Second Order Splitting Functions and Infrared Safe Crossection in N = 4 SYM

Amlan Chakraborty
V Ravindran, Pulak Banerjee, Prasanna K Dhani, Satyajit Seth
arXiv:1810.07672
Appeared to be in JHEP

IMSc, Chennai

December 12, 2018

Motivation

- To understand the universal quanities in amplitude and crossection level in different quantum field theories
- Direct comparison of observable quantities with QCD

Outline of Talk

- **Introduction to N = 4 SYM**
- **Infrared Structure**
- **Splitting Functions**
- **Strategy Of Computation**

Introduction to N = 4 SYM

- Cousin Of QCD with Simpler UV and Infrared Structure
- Beta function Zero all order in Perturbation Theory i.e no UV charge renormalization requires(UV Conformal)
- Certain Composite operators (Konishi) not protected by Supersymmetric current conservation requires UV renormalization
- Consists of 4 majorana fermion(A^{μ}) ,1 gluon(λ_n), 3 Scalar(ϕ_i) and 3 pseudo scalar(χ_i) in adjoint representaion of SU(N) group

Introduction to N = 4 SYM

- N = 4 SYM Lagrangian -

$$\mathcal{L}_{\text{SYM}}^{\mathcal{N}=4} = -\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu a} - \frac{1}{2\xi}(\partial_\mu A^{a\mu})^2 + \partial_\mu \bar{\eta}^a D^\mu \eta_a + \frac{i}{2}\bar{\lambda}^a_m \gamma^\mu D_\mu \lambda^a_m + \frac{1}{2}(D_\mu \phi^a_i)^2$$
$$+ \frac{1}{2}(D_\mu \chi^a_i)^2 - \frac{g}{2} f^{abc} \bar{\lambda}^a_m [\alpha^i_{m,n} \phi^b_i + \gamma_5 \beta^i_{m,n} \chi^b_i] \lambda^c_n - \frac{g^2}{4}\Big[(f^{abc}\phi^b_i \phi^c_j)^2$$
$$+ (f^{abc}\chi^b_i \chi^c_j)^2 + 2(f^{abc}\phi^b_i \chi^c_j)^2\Big]$$

(Tarasov)

- Ads/CFT duality suggests that weak coupling perturbation series for planner(large Nc) N = 4 SYM should have special properties - certain quantities in stong coupling limit equivalent to weakly coupled supergravity theory. (Maldacena,1998)

Infrared Structure

- Expand the multiloop amplitude in D=4+ϵ
- Infrared divergences consists of Soft and Collinear divergences
- Overlapping soft + collinear divergences at each loop order imply leading poles are $1/\epsilon^{2L}$ at L loops
- Pole terms are predictable

Soft-Collinear Factorization

$$\mathcal{M}_n = S(k_i, \mu, \alpha_s(\mu), \epsilon)[\prod_{i=1}^{n} J_i(\mu, \alpha_s(\mu), \epsilon)] \times h_n(k_i, \mu, \alpha_s(\mu), \epsilon)$$

- S = soft function(only depends on color of i^{th} particle)
- J = jet or collinear function (color diagonal depends on i^{th} spin)
- H = hard function(finite after UV renormalization)

L.Magnea,E.Gardi,L.Dixon

Sudakov Form Factor

- Final state colorless particle
- UV renormalized form factor satisfies K + G equation as a consequence of factorization, gauge and renormalization group invariances

Sudakov Form Factor

$$\frac{d}{d \ln Q^2} \ln \mathcal{F}_f^\rho = \frac{1}{2}\left[K_f^\rho + G_f^\rho\right]$$

- The Q^2 independent $K_f^\rho(a, \epsilon)$ contains all the poles in ϵ, whereas $G_f^\rho\left(a, Q^2/\mu^2, \epsilon\right)$ involves only the finite terms in $\epsilon \to 0$.
- Clearly dictates UV-IR factorisation

Sudakov,Mueller,Ashoke Sen,V Ravindran,G.Sterman ...

Splitting Functions

Anamolous dimention of wilson operator -

- Mellin moments of the Splitting functions are the UV anamolous dimention of certain wilson operator
- Wilson operator can be found by applying OPE(operator product expantion)
- They are the operator definition of nonperturbative parton distribution functions (PDF)
- Splitting functions are universal quantities of the theory ,does not depend on particular processes
- often interpreted as the probabilty distribution functions of the partons.
- parton distribution function satisty a RG equation namely DGLAP equation

Vermaseren,Vogt,Lipatov,Altareli,Parisi ...

- We devised a method to obtain splitting functions with taking it as unknown and therefore demanding finiteness of crossection as final state with certain UV protected operator.

