Wesley (1990).

Constraining DM fermions from relic density

Subsequently we get the following form of the Boltzmann equation [9]

$$\frac{dY}{dx} = -\left(\frac{45}{\pi}G\right)^{-\frac{1}{2}} \frac{m}{x^2}\, g_*^{\frac{1}{2}}\, \langle\sigma v\rangle\, \left(Y^2 - Y_{eq}^2\right);$$

$$g_*^{\frac{1}{2}} = \frac{h_{eff}}{g_{eff}^{\frac{1}{2}}}\left(1 + \frac{1}{3}\frac{T}{h_{eff}}\frac{dh_{eff}}{dT}\right)$$

Total effective degree of freedom for all species $g_{eff}(T) = \sum_i g_i(T)$ and $h_{eff}(T) = \sum_i h_i(T)$. The effective degrees of freedom for each species

$$g_i(T) = \frac{15 g_i}{\pi^4}\, x_i^4 \int_1^\infty \frac{z\sqrt{z^2-1}}{\exp(x_i z) + \eta_i} z\, dz$$

$$h_i(T) = \frac{45 g_i}{4\pi^4}\, x_i^4 \int_1^\infty \frac{z\sqrt{z^2-1}}{\exp(x_i z) + \eta_i} \frac{4z^2-1}{3z} dz$$

with $x_i = m_i/T$; $z = E_i/m_i$; $m_i =$ mass of that particular species. $\eta_i = \pm 1$ for Fermi-Dirac and Bose-Einstein statistics.

[9] P. Gondolo and G. Gelmini, Nucl. Phys. B **360**, 145 (1991).

Constraining DM fermions from relic density

Integrating from the freeze-out period to the present epoch we get [10]

$$\frac{1}{Y_0} = \left(\frac{45}{\pi}G\right)^{-\frac{1}{2}} \int_{T_0}^{T_f} g_*^{\frac{1}{2}} \langle \sigma v \rangle \, dT$$

At freeze-out the number density of the concerned particle is considered to be very high. So at $T = T_f$, $\frac{1}{Y_f} = \frac{s_f}{n_f}$ to be very small compared to the other terms. Relic density of the concerned particle at present day is obtained evaluating Y_0 in the units of the critical density

$$\Omega_\chi = \frac{\rho_\chi^0}{\rho_{crit}} = \frac{m_\chi s_0 Y_0}{\rho_{crit}}$$

with the critical density $\rho_{crit} = \frac{3H^2}{8\pi G}$, s_0 in the entropy density today. With the knowledge of the present day background radiation temperature $T_0 = 2.726 \text{ K} = 2.35 \times 10^{-16} \text{ TeV}$ we obtain

$$\Omega_\chi h^2 = 2.755 \times 10^{11} \; Y_0 \; \left(\frac{m_\chi}{\text{TeV}}\right)$$

[10] Debasish Majumdar, *DARK MATTER: An Introduction*, CRC Press, (2014).

Constraining DM fermions from relic density

The total cross section for the process $\chi(p_1)\bar{\chi}(p_2) \xrightarrow{\gamma} e^-(p_3)e^+(p_4)$ is given by

$$\sigma(\chi\bar{\chi} \xrightarrow{\gamma} e^-e^+) = \frac{\alpha}{6s} \cdot \left(\sqrt{1-\frac{4m_\chi^2}{s}}\right)^{-1} \cdot \left[\mu_\chi^2(s+8m_\chi^2) + d_\chi^2(s-4m_\chi^2)\right]$$

$=m_\chi$ mass of DM fermions, $\alpha = \frac{e^2}{4\pi}$ and $s = (p_1+p_2)^2 = (p_3+p_4)^2$.
Thermal averaged cross-section times velocity with the dimensionless variables $x_i = E_i/T$ $(i = 1, 2)$

