Predictions of Angular Observables of $B \to \rho\ell\ell$ and $B_s \to \bar{K}^*\ell\ell$ in Standard Model

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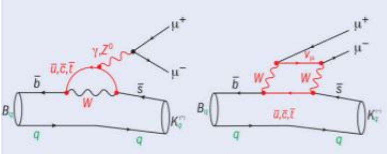
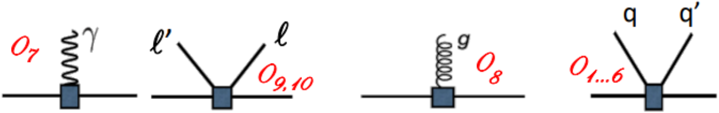
Theoretical Overview: Quark level

$b \to s, d$ quark transitions are Flavor Changing Neutral Currents

- in SM they occur through loop at leading order



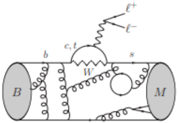
- Heavy particles are integrated out to write the effective Hamiltonian $H_{eff} \propto C_i O_i$
- Operators with non-zero contribution in Standard model are:



Theoretical Overview: Hadronic level

- The amplitude for hadronic transition is

$$\mathcal{A}(B \to F\ell\ell) = \langle F|\mathcal{A}(b \to q\ell\ell)|B\rangle \propto C_i L_{i,\mu} \langle F|H_i^{\mu}|B\rangle$$



- Hadronic matrix elements are written in terms of form factors.

Theoretical Overview: Hadronic level

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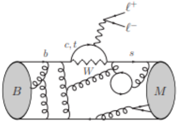


- Hadronic matrix elements are written in terms of form factors.
- $b \to d$ vs $b \to s$

CKM Factor	i= s	i= d
$V_{ti}^* V_{tb}$	λ^2	λ^3
$V_{ci}^* V_{cb}$	λ^2	λ^3
$V_{ui}^* V_{ub}$	$\lambda^4(\rho - i\eta)$	$\lambda^3(\rho - i\eta)$

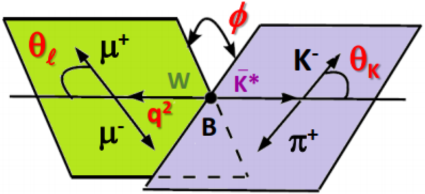
Experimentally Accessible

More sensitive to new physics

CP Violation

Theoretical Overview: Decay distribution

- Angular distribution depends on q^2 and three angles



- Theoretically, decay distribution is given as,

$$\frac{d^4\Gamma}{dcos\theta_l \; dcos\theta_K \; d\phi \; dq^2} = \frac{9}{32\pi}\sum_{i=1}^{12} I_i \; \Omega(\theta_l, \theta_K, \phi)$$

- $I_i \propto A_i A_j^*$; $A_i \propto \sum_j C_j O_j$
- Theoretically, $I_i s$ can be isolated by integrating over different ranges of parameters.

Experimental Status

- Experimental Status: $b \to s\ell\ell$
  - Branching ratio of $B \to K, K^*\mu\mu$, $B_s \to \phi\mu\mu$ is too low wrt SM.
  - Discrepancy in angular observables for $B \to K^*\mu\mu$ (P_5', R_{K,K^*}) has been confirmed by many experiments.
- Potential sources of anomalies
  - due to lack of knowledge of form factors and non-factorizable contribution
  - New physics Global Analysis $\Longrightarrow$ $C_9^{\text{NP}} = -1$
- Experimental status: $b \to d\ell\ell$: Branching ratio of $B \to \pi\ell\ell$ is in good agreement with expected value in SM.
- **In this talk, I will discuss angular observables for semileptonic decays based on $b \to d\ell\ell$: $B_s \to \bar{K}^*\ell\ell$ and $B \to \rho\ell\ell$.**

