Neutron Star Cooling via Axion Emission by Nucleon-Nucleon Axion Bremsstrahlung

Avik Paul
Saha Institute of Nuclear Physics, HBNI, Kolkata

Collaborator
Debasish Majumdar
Saha Institute of Nuclear Physics, HBNI, Kolkata

XXIII DAE-BRNS HIGH ENERGY PHYSICS SYMPOSIUM 2018

December 13, 2018

Axion

- Axions are introduced to circumvent the strong CP problem that refers to the presence of CP violation term in QCD Lagrangian arising from the non-Abelian nature of QCD gauge symmetry.
- The axion which is a pseudo Nambu-Goldstone boson that arises out of the Peccei-Quinn (PQ) solution where an anomalous chiral symmetry $U(1)_A$ is introduced and which is spontaneously broken at the PQ energy scale.
- The PQ solution of strong CP problem results in the prediction of new particle namely axion.

Axion Models

- **KSVZ Model**: A new heavy quark field introduced in this model. In this model axions couple only to the photons and hadrons but do not couple to leptons at tree level.
- **DFSZ Model**: Two Higgs doublet extension of the SM. In this model axions couple to the charged leptons in addition to the hadrons and photons.

Cooling of neutron star through Axion emission

- **Neutron star**: A neutron star with a typical radius of 10-12 km and generally having a mass range of 1-2 solar mass ($\mathrm{M}_{\odot}$) is created as an aftermath of a massive supernova explosion.
- Neutron stars may undergo a cooling process through photon and neutrino emission.
- In this work we study the rate of neutron star cooling if it cools via axion emission in addition to the emission of photon and neutrino.
- Neutron star can emit Axions via nucleon nucleon bremsstrahlung processes.

Nucleon Nucleon Axion Bremsstrahlung

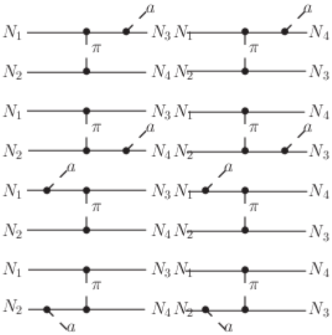

Figure 1: The direct and exchange diagrams for nucleon,nucleon Axion bremsstrahlung

- **Matrix Element for** $N + N \to N + N + a$: The interaction Lagrangian axion-nucleon interaction can be written as

$$\mathcal{L}_{int} = \frac{C_N}{2f_a}\overline{\psi}_N \gamma_\mu \gamma_5 \psi_N \partial^\mu a \quad (1)$$

- For the processes, $nn \to nn + a$ and $pp \to pp + a$, ("pure processes") the spin-summed squared matrix element is of the form

$$\sum_{spins} \mid \mathcal{M} \mid^2_{NN} = \frac{16(4\pi)^3 \alpha_\pi^2 \alpha_a}{3{m_N}^2} \Big[\Big(\frac{\mathbf{k}^2}{\mathbf{k}^2 + m_\pi^2} \Big)^2 + \Big(\frac{\mathbf{l}^2}{\mathbf{l}^2 + m_\pi^2} \Big)^2 + \frac{\mathbf{k}^2 \mathbf{l}^2 - 3\Big(\mathbf{k}.\mathbf{l}\Big)^2}{(\mathbf{k}^2 + m_\pi^2)(\mathbf{l}^2 + m_\pi^2)} \Big] \qquad (2)$$

where $\alpha_a \equiv (C_N m_N / f_a)^2 / 4\pi = g_{aN}^2 / 4\pi$, $g_{aN} = (C_N m_N / f_a)$, $(N = p, n)$, $\alpha_\pi \equiv (f 2 m_N / m_\pi)^2 / 4\pi \approx 17$, $f \approx 1.05$, $\mathbf{k} = \mathbf{p}_2 - \mathbf{p}_4$ and $\mathbf{l} = \mathbf{p}_2 - \mathbf{p}_3$.

- For the "mixed" process $np \to np + a$, spin-summed squared matrix element can be written as

$$\sum_{spins} \mid \mathcal{M} \mid^2_{np} = \frac{256\pi^2 \alpha_\pi^2}{3{m_N}^2} \frac{(g_{an} + g_{ap})^2}{4} \Big[2\Big(\frac{\mathbf{l}^2}{\mathbf{l}^2 + m_\pi^2} \Big)^2 - \frac{4\Big(\mathbf{k}.\mathbf{l}\Big)^2}{(\mathbf{k}^2 + m_\pi^2)(\mathbf{l}^2 + m_\pi^2)} \Big]$$
$$+ \frac{256\pi^2 \alpha_\pi^2}{3{m_N}^2} \frac{(g_{an}^2 + g_{ap}^2)}{2} \Big[\Big(\frac{\mathbf{k}^2}{\mathbf{k}^2 + m_\pi^2} \Big)^2 + 2\Big(\frac{\mathbf{l}^2}{\mathbf{l}^2 + m_\pi^2} \Big)^2 \qquad (3)$$
$$+2 \frac{\mathbf{k}^2 \mathbf{l}^2 - \Big(\mathbf{k}.\mathbf{l}\Big)^2}{(\mathbf{k}^2 + m_\pi^2)(\mathbf{l}^2 + m_\pi^2)} \Big].$$

- **Energy Loss Rate Expression**: The Axion energy-loss rate per unit volume is given by

$$Q_a = \int \frac{d^3\mathbf{K}_a}{2\omega_a(2\pi)^3}\omega_a \int \prod_{i=1}^{4} \frac{d^3\mathbf{P}_i}{2E_i(2\pi)^3} f_1 f_2 (1-f_3)(1-f_4) \\ \times (2\pi)^4 \delta^4(P_1+P_2-P_3-P_4-K_a) S \sum_{spins} |\mathcal{M}|^2_{NN} \tag{4}$$

