QCD-QED mixed corrections for inclusive Higgs production in $b\bar{b}$-channel

Ajjath A H

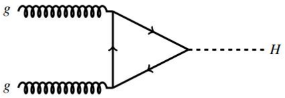
13-10-2018

Institute of Mathematical Sciences, Chennai

Motivation

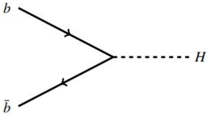
- Discovery of the Higgs boson opened up a new era in the high energy particle physics

- For the Higgs production,
  - dominant channel is $gg \to H$ through top quark loop
  - Sub-dominant ones
    - vector boson fusion
    - with $t\bar{t}$ pairs
    - $b\bar{b}$ annihilation channel *etc.*



$\downarrow$

Y_t



$\downarrow$

Y_b

Motivation

- Why we need the $b\bar{b} \to H$ contribution even it is less

  - More accurate experimental data leads to the necessity of more precise theoretcal predictions

    - to decrease the uncertainities on the measured Higgs properties
    - search for possible deviations from the standard model predictions

  - Yukawa coupling (Y_b) is less in SM, but can be enhanced in MSSM

- For the inclusive Higgs production, the state of art till today is,
  - via gg fustion channel at NNLO : Harlander, Kilgore ('02); Anastasiou, Melnikov ('02); Ravindran, Smith, van Neerven ('03)
  - via gg fusion channel at N^{3}LO : Anastasiou, Duhr, Dulat, Herzog, Mistlberger
  - via vector boson fusion at NNLO : Bolzoni, Maltoni, Moch, Zaro ('10)
  - via $b\bar{b}$ annihilation channel at NNLO : Harlander, Kilgore ('03)
  - via $b\bar{b}$ annihilation channel at N^{3}LO$_{sv}$: Taushif, Rana, Ravindran

Motivations

- One of the way to do the theoretical calculations are based on the perturbative QCD
- Results from higher order calculations shows rapid decrease in the uncertainities in the theoretical predictions
- For $b\bar{b} \rightarrow H$, with the energy scale 14 TeV and $M_H = 120GeV$, the theoretical accuracy improved

  from 70% at LO, 40% at NLO to 15% at NNLO.
- Since $\mathcal{O}(a_s^3) = \mathcal{O}(a_s a_e)$ for LHC processes, it becomes necessary to take into account the mixed QCD-QED corrections also in the perturbative expansion.

$a_s \rightarrow$ qcd coupling constant

$a_e \rightarrow$ qed coupling constant

Plan of the talk

- Introduction
- Methodology
- Challenges
- IBP and LI identities
- Reverse Unitarity
- UV divergence
- IR divergence
- Future outlook

Introduction

Collinear factorization theorem,

$$\sigma_{tot}(h_1 h_2 \to H) = \sum_{a,b} \int dx_1 dx_2 \tilde{f}_{a,h_1}(x_1, \mu_F) \tilde{f}_{b,h_2}(x_2, \mu_F)\ \sigma_{tot}(ab \to H)$$

where, $a, b = q, \bar{q}, g, \gamma$, $\tilde{f}_{a,h_1} \to$ PDF (*probability to get the parton'a' from hadron* h_1)

Partonic cross section for QCD

$$\hat{\sigma} = \hat{\sigma}^{(0,0)} + a_s \hat{\sigma}^{(1,0)} + a_s^2 \hat{\sigma}^{(2,0)} + \mathcal{O}(a_s)^3$$

Partonic cross section for QCD-QED

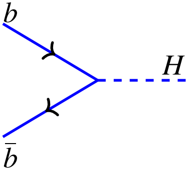
$$\sigma = \sigma^{(0,0)} + a_s \sigma^{(1,0)} + a_s^2 \sigma^{(2,0)} + \cdots + a_e \sigma^{(0,1)} + a_e^2 \sigma^{(0,2)} + \cdots + a_s a_e \sigma^{(1,1)} + h.t$$

- Our motive $\Rightarrow b\bar{b} \to H$

$$\mathcal{L} = -\lambda_b \bar{\psi}_b(x) \psi(x) \phi(x)$$

Introduction

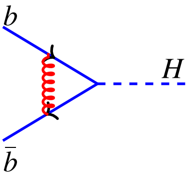
- The lowest order $\Rightarrow$ $b\bar{b}$ annihilation



Introduction

- The lowest order $\Rightarrow b\bar{b}$ annihilation
- higher orders $\Rightarrow$ Virtual + Radiative processes

