Fragmentation of Pseudo-Scalar Mesons

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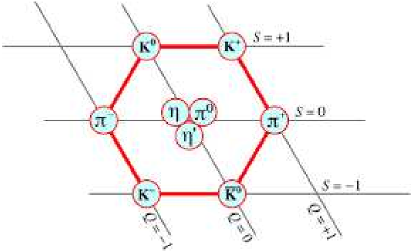
Outline of Talk

- Introduction
  1. Pseudo-Scalar Meson Nonet
  2. Fragmentation Process
- Model - Broken SU(3) Symmetry
- Results
  - e^-- e^+ process at Next-to-Leading Order (NLO)
- Conclusion and future work

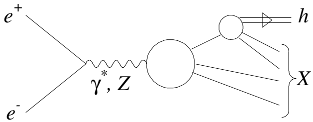
Mesons

- Mesons have a quark and an antiquark as their valence content ($q_i\overline{q_j}$ with i,j = u,d,s,..).
- Vector Meson - spin 1 odd parity
- Pseudoscalar Meson - spin 0 odd parity
  ($\pi(\pi^+, \pi^-, \pi^0)$, $K(K^+, K^-, K^0, K^{\bar{0}})$, η and η')



Fragmentation Process

- Quarks and antiquarks cannot be seen individually due to their confinement property.
- The quark-antiquark pair obtained in the $e^+ e^-$, $p p$ processes are observed only through the hadrons they produce. This is known to be *hadronisation or fragmentation process*.



- These fragmentation processes are characterised by fragmentation functions. pQCD can only explain this process at the quark production level but it cannot explain the origin of these fragmentation functions. However, QCD can explain their scale (momentum transfer squared Q^2) dependence.
- A comparison with experimental data allows the fragmentation functions to be determined at a given initial scale, Q_0^2. Their evolution is then completely determined by pQCD with the help of DGLAP evolution equations.
- A model is needed to determine these fragmentation functions from the observed event rates!

A Simple Model with SU(3) Symmetry

- Our model uses (a broken) SU(3) symmetry as SU(3) is a good description of octet of pseudo-scalar mesons: the $\pi(\pi^+,\, \pi^-, \pi^0)$, the $K(K^+,\, K^-,\, K^0,\, \overline{K}^0)$, and the η mesons.
- Under SU(3) process, a light quark ($q \to M^8 + X$) goes to an octet meson with the remainder X being a triplet (3), an anti-sixplet (6) or fifteenplet (15), for which $\alpha(x, Q^2)$, $\beta(x, Q^2)$, $\gamma(x, Q^2)$ are the corresponding fragmentation probabilities that have to be determined.
- In a similar way, an antiquark also produces an octet meson with X being $\overline{3}, 6$ or $\overline{15}$, for which $\overline{\alpha}(x, Q^2)$, $\overline{\beta}(x, Q^2)$ and $\overline{\gamma}(x, Q^2)$ have to be determined.
- Thus a single meson has *seven* unknown fragmentation functions $D_q^h(x, Q^2)$, $D_{\overline{q}}^h(x, Q^2)$ and $D_g^h(x, Q^2)$ associated with its production, while the heavier quark contributions are zero at the starting scale (below the charm threshold).
- The SU(3) symmetry reduces the complexity of $(7 \times 8 =)56$ unknown fragmentation functions into just seven fragmentation functions for all the mesons in the octet.

Model contd....

- Symmetries like *isospin invariance and charge conjugation* reduces the functions further.

$$q \leftrightarrow \overline{q} \quad : \quad D_q^{\pi^+} = D_{\overline{q}}^{\pi^-} (\text{charge}) \; ;$$
$$u \leftrightarrow d \quad : \quad D_u^{\pi^+} = D_d^{\pi^-} (\text{isospin}) \; .$$

