Angularity Distributions at Next-to-Leading Order

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Event shapes are Infrared and Collinear (IRC) safe jet observables that measure the geometric properties of energy flow in QCD events.

Bell, et.al. 1808.07867

Thrust- \[ \tau = \max_{\hat{n}} \frac{1}{Q} \sum_{i \in X} |\vec{p}_i \cdot \hat{n}| \]

Jet Broadening- \[ B = \frac{1}{Q} \sum_{i \in X} |p_{i\perp}| \]
Event shapes are Infrared and Collinear (IRC) safe jet observables that measure the geometric properties of energy flow in QCD events.

- **Thrust-** \[ \tau = \frac{1}{Q} \left[ \sum_{i \in L} |p_i^+| + \sum_{i \in R} |p_i^-| \right] \]

- **Jet Broadening-** \[ B = \frac{1}{Q} \left[ \sum_{i \in L} \sqrt{p_i^+ p_i^-} + \sum_{i \in R} \sqrt{p_i^- p_i^+} \right] \]

\[ p^\pm = p^0 \mp p^3 \]
Jet Angularities

**Thrust:**
\[ \tau = \frac{1}{Q} \left[ \sum_{i \in L} |p_i^+| + \sum_{i \in R} |p_i^-| \right] \]

**Broadening:**
\[ B = \frac{1}{Q} \left[ \sum_{i \in L} \sqrt{p_i^+ p_i^-} + \sum_{i \in R} \sqrt{p_i^- p_i^+} \right] \]

- Berger, Kucs, Sterman, 03

\[ \tau_b = \frac{1}{Q} \left[ \sum_{i \in L} (p_i^+)^\frac{1+b}{2} (p_i^-)^\frac{1-b}{2} + \sum_{i \in R} (p_i^+)^\frac{1-b}{2} (p_i^-)^\frac{1+b}{2} \right] \quad (1) \]

- For Infrared safety : \(-1 < b < \infty\).
- Generalization to ‘thrust’ \((b = 1)\) and jet ‘broadening’ \((b = 0)\).
- Varying ‘\(b\)’ changes the sensitivity to the substructure of jets.
Jet Angularities are novel observables that allow us to transform between recoil-insensitive to recoil-sensitive observables in a continuous manner.
Status of event shapes

Thrust*Broadening# Angularities@
b>1
(NLL) (N3LL')
b=-1 b=0 b=1

*Catani, Trentadue, Turnock, Webber, 93; Florian, Grazzini, 04; Schwartz, 07; Becher, Schwartz, 08; Abbate, Fickinger, Hoang, Mateu, Stewart, 10

#Dokshitzer, Lucenti, Marchesini, Salam, 98; Becher, Bell, Neubert, 11; Chiu, Jain, Neill, Rothstein, 11

@Hornig, Lee, Ovanesyan, 09

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Figure: Factorization of the hard scattering process into individual hard, jet and soft functions.

\[ \mathcal{L}_{SCET} = \mathcal{L}_{n\text{-coll}} J + \mathcal{L}_{\bar{n}\text{-coll}} \bar{J} + \mathcal{L}_{\text{soft}} S + \text{power-correction} \]

\[ d\sigma = \text{Hard} \cdot J_n \otimes \bar{J}_{\bar{n}} \otimes S \]  

- Factorization properties of QCD in the soft/collinear limit allows for the separation of the process into hard, jet and soft sectors.
- All factorized sectors depend only on a single dynamical scale and the scale of factorization.
Jet Angularity Cross-Section

- \( b > 0 \)

\[
\left[ \frac{1}{\sigma_0} \frac{d\sigma}{d\tau} \right]_{\text{sing}}^{\text{NLO}} = \frac{\alpha_s \ C_F}{\pi} \left\{ -\frac{3}{(1+b)} \frac{1}{\tau} - \frac{4}{1+b} \frac{\ln \tau}{\tau} \right\} + \frac{4}{b (1+b)} \sum_{n=1}^{N=[1/b]} \frac{c_n}{\tau^{1-nb}} \\
\text{with,} \\
\quad c_1 = b, \quad c_2 = -\frac{1}{2} b(1+2b), \quad c_3 = \frac{1}{6} b(2+9b+9b^2), \ldots
\]

- \( b < 0 \)

\[
\left[ \frac{1}{\sigma_0} \frac{d\sigma}{d\tau} \right]_{\text{sing}}^{\text{NLO}} = \frac{\alpha_s \ C_F}{\pi} \left\{ -\frac{3}{(1+b)} \frac{1}{\tau} - \frac{4}{(1+b)^2} \frac{\ln \tau}{\tau} \right\} - \frac{4}{b (1+b)} \sum_{n=1}^{N=[1/|b|]-1} \frac{c'_n}{\tau^{1+nb}} \\
\text{with,} \\
\quad c'_1 = -\frac{b}{1+b}, \quad c'_2 = \frac{b(1-b)}{2(1+b)^2}, \quad c'_3 = \frac{b(-2+5b-2b^2)}{6(1+b)^3}, \ldots
\]

where, \([\ldots]\) signifies an integer strictly less than \(1/b\) for \(b > 0\) and \(1/|b| - 1\) for \(b < 0\) case.
Estimation of the size of the corrections

<table>
<thead>
<tr>
<th>b</th>
<th>N = max(n)</th>
<th>% correction for $\tau \sim 0.05$</th>
<th>% correction for $\tau \sim 0.1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0.8</td>
<td>1</td>
<td>4</td>
<td>10</td>
</tr>
<tr>
<td>0.5</td>
<td>1</td>
<td>10</td>
<td>20</td>
</tr>
<tr>
<td>0.25</td>
<td>3</td>
<td>17</td>
<td>30</td>
</tr>
<tr>
<td>0</td>
<td>$\infty$</td>
<td>31</td>
<td>45</td>
</tr>
<tr>
<td>-0.2</td>
<td>3</td>
<td>16</td>
<td>27</td>
</tr>
<tr>
<td>-0.3</td>
<td>2</td>
<td>8</td>
<td>14</td>
</tr>
<tr>
<td>-0.5</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table: Numerical estimation of the size of subleading corrections in the peak region $\sim 0.05 - 0.1$, for different values of $b$. For a given $b$ value, the maximum value of $n$ up-to which the correction terms are singular is represented by the values given in the second column of the table.
Comparison to numerical data from \texttt{EVENT2} generator

\textbf{Figure:} Differences between \texttt{EVENT2} and our results from broadening-like factorization at NLO for $d\sigma/d\log_{10}\tau$ for different $b$ values.
• Jet angularities provide a novel way of looking into the substructure which remains unexposed while looking at a single event shape observable.

• A broadening-like factorization for angularities provides the correct distribution for all $b > -1$ angularities while a thrust-like factorization works only in a certain range.

• The fixed order angularity distributions with a broadening-like factorization suggest that the recoil effects are always important for $b < 1$ angularities.

• The subleading singular contribution in our analysis for $0 < b < 1$ provide a significant contribution in the peak region. This is expected to effect the resummation of these observables and hence the extraction of the strong coupling.

