Mixing dynamics of Dim-5 interactions (Scalar/ pseudoscalar-photon) in Magnetised medium

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Plan of Talk

1. Motivation

2. Formalism
   Interaction, $\phi - \gamma$ and $a - \gamma$:
   a) In magnetized vacuum
   b) In magnetized media.

3. Conclusion

Motivation

- Scalars ($\phi(x)$, associated with breaking of scale symmetry) and pseudo-scalars(a(x), strong CP problem of QCD and $U_A(1)$ problem) emerged as one of the suitable candidates for DM . Their low masses m, and high symmetry breaking scale M_F made them to interact weakly with ordinary matter , which has grabbed the attention.

- The vertex is $g_{\phi\gamma\gamma}\phi FF$ (for scalar-photon interaction) and $g_{a\gamma\gamma}a(x)\tilde{F}F$(for pseudoscalar-photon interaction), with coupling constant $g_{\phi\gamma\gamma}$ and $g_{a\gamma\gamma}$.

- Our one of the motivation is to discriminate between them through their interaction in magnetized media.

In magnetized vacuum

Models: Action for coupled photon-scalar system

$$S = \int d^4x \left[\frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \frac{1}{4} g_{\phi\gamma\gamma} \phi F_{\mu\nu} F^{\mu\nu} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \right] \tag{1}$$

Here we decompose the EM field into two parts, $F_{\mu\nu} = \bar{F}_{\mu\nu} + f_{\mu\nu}$.

Equations of Motion

The equations of motion for the fluctuations turn out to be:

$$\partial_\mu f^{\mu\nu} = -g_{\phi\gamma\gamma} \partial_\mu \phi \bar{F}^{\mu\nu}. \tag{2}$$

$$\partial_\mu \partial^\mu \phi = -\frac{1}{2} g_{\phi\gamma\gamma} \bar{F}^{\mu\nu} f_{\mu\nu} \tag{3}$$

Propagation Eigenstates

In external magnetic field, the natural variales to discuss the equations of motion are:

$$\Psi(k) = \left(\frac{f_{\mu\nu}(k)\bar{F}_{\mu\nu}}{2}\right) \tag{4}$$

$$\& \tag{5}$$

$$\tilde{\Psi}(k) = \left(\frac{f_{\mu\nu}(k)\tilde{\bar{F}}_{\mu\nu}}{2}\right) \tag{6}$$

The propagation eigenstates Ψ and $\tilde{\Psi}$, transform differently under **CP**.

EOM in matrix form:

$$\begin{bmatrix} k^2 & 0 & 0 \\ 0 & k^2 & -g_{\phi\gamma\gamma}(\omega Sin\Theta\mathscr{B}) \\ 0 & -g_{\phi\gamma\gamma}(\omega Sin\Theta\mathscr{B}) & k^2 \end{bmatrix} \begin{bmatrix} \tilde{\psi} \\ \psi \\ \Phi \end{bmatrix} = 0 \tag{7}$$

- The 3x3 matrix corresponding to total three degrees of freedom (2 for photon + 1 for scalar,$\phi(x)$.
- ψ is even under **CP** and mix with scalar $(\phi(x))$.
- $\tilde{\psi}$ is odd under CP, and propagates freely.
- For axion-photon system, $\tilde{\psi}$ would have mixed with a(x) and ψ would have propagated freely. This follows from the CP argument.

Including medium effect (magnetized medium)

Coupled photon-scalar system including medium effect and mass

Here we consider a medium effect as plasma, we include polarization tensor, Faraday term in action. The resulting action turns out to be

$$\begin{aligned} S &= \int d^4k \left[\frac{1}{2} \left(-k^2 \tilde{g}_{\mu\nu} + \Pi_{\mu\nu}(k) + \Pi^p_{\mu\nu}(k) \right) A^\mu(k) A^\nu(-k) \right. \\ &\quad \left. + i g_{\phi\gamma\gamma} \phi(k) \bar{F}_{\mu\nu} k^\mu A^\nu(-k) + \frac{1}{2} \phi(-k) [k^2 - m_\phi^2] \phi(k) \right]. \end{aligned} \tag{8}$$

- Here $\Pi_{\mu\nu}(k) = \Pi_T(k) R_{\mu\nu} + \Pi_L(k) Q_{\mu\nu}$ is polarization tensor
- And $\Pi^p_{\mu\nu}(k) = \Pi_p(k) P_{\mu\nu}$, is (Faraday term) Photon self energy tensor evaluated in a finite density magnetized medium upto order eB. $P_{\mu\nu} = i\varepsilon_{\mu\nu\alpha\beta_\parallel} \frac{k^\alpha}{|k|} u^{\beta_\parallel}$ is projection operator and $\tilde{g}^{\mu\nu} = (g^{\mu\nu} - \frac{k^\mu k^\nu}{k^2})$.