- As a consequence of these symmetries, it is seen that the $\overline{\alpha}$, $\overline{\beta}$ and $\overline{\gamma}$ are not independent of α, β and γ. Thus 6 functions are now reduced into just *three* independent fragmentation functions.
- Application of SU(3) invariance leads to $\beta = \gamma/2$ (flavour symmetry of sea fragmentation). Thus all the sea flavour fragmentation functions are equal to $S(x, Q^2) = 2\gamma(x, Q^2)$.
- All valence fragmentation functions can be expressed in terms of the function $V(x, Q^2)$, where V is given, say for π^+, by the difference $D_u^{\pi^+} - D_{\overline{u}}^{\pi^+}$.
- Hence there are *two* independent combinations: $V(x, Q^2) \equiv \alpha + \beta - (5/4)\gamma$ and $\gamma(x, Q^2)$, apart from the gluon fragmentation function.
- The symmetry is broken due to the introduction of a x independent factor λ in order to explain the suppression of strangeness in the K meson.

Table 1: Quark fragmentation functions in terms of the functions, V and γ for pseudo-scalar mesons with their valence content.

fragmenting quark		$K^+(u\overline{s})$	fragmenting quark		$K^0(d\overline{s})$
u	:	$V+2\gamma$	u	:	2γ
d	:	2γ	d	:	$V+2\gamma$
s	:	2γ	s	:	2γ
fragmenting quark		η'	fragmenting quark		π^0
u	:	$\frac{1}{6}V+2\gamma$	u	:	$\frac{1}{2}V+2\gamma$
d	:	$\frac{1}{6}V+2\gamma$	d	:	$\frac{1}{2}V+2\gamma$
s	:	$\frac{4}{6}V+2\gamma$	s	:	2γ
fragmenting quark		$\pi^+(u\overline{d})$	fragmenting quark		$\pi^-(\overline{u}d)$
u	:	$V+2\gamma$	u	:	2γ
d	:	2γ	d	:	$V+2\gamma$
s	:	2γ	s	:	2γ
fragmenting quark		$\overline{K^0}(\overline{u}s)$	fragmenting quark		$K^-(\overline{d}s)$
u	:	2γ	u	:	2γ
d	:	2γ	d	:	2γ
s	:	$V+2\gamma$	s	:	$V+2\gamma$

Fragmentation Functions of Pure Octet

- The π and K fragmentation functions can be written in terms of *sea and valence parts at* Q_0^2 *as,*

$$
\begin{aligned}
D_0^{\pi^+} &= 2V + 12\gamma \ , \\
D_0^{K^0} &= V(1+\lambda) + 12\lambda\gamma \ .
\end{aligned}
$$

- The functional form of V, γ, D_g at Q_0^2 needed to fit the data is

$$
F_i(x) = a_i x^{b_i} (1-x)^{c_i} (1 + d_i \sqrt{x} + e_i x) \ ;
$$

  where a, b, c, d and e are the parameters to be determined from the fit. • The function should go to zero as $x \to 1$; this is governed by c_i. The small-x behaviour is governed by b_i.

- Hence the octet mesons are described in terms of valence V and sea S parts with light quarks (u, d, s) at a starting scale of Q_0^2 =1.5 GeV^2 where the fragmentation of charm and bottom flavours are zero. As the fragmentation functions were evolved according to *NLO* evolution to the Z pole, these heavier quarks are included at appropriate thresholds.

Frag. Functions of Octet-Singlet Mixing- Octet Part

- This model was extended to the singlet sector with the inclusion of a few parameters. This is required for η and η' as they are mixtures of singlet and octet;

$$
\begin{aligned}
|\eta\rangle &= \cos\theta|\eta_8\rangle - \sin\theta|\eta_1\rangle \ , \\
|\eta'\rangle &= \sin\theta|\eta_8\rangle + \cos\theta|\eta_1\rangle \ ,
\end{aligned}
$$

  where $\eta_8 = (u\overline{u} + d\overline{d} - 2s\overline{s})/\sqrt{6}$, $\eta_1 = (u\overline{u} + d\overline{d} + s\overline{s})/\sqrt{3}$ are the corresponding orthogonal states and θ is the mixing angle.
- We have to assume some more parameters for η and η' since they are mixtures of singlet and octet.
- Using the functional forms of valence and sea part of π and K mesons we can fit the octet part of η and η'. Now for the pure octet part (η_8) the $u(=d), s$ fragmentation functions are