$$\langle \sigma_{\chi\bar{\chi}\to e^-e^+} \; V_{rel} \rangle = \frac{\alpha}{6} \frac{\int_{\frac{m_\chi}{T}}^{\infty} \int_{\frac{m_\chi}{T}}^{\infty} dx_1 \; dx_2 \; \sqrt{x_1^2 - \frac{m_\chi^2}{T^2}} \; \sqrt{x_2^2 - \frac{m_\chi^2}{T^2}} \; \mathcal{F} \; f_1 \; f_2}{\int_{\frac{m_\chi}{T}}^{\infty} \int_{\frac{m_\chi}{T}}^{\infty} dx_1 \; dx_2 \; x_1 \; x_2 \; \sqrt{x_1^2 - \frac{m_\chi^2}{T^2}} \; \sqrt{x_2^2 - \frac{m_\chi^2}{T^2}} \; f_1 \; f_2}$$

where the function $\mathcal{F}$ is given by

$$\mathcal{F} = \left(\sqrt{1-\frac{4m_\chi^2}{T^2(x_1+x_2)^2}}\right)^{-1} \cdot \left[\mu_\chi^2\left\{(x_1+x_2)^2 + \frac{8m_\chi^2}{T^2}\right\} + d_\chi^2\left\{(x_1+x_2)^2 - \frac{4m_\chi^2}{T^2}\right\}\right]$$

$$\mu_\chi \sim d_\chi \sim \frac{1}{\Lambda}$$

Constraining DM fermions from relic density

- The experimental value of the non-baryonic relic density i.e. 0.1186 ± 0.0020 (obtained from the measurement of CMB anisotropy and the spatial distribution of galaxies).
- In the limiting case:

$$\Omega_\chi h^2 = 0.1186 = 2.755 \times 10^{11}\ m_\chi\ Y_0$$

$$\Longrightarrow\ 0.1186 = \frac{2.755 \times 10^{11}\ m_\chi}{\left(\frac{45}{\pi}G\right)^{-\frac{1}{2}} \int_{T_0}^{T_f} g_*^{\frac{1}{2}}\ \langle\sigma v\rangle\ dT}$$

- The upper bound on Λ has been obtained as

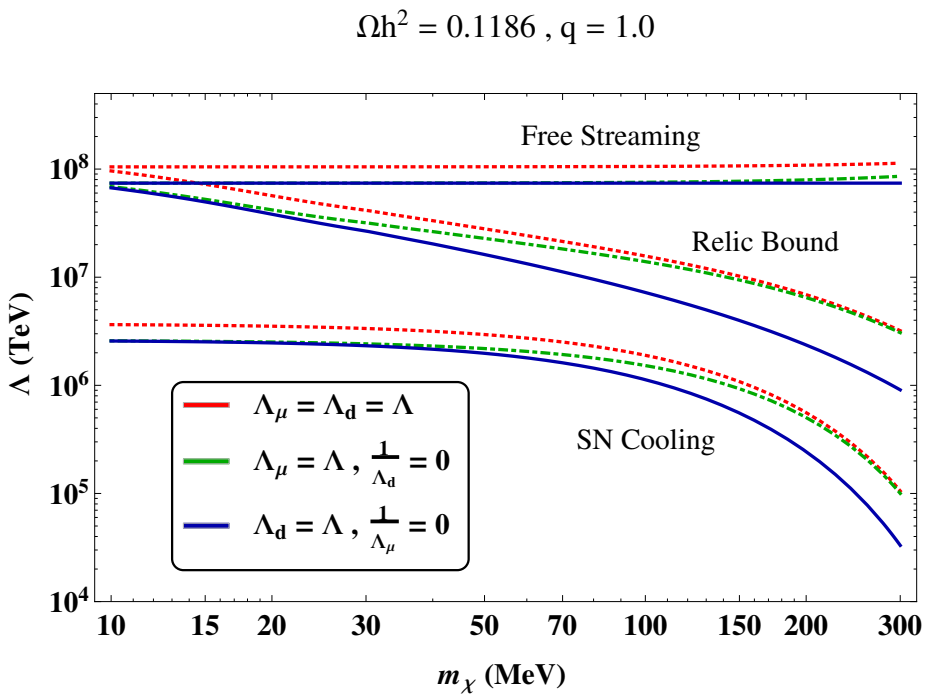
$$\langle\sigma v\rangle\ \propto \frac{1}{\Lambda^2};\quad \Omega_\chi h^2\ \propto \frac{1}{\langle\sigma v\rangle}$$