- **Nondegenerate Limit**: $T \lesssim 10$ MeV, $\rho_B \approx 10^{12}$ g/cm^3
- Nucleon Fermi energy

$$\epsilon_{F,N} = \frac{p_{F,N}^2}{2m_N} = \frac{\left(3\pi^2 Y_N n_B\right)^{2/3}}{2m_N} \ll T \tag{5}$$

- **Degenerate Limit**: $T \approx 100$ KeV, $\rho_B \approx 2\rho_{nuc} \approx 5.6 \times 10^{14}$ g/cm^3

$$\epsilon_{F,N} \gg T \tag{6}$$

Nondegenerate Limit

- For simplicity, we first neglect the pion mass contribution in Eqs. 2 and 3. With this approximation the squared matrix element reduces to
- $$\sum_{spins} \mid \mathcal{M} \mid^2_{NN} = \frac{256\pi^2\alpha_\pi^2}{3{m_N}^2}\tilde{g}^2_{NN} \tag{7}$$

$$\tilde{g}^2_{NN} \equiv \begin{cases} g^2_{an}(3-\beta) & nn \to nn + a \\ g^2_{ap}(3-\beta) & pp \to pp + a \\ \left(\frac{g_{an}+g_{ap}}{2}\right)^2(2-4\beta/3) + \frac{g^2_{an}+g^2_{ap}}{2}(5-2\beta/3) & np \to np + a. \end{cases} \tag{8}$$

- Here, $\tilde{g}_{NN} = \tilde{C}_N m_N / f_a$ is an effective coupling and $\beta \equiv 3\langle(\hat{\mathbf{k}}.\hat{\mathbf{l}})^2\rangle = 1.3078$.
- The Axion-nucleon coupling can be found numerically

$$\mid \tilde{C}_N \mid = \begin{cases} 0.013 & nn \rightarrow nn + a \\ 0.442 & pp \rightarrow pp + a \\ 0.495 & np \rightarrow np + a. \end{cases} \tag{9}$$

- The total energy loss rate per unit volume as

$$\begin{aligned} Q_a^{ND} &= \frac{\xi(T)}{280} \frac{n_B^2 T^{7/2}}{m_N^{5/2} \pi^{7/2}} \left(Y_n^2 \sum_{spins} \mid \mathcal{M} \mid_{nn}^2 + Y_p^2 \sum_{spins} \mid \mathcal{M} \mid_{pp}^2 + Y_n Y_p \sum_{spins} \mid \mathcal{M} \mid_{np}^2 \right) \\ &= \frac{32}{105} \xi(T) \frac{\alpha_\pi^2 n_B^2 T^{7/2}}{m_N^{9/2} \pi^{3/2}} \left(Y_n^2 \tilde{g}_{nn}^2 + Y_p^2 \tilde{g}_{pp}^2 + 4 Y_n Y_p \tilde{g}_{np}^2 \right) \\ &= \frac{32}{105} \xi(T) \frac{\alpha_\pi^2 n_B^2 T^{7/2}}{m_N^{9/2} \pi^{3/2}} g_{ND}^2 \end{aligned} \tag{10}$$

- With $Y_p \approx 0.1$, $Y_n \approx 0.9$, $\xi(T) \approx 0.5$, using Eqs. 8, 9 and $m_a = 6\mu\mathrm{eV}\left(\frac{10^{12}\mathrm{GeV}}{\mathrm{f_a}}\right)$ relation

$$g_{ND} = 4.71 \times 10^{-8} \left(\frac{m_a}{eV}\right) \qquad \text{and} \qquad \mathrm{C_N^{ND}} = 0.30. \tag{11}$$

- In this nondegenerate case the Axion energy loss rate per unit volume reduces to

$$Q_a^{ND} = 2.90166 \times 10^{31}\ erg\ cm^{-3}\ yr^{-1}\ T_9^{3.5}\ \rho_{12}^2\ m_{\mathrm{eV}}^2 \tag{12}$$

where $m_{eV} \equiv m_a/eV$ and $T_9 \equiv T/10^9$ and $\rho_{12} \equiv \frac{\rho}{10^{12}\mathrm{gm/cm^3}} = \left(\frac{n_B}{m_N}\right)$.

Degenerate Limit

- In the degenerate limit, the squared matrix elements simplies to

$$\sum_{spins} \mid \mathcal{M} \mid^2_{NN} = \frac{256\pi^2\alpha_\pi^2}{{m_N}^2}\tilde{g}^2_{NN} \qquad \text{here } \beta = 0. \tag{13}$$

- With

$$\tilde{g}^2_{NN} \equiv \begin{cases} g^2_{an} & nn \to nn + a \\ g^2_{ap} & pp \to pp + a \\ \left(g^2_{an} + g^2_{ap} + (g_{an}g_{ap})/3\right) & np \to np + a. \end{cases} \tag{14}$$

-

$$\mid \tilde{C}_N \mid = \begin{cases} 0.01 & nn \to nn + a \\ 0.34 & pp \to pp + a \\ 0.338 & np \to np + a. \end{cases} \tag{15}$$

- In this case the integration in Eq. 4 can be simplified analytically without neglecting the pion masses. Here the pionic contribution $F(u)$ is given by

$$F(u) = 1 - \frac{5u}{6}\ \arctan(\frac{2}{u}) + \frac{u^2}{3(u^2+4)} + \frac{u^2}{6\sqrt{2u^2+4}} \times \arctan\left(\frac{2\sqrt{2u^2+4}}{u^2}\right) \tag{16}$$

where $u = m_\pi / p_{F,N}$.

