INFLATION WITH AN ANTISYMMETRIC TENSOR FIELD

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Outline

Introduction

- Inflation is a period of accelarated expansion of the observable universe before the formation of CMB.
- Inflation explains the *Horizon Problem* and the *Flatness Problem* for which the Standard Bigbang Models have no explanation .
- Ordinary matter and Radiation can not be the cause for inflation as they don't meet the requirement of negative pressure for inflation. A new field could be introduced or the gravity can be modified to describe inflation.

- *Scalar field model* : Most of them (who have simple form of potential) don't fit into the observational constraints.
- *Vector field model* : The longitudinal mode of perturbation suffers from ghost instability.
- *major requirements for a stable inflationary model* :
  a. A de-Sitter space solution should exist
  b. Slow roll of the field must be established.
  c. The model should be free from instabilities.
  d. It should meet the observational requirements.

The Model with nonminimal curvature couplings

- *Action*: $S = \int d^4x\sqrt{-g}(\mathscr{L}_M + \mathscr{L}_{NM})$

$$\mathscr{L}_M = \frac{R}{2\kappa} - \frac{1}{2\kappa}H_{\lambda\mu\nu}(B)H^{\lambda\mu\nu}(B) - V(B)$$

$$\mathscr{L}_{NM} = \frac{\xi}{2\kappa}B^{\mu\nu}B_{\mu\nu}R + \frac{\zeta}{2\kappa}B^{\lambda\nu}B^{\mu}_{\nu}R_{\lambda\mu} + \frac{\gamma}{2\kappa}B^{\sigma\lambda}B^{\mu\nu}R_{\sigma\lambda\mu\nu}$$

- $H_{\lambda\mu\nu}(B) = \nabla_\lambda B_{\mu\nu} + \nabla_\mu B_{\nu\lambda} + \nabla_\nu B_{\lambda\mu}$ is the kinetic term which stays invariant under the gauge transformation

$$B_{\mu\nu} \to B_{\mu\nu} + \partial_\mu\Lambda_\nu - \partial_\nu\Lambda_\mu \ , \ \Lambda_\mu \to \Lambda_\mu + \partial_\mu\Sigma$$

- *Antisymmetric Tensor Field* : $(B_{\mu\nu} = -B_{\nu\mu})$

$$B_{0j} = -\Sigma^j \quad , \quad B_{jk} = \varepsilon_{jkl}\,\Xi^l$$

- *Einstein's Equation*: $G_{\mu\nu} = \kappa T_{\mu\nu} = \kappa(T^M_{\mu\nu} + T^{NM}_{\mu\nu})$
- *Energy Momentum Tensor of the minimally coupled action* :

$$T^M_{\mu\nu} = \frac{1}{2} H^{\alpha\beta}_{\mu} H_{\nu\alpha\beta} + m^2 B^{\alpha}_{\mu} B_{\alpha\nu} - g_{\mu\nu}\left(\frac{1}{12} H_{\alpha\beta\gamma} H^{\alpha\beta\gamma} + \frac{1}{4} m^2 B_{\alpha\beta} B^{\alpha\beta}\right)$$

- *Energy Momentum Tensor due to the nonminimal coupling term* :

$$T^{NM}_{\mu\nu} = \frac{\xi}{\kappa}\left[(\nabla_\mu \nabla_\nu - g_{\mu\nu} \nabla^\lambda \nabla_\lambda - G_{\mu\nu})(B_{\alpha\beta} B^{\alpha\beta}) - 2RB^{\alpha}_{\mu} B_{\alpha\nu}\right]$$

- *field equation*:

$$\nabla_\lambda H^{\lambda\mu\nu} + (\frac{2\xi}{\kappa} - m^2) B^{\mu\nu} = 0$$

Background Dynamics

- *Choice 1* : Background metric is the FLRW metric.

$$g_{00} = -1 \ , \ g_{oi} = 0 \ , \ g_{ij} = a(t)^2 \delta_{ij}$$

- *Choice 2* : $\Sigma^j = 0 \ , \ \Xi^l = B(t) \ , \ l = 1, 2, 3$

$$B_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & B(t) & -B(t) \\ 0 & -B(t) & 0 & B(t) \\ 0 & B(t) & -B(t) & 0 \end{pmatrix}$$