$$
\begin{aligned}
D_u^8 &= \frac{1}{6}\alpha + \frac{9}{6}\beta + \frac{9}{8}\gamma = \frac{V}{6} + 2f_{sea}\gamma \ , \\
D_s^8 &= \frac{4}{6}\alpha + \frac{9}{6}\gamma = \frac{2}{3}\lambda V + 2f_{sea}\gamma \ .
\end{aligned}
$$

  where f_{sea} is the sea quark suppression factor,different for two mesons.

Singlet Part

- There is a *single* fragmentation function, $\delta(x, Q^2)$ for the singlet meson $(q \to M^1 + X)$ with X can only be a triplet.
- In octet case, the probability of a parton to fragment into an octet hadron with X being triplet is $\alpha(x, Q^2)$.
- Ansatz: Hence,we assumed that the unknown function $\delta(x, Q^2)$ can be related to $\alpha(x, Q^2)$, the fragmentation function for members of octet meson. That is,

$$\frac{\delta}{3} = \frac{f_1 \alpha}{3} = \frac{f_1}{3}\left(V + \frac{3}{4}\gamma\right).$$

- Here f_1 is the proportionality constant. The u $(= d)$ and s fragmentation functions for singlet part can be written as follows:

$$\begin{aligned} D_u^1 &= \frac{f_1^u}{3}\left(V + \frac{3}{4} f_{sea}\gamma\right), \\ D_s^1 &= \frac{f_1^s}{3}\left(\lambda V + \frac{3}{4} f_{sea}\gamma\right). \end{aligned}$$

- Thus with these four equations in hand, we express the fragmentation functions for η and η' mesons, at the input scale as,

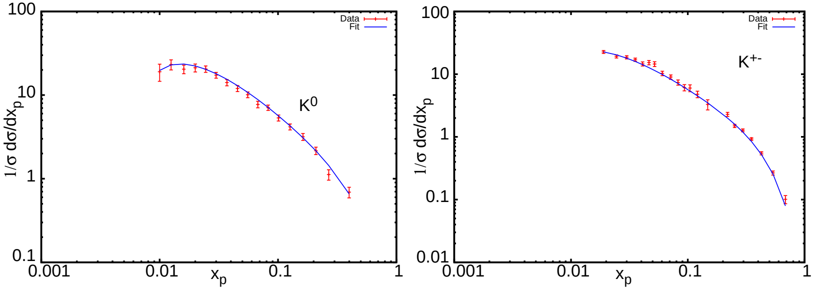
$$
\begin{aligned}
D_i^{\eta} &= (c_i^{\eta})^2 \left(\cos^2\theta \frac{D_i^8}{(c_i^8)^2} + \sin^2\theta \frac{D_i^1}{(c_i^1)^2} \right) ; \\
D_i^{\eta'} &= (c_i^{\eta'})^2 \left(\sin^2\theta \frac{D_i^8}{(c_i^8)^2} + \cos^2\theta \frac{D_i^1}{(c_i^1)^2} \right) .
\end{aligned}
$$

- Here, i refers to the three light quarks $(u,\, d,\, s)$; the co-efficients of η are $c_u^{\eta} = c_d^{\eta} = (\cos\theta - \sqrt{2}\sin\theta)$, $c_s^{\eta} = (-2\cos\theta - \sqrt{2}\sin\theta)$ and $c_u^8 = 1$, $c_s^8 = 2$, $c_u^1 = c_s^1 = \sqrt{2}$ and θ is the mixing angle.