- With $u \approx 0.32 Y_N^{-1/3}$ and consider $\rho_B \approx 2\rho_{nuc}$, $F(u)$ can be replaced by $F(Y_N)$ and the total emission rate is given as

$$\begin{aligned} Q_a^D &= \frac{31}{967680}\left(\frac{3n_B}{\pi}\right)^{1/3} T^6 \Big(Y_n^{1/3} F(Y_n) \sum_{spins} \mid \mathcal{M} \mid_{nn}^2 + Y_p^{1/3} F(Y_p) \sum_{spins} \mid \mathcal{M} \mid_{pp}^2 \\ &\quad + 4Y_{np}^{1/3} F(Y_{np}) \sum_{spins} \mid \mathcal{M} \mid_{np}^2 \Big) \\ &= \frac{31\pi^{5/3}(3n_B)^{1/3}\alpha_\pi^2 T^6}{3780 m_N^2}\left(Y_n^{1/3} F(Y_n)\tilde{g}_{an}^2 + Y_p^{1/3} F(Y_p)\tilde{g}_{ap}^2 + Y_{np}^{1/3} F(Y_{np})\tilde{g}_{np}^2 \right) \\ &= \frac{31\pi^{5/3}(3n_B)^{1/3}\alpha_\pi^2 T^6}{3780 m_N^2} g_D^2 \ , \end{aligned} \tag{17}$$

- The effective nucleon fraction for the mixed processes are given by

$$Y_{np}^{1/3} = \frac{1}{2\sqrt{2}}(Y_n^{2/3} + Y_p^{2/3})^{1/2}\Big[2 - \frac{\mid Y_n^{2/3} - Y_p^{2/3} \mid}{Y_n^{2/3} + Y_p^{2/3}}\Big] \tag{18}$$

- Using the values of nucleon number fractions $Y_p = 0.01$, $Y_n = 0.99$ and $Y_{np} = 0.06$, we can calculate $F(Y_n) \approx 0.64, F(Y_p) \approx 0.12$ and $F(Y_{np}) \approx 0.31$ for the pure and mixed processes. Also using the above values one can attain the effective coupling constants g_D and C_N^D for degenerate case as

$$g_D = 2.04 \times 10^{-8} \left(\frac{m_a}{eV}\right) \qquad \text{and} \qquad \mathrm{C_N^D} = 0.13. \tag{19}$$

- In this Degenerate case using Eqs. 17 and 19 the Axion emission rate per unit volume is obtained as

$$Q_a^D = 4.84244 \times 10^{30}\ erg\ cm^{-3}\ yr^{-1}\ T_9^6\ m_{\mathrm{eV}}^2 \left(\frac{\rho_{NS}}{\rho_{nuc}}\right)^{1/3}. \tag{20}$$

Neutron star cooling

- In Newtonian formulation the energy balance equation for the neutron star is given by

$$\frac{dE_{th}}{dt} = C_v \frac{dT}{dt} = -L_\nu(T) - L_a(T) - L_\gamma(T_e) + H(T). \tag{21}$$

- It is assumed here that $H(T) = 0$.
- The photon luminosity is given by the Stefan-Boltzmann law

$$L_\gamma = 4\pi\,\sigma\,R^2\,T_e^4 = S\,T^{2+4\alpha} \;\text{ using }\; \mathrm{T_e} \propto \mathrm{T}^{0.5+\alpha}\;(\alpha \ll 1). \tag{22}$$

- Various neutrino processes are involved in the cooling of neutron stars. Dominant neutrino emitting processes are direct Urca processes and modified Urca processes.
- The other neutrino emitting processes are electron-positron pair annihilation, electron synchrotron, photoneutrino emission, electron-nucleus bremsstrahlung, cooper pairing of neutrons, neutron-neutron bremsstrahlung, neutron-nucleus bremsstrahlung.

Results

- We adopt APR equation of state for our work.
- We use the NSCool code for calculating the neutrino and axion luminosities.

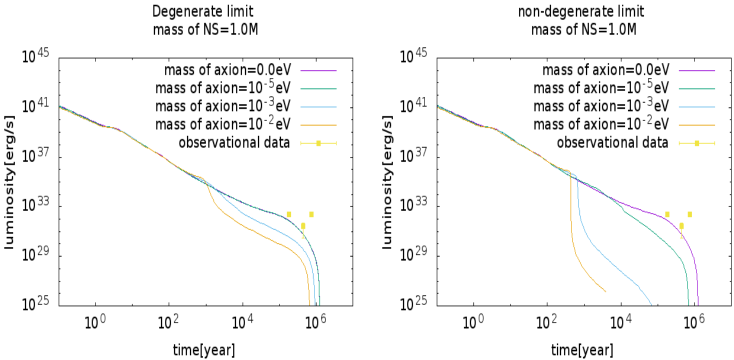

Figure 2: Luminosity (L) vs time (t) graph for both degenerate (left panel) and non-degenerate (right panel) limits with $\mathrm{M} = 1.0\mathrm{M}_{\odot}$ and $m_a = 0\mathrm{eV}, 10^{-5}\mathrm{eV}, 10^{-3}\mathrm{eV}, 10^{-2}\mathrm{eV}$ (from top to bottom). The observational data for three pulsars PSR B0656+14, Geminga and PSR B1055-52 are shown by dots with error bars from left to right.