- Due to KLN theorem in non abelian gauge theory all final state soft and collinear poles goes away when all of the final states summed up.
- initial state collinear divergences still remains.

remedy

- use mass factorisation prescription at Cross Section level to remove initial state divergences at the factorisation scale μ_F

$$\hat{\Delta}^i_{ab}(z, Q^2, 1/\epsilon) = \sum_{c,d=\lambda,\phi,\chi,g} \Gamma_{ca}(z, \mu_F^2, 1/\epsilon) \otimes \Gamma_{db}(z, \mu_F^2, 1/\epsilon) \otimes \Delta^i_{cd}(z, Q^2, \mu_F^2)$$

- $\hat{\Delta} \equiv \hat{\sigma}/z$
- $\hat{\sigma}_{ab}$ = UV renormalized cross section
- Δ_{lm} = finite when eps - 0
- Γ_{ij}= massfactorisation kernel

Splitting Functions

$$\Gamma_{ab}(z, \mu_F^2, 1/\epsilon) = \sum_{k=0}^{\infty} a_s^k(\mu_F^2)\Gamma_{ab}^{(k)}(z, \mu_F^2, 1/\epsilon)$$

$$\Gamma_{ab}^{(0)} = \delta_{ab}\delta(1-z)$$

$$\Gamma_{ab}^{(1)} = \frac{1}{\epsilon}P_{ab}^{(0)}(z)$$

$$\Gamma_{ab}^{(2)} = \frac{1}{\epsilon^2}\left(\frac{1}{2}P_{ac}^{(0)} \otimes P_{cb}^{(0)} + \beta_0 P_{ab}^{(0)}\right) + \frac{1}{\epsilon}\left(\frac{1}{2}P_{ab}^{(1)}\right)$$

- $P_{ab}^{(i)}$ are the Altarelli-Parisi splitting functions. The symbol $\otimes$ stands for the convolution

$$(f \otimes g)(z) \equiv \int_z^1 \frac{dx}{x} f(x) g\left(\frac{z}{x}\right)$$

Splitting Functions

- Expanding the unrenormalised coefficient function and the mass factorised one in powers of strong coupling constant as

$$\hat{\Delta}^{i}_{ab} = \sum_{k=0}^{\infty} \hat{a}_s^k S_\epsilon^k (\frac{Q^2}{\mu^2})^{k\frac{\epsilon}{2}} \hat{\Delta}^{i,(k)}_{ab}$$

$$\Delta^{i}_{ab} = \sum_{k=0}^{\infty} a_s^k(\mu_F^2) \Delta^{i,(k)}_{ab}$$

- Massfactorisation theorem garantees that all initial collinear divergences can be absorbed into Γ_{ab}.
- By demanding finiteness of the Δ^{i}_{ab} we can find the splitting functions.

DGLAP evolution equation for wilson operator

$$\frac{df_c(x,Q)}{d\ln Q} = \sum_{n=1}^{\infty} a_s^n \int_x^1 \frac{dy}{y} \sum_{b=\{\lambda,g,\phi,\chi\}} P_{cb}^{(n-1)}(x/y) f_b(y,Q).$$

- If we interpret splitting functions as probability distribution for the partons then we have momentam consevation equations that have to be satisfied by splitting functions.

$$\sum_{n=1}^{\infty} \int_0^1 dx\, x \sum_{b=\{\lambda,g,\phi,\chi\}} P_{b\lambda}^{(n-1)} = 0$$

$$\sum_{n=1}^{\infty} \int_0^1 dx\, x \sum_{b=\{\lambda,g,\phi,\chi\}} P_{bg}^{(n-1)} = 0$$

$$\sum_{n=1}^{\infty} \int_0^1 dx\, x \sum_{b=\{\lambda,g,\phi,\chi\}} P_{b\phi}^{(n-1)} = 0$$

DGLAP evolution equation for wilson operator

$$\sum_{n=1}^{\infty} \int_0^1 dx\, x \sum_{b=\{\lambda,g,\phi,\chi\}} P_{b\chi}^{(n-1)} = 0$$

- Following identities are satisfied at each order of perturbation theory
- A crucial crosscheck of our calculation after expicitly evaluating the splitting functions

Strategy Of Computation

- The strategy for doing the computation was to evaluate certain cross sections, whose final state contains heavy particle sharing effective vertices with N=4 SYM. We have taken the graviton coupled with the energy-momentum tensor in N=4 SYM and a another massive particle coupled with a BPS operator.
- The above computed cross section $\hat{\sigma}_{ab}$ contains only IR divergences (no UV divergences as $\beta = 0$ and $T_{\mu\nu}^{\mathcal{N}=4\,\mathrm{SYM}}$ is conserved).