The Amplitude

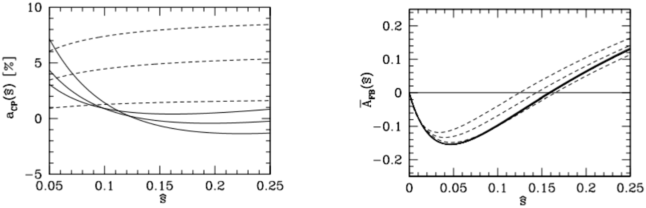
$$\mathcal{M} = \frac{G_F \alpha}{\sqrt{2\pi}} V_{tb} V_{td}^{*} \{ [C_9^{eff} \langle V | O_9^{\mu} | B \rangle - \frac{2 m_b}{q^2} C_9^{eff} \langle V | O_7^{\mu} | B \rangle] (\bar{\ell} \gamma_{\mu} \ell) \\ + C_{10} \langle V | O_{10}^{\mu} | B \rangle (\bar{\ell} \gamma_{\mu} \gamma_5 \ell) - 16 \pi^2 \frac{\bar{\ell} \gamma^{\mu} \ell}{q^2} H_{\mu}^{non-fac} \}$$

Sources of error:

- Wilson Coefficients
- Form factors
- Non-factorizable contribution

Wilson Coefficients

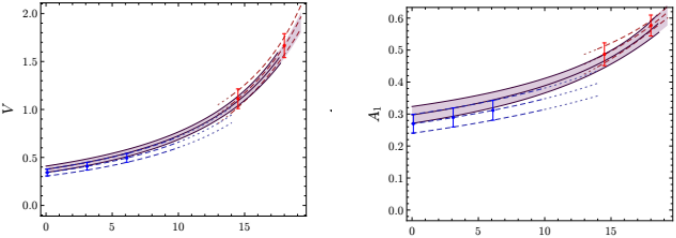
- Wilson Coefficients have been computed at next-to-next-to leading order (NNLO). Matching has been performed at two loop level.
  [K.Bieri, C.Greub, M.Walker, H.M.Asatrian] Phys.Rev.D (2004)



Solid Lines: NNLO, Dashed Lines: NLO

Form Factors

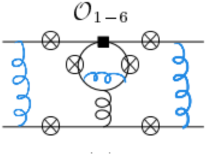
- Hadronic matrix elements are expressed in terms of Form factors which are computed using various QCD techniques like LCSR, lattice QCD, pQCD.. $\langle M(p', \epsilon)|\mathcal{H}_{eff}|B(p)\rangle = f(p, p', \epsilon)_i F_i$
- LCSR and Lattice QCD provide form factors at low q^2 ($<$14 GeV^2) and high q^2 ($>$ 16 GeV^2) respectively.



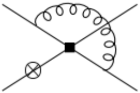
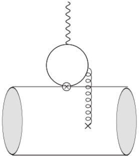
- We consider two set of form factors [A.Bharucha, D.M.Straub, R.Zwicky] JHEP (2016) :
  1. LCSR
  2. LCSR + Lattice QCD

Non-Factorizable corrections

- Some corrections can not be absorbed in Wilson coefficients which break factorization. $\implies A(B \to M\ell\ell) = \sum_i C_i O_i + nf$



- Non-factorization corrections considered are:
  1. Spectator Scattering [M.Beneke, T.Feldmann, D. Seidel] Nucl. Phys. B (2001)
  2. Weak Annihilation
  3. Soft gluon emission [A.Khodjamirian, T. Mannel, A.Pivovarov, Y. Wang] JHEP (2010)