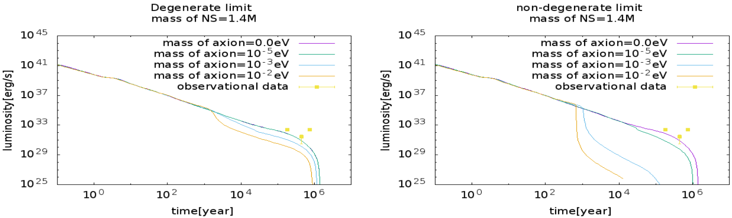

Figure 3: Same as in Figure 2 but for $\mathrm{M} = 1.4\mathrm{M}_{\odot}$

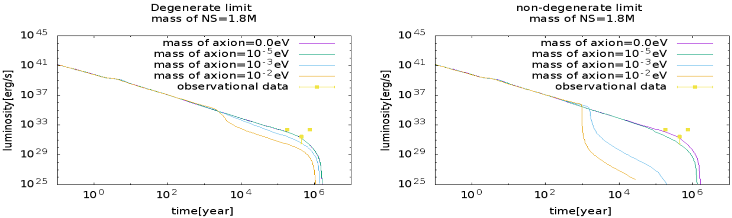

Figure 4: Same as in Figure 2 but for $\mathrm{M} = 1.8\mathrm{M}_{\odot}$

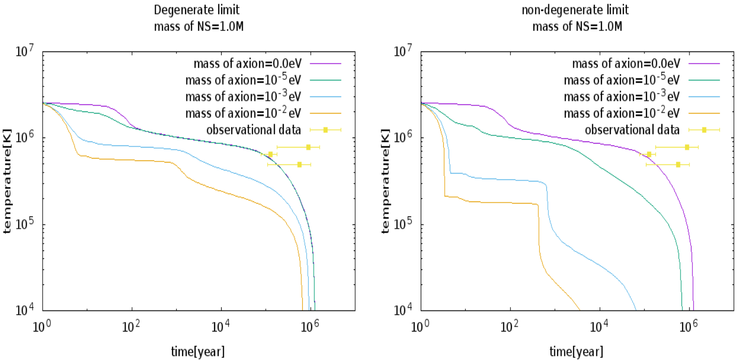

Figure 5: Surface temperature (T) vs time (t) graph for both degenerate (left panel) and non-degenerate (right panel) limits with $\mathrm{M} = 1.0\mathrm{M}_\odot$ and $m_a = 0\mathrm{eV}, 10^{-5}\mathrm{eV}, 10^{-3}\mathrm{eV}, 10^{-2}\mathrm{eV}$ (from top to bottom). The observational data for three pulsars PSR B0656+14, Geminga and PSR B1055-52 are shown by dots with error bars from left to right.

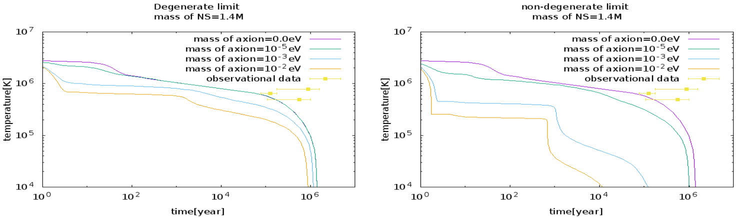

Figure 6: Same as in Figure 5 but for $\mathrm{M} = 1.4\mathrm{M}_{\odot}$

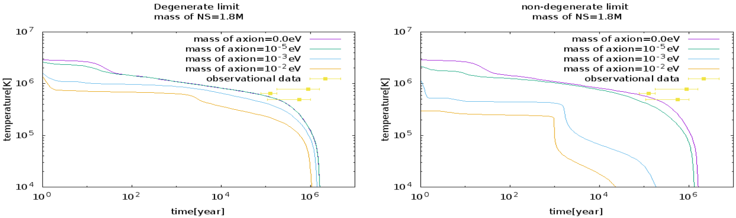

Figure 7: Same as in Figure 5 but for $\mathrm{M} = 1.8\mathrm{M}_{\odot}$

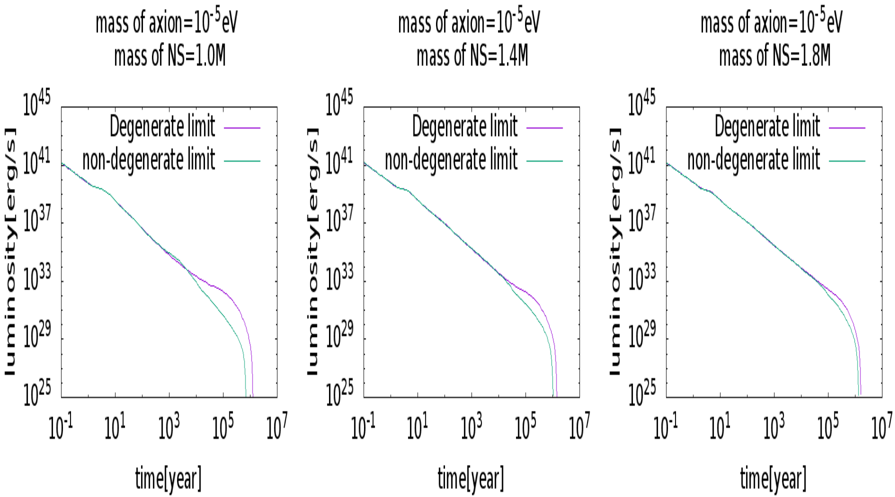

Figure 8: Comparison between degenerate and non-degenerate limits for L vs t plot with m_a=10^{-5}eV and $\mathrm{M} = 1.0\mathrm{M}_\odot$, $\mathrm{M} = 1.4\mathrm{M}_\odot$, $\mathrm{M} = 1.8\mathrm{M}_\odot$ (from left to right).

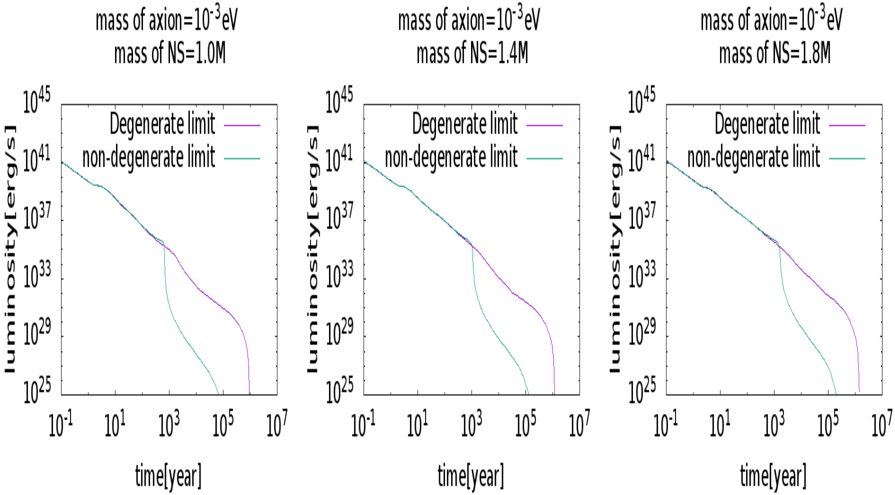

Figure 9: Same as in Figure 8 but for $m_a = 10^{-3}\mathrm{eV}$.