Results

We find that the both LO and NLO splitting functions satisfy the following relations:

$$\sum_{a=\lambda,g,\phi,\chi} P^{(i)}_{a\lambda} = \sum_{a=\lambda,g,\phi,\chi} P^{(i)}_{ag} = \sum_{a=\lambda,g,\phi,\chi} P^{(i)}_{a\phi} = \sum_{a=\lambda,g,\phi,\chi} P^{(i)}_{a\chi} = I^{(i)}(x),$$

where

$$\begin{aligned} I^{(0)}(x) &= 8\left[\frac{1}{(1-x)_+} + \frac{1}{x}\right], \\ I^{(1)}(x) &= 24\zeta_3\delta(1-x) + 32\frac{1}{x}\left[\mathrm{Li}_2(-x) + \log(x)\log(1+x) \right. \\ &\qquad \left. - \log(x)\log(1-x)\right] \\ &\quad + \frac{1}{(1-x)_+}\left[-32\log(x)\log(1-x) + 8\log^2(x) - 16\zeta_2\right] \\ &\quad + \frac{1}{1+x}\left[-32\mathrm{Li}_2(-x) - 32\log(x)\log(1+x) + 8\log^2(x) - 16\zeta_2\right]. \end{aligned}$$

Results

$$\sum_{a=\lambda,g,\phi,\chi} \int_0^1 dx\, x P_{ab}^{(i)} = \int_0^1 dx\, x I^{(i)}(x) = 0\,,$$
$$i = 0, 1 \text{ and } b = \{\lambda, g, \phi, \chi\}.$$

We find that both at NLO and NNLO, only the diagonal splitting functions contain "+" distributions. In addition, at NNLO level, terms proportional to $\delta(1-x)$ start contributing to diagonal splitting functions. Hence, in the limit x $\rightarrow$ 1, the diagoinal splitting functions can be parametrized as

$$P_{aa}^{(i)}(x) = 2A_{i+1}\frac{1}{(1-x)_+} + 2B_{i+1}\delta(1-x) + R_{aa}^{(i)}(x),$$

where A_{i+1} and B_{i+1} are the cusp and collinear anomalous dimensions respectively. $R_{aa}^{(i)}(x)$ is the regular function as $x \rightarrow 1$. We find that

$$A_1 \;=\; 4, A_2 = -8\zeta_2\,, \qquad \text{and} \qquad B_1 = 0, B_2 = 12\zeta_3,$$

Matches with earlier Sudakov Form Factor Computation(Neerven,Henn,Gehrman,Brandhuber)

General factorisation equation of soft virtual crossection:

$$\Delta^{I,\mathrm{SV}}_{aa} = \left(Z^I(a,\epsilon)\right)^2 |\hat{F}^I_{aa}(Q^2,\epsilon)|^2 \delta(1-z) \otimes \mathcal{C}\exp\left(2\Phi^I_{aa}(z,Q^2,\epsilon)\right) \otimes \Gamma^{-1}_{aa}(z,\mu_F^2,\epsilon) \otimes \Gamma^{-1}_{aa}(z,\mu_F^2,\epsilon).$$

(Ravindran,2005)

Z^I = Overall UV renormalisation constant of the final state operator

$\Phi^I_{aa}(z,Q^2,\epsilon)$ = Process independent soft function if the probing particle is colorless (Higgs,Weak Bosons)

Soft virtual crossection of protected operator in N=4SYM

$$\begin{aligned}
\Delta_{aa}^{I,(0),\mathrm{SV}} &= \delta(1-z)\,, \\
\Delta_{aa}^{I,(1),\mathrm{SV}} &= 8\zeta_2\delta(1-z) + 16\mathcal{D}_1(z)\,, \\
\Delta_{aa}^{I,(2),\mathrm{SV}} &= -\frac{4}{5}\zeta_2^2\delta(1-z) + 312\zeta_3\mathcal{D}_0(z) - 160\zeta_2\mathcal{D}_1(z) + 128\mathcal{D}_3(z) \\
\Delta_{aa}^{I,(3),\mathrm{SV}} &= \left[-\frac{8012}{3}\zeta_6\right]\delta(1-z) \\
&\quad + \left[11904\zeta_5 - \frac{23200}{3}\zeta_2\zeta_3\right]\mathcal{D}_0 + \left[-\frac{9856}{5}\zeta_2^2\right]\mathcal{D}_1 \\
&\quad + 11584\zeta_3\mathcal{D}_2 + \left[-3584\zeta_2\right]\mathcal{D}_3 + 512\mathcal{D}_5
\end{aligned}$$

After appropriately defining $C_a = C_F$ =N we find that leading Trancendental Part of Soft Vitual Higgs crossection(Anastasiou,Melnikov,Harlander,Ravindran,Smith,van Neerven) is exactly equal to above.

THANK YOU