Results: $\bar{B}_s \to K^* \mu^+ \mu^-$

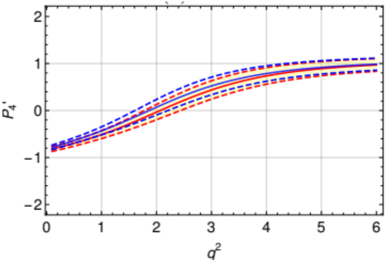
- Observables:
  - $$P_1 = \frac{I_3}{2I_2^s} \qquad P_2 = \beta_l \frac{I_6^s}{8I_2^s}, \qquad P_3 = \frac{I_9}{4I_2^s}, \qquad P_4' = \frac{I_4}{\sqrt{I_2^s I_2^c}},$$
    $$P_5' = \frac{I_5}{2\sqrt{I_2^s I_2^c}}, \quad P_6' = \frac{I_7}{2\sqrt{I_2^s I_2^c}}, \quad P_8' = \frac{I_9}{2\sqrt{I_2^s I_2^c}}$$
  - Branching Ratio: $d\Gamma/dq^2 = \frac{1}{4}(3I_1^c + 6I_1^s - I_2^c - 2I_2^s)$
  - $A_{FB} = -\frac{3}{4} I_6^s d\Gamma/dq^2$, $F_L = \frac{3I_1^c - I_2^c}{d\Gamma/dq^2}$
  - LFUV: $R_{K^*}^{B_s} = \frac{\mathcal{BR}(B_s \to \bar{K}^* \mu^+ \mu^-)}{\mathcal{BR}(B_s \to \bar{K}^* e^+ e^-)}$
- Computed in two bins: $[0.1 - 1]\mathrm{GeV}^2$ and $[1 - 6]\mathrm{GeV}^2$
- SM prediction for branching ratio using LCSR+lattice and LCSR are [B.Kindra, N.Mahajan] Phys.Rev.D (2018):

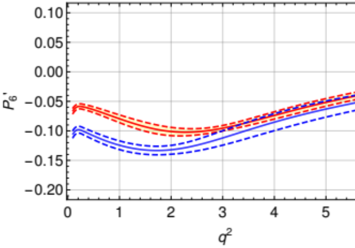
$$\mathcal{BR}(B_s \to \bar{K}^* \mu^+ \mu^-) = (3.356 \pm 0.814) \times 10^{-8}$$
$$\mathcal{BR}(B_s \to \bar{K}^* \mu^+ \mu^-) = (2.849 \pm 0.719) \times 10^{-8}$$

is consistent with experimental value [LHCb] JHEP (2018) $\mathcal{BR}(B_s \to \bar{K}^* \mu^+ \mu^-) = (3.0 \pm 1.0 \pm 0.2 \pm 0.3) \times 10^{-8}$

Results: $\bar{B}_s \to K^* \mu^+ \mu^-$

- P_6' and P_8' depend on $Im(A_i A_j^*)$





- Red: $B_s \to \bar{K}^* \ell\ell$; Blue: $\bar{B}_s \to K^* \ell\ell$

$B \to \rho\ell\ell$

- ρ is a self conjugate state.
- In $B_s \to \bar{K}^*\ell\ell$ final state tags the decaying meson.
- For $B \to \rho\ell\ell$, the information is available at some time of tagging at Belle and not available at LHCb at all.
- Study of time evolution of B meson is important.

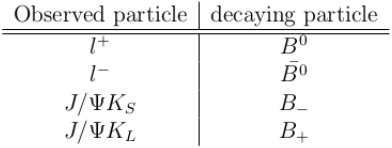
Theoretist's eye view of Experiments

Production Mechanism: LHCb

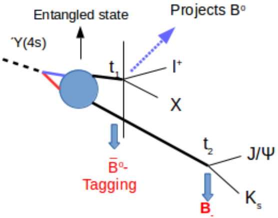
- p-p beams collide to create $b\bar{b}$ quarks.
- Quarks then hadronize with other quarks to form hadrons.
- B decays are then studied through reconstruction of final states.

Production Mechanism: Belle

- At Belle, $e^+ - e^-$ beams collide at center of mass energy $\sim 2M_b$ to create $\Upsilon(4S)$ which is a bound state of $b\bar{b}$.
- $\Upsilon(4S)$ decays to $B\bar{B}$ pairs which are in an entangled state.
- Tagging Mechanism is used to find the flavor of decaying particle.



