Compatibility of A_4 Flavour Symmetric Minimal Extended Seesaw with $(3+1)$ Neutrino Data

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Plan of talk

Introduction

• Non-zero neutrino masses and large leptonic mixing have now become a well established fact, thanks to a series of results from several experiments over the last twenty years.

• While the solar and atmospheric mixing angles plus mass squared difference measurements have become more precise with time, the evidence for a non-zero reactor mixing angle emerged with the relatively recent experiments like MINOS, T2K, NOνA, Double ChooZ, Daya-Bay and RENO.

• Apart from the currently unknown parameters in the neutrino sector, like mass hierarchy, Dirac CP violating phase etc, another interesting question in the neutrino sector is the possibility of additional neutrino species with eV scale mass.

• We adopt a minimal framework known as the minimal extended seesaw and study different possible realisations within the framework of non-abelian discrete flavour symmetry A_4.

Our Model

$$
\begin{aligned}
\mathcal{L}_Y = {} & \frac{y_e}{\Lambda}(\bar{l}H\phi)_{\underline{1}}e_R + \frac{y_\mu}{\Lambda}(\bar{l}H\phi)_{\underline{1'}}\mu_R + \frac{y_\tau}{\Lambda}(\bar{l}H\phi)_{\underline{1''}}\tau_R + \frac{y_1}{\Lambda}(\bar{l}H\phi)_{\underline{1}}\nu_{R1} \\
& + \frac{y_2}{\Lambda}(\bar{l}H\phi')_{\underline{1''}}\nu_{R2} + \frac{y_3}{\Lambda}(\bar{l}H\phi'')_{\underline{1}}\nu_{R3} + \frac{1}{2}\lambda_1\xi\overline{\nu^c_{R1}}\nu_{R1} + \frac{1}{2}\lambda_2\xi'\overline{\nu_{R2}}^c\nu_{R2} \\
& + \frac{1}{2}\lambda_3\xi\overline{\nu_{R3}}^c\nu_{R3} + \frac{1}{2}\rho\chi\overline{S^c}\nu_{R1} + y_4\xi\overline{S^c}\nu_{R2} + y_5\chi^\dagger\overline{S^c}\nu_{R3} + \text{h.c.}
\end{aligned} \tag{1}
$$

where Λ is the cut-off scale, $y_e, y_\mu, y_\tau, y_1, y_2, y_3, y_4, y_5, \lambda_1, \lambda_2, \lambda_3, \rho$ are the dimensionless Yukawa couplings.

• The particle content of the model along with their transformations under the symmetries of the model are shown in table.

	l	e_R	μ_R	τ_R	H	ϕ	ϕ'	ϕ''	ξ	ξ'	χ	ν_{R1}	ν_{R2}	ν_{R3}	S
$SU(2)_L$	2	1	1	1	2	1	1	1	1	1	1	1	1	1	1
A_4	3	1	$1''$	$1'$	1	3	3	3	1	$1'$	1	1	$1'$	1	1
Z_4	1	1	1	1	1	1	i	-1	1	-1	$-i$	1	$-i$	-1	i

Table 1 : Fields and their transformations under the chosen symmetries.

• We denote a generic vacuum alignment of the flavon fields as follows
$\langle\phi\rangle = v(n_1, n_2, n_3)$, $\langle\phi'\rangle = v(n_4, n_5, n_6)$, $\langle\phi''\rangle = v(n_7, n_8, n_9)$,

$$
\langle\xi\rangle = \langle\xi'\rangle = v, \quad \langle\chi\rangle = u \tag{2}
$$

where n_i $i = 1 - 9$ are the dimensionless numbers

• We have chosen to take values as $n_i \in (1, 0, -1)$ which are the natural choices for alignment.

The charged lepton mass matrix can be written as

$$m_l = \frac{\langle H \rangle v}{\Lambda} \begin{pmatrix} n_1 y_e & n_2 y_\mu & n_3 y_\tau \\ n_3 y_e & n_1 y_\mu & n_2 y_\tau \\ n_2 y_e & n_3 y_\mu & n_1 y_\tau \end{pmatrix}. \tag{3}$$

The neutral fermion mass matrix in the basis (ν_L, ν_R, S) can be written as

$$\mathcal{M} = \begin{pmatrix} 0 & M_D & 0 \\ M_D^T & M_R & M_S^T \\ 0 & M_S & 0 \end{pmatrix} \tag{4}$$

where M_D, the Dirac neutrino mass matrix is

$$M_D = \frac{\langle H \rangle v}{\Lambda} \begin{pmatrix} y_1 n_1 & y_2 n_5 & y_3 n_7 \\ y_1 n_3 & y_2 n_4 & y_3 n_9 \\ y_1 n_2 & y_2 n_6 & y_3 n_8 \end{pmatrix} \tag{5}$$

The right-handed neutrino mass matrix takes the diagonal form

$$M_R = \begin{pmatrix} \lambda_1 v & 0 & 0 \\ 0 & \lambda_2 v & 0 \\ 0 & 0 & \lambda_3 v \end{pmatrix}, \tag{6}$$

and M_S in the basis (S, ν_R) is given by

$$M_S = (\rho u, y_4 v, y_5 u). \tag{7}$$

The effective 4×4 light neutrino mass matrix in the basis (ν_L, ν_s) can be written as

$$M_\nu = \begin{pmatrix} M_D M_R^{-1} M_D^T & M_D M_R^{-1} M_S^T \\ M_S (M_R^{-1})^T M_D^T & M_S M_R^{-1} M_S^T \end{pmatrix} \tag{8}$$

The 4×4 active-sterile mass matrix can be written as

$$m_\nu^{4\times4} = \begin{pmatrix} Aa_7 & Aa_8 & Aa_9 & \sqrt{A}a_1 \\ Aa_{10} & Aa_{11} & Aa_{12} & \sqrt{A}a_2 \\ Aa_{13} & Aa_{14} & Aa_{15} & \sqrt{A}a_3 \\ \sqrt{A}a_4 & \sqrt{A}a_5 & \sqrt{A}a_6 & a_0 \end{pmatrix} \tag{9}$$

where

$$a_0 = (\frac{\rho^2 u^2}{\lambda_1 v} + \frac{y_4^2 v}{\lambda_2} + \frac{y_5^2 u^2}{v \lambda_3}), \tag{10}$$

$$a_1 = a_4 = (\frac{\rho u y_1 n_1}{v \lambda_1} + \frac{y_4 y_2 n_5}{\lambda_2} + \frac{u y_5 y_3 n_7}{v \lambda_3}), \tag{11}$$

$$a_2 = a_5 = (\frac{\rho u y_1 n_3}{v \lambda_1} + \frac{y_4 y_2 n_4}{\lambda_2} + \frac{u y_5 y_3 n_9}{v \lambda_3}), \tag{12}$$

$$a_3 = a_6 = (\frac{\rho u y_1 n_2}{v \lambda_1} + \frac{y_4 y_2 n_6}{\lambda_2} + \frac{u y_5 y_3 n_8}{v \lambda_3}), \tag{13}$$

$$a_7 = (\frac{y_1^2 n_1^2}{v\lambda_1} + \frac{y_2^2 n_5^2}{v\lambda_2} + \frac{y_3^2 n_7^2}{v\lambda_3}), \tag{14}$$

$$a_8 = a_{10} = (\frac{y_1^2 n_3 n_1}{v\lambda_1} + \frac{y_2^2 n_4 n_5}{v\lambda_2} + \frac{y_3^2 n_7 n_9}{v\lambda_3}), \tag{15}$$

$$a_9 = a_{13} = (\frac{y_1^2 n_1 n_2}{v\lambda_1} + \frac{y_2^2 n_5 n_6}{v\lambda_2} + \frac{y_3^2 n_7 n_8}{v\lambda_3}), \tag{16}$$

$$a_{11} = (\frac{y_1^2 n_3^2}{v\lambda_1} + \frac{y_2^2 n_4^2}{v\lambda_2} + \frac{y_3^2 n_9^2}{v\lambda_3}), \tag{17}$$

$$a_{12} = a_{14} = (\frac{y_1^2 n_2 n_3}{v\lambda_1} + \frac{y_2^2 n_4 n_6}{v\lambda_2} + \frac{y_3^2 n_8 n_9}{v\lambda_3}), \tag{18}$$

$$a_{15} = (\frac{y_1^2 n_2^2}{v\lambda_1} + \frac{y_2^2 n_6^2}{v\lambda_2} + \frac{y_3^2 n_8^2}{v\lambda_3}). \tag{19}$$

Disallowed cases:

1. Texture zero in entire second row and column. Total number of such textures is 71.
2. Texture zero in entire third row and column. Total number of such textures is 73.
3. Texture zero in entire second and third rows and columns. Total number of such textures is 9.
4. $\mu - \tau$ symmetry in the entire 4×4 block. Total number of such textures is 72.

Allowed cases:

1. $\mu - \tau$ symmetry in 3×3 active neutrino block. Total number of such textures is 40.
2. One zero texture mass matrix. Total number of such textures is 96.
3. Two zero texture mass matrix. Total number of such textures is 64.
4. Three zero texture mass matrix. Total number of such textures is 8.
5. Hybrid texture mass matrix with no zeros but some constraints relating different elements. Total number of such textures is 296.

Total number of such allowed mass matrices is 504.