- *Choice 3* : $V(B) = \frac{1}{4} m^2 B_{\mu\nu} B^{\mu\nu}$
- *Choice 4* : $B(t) = a(t)^2 \phi(t)$ (to get e.o.m resembling Scalar field model)

- *Equation of motion in the chosen background* :

$$G_{00} = \kappa T_{00} \implies H^2 + 6\xi(2H\phi\dot{\phi} + H^2\phi^2) = \frac{\kappa}{2}\left[(\dot{\phi} + 2H\phi)^2 + m^2\phi^2\right]$$

$$G_{ij} = \kappa T_{ij}^M \implies$$

for $i = j$:

$$2\dot{H} + 3H^2 + 6\xi(2\phi\ddot{\phi} + 2\dot{\phi}^2 - 2\dot{H}\phi^2 - 5H^2\phi^2 + 4H\phi\dot{\phi}) = \frac{\kappa}{2}\left[(\dot{\phi} + 2H\phi)^2 - m^2\phi^2\right]$$

for $i \neq j$:

$$\frac{\kappa}{2}\left[(\dot{\phi} + 2H\phi)^2 - m^2\phi^2\right] = -6\xi(\dot{H} + 2H^2)\phi^2$$

- The $B_{\mu\nu}$ field equation is not an independent equation in the chosen background.

- *Minimal Model* $(\xi = 0)$: $\frac{\ddot{a}}{a} = -\frac{H^2}{2} < 0 \Rightarrow$ No acceleration $\Rightarrow$ No Inflation
- *Nonminimal Model* : In exact de-Sitter limit $\dot{H} = \dot{\phi} = 0$

$$\phi_0^2 = \frac{1}{6\xi} \qquad H_0^2 = \frac{\kappa m^2}{4(6\xi - \kappa)}$$

- *Condition for exact de-Sitter Inflation* : $\boxed{\xi > \frac{\kappa}{6}}$

Slow Roll Analysis

- *slow roll parameter 1* : $\varepsilon = -\frac{\dot{H}}{H^2}$, $\frac{\ddot{a}}{a} = H^2(1-\varepsilon)$
- *slow roll parameter 2(model dependent)* : $\delta = \frac{\dot{\phi}}{H\phi}$
- *equation of motion in terms of slow roll parameter* :

$$3 - 18\xi\phi^2 - 2\varepsilon + 12\xi\phi^2\left[\frac{\dot{\delta}}{H} + 2\delta^2 + (2-\varepsilon)\delta + \frac{\varepsilon}{2}\right] = 0$$

$$\varepsilon = \delta\left[\frac{(6\xi - 2\kappa)\phi^2}{1 + (6\xi - 2\kappa)\phi^2 + \delta(12\xi - 2\kappa)\phi^2} - \frac{\phi V_\phi}{2V}\right]$$

- *conditions for slow roll* : $\dot{\phi}^2 < V$ and $V_\phi << V$
- $\varepsilon \approx \dfrac{\delta}{(6\xi - 2\kappa)^{-1}\phi^{-2} + 1} \sim \delta$, *small* $\varepsilon \implies$ *small* δ
- *number of e-folds* : $N = \int_{t_i}^{t} H dt = \int_{\phi_i}^{\phi} d\phi \frac{H}{\dot{\phi}} = \frac{1}{\delta} ln\left(\frac{\phi}{\phi_i}\right)$
- $N > 60$ is required for inflation. Smallness of δ ensures sufficient duration of slow-roll inflation.

Stability analysis of Perturbation

- Leaving the metric unperturbed, only the antisymmetric tensor field is perturbed .

$$B_{\mu\nu} \rightarrow B_{\mu\nu} + \delta B_{\mu\nu}$$

- $\delta B_{0i} = -E_i \qquad \delta B_{ij} = \varepsilon_{ijk} M_k$

$$S_2 = \int d^4x \Big[\frac{1}{2a} \Big(\dot{\vec{M}}.\dot{\vec{M}} + 2\dot{\vec{M}}.(\vec{\nabla} \times \vec{E}) + (\vec{\nabla} \times \vec{E}).(\vec{\nabla} \times \vec{E}) \Big) - \frac{1}{2a^3} (\vec{\nabla}.\vec{M})^2 + \left(\frac{m^2}{2} - \frac{6\xi}{\kappa} (\dot{H} + 2H^2) \right) (a\vec{E}.\vec{E} - \frac{\vec{M}.\vec{M}}{a}) \Big]$$