And

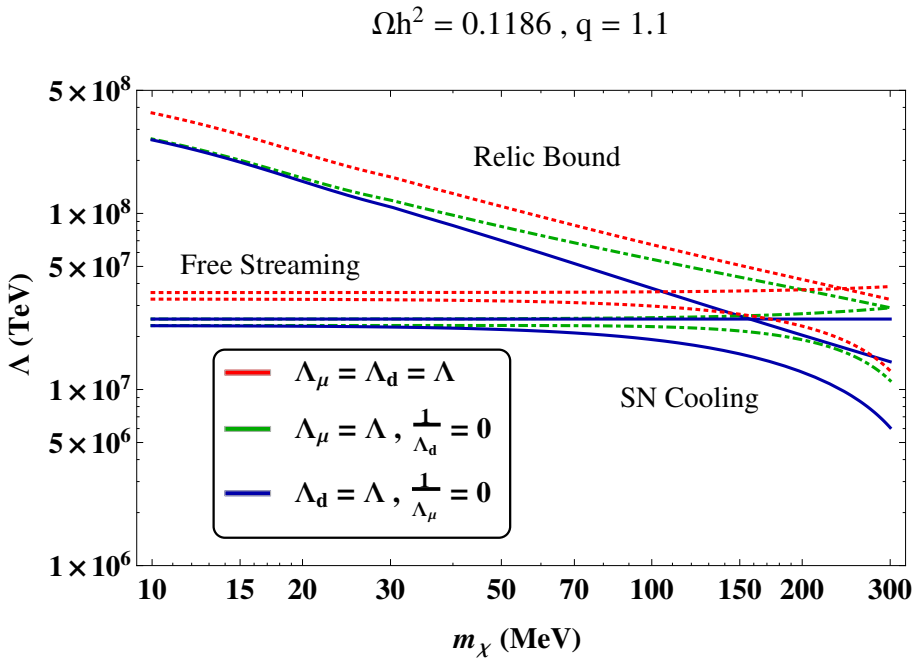
$$\Omega_\chi h^2 \leqslant 0.1186$$

Astrophysical Bounds on effective scale Λ

- SN Cooling bound using Raffelt's criteria: $\dot{\varepsilon}_{e^-e^+\to\chi\overline{\chi}} \leq 10^{19}\ \mathrm{erg\ g^{-1}\ s^{-1}}$.
- Free Streaming Bound using Optical Depth Criterion: $\int_{r_0}^{R_c} n_e\ \sigma_{scatt}\ dr \leq \frac{2}{3}$.
- Relic Bound using $\Omega_\chi h^2 \leqslant 0.1186$.



$T_{SN} = 30$ MeV



$T_{SN} = 50$ MeV

Summary

- Existing literature: Our results for the case $q = 1.0, \Lambda = \Lambda_\mu,\ d_\chi = 0$ is in agreement with **Kadota and Silk, Phys. Rev. D, 89, 103528 (2014)**.
- We worked within the formalism of Tsallis statistics, where we took care of the non-equilibrium conditions which may arise due to temperature fluctuations inside supernova.
- $q \to 1.0$ denotes the undeformed case and for q-deformed case we took a benchmark value $q = 1.1$.
  For the supernovae temperature(average) $T_{SN} = 30(50)\ \mathrm{MeV}$ and dark matter mass $m_\chi = 30\ \mathrm{MeV}$, the lower and the upper bound on Λ

	$\Lambda = \Lambda_\mu = \Lambda_d$ (TeV)		
	Free streaming	SN cooling	Relic bound
$q = 1.0$	1.05×10^8	3.34×10^6	4.92×10^7
$q = 1.1$	3.6×10^7	3.23×10^7	1.58×10^8

Summary

- In undeformed scenario ($q = 1.0$) the free-streaming bound and SN cooling bound are different. But if we consider that most of the DM fermions are producing at the outer region of the SN core (at temperature ~ 50 MeV) [11] and takes into account the non-equilibrium conditions, they match with each other.
- The value of the deformation parameter ($q = 1.1$) is consistent with the spectrum [12] of the emitted neutrinos or anti-neutrinos ($q \leqslant 1.27$) [13] and has been chosen for pure phenomenological purposes.