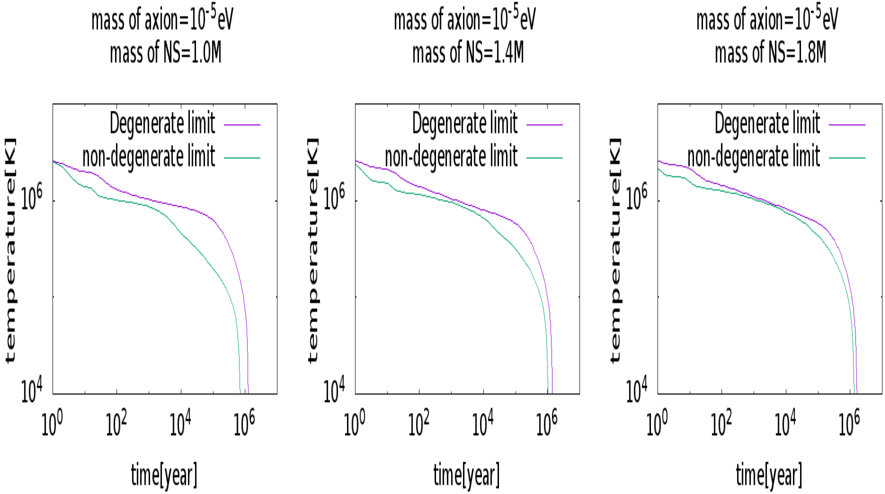

Figure 10: Comparison between degenerate and non-degenerate limits for T vs t plot with m_a=10^{-5}eV and $\mathrm{M} = 1.0\mathrm{M}_\odot$, $\mathrm{M} = 1.4\mathrm{M}_\odot$, $\mathrm{M} = 1.8\mathrm{M}_\odot$ (from left to right).

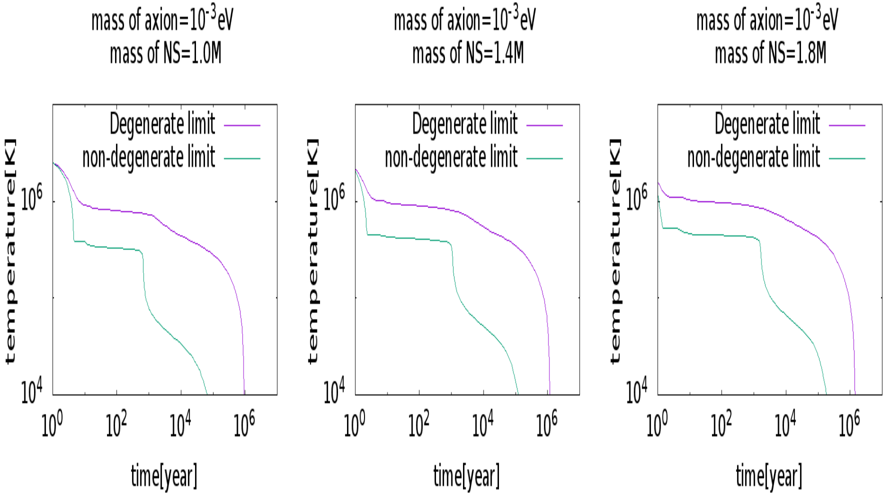

Figure 11: Same as in Figure 10 but for $m_a = 10^{-3}$eV.

Conclusions

- From these analyses we derive a bound on axion masses $m_a \leq 10^{-3}\mathrm{eV}$ which implies that the axion decay constant $f_a \geq 0.6 \times 10^{10}\mathrm{GeV}$.
- We also find that the cooling patterns for degenerate and non-degenerate cases are different. This difference increases with the mass of the emitted axion.

Thank You

Summary and Discussions

- In this work we have explored the effect of axion emission on the cooling of neutron stars. For the axion emission we consider nucleon-nucleon axion bremsstrahlung process. We have made our analysis for the stars considering degenerate and non-degenerate limits.
- We demonstrate our results by calculating the variations of luminosity and temperature of neutron stars with time.
- We adopt APR equation of state for our work.
- We have demonstrated this by considering three axion masses namely $10^{-5}\mathrm{eV}$, $10^{-3}\mathrm{eV}$, $10^{-2}\mathrm{eV}$ and the neutron star masses of $1.0 M_\odot$, $1.4 M_\odot$, $1.8 M_\odot$.
- For comparison with observational results we used luminosity and temperature of three pulsars namely PSR B0656+14, Geminga and PSR B1055-52.
- Finally we derive a bound on axion masses $m_a \leq 10^{-3}\mathrm{eV}$ which implies that the axion decay constant $f_a \geq 0.6 \times 10^{10}\mathrm{GeV}$.

QCD Lagrangian

- The most general renormalisable QCD Lagrangian is

$$\mathcal{L}_{QCD} = -\frac{1}{4} G^{a\dagger}_{\mu\nu} G^{a}_{\mu\nu} + \sum_{A} \overline{q}_A (i\gamma^\mu D_\mu - m_A) q_A \ . \tag{23}$$

- The field strength tensor is given by

$$G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu - g f_{abc} G^b_\mu G^c_\nu \tag{24}$$

- The covariant derivative has the form

$$D_\mu = \partial_\mu - ig\lambda_a G^a_\mu \tag{25}$$

- One can add a gauge invariant and renormalisable term in the Lagrangian as

$$(constant) \times \widetilde{G}^{a\dagger}_{\mu\nu} \widetilde{G}^{\mu\nu}_a = (constant) \times \widetilde{G}^{\overline{a}}_{\mu\nu} \widetilde{G}^{\mu\nu}_a \ , \tag{26}$$

where $\widetilde{G}^a_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\lambda\rho} G^{\lambda\rho}_a$ is the dual to the field-strength tensor.