$\mu - \tau$ **symmetric textures**

(i) 16 matrices with 5 complex constraints: $M_{e\mu} = 0, \quad M_{e\tau} = 0, \quad M_{\mu\mu} = M_{\tau\tau}, \, M_{\mu\tau} = -M_{\tau\tau}, \quad M_{\mu s} = -M_{\tau s}$

- $n_1 = 1; n_2 = 0; n_3 = 0; n_4 = 1; n_5 = 0; n_6 = -1; n_7 = 0; n_8 = 1; n_9 = -1;$

$$\begin{pmatrix} -\frac{H^2 v y_1^2}{\Lambda^2 \lambda_1} & 0 & 0 & -\frac{u\sqrt{\frac{H^2 v^2}{\Lambda^2}}\rho y_1}{v\lambda_1} \\ 0 & -\frac{H^2 v^2\left(\frac{y_2^2}{v\lambda_2}+\frac{y_3^2}{v\lambda_3}\right)}{\Lambda^2} & -\frac{H^2 v^2\left(-\frac{y_2^2}{v\lambda_2}-\frac{y_3^2}{v\lambda_3}\right)}{\Lambda^2} & -\sqrt{\frac{H^2 v^2}{\Lambda^2}}\left(\frac{y_2 y_4}{\lambda_2}-\frac{u y_3 y_5}{v\lambda_3}\right) \\ 0 & -\frac{H^2 v^2\left(-\frac{y_2^2}{v\lambda_2}-\frac{y_3^2}{v\lambda_3}\right)}{\Lambda^2} & -\frac{H^2 v^2\left(\frac{y_2^2}{v\lambda_2}+\frac{y_3^2}{v\lambda_3}\right)}{\Lambda^2} & -\sqrt{\frac{H^2 v^2}{\Lambda^2}}\left(-\frac{y_2 y_4}{\lambda_2}+\frac{u y_3 y_5}{v\lambda_3}\right) \\ -\frac{u\sqrt{\frac{H^2 v^2}{\Lambda^2}}\rho y_1}{v\lambda_1} & -\sqrt{\frac{H^2 v^2}{\Lambda^2}}\left(\frac{y_2 y_4}{\lambda_2}-\frac{u y_3 y_5}{v\lambda_3}\right) & -\sqrt{\frac{H^2 v^2}{\Lambda^2}}\left(-\frac{y_2 y_4}{\lambda_2}+\frac{u y_3 y_5}{v\lambda_3}\right) & -\frac{u^2\rho^2}{v\lambda_1}-\frac{v y_4^2}{\lambda_2}-\frac{u^2 y_5^2}{v\lambda_3} \end{pmatrix}$$

(ii) 8 matrices with 3 complex constraints: $M_{e\mu} = 0, \quad M_{e\tau} = 0, \quad M_{\mu\mu} = M_{\tau\tau}$

- $n_1 = 1; n_2 = 0; \; n_3 = 0; n_4 = 1; n_5 = 0; \; n_6 = 1; n_7 = 0; n_8 = 1; \; n_9 = -1;$

$$\begin{pmatrix} -\frac{H^2 v y_1^2}{\Lambda^2 \lambda_1} & 0 & 0 & -\frac{u\sqrt{\frac{H^2 v^2}{\Lambda^2}}\rho y_1}{v\lambda_1} \\ 0 & -\frac{H^2 v^2\left(\frac{y_2^2}{v\lambda_2}+\frac{y_3^2}{v\lambda_3}\right)}{\Lambda^2} & -\frac{H^2 v^2\left(\frac{y_2^2}{v\lambda_2}-\frac{y_3^2}{v\lambda_3}\right)}{\Lambda^2} & -\sqrt{\frac{H^2 v^2}{\Lambda^2}}\left(\frac{y_2 y_4}{\lambda_2}-\frac{u y_3 y_5}{v\lambda_3}\right) \\ 0 & -\frac{H^2 v^2\left(\frac{y_2^2}{v\lambda_2}-\frac{y_3^2}{v\lambda_3}\right)}{\Lambda^2} & -\frac{H^2 v^2\left(\frac{y_2^2}{v\lambda_2}+\frac{y_3^2}{v\lambda_3}\right)}{\Lambda^2} & -\sqrt{\frac{H^2 v^2}{\Lambda^2}}\left(\frac{y_2 y_4}{\lambda_2}+\frac{u y_3 y_5}{v\lambda_3}\right) \\ -\frac{u\sqrt{\frac{H^2 v^2}{\Lambda^2}}\rho y_1}{v\lambda_1} & -\sqrt{\frac{H^2 v^2}{\Lambda^2}}\left(\frac{y_2 y_4}{\lambda_2}-\frac{u y_3 y_5}{v\lambda_3}\right) & -\sqrt{\frac{H^2 v^2}{\Lambda^2}}\left(\frac{y_2 y_4}{\lambda_2}+\frac{u y_3 y_5}{v\lambda_3}\right) & -\frac{u^2\rho^2}{v\lambda_1}-\frac{v y_4^2}{\lambda_2}-\frac{u^2 y_5^2}{v\lambda_3} \end{pmatrix}$$

(iii) 8 matrices with 3 complex constraints: $M_{e\mu} = M_{e\tau}\,, \quad M_{\mu\mu} = M_{\tau\tau}\,, \quad M_{\tau\tau} + M_{\mu\tau} = 2M_{e\tau}$

- $n_1 = 1;\, n_2 = 0;\;\; n_3 = 0;\, n_4 = 1;\, n_5 = 0;\;\; n_6 = -1;\, n_7 = 1;\, n_8 = 1;\;\; n_9 = 1;$

$$\begin{pmatrix} -\frac{H^2 v^2 \left(\frac{y_1^2}{v\lambda_1}+\frac{y_3^2}{v\lambda_3}\right)}{\Lambda^2} & -\frac{H^2 v y_3^2}{\Lambda^2\lambda_3} & -\frac{H^2 v y_3^2}{\Lambda^2\lambda_3} & -\sqrt{\frac{H^2v^2}{\Lambda^2}}\left(\frac{u\rho y_1}{v\lambda_1} + \cdots\right. \\ -\frac{H^2 v y_3^2}{\Lambda^2\lambda_3} & -\frac{H^2 v^2 \left(\frac{y_2^2}{v\lambda_2}+\frac{y_3^2}{v\lambda_3}\right)}{\Lambda^2} & -\frac{H^2 v^2 \left(-\frac{y_2^2}{v\lambda_2}+\frac{y_3^2}{v\lambda_3}\right)}{\Lambda^2} & -\sqrt{\frac{H^2v^2}{\Lambda^2}}\left(\frac{y_2 y_4}{\lambda_2} + \cdots\right. \\ -\frac{H^2 v y_3^2}{\Lambda^2\lambda_3} & -\frac{H^2 v^2 \left(-\frac{y_2^2}{v\lambda_2}+\frac{y_3^2}{v\lambda_3}\right)}{\Lambda^2} & -\frac{H^2 v^2 \left(\frac{y_2^2}{v\lambda_2}+\frac{y_3^2}{v\lambda_3}\right)}{\Lambda^2} & -\sqrt{\frac{H^2v^2}{\Lambda^2}}\left(-\frac{y_2 y_4}{\lambda_2} + \cdots\right. \\ -\sqrt{\frac{H^2v^2}{\Lambda^2}}\left(\frac{u\rho y_1}{v\lambda_1} + \frac{u y_3 y_5}{v\lambda_3}\right) & -\sqrt{\frac{H^2v^2}{\Lambda^2}}\left(\frac{y_2 y_4}{\lambda_2} + \frac{u y_3 y_5}{v\lambda_3}\right) & -\sqrt{\frac{H^2v^2}{\Lambda^2}}\left(-\frac{y_2 y_4}{\lambda_2} + \frac{u y_3 y_5}{v\lambda_3}\right) & -\frac{u^2\rho^2}{v\lambda_1} - \frac{v y_4^2}{\lambda_2} - \cdots \end{pmatrix}$$

(iv) 8 matrices with 3 complex constraints: $M_{e\mu} = M_{e\tau}\,, \quad M_{\mu\mu} = M_{\tau\tau}\,, \quad M_{\tau\tau} + M_{\mu\tau} = -2M_{e\tau}$

- $n_1 = 1;\, n_2 = 0;\;\; n_3 = 0;\, n_4 = 1;\, n_5 = 0;\;\; n_6 = -1;\, n_7 = 1;\, n_8 = -1;\;\; n_9 = -1;$

$$\begin{pmatrix} -\frac{H^2 v^2 \left(\frac{y_1^2}{v\lambda_1}+\frac{y_3^2}{v\lambda_3}\right)}{\Lambda^2} & \frac{H^2 v y_3^2}{\Lambda^2\lambda_3} & \frac{H^2 v y_3^2}{\Lambda^2\lambda_3} & -\sqrt{\frac{H^2v^2}{\Lambda^2}}\left(\frac{u\rho y_1}{v\lambda_1} + \cdots\right. \\ \frac{H^2 v y_3^2}{\Lambda^2\lambda_3} & -\frac{H^2 v^2 \left(\frac{y_2^2}{v\lambda_2}+\frac{y_3^2}{v\lambda_3}\right)}{\Lambda^2} & -\frac{H^2 v^2 \left(-\frac{y_2^2}{v\lambda_2}+\frac{y_3^2}{v\lambda_3}\right)}{\Lambda^2} & -\sqrt{\frac{H^2v^2}{\Lambda^2}}\left(\frac{y_2 y_4}{\lambda_2} - \cdots\right. \\ \frac{H^2 v y_3^2}{\Lambda^2\lambda_3} & -\frac{H^2 v^2 \left(-\frac{y_2^2}{v\lambda_2}+\frac{y_3^2}{v\lambda_3}\right)}{\Lambda^2} & -\frac{H^2 v^2 \left(\frac{y_2^2}{v\lambda_2}+\frac{y_3^2}{v\lambda_3}\right)}{\Lambda^2} & -\sqrt{\frac{H^2v^2}{\Lambda^2}}\left(-\frac{y_2 y_4}{\lambda_2} - \cdots\right. \\ -\sqrt{\frac{H^2v^2}{\Lambda^2}}\left(\frac{u\rho y_1}{v\lambda_1} + \frac{u y_3 y_5}{v\lambda_3}\right) & -\sqrt{\frac{H^2v^2}{\Lambda^2}}\left(\frac{y_2 y_4}{\lambda_2} - \frac{u y_3 y_5}{v\lambda_3}\right) & -\sqrt{\frac{H^2v^2}{\Lambda^2}}\left(-\frac{y_2 y_4}{\lambda_2} - \frac{u y_3 y_5}{v\lambda_3}\right) & -\frac{u^2\rho^2}{v\lambda_1} - \frac{v y_4^2}{\lambda_2} - \cdots \end{pmatrix}$$