[11] P. Char, S. Banik and D. Bandyopadhyay, Astrophys.J., 809:116 (2015).
[12] A. Nikrant, R. Laha and S. Horiuchi, Phys. Rev. D 97, 023019 (2018); I. Tamborra, B. Muller, L. Hudepohl, H. T. Janka and G. Raffelt, Phys. Rev. D 86, 125031 (2012)
[13] AG, P.S. Bhupal Dev, P. K. Das, arXiv:1810.00399 [hep-ph]

Ongoing Work

- To work with other operators like leptophilic operators.

$$\mathcal{L}_{\rm eff} \;=\; \frac{1}{\Lambda^2}\sum_j \left(\bar{\chi}\Gamma^j_\chi \chi\right)\left(\bar{e}\Gamma^j_e e\right)$$

S-P type :	$\Gamma_\chi \;=\; c^\chi_S + i c^\chi_P \gamma_5\,,$	$\Gamma_e = c^e_S + i c^e_P \gamma_5$
V-A type :	$\Gamma^\mu_\chi = \left(c^\chi_V + c^\chi_A \gamma_5\right)\gamma^\mu\,,$	$\Gamma_{e\mu} = \left(c^e_V + c^e_A \gamma_5\right)\gamma_\mu$
T-AT type :	$\Gamma^{\mu\nu}_\chi = \left(c^\chi_T + i c^\chi_{AT}\gamma_5\right)\sigma^{\mu\nu}\,,$	$\Gamma_{e\mu\nu} = \sigma_{\mu\nu}$

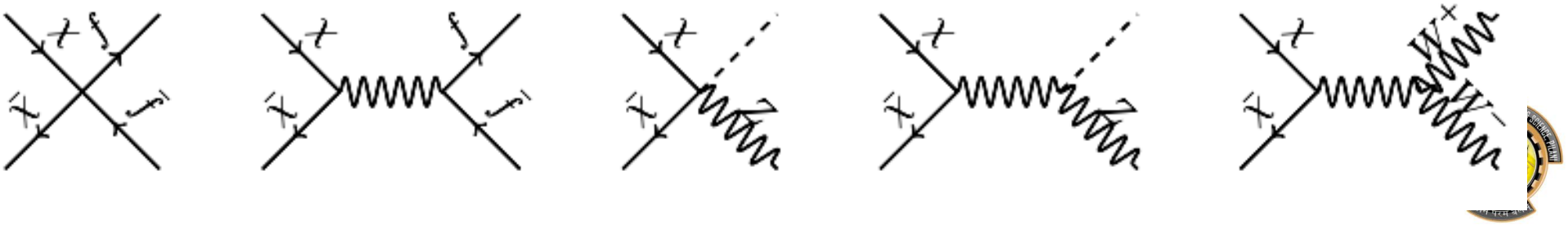
- Taking care of the uncertainty in the progenitor mass using a SN profile

$$\rho(r) = \rho_c \times \begin{cases} 1 + k_\rho\left(1 - r/R_c\right);\; r < R_c \\ \left(r/R_c\right)^{-\nu};\; r \geq R_c \end{cases} \quad ; \quad T(r) = T_c \times \begin{cases} 1 + k_T\left(1 - r/R_c\right);\; r < R_c \\ \left(r/R_c\right)^{-\nu/3};\; r \geq R_c \end{cases}$$