$e^+ e^-$ process at Next-to-Leading Order

K Fits for $e^+ e^-$ Process at NLO

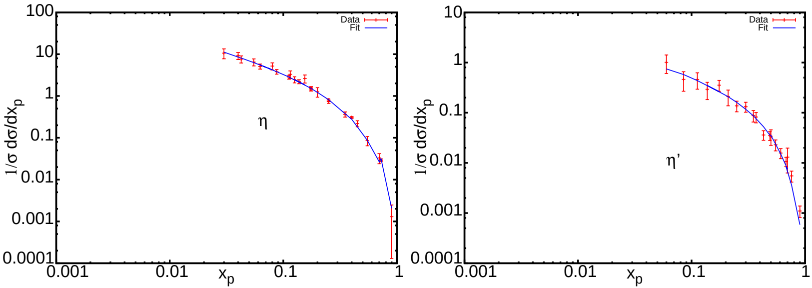
- Fits for $K^0 (= K^0 + \overline{K^0})$ (L) and $K^{+-} (= K^+ + K^-)$ (R) meson in terms of fragmentation functions with SLD uds data at $\sqrt{s} = 91$ GeV at NLO.



- The SLD uds data is being used to remove contamination from heavy flavours that contribute both directly through fragmentation as well as indirectly through production and decay of heavy flavour mesons.

η&η' Fits for $e^+ e^-$ Process at NLO

Fit for η and η' in terms of fragmentation functions with LEP data at $\sqrt{s} = 91$ GeV at NLO.



π Fits for $e^+ e^-$ Process at NLO

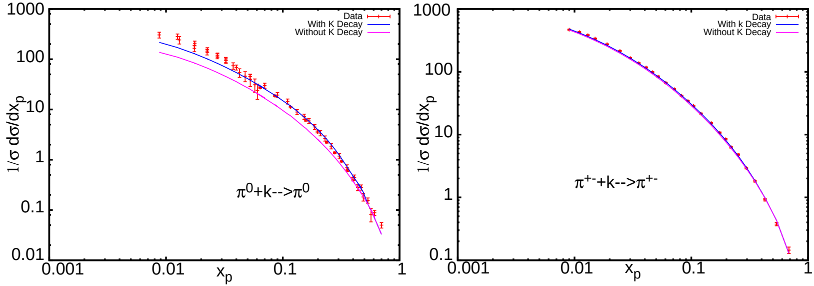


Figure 1: Fits for π meson in terms of fragmentation functions with π^0 LEP data (L) and $\pi^{+,-}$ SLD data (R) at $\sqrt{s} = 91$ GeV at NLO.

- Being lightest meson, π data is challenging to study because most of the heavy (strange,charm..) mesons decay chain ends up in π meson.

Various Input Parameters at NLO

Best fit values of various input parameters defining the input fragmentation functions at the starting scale of $Q_0^2 = 1.5$ GeV^2.

		Central Value	Error Bars
V	a	3.18	0.12
	b	-0.31	0.03
	c	1.47	0.13
	d	-0.13	0.28
	e	-0.78	0.08
γ	a	9.26	0.25
	b	-0.47	0.01
	c	9.79	0.14
	d	-3.15	0.10
	e	3.69	0.22
D_g	a	4.47	0.56
	b	-0.32	0.07
	c	5.92	0.57
	d	-10.84	1.20
	e	-19.92	2.89

	Central Value	Error Bars
λ	0.10	0.01
θ	-17.19	1.27
f_{sea}^{η}	0.10	0.02
$f_{sea}^{\eta'}$	0.36	0.13
$f_1^u(\eta)$	2.30	0.42
$f_1^s(\eta')$	0.43	0.04
f_g^K	0.02	0.04
f_g^{η}	0.89	0.12
$f_g^{\eta'}$	0.0002	0.06

The χ^2 is calculated by,

$$\chi^2 = \sum_{i=1}^{n} \frac{(N_i^{data} - N_i^{theo}(p_k))^2}{(\sigma_i^{data})^2} \ ,$$

where N_i^{data} and N_i^{theo} are measured and theoretically predicted cross section values and n is the number of data points. The experimental errors are calculated from systematic and statistical errors by

$$(\sigma_i^{data})^2 = (\sigma_i^{sys})^2 + (\sigma_i^{stat})^2 \ .$$

Here $N_i^{theo}(p_k)$ is expressed in terms of the unknown parameters p_k. The parameters are determined by minimising the χ^2.
χ^2 for fits to inclusive pseudo-scalar meson production data on the Z-pole from LEP and SLD experiments.