- It can be shown that

$$\widetilde{G}^{\overline{a}}_{\mu\nu} \widetilde{G}^{\mu\nu}_a = G^{\overline{a}}_{\mu\nu} G^{\mu\nu}_a = G^{a\dagger}_{\mu\nu} G^{\mu\nu}_a \ . \tag{27}$$

- It is also possible to consider a term like

$$\mathcal{L}_{new} = (constant) \times G^{\overline{a}}_{\mu\nu} \widetilde{G}^{\mu\nu}_{a} \tag{28}$$

- It can be shown that

$$G^{\overline{a}}_{\mu\nu} \widetilde{G}^{\mu\nu}_{a} = \partial_\mu K_\mu \ , \tag{29}$$

where,

$$K^\mu = 2\epsilon^{\mu\nu\lambda\rho}(G^a_\nu \partial_\lambda G^{\overline{a}}_\rho - \frac{1}{3} g f_{bc\overline{a}} G^a_\nu G^b_\lambda G^c_\rho) \ , \tag{30}$$

or,

$$K^\mu = \epsilon^{\mu\nu\lambda\rho} G^a_\nu (G^{\overline{a}}_{\lambda\rho} + \frac{1}{3} g f_{bc\overline{a}} G^b_\lambda G^c_\rho) \ . \tag{31}$$

- Taking surface integral of Eq. 29, and using Eq. 31, we get

$$\int d^4x \ G^{a\dagger}_{\mu\nu} \widetilde{G}^{\mu\nu}_{a} = \int dS_\mu K^\mu \tag{32}$$

or,

$$\int d^4x \ G^{a\dagger}_{\mu\nu} \widetilde{G}^{\mu\nu}_{a} = \frac{1}{3} g \epsilon^{\mu\nu\lambda\rho} f_{bc\overline{a}} \int dS_\mu G^a_\nu G^b_\lambda G^c_\rho = \frac{32\pi^2 n}{g^2} \ . \tag{33}$$

QCD Vacuum

- A gauge invariant QCD vacuum state can be represented as

$$|\,\theta\rangle = \sum_{n=-\infty}^{\infty} e^{-in\theta}\,|\,n\rangle\ . \tag{34}$$

- The θ paramerer of Eq. 34 is an eigenstate of the Hamiltonian and it can be proved by considering a matrix

$$\big\langle\theta'\,|\,e^{-iHt}\,|\,\theta\big\rangle = \sum_{n,n'} e^{i(n'\theta'-n\theta)}\big\langle n'\,|\,e^{-iHt}\,|\,n\big\rangle\ . \tag{35}$$

- Putting $n' = n + k$ and simplifying

$$\begin{aligned}\big\langle\theta'\,|\,e^{-iHt}\,|\,\theta\big\rangle &= \sum_{n,k} e^{in(\theta'-\theta)}e^{ik\theta'}\big\langle k\,|\,e^{-iHt}\,|\,0\big\rangle \\ &= \delta(\theta'-\theta)\sum_{k} e^{ik\theta}\big\langle k\,|\,e^{-iHt}\,|\,0\big\rangle.\end{aligned} \tag{36}$$

- Now, writing the matrix element of the time-evolution operator

$$\left\langle k \mid e^{-iHt} \mid 0 \right\rangle = \int \mathcal{D}\Psi \int \mathcal{D}\overline{\Psi} \int \mathcal{D}G^{a}_{\mu} \ \exp \ \left(i \int d^4x \ \mathcal{L}_{\mathcal{QCD}} \right) \tag{37}$$

- Using Eqs. 33 and 37 into Eq., we obtain

$$\left\langle \theta' \mid e^{-iHt} \mid \theta \right\rangle = \int \mathcal{D}\Psi \int \mathcal{D}\overline{\Psi} \int \mathcal{D}G^{a}_{\mu} \ exp \ \left(i \int d^4x (\mathcal{L}_{\mathcal{QCD}} + \frac{\theta g^2}{32\pi^2} G^{a\dagger}_{\mu\nu} \widetilde{G}^{\mu\nu}_{a} \right) \tag{38}$$

- The effective value of the θ parameter is given by

$$\theta_{eff} = \overline{\theta} = \theta_{QCD} + arg \ det \ (M) \tag{39}$$

where M is quark Mass Matrix.

- So, finally we get an extra vacuum term that adds to the QCD Lagrangian given by

$$\mathcal{L}_{new} = \frac{\overline{\theta}_{QCD} g^2}{32\pi^2} G^{a\dagger}_{\mu\nu} \widetilde{G}^{\mu\nu}_{a}. \tag{40}$$

The Strong CP Problem

- The $\overline{\theta}$ term violates P and CP invariance, so the QCD Lagrangian contains a CP violating term.
- An electric dipole moment for the neutron is the observed consequence of QCD or Strong CP violation. Neutron electric dipole moment is expressed as

$$\mid d_n \mid \sim 10^{-16} \, \overline{\theta} \, e \text{ cm} \,, \tag{41}$$

- The experimental limit is $\mid d_n \mid < 6.3 \times 10^{-26} \; e \text{ cm}$ and from this the upper bound for $\overline{\theta}$ is obtained as $\mid \overline{\theta} \mid \lesssim 10^{-9}$.
- There is no natural reason to expect dimensionless parameter $\overline{\theta}$ to be this small. CP violation occurs in the Standard Model by considering the quark masses to be complex and thus the natural value of $\overline{\theta}$ is expected to be of order 1 (between zero to 2π).
- This is the **Strong CP Problem**.