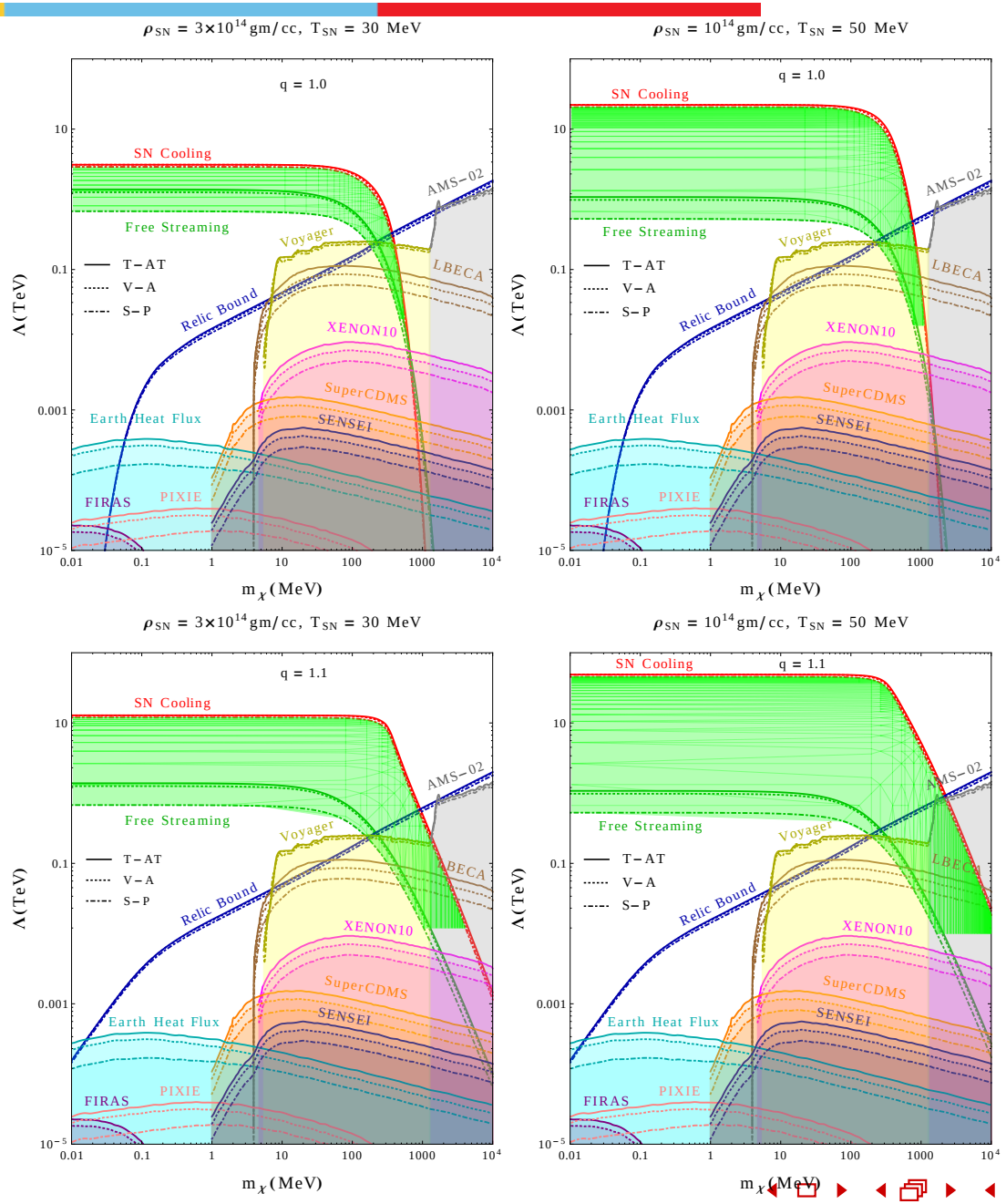
$$R_c = 10\ \mathrm{km}, \mathrm{T_c} = 30\ \mathrm{MeV}, \rho_\mathrm{c} = 3\times 10^{14}\ \mathrm{gm\,cm^{-3}}.$$

Effective vertices for DM interaction with SM scalars, gauges bosons and fermions:

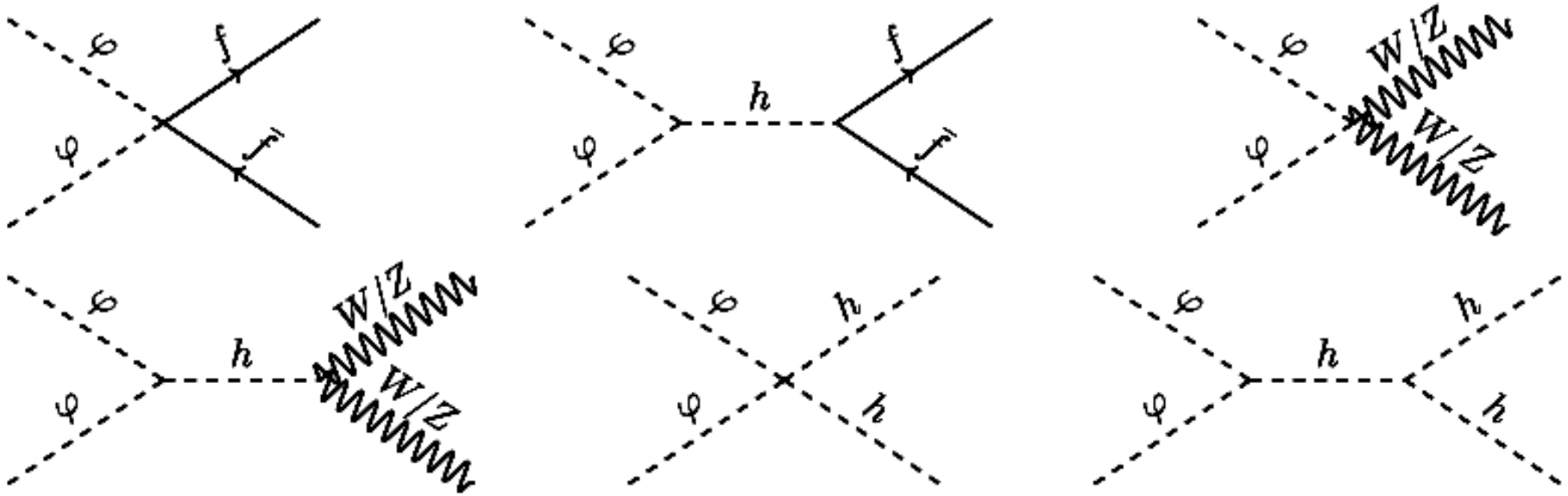
Annihilation processes of DM fermions:



Preliminary result



The annihilation processes we can construct using the effective vertices:



This leads to interesting phenomenological studies both in collider (Linear collider and LHC) and astrophysical contexts.

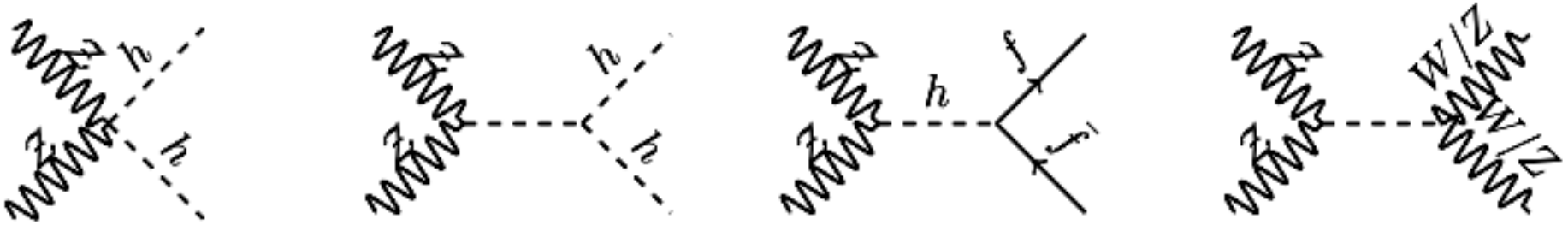
The dimension-6 effective operator for $SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_\chi$ invariant Lagrangian:

$$\mathcal{O}_{\phi\mathcal{Z}} = \frac{1}{2}(\phi^\dagger\phi)C_{\mu\nu}C^{\mu\nu}$$

where we assumed that SM Quarks, Leptons and the Higgs field don't have any $U(1)_\chi$ quantum number and $C_{\mu\nu} = \partial_\mu \mathcal{Z}_\nu - \partial_\nu \mathcal{Z}_\mu$. So,

$$\mathcal{O}_{\phi\mathcal{Z}} = \frac{1}{4}(v^2 + 2vh + h^2)C_{\mu\nu}C^{\mu\nu}$$

The annihilation processes we can construct using the effective vertices:



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"No point is more central than this, that empty space is not **empty**. It is the seat of the most violent **Physics**."

—John **Archibald Wheeler**