- *transforming to momentum space, choosing* $\vec{K} = K\hat{z}$*, replacing the nondynamical mode with dynamical modes*

- The kinetic part of the action is given by

$$(S_{eff})_{Kin} = \int dt\, d^3k \left[\frac{N}{2a(N - \kappa k^2)} (\dot{\tilde{M}}_x^\dagger \dot{\tilde{M}}_x + \dot{\tilde{M}}_y^\dagger \dot{\tilde{M}}_y) + \frac{1}{2a} \dot{\tilde{M}}_z^\dagger \dot{\tilde{M}}_z \right]$$

  Where $N = \kappa(2k^2 + m^2 a^2) - 12\xi(\dot{H} + 2H^2)a^2$
- *no ghost condition* :

$$\frac{k^2}{a^2} + m^2 > \frac{(2-\varepsilon)}{\kappa} 12\xi H^2$$

- *Special case of exact de-Sitter space*: $k^2 > 4a^2 H_0^2$
- There will be no ghost in the action for sub-Horizon modes only. While for close to horizon and super-horizon modes the action will encounter ghost.
- Even if when the nonminimal coupling with other curvature terms are considered, the ghost instability still remains in the action.

Removing the Ghost Instability

- It is noticed that the nondynamical modes (E_i) are leading to the ghost instability.
- *Resolution* : Adding dynamical terms of E_i
- One such possible term is (Breaks Gauge symmetry)
  $S_\tau = \int d^4x\sqrt{-g}(\tau(\nabla_\lambda B^{\lambda\nu})(\nabla_\mu B^{\mu}_{\ \nu}))$

$$T^{\tau}_{\mu\nu} = \tau\Big[g_{\mu\nu}\left((\nabla_\lambda B^{\sigma\lambda})(\nabla_\rho B^{\rho}_{\ \sigma}) + 2B^{\sigma\lambda}\nabla_\lambda\nabla_\rho B^{\rho}_{\ \sigma}\right) + 2(\nabla_\lambda B^{\lambda}_{\ \mu})(\nabla_\rho B^{\rho}_{\ \nu}) + 2\left(B_\mu^{\ \lambda}\nabla_\lambda\nabla_\rho B_\nu^{\ \rho} + B_\nu^{\ \lambda}\nabla_\lambda\nabla_\rho B_\mu^{\ \rho}\right)\Big]$$

- *effective action* :

$$S_2 = \int d^4x\left[\frac{1}{2a}\left(\dot{\vec{M}}.\dot{\vec{M}} + \tau a^2\dot{\vec{E}}.\dot{\vec{E}}\right) + \mathscr{L}_{nonkinetic}\right]$$

- This term plays no role in the background dynamics. $T^{\tau}_{\mu\nu}(\phi) = 0$

Summary

- Inflation can be sourced by the antisymmetric tensor field $B_{\mu\nu}$ if the nonminimal coupling with curvature is taken into consideration.
- Slow roll parameter for this model is defined and slow roll conditions are established.
- When only $B_{\mu\nu}$ is perturbed, keeping the metric unperturbed, it is observed that the action can have ghost instability for certain modes. However it turns out that this instability can be evaded by considering an U(1) nonvariant kinetic term for antisymmetric tensor.

future plan

- Full perturbation Analysis
- Dynamics of this model for anisotropic metric.
- post inflationary physics

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Thank you

Appendix

- *metric* : $ds^2 = g_{\mu\nu}\, dx^\mu dx^\nu$
- *Action* : $S = \int d^4x \sqrt{-g} \left(\frac{R}{2\kappa} + \mathscr{L}_m \right)$
- *Energy Momentum Tensor* : $T_{\mu\nu} = -\frac{2}{\sqrt{-g}} \frac{\delta(\sqrt{-g}\mathscr{L}_m)}{\delta g^{\mu\nu}}$
- *Einstein Equation* : $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \kappa T_{\mu\nu},$
  $$\kappa = \frac{1}{M^2_{planck}}$$
- *Scale Factor* : $a(t)$
- *Hubble Parameter* : $H = \frac{\dot{a}}{a}$