Observed particle	decaying particle
l^+	B^0
l^-	$\bar{B}^0$
$J/\Psi K_S$	B_-
$J/\Psi K_L$	B_+



$B \to \rho\mu^+\mu^-$

- The tagged meson undergoes $B - \bar{B}$ mixing.
- Time evolution of transversity amplitudes:

$$A_i(t) = A_i(B(t) \to \rho(\pi\pi)\ell\ell) = g_+(t)A_i(0) + \frac{q}{p}g_-(t)\tilde{A}_i(0)$$

$$\tilde{A}_i(t) = A_i(\bar{B}(t) \to \rho(\pi\pi)\ell\ell) = \frac{p}{q}g_-(t)A_i(0) + g_+(t)\tilde{A}_i(0)$$

- q/p=$e^{-i\phi}$; $\phi_{B_d} \approx \pi/4$, $\phi_{B_s} \approx 0$
- The number of final state in time interval [t,t+δt] is $\langle f|T|i\rangle^2\,\delta t$. Time-Integrated observables
- $\{t_1, t_2\} \to \{t_+, t_-\}$, where $t_+ \in [|t_-|, \infty]$ and $|t_-| \in [-\infty, \infty]$.
- At LHCb, there is no correlation between the meson pair and no of events is $\propto |\mathcal{M}|^2 dt$ where $t \in \{0, \infty\}$.

$B \to \rho\mu^+\mu^-$

- Thus, angular coefficients are integrated over time and given as [Genon, S and Virto, J JHEP 1504 (2015)],

$$\left\langle J_i + \tilde{J}_i \right\rangle_{LHCb} = \frac{1}{\Gamma}\left(\frac{J_i + \tilde{J}_i}{1-y^2} - \frac{y}{1-y^2} \times h_i\right)$$

$$\left\langle J_i - \tilde{J}_i \right\rangle_{LHCb} = \frac{1}{\Gamma}\left(\frac{1}{1+x^2} \times (J_i - \tilde{J}_i) - \frac{x}{1+x^2} \times s_i\right)$$

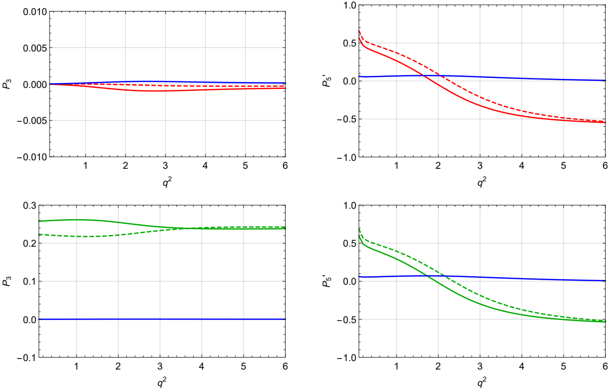
$$\left\langle J_i + \tilde{J}_i \right\rangle_{Belle} = \frac{2}{\Gamma}\frac{1}{1-y^2}(J_i + \tilde{J}_i)$$

$$\left\langle J_i - \tilde{J}_i \right\rangle_{Belle} = \frac{2}{\Gamma}\frac{1}{1+x^2} \times (J_i - \tilde{J}_i)$$

- $y = \Delta\Gamma/(2\Gamma) = 0$; $x = \Delta m/\Gamma = 0.77$; $h_i \propto Re(q/p\tilde{A}_i A_j{}^*)$; $s_i \propto Im(q/p\tilde{A}_i A_j{}^*)$
- Observables are obtained by replacing $I_i \to \langle J_i \rangle$

$B \to \rho\mu^+\mu^-$: Results

$$P'_3 = \frac{I_9}{2\sqrt{-I_2^s I_2^c}} \qquad P'_5 = \frac{I_5}{2\sqrt{-I_2^s I_2^c}}$$



Red(Dashed): $B(\bar{B}) \to \rho\mu^+\mu^-$ at Belle, Green(Dashed): $B(\bar{B}) \to \rho\mu^+\mu^-$ at LHCb with tagging, Blue: LHCb without tagging.