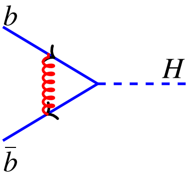
NLO-Virtual

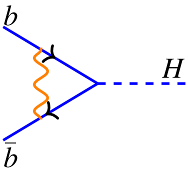


Introduction

- The lowest order $\Rightarrow b\bar{b}$ annihilation
- higher orders $\Rightarrow$ Virtual + Radiative processes

NLO-Virtual

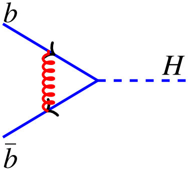


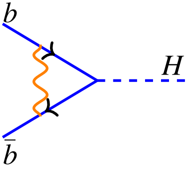


Introduction

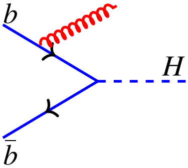
- The lowest order ⇒ $b\bar{b}$ annihilation
- higher orders ⇒ Virtual + Radiative processes

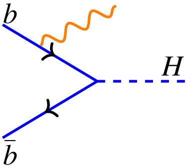
NLO-Virtual





NLO-Real

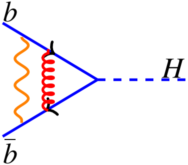


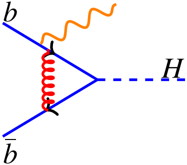


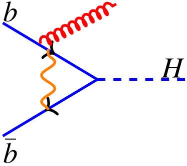
Introduction

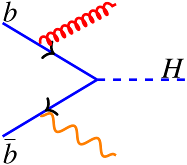
- The lowest order $\Rightarrow$ $b\bar{b}$ annihilation
- higher orders $\Rightarrow$ Virtual + Radiative processes

Double virual | *Real virtual* | *Double real*









Introduction

- For inclusive cross section, in higher orders , in addition to the $b\bar{b}$,we have contributions from gg, bg, $b\gamma$, $g\gamma$ & bb initiated processes

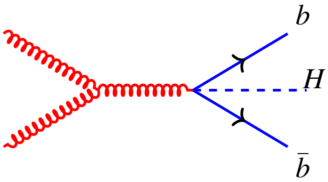
Introduction

- For inclusive cross section, in higher orders , in addition to the $b\bar{b}$,we have contributions from gg, bg, $b\gamma$, $g\gamma$ & bb initiated processes
- For each of these, we have to take into account the DV, RV, RR contibutions, if there is.

Introduction

- For inclusive cross section, in higher orders , in addition to the $b\bar{b}$,we have contributions from gg, bg, $b\gamma$, $g\gamma$ & bb initiated processes
- For each of these, we have to take into account the DV, RV, RR contibutions, if there is.

  For instance, for $g\ b \to b\ H$,



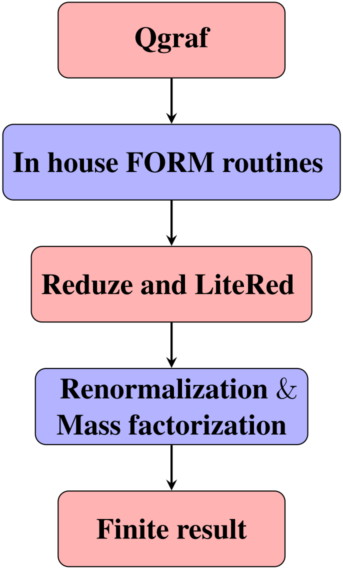
at NLO,

$$b\ \bar{b} \to H+\ \text{1-loop}$$
$$b\ \bar{b} \to H\ g$$
$$b\ \bar{b} \to H\ \gamma$$
$$g\ b \to b\ H$$
$$\gamma\ b \to b\ H$$

at NNLO,

$$b\ \bar{b} \to H+\ \text{2- loop}$$
$$b\ \bar{b} \to \gamma\ H+\ \text{1-loop}$$
$$b\ \bar{b} \to g\ H+\ \text{1-loop}$$
$$g\ b \to b\ H+\ \text{1-loop}$$
$$\gamma\ b \to b\ H+\ \text{1-loop}$$
$$b\ \bar{b} \to b\ b\ H$$
$$b\ \bar{b} \to \gamma\ g\ H$$
$$g\ b \to b\gamma\ H$$

Methodology



$\Longrightarrow$ to generate diagrams

$\Longrightarrow$ Use in-house FORM routines to convert symbolic raw output to a format usable in FORM

$\Longrightarrow$ Algebraic simplification using trace,dirac & color algebra
$\Longrightarrow$ cross section in form of numerous Feynman integrals