Texture 1 zero case

(i) 8 matrices with 3 complex constraints: $M_{e\mu} = 0, \quad M_{\mu\mu} = -M_{\mu\tau}, \quad M_{e\tau} + M_{\mu\tau} = -M_{\tau\tau}$

- $n_1 = 1; n_2 = 0; n_3 = 0; n_4 = 0; n_5 = 1; n_6 = -1; n_7 = 0; n_8 = 1; n_9 = -1;$

$$\begin{pmatrix} -\frac{H^2 v^2\left(\frac{y_1^2}{v\lambda_1}+\frac{y_2^2}{v\lambda_2}\right)}{\Lambda^2} & 0 & \frac{H^2 v y_2^2}{\Lambda^2\lambda_2} & -\sqrt{\frac{H^2v^2}{\Lambda^2}}\left(\frac{u\rho y_1}{v\lambda_1}+\frac{y_2y_4}{\lambda_2}\right) \\ 0 & -\frac{H^2 v y_3^2}{\Lambda^2\lambda_3} & \frac{H^2 v y_3^2}{\Lambda^2\lambda_3} & \frac{u\sqrt{\frac{H^2v^2}{\Lambda^2}}y_3y_5}{v\lambda_3} \\ \frac{H^2 v y_2^2}{\Lambda^2\lambda_2} & \frac{H^2 v y_3^2}{\Lambda^2\lambda_3} & -\frac{H^2 v^2\left(\frac{y_2^2}{v\lambda_2}+\frac{y_3^2}{v\lambda_3}\right)}{\Lambda^2} & -\sqrt{\frac{H^2v^2}{\Lambda^2}}\left(-\frac{y_2y_4}{\lambda_2}+\frac{uy_3y_5}{v\lambda_3}\right) \\ -\sqrt{\frac{H^2v^2}{\Lambda^2}}\left(\frac{u\rho y_1}{v\lambda_1}+\frac{y_2y_4}{\lambda_2}\right) & \frac{u\sqrt{\frac{H^2v^2}{\Lambda^2}}y_3y_5}{v\lambda_3} & -\sqrt{\frac{H^2v^2}{\Lambda^2}}\left(-\frac{y_2y_4}{\lambda_2}+\frac{uy_3y_5}{v\lambda_3}\right) & -\frac{u^2\rho^2}{v\lambda_1}-\frac{vy_4^2}{\lambda_2}-\frac{u^2y_5^2}{v\lambda_3} \end{pmatrix}$$

(ii) 8 matrices with 3 complex constraints: $M_{e\tau} = 0, \quad M_{\tau\tau} = -M_{\mu\tau}, \quad M_{e\mu} + M_{\mu\tau} = -M_{\mu\mu}$

- $n_1 = 1; n_2 = 0;_3 = 0; n_4 = 1; n_5 = 0; n_6 = -1; n_7 = 1; n_8 = 0; n_9 = -1;$

$$\begin{pmatrix} -\frac{H^2 v^2\left(\frac{y_1^2}{v\lambda_1}+\frac{y_3^2}{v\lambda_3}\right)}{\Lambda^2} & \frac{H^2 v y_3^2}{\Lambda^2\lambda_3} & 0 & -\sqrt{\frac{H^2v^2}{\Lambda^2}}\left(\frac{u\rho y_1}{v\lambda_1}+\frac{uy_3y_5}{v\lambda_3}\right) \\ \frac{H^2 v y_3^2}{\Lambda^2\lambda_3} & -\frac{H^2 v^2\left(\frac{y_2^2}{v\lambda_2}+\frac{y_3^2}{v\lambda_3}\right)}{\Lambda^2} & \frac{H^2 v y_2^2}{\Lambda^2\lambda_2} & -\sqrt{\frac{H^2v^2}{\Lambda^2}}\left(\frac{y_2y_4}{\lambda_2}-\frac{uy_3y_5}{v\lambda_3}\right) \\ 0 & \frac{H^2 v y_2^2}{\Lambda^2\lambda_2} & -\frac{H^2 v y_2^2}{\Lambda^2\lambda_2} & \frac{\sqrt{\frac{H^2v^2}{\Lambda^2}}y_2y_4}{\lambda_2} \\ -\sqrt{\frac{H^2v^2}{\Lambda^2}}\left(\frac{u\rho y_1}{v\lambda_1}+\frac{uy_3y_5}{v\lambda_3}\right) & -\sqrt{\frac{H^2v^2}{\Lambda^2}}\left(\frac{y_2y_4}{\lambda_2}-\frac{uy_3y_5}{v\lambda_3}\right) & \frac{\sqrt{\frac{H^2v^2}{\Lambda^2}}y_2y_4}{\lambda_2} & -\frac{u^2\rho^2}{v\lambda_1}-\frac{vy_4^2}{\lambda_2}-\frac{u^2y_5^2}{v\lambda_3} \end{pmatrix}$$

(iii) 8 matrices with 3 complex constraints:
$M_{e\mu} = 0, \quad M_{\mu\mu} = M_{\mu\tau}, \quad M_{e\tau} + M_{\mu\tau} = M_{\tau\tau}$

• $n_1 = 1; n_2 = 0; \ n_3 = 0; n_4 = 0; n_5 = 1; \ n_6 = 1; n_7 = 0; n_8 = 1; \ n_9 = 1;$

$$\begin{pmatrix} -\frac{H^2 v^2\left(\frac{y_1^2}{v\lambda_1}+\frac{y_2^2}{v\lambda_2}\right)}{\Lambda^2} & 0 & -\frac{H^2 v y_2^2}{\Lambda^2 \lambda_2} & -\sqrt{\frac{H^2 v^2}{\Lambda^2}}\left(\frac{u\rho y_1}{v\lambda_1}+\frac{y_2 y_4}{\lambda_2}\right) \\ 0 & -\frac{H^2 v y_3^2}{\Lambda^2 \lambda_3} & -\frac{H^2 v y_3^2}{\Lambda^2 \lambda_3} & -\frac{u\sqrt{\frac{H^2 v^2}{\Lambda^2}} y_3 y_5}{v\lambda_3} \\ -\frac{H^2 v y_2^2}{\Lambda^2 \lambda_2} & -\frac{H^2 v y_3^2}{\Lambda^2 \lambda_3} & -\frac{H^2 v^2\left(\frac{y_2^2}{v\lambda_2}+\frac{y_3^2}{v\lambda_3}\right)}{\Lambda^2} & -\sqrt{\frac{H^2 v^2}{\Lambda^2}}\left(\frac{y_2 y_4}{\lambda_2}+\frac{u y_3 y_5}{v\lambda_3}\right) \\ -\sqrt{\frac{H^2 v^2}{\Lambda^2}}\left(\frac{u\rho y_1}{v\lambda_1}+\frac{y_2 y_4}{\lambda_2}\right) & -\frac{u\sqrt{\frac{H^2 v^2}{\Lambda^2}} y_3 y_5}{v\lambda_3} & -\sqrt{\frac{H^2 v^2}{\Lambda^2}}\left(\frac{y_2 y_4}{\lambda_2}+\frac{u y_3 y_5}{v\lambda_3}\right) & -\frac{u^2\rho^2}{v\lambda_1}-\frac{v y_4^2}{\lambda_2}-\frac{u^2 y_5^2}{v\lambda_3} \end{pmatrix}$$

(iv) 8 matrices with 3 complex constraints:

$$M_{e\mu} = 0, \quad M_{\mu\mu} = -M_{\mu\tau}, \quad M_{e\tau} - M_{\mu\tau} = M_{\tau\tau}$$

• $n_1 = 1; n_2 = 0; \ n_3 = 0; n_4 = 0; n_5 = 1; \ n_6 = 1; n_7 = 0; n_8 = 1; \ n_9 = -1;$

$$\begin{pmatrix} -\frac{H^2 v^2\left(\frac{y_1^2}{v\lambda_1}+\frac{y_2^2}{v\lambda_2}\right)}{\Lambda^2} & 0 & -\frac{H^2 v y_2^2}{\Lambda^2 \lambda_2} & -\sqrt{\frac{H^2 v^2}{\Lambda^2}}\left(\frac{u\rho y_1}{v\lambda_1}+\frac{y_2 y_4}{\lambda_2}\right) \\ 0 & -\frac{H^2 v y_3^2}{\Lambda^2 \lambda_3} & \frac{H^2 v y_3^2}{\Lambda^2 \lambda_3} & \frac{u\sqrt{\frac{H^2 v^2}{\Lambda^2}} y_3 y_5}{v\lambda_3} \\ -\frac{H^2 v y_2^2}{\Lambda^2 \lambda_2} & \frac{H^2 v y_3^2}{\Lambda^2 \lambda_3} & -\frac{H^2 v^2\left(\frac{y_2^2}{v\lambda_2}+\frac{y_3^2}{v\lambda_3}\right)}{\Lambda^2} & -\sqrt{\frac{H^2 v^2}{\Lambda^2}}\left(\frac{y_2 y_4}{\lambda_2}+\frac{u y_3 y_5}{v\lambda_3}\right) \\ -\sqrt{\frac{H^2 v^2}{\Lambda^2}}\left(\frac{u\rho y_1}{v\lambda_1}+\frac{y_2 y_4}{\lambda_2}\right) & \frac{u\sqrt{\frac{H^2 v^2}{\Lambda^2}} y_3 y_5}{v\lambda_3} & -\sqrt{\frac{H^2 v^2}{\Lambda^2}}\left(\frac{y_2 y_4}{\lambda_2}+\frac{u y_3 y_5}{v\lambda_3}\right) & -\frac{u^2\rho^2}{v\lambda_1}-\frac{v y_4^2}{\lambda_2}-\frac{u^2 y_5^2}{v\lambda_3} \end{pmatrix}$$