Conclusion

- Deviations from SM have been observed in $b \to s\ell\ell$.
- There are two measurements for $b \to d\ell\ell$ decays. Both in consistent with SM.
- In our work, we have provided predictions for observables which can be tested when data is available.
- Better determination of form factors in future will reduce the uncertainty in theoretical predictions.

Thank you!

	$B \to \rho\mu^+\mu^-$		$\bar{B} \to \rho\mu^+\mu^-$	
	[0.1-1] GeV^2	[1-6] GeV^2	[0.1-1] GeV^2	[1-6] GeV^2
$\langle P_1 \rangle$(Belle)	$0.011 \pm 0.181 \pm 0.001$	$-0.059 \pm 0.110 \pm 0.003$	$0.112 \pm 0.179 \pm 0.001$	$-0.063 \pm 0.113 \pm 0.003$
(LHCb)	0.050 ± 0.181	-0.044 ± 0.110	-0.034 ± 0.179	-0.061 ± 0.111
$\langle P_2 \rangle$(Belle)	$0.083 \pm 0.010 \pm 0.001$	$0.073 \pm 0.053 \pm 0.023$	$0.008 \pm 0.009 \pm 0.0$	$0.0105 \pm 0.050 \pm 0.024$
(LHCb)	0.083 ± 0.010	0.074 ± 0.053	0.078 ± 0.009	0.002 ± 0.050
$\langle P_3 \rangle$(Belle)	$0 \pm 0.005 \pm 0.0$	$0.001 \pm 0.005 \pm 0.002$	$0 \pm 0.001 \pm 0.0$	$0.001 \pm 0.005 \pm 0.002$
(LHCb)	-0.228 ± 0.044	-0.229 ± 0.028	0.261 ± 0.050	0.240 ± 0.030
$\langle P'_4 \rangle$(Belle)	$-0.618 \pm 0.076 \pm 0.047$	$0.467 \pm 0.161 \pm 0.029$	$-0.586 \pm 0.076 \pm 0.046$	$0.529 \pm 0.155 \pm 0.017$
(LHCb)	-0.591 ± 0.077	0.470 ± 0.161	-0.616 ± 0.075	0.526 ± 0.155
$\langle P'_5 \rangle$(Belle)	$0.332 \pm 0.043 \pm 0.027$	$-0.211 \pm 0.084 \pm 0.085$	$0.300 \pm 0.040 \pm 0.030$	$-0.263 \pm 0.075 \pm 0.098$
(LHCb)	0.368 ± 0.043	-0.178 ± 0.084	0.331 ± 0.039	-0.228 ± 0.075
$\langle P'_6 \rangle$(Belle)	$-0.050 \pm 0.004 \pm 0.001$	$-0.064 \pm 0.004 \pm 0.002$	$-0.088 \pm 0.006 \pm 0.001$	$-0.076 \pm 0.009 \pm 0.002$
(LHCb)	-0.030 ± 0.003	-0.042 ± 0.003	-0.109 ± 0.008	-0.099 ± 0.010
$\langle P'_8 \rangle$(Belle)	$0.013 \pm 0.002 \pm 0.016$	$0.012 \pm 0.001 \pm 0.018$	$0.014 \pm 0.005 \pm 0.002$	$0.020 \pm 0.003 \pm 0.017$
(LHCb)	-0.133 ± 0.021	0.113 ± 0.013	-0.145 ± 0.024	0.124 ± 0.014
$\langle R_\rho \rangle$(Belle)	$0.938 \pm 0.203 \pm 0.001$	$0.997 \pm 0.278 \pm 0.0$	$0.939 \pm 0.167 \pm 0.002$	$0.998 \pm 0.362 \pm 0.0$
(LHCb)	0.955 ± 0.194	1.036 ± 0.289	0.954 ± 0.192	1.033 ± 0.289
$\langle BR \rangle \times 10^9$(Belle)	$4.078 \pm 0.585 \pm 0.080$	$7.908 \pm 1.549 \pm 0.366$	$3.653 \pm 0.529 \pm 0.077$	$7.626 \pm 1.504 \pm 0.365$
(LHCb)	2.165 ± 0.302	4.064 ± 0.778	1.977 ± 0.273	3.943 ± 0.756
$\langle A_{FB} \rangle$(Belle)	$-0.045 \pm 0.005 \pm 0.001$	$-0.023 \pm 0.018 \pm 0.007$	$-0.041 \pm 0.005 \pm 0.001$	$-0.003 \pm 0.016 \pm 0.006$
(LHCb)	-0.046 ± 0.005	-0.024 ± 0.018	-0.041 ± 0.001	-0.011 ± 0.002
$\langle F_L \rangle$(Belle)	$0.414 \pm 0.067 \pm 0.014$	$0.822 \pm 0.039 \pm 0.007$	$0.431 \pm 0.069 \pm 0.014$	$0.832 \pm 0.038 \pm 0.006$
(LHCb)	0.409 ± 0.067	0.822 ± 0.039	0.437 ± 0.068	0.832 ± 0.037