Peccei-Quinn Solution and the Axion

- The Peccei-Quinn (PQ) solution of strong CP problem results in the prediction of a new particle namely Axion.
- In the PQ mechanism, an additional scalar S is introduced into the theory which respects an additional $U(1)$ ($U(1)_{PQ}$) symmetry.

$$\mathcal{L} = \partial_\mu S^* \partial^\mu S - V(S) \tag{42}$$

- where $S(x) = \frac{v_a}{\sqrt{2}}\, e^{ia(x)/v_a}$ and $v_a = \langle 0 \mid S \mid 0 \rangle$.
- Under this symmetry the Axion field "$a(x)$" translates as $a \to a + \alpha v_a$. The Lagrangian is now written as

$$\begin{aligned}\mathcal{L}_{QCD,tot} &= \mathcal{L}_{QCD} + \mathcal{L}_{\overline{\theta}} + \mathcal{L}_a \\ &= \mathcal{L}_{QCD} + \frac{\overline{\theta} g^2}{32\pi^2} G^a_{\mu\nu} \widetilde{G}^{\mu\nu}_a + \frac{1}{2}\partial_\mu a \partial^\mu a + \frac{a}{v_a}\frac{g^2}{32\pi^2} G^a_{\mu\nu}\widetilde{G}^{\mu\nu}_a\end{aligned} \tag{43}$$

- Now the effective potential is of the form

$$V_{eff} \sim \cos\left(\overline{\theta} + \frac{a}{v_a}\right). \tag{44}$$

- Minimising this potential with respect to $\langle a \rangle$ gives the PQ solution as

$$\langle a \rangle = -v_a \overline{\theta}. \tag{45}$$

- Due to the periodicity of $\overline{\theta}$, $\langle a \rangle$ is periodic with $\langle a \rangle = 2\pi k v_a$
- Now, from Eq. 44 the $\overline{\theta}$ term can be eliminated by substituting the physical Axion field as $a_{phys} = a - \langle a \rangle = a + v_a \overline{\theta}$.
- Therefore, $\frac{g^2}{32\pi^2}(\overline{\theta} + \frac{a}{v_a}) = \frac{g^2}{32\pi^2}\frac{a_{phys}}{v_a}$.
- The effective potential for the Axion field has a minimum at $\langle a \rangle = -v_a \overline{\theta}$.

$$\left\langle \frac{\partial V_{eff}}{\partial a} \right\rangle = \frac{1}{v_a}\frac{g^2}{32\pi^2}\left\langle G^a_{\mu\nu}\widetilde{G}^{\mu\nu}_a \right\rangle\Big|_{\langle a \rangle = -v_a\overline{\theta}} = 0 \tag{46}$$

where $\langle a \rangle = \langle 0 \mid a \mid 0 \rangle$.

- Since at the minimum, the $\overline{\theta}$ term is cancelled out, this gives a dynamical solution to the strong CP problem.
- Expanding V_{eff} at the minimum gives the Axion a mass

$$\left\langle \frac{\partial^2 V_{eff}}{\partial a^2} \right\rangle = m_a^2 = \frac{1}{v_a}\frac{g^2}{32\pi^2}\frac{\partial}{\partial a}\left\langle G^a_{\mu\nu}\widetilde{G}^{\mu\nu}_a \right\rangle\Big|_{\langle a \rangle = -v_a\overline{\theta}}. \tag{47}$$

Solving CP Problem

- In the PQ solution, $\overline{\theta}$ is promoted from a parameter to a dynamical variable.
- This varible relaxes to the minimum of its potential and hence is small.
- Non-perturbative effects which make QCD, $\overline{\theta}$ dependent, results in a potential for the Axion.
- Although initially massless, this potential causes the Axion to acquire a mass and relax to the CP-conserving minimum.
- Hence the strong CP problem is solved.

Axion Models

- **The Visible Axion Model**: Predicts an Axion with a mass of the electroweak (EW) scale. The model was quickly ruled out, because the Axion did not show up in experimental data and is replaced by the "invisible" Axion models, which predict a very light Axion.
- **The Invisible Axion Model**: The scale of f_a is not taken to be that of weak scale v. The dynamical adjustment of the strong CP angle, $\overline{\theta} \to 0$ works for any scale for f_a. If $f_a >> v$, the resulting Axions are very light ($m_a \sim \frac{1}{f_a}$), very weakly coupled (coupling $\sim \frac{1}{f_a}$) and very long lived ($\tau(a \to 2\gamma) \sim f_a^5$). Thus these Axions are, apparently, invisible.
- There are two types of invisible axion models – the KSVZ model and the DFSZ model. In the KSVZ model, the axion couples only to the photons and hadrons, while in the DFSZ model the axion couples to the charged leptons as well.

KSVZ Model

- In the KSVZ model, the symmetry breaking scale is much above the electroweak scale.
- A heavy Dirac quark field Q −a colour triplet in the fundamental representation with no bare mass and a new complex scalar singlet S field are added to the SM.
- The Lagrangian in the KSVZ model is

$$\mathcal{L}_{KSVZ} = -\frac{1}{4}G^a_{\mu\nu}G^{\mu\nu}_a + \frac{\overline{\theta}g^2}{32\pi^2}G^a_{\mu\nu}\widetilde{G}^{\mu\nu}_a + \overline{Q}i\gamma^\mu\partial\mu Q + gG^a_\mu\overline{Q}\gamma^\mu\lambda_a Q + \frac{1}{2}\partial_\mu S^*\partial^\mu S - \lambda_S(S^*S - v_a^2)^2 - \lambda_{QS}(Q_L^\dagger SQ_R + h.c). \quad (48)$$

where $v_a = \langle 0 \mid S \mid 0 \rangle$

- This Lagrangian is invariant at the classical level under the global chiral $U(1)_{PQ}$ symmetry

$$Q_L \to e^{i\alpha/2}Q_L, \quad Q_R \to e^{-i\alpha/2}Q_R, \quad S \to e^{i\alpha}S \quad (49)$$