(v) 8 matrices with 3 complex constraints:
$M_{e\mu} = 0, \quad M_{\mu\mu} = M_{\mu\tau}, \quad M_{e\tau} - M_{\mu\tau} = -M_{\tau\tau}$

- $n_1 = 1; n_2 = 0; \; n_3 = 0; n_4 = 0; n_5 = 1; \; n_6 = -1; n_7 = 0; n_8 = 1; \; n_9 = 1;$

$$\begin{pmatrix} -\frac{H^2 v^2\left(\frac{y_1^2}{v\lambda_1}+\frac{y_2^2}{v\lambda_2}\right)}{\Lambda^2} & 0 & \frac{H^2 v y_2^2}{\Lambda^2\lambda_2} & -\sqrt{\frac{H^2v^2}{\Lambda^2}}\left(\frac{u\rho y_1}{v\lambda_1}+\frac{y_2y_4}{\lambda_2}\right) \\ 0 & -\frac{H^2 v y_3^2}{\Lambda^2\lambda_3} & -\frac{H^2 v y_3^2}{\Lambda^2\lambda_3} & -\frac{u\sqrt{\frac{H^2v^2}{\Lambda^2}}y_3y_5}{v\lambda_3} \\ \frac{H^2 v y_2^2}{\Lambda^2\lambda_2} & -\frac{H^2 v y_3^2}{\Lambda^2\lambda_3} & -\frac{H^2 v^2\left(\frac{y_2^2}{v\lambda_2}+\frac{y_3^2}{v\lambda_3}\right)}{\Lambda^2} & -\sqrt{\frac{H^2v^2}{\Lambda^2}}\left(-\frac{y_2y_4}{\lambda_2}+\frac{uy_3y_5}{v\lambda_3}\right) \\ -\sqrt{\frac{H^2v^2}{\Lambda^2}}\left(\frac{u\rho y_1}{v\lambda_1}+\frac{y_2y_4}{\lambda_2}\right) & -\frac{u\sqrt{\frac{H^2v^2}{\Lambda^2}}y_3y_5}{v\lambda_3} & -\sqrt{\frac{H^2v^2}{\Lambda^2}}\left(-\frac{y_2y_4}{\lambda_2}+\frac{uy_3y_5}{v\lambda_3}\right) & -\frac{u^2\rho^2}{v\lambda_1}-\frac{vy_4^2}{\lambda_2}-\frac{u^2y_5^2}{v\lambda_3} \end{pmatrix}$$

(vi) 8 matrices with 3 complex constraints:

$$M_{e\tau} = 0, \quad M_{\tau\tau} = M_{\mu\tau}, \quad M_{e\mu} + M_{\mu\tau} = M_{\mu\mu}$$

- $n_1 = 1; n_2 = 0; \; n_3 = 0; n_4 = 1; n_5 = 0; \; n_6 = 1; n_7 = 1; n_8 = 0; \; n_9 = 1;$

$$\begin{pmatrix} -\frac{H^2 v^2\left(\frac{y_1^2}{v\lambda_1}+\frac{y_3^2}{v\lambda_3}\right)}{\Lambda^2} & -\frac{H^2 v y_3^2}{\Lambda^2\lambda_3} & 0 & -\sqrt{\frac{H^2v^2}{\Lambda^2}}\left(\frac{u\rho y_1}{v\lambda_1}+\frac{uy_3y_5}{v\lambda_3}\right) \\ -\frac{H^2 v y_3^2}{\Lambda^2\lambda_3} & -\frac{H^2 v^2\left(\frac{y_2^2}{v\lambda_2}+\frac{y_3^2}{v\lambda_3}\right)}{\Lambda^2} & -\frac{H^2 v y_2^2}{\Lambda^2\lambda_2} & -\sqrt{\frac{H^2v^2}{\Lambda^2}}\left(\frac{y_2y_4}{\lambda_2}+\frac{uy_3y_5}{v\lambda_3}\right) \\ 0 & -\frac{H^2 v y_2^2}{\Lambda^2\lambda_2} & -\frac{H^2 v y_2^2}{\Lambda^2\lambda_2} & -\frac{\sqrt{\frac{H^2v^2}{\Lambda^2}}y_2y_4}{\lambda_2} \\ -\sqrt{\frac{H^2v^2}{\Lambda^2}}\left(\frac{u\rho y_1}{v\lambda_1}+\frac{uy_3y_5}{v\lambda_3}\right) & -\sqrt{\frac{H^2v^2}{\Lambda^2}}\left(\frac{y_2y_4}{\lambda_2}+\frac{uy_3y_5}{v\lambda_3}\right) & -\frac{\sqrt{\frac{H^2v^2}{\Lambda^2}}y_2y_4}{\lambda_2} & -\frac{u^2\rho^2}{v\lambda_1}-\frac{vy_4^2}{\lambda_2}-\frac{u^2y_5^2}{v\lambda_3} \end{pmatrix}$$

(vii) 7 matrices with 3 complex constraints:
$M_{e\tau} = 0, \quad M_{\tau\tau} = M_{\mu\tau}, \quad M_{e\mu} - M_{\mu\tau} = -M_{\mu\mu}$

- $n_1 = 1; n_2 = 0; \; n_3 = 0; n_4 = 1; n_5 = 0; \; n_6 = 1; n_7 = 1; n_8 = 0; \; n_9 = -1;$

$$\begin{pmatrix} -\frac{H^2 v^2\left(\frac{y_1^2}{v\lambda_1}+\frac{y_3^2}{v\lambda_3}\right)}{\Lambda^2} & \frac{H^2 v y_3^2}{\Lambda^2\lambda_3} & 0 & -\sqrt{\frac{H^2v^2}{\Lambda^2}}\left(\frac{u\rho y_1}{v\lambda_1}+\frac{u y_3 y_5}{v\lambda_3}\right) \\ \frac{H^2 v y_3^2}{\Lambda^2\lambda_3} & -\frac{H^2 v^2\left(\frac{y_2^2}{v\lambda_2}+\frac{y_3^2}{v\lambda_3}\right)}{\Lambda^2} & -\frac{H^2 v y_2^2}{\Lambda^2\lambda_2} & -\sqrt{\frac{H^2v^2}{\Lambda^2}}\left(\frac{y_2 y_4}{\lambda_2}-\frac{u y_3 y_5}{v\lambda_3}\right) \\ 0 & -\frac{H^2 v y_2^2}{\Lambda^2\lambda_2} & -\frac{H^2 v y_2^2}{\Lambda^2\lambda_2} & -\frac{\sqrt{\frac{H^2v^2}{\Lambda^2}}\,y_2 y_4}{\lambda_2} \\ -\sqrt{\frac{H^2v^2}{\Lambda^2}}\left(\frac{u\rho y_1}{v\lambda_1}+\frac{u y_3 y_5}{v\lambda_3}\right) & -\sqrt{\frac{H^2v^2}{\Lambda^2}}\left(\frac{y_2 y_4}{\lambda_2}-\frac{u y_3 y_5}{v\lambda_3}\right) & -\frac{\sqrt{\frac{H^2v^2}{\Lambda^2}}\,y_2 y_4}{\lambda_2} & -\frac{u^2\rho^2}{v\lambda_1}-\frac{v y_4^2}{\lambda_2}-\frac{u^2 y_5^2}{v\lambda_3} \end{pmatrix}$$

(viii) 8 matrices with 3 complex constraints: $M_{e\tau} = 0, \quad M_{\tau\tau} = -M_{\mu\tau}, \quad M_{e\mu} - M_{\mu\tau} = M_{\mu\mu}$

- $n_1 = 1; n_2 = 0; \; n_3 = 0; n_4 = 1; n_5 = 0; \; n_6 = -1; n_7 = 1; n_8 = 0; \; n_9 = 1;$

$$\begin{pmatrix} -\frac{H^2 v^2\left(\frac{y_1^2}{v\lambda_1}+\frac{y_3^2}{v\lambda_3}\right)}{\Lambda^2} & -\frac{H^2 v y_3^2}{\Lambda^2\lambda_3} & 0 & -\sqrt{\frac{H^2v^2}{\Lambda^2}}\left(\frac{u\rho y_1}{v\lambda_1}+\frac{u y_3 y_5}{v\lambda_3}\right) \\ -\frac{H^2 v y_3^2}{\Lambda^2\lambda_3} & -\frac{H^2 v^2\left(\frac{y_2^2}{v\lambda_2}+\frac{y_3^2}{v\lambda_3}\right)}{\Lambda^2} & \frac{H^2 v y_2^2}{\Lambda^2\lambda_2} & -\sqrt{\frac{H^2v^2}{\Lambda^2}}\left(\frac{y_2 y_4}{\lambda_2}+\frac{u y_3 y_5}{v\lambda_3}\right) \\ 0 & \frac{H^2 v y_2^2}{\Lambda^2\lambda_2} & -\frac{H^2 v y_2^2}{\Lambda^2\lambda_2} & \frac{\sqrt{\frac{H^2v^2}{\Lambda^2}}\,y_2 y_4}{\lambda_2} \\ -\sqrt{\frac{H^2v^2}{\Lambda^2}}\left(\frac{u\rho y_1}{v\lambda_1}+\frac{u y_3 y_5}{v\lambda_3}\right) & -\sqrt{\frac{H^2v^2}{\Lambda^2}}\left(\frac{y_2 y_4}{\lambda_2}+\frac{u y_3 y_5}{v\lambda_3}\right) & \frac{\sqrt{\frac{H^2v^2}{\Lambda^2}}\,y_2 y_4}{\lambda_2} & -\frac{u^2\rho^2}{v\lambda_1}-\frac{v y_4^2}{\lambda_2}-\frac{u^2 y_5^2}{v\lambda_3} \end{pmatrix}$$