$\Longrightarrow$ Exploiting the symmetry
$\Longrightarrow$ Classify Feynman integrals into a few set of integral family
$\Longrightarrow$ Minimize the integrals to be computed

$\Longrightarrow$ Reguarization and cancellation of UV divergences

$\Longrightarrow$ Cancellation of soft and collinear divergences

- **Challenges**
  - Large number of diagrams
  - Numerous integrals
  - 3-body phase space integration
  - UV and IR singularities

IBP and LI identities

- Most of the integrals are not independent.
- Use IBP and LI identites : to get irreducble master integrals(MI)

Integration by Parts(IBP) identity *for an n-loop integral*,

$$\int \frac{d^d k_1}{(2\pi)^d} \cdots \frac{d^d k_n}{(2\pi)^d} \frac{\partial}{\partial k_j^\mu} v^\mu J(k_1, \cdots, k_n, p_i) = 0$$

J is a function of scalar products of momenta and propagators
$v^\mu \to$ external/loop momentum [Chetyrkin-Tkachov]

IBP and LI identities

Lorentz Invariance (LI) identity *for a 4-point function,*

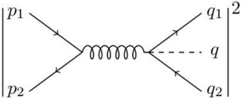
$$p_i^{\mu} p_j^{\nu} \quad \left(p_{k,\mu} \frac{\partial}{\partial p_k^{\nu}} - p_{k,\nu} \frac{\partial}{\partial p_k^{\mu}} \right) I(p_1, p_2, p_3) = 0$$

[Gehrmann-Remiddi]

- These relations gives recursion relation among the integrals, thus reduces the number of integrals to be evaluated, these final set of integrals are called master integrals (MI)
- The number of MIs depends on the number of external legs and if the legs are massive or not.
- All the master integrals appeared in our problem is known in literature

Phase space integration : Reverse Unitarity

- *Another challenge is phase space integration* : 3-body phase space for RR process!
- People has come up with an efficient technique : Reverse unitarity

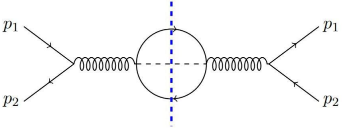


$$\propto \int \frac{d^n q_1}{(2\pi)^{n-1}} \frac{d^n q_2}{(2\pi)^{n-1}} \delta_+(q_1^2)\delta_+(q_2^2)\delta_+(q^2 - m_h^2)[\cdots]$$

$$\delta_+(q^2 - m^2) \sim \frac{1}{q^2 - m^2 + i\varepsilon} - \frac{1}{q^2 - m^2 - i\varepsilon}$$

Reverse unitarity

Anastasiou, Melnikov



$\Longrightarrow$ *two-loop virtual diagram*

- Exploit symmetries and reduce to MIs
- Put back to delta function at the end

UV divergence

- The cross section obtained is **UV** and **IR divergent** in **4-dimension**
  - UV divergence : loop integrals at high momentum limit
  - IR divergence :

    - soft gluons, at the zero momentum limit: *soft divergence*

    - massless particle, when collinear to massless external leg: *collinear divergence*

- UV renormalization

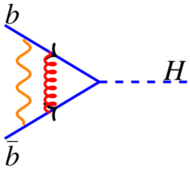
  - **Coupling constant renormalization** :

$$\hat{a}_s = Z_{a_s} a_s \left(\frac{\mu_0}{\mu_R} \right)^{\epsilon} ; \;\; Z_{a_s} = 1 + \mathcal{O}(a_s)$$
$$\hat{a}_e = Z_{a_e} a_e \left(\frac{\mu_0}{\mu_R} \right)^{\epsilon} ; \;\; Z_{a_e} = 1 + \mathcal{O}(a_e)$$

  - Yukawa coupling renormalization

Yukawa coupling renormalization

- We encounter a new vertex renormalization(yukawa coupling renormalization) : need of $\Rightarrow Z_\lambda^{(\mathrm{qcd-qed})}$-expansion



- Solving the renormalization group equation in $d = 4 + \epsilon$,

$$\mu_R^2 \frac{d}{d\mu_R^2} \ln Z_\lambda^{(\mathrm{qcd-qed})}(a_s, a_e, \epsilon) = \sum_{i,j=1}^{\infty} a_s^i \, a_e^j \, \gamma_{(i,j)}$$

$$Z_\lambda = 1 + a_s \frac{2\gamma_{(1,0)}}{\epsilon} + a_e \frac{2\gamma_{(0,1)}}{\epsilon} + a_s a_e \left(\frac{4\gamma_{(1,0)}\gamma_{(0,1)}}{\epsilon^2} + \frac{\gamma_{(1,1)}}{\epsilon} \right) + \mathrm{h.t}$$