	$\bar{B}_s \to K^* \mu^+ \mu^-$		$B_s \to \bar{K}^* \mu^+ \mu^-$	
Observable	[0.1-1] GeV^2	[1-6] GeV^2	[0.1-1] GeV^2	[1-6] GeV^2
P_1	$0.012 \pm 0.129 \pm 0.001$	$-0.081 \pm 0.111 \pm 0.005$	$0.011 \pm 0.135 \pm 0.001$	$-0.075 \pm 0.108 \pm 0.005$
P_2	$0.118 \pm 0.013 \pm 0.001$	$0.112 \pm 0.072 \pm 0.036$	$0.112 \pm 0.013 \pm 0.001$	$0.142 \pm 0.071 \pm 0.034$
P_3	$0.001 \pm 0.002 \pm 0.0$	$0.004 \pm 0.010 \pm 0.002$	$0.001 \pm 0.007 \pm 0.0$	$0.003 \pm 0.010 \pm 0.002$
P_4'	$-0.593 \pm 0.057 \pm 0.009$	$0.464 \pm 0.164 \pm 0.014$	$-0.650 \pm 0.060 \pm 0.008$	$0.379 \pm 0.171 \pm 0.016$
P_5'	$0.547 \pm 0.051 \pm 0.016$	$-0.286 \pm 0.125 \pm 0.046$	$0.543 \pm 0.053 \pm 0.016$	$-0.273 \pm 0.132 \pm 0.047$
P_6'	$-0.104 \pm 0.006 \pm 0.016$	$-0.095 \pm 0.011 \pm 0.002$	$-0.069 \pm 0.005 \pm 0.001$	$-0.078 \pm 0.004 \pm 0.002$
P_8'	$0.015 \pm 0.003 \pm 0.016$	$0.040 \pm 0.004 \pm 0.017$	$0.044 \pm 0.003 \pm 0.016$	$0.034 \pm 0.002 \pm 0.019$
$R_{K^*}^{B_s}$	$0.940 \pm 0.009 \pm 0.001$	$0.998 \pm 0.004 \pm 0.0$	$0.942 \pm 0.008 \pm 0.001$	$0.998 \pm 0.004 \pm 0.0$
$BR \times 10^9$	$3.812 \pm 0.450 \pm 0.086$	$7.803 \pm 1.758 \pm 0.357$	$4.411 \pm 0.560 \pm 0.101$	$8.391 \pm 1.856 \pm 0.375$
A_{FB}	$-0.060 \pm 0.008 \pm 0.001$	$-0.029 \pm 0.020 \pm 0.009$	$-0.056 \pm 0.008 \pm 0.001$	$-0.036 \pm 0.020 \pm 0.009$
F_L	$0.453 \pm 0.067 \pm 0.014$	$0.853 \pm 0.038 \pm 0.007$	$0.464 \pm 0.064 \pm 0.014$	$0.851 \pm 0.038 \pm 0.007$