(ix) 8 matrices with 3 complex constraints: $M_{\mu\tau} = 0, \quad M_{e\mu} = -M_{\mu\mu}, \quad M_{e\tau} = -M_{\tau\tau}$

- $n_1 = 1; n_2 = 0; n_3 = 0; n_4 = 1; n_5 = -1; n_6 = 0; n_7 = 1; n_8 = -1; n_9 = 0;$

$$\begin{pmatrix} -\frac{H^2 v^2\left(\frac{y_1^2}{v\lambda_1}+\frac{y_2^2}{v\lambda_2}+\frac{y_3^2}{v\lambda_3}\right)}{\Lambda^2} & \frac{H^2 v y_2^2}{\Lambda^2\lambda_2} & \frac{H^2 v y_3^2}{\Lambda^2\lambda_3} & -\sqrt{\frac{H^2v^2}{\Lambda^2}}\left(\frac{u\rho y_1}{v\lambda_1} - \frac{y_2 y_4}{\lambda_2} + \frac{u y_3 y_5}{v\lambda_3}\right) \\ \frac{H^2 v y_2^2}{\Lambda^2\lambda_2} & -\frac{H^2 v y_2^2}{\Lambda^2\lambda_2} & 0 & -\frac{\sqrt{\frac{H^2v^2}{\Lambda^2}}y_2 y_4}{\lambda_2} \\ \frac{H^2 v y_3^2}{\Lambda^2\lambda_3} & 0 & -\frac{H^2 v y_3^2}{\Lambda^2\lambda_3} & \frac{u\sqrt{\frac{H^2v^2}{\Lambda^2}}y_3 y_5}{v\lambda_3} \\ -\sqrt{\frac{H^2v^2}{\Lambda^2}}\left(\frac{u\rho y_1}{v\lambda_1} - \frac{y_2 y_4}{\lambda_2} + \frac{u y_3 y_5}{v\lambda_3}\right) & -\frac{\sqrt{\frac{H^2v^2}{\Lambda^2}}y_2 y_4}{\lambda_2} & \frac{u\sqrt{\frac{H^2v^2}{\Lambda^2}}y_3 y_5}{v\lambda_3} & -\frac{u^2\rho^2}{v\lambda_1} - \frac{v y_4^2}{\lambda_2} - \frac{u^2 y_5^2}{v\lambda_3} \end{pmatrix}$$

(x) 8 matrices with 3 complex constraints: $M_{\mu\tau} = 0, \quad M_{e\mu} = M_{\mu\mu}, \quad M_{e\tau} = M_{\tau\tau}$

- $n_1 = 1; n_2 = 0; n_3 = 0; n_4 = 0; n_5 = 1; n_6 = 1; n_7 = 1; n_8 = 0; n_9 = 1;$

$$\begin{pmatrix} -\frac{H^2 v^2\left(\frac{y_1^2}{v\lambda_1}+\frac{y_2^2}{v\lambda_2}+\frac{y_3^2}{v\lambda_3}\right)}{\Lambda^2} & -\frac{H^2 v y_3^2}{\Lambda^2\lambda_3} & -\frac{H^2 v y_2^2}{\Lambda^2\lambda_2} & -\sqrt{\frac{H^2v^2}{\Lambda^2}}\left(\frac{u\rho y_1}{v\lambda_1} + \frac{y_2 y_4}{\lambda_2} + \frac{u y_3 y_5}{v\lambda_3}\right) \\ -\frac{H^2 v y_3^2}{\Lambda^2\lambda_3} & -\frac{H^2 v y_3^2}{\Lambda^2\lambda_3} & 0 & -\frac{u\sqrt{\frac{H^2v^2}{\Lambda^2}}y_3 y_5}{v\lambda_3} \\ -\frac{H^2 v y_2^2}{\Lambda^2\lambda_2} & 0 & -\frac{H^2 v y_2^2}{\Lambda^2\lambda_2} & -\frac{\sqrt{\frac{H^2v^2}{\Lambda^2}}y_2 y_4}{\lambda_2} \\ -\sqrt{\frac{H^2v^2}{\Lambda^2}}\left(\frac{u\rho y_1}{v\lambda_1} + \frac{y_2 y_4}{\lambda_2} + \frac{u y_3 y_5}{v\lambda_3}\right) & -\frac{u\sqrt{\frac{H^2v^2}{\Lambda^2}}y_3 y_5}{v\lambda_3} & -\frac{\sqrt{\frac{H^2v^2}{\Lambda^2}}y_2 y_4}{\lambda_2} & -\frac{u^2\rho^2}{v\lambda_1} - \frac{v y_4^2}{\lambda_2} - \frac{u^2 y_5^2}{v\lambda_3} \end{pmatrix}$$

(xi) 9 matrices with 3 complex constraints: $M_{\mu\tau} = 0, \quad M_{e\mu} = -M_{\mu\mu}, \quad M_{e\tau} = M_{\tau\tau}$

- $n_1 = 1; n_2 = 0; \ n_3 = 0; n_4 = 0; n_5 = 1; \ n_6 = 1; n_7 = 1; n_8 = 0; \ n_9 = -1;$

$$\begin{pmatrix} -\frac{H^2 v^2\left(\frac{y_1^2}{v\lambda_1}+\frac{y_2^2}{v\lambda_2}+\frac{y_3^2}{v\lambda_3}\right)}{\Lambda^2} & \frac{H^2 v y_3^2}{\Lambda^2\lambda_3} & -\frac{H^2 v y_2^2}{\Lambda^2\lambda_2} & -\sqrt{\frac{H^2v^2}{\Lambda^2}}\left(\frac{u\rho y_1}{v\lambda_1}+\frac{y_2y_4}{\lambda_2}+\frac{uy_3y_5}{v\lambda_3}\right) \\ \frac{H^2 v y_3^2}{\Lambda^2\lambda_3} & -\frac{H^2 v y_3^2}{\Lambda^2\lambda_3} & 0 & \frac{u\sqrt{\frac{H^2v^2}{\Lambda^2}}y_3y_5}{v\lambda_3} \\ -\frac{H^2 v y_2^2}{\Lambda^2\lambda_2} & 0 & -\frac{H^2 v y_2^2}{\Lambda^2\lambda_2} & -\frac{\sqrt{\frac{H^2v^2}{\Lambda^2}}y_2y_4}{\lambda_2} \\ -\sqrt{\frac{H^2v^2}{\Lambda^2}}\left(\frac{u\rho y_1}{v\lambda_1}+\frac{y_2y_4}{\lambda_2}+\frac{uy_3y_5}{v\lambda_3}\right) & \frac{u\sqrt{\frac{H^2v^2}{\Lambda^2}}y_3y_5}{v\lambda_3} & -\frac{\sqrt{\frac{H^2v^2}{\Lambda^2}}y_2y_4}{\lambda_2} & -\frac{u^2\rho^2}{v\lambda_1}-\frac{vy_4^2}{\lambda_2}-\frac{u^2y_5^2}{v\lambda_3} \end{pmatrix}$$

(xii) 8 matrices with 3 complex constraints: $M_{\mu\tau} = 0, \quad M_{e\mu} = M_{\mu\mu}, \quad M_{e\tau} = -M_{\tau\tau}$

- $n_1 = 1; n_2 = 0; \ n_3 = 0; n_4 = 0; n_5 = 1; \ n_6 = -1; n_7 = 1; n_8 = 0; \ n_9 = 1;$

$$\begin{pmatrix} -\frac{H^2 v^2\left(\frac{y_1^2}{v\lambda_1}+\frac{y_2^2}{v\lambda_2}+\frac{y_3^2}{v\lambda_3}\right)}{\Lambda^2} & -\frac{H^2 v y_3^2}{\Lambda^2\lambda_3} & \frac{H^2 v y_2^2}{\Lambda^2\lambda_2} & -\sqrt{\frac{H^2v^2}{\Lambda^2}}\left(\frac{u\rho y_1}{v\lambda_1}+\frac{y_2y_4}{\lambda_2}+\frac{uy_3y_5}{v\lambda_3}\right) \\ -\frac{H^2 v y_3^2}{\Lambda^2\lambda_3} & -\frac{H^2 v y_3^2}{\Lambda^2\lambda_3} & 0 & -\frac{u\sqrt{\frac{H^2v^2}{\Lambda^2}}y_3y_5}{v\lambda_3} \\ \frac{H^2 v y_2^2}{\Lambda^2\lambda_2} & 0 & -\frac{H^2 v y_2^2}{\Lambda^2\lambda_2} & \frac{\sqrt{\frac{H^2v^2}{\Lambda^2}}y_2y_4}{\lambda_2} \\ -\sqrt{\frac{H^2v^2}{\Lambda^2}}\left(\frac{u\rho y_1}{v\lambda_1}+\frac{y_2y_4}{\lambda_2}+\frac{uy_3y_5}{v\lambda_3}\right) & -\frac{u\sqrt{\frac{H^2v^2}{\Lambda^2}}y_3y_5}{v\lambda_3} & \frac{\sqrt{\frac{H^2v^2}{\Lambda^2}}y_2y_4}{\lambda_2} & -\frac{u^2\rho^2}{v\lambda_1}-\frac{vy_4^2}{\lambda_2}-\frac{u^2y_5^2}{v\lambda_3} \end{pmatrix}$$