We can find the equations of motion in momentum space by standard variational principle.

Expanding Gauge potential $A_\mu(k)$, in orthogonal basis vectors

- In order to capture the dynamics of available DOF in a medium, we need to expand the four-vector potential A_μ (k), by constructing set of orthogonal vectors, defined as:

$$
\begin{aligned}
e_\mu^{(1)} &= N_1 \varepsilon_{\mu\gamma\lambda\alpha} k^\gamma \tilde{U}^\lambda B^\alpha \\
e_\mu^{(2)} &= N_2 \varepsilon_{\mu\gamma\lambda\alpha} k^\gamma \tilde{U}^\lambda e_{(1)}^\alpha \\
\tilde{U}_\mu &= N_L (u_\mu - \frac{(k.u)}{k^2} k_\mu) \\
\hat{k}^\mu &= \frac{k^\mu}{k^2}
\end{aligned}
\tag{9}
$$

where,

$$
\begin{aligned}
B_\lambda &= (0,0,0,1) \\
u_\rho &= (1,0,0,0)
\end{aligned}
$$

Hence, we can write A_μ as:

$$
A_\mu = A_\parallel e_\mu^{(1)} + A_\perp e_\mu^{(2)} + A_L \tilde{U}_\mu + A_{gf} \hat{k}_\mu \tag{10}
$$

These vectors are orthogonal,

$$U_\mu.k^\mu = e^{(1)}_\mu.k^\mu = e^{(2)}_\mu.k^\mu = e^{(1)}_\mu.\tilde{U}^\mu = e^{(2)}_\mu.\tilde{U}^\mu = e^{(1)}_\mu.e^{(2)\mu} = 0$$

In absence of charge, $\nabla.E = 0$ (Gauss Law). The same can be written as :

$$\begin{aligned}\partial_i(F^{0i}) =& \partial_i(\partial^0 A^i - \partial^i A^0)\\ =& \partial^0(\partial_\mu A^\mu) - \partial_\mu\partial^\mu(A.u)\end{aligned}$$

$$\partial_i(F^{0i}) = \xi\,\partial_\mu\partial^\mu(A_L) = \quad 0. \tag{11}$$

- The above relation infers that A_L is the only form factor, that is involved in the description of the Gauss law.
- Hence, out of three remaining form factors associated with gauge field(after fixing the gauge), the constraint equation takes care of A_L part, and hence we left with two transverse form factors.

EOM in matrix form ($\phi - \gamma$ system in magnetized medium)

We can write all equations of motion in matrix form, given as:

$$\begin{bmatrix} (k^2 - \Pi_L) & iF & 0 & 0 \\ -iF & (k^2 - \Pi_T) & 0 & i\frac{g_{\phi\gamma\gamma}}{N_1} \\ 0 & 0 & (k^2 - \Pi_T) & 0 \\ 0 & -i\frac{g_{\phi\gamma\gamma}}{N_1} & 0 & (k^2 - m_\phi^2) \end{bmatrix} \begin{bmatrix} A_{\parallel}(k) \\ A_{\perp}(k) \\ A_L(k) \\ \phi(k) \end{bmatrix} = 0. \tag{12}$$

Here, $F = \Pi^p(k) N_1 N_2 [\varepsilon_{\alpha\nu\delta\beta} \frac{k^\beta}{|k|} u^{\bar{\delta}_{\parallel}}$

Equation (12) shows

- Total degrees of freedom (DOF) is four, for coupled photon-scalar system.
- One DOF corresponds to scalar field and three DOF correspond to gauge field.
- A_L decouples and evolves separately. We have found that for axions this is not true.

Coupled photon-Axion system in Magnetized media

EOM in matrix form ($a(x)-\gamma$, system in magnetized medium)

$$\begin{pmatrix} (k^2-\Pi_T) & -iF & 0 & 0 \\ iF & (k^2-\Pi_T) & 0 & -ig_{a\gamma\gamma}N_2 b^{(2)}_{\mu} I^{\mu} \\ 0 & 0 & (k^2-\Pi_L) & -ig_{a\gamma\gamma}N_L b^{(2)}_{\mu}\tilde{u}^{\mu} \\ 0 & ig_{a\gamma\gamma}N_2 b^{(2)}_{\mu} I^{\mu} & ig_{a\gamma\gamma}N_L b^{(2)}_{\mu}\tilde{u}^{\mu} & (k^2-m_a^2) \end{pmatrix} \begin{pmatrix} A_{\parallel}(k) \\ A_{\perp}(k) \\ A_L(k) \\ a(k) \end{pmatrix} = 0. \tag{13}$$

Here $F = \Pi_p N_1 N_2 P_{\mu\nu} b^{(1)\mu} I^{\nu}$

- Previously, we saw that in case of $\phi-\gamma$ mixing, the longitudinal DOF of photon is de-coupled and therefore we drop that part, but in this case (a-γ mixing), the longitudinal part is coupled with pseudoscalar field and can not be ignored.
- Consequently, the physics would be change here because of this type of coupling.