Data Set	No. of data points	χ^2
π^0	49	190.0
π^{+-}	27	26.3
K^0	17	12.3
K^{+-}	21	26.9
η	24	12.3
η'	19	14.3

Conclusion

- Using simple SU(3) model and NLO QCD evolution, we fitted the quark fragmentation functions for pseudo-scalar mesons π K η and η' for $e^+ e^-$ process .
- The SU(3) model with three light flavours uses universal functions, the valence $V(x, Q^2)$, sea $\gamma(x, Q^2)$ quark fragmentation functions and a gluon fragmentation function $D_g(x, Q^2)$.
- No new additional fragmentation function is introduced in order to explain the singlet sector which shows the efficiency of the model.
- Extending this NLO study to $p\,p$ collision in future will help to understand the η meson production in particular which forms the base-line to understand the nature of QGP (quark-gluon plasma) studies.

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9. DELPHI Collab:
   (Abreu et al) Eur. Phys. J. C5, 585 (1998) for π^{+-}, K^{+-} ; (Abreu et al) Zeit. Phys. C65, 587 (1995) for K^0 at Z- pole.

Thank You

Backup Slides

DGLAP Evolution Equations

- The DGLAP evolution equations, named after the people who introduced them —Dokshitser[1], Gribov and Lipatov[2], Altarelli and Parisi[3]— form the baseline to understand the fragmentation functions, independent of processes involved.
- The general form of the DGLAP equation, a $(2N_f + 1)$ dimensional matrix equation in the space of quarks and gluons for the fragmentation process is given by:

$$t\frac{\partial}{\partial t}\begin{pmatrix} D^h_{q_j}(x,t) \\ D^h_g(x,t) \end{pmatrix} = \frac{\alpha_s(t)}{2\pi}\sum_{q_i,\overline{q}_i}\int_x^1 \frac{dz}{z}\begin{pmatrix} \mathcal{P}_{q_j q_i}(z,\alpha_s(t)) & \mathcal{P}_{g q_i}(z,\alpha_s(t)) \\ \mathcal{P}_{q_j g}(z,\alpha_s(t)) & \mathcal{P}_{gg}(z,\alpha_s(t)) \end{pmatrix} \times \begin{pmatrix} D^h_{q_j}(z,t) \\ D^h_g(z,t) \end{pmatrix} .$$

Here the subscripts i,j run over different flavours.

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Quark frag. fns. into members of meson octet in terms of the SU(3) functions, α, β and γ

fragmenting quark	K^+	fragmenting quark	K^0
u	: $\alpha+\beta+\frac{3}{4}\gamma$	u	: $2\beta+\gamma$
d	: $2\beta+\gamma$	d	: $\alpha+\beta+\frac{3}{4}\gamma$
s	: 2γ	s	: 2γ
fragmenting quark	η/η'	**fragmenting quark**	π^0
u	: $\frac{1}{6}\alpha+\frac{9}{6}\beta+\frac{9}{8}\gamma$	u	: $\frac{1}{2}\alpha+\frac{1}{2}\beta+\frac{11}{8}\gamma$
d	: $\frac{1}{6}\alpha+\frac{9}{6}\beta+\frac{9}{8}\gamma$	d	: $\frac{1}{2}\alpha+\frac{1}{2}\beta+\frac{11}{8}\gamma$
s	: $\frac{4}{6}\alpha+\frac{9}{6}\gamma$	s	: $2\beta+\gamma$
fragmenting quark	π^+	**fragmenting quark**	π^-
u	: $\alpha+\beta+\frac{3}{4}\gamma$	u	: 2γ
d	: 2γ	d	: $\alpha+\beta+\frac{3}{4}\gamma$
s	: $2\beta+\gamma$	s	: $2\beta+\gamma$
fragmenting quark	$\overline{K^0}$	**fragmenting quark**	K^-
u	: $2\beta+\gamma$	u	: 2γ
d	: 2γ	d	: $2\beta+\gamma$
s	: $\alpha+\beta+\frac{3}{4}\gamma$	s	: $\alpha+\beta+\frac{3}{4}\gamma$