Texture 2 zero case

(i) 8 matrices with 3 complex constraints: $M_{e\mu}=0, \quad M_{e\tau}=0, \quad M_{\mu\mu}=M_{\mu\tau}$

- $n_1=1;\ n_2=0;\ n_3=0;\ n_4=0;\ n_5=0;\ n_6=1;\ n_7=0;\ n_8=1;\ n_9=1;$

$$\begin{pmatrix} -\frac{H^2 v y_1^2}{\Lambda^2\lambda_1} & 0 & 0 & -\frac{u\sqrt{\frac{H^2v^2}{\Lambda^2}}\rho y_1}{v\lambda_1} \\ 0 & -\frac{H^2 v y_3^2}{\Lambda^2\lambda_3} & -\frac{H^2 v y_3^2}{\Lambda^2\lambda_3} & -\frac{u\sqrt{\frac{H^2v^2}{\Lambda^2}}y_3y_5}{v\lambda_3} \\ 0 & -\frac{H^2 v y_3^2}{\Lambda^2\lambda_3} & -\frac{H^2v^2\left(\frac{y_2^2}{v\lambda_2}+\frac{y_3^2}{v\lambda_3}\right)}{\Lambda^2} & -\sqrt{\frac{H^2v^2}{\Lambda^2}}\left(\frac{y_2y_4}{\lambda_2}+\frac{uy_3y_5}{v\lambda_3}\right) \\ -\frac{u\sqrt{\frac{H^2v^2}{\Lambda^2}}\rho y_1}{v\lambda_1} & -\frac{u\sqrt{\frac{H^2v^2}{\Lambda^2}}y_3y_5}{v\lambda_3} & -\sqrt{\frac{H^2v^2}{\Lambda^2}}\left(\frac{y_2y_4}{\lambda_2}+\frac{uy_3y_5}{v\lambda_3}\right) & -\frac{u^2\rho^2}{v\lambda_1}-\frac{vy_4^2}{\lambda_2}-\frac{u^2y_5^2}{v\lambda_3} \end{pmatrix}$$

(ii) 8 matrices with 3 complex constraints: $M_{e\mu}=0, \quad M_{e\tau}=0, \quad M_{\mu\mu}=-M_{\mu\tau}$

- $n_1=1;\ n_2=0;\ n_3=0;\ n_4=0;\ n_5=0;\ n_6=1;\ n_7=0;\ n_8=1;\ n_9=-1;$

$$\begin{pmatrix} -\frac{H^2 v y_1^2}{\Lambda^2\lambda_1} & 0 & 0 & -\frac{u\sqrt{\frac{H^2v^2}{\Lambda^2}}\rho y_1}{v\lambda_1} \\ 0 & -\frac{H^2 v y_3^2}{\Lambda^2\lambda_3} & \frac{H^2 v y_3^2}{\Lambda^2\lambda_3} & \frac{u\sqrt{\frac{H^2v^2}{\Lambda^2}}y_3y_5}{v\lambda_3} \\ 0 & \frac{H^2 v y_3^2}{\Lambda^2\lambda_3} & -\frac{H^2v^2\left(\frac{y_2^2}{v\lambda_2}+\frac{y_3^2}{v\lambda_3}\right)}{\Lambda^2} & -\sqrt{\frac{H^2v^2}{\Lambda^2}}\left(\frac{y_2y_4}{\lambda_2}+\frac{uy_3y_5}{v\lambda_3}\right) \\ -\frac{u\sqrt{\frac{H^2v^2}{\Lambda^2}}\rho y_1}{v\lambda_1} & \frac{u\sqrt{\frac{H^2v^2}{\Lambda^2}}y_3y_5}{v\lambda_3} & -\sqrt{\frac{H^2v^2}{\Lambda^2}}\left(\frac{y_2y_4}{\lambda_2}+\frac{uy_3y_5}{v\lambda_3}\right) & -\frac{u^2\rho^2}{v\lambda_1}-\frac{vy_4^2}{\lambda_2}-\frac{u^2y_5^2}{v\lambda_3} \end{pmatrix}$$

(iii) 8 matrices with 3 complex constraints: $M_{e\tau} = 0, \quad M_{\mu\tau} = 0, \quad M_{e\mu} = -M_{\mu\mu}$

- $n_1 = 1; n_2 = 0; \ n_3 = 0; n_4 = 0; n_5 = 0; \ n_6 = 1; n_7 = 1; n_8 = 0; \ n_9 = -1;$

$$\begin{pmatrix} -\frac{H^2 v^2\left(\frac{y_1^2}{v\lambda_1}+\frac{y_3^2}{v\lambda_3}\right)}{\Lambda^2} & \frac{H^2 v y_3^2}{\Lambda^2\lambda_3} & 0 & -\sqrt{\frac{H^2v^2}{\Lambda^2}}\left(\frac{u\rho y_1}{v\lambda_1}+\frac{u y_3 y_5}{v\lambda_3}\right) \\ \frac{H^2 v y_3^2}{\Lambda^2\lambda_3} & -\frac{H^2 v y_3^2}{\Lambda^2\lambda_3} & 0 & \frac{u\sqrt{\frac{H^2v^2}{\Lambda^2}}\,y_3y_5}{v\lambda_3} \\ 0 & 0 & -\frac{H^2 v y_2^2}{\Lambda^2\lambda_2} & -\frac{\sqrt{\frac{H^2v^2}{\Lambda^2}}\,y_2y_4}{\lambda_2} \\ -\sqrt{\frac{H^2v^2}{\Lambda^2}}\left(\frac{u\rho y_1}{v\lambda_1}+\frac{u y_3 y_5}{v\lambda_3}\right) & \frac{u\sqrt{\frac{H^2v^2}{\Lambda^2}}\,y_3y_5}{v\lambda_3} & -\frac{\sqrt{\frac{H^2v^2}{\Lambda^2}}\,y_2y_4}{\lambda_2} & -\frac{u^2\rho^2}{v\lambda_1}-\frac{v y_4^2}{\lambda_2}-\frac{u^2y_5^2}{v\lambda_3} \end{pmatrix}$$

(iv) 8 matrices with 3 complex constraints: $M_{e\tau} = 0, \quad M_{\mu\tau} = 0, \quad M_{e\mu} = M_{\mu\mu}$

- $n_1 = 1; n_2 = 0; \ n_3 = 0; n_4 = 0; n_5 = 0; \ n_6 = 1; n_7 = 1; n_8 = 0; \ n_9 = 1;$

$$\begin{pmatrix} -\frac{H^2 v^2\left(\frac{y_1^2}{v\lambda_1}+\frac{y_3^2}{v\lambda_3}\right)}{\Lambda^2} & -\frac{H^2 v y_3^2}{\Lambda^2\lambda_3} & 0 & -\sqrt{\frac{H^2v^2}{\Lambda^2}}\left(\frac{u\rho y_1}{v\lambda_1}+\frac{u y_3 y_5}{v\lambda_3}\right) \\ -\frac{H^2 v y_3^2}{\Lambda^2\lambda_3} & -\frac{H^2 v y_3^2}{\Lambda^2\lambda_3} & 0 & -\frac{u\sqrt{\frac{H^2v^2}{\Lambda^2}}\,y_3y_5}{v\lambda_3} \\ 0 & 0 & -\frac{H^2 v y_2^2}{\Lambda^2\lambda_2} & -\frac{\sqrt{\frac{H^2v^2}{\Lambda^2}}\,y_2y_4}{\lambda_2} \\ -\sqrt{\frac{H^2v^2}{\Lambda^2}}\left(\frac{u\rho y_1}{v\lambda_1}+\frac{u y_3 y_5}{v\lambda_3}\right) & -\frac{u\sqrt{\frac{H^2v^2}{\Lambda^2}}\,y_3y_5}{v\lambda_3} & -\frac{\sqrt{\frac{H^2v^2}{\Lambda^2}}\,y_2y_4}{\lambda_2} & -\frac{u^2\rho^2}{v\lambda_1}-\frac{v y_4^2}{\lambda_2}-\frac{u^2y_5^2}{v\lambda_3} \end{pmatrix}$$

(v) 8 matrices with 3 complex constraints: $M_{e\mu} = 0, \quad M_{\mu\tau} = 0, \quad M_{e\tau} = M_{\tau\tau}$