$\gamma_{(1,0)} = 3C_F$ $\quad$ *$\gamma_{(0,1)}$ and $\gamma_{(1,1)}$ are unknown* $\quad$ $\gamma(i,j) \rightarrow UV$ *anomalous dimension*

Yukawa coupling renormalization

- How to find $\gamma_{(0,1)} \& \gamma_{(1,1)}$: Exploit the universality of soft (f_q) and collinear (B_q) anomalous dimension!
- The coefficient of $1/\epsilon$ of logarithmic form factor can be written as,

$$G^q_{(i,j)}|_{\epsilon\to 0} = 2B^q_{(i,j)} - 2\gamma^q_{(i,j)} + f^q_{(i,j)}$$

[Ravindran('05)]

f^q & B^q are universal. Independent of flavor or final leg of the process

$\Rightarrow f^q_{(0,1)} \& f^q_{(1,1)}$ are extracted from the known result of Drell-Yan, exploiting the fact that $\gamma^q_{(i,j)} = 0$ for DY upto $(i,j) = (1,1)$ [de Florian, Der, Fabre ('18)]

$\Rightarrow B^q_{(0,1)} \& B^q_{(1,1)}$ is obtained from the splitting kernels [de Florian, Sborlini, Rodrigo ('17)]

Thus, we obtain,

⇒ Yukawa coupling renormalization,

$$\hat{\lambda} = Z_\lambda \, \lambda \left(\frac{\mu_0}{\mu_R} \right)^{\epsilon} ,$$

$$Z_\lambda = 1 + a_s \frac{6C_F}{\epsilon} + a_e \frac{6Q_b^2}{\epsilon} + a_s a_e \left(\frac{36 C_F Q_b^2}{\epsilon^2} + \frac{3 C_F Q_b^2}{\epsilon} \right)$$

IR divergence-soft

- Removing soft divergence

$$\left.\begin{array}{l}\text{quark with a virtual gluon}\\ \text{quark accompanied by a real gluon}\end{array}\right\}\text{indistiguishable: degenerate states}$$

- summation over these degenerate states eliminate the soft - divergence (KLN theorem).

$$\sum_{\text{deg.final state}} \text{virtual} + \text{real} \rightarrow \text{soft free}$$

IR divergence-collinear

- The same way we can eliminate the final state collinear divergence too.

$$\left.\begin{array}{l}\text{massless quark with a virtual gluon}\\ \text{massless quark accompanied by a real collinear gluon}\end{array}\right\}\text{degenerate states}$$

- Unlike to soft, here, the external massless partons are involved : massless virtual or real partons collinear to external massless partons

$$\sum_{\text{initial state}}\sum_{\text{deg.final state}} \text{virtual} + \text{real} \rightarrow \text{collinear free}$$

- This can be accomplished by Mass factorization, where we redefine the PDFs in such a way that the collinear divergence of bare partonic cross section is removed.

Mass factorization

$$\hat{\sigma}_{ab}(z, Q^2, 1/\epsilon) = \sum_{c,d=b,\bar{b},g,\gamma} \Gamma_{ca}(z,^2, 1/\epsilon) \otimes \sigma_{cd}(z, Q^2, \mu_F^2) \otimes \Gamma_{db}(z,^2, 1/\epsilon)$$

Collinear factorization theorem,

$$\sigma_{tot}(h_1 h_2 \to H) = \sum_{i,j} \int dx_1 dx_2 f_{i,h_1}(x_1, \mu_F) f_{j,h_2}(x_2, \mu_F) \hat{\sigma}_{tot}(ij \to H)$$

$$= \sum_{i,j} \int dx_1 dx_2 \tilde{f}_{i,h_1}(x_1, \mu_F) \tilde{f}_{j,h_2}(x_2, \mu_F) \sigma_{tot}(ij \to H)$$

where $\tilde{f} = f \otimes \Gamma$ $\quad$ $\tilde{f}$ & σ *are finite,* $\quad$ $\Gamma \to$ *splitting function*

- Thus we obtain the *finite partonic cross section*
- We can perform the numerical analysis using PDFs which are experimentally obtained, and thus predict the contribution

Future Outlook

- This is first step towards the full EW corrections for the single Higgs production
- Extend the work to compute the fully differential distributions using q_t-substraction method at NNLO mixed QCD-QED

Future Outlook

- This is first step towards the full EW corrections for the single Higgs production
- Extend the work to compute the fully differential distributions using q_t-substraction method at NNLO mixed QCD-QED

Thank You