Quark fragmentation functions in terms of α, β and γ SU(3) functions

Let q^i be a quark triplet and M^i_j a member of the pseudo-scalar meson octet, where $i, j, k = 1, 2, 3$. **Case 1 :** X being triplet, X^i: The invariant amplitude for a process

$$q^i \rightarrow V^i_j + X^j;$$

is $q_i V^i_j X^j$, where X^i and q^i are normalised. It is to be remembered throughout the calculation that the indices of the amplitude should contract to tensor of rank zero since cross section is a scalar quantity. Hence in this case, $X^i = q^i$, M^i_j are the elements of the pseudo-scalar meson matrix . For example, the rate for $u \rightarrow \pi^+ + X$ is $\alpha|q_1 V^1_2 X^2|^2$ which is equal to α. Here α is defined as the relevant fragmentation function for the process. In a similar way, the rates for d and s quarks that fragments into π^+ meson can be done and for all the three light quarks into other pseudo-scalar mesons as well.
Likewise, the other two SU(3) probability functions β and γ for $X(=\overline{6}, 15)$ are also calculated.

Charge Factors of Cross Section at LO of $e^+ e^-$

The charge factors c_q are associated with the quark q_i with flavour i, written in terms of the electromagnetic charge e_i, vector and axial vector electroweak couplings, $v_i = T_{3i} - 2e_i \sin^2\theta_\mathrm{w}$ and $a_i = T_{3i}$, and similarly those of electrons, v_e and a_e as

$$\begin{aligned}
c_q &= c_q^V + c_q^A \ , \\
c_q^V &= \frac{4\pi\alpha^2}{s}[e_q^2 + 2e_q v_e v_q \,\rho_1(s) + (v_e^2 + a_e^2)v_q^2 \,\rho_2(s)] \ , \\
c_q^A &= \frac{4\pi\alpha^2}{s}(v_e^2 + a_e^2)\; a_q^2 \;\rho_2(s) \ , \\
\rho_1(s) &= \frac{1}{4\sin^2\theta_\mathrm{w}\cos^2\theta_\mathrm{w}} \; \frac{s(m_Z^2 - s)}{(m_Z^2 - s)^2 + m_Z^2\Gamma_Z^2} \ , \\
\rho_2(s) &= \frac{1}{(4\sin^2\theta_\mathrm{w}\cos^2\theta_\mathrm{w})^2} \; \frac{s^2}{(m_Z^2 - s)^2 + m_Z^2\Gamma_Z^2} \ .
\end{aligned}$$

For values of e_i, charge of the particle, a_i, the third component of weak isospin, and v_i with weak mixing angle θ_w see appendix of thesis. Γ_Z and m_Z are the decay width and mass of the Z-intermediate gauge boson for high energy scale.

Co-efficients of Cross Section at LO of $e^+ e^-$

- The singlet D_0 and the non-singlet D_3, D_8, D_{15} and D_{24} combinations are:

$$
\begin{aligned}
D_0 &= D_u + D_d + D_s + D_c + D_b + \text{anti-quarks} \\
D_3 &= D_u - D_d + \text{ anti-quarks} \\
D_8 &= D_u + D_d - 2D_s + \text{ anti-quarks} \\
D_{15} &= D_u + D_d + D_s - 3D_c + \text{ anti-quarks} \\
D_{24} &= D_u + D_d + D_s + D_c - 4D_b + \text{ anti-quarks}
\end{aligned}
$$

- The corresponding co-efficients involved in the cross section equation are

$$
\begin{aligned}
a_0 &= (c_u + c_d + c_s + c_c + c_b)/5 \ , \\
a_3 &= (c_u - c_d)/2 \ , \\
a_8 &= (c_u + c_d - 2c_s)/6 \ , \\
a_{15} &= (c_u + c_d + c_s - 3c_c)/12 \ , \\
a_{24} &= (c_u + c_d + c_s + c_c - 4c_b)/20 \ ,
\end{aligned}
\tag{1}
$$