- $n_1 = 1; n_2 = 0; \; n_3 = 0; n_4 = 0; n_5 = 1; \; n_6 = 1; n_7 = 0; n_8 = 0; \; n_9 = -1;$

$$\begin{pmatrix} -\frac{H^2 v^2\left(\frac{y_1^2}{v\lambda_1}+\frac{y_2^2}{v\lambda_2}\right)}{\Lambda^2} & 0 & -\frac{H^2 v y_2^2}{\Lambda^2\lambda_2} & -\sqrt{\frac{H^2v^2}{\Lambda^2}}\left(\frac{u\rho y_1}{v\lambda_1}+\frac{y_2 y_4}{\lambda_2}\right) \\ 0 & -\frac{H^2 v y_3^2}{\Lambda^2\lambda_3} & 0 & \frac{u\sqrt{\frac{H^2v^2}{\Lambda^2}}y_3y_5}{v\lambda_3} \\ -\frac{H^2 v y_2^2}{\Lambda^2\lambda_2} & 0 & -\frac{H^2 v y_2^2}{\Lambda^2\lambda_2} & -\frac{\sqrt{\frac{H^2v^2}{\Lambda^2}}y_2y_4}{\lambda_2} \\ -\sqrt{\frac{H^2v^2}{\Lambda^2}}\left(\frac{u\rho y_1}{v\lambda_1}+\frac{y_2 y_4}{\lambda_2}\right) & \frac{u\sqrt{\frac{H^2v^2}{\Lambda^2}}y_3y_5}{v\lambda_3} & -\frac{\sqrt{\frac{H^2v^2}{\Lambda^2}}y_2y_4}{\lambda_2} & -\frac{u^2\rho^2}{v\lambda_1}-\frac{vy_4^2}{\lambda_2}-\frac{u^2y_5^2}{v\lambda_3} \end{pmatrix}$$

(vi) 8 matrices with 3 complex constraints: $M_{e\mu} = 0, \quad M_{\mu\tau} = 0, \quad M_{e\tau} = -M_{\tau\tau}$

- $n_1 = 1; n_2 = 0; \; n_3 = 0; n_4 = 0; n_5 = 1; \; n_6 = -1; n_7 = 0; n_8 = 0; \; n_9 = 1;$

$$\begin{pmatrix} -\frac{H^2 v^2\left(\frac{y_1^2}{v\lambda_1}+\frac{y_2^2}{v\lambda_2}\right)}{\Lambda^2} & 0 & \frac{H^2 v y_2^2}{\Lambda^2\lambda_2} & -\sqrt{\frac{H^2v^2}{\Lambda^2}}\left(\frac{u\rho y_1}{v\lambda_1}+\frac{y_2 y_4}{\lambda_2}\right) \\ 0 & -\frac{H^2 v y_3^2}{\Lambda^2\lambda_3} & 0 & -\frac{u\sqrt{\frac{H^2v^2}{\Lambda^2}}y_3y_5}{v\lambda_3} \\ \frac{H^2 v y_2^2}{\Lambda^2\lambda_2} & 0 & -\frac{H^2 v y_2^2}{\Lambda^2\lambda_2} & \frac{\sqrt{\frac{H^2v^2}{\Lambda^2}}y_2y_4}{\lambda_2} \\ -\sqrt{\frac{H^2v^2}{\Lambda^2}}\left(\frac{u\rho y_1}{v\lambda_1}+\frac{y_2 y_4}{\lambda_2}\right) & -\frac{u\sqrt{\frac{H^2v^2}{\Lambda^2}}y_3y_5}{v\lambda_3} & \frac{\sqrt{\frac{H^2v^2}{\Lambda^2}}y_2y_4}{\lambda_2} & -\frac{u^2\rho^2}{v\lambda_1}-\frac{vy_4^2}{\lambda_2}-\frac{u^2y_5^2}{v\lambda_3} \end{pmatrix}$$

(vii) 9 matrices with 3 complex constraints: $M_{e\mu} = 0, \quad M_{e\tau} = 0, \quad M_{\mu\tau} = M_{\tau\tau}$

- $n_1 = 1; n_2 = 0;\ n_3 = 0; n_4 = 1; n_5 = 0;\ n_6 = 0; n_7 = 0; n_8 = 1;\ n_9 = 1;$

$$\begin{pmatrix} -\frac{H^2 v y_1^2}{\Lambda^2 \lambda_1} & 0 & 0 & -\frac{u\sqrt{\frac{H^2 v^2}{\Lambda^2}}\rho y_1}{v\lambda_1} \\ 0 & -\frac{H^2 v^2\left(\frac{y_2^2}{v\lambda_2}+\frac{y_3^2}{v\lambda_3}\right)}{\Lambda^2} & -\frac{H^2 v y_3^2}{\Lambda^2\lambda_3} & -\sqrt{\frac{H^2 v^2}{\Lambda^2}}\left(\frac{y_2 y_4}{\lambda_2}+\frac{u y_3 y_5}{v\lambda_3}\right) \\ 0 & -\frac{H^2 v y_3^2}{\Lambda^2\lambda_3} & -\frac{H^2 v y_3^2}{\Lambda^2\lambda_3} & -\frac{u\sqrt{\frac{H^2 v^2}{\Lambda^2}}y_3 y_5}{v\lambda_3} \\ -\frac{u\sqrt{\frac{H^2 v^2}{\Lambda^2}}\rho y_1}{v\lambda_1} & -\sqrt{\frac{H^2 v^2}{\Lambda^2}}\left(\frac{y_2 y_4}{\lambda_2}+\frac{u y_3 y_5}{v\lambda_3}\right) & -\frac{u\sqrt{\frac{H^2 v^2}{\Lambda^2}}y_3 y_5}{v\lambda_3} & -\frac{u^2\rho^2}{v\lambda_1}-\frac{v y_4^2}{\lambda_2}-\frac{u^2 y_5^2}{v\lambda_3} \end{pmatrix}$$

(viii) 7 matrices with 3 complex constraints: $M_{e\mu} = 0, \quad M_{e\tau} = 0, \quad M_{\mu\tau} = -M_{\tau\tau}$

- $n_1 = 1; n_2 = 0;\ n_3 = 0; n_4 = 1; n_5 = 0;\ n_6 = 0; n_7 = 0; n_8 = -1;\ n_9 = 1;$

$$\begin{pmatrix} -\frac{H^2 v y_1^2}{\Lambda^2 \lambda_1} & 0 & 0 & -\frac{u\sqrt{\frac{H^2 v^2}{\Lambda^2}}\rho y_1}{v\lambda_1} \\ 0 & -\frac{H^2 v^2\left(\frac{y_2^2}{v\lambda_2}+\frac{y_3^2}{v\lambda_3}\right)}{\Lambda^2} & \frac{H^2 v y_3^2}{\Lambda^2\lambda_3} & -\sqrt{\frac{H^2 v^2}{\Lambda^2}}\left(\frac{y_2 y_4}{\lambda_2}+\frac{u y_3 y_5}{v\lambda_3}\right) \\ 0 & \frac{H^2 v y_3^2}{\Lambda^2\lambda_3} & -\frac{H^2 v y_3^2}{\Lambda^2\lambda_3} & \frac{u\sqrt{\frac{H^2 v^2}{\Lambda^2}}y_3 y_5}{v\lambda_3} \\ -\frac{u\sqrt{\frac{H^2 v^2}{\Lambda^2}}\rho y_1}{v\lambda_1} & -\sqrt{\frac{H^2 v^2}{\Lambda^2}}\left(\frac{y_2 y_4}{\lambda_2}+\frac{u y_3 y_5}{v\lambda_3}\right) & \frac{u\sqrt{\frac{H^2 v^2}{\Lambda^2}}y_3 y_5}{v\lambda_3} & -\frac{u^2\rho^2}{v\lambda_1}-\frac{v y_4^2}{\lambda_2}-\frac{u^2 y_5^2}{v\lambda_3} \end{pmatrix}$$

Texture 3 zero case
(i) 8 matrices with 3 complex constraints: $M_{e\mu} = 0, \quad M_{e\tau} = 0, \quad M_{\mu\tau} = 0$

- $n_1 = 1; n_2 = 0; \; n_3 = 0; n_4 = 0; n_5 = 0; \; n_6 = 1; n_7 = 0; n_8 = 0; \; v_9 = -1;$

$$\begin{pmatrix} -\frac{H^2 v y_1^2}{\Lambda^2 \lambda_1} & 0 & 0 & -\frac{u\sqrt{\frac{H^2 v^2}{\Lambda^2}}\rho y_1}{v\lambda_1} \\ 0 & -\frac{H^2 v y_3^2}{\Lambda^2 \lambda_3} & 0 & \frac{u\sqrt{\frac{H^2 v^2}{\Lambda^2}} y_3 y_5}{v\lambda_3} \\ 0 & 0 & -\frac{H^2 v y_2^2}{\Lambda^2 \lambda_2} & -\frac{\sqrt{\frac{H^2 v^2}{\Lambda^2}} y_2 y_4}{\lambda_2} \\ -\frac{u\sqrt{\frac{H^2 v^2}{\Lambda^2}}\rho y_1}{v\lambda_1} & \frac{u\sqrt{\frac{H^2 v^2}{\Lambda^2}} y_3 y_5}{v\lambda_3} & -\frac{\sqrt{\frac{H^2 v^2}{\Lambda^2}} y_2 y_4}{\lambda_2} & -\frac{u^2\rho^2}{v\lambda_1} - \frac{v y_4^2}{\lambda_2} - \frac{u^2 y_5^2}{v\lambda_3} \end{pmatrix}$$

- We present the method adopted for numerical analysis for $(\mu - \tau)$ symmetric textures, texture 1, 2 , 3 zero cases, in order to check their consistency with $3+1$ neutrino data. It is well known that 4×4 unitary mixing matrix can be parametrised as

$$U = R_{34}\tilde{R}_{24}\tilde{R}_{14}R_{23}\tilde{R}_{13}R_{12}P \tag{20}$$

where

$$R_{34} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & c_{34} & s_{34} \\ 0 & 0 & -s_{34} & c_{34} \end{pmatrix} \tag{21}$$

$$\tilde{R}_{14} = \begin{pmatrix} c_{14} & 0 & 0 & s_{14}e^{-i\delta_{14}} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -s_{14}e^{i\delta_{14}} & 0 & 0 & c_{14} \end{pmatrix} \tag{22}$$

with $c_{ij} = \cos\theta_{ij}$, $s_{ij} = \sin\theta_{ij}$, δ_{ij} being the Dirac CP phases, and

$$P = \mathrm{diag}(1, e^{-i\frac{\alpha}{2}}, e^{-i(\frac{\beta}{2}-\delta_{13})}, e^{-i(\frac{\gamma}{2}-\delta_{14})})$$