Stokes parameters

The Stokes parameters are for this system is given by:

$$\begin{aligned} \mathrm{I} &= <A_{\parallel}^{*}A_{\parallel}> + <A_{\perp}^{*}A_{\perp}>, \\ \mathrm{Q} &= <A_{\parallel}^{*}A_{\parallel}> - <A_{\perp}^{*}A_{\perp}>, \\ \mathrm{U} &= 2\mathrm{Re} < \mathrm{A}_{\parallel}^{*}\mathrm{A}_{\perp} >, \\ \mathrm{V} &= 2\,\mathrm{Im} < \mathrm{A}_{\parallel}^{*}\mathrm{A}_{\perp} > . \end{aligned} \tag{14}$$

We can find the ellipticity angle(χ) and polarization angle(Ψ) defined by

$$\begin{aligned} \tan 2\chi &= \frac{\mathrm{V}}{\sqrt{\mathrm{Q}^2 + \mathrm{U}^2}}, \\ \tan 2\Psi &= \left(\frac{\mathrm{U}}{\mathrm{Q}}\right). \end{aligned} \tag{15}$$

Ellipticity angle

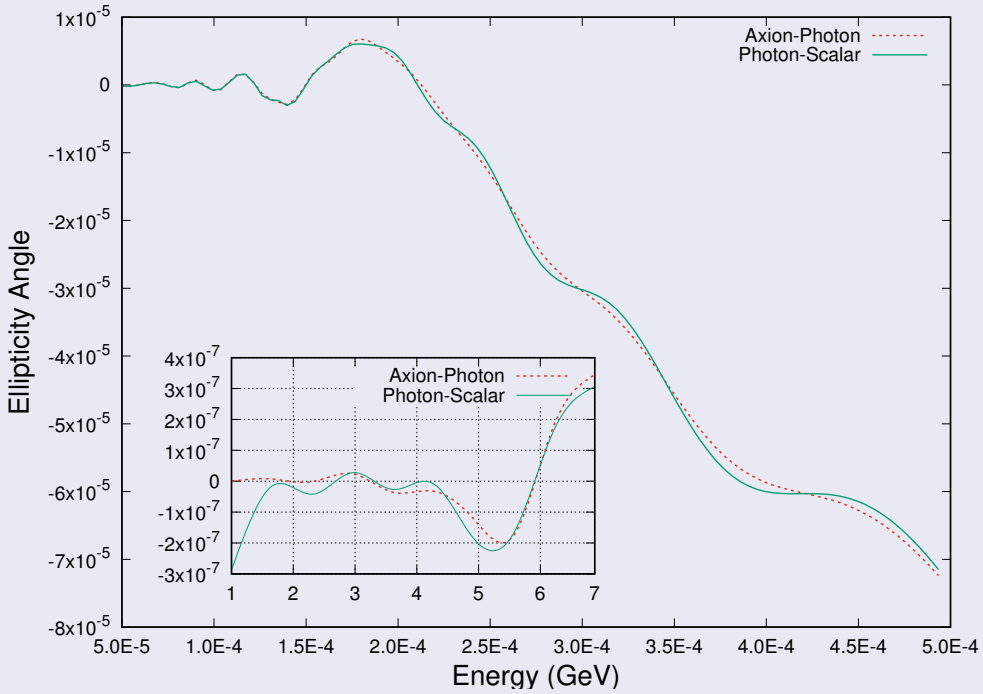

Figure: Ellipticity versus Energy. Solid line for scalar-photon and dotted line for axion-photon case.

Conclusion

- We have analysed the dynamics of scalar-photon and pseudo-scalar photon, interaction arising out of dim-5 operators in a magnetized vacuum and medium.
- In magnetized vacuum : Photons in these systems have two transverse degrees of freedom. One of them is CP preserving and the other CP violating. Scalars couple with the CP preserving part and the pseudo -scalars couple with the CP violating part. The mixing matrix however remains 2X2. Therefore the mixing scenario remains almost same.
- In magnetized media: One has a distinguishable mixing scenario, only when one includes the parity violating self energy part of photon polarization tensor (self energy) – that arises from magnetized medium effects – in the effective Lagrangian of the system.
- As a result the ellipticity angle of one becomes distinguishable from the other (even when the other parameters of the two theories like mass and coupling constants are same).

References

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Thanks