- The scalar singlet S can be written in polar form as

$$S(x) = \frac{v_a}{\sqrt{2}}\, e^{ia(x)/v_a} \quad (50)$$

- For the Axion field the PQ-transformation given in Eq. (50) corresponds to the shift $a \to a + \alpha v_a$.
- After the spontaneous breaking of $U(1)_{PQ}$ the Lagrangian is now

$$\begin{aligned}\mathcal{L}'_{KSVZ} = -\frac{1}{4}G^a_{\mu\nu}G^{\mu\nu}_a + \frac{\overline{\theta}g^2}{32\pi^2}G^a_{\mu\nu}\widetilde{G}^{\mu\nu}_a + \overline{Q}i\gamma^\mu\partial\mu Q + gG^a_\mu\overline{Q}\gamma^\mu\lambda_a Q \\ +\frac{1}{2}\partial_\mu a\partial^\mu a - v_a\lambda_{QS}(Q^\dagger_L e^{ia(x)/v_a}Q_R + Q^\dagger_R e^{-ia(x)/v_a}Q_L)\end{aligned} \tag{51}$$

- Redefine the chiral heavy quark field as

$$Q_L \to e^{ia/2v_a}Q_L, \qquad Q_R \to e^{-ia/2v_a}Q_R \, . \tag{52}$$

- Under the above transformations, the spontaneous breaking of the PQ symmetry produces an effective mass term for the quark field as $m_Q \sim v_a\lambda_{QS}$.
- The above transformations introduce two additional terms in the Lagrangian

$$\delta\mathcal{L} = \frac{\partial_\mu a}{2v_a}\overline{Q}\gamma_5\gamma^\mu Q + \frac{g^2}{32\pi^2}\frac{a}{v_a}G^a_{\mu\nu}\widetilde{G}^{\mu\nu}_a \tag{53}$$

- At energies much lower than m_Q, the Lagrangian is

$$\mathcal{L}_{KSVZ} = -\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a + (\bar{\theta} + \frac{a}{v_a}) \frac{g^2}{32\pi^2} G^a_{\mu\nu} \widetilde{G}^{\mu\nu}_a + \frac{1}{2} \partial_\mu a \partial^\mu a \ . \qquad (54)$$

- In the low-energy state the two-photon coupling for the Axion is given by

$$\mathcal{L}_{a\gamma\gamma} = -\frac{g_{a\gamma\gamma}}{4} a G \widetilde{G}, \qquad (55)$$

$G_{\mu\nu}$ being the photon field strength tensor and $\tilde{G}^{\mu\nu}$ its dual.

- The coupling constant $g_{a\gamma\gamma}$ is

$$g_{a\gamma\gamma} = \frac{\alpha C_\gamma}{2\pi f_a} \qquad (56)$$

DFSZ Model

- In this model SM is extended by two Higgs doublets, H_u and H_d and a Higgs-like scalar singlet S.
- The most general potential for two Higgs doublets and a singlet Higgs fields can be written as

$$V(H_u, H_d, S) = \lambda_u(H_u^\dagger H_u - v_u^2)^2 + \lambda_d(H_d^\dagger H_d - v_d^2)^2 + \lambda_S(S^* S - v_S^2)^2 + \lambda_{uu}(H_u^\dagger H_u)(H_d^\dagger H_d) + \lambda_{ud}(H_u^\dagger H_d)(H_d^\dagger H_u) + [\lambda_{uS}(H_u^\dagger H_u) + \lambda_{dS}(H_d^\dagger H_d)]S^* S + \lambda[(H_u^\dagger H_d)S^2 + h.c.] \quad (57)$$

where λ_i's are dimensionless parameters.
- The above Higgs-potential Eq. (36) is invariant under the transformation

$$H_u \to e^{iX_1} H_u, \quad H_d \to e^{iX_2} H_d, \quad S \to e^{i(X_1 - X_2)/2} S. \quad (58)$$

- The corresponding Yukawa Lagrangian is

$$\mathcal{L}_Y = \Gamma^u_{ij} \overline{q}_{Li} H_u u_{Rj} + \Gamma^d_{ij} \overline{q}_{Li} H_d d_{Rj} + \Gamma^l_{ij} \overline{L}_{Li} H_d l_{Rj} + h.c. \quad (59)$$

- Yukawa Lagrangian remains invariant under the transformation

$$u_R \to e^{-iX_1} u_R, \quad d_R \to e^{-iX_2} d_R, \quad l_R \to e^{-iX_2} l_R. \quad (60)$$

The left-handed fields remain unchanged.

- The vacuum expectation values (VEV) of the Higgs fields can be written as

$$\langle H_u \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_u \end{pmatrix} \qquad \langle H_d \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_d \end{pmatrix} \qquad \langle S \rangle = v_a. \tag{61}$$

- After the electroweak phase transition at the phenomenologically required scale $v_{EW} = \sqrt{v_u^2 + v_d^2} << v_a$, there exist additional physical neutral Higgs fields and orthogonal Nambu-Goldstone bosons as the phase degrees of freedom.
- In the range between E_{EW} and Λ_{QCD} the Lagrangian is

$$\mathcal{L}_a = \frac{1}{2}\partial_\mu a \partial^\mu a + \frac{a}{f_a}\frac{g^2}{32\pi^2}G\widetilde{G} - \frac{g_{a\gamma\gamma}}{4}aF\widetilde{F} - \frac{\partial_\mu a}{2f_a}\sum_f C_f \overline{\psi}_f \gamma_5 \gamma^\mu \psi_f \tag{62}$$

- Below Λ_{QCD} gluon and quarks confine, so that one can write an effective Lagrangian including the couplings to nucleons and mesons

$$\mathcal{L}_a = \frac{1}{2}\partial_\mu a \partial^\mu a + m_a^2 f_a^2[1 - \cos(\frac{a}{f_a})] - \frac{g_{a\gamma\gamma}}{4}aF\widetilde{F} - \frac{\partial_\mu a}{2f_a}\sum_f C_f \overline{\psi}_f \gamma_5 \gamma^\mu \psi_f + \mathcal{L}_{a\pi} \tag{63}$$

- Here $m_a^2 f_a^2[1 - \cos(\frac{a}{f_a})]$ is an effective periodic potential like Eq. (23).