- Using the above form of mixing matrix, the 4×4 complex symmetric Majorana light neutrino mass matrix can be written as

$$M_\nu = UM_\nu^{\mathrm{diag}}U^T \tag{23}$$

$$= \begin{pmatrix} m_{ee} & m_{e\mu} & m_{e\tau} & m_{es} \\ m_{\mu e} & m_{\mu\mu} & m_{\mu\tau} & m_{\mu s} \\ m_{\tau e} & m_{\tau\mu} & m_{\tau\tau} & m_{\tau s} \\ m_{se} & m_{s\mu} & m_{s\tau} & m_{ss} \end{pmatrix}, \tag{24}$$

where $M_\nu^{diag} = \mathrm{diag}(m_1, m_2, m_3, m_4)$

- For normal hierarchy (NH) of active neutrinos i.e., $m_4 > m_3 > m_2 > m_1$, the neutrino mass eigenvalues can be written in terms of the lightest neutrino mass m_1 as

$$m_2 = \sqrt{m_1^2 + \Delta m_{21}^2}, \quad m_3 = \sqrt{m_1^2 + \Delta m_{31}^2}, m_4 = \sqrt{m_1^2 + \Delta m_{41}^2}.$$

Results for $\mu - \tau$ symmetric textures.

• Only three out of four different classes of $\mu - \tau$ symmetric textures are allowed for NH of active neutrino masses, namely subclasses (ii), (iii) , (iv).

• Figure 1 shows the correlations between active-sterile mixing angles $\theta_{34} - \theta_{14}$ and between Majorana CP phases $\alpha - \gamma$.

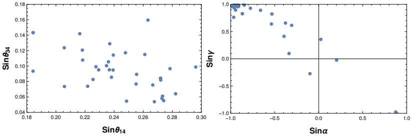

Figure 1 : Neutrino oscillation parameters in active-sterile sector for case (ii) from $\mu - \tau$ symmetric category for NH.

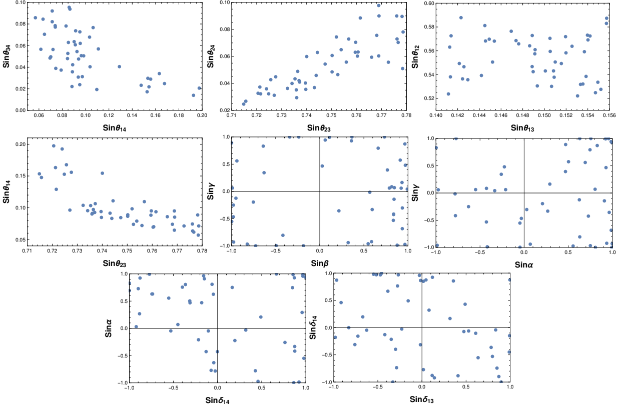

Figure 2 : Neutrino oscillation parameters in active-sterile sector for case (iii) from $\mu - \tau$ symmetric category for NH.

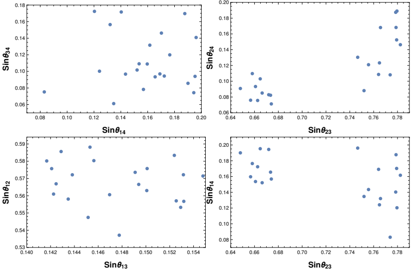

Figure 3 : Neutrino oscillation parameters in active-sterile sector for case (iv) from $\mu - \tau$ symmetric category for NH

Results for texture 1 zero symmetric cases.

- Among the one zero texture category, only two subclasses namely (ix), (x) with NH are allowed.

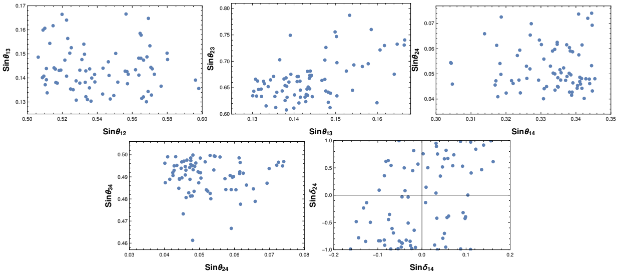

Figure 4 : Neutrino oscillation parameters in active-sterile sector for case (ix) from texture 1 zero category for NH.

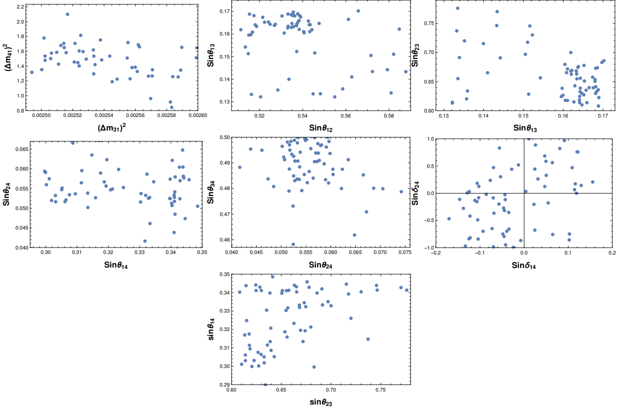

Figure 5 : Neutrino oscillation parameters in active-sterile sector for case (x) from texture 1 zero category for NH.

Results for texture 2 zero symmetric cases.

- Among the two zero texture category, only two subclasses namely (i), (ii) with NH are allowed.

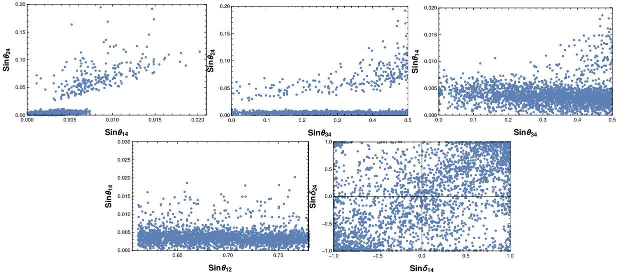

Figure 6 : Neutrino oscillation parameters in active-sterile sector for case (i) from texture 2 zero category for NH.

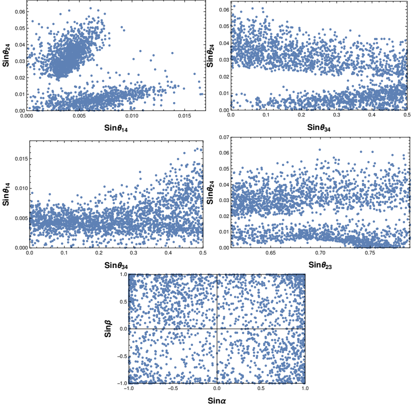

Figure 7 : Neutrino oscillation parameters in active-sterile sector for case (ii) from texture 2 zero category for NH.

Results for texture 3 zero symmetric cases.

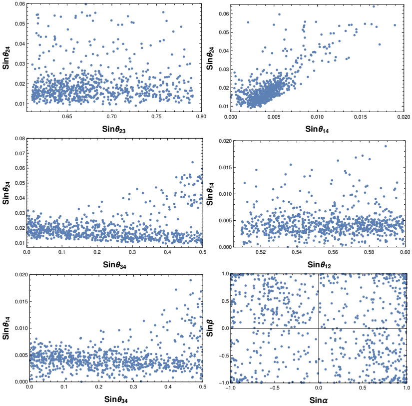

Figure 8 : Neutrino oscillation parameters in active-sterile sector for texture 3 zero case for NH.

Conclusion

- To summarise, we have studied the viability of different possible textures in light neutrino mass matrix within the framework of 3 + 1 light neutrino scenario by considering a A_4 flavour symmetric minimal extended seesaw mechanism.

- Considering generic A_4 flavon alignments where a triplet flavon acquires VEV like $\langle \phi \rangle = v(n_1, n_2, n_3)$, $n_i \in (-1, 0, 1)$, we first consider all possible combinations of such alignments and find the analytical form of the light neutrino mass matrix for each such case.

- We discarded the disallowed textures i.e. 225 mass matrices out of the total 729 mass matrices from our analysis. From the remaining cases, we classified 96 of them as one zero texture, 64 as two zero texture, 8 as three zero texture and 296 of them as hybrid textures (which do not contain any zeros). The remaining 40 mass matrices correspond to an interesting category where the 3×3 active neutrino block of the 3 + 1 light neutrino mass matrix possess $\mu - \tau$ symmetry whereas the active-sterile block breaks it explicitly.

- We then analyse the mass matrices with texture zeros and $\mu - \tau$ symmetry by numerically solving the constraint equations in each case and comparing the resulting solution with the 3 + 1 neutrino data for consistency.

- We therefore numerically solved the constraint equations for these 25 cases in total. We found that only 8 out of 25 subclasses are allowed by the 3 + 1 global fit data and all of them have normal hierarchical pattern of light neutrino masses.

- For Inverted hierarchy of active neutrino masses, none of textures are allowed.

- While the fate of an additional light neutrino having mass around the eV scale is yet to be confirmed by other neutrino experiments, our analysis show how difficult it is to realise such a scenario in the minimal extended seesaw if A_4 flavour symmetry with generic vacuum alignment is present.

- If the existence of such light sterile neutrino gets well established later, the predictions for unknown neutrino parameters obtained in our analysis can be tested for further scrutiny of the model, in a way similar to Ref [3] where the possibility of probing texture zeros in three neutrino scenarios at neutrino oscillation experiments was studied.

Acknowledgements

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Thank you.